



Universidad Pública de Navarra

Máster en Ingeniería Mecánica Aplicada y Computacional

# ***Modeling, Simulation and Identification of a Vacuum Pump***

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September 2014

*Public University of Navarra*



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**Note: In order to get the code corresponding to 3d-mec and Matlab files, please contact with Jesus Sanz García, ([jesus.sanz@entecnia.com](mailto:jesus.sanz@entecnia.com)).**

## 1 Introduction

Entecnia Consulting, S.L. is an Engineering Company incorporated to provide enduring comprehensive Engineering Services to the automotive industry, in a global market, in order to develop new products and engineering processes. The present project is involved into the frame of Safety Brake System. Specifically, it consists of the development of an Electric Vacuum Pump (EVP).

### 1.1 Automotive brake system

Brake system arises as one of the important applications based on vacuum to provide brake assistance. The input force made by the driver on the brake pedal is increased due to vacuum assistant into the booster. The output force drives the hydraulic brake system to control the speed wheels.

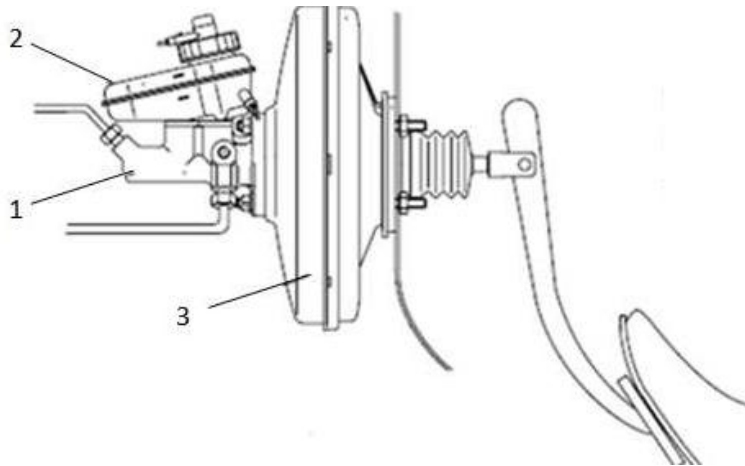


Figure 1. Drive unit is composed by the master cylinder (1), the fluid brake reservoir (2) and the booster (3).

During many years, vacuum generation on vehicles was not a problem. Combustion engines generate enough vacuum at the same time they were working. The technology has advanced in the last years with more efficient combustion engines (injection), diesel motor, electric cars and hybrid technology. All this solutions affect to the brake system by reducing the vacuum generated by the engine. The EVP is an alternative vacuum source to solve the vacuum absence in these cases. It consists of an electric engine and a rotary pump.

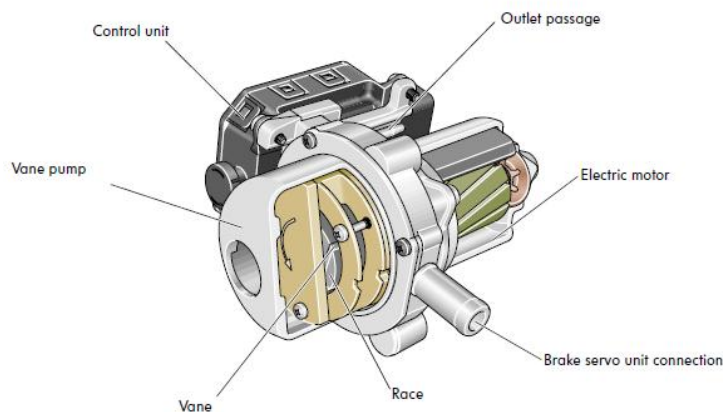


Figure 2. Rotary Electro Vacuum Pump.

## 1.2 Rotary Vacuum Pump

The Rotary Vacuum Pump is composed mainly of Vanes (minimum of two), Rotor and Housing. The Rotor involves slots where the Vanes slide going outward and inward of the slot. The Rotor turns eccentrically into the stator (Housing). The Vane should be always in contact with Housing due to centrifugal force generated by the angular speed. The movement during a lap defines a variable volume chamber between Rotor and Housing. The increase in volume into the chamber causes under pressure. The air is suctioned from the booster through the inlet valve, that is, generation of vacuum. If volume decrease, causes over pressure to exhaust the air through the outlet valve to atmosphere.

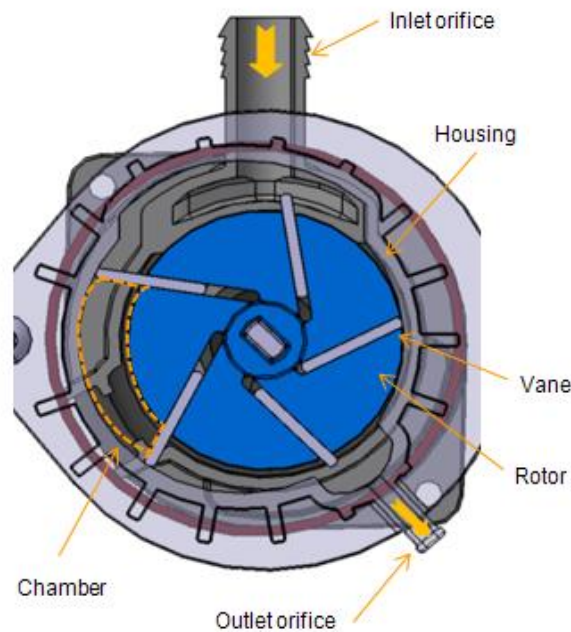


Figure 3. Parts of a EVP

The property material of the Vane is critical to obtain a proper operation and endurance product. It should have low friction coefficient, wear resistance, dimensional stability, and high stiffness. The friction coefficient arises as an important property, and it must be controlled to guarantee the pump operation. A high friction coefficient could cause excessive wear, high consumption of torque and high temperature.

## 1.3 Test rig for measuring the COF

One of the activities carried out in this project is about identification of coefficient of friction (COF) based on experimental data. Entecnia has developed a test rig to analyze the COF generated between Vane and Housing. As it is shown on the Figure 4, the test device is a simplified system of the real pump. The test device is composed mainly of Housing, Vane, Rotor, torque sensor and a controlled speed motor.

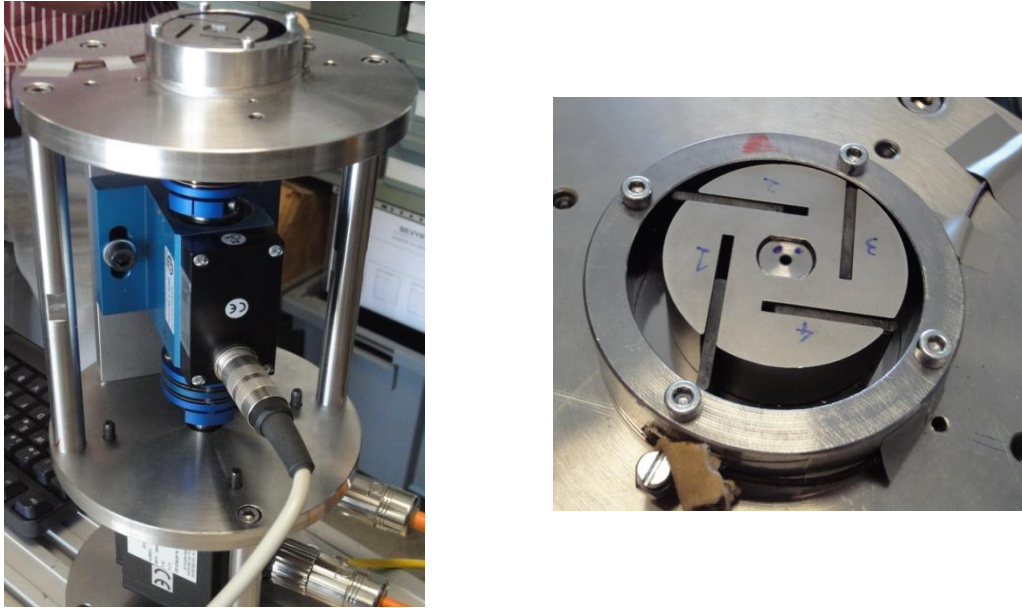


Figure 4. Test rig

## **2 Objectives and Scope**

The presented objectives are focused on acquiring knowledge about dynamic modeling and identification to be applied on a Vacuum Pump product, system under development stage. The objectives of this project are the next.

### **-Development of the dynamic model of a Vacuum Pump.**

The model is developed to analyze the contact between Vane and, Housing and Rotor. Specifically, the analysis is focus on modeling the contact taking into account the friction Coulomb's law. The contact has been defined in two ways. First, contact model based on a spring-dashpot system with linear and no-linear spring. Second, contact model based on a constraint between Vane and Housing.

For the present project, the dynamic model has not included other parts of the whole EVP model such as motor, pneumatic system (valves, leaks, chambers) and thermal system. Once tested and validate, the dynamic model and modeling procedure will be use at the design stage to analyze the operation under different geometries and to determine the most efficient design according to specifications. This analysis is not included in this report.

### **-Development of the identification tool to determine the COF based on experimental data.**

This tool is based in the aforementioned dynamic model, and it can be used to identify different parameters such as COF, signal phase or speed offset.

The friction coefficient of the EVP product acquires critical importance in terms of power consumption, operation, efficiency and durability, among others. So, the identification of the unknown data provides us greater knowledge and guidelines to improve the product designs and to save cost and time in other stages of the project such as prototype stage.

### **3 Activities & Methodology**

#### **3.1 Dynamic model**

The activities to develop a dynamic model are detailed below.

1-Definition of basic models

- First step was focus on to improve the knowledge about kinematic and dynamic theory, and use of 3d-mec software. This activity is not included in the report.

2-Development of advanced contact model.

*Modeling*

- The system is modeled with vector expressions, and it is programmed with 3d-mec software (Public University of Navarra) to get part of the system equations.
- The rest of the system is modeled with Matlab software. That is, kinematic equations and contact mechanics. It is defined the structure for the numerical simulation.

*Simulation*

- Integration procedure to solve the system equations with Matlab software.

*Post-process of plots and results*

3-Analysis of type of solver, maximum step and time computing.

4-Analysis of corrections methods.

5-Comparison between models.

6-Development of model for  $n^0$  Vanes.

#### **3.2 Identification tool**

The activities to develop the identification tool are detailed below.

1-Development of an identification tool of friction coefficient and signal phase

- Programming of a solver to calculate the desired variables. The identification is based on toque signal.

2-Validation with reference signals

3-Application to experimental data

### **4 Evaluation**

The results obtained with the dynamic model have been checked in two ways. The position variables have been compared with CATIA software. Velocity and force variables have been checked with Inventor software results (dynamic application). In both cases, the results are accurate enough.

This way lets us to fix some settings such as the integration tolerance, integration maximum step.

## 5 Dynamic Modeling of a Rotary Vacuum Pump

### 5.1 System

As it has been mentioned, the system to be modeled consists of Housing, Vane and Rotor, as Figure 5 shows. The model is parametric to allow the analysis of different geometries, such as circular or elliptical Housing, concentric or eccentric Rotor (position of Rotor respect to Housing), number of vanes, tilt vanes and other dimensions. As first step, the system is modeled for one vane.

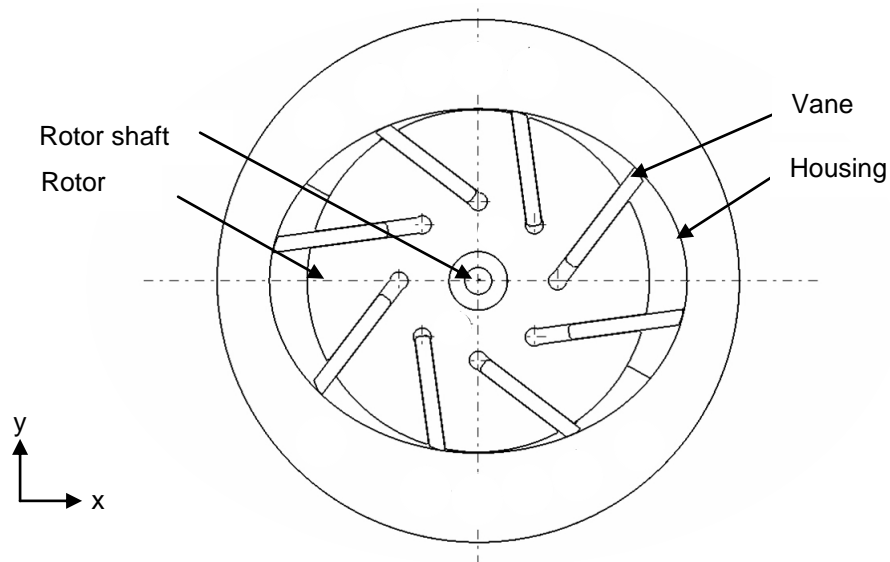


Figure 5. EVP system

### 5.2 Geometry

The Housing geometry used in the present report to check and validate the dynamic model and identification tool corresponds to elliptical Housing. The identification of COF based on experimental data has been carried out with a cylindrical Housing. Geometry and dimension of each assembly are not included in the present report.

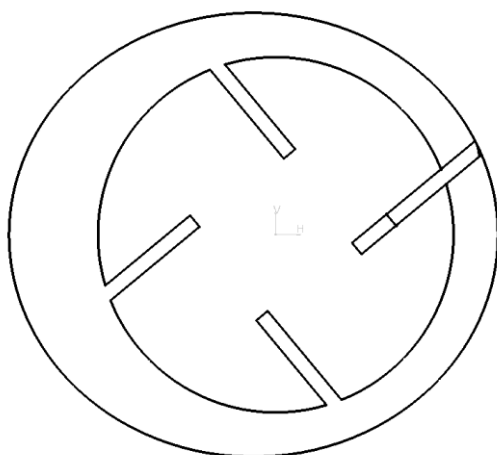


Figure 6. Elliptical Housing.

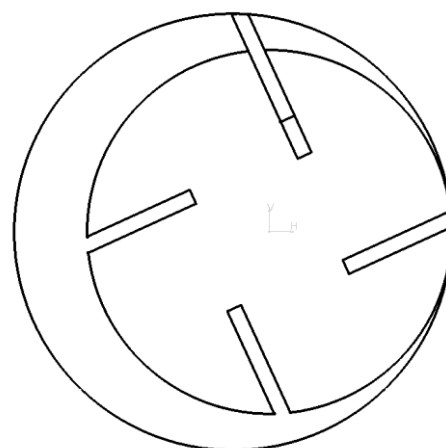


Figure 7. Cylindrical Housing.



### 5.3 Assumptions

The next assumptions have been taking into account.

#### Modeling

- The absolute axis (0, 0, 0) is fixed at the center of the Rotor.
- The model is defined in X-Y plane.
- The Rotor turns at Z axis.
- Rotation in X or Y is not allowed.
- All bodies are considered as rigid.
- The Model is defined for one vane.
- The Vane mass is concentrate in the geometric center of the Vane.
- The vane can move along the slot by means of a prismatic support. The gap between Rotor and Vane is not taken into account.
- The gravity is defined in Z axis.

#### Contact model

- The friction force is based on Coulomb's law.
- The transition between static and dynamic friction force is taken into account by Impact Model (Adams/Solver model).
- The friction takes place due to contact of Vane with Housing and Rotor.
- The contact between Vane and Housing and bottom of the Rotor slot is modeled by a spring-dashpot.
- The contact between Vane and Rotor slot (parallel wall) are modeled as forces (normal and frictional) and moment applied at the geometric center of the vane.
- The friction model takes into account the side of contact between Vane and slot Rotor.

#### Input variable

- The input variables correspond to torque or angular acceleration applied at Rotor shaft.

### 5.4 Models

Two types of models have been defined according to the contact between Vane and Housing.

The Model 1 has two degree of freedom,  $\theta_1$  and  $s_1$ . The angle,  $\theta_1$ , is the angle of the Rotor shaft. The dimension,  $s_1$ , represents the distance between the bottom of the slot and Vane. The contact between Vane and Housing is driven by a spring-dashpot. Further information about contact mechanics can be found in Section 5.5.

The Model 2 has an additional geometric constrain. The Vane is always at contact with the Housing. The system has one degree of freedom,  $\theta_1$ .

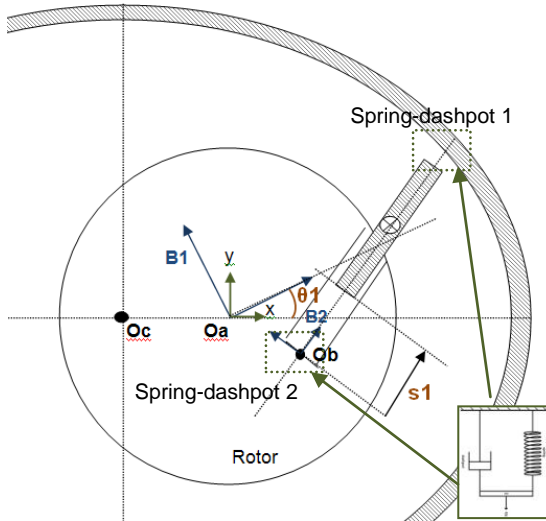


Figure 8. Model 1: Two degrees of freedom,  $\theta_1$  and  $s_1$ .

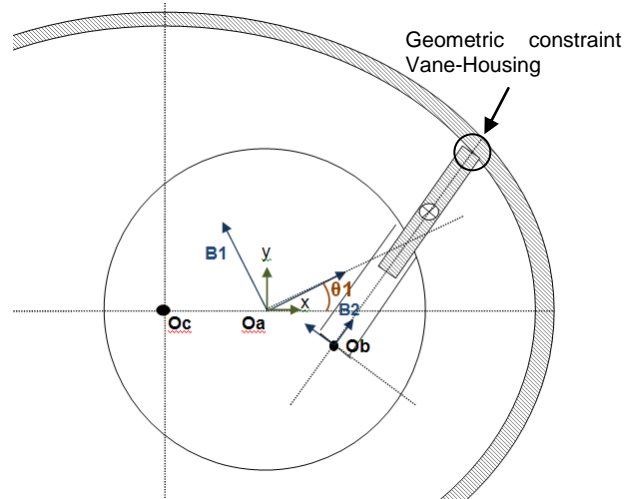


Figure 9. Model 2: One degree of freedom,  $\theta_1$ .

#### 5.4.1 Generalized coordinates

The next description is valid for both models. In case of differences, it will be specified.

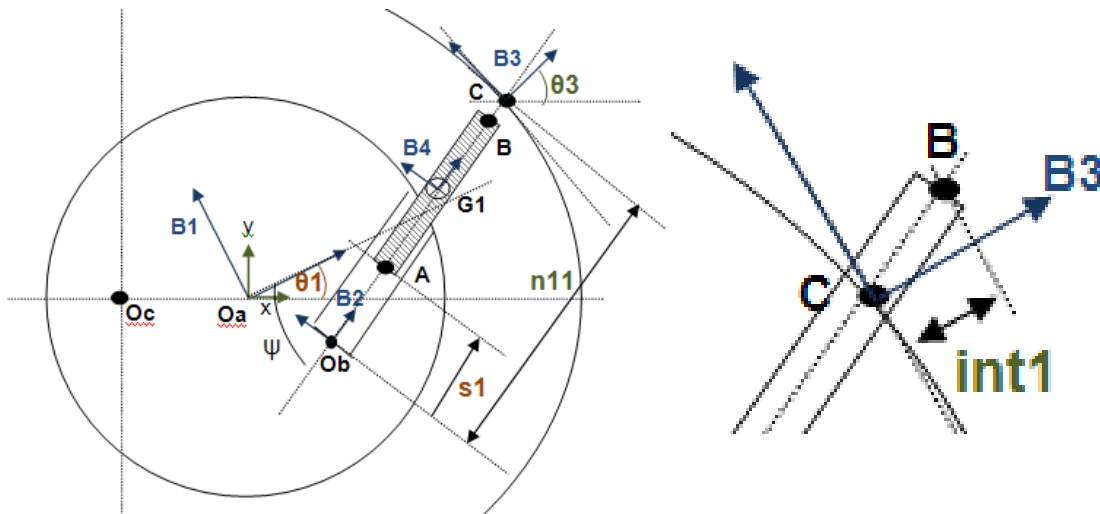


Figure 10. Generalized coordinates

Angle Theta1,  $\theta_1$ , defines the rotation of the rotor. It involves a degree of freedom for both models.

Dimension,  $s_1$ , defines the distance between points Ob (bottom of the slot) and A (end of Vane). It is the second degree of freedom for Model 1.

Point C ( $x_3, y_3$ ) belongs to Housing. It defines the contact point between Vane and Housing.

Dimension "n11" is the distance between points C and Ob (bottom of the Vane slot).

Angle Theta3,  $\theta_3$ , defines the orientation of contact forces between Housing and Vane along normal direction to Housing.

Dimension "int1" defines the penetration of the Vane into the Housing. Positive value means penetration exists.

### 5.4.2 Points, bases and references

The bodies are defined according to the next points, bases and references.

#### Rotor

- Base 1 (B1) is defined by turning the absolute base (x, y, z) the angle  $\theta_1$ .
- Base 2 (B2) is defined by turning the base B1 (x, y, z) the angle  $\varphi$ .
- Point Oa (0, 0, 0) defined from point O (origin of the absolute reference). It defines the gravity center of the Rotor.
- Point Ob defined from point Oa in base B1. It determines the bottom of the slot.
- Reference R1 is defined by base B1 and point Oa. The motion of the Rotor is defined by R1.
- Reference R2 is defined by point Ob and base B2.

#### Housing

- Base 3 (B3) defined by turning the absolute base (x, y, z) the angle  $\theta_3$ .
- Point Oc ( $x_2, y_2$ ) defined from point Oa with respect to absolute base (x, y, z). It defines the center of the Housing.
- Point C ( $x_3, y_3$ ) defined from point O (origin of the absolute reference) with respect to absolute base (x, y, z). It defines the contact point between Vane and Housing.
- Reference R3 is defined by point C and base B3.

#### Vane

- Base 4 (B4) is defined by turning the base B2 the angle  $\theta_4$ . It defines the tilt of the Vane.
- Point G1 defined from point Ob in base B2. It is the gravity center of the Rotor.
- Point B defined from point G1 in base B4. It defines the contact point between Vane and Housing.
- Point A defined from point G1 in base B4. It defines the contact point between Vane and bottom of the Rotor slot.
- Reference R4 is defined by point G1 and base B4.

### 5.4.3 Variables for Model 1

Independent variables

$$z = \begin{bmatrix} \theta_1 \\ s_1 \end{bmatrix}$$

Dependent variables

$$qd = \begin{bmatrix} x_3 \\ y_3 \\ n11 \\ int1 \\ \theta_3 \end{bmatrix}$$

Variables vector

$$q = \begin{bmatrix} z \\ qd \end{bmatrix}$$

#### 5.4.4 Variables for Model 2

Independent variables

$$z = [\theta_1]$$

Dependent variables

$$qd = \begin{bmatrix} x_3 \\ y_3 \\ s_1 \\ \theta_3 \end{bmatrix}$$

Variables vector

$$q = \begin{bmatrix} z \\ qd \end{bmatrix}$$

#### 5.4.5 External moment

As it has been explained, the input variable could be the torque or angular acceleration applied at Rotor shaft. In case of torque as input, the moment is external. In case of angular acceleration as input, the torque is a joint force.

$$Torque = [0, 0, Tor_z]$$

#### 5.4.6 Constitutive forces

It depends of  $q$  and  $\dot{q}$  (gravity forces and spring forces).

F1B\_B3\_1 (Model 1) is applied at point C, and it is defined according to B3 (1,0,0). It represents the contact force between Vane and Housing. In case of Model 2, this force is a joint force due to the geometric constraint.

F1A\_B2\_1 is applied at point A and Ob. It is defined according to B2 (1,0,0). It represents the contact force between Vane and bottom of the slot.

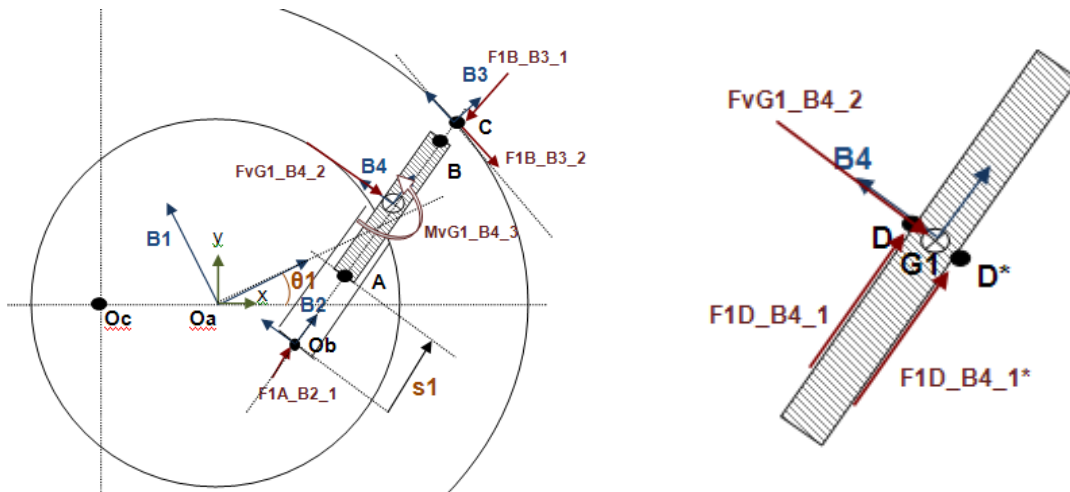


Figure 11. Forces.

#### 5.4.7 Joint forces

$$\varepsilon = \begin{bmatrix} FvG1_B4_2 \\ MvG1_B4_3 \end{bmatrix}$$

Force FvG1\_B4\_2 is applied at point G1 according to B4 (0,1,0). It represents the contact force between Vane and lateral side of the Rotor slot.

Moment  $MvG1\_B4\_3$  is applied on point G1 according to B4 (0,0,1). It represents the contact moment between Vane and lateral side of the Rotor slot.

The joint forces and moments involve an action and reaction force. That is, they are applied at both bodies with the same magnitude and opposite sign.

#### 5.4.8 Frictional forces

$F1B\_B3\_2$  is the frictional force of the normal contact force  $F1B\_B3\_1$  applied at point C, and it is defined according to B3 (0,1,0).

$$F1B\_B3 = \{[F1B\_B3\_1, \quad F1B\_B3\_2, \quad 0]\}B3$$

$$F1B\_B3\_2 = F1B\_B3\_1 \cdot \mu_{Housing-Vane}$$

Force  $F1D\_B4\_1$  is the frictional force of the normal force of  $FvG\_B4\_2$  applied at point D, and it is defined according to B4 (1,0,0). The point D is defined according to the contact side of the Vane with Rotor slot. The point D is determined by means of the sign of the force  $FvG1\_B4\_2$ .

$$F1D\_B4\_1 = FvG\_B4\_2 \cdot \mu_{Rotor-Vane} \cdot \{1, \quad 0, \quad 0\}B4$$

#### 5.4.9 Inertial forces and moments

$$Fi_{Rotor} = -m_{Rotor} \cdot \frac{d}{dt} \left( \frac{d}{dt} (P(O, Oa))_{abs} \right)_{abs}$$

$$Mi_{Rotor}(gc) = -m_{Rotor} \cdot \frac{d}{dt} (I_2 \cdot \Omega(abs, R1))_{abs}$$

$$Fi_{Vane} = -m_{Vane} \cdot \frac{d}{dt} \left( \frac{d}{dt} (P(O, G1))_{abs} \right)_{abs}$$

$$Mi_{Vane}(gc) = -m_{Rotor} \cdot \frac{d}{dt} (I_1 \cdot \Omega(abs, R4))_{abs}$$

$I_1$  and  $I_2$  are the inertia tensor of the Vane and Rotor defined in the gravity center of each part.

#### 5.4.10 Geometric equations (Model 1)

The equation system consists of five equations and it is no-linear. Some of them have been obtained by programming the vector system in the software 3d-mec (Public University of Navarra).

The equation system shown in this report takes the form  $\phi(q) = 0$ .

$$\phi(q, t) = \begin{bmatrix} P(Oc, Oa)\{xyz\} + \{P(Oa, Ob)\}xyz + \{n11\}xyz - \{P(Oc, C)\}xyz \\ \frac{(C_x - O_{ax})^2}{a^2} + \frac{(C_y - O_{ay})^2}{b^2} - 1 \\ P(C, B)\{B3\} * [1,0,0] - int1 \\ \frac{J_1(2)}{J_1(1)} - \tan \theta_3 \end{bmatrix}$$

Being  $J_1$  the next expression.

$$J_1 = \frac{\partial}{(\partial C_x, \partial C_y)} \left( \frac{(C_x - O_{ax})^2}{a^2} + \frac{(C_y - O_{ay})^2}{b^2} - 1 \right)$$

The expression,  $\{P(Oc, Oa)\}xyz$ , represents the position vector from Oc to Oa reported

in base (xyz).

The first equation involves the variables  $x_3$ ,  $y_3$  and  $n_{11}$ . The development of this vector equation yields two equations, in "x" and "y" axis.

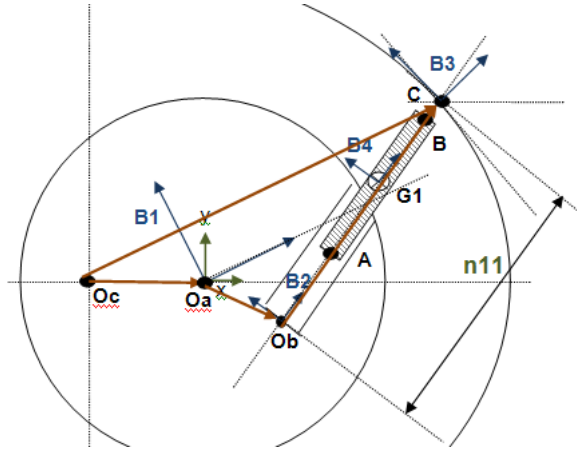


Figure 12. First equation

The second equation corresponds to ellipse equation.

The third equation represents the interference between Vane and Housing measured as the distance between point C and B along base B3 (1,0,0). See Figure 10.

The fourth equation involves the angle  $\theta_3$ . It involves the angle of the normal vector to the Housing at point C( $x_3$ ,  $y_3$ ). See Figure 10.

#### 5.4.11 Geometric equations (Model 2)

The equation system consists of four equations and it is no-linear.

$$\phi(q, t) = \begin{bmatrix} \{P(O, B)\}_{xyz} - \{P(O, C)\}_{xyz} \\ \frac{(O_{cx} - O_{ax})^2}{a^2} + \frac{(O_{cy} - O_{ay})^2}{b^2} - 1 \\ \frac{J_1(2)}{J_1(1)} - \tan \theta_3 \end{bmatrix}$$

The first equation involves the coincidence of points B and C. The development of this vector equation yields two equations, in "x" and "y" axis.

The second equation corresponds to ellipse equation.

The fourth equation involves the angle  $\theta_3$ . It involves the angle of the normal vector to the Housing at point C( $x_3$ ,  $y_3$ ). See Figure 10.

#### 5.4.12 Velocity equations

The equations are obtained through the differentiation of the geometric equations with respect to time. The resultant system is linear. It depends on, in general, position variables and time.

$$\frac{d\phi(q, t)}{dt} \equiv \frac{\partial \phi}{\partial q} \cdot \frac{dq}{dt} + \frac{\partial \phi}{\partial t} = 0$$

It can be also write as follow.

$$\phi_q(q, t) \cdot \dot{q} + \phi_t = 0$$

In the present case, none of the constraints depends explicitly on time, then  $\phi_t = 0$ .

### 5.4.13 Acceleration equations

The equations are obtained by differentiating the equation of velocity with respect to time. The resultant system is linear. It depends on, in general, position, velocity variables and time.

$$\frac{d^2\phi(q, t)}{dt^2} \equiv \dot{\phi}_q \cdot \dot{q} + \phi_q \cdot \ddot{q} + \dot{\phi}_t = 0$$

### 5.4.14 Dynamic equations

The dynamic equations have been obtained by programming the vector system in 3d-mec software (Public University of Navarra). The equations have been formulated with virtual power method.

### 5.4.15 Virtual Power method

The dynamic equation of Newton-Euler can be written as follows.

$$\sum_{i=1}^{N_{Sol}} (\mathcal{F}_i \cdot V_{ik}^* + \mathcal{M}_i \cdot \Omega_{ik}^*) + \sum_{i=1}^{N_{Sol}} \sum_{j=1}^{N_{Acc}} (F_{ij} \cdot V_{ik}^* + M_{ij} \cdot \Omega_{ik}^*) = 0$$

$F_{ij}$  and  $M_{ij}$  are the external forces and moments number  $j$ , applied on body  $i$ .  $N_{sol}$  is the number of bodies of the system.  $N_{acc}$  is the number of external forces over each body.  $V^*$  and  $\Omega^*$  are virtual velocities.

### 5.4.16 Vector of virtual velocities

The virtual velocity vector can be defined as velocity vectors at the points of the bodies where the wrench (force and moments) and angular velocity are defined. The dynamic equations are obtained by including as many independent virtual velocity vectors as the number of degree of freedom of the mechanism. Joint forces do not appear in these equations. In order to include the desired joint forces in the dynamic equations, it is necessary to include the proper virtual velocities over a disallowed movement of the mechanism. That is, it is included the virtual velocities incompatible with geometric constraints.

There are two joint forces in the system under analysis. So, two virtual velocities are necessary to include into the dynamic equations to get the contact wrench between Vane and Rotor,  $FvG1\_B4\_2$  and  $MvG1\_B4\_3$ . The virtual velocities are written in terms of  $v1$  and  $\theta_4$ .

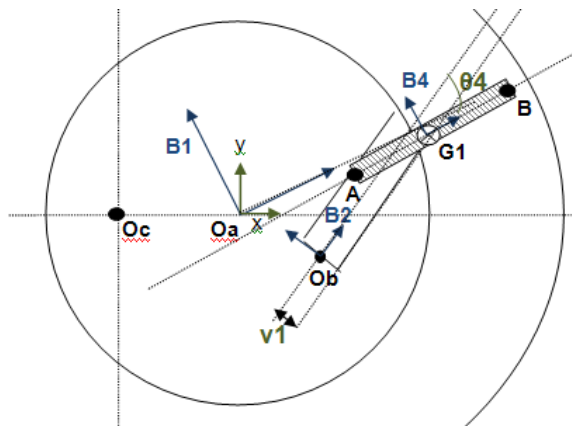


Figure 13. Virtual velocities.

The joint forces are necessary in order to include the frictional forces into the dynamic equations.

## 5.5 Contact mechanics

The next explanation is detailed for the Model 1. The Model 2 has an additional geometric constraint. The Vane is always in contact with the Housing. So, it is not necessary to include a spring-dashpot due to the force between Vane and Housing is a joint force and, as it has been explained in Section 5.4.15, it can be obtained by applying virtual power method.

The contacts among Vane-Housing and Vane-bottom of the slot are characterized by means of a spring-dashpot (spring and shock absorber). Three states have been defined according to the Vane position.

- State 1: Contact with Housing.
- State 2: No-contact with Housing.
- State 3: Vane at bottom of the Rotor slot.

The states have been implemented into the dynamic equations by means of a spring-dashpot equation. The activation or deactivation of each state is carried out by means of boolean values (1/0), named "aux" values. The equation of a spring-dashpot is the next.

$$F = k * x + c * \dot{x}$$

Being:

k, spring rate (N/m)

c, viscosity rate (N/m/s)

This equation has been set for the different contacts and states.

The values "k", "c" can be defined as constant or functions. See section 8 Analysis of Spring-Dashpot Parameters.

### 5.5.1 Contact Vane-Housing

"F1B B3 1" is the normal force to Housing according to base B3 (1,0,0). It is applied at point B (Vane).

$$F1B B3 1 = -((k \cdot int1 + c \cdot \dot{s}_1) * aux1)$$

$$F1B B3 2 = F1B B3 1 \cdot \mu_{Housing-vane} \cdot aux3$$

### 5.5.2 Contact Vane- bottom of the Rotor slot

"F1A B2 1" is the force applied at point A according to base B2 (1,0,0). This force affects to both Vane and Rotor with different sign.

$$F1A B2 1 = -((k \cdot int1 + c \cdot \dot{s}_1) * aux2)$$

### 5.5.3 Contact Vane - Rotor (along the side of the slot)

"F1D B4 1" is the force applied at point D according to base B4 (1,0,0). This force affects to both Vane and Rotor with different sign. The direction of this force is driven by parameter "aux4". This parameter, changes the sign as function of the relative velocity between Vane and Rotor.

$$F1D B4 1 = FvG1 B4 2 \cdot \mu_{Rotor-vane} \cdot aux4$$

### 5.5.4 Aux parameters

The equation forces are controlled by the parameters "aux" according to the conditions shown in the Table 2 sorted by priority. The contact of Vane with Housing is detected through variable int1, interference Vane-Housing (see Figure 10). The contact of Vane



with bottom of the Rotor slot is detected with  $s_1$  variable. The friction force direction is detected taking into account the relative movement of the parts.

State	Condition	Parameters
Contact with Housing	$int1 = \{P(C, B)\}B3 * \{[1,0,0]B3\} > 0$	aux1=1 aux2=0
Vane at bottom of the slot (Rotor)	$s_1 < 0$	aux1=0 aux2=1
No-contact with Housing	$int1 = \{P(C, B)\}B3 * \{[1,0,0]B3\} \leq 0$	aux1=0 aux2=0

Table 1. Aux1 and Aux2 parameters.

### 5.5.5 Friction model

The friction model has been developed according to Adams/Solver Impact model. The transition between static and dynamic friction coefficient has been characterized based on the velocity-friction relation. The figure below shows how the coefficient varies with slip velocity.

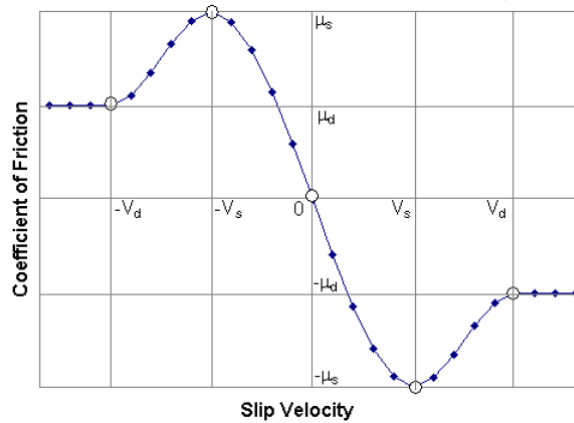


Figure 14. Friction Model

Being:

$V_s$ : Static transitory velocity.

$V_d$ : Friction transitory velocity.

$\mu_s$ : Static friction coefficient.

$\mu_d$ : Dynamic friction coefficient.

The mathematical implementation has been carried out through the variables "aux3" and "aux4". The objective is to fix the proper friction coefficient according to slip velocity and direction.

$$F1D B4 1 = FvG1 B4 2 \cdot \mu_{Rotor-Vane (Static)} \cdot aux4$$

$$F1B B3 1 = -((k \cdot int1 + c \cdot \dot{s}_1) * aux1)$$

$$F1B B3 2 = F1B B3 1 \cdot \mu_{Housing-Vane(Static)} \cdot aux3$$

The variables "aux3" and "aux4" are defined according to the next laws. It takes into account the relation static-dynamic friction coefficient and the sign of the slip velocity.

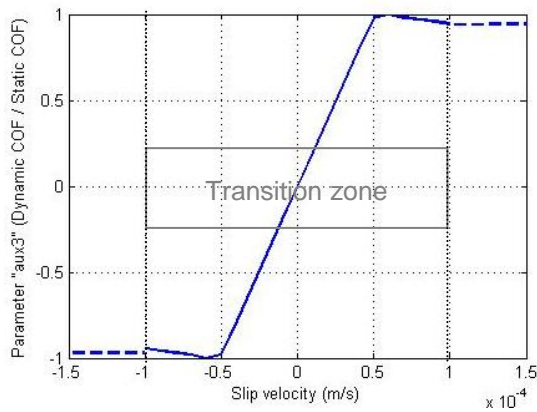


Figure 15. Law for aux3.

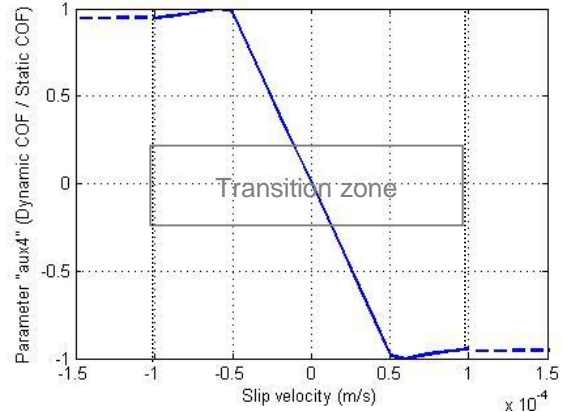


Figure 16. Law for aux4.

According to the slip velocity the variables "aux3" and "aux4" take the next values.

State	Condition	Parameters
Friction force direction "F1D B4 1"	$\dot{s}_1 > \text{positive slip vel. limit}$	$aux4 = -\frac{\mu_{Housing-Vane(Dynamic)}}{\mu_{Housing-Vane(Static)}}$
	$\dot{s}_1 < \text{negative slip vel. limit}$	$aux4 = \frac{\mu_{Housing-Vane(Dynamic)}}{\mu_{Housing-Vane(Static)}}$
	$\dot{s}_1 \in \text{transition zone}$	$aux4 = f(\text{slip vel.})$
Friction force direction "F1B B3 2"	$\{V(Oc, B)\}B3 * [0,1,0] > \text{positive slip vel. limit}$	$aux3 = \frac{\mu_{Housing-Vane(Dynamic)}}{\mu_{Housing-Vane(Static)}}$
	$\{V(Oc, B)\}B3 * [0,1,0] < \text{negative slip vel. limit}$	$aux3 = -\frac{\mu_{Housing-Vane(Dynamic)}}{\mu_{Housing-Vane(Static)}}$
	$\{V(Oc, B)\}B3 * [0,1,0] \in \text{transition zone}$	$aux3 = f(\text{slip vel.})$

Table 2. "Aux" parameters for friction direction.

The friction model result is available in section 10 Friction Model Results.

### 5.6 Contact model Vane-Rotor

As it has been commented, the contact between Vane and Rotor slot (parallel wall) are modeled as forces (normal and frictional) and moment applied at the center of the vane. The friction force is calculated at the side of contact between Vane and Rotor slot. This characterization is useful at mathematical level due to it is equivalent to any other force distribution taking into account the previous boundary conditions. So, it does not represent strictly the real contact. The first step to have a closer view of the real contact is to define a new contact model, that is, a new distribution of forces. The second step involves the calculation of the new forces from the wrench calculated in the gravity center of the Vane.

The new contact model between Rotor and Vane is defined in two points by normal and friction forces according to base B4. Normal force  $F_{x1}$  is defined at the end of the Vane (point A) and normal force  $F_{x2}$  is defined at Rotor radius. The model takes into account the contact side between Vane and Rotor slot to place properly the friction force. The force F1A\_B2\_1 is not taken into account. The rest of the conditions remain as they were explained in section 5.3.

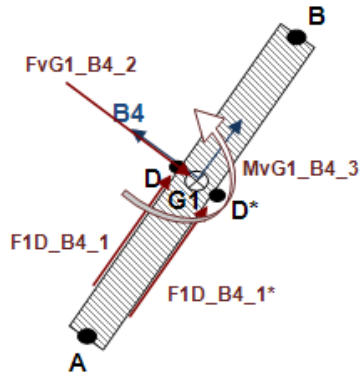


Figure 17. Mathematical model.

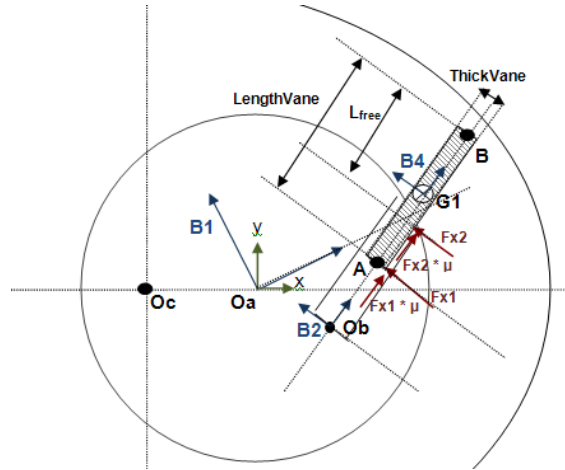


Figure 18. Contact model Vane-Rotor.

The equations of the contact model are the next.

$$FvG1\ B4\ 2 = F_{x1} + F_{x2}$$

$$F1D\ B4\ 1 = FvG\ B4\ 2 \cdot \mu_{Rotor-Vane} \cdot \{1, 0, 0\}B4$$

$$F1D\ B4 = \{[F1D\ B4\ 1, 0, 0]\}B4$$

$$MvG1\ B4\ 3 + \{P(G1, D)\}B4^{\wedge}F1D\ B4\ 1$$

$$= \left( \frac{-LengthVane}{2} + aux3 \cdot \mu_{Rotor-Vane} \cdot \frac{ThickVane}{2} \right) \cdot F_{x1}$$

$$+ \left( \frac{LengthVane}{2} - L_{free} + aux4 \cdot \mu_{Rotor-Vane} \cdot \frac{ThickVane}{2} \right) \cdot F_{x2}$$

The forces Fx1 and Fx2 can be obtained by solving the system of two linear equations.

As it has been commented, the point D is defined according to the contact side of the Vane with Rotor slot. The point D is determined by means of the sign of the force FvG1\_B4\_2.

## 6 Dynamic Simulation Procedure

The numerical simulation involves the next steps.

- Calculation of initial values
- Resolution of position problem
- Resolution of velocity problem
- Loop of integration
  - Position correction
  - Velocity correction
  - Determination of the dynamic system according to contact constraints.
  - Calculation of constitutive forces. That is, input torque as function of time and spring forces.
  - Resolution of dynamic and acceleration equations to obtain generalized coordinates,  $\ddot{q}$ , and joint forces,  $\varepsilon$ .
  - Checking of friction force conditions.
  - Integration of the coordinates,  $\dot{q}$  and  $q$ , to obtain the velocity and position coordinates for the next time step.

### 6.1 Resolution of geometric equations

The resolution of position problems corresponds to the determination of dependent coordinates  $q_d$  as function of independent coordinates  $z$ . It is calculated by solving the

geometric equations. In this case, the equation is no-linear. The system can be solved by using Newton-Raphson iterative method.

$$\phi(q_i) + \phi_q(q_i) \cdot (q_{i+1} - q_i) = 0$$

Where  $\phi_q$  is the jacobian matrix of the system  $\phi$ . The method should start with an initial vector ( $q_0$ ) close to final solution ( $q$ ) in order to get a proper convergence.

## 6.2 Resolution of velocity equations

Once the geometric system is solved, the resolution of velocity problems corresponds to the determination of dependent velocities ( $\dot{q}_d$ ) as function of independent velocities ( $\dot{z}$ ). The velocity system is linear and it can be solved by means of a jacobian matrix.

$$\begin{aligned} \frac{\partial \phi}{\partial \dot{z}} \cdot \dot{z} &= -\lambda \\ \lambda &= \text{subs}(\phi, \dot{z}, [0]) \\ z &= \left( \frac{\partial \phi}{\partial \dot{z}} \right)^{-1} \cdot (-\lambda) \end{aligned}$$

## 6.3 Resolution of dynamic and acceleration equations

Dynamic and acceleration equations are also linear. It can be solved at the same time by mean of the calculation of mass matrix of acceleration and force vector. They are calculated by means of the jacobian.

$$\begin{aligned} M &= \frac{\partial (\text{Dynamic equations})}{\partial \ddot{q}} \\ V &= \frac{\partial (\text{Dynamic equations})}{\partial \varepsilon} \\ [M \quad V] \cdot \begin{bmatrix} \ddot{q} \\ \varepsilon \end{bmatrix} &= [\delta] \\ \ddot{\phi}_q &= \frac{\partial (\ddot{\phi})}{\partial \ddot{q}} \\ \begin{bmatrix} M & V \\ \ddot{\phi}_q & 0 \end{bmatrix} \cdot \begin{bmatrix} \ddot{q} \\ \varepsilon \end{bmatrix} &= \begin{bmatrix} \delta \\ (\ddot{\phi}_q \cdot \dot{q} + \dot{\phi}_t) \end{bmatrix} \end{aligned}$$

## 6.4 Integration

The integration method involves an important part to get a right solution of the problem. It depends on the structure of the equation system such as linear or non linear system, springs or dashpot for example. The calculation step also plays an important role. A proper small step can stabilize the numerical integration.

Matlab software provides a wide range of algorithm to solve the differential equations of first order with initial value. The differential equation takes the next form.

$$\dot{y} = f(t, y)$$

The solver Ode45 is based on Runge-Kuta (4,5) (Dormand-Price). It is one step method. The solver ode23s is based on Rosenbrock method of order two. It is used in case of stiff problems. The stiff problems involve very different eigenvalues in the matrix of the differential equation system, and so, stiff in the equations.

### 6.4.1 Correction

The integration is a numerical procedure always involves an error calculation. At each step calculation, the error is added making the error bigger with time. This tendency could finish without convergence of the problem. The position and velocity correction is a technical procedure used to correct the integration error since the model does not include the geometric and velocity equations in the integration process. Therefore, the correction is based on to determine the value of the integration variables by solving the position and velocity equation systems. It can be used at each step or several steps. In this project the correction is used at each step due to difficulties at convergence.

The system equations are solved by using the iterative method Newton-Raphson since the position problems is a non linear system. It takes as initial value the integration values, and usually, few iterations are enough to get an accurate solution. The correction procedure starts first with position problem, and second velocity problems. That is because the velocity equation depends on position variables.

The correction can be done in different ways, considering the minimum distance between integration point and exact solution or calculating the coordinates as function of a specific one. The minimum distance involves that all position/velocity coordinates are calculated.

### 6.4.2 Friction force dependences

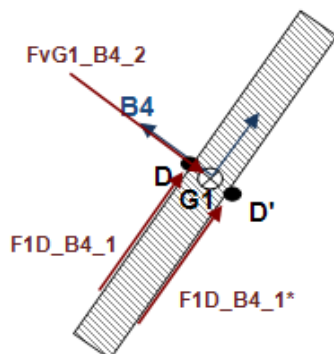
As it has been commented in the Section 5.5 Contact Mechanics, the equations of the system are fixed according to the state of the contact. That is, the constraints are imposed based on position and velocity of the Vane respect to Housing and Rotor.

The nature of the friction force imposes other conditions to the equations of the dynamic system. This is the case of the friction force  $F1D\_B4\_1$ .

$$F1D\_B4\_1 = FvG1\_B4\_2 \cdot \mu_{Rotor-Vane} \cdot aux4$$

The parameter  $aux4$  changes as function of two conditions. First, the direction and value of the speed between Rotor and Vane (Table 2). This condition is set after velocity calculation.

Second, the contact side between Vane and Rotor slot. A priori, it is not possible to know the contact side of the Rotor slot with the Vane, that is, the position of point D. So, it is estimated in the first loop of the calculation with  $aux5$  equal to "-1". The point D is fixed though the parameter  $aux5$ .



$$D = \left\{ \left[ 0, \frac{ThickVane}{2} \cdot aux5(+1), 0 \right] \right\}_{B4}$$

$$D' = \left\{ \left[ 0, \frac{ThickVane}{2} \cdot aux5(-1), 0 \right] \right\}_{B4}$$

Figure 19. Parameter  $aux5$ .

Once the forces are calculated, the first estimation is checked. The contact side between Vane and Rotor slot is determined by means of the sign of the normal force

direction of FvG1\_B4\_2.

In case of right estimation, the calculation goes to the next step. In case of failed estimation, aux4 and aux5 change the sign. That involves the point D is now in the other face of the Vane. It is necessary to change both parameters in order to match the proper friction direction and momentum over point G1.

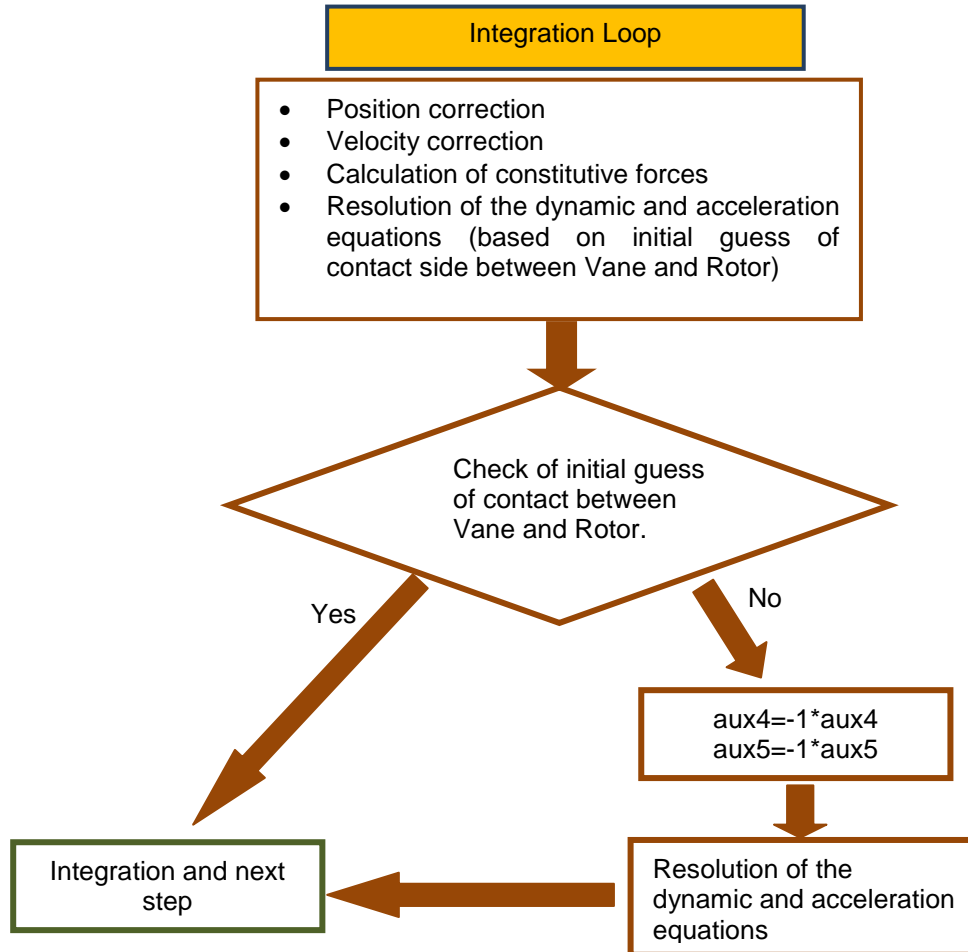


Figure 20. Integration loop scheme.

## 7 Analysis of Solver Parameters and Correction Method

In this section the behavior of the Model 1 is analyzed in terms of type of solver (ode45, ode23s), maximum step, step size, time computing, and correction method.

As it has been explained, the correction can be done in different ways. First procedure considers all position/velocity coordinates. It is named "C1". Second procedure takes into account some coordinates, not all of them. In this case the correction is done as function of  $\theta_1$ . That is, the value of  $\theta_1$  from integration procedure is not re-calculated. It is named "C2".

### 7.1 Simulation settings

Parameter	Initial values
ddt1 (rad/s <sup>2</sup> )	0
dt1(rad/s)	0
t1(rad)	0.5
Geometry	Elliptical housing
COF static / dynamic (Vane-Housing)	0.18 / 0.17
COF static / dynamic (Vane-Rotor)	0.18 / 0.17

Table 3. Initial values

Input	Value
Torque (N/m) (Constant)	0.00022

Table 4. Input value.

Parameters	Model 1
Tolerance (m)	1e-4
Max. step (s)	1e-3
Spring-dashpot 2- k (N/m) // c (Ns/m)	50000 // 15
Spring-dashpot 1	See Figure 23

Table 5. Solver parameters.

### 7.2 Comparison among correction methods and type of solver

The analysis has been carried out in terms of comparison among parameters settings.

- Comparison 1: C1 & ode45 Vs. C2 & ode45
- Comparison 2: C1 & ode45 Vs. C1 & ode23s
- Comparison 3: C2 & ode45 Vs. C2 & ode23s
- Comparison 4: C1 & ode23s Vs. C2 & ode23s
- Comparison 5: C1 & ode23s Vs. C2 & ode45

The evaluation of the comparison among simulations settings have been carried out by determining a “similar behavior”. A “similar behavior” must be understood as very similar results at the position and velocity coordinates. Small differences in force signals are allowed. The comparison of correction methods and type of solver are shown in Table 6.

	Ode45	Ode23s
C1	Similar	No Similar
C2	Similar	No Similar

Table 6. Comparison of correction methods and type of solver.

The comparison shows different results in case of ode23s and Correction 1 (C1). The next figures show the comparison 5 (C1 & ode23s Vs. C2 & ode45). The rest of setting combinations are quite similar among them.

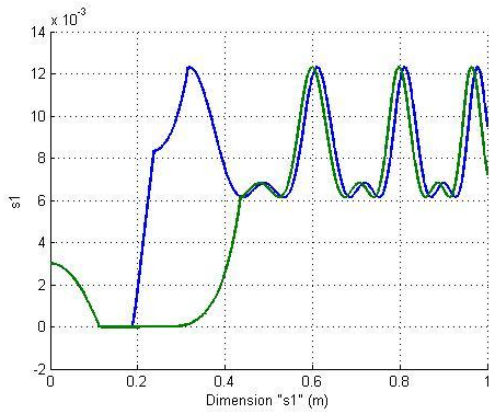


Figure 21. Dimension  $s_1$  (m). C1 & ode23s in green. C2 & ode45 in blue.

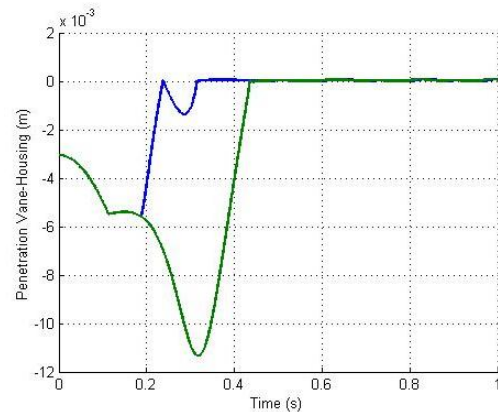


Figure 22. Penetration Vane-Housing (m). C1 & ode23s in green. C2 & ode45 in blue.

The results show mainly two differences. First, there is an offset between both simulations. Second, the bigger difference takes place in the unsteady state. The behavior of the Vane is clearly different. With "C2 & ode45", the Vane remains resting at the bottom of the slot, meanwhile, with "C1 & ode23s", the Vane takes off 0.2 seconds before. It can be seen in Figure 21 and Figure 22. This behavior ("C1 & ode23s") is not logic and unexpected.

With exception of this case, it can be concluded that there is not significant different between the type of solver, and correction method.

Computing time (s)		
	Ode45	Ode23s
<b>C1</b>	23.6	2256.9
<b>C2</b>	140.3	486.2

Table 7. Computing time.

The solver ode45 shows the lowest time computing in all cases.

## 8 Analysis of Spring-Dashpot Parameters

The contact model is based on a spring-dashpot between Vane and Housing. The spring-dashpot has been modeled as linear and non linear behavior. The objective of this section is to compare both configurations. The analysis has been carried out with the Model 1 (without geometric constraints).

The geometry used in this case corresponds to Elliptical Housing.

### 8.1 Simulation settings

Parameter	Initial values
<b>ddt1 (rad/s<sup>2</sup>)</b>	0
<b>dt1(rad/s)</b>	0
<b>t1(rad)</b>	0.5
<b>Geometry</b>	Elliptical housing
<b>COF static / dynamic (Vane-Housing)</b>	0.18 / 0.17
<b>COF static / dynamic (Vane-Rotor)</b>	0.18 / 0.17

Table 8. Initial values



Input	Value
Torque (N/m) (Constant)	0.00022

Table 9. Input value.

Parameters	Model 1
Solver	Ode45
Tolerance (m)	1e-4
Max. step (s)	1e-3

Table 10. Solver parameters.

Parameters	Spring-dashpot 2
k (N/m)	50000
c(Ns/m)	15

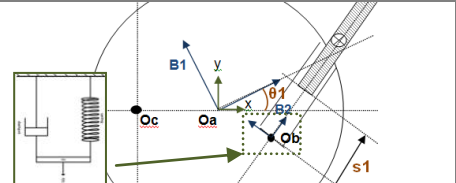


Table 11. Spring dashpot 2 parameters.

Case Simulation	Spring-dashpot 1	Values
1	Linear behavior k (N/m) // c (Ns/m)	100000 // 15
2	Linear behavior k (N/m) // c (Ns/m)	50000 // 15
3	Non linear behavior	(Variable)

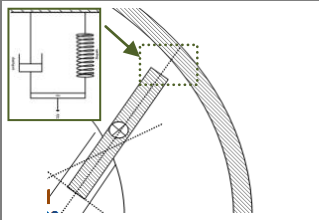


Table 12. Spring-dashpot parameters.

The non linear behavior has been defined by means of the next function.

$$F1B B3 1 = -((p1 \cdot int1^4 + 2 \cdot m_{vane} \cdot \sqrt{\frac{p1 \cdot int1^3}{m_{vane}} \cdot \dot{s}_1}) * aux1)$$

$$p1 = \frac{50000}{(1E - 3)^4}$$

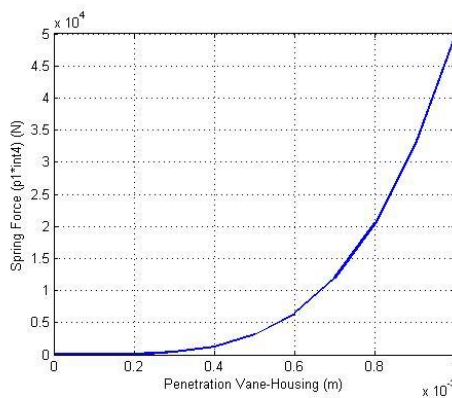


Figure 23. Spring non linear behavior.

## 8.2 Results

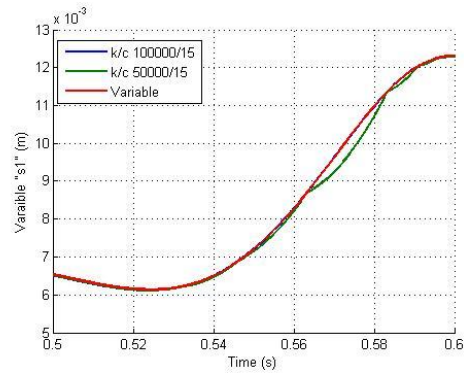
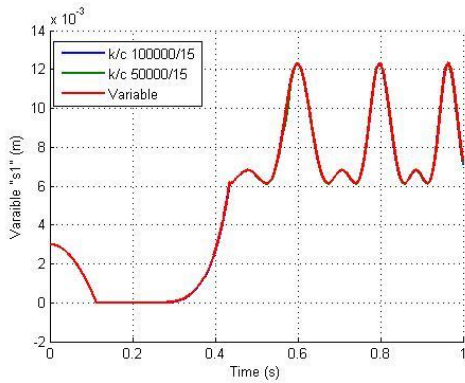


Figure 24. Variable  $s_1$ .

The different configurations cause small differences according to coordinate  $s_1$ , as it is shown in Figure 24.

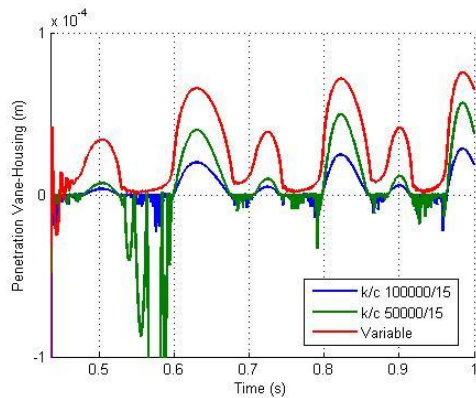


Figure 25. Penetration (m) Vane-Housing

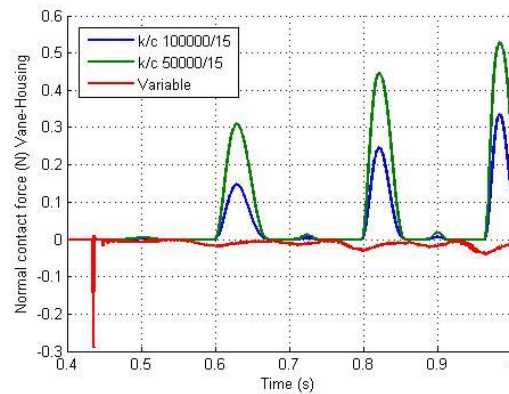


Figure 26. Normal contact force Vane-Housing.

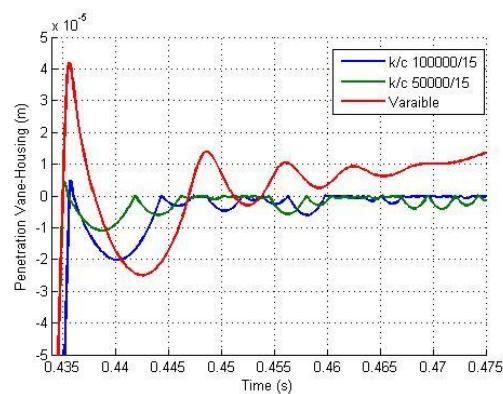


Figure 27. Penetration (m) Vane-Housing

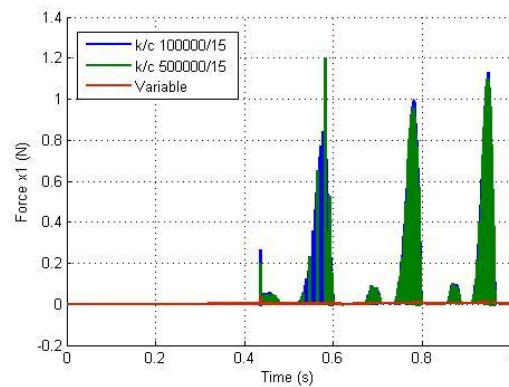


Figure 28. Force  $x_1$ .

The penetration Vane-Housing with the linear system shows a lot of rebounds of the Vane with the Housing. In some cases, the normal contact force has not physic sense. That is, positive forces (Figure 26) should not appear. That means the Housing pulls the Vane. The normal contact force must be negative according to base B3. This response takes place due to the values of  $k$ ,  $c$ ,  $int_1$  and  $\dot{s}_1$ . In case of very small positive value of  $int_1$ , and negative value of  $\dot{s}_1$ , the resultant force could be positive.

$$F1B B3 1 = -((k \cdot int_1(+)) + c \cdot \dot{s}_1(-)) * aux1$$

The spring-dashpot parameters should be adjusted according to the operation conditions. A high stiffness of the spring can cause a high frequency response, and high viscosity ratio can cause a non physic behavior of the model.

The non linear configuration shows higher penetration Vane-Housing, but it is able to keep the contact between parts. The stiffness and viscosity ratios change according to the penetration adapting the contact model to operation conditions.

The next table shows the numerical results from the solver. It can be seen that high stiffness causes higher computing time.

Case k // c	Successful steps	Failed steps	Function evaluations	Elapse time (s)
100000 // 15	4081	1296	32263	205
50000 // 15	4486	1288	34645	59.85
Variable	1715	165	11281	23.63

Table 13. Solver results.

## 9 Comparison of Models at Constant Angular Speed

The Models 1 & 2 are compared in order to analyze the differences between the contact models, with and without geometric constraints. It takes into account a constant angular speed, that is,  $\dot{\theta}_1$ , equal to zero. The response is shown in terms of force, torque,  $\theta_1$  and  $s_1$  parameters.

The Housing geometry used in this case corresponds to cylindrical Housing.

The Model 2 has geometric constraints between Housing and Vane. The contact is guaranteed in all cases. The Model 1 is driven by contact mechanics explained in section 5.5.

### 9.1 Simulation settings

Parameter	Initial values
ddt1 (rad/s <sup>2</sup> )	0
dt1(rad/s)	471
t1(rad)	0
Geometry	Cylindrical Housing
COF static / dynamic (Vane-Housing)	0.21 / 0.2
COF static / dynamic (Vane-Rotor)	0.21 / 0.2

Table 14. Simulation settings

Parameter	Model 1	Model 2
Solver	Ode23s	Ode45
Tolerance (m)	1e-4	1e-4
Max. step (s)	1e-5	1e-3
Spring-dashpot 2- k (N/m) // c (Ns/m)	50000 // 15	-
Spring-dashpot 1	See Figure 23	-

Table 15. Simulation settings

## 9.2 Results

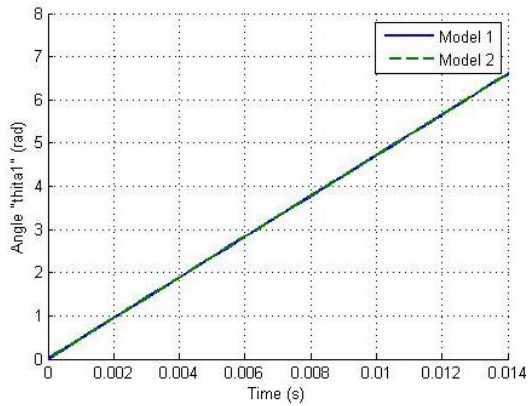


Figure 29. Angle  $\theta_1$

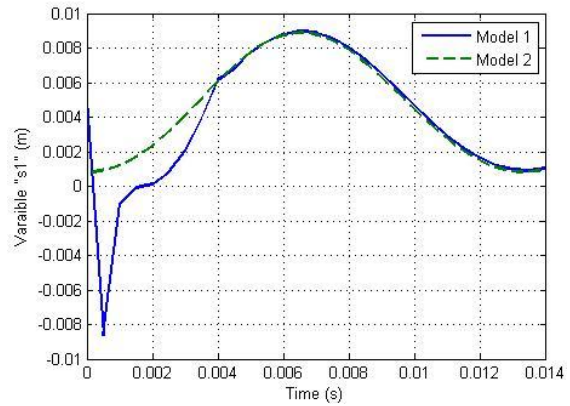


Figure 30. Variable  $s_1$

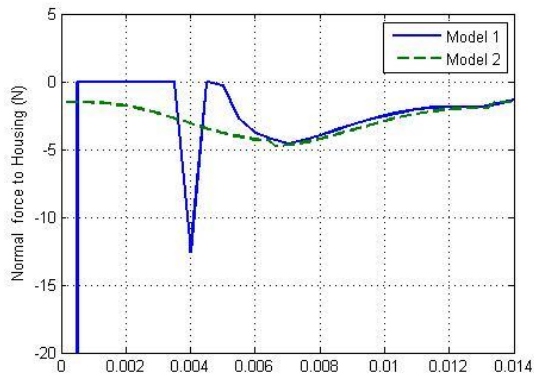


Figure 31. Normal force to Housing

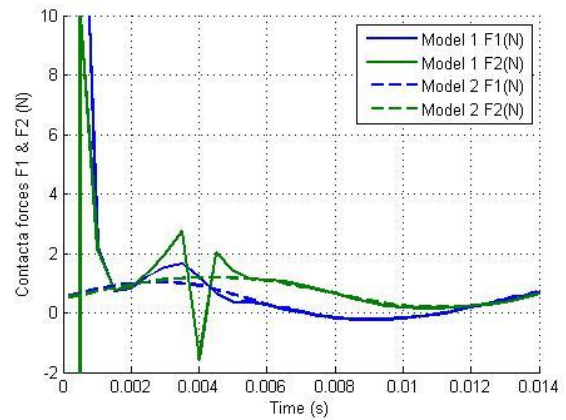


Figure 32. contact force  $F_{x1}$  &  $F_{x2}$ .

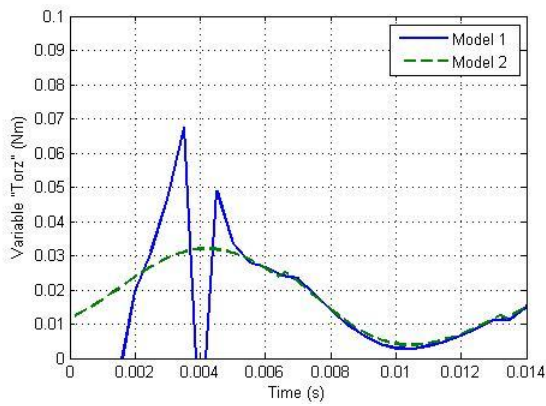


Figure 33. Torque

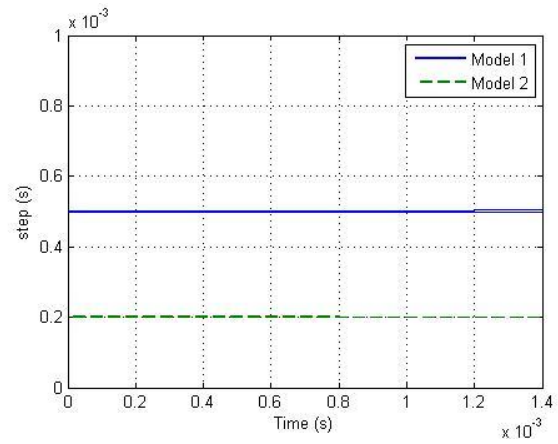
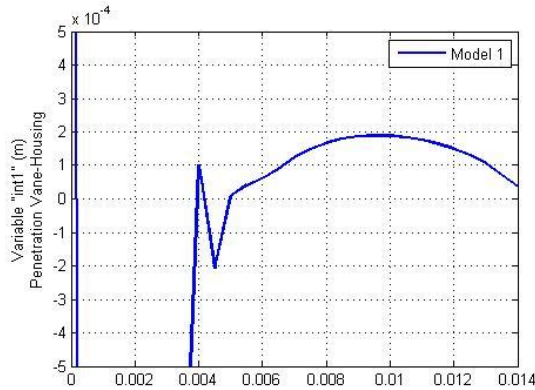


Figure 34. Step



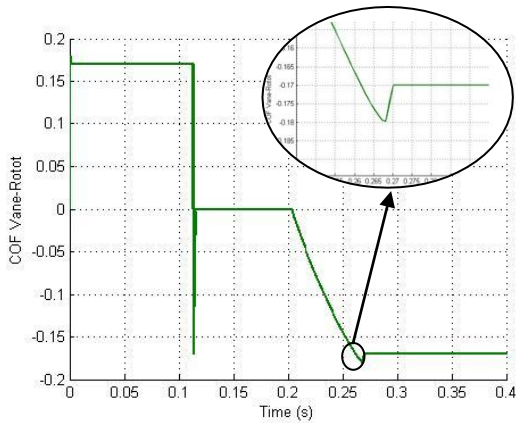
**Figure 35. Penetration Vane-Housing**

As it can be observed in figures, the Model 1 (without geometric constraints) shows a first stage of impacts and bounces until the steady state is reached. The variable “int1” involves the interference between Vane and Housing. Values higher than zero means interference, that is, contact between parts. Values lower than zero means non contact. This contact model requires a trade-off between accuracy and time convergence. The higher penetration, the easier convergence. As it has been mentioned, the penetration and convergence is controlled by the spring-dashpot parameters.

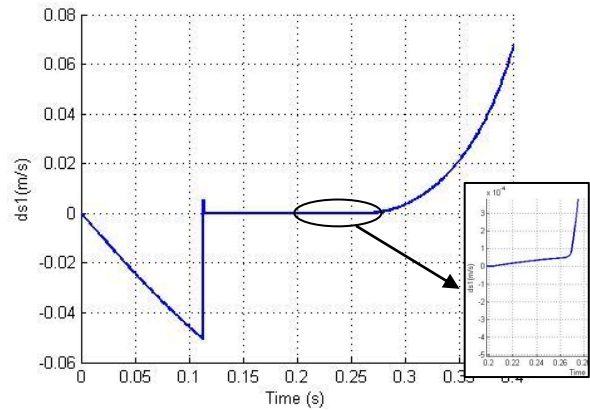
The results are quite similar between models considering both in contact with Housing.

## 10 Friction Model Results

In this section is shown the calculated friction coefficient according to section 5.5.5 Friction model.



**Figure 36. COF Vane-Rotor**



**Figure 37. Variable  $\dot{s}_1$**

The dimension  $\dot{s}_1$  is equal to zero involve that the Vane is stopped on the bottom of the slot. In this case, the COF is also zero according to the friction law. As the speed is higher, the friction coefficient increases until it reaches the dynamic coefficient.

## 11 Contact Model Vane-Rotor Results

In this section is shown the contact force Vane-Rotor according to section 5.6 Contact model Vane-Rotor. Simulation setting corresponds to Table 3, Table 4 and Table 10.

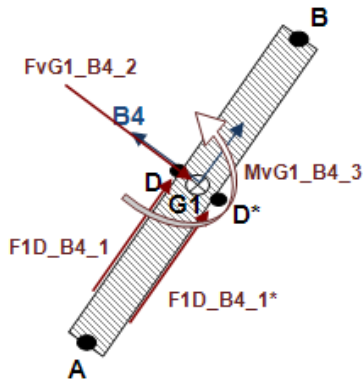


Figure 38. Mathematical model.

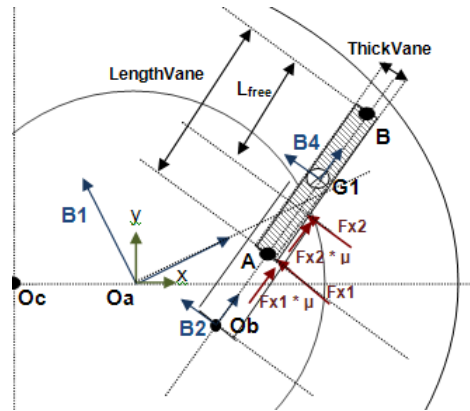


Figure 39. Contact model Vane-Rotor.

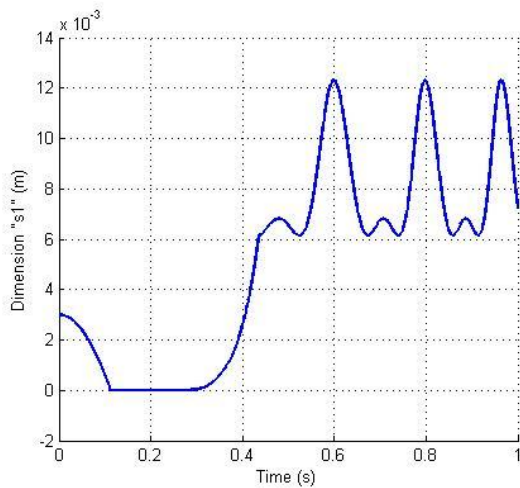


Figure 40. Dimension "s1"

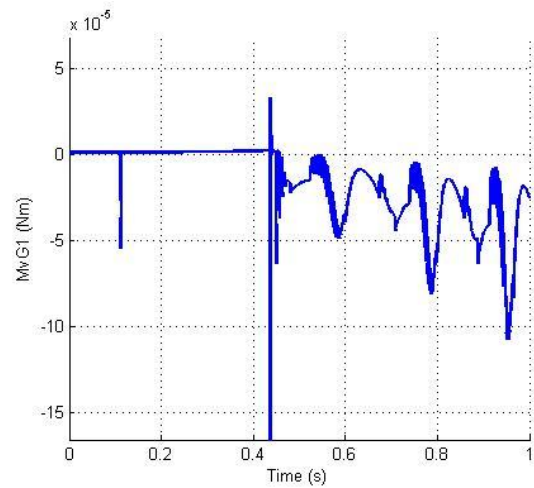


Figure 41. Moment MvG1

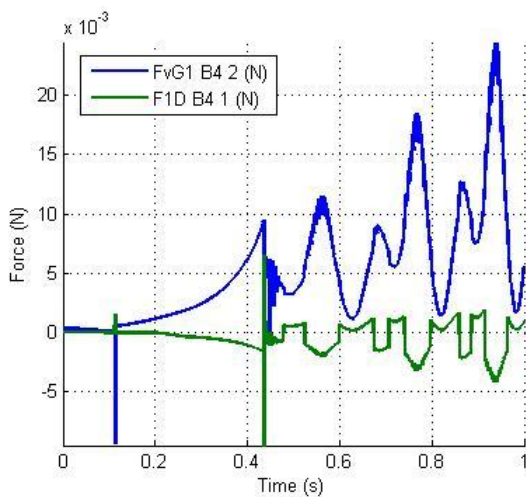


Figure 42. Mathematical Model.

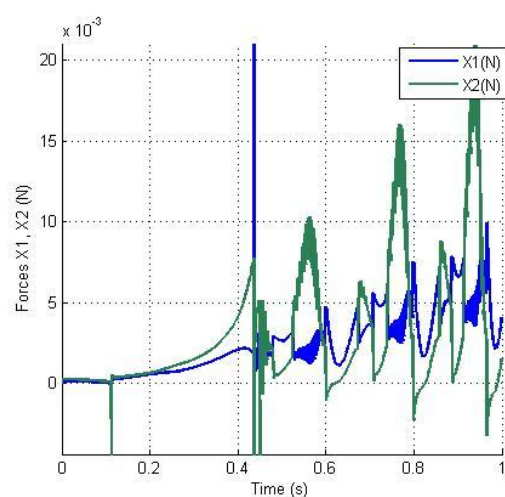


Figure 43. Contact Model.

As it can be seen in Figure 42, the force F1D changes the sign according to variation of  $s_1$ , Figure 40. As the Vane goes outward the slot, the friction force is negative. The moment MvG1 is negative due to the contact force between Vane and Housing.

Therefore, the forces  $x_1$  and  $x_2$ , Figure 43, shows higher force at  $x_2$  than  $x_1$ . The force  $x_1$  becomes negative when increasing the COF between Vane and Housing.

## 12 Identification Tool for COF

The system identification involves the mathematical modeling of the physic system, and the fitting of the unknowns to match the model with a specific behavior of the system.

In this case, the identification procedure is applied mainly to determine the friction coefficient, and the behavior of the system is characterized by the torque variable. The identification involves the search for the minimum error between calculated variables and experimental (or reference) variables. The identification procedure can be summarized as follows.

Considering a non linear model where  $W$  is the function of the system, it represents the behavior of the system (that is, the torque),  $\phi$  involves the variable to be identified and  $x$  involves the rest of parameters. The parameter “ $y$ ” represents the reference or measured torque data.

$$y = W(x, \phi)$$

The identification procedure requires linearizing the model, around initial estimation.

$$\phi = \phi_0 + \Delta\phi$$

$\Delta\phi$  is a small value, so  $\phi$  and  $\phi_0$  are close.

$$y = W(x, \phi) = W(\phi) = W(\phi_0 + \Delta\phi) \cong W(\phi_0) + \left. \frac{\partial W}{\partial \phi} \right|_{\phi=\phi_0} \cdot \Delta\phi$$

$$J^{(k)} = \left. \frac{\partial W}{\partial \phi} \right|_{\phi=\phi^{(k)}}$$

The objective of identification is to find  $\phi$  to meet the relation  $y = W(\phi)$ . That is equivalent to minimize  $(y - W(\phi))$  by using least square technique. The procedure is iterative by calculating new values of  $\Delta\phi$ , and updating the value of  $\phi$  and  $\phi_0$ .

The algorithm is detailed next.

$$\begin{aligned} \phi &= \phi_0 \\ J^{(0)} &= \left. \frac{\partial W}{\partial \phi} \right|_{\phi=\phi^{(0)}} \\ \Delta\phi^{(0)} &= (J' \cdot J)^{-1} \cdot J' \cdot (y - W(\phi^{(0)})) \\ \phi^{(1)} &= \phi^{(0)} + \Delta\phi^{(0)} \\ \text{While } \|\Delta\phi\| &> TOL \\ \phi^{(k)} &= \phi^{(k+1)} \\ J^{(k)} &= \left. \frac{\partial W}{\partial \phi} \right|_{\phi=\phi^{(k)}} \\ \Delta\phi^{(k)} &= (J' \cdot J)^{-1} \cdot J' \cdot (y - W(\phi^{(k)})) \\ \phi^{(k+1)} &= \phi^{(k)} + \Delta\phi^{(k)} \cdot f \\ \text{End} \end{aligned}$$

The parameter “ $f$ ” means the relaxation factor used to improve the convergence. The value of this factor ranges from 0 to 1.

### 13 Application of Identification Tool to Test Signals

The first step to check and validate the identification tool is to identify a reference signal. The identification is carried out to identify the COF, phase of the signal, and phase of Rotor speed, “ds1” velocity coordinate. In addition, it has been considering that the reference signal (torque in our case) could have noise or other effect not included in the mathematical model. This problem has been deal including the identification of several phases for the signal and Rotor speed. These settings are not intended to be a final solution. It is just an example of the system flexibility and efficiency. The identification tool must be fitting for each measure (signal to identify). Thus, it has been defined two signals (torque) to identified different parameters.

#### 13.1 Signal 1

Parameters	Value
COF	0.17
ddt1 (rad/s <sup>2</sup> )	0
dt1 (rad/s)	471
Phase 1 (rad)	$\pi/2$
Phase 2 (rad)	$\pi$
Geometry	Elliptical Geometry Test
Number of Vanes	1
Mathematical Model	2

Table 16. Signal 1 parameters.

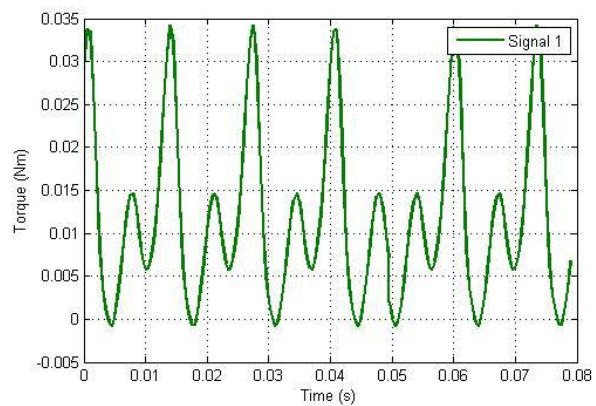


Figure 44. Signal 1.

#### 13.2 Signal 2

Parameters	Value
COF	0.17
ddt1 (rad/s <sup>2</sup> )	0
dt1 (rad/s)	471
Phase (rad)	0.15
Geometry	Elliptical Geometry Test
Number of Vanes	4
Mathematical Model	2

Table 17. Signal 2 Parameters

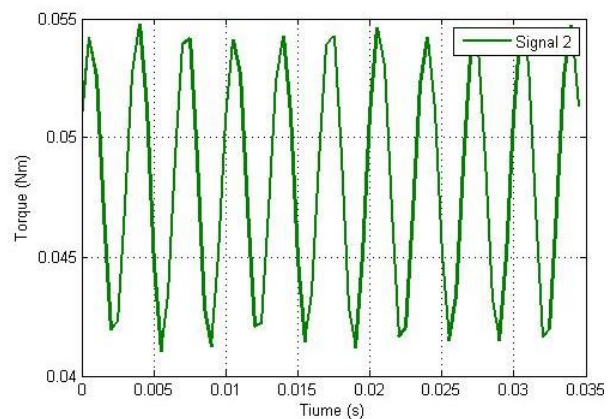


Figure 45. Signal 2



### 13.3 Identification of Signal 1

The initial conditions for the identification of Signal 1 are the next.

Parameters	Signal 1	Initial guess
COF	0.17	0.1
ddt1 (rad/s <sup>2</sup> )	0	0
dt1 (rad/s)	471	471
Phase 1 (rad)	pi/2	0.1
Phase 2 (rad)	pi	0.1

Table 18. Initial guess.

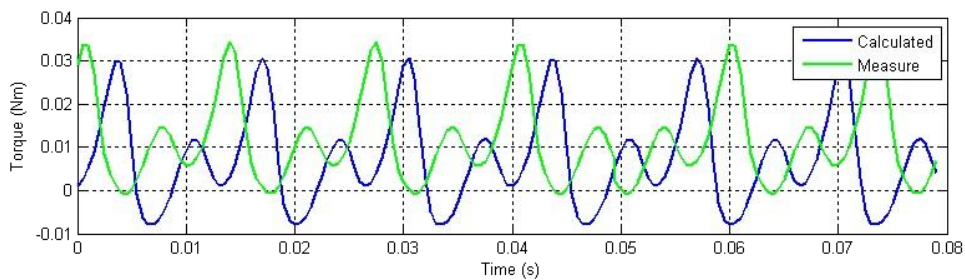


Figure 46. Comparison Signal 1 vs. initial conditions.

The next figure shows the identification procedure through the iterations.

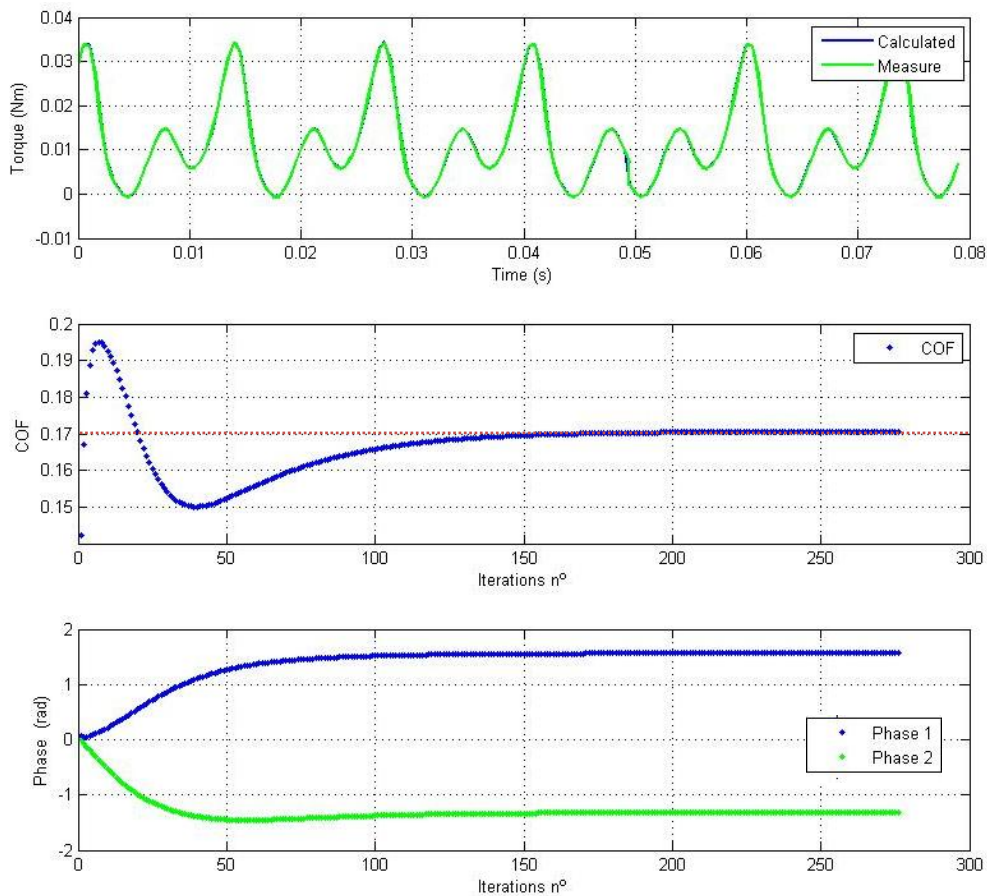


Figure 47. Identification procedure for Signal 1. Target COF in red.

### 13.4 Identification of Signal 2

The initial conditions for the identification of example 2 are the next.

Parameters	Signal 2	Initial guess
COF	0.17	0.05
ddt1 (rad/s <sup>2</sup> )	0	0
dt1 (rad/s)	471	400
Phase (rad)	0.15	0.15
Number of Vanes	4	4

Table 19. Initial guess.

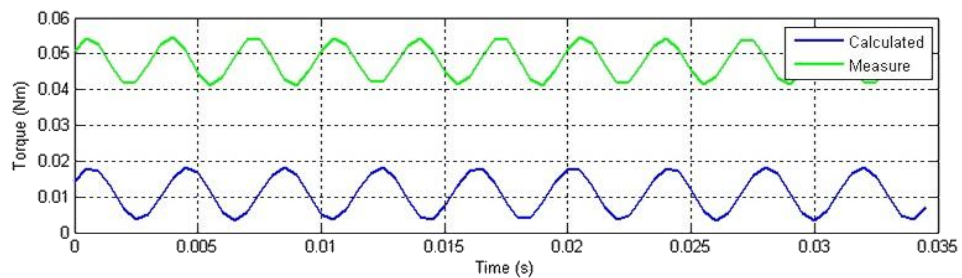


Figure 48. Comparison Signal 2 vs. initial conditions.

The next figures show the identification procedure through the iterations.

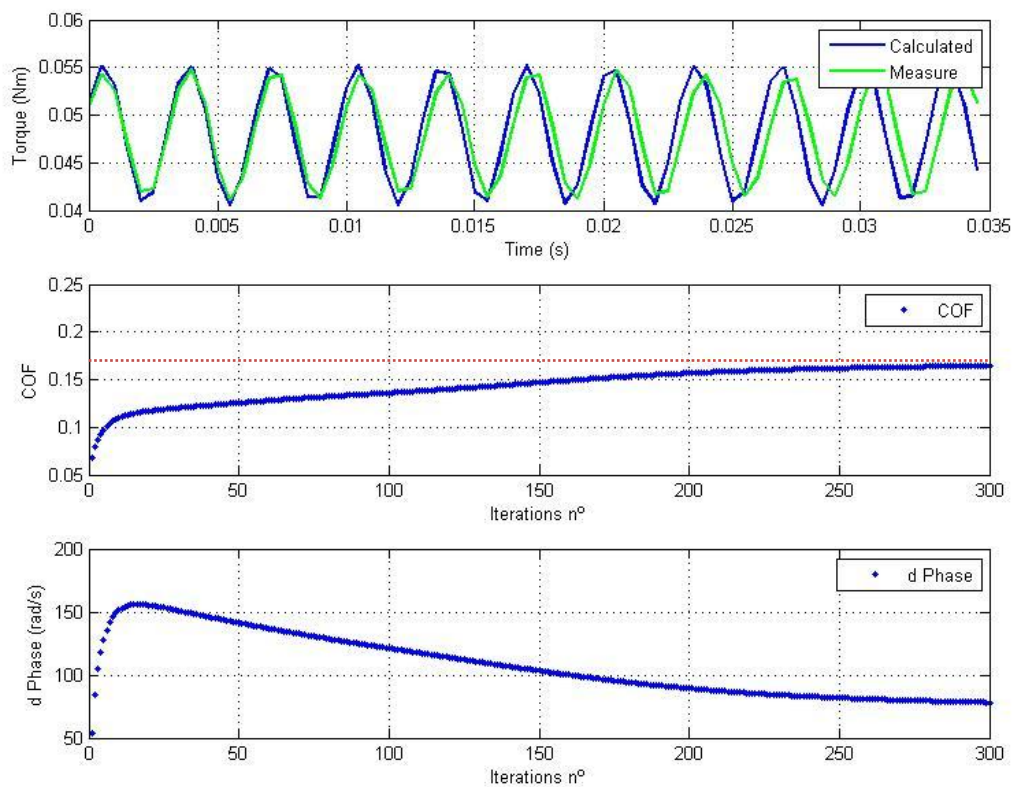


Figure 49. Identification procedure for Signal 2. Target COF in red.

## 14 Application of Identification Tool to Experimental Data

In this section, the identification tool is applied to experimental data to identify the COF. The phase of the signal or the Rotor speed has not been identified due to the geometric conditions of the test rig. The Housing is cylindrical, and the Rotor is concentric with the Housing.



Figure 50. Test Rig.

### 14.1 Experimental data

According to the geometric conditions, and considering constant speed, the torque signal should be a horizontal line. As it can be observed in Figure 51, the signal shows scattering. It could take place due to speed control effect or other phenomenon not considered. The Rotor speed keeps constant at 607.37 rad/s (5800 rpm). The model involves four vanes. The identification tool has been applied with a short length of the signal at constant angular speed.

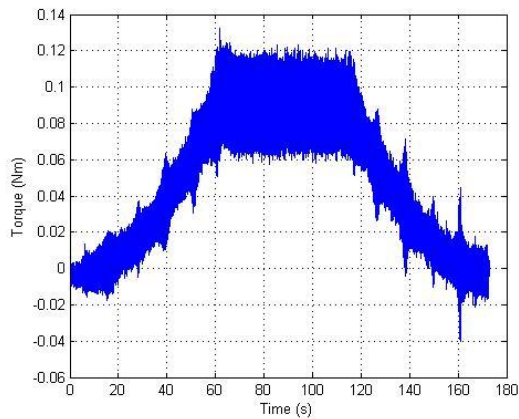


Figure 51. Experimental data.

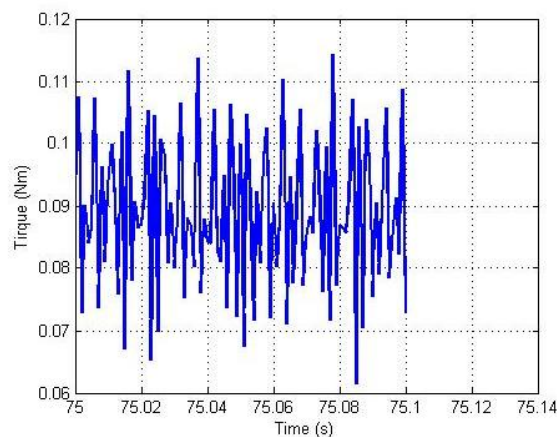
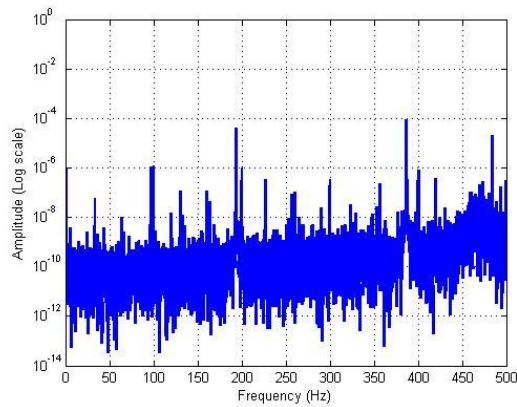


Figure 52. Sample data.



**Figure 53. Fast Fourier transform of the experimental data at constant angular speed.**

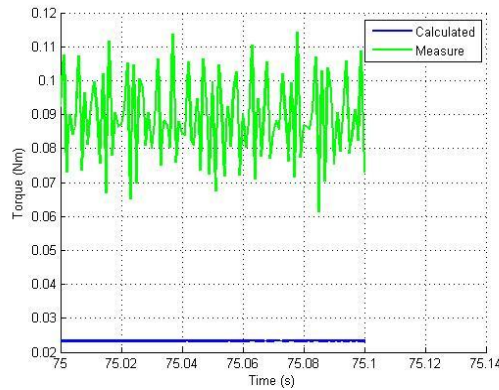
Figure 53 shows the fast Fourier transform of the signal at constant angular speed (horizontal section). The signal shows frequencies at multiples of the speed frequency (96.66 Hz) or equivalent to frequency of one Vane. That is, 193.33, 289.99 and 386.66 Hz.

#### 14.2 Identification of experimental data

The initial conditions for the identification are the next.

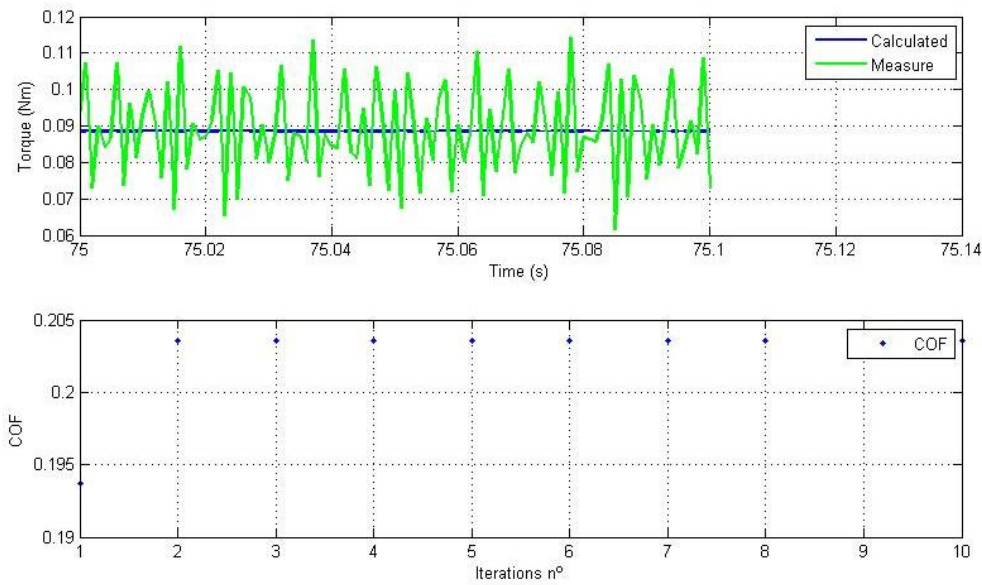
Parameters	Experimental data	Initial guess
COF	-	0.1
ddt1 (rad/s <sup>2</sup> )	0	0
dt1 (rad/s)	607.37	607.37

**Table 20. Initial guess.**



**Figure 54. Comparison experimental data vs. initial conditions.**

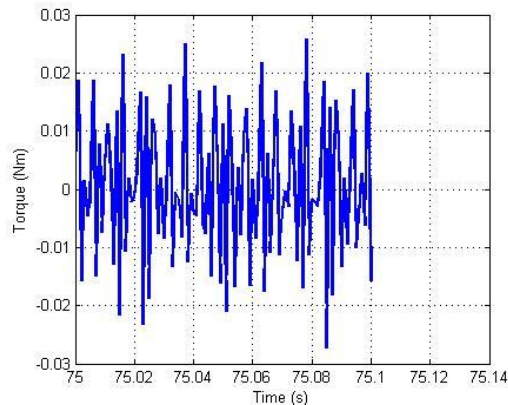
The next figures show the identification procedure through the iterations.



**Figure 55. Identification procedure for experimental data.**

The identification tool is able to identify the COF, 0.203, at few iterations.

In order to identify the nature of the noise, the white noise (defined as random signal) has been checked by applying the algorithm "runtest" available in Matlab. The noise has been calculated by extracting the calculated signal to the measure signal.



**Figure 56. Noise.**

The test has identified the noise as no random. That involves the model is not able to capture all the physic phenomena of the real system such as misalignment or geometric imperfections.

## 15 Conclusions

This project has been focused on acquiring new knowledge and techniques about dynamic modeling and identification procedure to be applied on a Vacuum Pump product, system under development stage by Entecnia Consulting S.L.

### 15.1 Modeling

A dynamic model has been developed to analyze the contact between Vane and, Housing and Rotor. Specifically, the analysis has been focused on modeling the contact taking into account the friction Coulomb's law.

The contact mechanism has been defined in two ways. First, the contact model is

based on a spring-dashpot system (Model 1). Second, the contact model is based on imposed constraint between Vane and Housing (Model 2). The Model 1 requires a trade-off between accuracy and time convergence. The higher penetration, the easier convergence. The penetration and convergence is controlled by the spring-dashpot parameters. The comparison between models shows similar solution for both models considering both in contact with Housing.

The solver, maximum step and correction have been tested and compared. Only one configuration (C1 & ode23s) show an unexpected behavior. With exception of this case, it can be concluded that there is not significant difference between the type of solver, and correction method.

The spring-dashpot has been modeled as linear and non linear behavior. The system response changes due to the combination of “k” and “c”. These parameters should be adjusted according to the operation conditions. A high stiffness of the spring can cause a high frequency response, and high viscosity ratio can cause a non physic behavior of the model. The non linear configuration shows higher penetration Vane-Housing, but it is able to keep the contact between parts. The stiffness and viscosity ratios change according to the penetration adapting the contact model to operation conditions.

The system is modeled with vector expressions, and it is programmed with 3d-mec software (Public University of Navarra) and Matlab software to get the system equations and to define the matrix structure for the numerical simulation. The dynamic equations have been formulated with virtual power method.

The friction model has been developed according to Adams/Solver Impact model. The transition between static and dynamic friction coefficient has been characterized based on the velocity-friction law. This transition is detected only with small integration step.

The equations of the system are fixed according to the state of the contact. The nature of the friction force imposes other conditions to the equations of the dynamic system, that, a priory, it is not possible to know. That is, the contact side of the Rotor slot with the Vane. So, a first estimation is made in the first loop and checked and re-calculated in case of failure prediction.

The model and modeling procedure will be used in further stage to analyze the operation under different geometries and to determine the most efficient design.

## **15.2 Identification tool for COF**

The system identification involves the mathematical modeling of the physic system, and the fitting of the unknowns to match the model with a specific behavior of the system. In this case, the identification procedure is applied mainly to determine the friction coefficient, and the behavior of the system is characterized by the torque variable.

The first step to check and validate the identification tool is to identify a test signal. The results are accurate enough.

The identification tool has been applied to experimental data to identify the COF being able to identify the COF in few iterations.

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