Sharing the costs of cleaning a river: the Upstream Responsibility rule

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Abstract

The cleaning up of waste present in transboundary rivers, which requires the cooperation of different authorities, is a problematic issue, especially when responsibility for the discharge of the waste is not well-defined. Following Ni and Wang (2007) we assume that a river is a segment divided into several regions from upstream to downstream. We show that when the transfer rate of the waste is unknown, the clean-up cost vector provides useful information for estimating some limits in regard to the responsibility of each region. We propose a cost allocation rule, the *Upstream Responsibility rule*, which takes into account these limits in distributing costs "fairly" and we provide an axiomatic characterization of this rule via certain properties based on basic ideas concerning the responsibility of regions.

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1 Introduction

Motivation The presence of waste in river channels is a major environmental problem faced by authorities since, on the one hand, waste can pollute water, which can be harmful for people, plants and animals, causing serious diseases and affecting ecosystems. As a consequence, the inhabitants of a region with more waste in its part of the river confront a cost: they consume lower quality water and/or face higher water depolluting costs. On the other hand, the presence of accumulated waste in a river is known to have a substantial effect on the probability of flooding when there is heavy rain, so it seems advisable to clean river channels regularly to reduce this danger. Around the world, about 200 rivers (see Ambec and Sprumont, 2002 and Barrett, 1994) flow across national borders, and a much greater number across borders between regions or municipalities. All the activities for cleaning transboundary rivers may require cooperation on the part of the different authorities involved and coordination of efforts if they are to be effective. However, the distribution of the costs of these activities among the different regions may be a problematic issue, particularly when the extent to which each region is responsible for the waste discharged is not well-defined.

As far as we know, the first paper to analyze the problem of sharing the costs of cleaning a river among different regions from a theoretical point of view is that of Ni and Wang (2007). They model a river as a segment which is divided into subsegments from upstream to downstream such that each region is located in one of them. They assume that there is a central agency that determines the cost of cleaning each of these segments and they axiomatically propose two methods for allocating the total cleaning costs among all regions along the river. The first method, called *Local Responsibility Sharing*, establishes that the total cost that each region should pay is directly the cost of cleaning the segment in which this region is located. The second method, called *Upstream Equal Sharing*, states that the total cost that each region should pay is obtained by distributing equally the cost of cleaning each segment among the region in that segment and all the regions situated upstream from it.[1] We show that neither

[1]These methods are based on the theories or principles of Absolute Territorial Sovereignty and Unlimited Territorial Integrity, respectively (see Godana, 1985 and Kilgour and Dinar, 1996).

of these methods allocates the costs in a way that reflects the responsibility of each region in producing the waste present in river channels. The first does not take into consideration that the water of a river flows from one segment to another, taking part of the waste with it. The second implicitly assumes that the region in a segment and all the regions situated upstream from this have the same degree of responsibility for the waste present in the segment in question. However, this would only be "fair" if all regions have discharged exactly the same quantity of waste of the one present in that segment, which is not necessarily the case.

Overview of results In this paper, we seek to develop an alternative rule to the methods proposed by Ni and Wang (2007) which takes into account the responsibility of the regions for the presence of the waste. We explicitly introduce into our model the fact that the waste is transferred, with the water, from upstream to downstream at a particular rate, an idea that is implicitly assumed in Ni and Wang (2007). If the social planner knew this rate, she could use the cost vector to accurately calculate the amount of waste discharged by each region into the river, and the costs could thus be distributed according to their actual responsibilities. However, in practice, the transfer rate may be unknown.[2] In that case, we show that the social planner could estimate certain limits of that rate from the cost vector. Those limits provide useful information for distributing the costs fairly, since they enable certain limits of responsibility to be inferred for each region. We show that the rules that naturally adapt the methods proposed by Ni and Wang (2007) do not always assign costs in the intervals constructed with these limits, thus violating this basic principle of fairness.

We introduce a set of desirable properties taking into account this information concerning the responsibility of each region in discharging the waste. Those properties are: (i) *Limits of Responsibility*, which requires the cost paid by each region for cleaning its own segment always to be within its limits of responsibility; (ii) *No Downstream Responsibility*, which states that a region j situated downstream from another region i has no responsibility for the waste present in i and therefore does not have to pay anything towards the cost of cleaning it up;

[2] This uncertainty about the transfer rate is usually assumed in the literature on non-point source pollution (see Segerson (1988)).

(*iii*) *Consistent Responsibility*, which ensures that the part of the cost of cleaning a segment paid by one region relative to the part paid by another region is consistent throughout all the segments situated downstream from both regions; and (*iv*) *Monotonicity with respect to Information on the Transfer Rate*, which states that when information on the transfer rate improves in such a way that it becomes natural to induce a higher (lower) estimated value for the real transfer rate, the amount of waste in any segment for which all its upstream regions are responsible must not be lower (higher) than before.

That set of properties characterize a new cost allocation rule, the *Upstream Responsibility rule*, which works as follows: first, it assigns to the region situated in a given segment the value of its responsibility taking as the transfer rate the mid-point in the interval between its lower and its higher limits. The remaining cost of cleaning the segment in question is divided among the upstream regions, maintaining the proportions of the allocation of the cost of cleaning the previous segment.

Related literature The study of allocation problems using game theoretical and/or axiomatic models to solve issues related to transboundary rivers has developed in two directions. On the one hand (the *harmful side*) some authors have developed models for studying how to share the costs of cleaning a river among the regions located along it. On the other hand (the *beneficial side*) some papers have analyzed models for determining how to share water resources among the different regions along a river.

Among the papers dealing with the harmful side, which is the body of literature into which our paper fits, there are two main approaches. Several papers, starting with Ni and Wang (2007) and including ours, consider a river as a segment divided into different regions and assume that the cost of cleaning each region is exogenously given. Along these lines, Ni and Wang (2007) propose and characterize the two methods - Local Responsibility Sharing and Upstream Equal Sharing - described above. They also defend these methods as the Shapley values of two appropriately defined TU games and as solutions belonging to the core of this problem. Van den Brink and van der Laan (2008) show that these additional results are particularizations of certain well-known results of cooperative game theory (in particular, the problem is essentially an airport cost game, see Littlechild and Owen (1973)) and they provide an alternative axiomatic characterization of these methods. This model is extended by Dong et al. (2012) by considering a river as a network. Based on a different principle (the "polluter-pays" principle), Gómez-Rúa (2013) defines water taxes according to regions' responsibilities for pollution and characterizes several cost allocation methods based on properties of those taxes. Other papers such as Gengenbach et al. (2010) and van der Laan and Moes (2012) take a substantially different approach by assuming that the cost allocation method adopted may affect the decision of each region about how much waste to discharge.

On the beneficial side, papers generally analyze water allocation problems and the fair distribution of the welfare resulting from distributing the water of a river among different regions. Based on cooperative game theory, Ambec and Sprumont (2002) model this situation by defining a coalitional form game. They analyze how water should be allocated across the agents and propose what monetary transfers should be made. Along these lines, Ambec and Ehlers (2008) generalize the aforesaid model by allowing for satiable agents. Wang (2011), using a similar model but with a market-based approach, analyzes efficient allocations when trade is restricted to neighboring agents along the river. Khmelnitskaya (2010), and van den Brink et al. (2012) extend the previous models by considering rivers with multiple springs.[3] Rebille and Richefort (2012) analyze the problem of water allocation from a non-cooperative point of view.

Remainder The rest of this paper is organized as follows. Section 2 describes the basic model, adapts the existing solutions of the literature to our framework and introduces a result that shows that the cost vector can provide useful information worth considering when constructing a cost allocation rule based on responsibility. Section 3 discusses some axioms for cost allocation rules reflecting basic ideas of responsibility, defines the Upstream Responsibility rule, provides a characterization of it based on the axioms defined previously and compares it with the other solutions. Section 4 contains several extensions of the basic model covering more complex situations and, finally, Section 5 concludes. The Appendix contains the proofs of the results.

[3] For more details on the use of cooperative game theory to model water allocation problems, readers are referred to any of the numerous surveys on the matter. See for instance, Béal et al. (2013), Beard (2011) and Parrachino et al. (2006).

2 The basic model

Notation and definitions

Consider a river which is divided into n segments of the same size from upstream to downstream. There is a set of regions, each of which is located in one of the segments, which have discharged waste into the river.[4] This river has a transfer rate t that measures the proportion of waste that is transferred from one segment of the river to the next. This transfer rate may not be exactly known. Consider a general case in which the social planner knows that t is situated within an interval $[\underline{t}, \bar{t}]$, where $\underline{t} \in [0, 1)$ and $\bar{t} \in (0, 1]$.[5] Cases in which $\underline{t} = \bar{t}$ are situations in which the social planner knows the actual transfer rate, while the cases in which $\underline{t} = 0$ and $\bar{t} = 1$ are situations in which there is no information at all about t.

There is a central agency that determines the cost of cleaning the river in each segment. We assume that this cost is exactly the amount of waste present in the segment in question.[6] The agency has to allocate the costs of the cleaning process to the different regions in a fair way. Our main objective is to find rules for allocating those costs in a way that reflects the responsibility of each region in the discharging of the waste.

Formally, let $N = \{1, ..., n\} \subset \mathbb{N}$ be a finite set of regions such that i is situated upstream of $i + 1$ for all $i \in \{1, \ldots, n - 1\}$. Let $C = (c_1, ..., c_n) \in \mathbb{R}^n_+$ be the cleaning cost vector, where c_i represents the cost incurred to clean the river in region i. Then, a *cost allocation problem* is a tuple $(N, C, \underline{t}, \bar{t})$.[7]

[4] To make the results clearer, we start in the basic model with the strong assumption that all segments are the same size. In Section 4 we explain how the results can be generalized to cover the cases in which the segments may be of different sizes.

[5] We exclude from the basic analysis the extreme cases in which either $\bar{t} = 0$ or $\underline{t} = 1$. Similarly, this basic model assumes a uniform transfer rate along the river. We explain in Section 4 how the results can be adapted when these assumptions are dropped.

[6] This assumption is made for the sake of fluency. We could have assumed instead that the cost of cleaning each segment is an increasing linear function of the amount of waste present in it, without essentially altering the results.

[7] A problem can also be defined by a triple $(C, \underline{t}, \bar{t})$ given that the information on N is included in C. However, we prefer to maintain both to be consistent with the notation used by Ni and Wang (2007).

A *cost allocation rule* is a mapping x that assigns to each problem $(N, C, \underline{t}, \overline{t})$ a matrix of size $n \times n$, $(x_i^j)_{i,j \in N}(N, C, \underline{t}, \overline{t})$ such that all its components are non–negative and $\sum_{i \in N} x_i^j(N, C, \underline{t}, \overline{t}) = c_j$. With this interpretation, $x_i^j(N, C, \underline{t}, \overline{t})$ represents the part of the cost of cleaning segment j that region i pays. When there is no risk of confusion about the description of the problem, we will only write $x_i^j(\cdot)$. We will denote by $x_i(N, C, \underline{t}, \overline{t})$ the total cost allocated to region i by the rule x in the problem $(N, C, \underline{t}, \overline{t})$; i.e. $x_i(N, C, \underline{t}, \overline{t}) = \sum_{j \in N} x_i^j(\cdot)$. Note that the definition of a rule implies that $\sum_{i \in N} x_i(\cdot) = \sum_{i \in N} c_i$.[8] A different solution concept to cost allocation problems that was proposed by Ni and Wang (2007) and also studied by van den Brink and van der Laan (2008) is a *cost allocation method*. This solution concept is a function x that assigns to each cost allocation problem $(N, C, \underline{t}, \overline{t})$ the vector $(x_i(N, C, \underline{t}, \overline{t}))_{i \in N} \in \mathbb{R}_+^n$; i.e. the total cost that each region pays for cleaning the entire river.[9]

The LRS and UES rules

In this subsection we discuss the solutions proposed by Ni and Wang (2007) and also studied by van den Brink and van der Laan (2008): the Local Responsibility Sharing method, $\bar{x}$, defined by $\bar{x}_i(N, C, \underline{t}, \overline{t}) = c_i$ for all $i \in N$; and the Upstream Equal Sharing method, $\hat{x}$, defined by $\hat{x}_i(N, C, \underline{t}, \overline{t}) = \sum_{j \geq i} \frac{c_j}{j}$ for all $i \in N$.

A *cost allocation method* is a less precise solution concept than a *cost allocation rule* given that the latter also makes it explicit how the total cost that each region pays to clean the entire river is attributed to each segment of the river (i.e. how each $x_i(\cdot)$ is decomposed into $x_i^j(\cdot)$ for each j). We consider that a cost allocation rule is not only more informative but is also a better solution concept if the social planner decides to take into account the responsibility held by each region for the discharging of the waste present in each segment of the river. This is because a cost allocation rule explains explicitly how the cost of cleaning each segment has to be shared (i.e. how each c_j is allocated

[8] This condition is imposed in the studies of Ni and Wang (2007) and van den Brink and van der Laan (2008) as an axiom called Efficiency. We consider that this property should be included in the definition of a rule.

[9] Notice that, as mentioned in the Introduction, these papers do not explicitly consider information about the transfer rate ($\underline{t}$ and $\overline{t}$). However, we prefer to maintain it in the definition of their framework to highlight the main difference between the solution concepts.

to $x_i^j(\cdot)$ for each i). Note that each cost allocation method, and $\bar{x}$ and $\hat{x}$ in particular, corresponds to multiple cost allocation rules. It could be argued that this makes it hard to compare these solutions to those formulated in terms of cost allocation rules. Thus, it is convenient to determine a matching between each of those method and a particular rule. To that end, we adapt the set of axioms that characterize each method to the case of cost allocation rules and are able to isolate one particular rule for each particular method. We use the characterizations proposed by van den Brink and van der Laan (2008).[10]

The first axiom is No Blind Cost, which states that the total cost paid by a region in which there is no waste should be zero. We maintain this property invariant with respect to its original version.

No Blind Cost (NBC): For all problems $(N, C, \underline{t}, \overline{t})$ and all $i \in N$ such that $c_i = 0$,

$$x_i(\cdot) = 0.$$

The second property is Cost Symmetry, which states that region i and all its upstream regions should pay the same total cost if there is no waste in any segment upstream from i. We adapt the property to our framework by requiring not only that the total cost paid by each of these regions should be the same, but also its decomposition in terms of each segment.

Cost Symmetry (CS): For all problems $(N, C, \underline{t}, \overline{t})$ and all $i, j, k \in N$ such that $j, k \leq i$ and $c_k = 0$ for all $k < i$,

$$x_j^l(\cdot) = x_k^l(\cdot) \text{ for all } l \in N.$$

The third and last property needed for these characterizations is Independence of Upstream Costs, which states that the total cost paid by a region should depend only on the waste present in its segment and in all the regions situated downstream from it. Similarly to CS, we also adapt this property to our

[10] Unlike those proposed by Ni and Wang (2007), the characterizations proposed by van den Brink and van der Laan (2008) do not require an additivity property. They show that this axiom becomes unnecessary for the characterizations when the set of the other properties is slightly modified.

framework by requiring not only that the total cost paid by each region be independent of the waste upstream from it, but also its decomposition in terms of each segment.

Independence of Upstream Costs (IUC): For all problems $(N, C, \underline{t}, \overline{t})$ and $(N, C', \underline{t}, \overline{t})$ and all $i \in N$ such that $c_h = c'_h$ for all $h > i$,

$$x_j^k(N, C, \underline{t}, \overline{t}) = x_j^k(N, C', \underline{t}, \overline{t}) \text{ for all } j, k \in N \text{ such that } j > i.$$

We consider that these additional requirements in CS and IUC appropriately complement the original idea of the axioms when the solution concept adopted is a cost allocation rule.

Now we introduce two cost allocation rules that are particular extensions of the two methods proposed and studied by Ni and Wang (2007) and van den Brink and van der Laan (2008).

Definition 1 *The* Local Responsibility Sharing *(LRS) rule, α, is given by*

$$\alpha_i^j(\cdot) = \begin{cases} 0 & \text{if } i \neq j \\ c_i & \text{if } i = j. \end{cases}$$

Definition 2 *The* Upstream Equal Sharing *(UES) rule, β, is given by*

$$\beta_i^j(\cdot) = \begin{cases} 0 & \text{if } i > j \\ \frac{c_j}{j} & \text{if } i \leq j. \end{cases}$$

We show that these are the only cost allocation rules isolated by using the natural adaptations of the axioms that characterize the original methods.[11]

Proposition 1 *A cost allocation rule satisfies NBC and IUC if and only if it is the Local Responsibility Sharing rule α.*

Proposition 2 *A cost allocation rule satisfies CS and IUC if and only if it is the Upstream Equal Sharing rule β.*

[11] The proofs, which can be found in the Appendix, follow the same arguments used by van den Brink and van der Laan (2008).

Limits of responsibility

We have assumed that the transfer rate t may be not totally known a priori by the social planner. However, there is some information that can be deduced from the cleaning cost vector. Let us first explain this idea with an (extreme) case: Consider a river with a cleaning cost vector such that $c_1 > 0$, $c_2 = 0$ and $c_3 > 0$. On the one hand, the presence of waste in region 1 implies that $t \neq 1$, since otherwise there would be no waste left in this segment and c_1 would be 0. On the other hand, the absence of waste in region 2 implies that $t \notin (0,1)$, because those values of the transfer rate jointly with the presence of waste in region 1 would imply the presence of some waste in region 2 and c_2 would be strictly positive. Thus, we say that the unique value for the transfer rate compatible with this problem is 0.

In general, we say that a value $\hat{t}$ for the transfer rate is compatible with a cost allocation problem if the amounts of waste present in each segment described by the cost vector can occur given the value $\hat{t}$ for the transfer rate. Otherwise we say that $\hat{t}$ is incompatible with the problem. The following proposition determines what values for the transfer rate are compatible with each possible cost allocation problem.

Proposition 3 *A value $\hat{t}$ for the transfer rate is compatible with a cost allocation problem $(N, C, \underline{t}, \overline{t})$ if and only if $\hat{t} \in [\underline{t}, \overline{t}^*(\overline{t}, C)]$, where*

$$\overline{t}^*(\overline{t}, C) = \min\left\{ \min_{i \in \{2, \ldots n-1\}} \left\{ \frac{c_i}{c_{i-1}} \right\}, \frac{c_n}{c_{n-1} + c_n}, \overline{t} \right\}.^{12}$$

This result allows us to reduce the uncertainty over the transfer rate. In particular, the cost vector C provides, jointly with $\overline{t}$, a maximum limit for this rate that we denote $\overline{t}^*(\overline{t}, C)$. To see the capacity of this result, consider the following example.

Example 1 *Suppose a problem in which $N = \{1, 2, 3, 4\}$, the cost vector is $C = \{10, 16, 8, 24\}$ and the social planner has no information* a priori *about the*

[12] The possible quotients with the indeterminate form $\frac{0}{0}$ have not to be considered in the determination of $\overline{t}^*(\overline{t}, C)$. Obviously, $\overline{t}^*(\overline{t}, C)$ has to be not smaller than $\underline{t}$ because in other case the problem $(N, C, \underline{t}, \overline{t})$ would not be well-defined.

transfer rate, i.e. $\underline{t} = 0$ *and* $\bar{t} = 1$. *Then, focusing on the costs of cleaning the segments, the information about the transfer of the waste can be improved using Proposition 3. In this case, we obtain that* $t^*(\bar{t}, C) = \min\{\frac{8}{5}, \frac{1}{2}, \frac{3}{4}, 1\}$. *Therefore, Proposition 3 indicates that the transfer rate is* at most *one half and, then, the information about the transfer rate after observing the cost vector can be adapted.*

Given a problem $(N, C, \underline{t}, \bar{t})$, we will denote by $l_i^j(N, C, \underline{t}, \bar{t})$ the amount of waste present in segment j that has been discharged by region i. When there is no risk of confusion about the description of the problem, we simply write $l_i^j(\cdot)$. When the actual transfer rate t is unknown, $l_i^j(\cdot)$ cannot be precisely calculated. However, some limits of this value can be deduced from the information about the transfer rate held by the social planner and from what the planner can infer from the cost vector via Proposition 3. We will denote the lower and higher limits of $l_i^j(\cdot)$ by $\underline{l}_i^j(\cdot)$ and $\bar{l}_i^j(\cdot)$, respectively. The following proposition will provide formulas for $\underline{l}_i^i(\cdot)$ and $\bar{l}_i^i(\cdot)$ for all $i \in N$.[13]

Proposition 4 *Let* $(N, C, \underline{t}, \bar{t})$ *be a problem. Then,*

$$\underline{l}_i^i(\cdot) = \begin{cases} c_i & \text{if } i = 1 \\ c_i - c_{i-1} \cdot \bar{t}^*(\bar{t}, C) & \text{if } i \in \{2, \ldots, n-1\} \\ c_i - \frac{c_{i-1} \cdot \bar{t}^*(\bar{t}, C)}{1 - \bar{t}^*(\bar{t}, C)} & \text{if } i = n \text{ and } \bar{t}^*(\bar{t}, C) < 1 \\ 0 & \text{if } i = n \text{ and } \bar{t}^*(\bar{t}, C) = 1. \end{cases}$$

$$\bar{l}_i^i(\cdot) = \begin{cases} c_i & \text{if } i = 1 \\ c_i - c_{i-1} \cdot \underline{t} & \text{if } i \in \{2, \ldots, n-1\} \\ c_i - \frac{c_{i-1} \cdot \underline{t}}{1 - \underline{t}} & \text{if } i = n \end{cases}$$

It is natural to require that any rule that seeks to allocate costs in terms of each region's responsibility for producing the waste present in each segment should always respect the limits calculated in Proposition 4 when the costs are allocated. In the rest of this section, we discuss whether the LRS and UES rules fulfil this requirement.

On the one hand, the LRS rule meets the aforesaid requirement of responsibility only when $\underline{t} = 0$. However, it can only be accepted as a rule that allocates costs

[13] It is also possible, but extremely tedious, to construct formulas for the limits of any $l_i^j(\cdot)$ in a similar way, but these ones are sufficient for our purposes.

taking responsibilities into account if the real transfer rate, t, is 0 in all rivers. Nevertheless, this literature only makes sense when waste is transferred from one region to another, an idea that is realistic. On the other hand, the following example shows that, independently of the information about t ($\underline{t}$ and $\overline{t}$), the UES rule does not satisfy the requirement of allocating costs within the intervals of responsibility defined in Proposition 4.

Example 2 *Consider the family of cost allocation problems* $(N, C, \underline{t}, \overline{t})$ *such that* $N = \{1, 2, 3, 4\}$ *and* $C = \{10, 16, 8, 24\}$. *In all these problems, the UES rule assigns to region* 2 *only half of the cost of cleaning its own segment; i.e.* $\beta_2^2(\cdot) = 8$. *However, given that* $\overline{t}^*(\overline{t}, C) \leq \frac{1}{2}$, *it is easy to calculate from Proposition 4 that* $\underline{l}_2^2(\cdot) \geq 11$ *for all possible values of* $\underline{t}$ *and* $\overline{t}$. *Hence, region* 2 *should pay at least* 11 *to clean its own segment if responsibilities are considered.*

3 The Upstream Responsibility Rule

Axioms, definition and characterization

The axioms that we present for a rule are based on basic ideas about responsibility for the waste present in the river channel. The first axiom, Limits of Responsibility, seeks to avoid the problem found in the LRS and UES rules studied in the previous section. To that end, the property requires that the cost paid by each region for cleaning its own segment should always be within the limits calculated in Proposition 4.

Limits of Responsibility (LR): For all problems $(N, C, \underline{t}, \overline{t})$, and for all $i \in N$, $x_i^i(\cdot) \in [\underline{l}_i^i(\cdot), \overline{l}_i^i(\cdot)]$.

The second axiom, No Downstream Responsibility, states that a region j located downstream from another region i has no responsibility for the waste present in i, and should therefore not pay any part of the cost of cleaning it up.

No Downstream Responsibility (NDR): For all problems $(N, C, \underline{t}, \overline{t})$ and all $i, j \in N$ such that $i < j$, $x_j^i(\cdot) = 0$.

To introduce the next property, Consistent Responsibility, assume three regions i, j and k such that i is located upstream from j and j upstream from k. A rule decides how the cost of cleaning the river in region j should be divided among all the regions depending on their responsibility for the waste present in this region. In particular, it establishes the responsibility of region i relative to the responsibility of region j for producing that waste $\left(\frac{x_i^j(\cdot)}{x_j^j(\cdot)}\right)$. Observe that part of the waste that has at some time entered j, which comprises the waste discharged by j and some of the waste discharged by the regions upstream from it including i, remains in j and part flowed on to $j+1$. Given that regions i and j do not produce any waste other than that which arrived at some time at j, then the amount of waste that entered $j+1$ from j must contain the same ratio of waste discharged by region i to waste discharged by region j as exists in the waste present in j. By a similar argument, that ratio must also be maintained in the amount of waste that remains in region $j+1$ and did not flow on to $j+2$. Reasoning in the same way for the subsequent segments, it can be stated that this ratio must be maintained for any downstream segment k. Thus, the axiom states that the rule should establish the same degree of responsibility of region i relative to the responsibility of region j for the waste present in region k $\left(\frac{x_i^k(\cdot)}{x_j^k(\cdot)}\right)$ as the relative responsibilities established for these regions in the waste present in j. For example, if region i is responsible for twice as much waste as region j in j $\left(\frac{x_i^j(\cdot)}{x_j^j(\cdot)} = 2\right)$, the axiom establishes that region i is also responsible for twice as much as region j in k $\left(\frac{x_i^k(\cdot)}{x_j^k(\cdot)} = 2\right)$. In general, the axiom establishes that $\left(\frac{x_i^j(\cdot)}{x_j^j(\cdot)}\right)$ should be equal to $\left(\frac{x_i^k(\cdot)}{x_j^k(\cdot)}\right)$.[14]

Consistent Responsibility (CR): For all problems $(N, C, \underline{t}, \overline{t})$ and all $i, j, k \in N$ such that $i < j < k$,

$$x_j^j(\cdot) \cdot x_i^k(\cdot) = x_j^k(\cdot) \cdot x_i^j(\cdot).$$

The last property, Monotonicity with respect to Information on the Transfer Rate, refers to situations in which, *ceteris paribus*, the information on the transfer rate improves. Given a problem $(N, C, \underline{t}, \overline{t})$, it is known from Proposition 3 that the transfer rate t is within the interval $[\underline{t}, \overline{t}^*(\overline{t}, C)]$. Assume that information on the transfer rate becomes more precise in such a way that some previous possible values of t can now be ruled out; that is, consider a new problem

[14]The axiom is not expressed in terms of these quotients but in terms of products so as to avoid indeterminate forms.

$(N, C, \underline{u}, \overline{u})$ such that $[\underline{u}, \overline{u}^*(\overline{u}, C)] \subset [\underline{t}, \overline{t}^*(\overline{t}, C)]$. If this informational improvement is such that the values discarded are mainly from the lower (higher) part of the interval $[\underline{t}, \overline{t}^*(\overline{t}, C)]$, it would be natural to induce a not lower (not higher) *estimated* value for the real transfer rate.[15] Given that the cost vector is the same, the quantity of waste in any segment for which responsibility lies with all the upstream regions must be no lower (no higher) under the new estimation. Therefore, the axiom requires that for any segment the total amount paid by all its upstream regions for cleaning the segment in question should now be no lower (no higher).

Monotonicity with respect to Information on the transfer rate (MIT): For all problems $(N, C, \underline{t}, \overline{t})$ and $(N, C, \underline{u}, \overline{u})$ such that $[\underline{u}, \overline{u}^*(\overline{u}, C)] \subset [\underline{t}, \overline{t}^*(\overline{t}, C)]$ and for all $j \in N$,

$$\underline{u} - \underline{t} > \overline{t}^*(\overline{t}, C) - \overline{u}^*(\overline{u}, C) \Rightarrow \sum_{i<j} x_i^j(N, C, \underline{u}, \overline{u}) \geq \sum_{i<j} x_i^j(N, C, \underline{t}, \overline{t})$$

$$\underline{u} - \underline{t} < \overline{t}^*(\overline{t}, C) - \overline{u}^*(\overline{u}, C) \Rightarrow \sum_{i<j} x_i^j(N, C, \underline{u}, \overline{u}) \leq \sum_{i<j} x_i^j(N, C, \underline{t}, \overline{t}).$$

Observe that the above list of axioms includes, on the one hand, a basic principle of fairness in this context (LR) and, on the other hand, a set of three very weak properties (NDR, CR and MIT) that are satisfied by many possible rules which are very different one from another (for example, both α and β satisfy them). However, as we are going to show, the addition of LR to these three axioms isolates one new rule, the Upstream Responsibility rule. We begin by presenting it in an intuitive way. To assign the total cost of cleaning each segment i, this rule first imputes to the region situated in that segment the value of its responsibility obtained from Proposition 4 taking as the transfer rate the mid-point in the interval between $\underline{t}$ and $\overline{t}^*(\overline{t}, C)$. The remaining cost, if any, is allocated to the upstream regions in line with the proportions applied in the allocation of the cost of the previous segment. The formal definition of the rule is as follows.

[15]This deduction makes sense if the uncertainty of the social planner on the transfer rate takes the form of a symmetric random variable (for example, a uniform distribution). More general cases are analyzed in Section 4.

Definition 3 *The* Upstream Responsibility *rule, γ, is given by:*

$$\gamma_i^j(N, C, \underline{t}, \overline{t}) = \begin{cases} 0 & \text{if } i > j, \\ c_i \cdot s^{j-i} - c_{i-1} \cdot s^{j+1-i} & \text{if } i \leq j < n, \\ c_i - \frac{c_{i-1} \cdot s}{1-s} & \text{if } i = j = n, \\ \frac{c_i \cdot s^{j-i} - c_{i-1} \cdot s^{j+1-i}}{1-s} & \text{if } i < j = n, \end{cases}$$

where $s = \frac{\underline{t} + \overline{t}^*(\overline{t}, C)}{2}$, c_0 is set to 0 and the indeterminate form 0^0 is set to 1.

The Upstream Responsibility rule is based on the responsibility of the agents involved in discharging waste into a river. In particular, if the social planner knows the actual transfer rate ($\underline{t} = \overline{t}^*(\overline{t}, C) = t$), it can be shown that this rule establishes that the total cost that each region has to pay to clean the entire river exactly matches the total quantity of waste that it has discharged, which can be deduced from the cost vector. To express it formally, we have that the quantity of waste discharged by a region i, that we denote by V_i, can be deduced observing t and C. This amount of waste $V_i(t, C)$ is exactly the total cost that region i pays under the Upstream Responsibility rule. If, however, there is uncertainty over the transfer rate, the rule assigns to each region the total cost corresponding to the amount of waste that it is considered to have discharged, using $s = \frac{\underline{t} + \overline{t}^*(\overline{t}, C)}{2}$ as the *estimated* value of the transfer rate. That is, it assigns $V_i(s, C)$ to each region i. In other words, the cost allocation method corresponding to the Upstream Responsibility rule (which is uniquely determined and will be called the Upstream Responsibility method) assigns a distribution equal to the responsibility of each region, using s as the estimated transfer rate.

Proposition 5 *Let $(N, C, \underline{t}, \overline{t})$ be a cost allocation problem. Then, the Upstream Responsibility method Γ is*

$$\Gamma_i(N,C,\underline{t},\overline{t}) = V_i(s,C) = \begin{cases} \frac{c_i}{1-s} & \text{if } i=1, \\ \frac{c_i}{1-s} - \frac{c_{i-1}\cdot s}{1-s} & \text{if } i \in \{2,\ldots,n-1\}, \\ c_i - \frac{c_{i-1}\cdot s}{1-s} & \text{if } i=n, \end{cases}$$

where $s = \frac{\underline{t}+\overline{t}^*(\overline{t},C)}{2}$.

The following result states that the Upstream Responsibility rule is characterized by the combination of the four axioms introduced above.

Theorem 1 *A rule satisfies LR, NDR, CR and MIT if and only if it is the Upstream Responsibility rule* γ.

We also show that this characterization is tight.

Proposition 6 *Axioms LR, NDR, CR and MIT are independent.*

A comparison with the LRS and UES solutions

In Section 2 we have constructed cost allocation rules that maintain the spirit of the LRS and UES methods. In order to avoid the problems detected in those rules, we have defined and characterized a new rule: the Upstream Responsibility rule. In this subsection, we discuss the differences between the three solutions. First, to illustrate how the three rules behave, we apply them to a particular cost allocation problem.

Example 3 *Consider again the cost allocation problem defined in Example 1, where* $N = \{1,2,3,4\}$, $C = \{10,16,8,24\}$, $\underline{t} = 0$, $\overline{t} = 1$ *and* $\overline{t}^*(\overline{t},C) = \frac{1}{2}$. *On the one hand, the solutions proposed to this problem by* α *and* β *are, respectively:*

$$\alpha(\cdot) = \begin{pmatrix} 10 & 0 & 0 & 0 \\ 0 & 16 & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 24 \end{pmatrix}$$

$$\beta(\cdot) = \begin{pmatrix} 10 & 8 & \frac{8}{3} & 6 \\ 0 & 8 & \frac{8}{3} & 6 \\ 0 & 0 & \frac{8}{3} & 6 \\ 0 & 0 & 0 & 6 \end{pmatrix}.$$

On the other hand, the Upstream Responsibility rule assigns to this problem the following solution:

$$\gamma(\cdot) = \begin{pmatrix} 10 & \frac{5}{2} & \frac{5}{8} & \frac{5}{24} \\ 0 & \frac{27}{2} & \frac{27}{8} & \frac{9}{8} \\ 0 & 0 & 4 & \frac{4}{3} \\ 0 & 0 & 0 & \frac{64}{3} \end{pmatrix}.$$

To discuss the differences between all these rules, we use their axiomatic decompositions. Consider first the new rule, γ. It is characterized by the combination of four properties: one based on the responsibilities inferred from the cost vector, LR, and three more basic properties (NDR, CR and MIT). As mentioned above, α and β satisfy all these properties except LR. Hence, the property LR is the source of the discrepancies between our proposal γ and the other two proposals, α and β. This is because LR compels us to consider the information that can be inferred from the cost vector about the responsibility of each region in producing the waste.

Another way to see this divergence between the rules is to focus on the characterizations of α and β. Remember that α is characterized by a combination of NBC and IUC, while β is characterized by IUC and CS. However, our rule only satisfies NBC. Again, this happens because the other two properties ignore the information that can be inferred from the cost vector about responsibilities. In particular, γ does not satisfy IUC because the *estimated* responsibility of region i for waste in the river is not totally independent of the waste present in the regions upstream from it: the waste in region $i-1$ gives important information about the amount of waste that passed on to region i from its upstream regions and, therefore, γ takes it into account in estimating responsibilities. Similarly, γ does not satisfy CS because the responsibilities for the waste present in a segment are not symmetric across all its upstream regions: the cost vector gives information about these heterogeneous responsibilities that γ takes into account.

Besides comparing our cost allocation rule with the LRS and UES ones, we could compare their associated cost allocation methods directly. This analysis is important since, theoretically, it could happen that a cost allocation rule does not satisfy a particular property but its associated method does satisfy the adapted axiom for cost allocation methods. In the case of the Upstream Responsibility method, we can analyze whether or not it satisfies the original axioms that characterize the LRS and UES methods. It can be seen that it satisfies the original NBC axiom, but it does not satisfy the original IUC and CS properties for the same reasons expressed in the previous paragraph, so the direct analysis of our method does not differ in this aspect from the analysis of the rule. Unfortunately, since there is no direct way of adapting our new axioms for cost allocation methods, we cannot evaluate any method, and in particular the LRS and UES ones, on the basis of these properties.

4 Extensions of the basic model and related problems

We have shown that given a transboundary river with waste transfer, the costs of cleaning each region provide information about the responsibility of each one for producing the waste which can be used to construct a new cost allocation rule. To infer this information, we have followed a simplified model that has enabled us to obtain results in a simple manner. Although such a model may seem too simple to be applied to real cost allocation problems, it is not difficult to extend it to more general situations. Some of those extensions are discussed below, with an explanation of how the axioms and results can be adapted. We also include a subsection discussing an interesting dual problem to the one studied in this paper and how our results can be adapted to it.[16]

Extensions on the definition of the segments

One of the assumptions of the basic model is that a river is divided into segments of the same size. A river with segments of different sizes could be posited to reflect the fact that regions can occupy different extensions along a river. We can analyze this more general case from our framework by associating a cost

[16]We thank two anonymous referees for drawing our attention to some of these topics.

allocation problem in which segments have different sizes with a new problem in which all segments have the same size. The number of segments in this new problem should be the total length of the river divided by the maximum common divisor of the lengths of all regions in the original problem. Formally, let $(N, C, l, \underline{t}, \bar{t})$ be an extended cost allocation problem in which N, C, $\underline{t}$ and $\bar{t}$ have the same meaning as in the basic model and $l \in \mathbb{R}^n_{++}$ describes the (possibly different) lengths of each region. Then, the associated problem has a set of segments N^* with cardinality $|N^*| = \frac{\sum_{i \in N} l_i}{mcd(l)}$, where $mcd(l)$ refers to the maximum common divisor of the values of l. In this way, each region $i \in N$ is associated with a set of segments in the new problem $\{i_1, \ldots, i_k\} \subset N^*$, with $k = \frac{l_i}{mcd(l)}$. The cost vector C^* of the associated problem has to satisfy that $c_i = \sum_{j=1}^{k} c^*_{i_j}$. Thus, using this strategy, we construct an associated problem $(N^*, C^*, \underline{t}, \bar{t})$ to $(N, C, l, \underline{t}, \bar{t})$.[17] Now, given that all segments in the problem $(N^*, C^*, \underline{t}, \bar{t})$ are of the same size, this associated problem is included in the domain of our basic model and, therefore, our results would imply the use of the γ rule to assign the costs. We can thus use this solution to define an assignment for the original problem by allocating to each region $i \in N$ the sum of the costs allocated by γ to each of the segments in the associated problem: $\sum_{j=1}^{k} \gamma_{i_j}(N^*, C^*, \underline{t}, \bar{t})$.

An interesting aspect of this extension that deserves more discussion is the distribution of the cost over the subsegments of a particular region. Although there is only one way of constructing N^* from N, there are many different ways of constructing C^* from C. In contexts in which the social planner has information about the waste present in each subsegment of a region, it suffices to apply γ and there is no room for further research. However, if the social planner only knows the total cost of cleaning the entire region i but is unaware of how it breaks down by subsegments, the precise distribution of c_i into $c_{i_1}, \ldots, c_{i_k}$ could be relevant in the final allocation selected by the rule. We illustrate this with the following simple example.

Example 4 *Consider the extended cost allocation problem* $(N, C, l, \underline{t}, \bar{t})$ *such that* $N = \{1, 2, 3\}$, $C = \{10, 16, 24\}$, $l_2 = 2l_1 = 2l_3$, $\underline{t} = 0$ *and* $\bar{t} = \frac{2}{5}$. *Thus,*

[17] The meaning of upstream and downstream in N^* is similar than in N: i_h is upstream from j_m if $i < j$ or if $i = j$ and $h < m$.

a new problem must be considered in which region 2 *is decomposed into two subsegments,* 2_1 *and* 2_2*, in such a way that all subsegments are the same size. Then,* c_2 *must also be broken down into* c_{2_1} *and* c_{2_2} *and we have to work with the problems* $(\{1_1, 2_1, 2_2, 3_1\}, (10, c_{2_1}, c_{2_2}, 24), 0, \frac{2}{5})$ *such that* $c_{2_1} + c_{2_2} = 16$*. If the cost of cleaning region* 2 *is decomposed such that* $c_{2_1} = c_{2_2} = 8$*, then the total cost that region* 2 *pays to clean the entire river by applying* γ *is* 15.5*, while if the decomposition is* $c_{2_1} = 4$ *and* $c_{2_2} = 12$*, it is* 16.5.[18]

Example 4 points out the importance of having the most disaggregated data possible on the waste present in each segment of the river. If the social planner has no access to such data then, as this example shows, the regions could have incentives to misrepresent them.

Another natural extension of our model is to consider a river which is not a segment but a network divided into segments. This could be useful in modeling a river with tributaries and/or forks. In that case, all the results of the paper can be easily adapted by incorporating the number of outlets on each fork into the calculation of the limits of the transfer rate and extending the rule as Dong et al. (2012) extend the methods of Ni and Wang (2007).[19]

Extensions on the values of $\underline{t}$ and $\bar{t}$

We have assumed $\underline{t} \in [0, 1)$ and $\bar{t} \in (0, 1]$ in our basic model, thus excluding the cases in which the social planner knows for certain that the actual transfer rate is 0 ($\bar{t} = 0$) and those in which she knows that the actual transfer rate is 1 ($\underline{t} = 1$). Although these are extreme cases without much practical relevance, the results of the basic model can also be applied to them after some details are considered.

Cases in which the social planner knows that there is no transfer of waste between two adjacent regions ($\bar{t} = 0$) can be included in our basic model without changes, keeping the axioms and the characterization result invariant. Observe that in these extreme cases the Upstream Responsibility rule assigns the cost

[18]Notice that Proposition 3 implies a maximum limit for t of $\frac{2}{5}$ for both decompositions. However, other decompositions might affect this maximum limit, biasing also the allocation selected.

[19]More details on this extension can be provided upon request.

of cleaning each segment entirely to the region located there, thus coinciding in such cases with the LRS rule.

With respect to the cases in which the social planner knows that all the waste that enters a region is transferred to the next downstream region ($\underline{t} = 1$), observe that they are compatible only with cost vectors C such that $c_i = 0$ for all $i < n$.[20] In such cases no information can be deduced from the cost vector to infer responsibilities for the waste present in the last region, c_n. Thus, sharing this cost equally among all regions could be a reasonable possibility.

Consider the extended domain that includes these extreme cases; that is, the domain of all cost allocation problems $(N, C, \underline{t}, \overline{t})$, where $N = \{1, \ldots, n\} \subset \mathbb{N}$, $C \in \mathbb{R}^n_+$, $\underline{t}, \overline{t} \in [0, 1]$ with $\underline{t} \leq \overline{t}$. Our characterization result can be extended to this extended domain by adding a new property to the axioms of Theorem 1. This property is Weak Cost Symmetry, which applies the spirit of CS only to extreme cases in which there is total uncertainty about which region is responsible for each unit of waste.

Weak Cost Symmetry (WCS): For all problems $(N, C, 1, 1)$ and all $j, k \in N$ such that $c_k = 0$ for all $k < n$,

$$x^l_j(\cdot) = x^l_k(\cdot) \text{ for all } l \in N.$$

The introduction of WCS implies that the rule must coincide with the UES rule for the extreme cases in which $\underline{t} = 1$. The Upstream Responsibility rule can be therefore extended for this domain in the following way:

Definition 4 *The* Extended Upstream Responsibility *rule,* Γ*, is given by*

$$\Gamma^j_i(\cdot) = \begin{cases} \gamma^j_i(\cdot) & \textit{if } \underline{t} \in [0, 1) \textit{ and } \overline{t} \in (0, 1] \\ \gamma^j_i(\cdot) = \alpha^j_i(\cdot) & \textit{if } \overline{t} = 0 \\ \beta^j_i(\cdot) & \textit{if } \underline{t} = 1. \end{cases}$$

The general characterization can then be presented for this extension.[21]

[20] Note that these cost vectors are also compatible with other values of the transfer rate.

[21] The proof is straightforward using Theorem 1 for all the natural cases, applying LR to the extreme cases of $\overline{t} = 0$ (in which $\underline{l}^i_i(\cdot) = \overline{l}^i_i(\cdot) = c_i$) and applying WCS to the extreme cases in which $\underline{t} = 1$. We therefore omit it.

Theorem 2 *A rule satisfies LR, NDR, CR, MIT and WCS if and only if it is the Extended Upstream Responsibility rule* Γ.

Extension on the structure of the transfer rate

We have also assumed in the basic model that the transfer rate is uniform along the river. A possible extension of the model in regard to this assumption is to consider that the transfer rate changes in some areas of the river. This could be useful in modeling rivers that run through regions with different types of terrain, weather or biosystems. In such cases the model could be adapted by dividing the problem into subproblems with homogeneous characteristics. By applying Proposition 3 to each of them, different limits can be deduced for each particular transfer rate. Additionally, new limits can be deduced with the information of the entire river.

Below, we explain how the limits of each transfer rate must be calculated and, as a result, how the Upstream Responsibility rule is adapted in this extended model for a particular case.

Consider the cost allocation problems with five regions such that the terrain is homogeneous in the transitions from regions 1 to 3 and in the transitions from regions 3 to 5, but is different in these two parts. These problems can be formulated using two transfer rates: t, which measures the proportion of waste transferred from one segment to the next between segments 1 and 2 and between segments 2 and 3, and u, which does the same from segment 3 to 4 and from 4 to 5. Thus, the cost allocation problems are defined as $(N_t, N_u, C, \underline{t}, \overline{t}, \underline{u}, \overline{u})$, where $N_t = \{1, 2, 3\}$ and $N_u = \{3, 4, 5\}$ specify between the segments to which each of the transfer rates (t and u, respectively) apply. In such cases, following arguments similar to those in Proposition 3, an upper limit for u is obtained with the same form as in the basic case:

$$\overline{u}^*(\overline{u}, C) = \min\{\frac{c_4}{c_3}, \frac{c_5}{c_4 + c_5}, \overline{u}\}.$$

The upper limit for t is also similar to that in the basic model, but here a new restriction can be established for it based on the information for region 3, where the two transfer rates interact:[22]

[22]The proof of all the arguments of this subsection follows a similar path to the proofs of the basic model and can be found in Appendix B (Supplementary Material).

$$\overline{t}^*(\overline{u},\overline{t},C)=\min\{\frac{c_2}{c_1},\overline{t},h(\overline{u}^*(\overline{u},C))\},$$

where $h(u)=\frac{c_3}{c_3+c_2\cdot(1-u)}$.

Additionally, this new interaction enables a lower limit for u to be introduced:

$$\underline{u}^*(\underline{u},\underline{t},C)=\max\{h^{-1}(\underline{t}),\underline{u}\}.$$

Thus, in this extended model the results of Proposition 4 could be adapted according to the information about the transfer rates, adapting also axiom LR. On the other hand, axioms NDR and CR need no adaptation in this context, while MIT has to be applied to the information on both transfer rates. As a result, the characterized rule for these problems uses the values $s=\frac{\underline{t}+\overline{t}^*(\overline{u},\overline{t},C)}{2}$ and $v=\frac{\underline{u}^*(\underline{u},\underline{t},C)+\overline{u}^*(\overline{u},C)}{2}$ to estimate the transfer rates t and u, respectively, and considers them in allocating costs in a way similar to that in the basic model: s is used for transfers between segments upstream from 3 and v for transfers downstream from 3. The formal definition of the rule for the five player case is as follows:

$$\gamma_i^j(N_t, N_u, C, \underline{t}, \bar{t}, \underline{u}, \overline{u}) = \begin{cases} 0 & \text{if } i > j, \\ c_i \cdot s^{j-i} - c_{i-1} \cdot s^{j+1-i} & \text{if } i,j \in \{1,2\} \text{ and } i \leq j, \\ \frac{c_i \cdot s^{j-i} \cdot (1-v) - c_{i-1} \cdot s^{j+1-i} \cdot (1-v)}{1-s} & \text{if } i < j \text{ and } j = 3, \\ c_i \cdot v^{j-i} - c_{i-1} \cdot \frac{s}{1-s} \cdot (1-v) \cdot v^{j-i} & \text{if } i = 3 \text{ and } j \in \{3,4\}, \\ c_i - c_{i-1} \cdot v & \text{if } i = j = 4, \\ \frac{c_i \cdot s^{j-i-1} \cdot (1-v) \cdot v - c_{i-1} s^{j-i} \cdot (1-v) \cdot v}{1-s} & \text{if } i \in \{1,2\} \text{ and } j = 4. \\ c_i - \frac{c_{i-1} \cdot v}{1-v} & \text{if } i = j = 5, \\ \frac{c_i \cdot v - c_{i-1} \cdot v^2}{1-v} & \text{if } i = 4 \text{ and } j = 5, \\ c_i \cdot \frac{v^{j-i}}{1-v} - c_{i-1} \cdot \frac{s}{1-s} \cdot v^{j-i} & \text{if } i = 3 \text{ and } j = 5, \\ \frac{c_i \cdot (s^{j-i-2} \cdot v^2) - c_{i-1} \cdot (s^{j-i-1} \cdot v^2)}{1-s} & \text{if } i \in \{1,2\} \text{ and } j = 5, \end{cases}$$

where c_0 is set to 0 and the indeterminate form 0^0 is set to 1.

Extension on the information about the transfer rate

Another implicit assumption of the basic model is that the uncertainty of the social planner on the transfer rate takes the form of a symmetric random variable (for example, a uniform distribution) on the interval $[\underline{t}, \bar{t}^*(\bar{t}, C)]$, so the mean value between $\underline{t}$ and $\bar{t}^*(\bar{t}, C)$ is always a good *estimator* of t. However, other distributions that are not symmetric may be assumed *a priori* and, axiom MIT would have to be reformulated to adapt our results to those cases. To be more precise, a modified version of MIT would have to be considered in which the changes in the intervals are evaluated on the basis not of their lengths but their masses of probability of the specific random variable assumed. As a result, the characterized rule would change to one in which the transfer rate considered in allocating costs is the expected value of the random variable. This process is explained below for the case of any arbitrary continuous variable.

Consider the cost allocation problems defined by $5-$tuples $(N, C, \underline{t}, \overline{t}, f(t))$ such that the information of the social planner about t takes the form of a random variable with density function $f(t)$ with support in $[\underline{t}, \overline{t}^*(\overline{t}, C)]$. In this extended model, axiom MIT is thus formulated as follows:

Monotonicity with respect to Information on the transfer rate (MIT): For all problems $(N, C, \underline{t}, \overline{t}, f(t))$ and $(N, C, \underline{u}, \overline{u}, g(u))$ such that $[\underline{u}, \overline{u}^*(\overline{u}, C)] \subset [\underline{t}, \overline{t}^*(\overline{t}, C)]$ and $f(t)$ truncated at $[\underline{u}, \overline{u}^*(\overline{u}, C)]$ is equal to $g(u)$ [23] and for all $j \in N$,

$$F(\underline{u}) > 1 - F(\overline{u}^*(\overline{u}, C)) \Rightarrow \sum_{i<j} x_i^j(N, C, \underline{u}, \overline{u}) \geq \sum_{i<j} x_i^j(N, C, \underline{t}, \overline{t})$$

$$F(\underline{u}) < 1 - F(\overline{u}^*(\overline{u}, C)) \Rightarrow \sum_{i<j} x_i^j(N, C, \underline{u}, \overline{u}) \leq \sum_{i<j} x_i^j(N, C, \underline{t}, \overline{t}),$$

where F is the cumulative distribution function corresponding to f.

Using this new general formulation of axiom MIT and maintaining the other axioms as in the basic model, the characterized rule is found to have exactly the same structure as in the basic case, except that the value of the parameter s in the formula of γ is equal to the expected value of t: $s = \int_{\underline{t}}^{\overline{t}^*(\overline{t}, C)} t \cdot f(t)dt$.

A dual problem

We have studied a model in which there is already waste in a river and the clean-up costs have to be shared. However, there are situations in which certain identified polluters produce most of the residues and, in such cases, the social planner may be interested in implementing preventive policies to avoid subsequent problems. These policies may consist of giving such polluters incentives to reduce their levels of pollution. For instance, some public agencies have introduced contracts in which farmers commit to reducing their levels of pollution in return for a payment from the agencies (see also Barrett, 1994).[24]

[23] That is, $g(u) = \frac{f(u) \cdot I_{[\underline{u}, \overline{u}^*(\overline{u}, C)]}(u)}{F(\overline{u}^*(\overline{u}, C)) - F(\underline{u})}$, where F is the cumulative distribution function corresponding to f and $I_{[\underline{u}, \overline{u}^*(\overline{u}, C)]}(u) = 1$ if $u \in [\underline{u}, \overline{u}^*(\overline{u}, C)]$ and 0 otherwise.

[24] More recently, the OECD has also argued that it is necessary to assess the efficiency and effectiveness of water pollution abatement measures in the context of river basin management (OECD, 2008).

The model studied in this paper so far assumes that polluting regions must be required to pay a cost for their activities, but this other framework implicitly assumes that they have the right to pollute the water and must be compensated for not doing so. As a result, the question of where the necessary funds should come from is dual to the question of who has to pay the clean-up costs in the other cases. Thus, in these cases, the social planner should allocate the costs of the incentive program to the regions situated downstream from the polluter, which will be the beneficiaries of the pollution abatement.

The dual problem can be defined starting with a polluted river, defined as in our basic model $(N, C, \underline{t}, \overline{t})$, where C represents the amount of waste present in each segment. Consider also that there is an identified polluter in region $k \in N$ that will receive a payment of z from the social planner if it reduces its pollution emissions by z units. The dual problem is thus defined as $(N, C, \underline{t}, \overline{t}, k, z)$ and the adapted definition of a rule for these problems is a function x that assigns to each problem a matrix $(x_i^j(\cdot))_{i,j \in N}$ of non–negative numbers such that $\sum_{i \in N} x_i^k(\cdot) = z$ and $\sum_{i \in N} x_i^j(\cdot) = 0$ for all $j \neq k$.

To construct a rule, dual axioms to those proposed in our basic model can be used, with all references to upstream regions being replaced by downstream ones (and vice versa) in such a way that regions pay in line with how much less waste they will have in their respective segments with the incentive program. For example, axiom NDR would be substituted by a dual property, which can be called No Upstream Beneficiary, stating that no region situated upstream from k should pay any part of the program because they will not benefit from it. The other axioms can be adapted similarly and, as a result, the characterized rule π, which can be called *Downstream Beneficiary* rule, is dual to the Upstream Responsibility rule γ:

$$\pi_i^j(N, C, \underline{t}, \overline{t}, k, z) = \begin{cases} 0 & \text{if } j \neq k, \\ \\ \frac{\gamma_j^i(N, C, \underline{t}, \overline{t})}{\gamma_j(N, C, \underline{t}, \overline{t})} \cdot z & \text{if } j = k. \end{cases}$$

The interpretation of this rule for assigning the costs of the incentive program is also dual to the interpretation of the Upstream Responsibility rule. It assigns a total cost to each region equal to the amount of pollution that would be eliminated in that region if the polluter is paid to discharge less waste. In cases in which this amount cannot be precisely calculated because there is uncertainty over t, the rule uses the estimated transfer rate $s = \frac{\underline{t}+\bar{t}^*(\bar{t},C)}{2}$.

The reasoning behind the formal definition of the rule is simple. The proportion of waste discharged by region k that ends up in region i can be estimated using the Upstream Responsibility rule by quotient $\frac{\gamma_k^i(N,C,\underline{t},\bar{t})}{\gamma_k(N,C,\underline{t},\bar{t})}$. Thus, if the quantity discharged by k is reduced by z units thanks to the incentive, it is reasonable for region i to pay exactly that proportion of the program.

5 Concluding remarks

The presence of waste in rivers produces environmental problems and the costs of solving them may be quite high. In cases in which different municipalities, regions or even countries share a river, it is obvious that each region is not totally responsible for all the waste in its own segment, and it therefore seems desirable from a social point of view for these costs to be shared between all the regions responsible. This paper studies the possibility of constructing a rule for allocating those costs in line with the responsibilities of the regions for producing the waste. We have shown that the solutions previously proposed in the literature do not satisfy this objective because they fail to consider the information about the transfer rate that can be deduced from the cleaning cost vector. Additionally, we have provided normative foundations for the use of the new Upstream Responsibility rule to allocate such costs with this objective.

An interesting problem for further research would be to study how the implementation of this cost allocation rule can affect the incentives for agents to decide how much waste to discharge. This could be important because the *optimal* rule should incorporate not only a *fair* allocation of the costs for cleaning up the waste in the river but also incentives for establishing an equilibrium with less waste.

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Appendix

Proof of Proposition 1

First, it is easy to see that the LRS rule α satisfies NBC and IUC. To prove the other implication, consider a problem $(N, C, \underline{t}, \bar{t})$ and let x be a cost allocation rule that satisfies NBC and IUC. We will show that x equals α.

Let $\{(N, C^k, \underline{t}, \bar{t})\}_{k \in N}$ be a sequence of cost allocation problems such that $c_i^k = 0$ for all $i < k$ and $c_i^k = c_i$ for all $i \geq k$. Consider first the problem $(N, C^n, \underline{t}, \bar{t})$. By NBC we have that $x_i^j(N, C^n, \underline{t}, \bar{t}) = 0$ for all $i, j \in N$ such that $i < n$. Given that $c_j^n = 0$ for all $j < n$, we also have that $x_n^j(N, C^n, \underline{t}, \bar{t}) = 0$ for all $j < n$. Therefore, the unique possibility by the definition of a rule is that $x_n^n(N, C^n, \underline{t}, \bar{t}) = c_n$.

Now, the proof follows by induction. Assume that we have determined the solution for each problem $(N, C^k, \underline{t}, \bar{t})$ with $k \geq j + 1$ of the sequence and the resulting allocations for these problems are $x_i^i(N, C^k, \underline{t}, \bar{t}) = c_i$ for all $i \geq k$ and $x_i^l(N, C^k, \underline{t}, \bar{t}) = 0$ for the remaining elements of the matrix. We have to determine the solution for the $j-$th problem of the sequence, $(N, C^j, \underline{t}, \bar{t})$. First, by NBC we have that $x_i^l(N, C^j, \underline{t}, \bar{t}) = 0$ for all $i, l \in N$ such that $i < j$. Secondly, by IUC and the induction hypothesis we have that $x_i^i(N, C^j, \underline{t}, \bar{t}) = c_i$ and $x_i^l(N, C^j, \underline{t}, \bar{t}) = 0$ for all $i, l \in N$ such that $i > j$. Then, the unique possibility to satisfy the definition of a rule is that $x_j^j(N, C^j, \underline{t}, \bar{t}) = c_j$ and $x_j^l(N, C^j, \underline{t}, \bar{t}) = 0$ for all $l \neq j$. Therefore, we have determined the complete solution for the j-th problem of the sequence.

It is easy to see that the last problem of the sequence $(N, C^1, \underline{t}, \bar{t})$ corresponds with $(N, C, \underline{t}, \bar{t})$ and that the solution deduced by this induction argument corresponds to the solution of the α rule.

Proof of Proposition 2

First, it is easy to see that the UES rule β satisfies CS and IUC. To prove the other implication, consider a problem $(N, C, \underline{t}, \bar{t})$ and let x be a cost allocation rule that satisfies CS and IUC. We will show that x equals β.

Let $\{(N, C^k, \underline{t}, \bar{t})\}_{k \in N}$ be a sequence of cost allocation problems such that $c_i^k = 0$ for all $i < k$ and $c_i^k = c_i$ for all $i \geq k$. Consider first the problem $(N, C^n, \underline{t}, \bar{t})$. By CS we have that $x_i^l(N, C^n, \underline{t}, \bar{t}) = x_j^l(N, C^n, \underline{t}, \bar{t})$ for all $i, j, l \in N$. Then, we can deduce that $x_i^n(N, C^n, \underline{t}, \bar{t}) = \frac{c_n}{n}$ and $x_i^l(N, C^n, \underline{t}, \bar{t}) = 0$ for all $i \in N$ and $l < n$. As a result, we have determined the complete solution for the problem $(N, C^n, \underline{t}, \bar{t})$.

Now, the proof follows by induction. Assume that we have determined the solution for each problem $(N, C^k, \underline{t}, \bar{t})$ with $k \geq j+1$ of the sequence and the resulting allocations for these problems are $x_i^l(N, C^k, \underline{t}, \bar{t}) = \frac{c_l}{l}$ if $(i \leq l$ and $l \geq k)$ and $x_i^l(N, C^k, \underline{t}, \bar{t}) = 0$ otherwise. We have to determine the solution for the $j-$th problem of the sequence, $(N, C^j, \underline{t}, \bar{t})$. First, by IUC and the induction hypothesis we have that for all $i > j$, $x_i^l(N, C^j, \underline{t}, \bar{t}) = \frac{c_l}{l}$ if $i \leq l$ and $x_i^l(N, C^j, \underline{t}, \bar{t}) = 0$ otherwise. Secondly, by CS we have that $x_i^l(N, C^j, \underline{t}, \bar{t}) = x_k^l(N, C^j, \underline{t}, \bar{t})$ for all $i, k \leq j$ and all $l \in N$. Then, we can deduce that $x_i^l(N, C^j, \underline{t}, \bar{t}) = \frac{c_l}{l}$ for all $i \leq l$ and $l \geq j$ and $x_i^l(N, C^j, \underline{t}, \bar{t}) = 0$ for all $l < j$. Therefore, we have determined the complete solution for the j-th problem of the sequence.

It is easy to see that the last problem of the sequence $(N, C^1, \underline{t}, \bar{t})$ corresponds with $(N, C, \underline{t}, \bar{t})$ and that the solution deduced by this induction argument corresponds to the solution of the β rule.

Proof of Proposition 3

Let $(N, C, \underline{t}, \bar{t})$ be a problem. For any segment $i \in N \setminus \{n\}$, the cost that we observe, c_i, is the difference between all the waste entering the segment, denoted as V_i^*, and the amount transferred to the next segments, given by tV_i^*. Then, $c_i = V_i^* - tV_i^*$ for all $i \in \{1, \ldots, n-1\}$. If the actual transfer rate t is 1, we have that $c_i = 0$ for all $i \in \{1, \ldots, n-1\}$ and $\bar{t} = 1$. Therefore, $\bar{t}^*(\bar{t}, C) = 1$ and the proposition is proved for that case.

Let us assume now that $t < 1$. Given that the waste cannot be transferred far from the most downstream region[25], we have that $c_n = V_n^*$. Then,

$$V_i^* = \begin{cases} \frac{c_i}{1-t} & \text{if } i \in \{1, \ldots, n-1\} \\ c_i & \text{if } i = n. \end{cases} \tag{1}$$

Let V_i be the amount of waste thrown into the water by region i. It is immediate that $V_i \leq V_i^*$ given that upstream regions may transfer waste to region i. In particular, the amount thrown into the water by region i is the difference between the total amount entered segment i and the amount transferred from its immediate upstream segment. Then, for all $i \in \{2, \ldots, n\}$, $V_i = V_i^* - tV_{i-1}^*$.

However, for $i = 1$, since there is no upstream region, $V_1 = V_1^*$. Then,

$$V_i = \begin{cases} V_i^* & \text{if } i = 1 \\ V_i^* - tV_{i-1}^* & \text{if } i \in \{2, \ldots, n\}. \end{cases} \tag{2}$$

Using expressions (1) and (2), we can obtain an expression of V_i in terms of C and t:

$$V_i(t, C) = \begin{cases} \frac{c_i}{1-t} & \text{if } i = 1 \\ \frac{c_i}{1-t} - \frac{c_{i-1}}{1-t}t & \text{if } i \in \{2, \ldots, n-1\} \\ c_i - \frac{c_{i-1}}{1-t}t & \text{if } i = n. \end{cases} \tag{3}$$

Given that $V_i(t, C)$ is, by definition, non-negative and taking into account expression (3), the following conditions have to be satisfied:

- $\frac{c_i}{1-t} - \frac{c_{i-1}}{1-t}t \geq 0$ for all $i \in \{2, \ldots, n-1\}$. If $c_i = c_{i-1} = 0$, the condition is always satisfied. Otherwise, we deduce that $t \leq \frac{c_i}{c_{i-1}}$ for all $i \in \{2, \ldots, n-1\}$.
- $c_n - \frac{c_{n-1}}{1-t}t \geq 0$. If $c_i = c_{i-1} = 0$, the condition is always satisfied. Otherwise, we deduce that $t \leq \frac{c_n}{c_n + c_{n-1}}$.

[25] Note that the fact that the region furthest downstream accumulates all the waste that enters it, contrary to what occurs in the other regions, where part of the waste flows on to the next region downstream, introduces a particularity into the treatment of this region. This is compatible with the concept of the river ending in a lake which belongs to a single region. If, however, the river ends in the sea, the model can be easily adapted by dropping this differentiation between regions.

Additionally, it is easy to see from the previous reasoning that any value of $\hat{t}$ between $\underline{t}$ and $\bar{t}^*(\bar{t}, C)$ is compatible with $(N, C, \underline{t}, \bar{t})$. Then we have arrived at the desired result.

Proof of Proposition 4

Let $(N, C, \underline{t}, \bar{t})$ be a problem. First, take $i = 1$. Given that this region is the most upstream region in the river, it is straightforward that all the waste in this segment is of its own responsibility. Then, $\underline{l}_1^1(\cdot) = \bar{l}_1^1(\cdot) = c_1$.

Take now any $i \in \{2, \ldots, n-1\}$. In this case, if $t \in (0,1)$ we have that $\frac{c_{i-1}}{1-t}$ units of waste entered region $i-1$. Then, $\frac{c_{i-1} \cdot t}{1-t}$ units of waste entered region i from the immediate upstream region, $i-1$, and $\frac{c_{i-1} \cdot t^2}{1-t}$ of these units left region i to the immediate downstream region, $i+1$. Therefore, $c_{i-1} \cdot t$ units of the waste present in region i are responsibility of the regions situated upstream from i. Then, we have that $l_i^i(\cdot) = c_i - c_{i-1} \cdot t$. If $t = 1$, we have that c_i equals 0 for all $i \in \{1, \ldots, n-1\}$ and, therefore, $l_i^i(\cdot) = 0$. Finally, if $t = 0$, we have that all the waste present in region i is of its own responsibility and, thus, $l_i^i(\cdot) = c_i$. In situations in which there is uncertainty over t, $t \in [\underline{t}, \bar{t}^*(\bar{t}, C)]$, we can summarize all these expressions and we have that $\underline{l}_i^i(\cdot) = c_i - c_{i-1} \cdot \bar{t}^*(\bar{t}, C)$ and $\bar{l}_i^i(\cdot) = c_i - c_{i-1} \cdot \underline{t}$ for all $i \in \{2, \ldots, n-1\}$.

Finally, take $i = n$. Consider first the case in which $t \in (0,1)$. In this case, we have that $\frac{c_{n-1}}{1-t}$ units of waste entered region $n-1$. Then, $\frac{c_{n-1} \cdot t}{1-t}$ units of waste entered and remain in region n from its upstream regions and, then, $l_n^n(\cdot) = c_n - \frac{c_{n-1} \cdot t}{1-t}$, given that n is the most downstream region. Consider now the case in which $t = 1$. In this case, there is no information at all about how much of the waste is the responsibility of region n. Then, $l_n^n(\cdot) \in [0, c_n]$. Finally, if $t = 0$, we have that all the waste present in region n is of its own responsibility and, thus, $l_n^n(\cdot) = c_n$. It is easy to see that all these expressions can be summarized as in the proposition. Then, the result is proved.

Proof of Proposition 5

In the proof of Proposition 3 (Equation (3)), we have shown that, knowing the transfer rate t and the cost vector C, we can deduce the amount of waste discharged by each region. This amount, denoted by $V_i(t, C)$ is given by the following formula:

$$V_i(t,C) = \begin{cases} \frac{c_i}{1-t} & \text{if } i = 1, \\ \\ \frac{c_i}{1-t} - \frac{c_{i-1} \cdot t}{1-t} & \text{if } i \in \{2, \ldots, n-1\}, \\ \\ c_i - \frac{c_{i-1} \cdot t}{1-t} & \text{if } i = n. \end{cases}$$

Now, we have to show that $\gamma_i(\cdot) = V_i(s, C)$. The case of $i = n$ is straightforward given that $\gamma_n^j(\cdot) = 0$ for all $j < n$ and $\gamma_n^n(\cdot) = V_i(s, C)$ by definition. We focus now on the case of $i = 1$. By definition of γ, we have that $\gamma_1^j(\cdot) = c_1 \cdot s^{j-1}$ for all $j \in \{1, \ldots, n-1\}$ and $\gamma_1^n(\cdot) = \frac{c_1 \cdot s^{n-1}}{1-s}$. Note that

$$\gamma_1(\cdot) = \sum_{j=1}^{n-1} \gamma_1^j(\cdot) + \frac{c_1 \cdot s^{n-1}}{1-s}.$$

Given that

$$(1-s) \cdot \sum_{j=1}^{n-1} \gamma_1^j(\cdot) = c_1 - c_1 \cdot s^{n-1},$$

we obtain that $\gamma_1(\cdot) = \frac{c_1}{1-s} = V_1(s, C)$.

Finally, consider the case of $i \in \{2, \ldots, n-1\}$. By definition of γ, we have that $\gamma_i^j(\cdot) = 0$ for all $j < i$, $\gamma_i^j(\cdot) = c_i \cdot s^{j-i} - c_{i-1} \cdot s^{j+1-i}$ for all $j \in \{i, \ldots, n-1\}$ and $\gamma_i^n(\cdot) = \frac{c_i \cdot s^{n-i} - c_{i-1} \cdot s^{n+1-i}}{1-s}$. Note that

$$\gamma_i(\cdot) = \sum_{j=i}^{n-1} \gamma_i^j(\cdot) + \frac{c_i \cdot s^{n-i} - c_{i-1} \cdot s^{n+1-i}}{1-s}.$$

Given that

$$(1-s) \cdot \sum_{j=i}^{n-1} \gamma_i^j(\cdot) = c_i - c_i \cdot s^{n-i} - c_{i-1} \cdot s + c_{i-1} \cdot s^{n+1-i},$$

we obtain that $\gamma_i(\cdot) = \frac{c_i - c_{i-1} \cdot s}{1-s} = V_i(s, C)$ and the proposition is proved.

Proof of Theorem 1

First, it is easy to see that the Upstream Responsibility rule γ satisfies LR, NDR, CR and MIT. To prove the other implication, consider a problem $(N, C, \underline{t}, \bar{t})$ and its corresponding $\bar{t}^*(\bar{t}, C)$ inferred from Proposition 3. Let x be a rule satisfying LR, NDR, CR and MIT. We are going to show that x has to correspond to γ.

We will calculate the assignment given by x in n steps. In the $j-$th step, we calculate the values of $x_i^j(\cdot)$ for all $i \in \{1, \ldots, n\}$.

- Step 1: We distribute the cost c_1. In this case, by NDR, $x_i^1(\cdot) = 0$ for all $i > 1$. Then, by definition of a rule, $x_1^1(\cdot) = c_1$. If $n = 1$, the proof is finished. If $n > 1$, go to step 2.

- Step j, with $j \in \{2, \ldots, n\}$: We distribute the cost c_j. By the application of NDR, $x_i^j(N, C, \underline{t}, \bar{t}) = 0$ for all $i > j$. Consider other problem (N, C, s, s), where $s = \frac{\underline{t}+\bar{t}^*(\bar{t}, C)}{2}$. Now, we have two cases:

  - If $n > j$, we have by LR that $x_j^j(N, C, s, s) = c_j - c_{j-1} \cdot s$. We are going to prove that $x_j^j(N, C, s, s) = x_j^j(N, C, \underline{t}, \bar{t})$. If $\underline{t} = s = \bar{t}^*(\bar{t}, C)$, it is straightforward that they are equal. For the rest of the cases, consider all problems (N, C, r, r) such that $r \in [\underline{t}, s)$. Then, by LR we have that $x_j^j(N, C, r, r) = c_j - c_{j-1} \cdot r$. Given that $r - \underline{t} < \bar{t}^*(\bar{t}, C) - r$, we have by MIT that $\sum_{i<j} x_i^j(N, C, r, r) \leq \sum_{i<j} x_i^j(N, C, \underline{t}, \bar{t})$ and then, by definition, $x_j^j(N, C, r, r) \geq x_j^j(N, C, \underline{t}, \bar{t})$. Therefore, $x_j^j(N, C, \underline{t}, \bar{t}) \leq c_j - c_{j-1} \cdot (s - \varepsilon)$ for all $\varepsilon \geq 0$. Similarly, we can deduce that $x_j^j(N, C, u, u) \leq x_j^j(N, C, \underline{t}, \bar{t})$ for all $u \in (s, \bar{t}^*(\bar{t}, C)]$ and, then, $x_j^j(N, C, \underline{t}, \bar{t}) \geq c_j - c_{j-1} \cdot (s + \varepsilon)$ for all $\varepsilon \geq 0$. Then, the unique possibility is that $x_j^j(N, C, \underline{t}, \bar{t}) = x_j^j(N, C, s, s)$. Therefore, $x_j^j(N, C, \underline{t}, \bar{t}) = c_j - c_{j-1} \cdot s$.

    If $s = 0$, we have that $x_j^j(N, C, \underline{t}, \bar{t}) = c_j$ and the proof of step j is finished. Then, go to step $j + 1$.

    If, however, $s > 0$, let us concentrate first in the case of $j = 2$. Then, we have by definition that $x_1^2(N, C, \underline{t}, \bar{t}) = c_1 \cdot s$ and the proof of step 2 is finished. Now, go to step 3.

    If $s > 0$ and $j \geq 3$, we have that $\sum_{i=1}^{j-1} x_i^j(N, C, \underline{t}, \bar{t}) = c_{j-1} \cdot s$. By CR, $x_i^j(N, C, \underline{t}, \bar{t}) \cdot x_k^{j-1}(N, C, \underline{t}, \bar{t}) = x_k^j(N, C, \underline{t}, \bar{t}) \cdot x_i^{j-1}(N, C, \underline{t}, \bar{t})$ for all $i, k \in \{1, \ldots, j-1\}$. Or, equivalently, $x_i^j(N, C, \underline{t}, \bar{t}) \cdot \sum_{i=1}^{j-1} x_i^{j-1}(N, C, \underline{t}, \bar{t}) = x_i^{j-1}(N, C, \underline{t}, \bar{t}) \cdot \sum_{i=1}^{j-1} x_i^j(N, C, \underline{t}, \bar{t})$ for all $i \in \{1, \ldots, j-1\}$.

Given that $\sum_{i=1}^{j-1} x_i^{j-1}(N, C, \underline{t}, \bar{t}) = c_{j-1}$ and that we also know from step $j-1$ that $x_i^{j-1}(N, C, \underline{t}, \bar{t}) = c_i \cdot s^{j-1-i} - c_{i-1} \cdot s^{j-i}$, we have that for all $i \in \{1, \ldots, j-1\}$,

$$x_i^j(N, C, \underline{t}, \bar{t}) = \frac{c_i \cdot s^{j-1-i} - c_{i-1} \cdot s^{j-i}}{c_{j-1}} \cdot c_{j-1} \cdot s.$$

Therefore, for all $i \in \{1, \ldots, j-1\}$,

$$x_i^j(N, C, \underline{t}, \bar{t}) = c_i \cdot s^{j-i} - c_{i-1} \cdot s^{j+1-i}.$$

Now, go to step $j+1$.

- If $n = j$, we have by LR that $x_n^n(N, C, s, s) = c_n - \frac{c_{n-1} \cdot s}{1-s}$. We are going to prove that $x_n^n(N, C, s, s) = x_n^n(N, C, \underline{t}, \bar{t})$. If $\underline{t} = s = \bar{t}^*(\bar{t}, C)$, it is straightforward that they are equal. For the rest of the cases, consider all problems (N, C, r, r) such that $r \in [\underline{t}, s)$. Then, by LR we have that $x_n^n(N, C, r, r) = c_n - \frac{c_{n-1} \cdot r}{1-r}$. Given that $r - \underline{t} < \bar{t}^*(\bar{t}, C) - r$, we have by MIT that $\sum_{i<n} x_i^n(N, C, r, r) \leq \sum_{i<n} x_i^n(N, C, \underline{t}, \bar{t})$ and then, by definition, $x_n^n(N, C, r, r) \geq x_n^n(N, C, \underline{t}, \bar{t})$. Therefore, $x_n^n(N, C, \underline{t}, \bar{t}) \leq c_n - \frac{c_{n-1} \cdot (s-\varepsilon)}{1-(s-\varepsilon)}$ for all $\varepsilon \geq 0$. Similarly, we can deduce that $x_n^n(N, C, u, u) \leq x_n^n(N, C, \underline{t}, \bar{t})$ for all $u \in (s, \bar{t}^*(\bar{t}, C)]$ and, then, $x_n^n(N, C, \underline{t}, \bar{t}) \geq c_n - \frac{c_{n-1} \cdot (s+\varepsilon)}{1-(s+\varepsilon)}$ for all $\varepsilon \geq 0$. Then, the unique possibility is that $x_n^n(N, C, \underline{t}, \bar{t}) = x_n^n(N, C, s, s)$. Therefore, $x_n^n(N, C, \underline{t}, \bar{t}) = c_n - \frac{c_{n-1} \cdot s}{1-s}$ and, by definition, $\sum_{i=1}^{n-1} x_i^n(N, C, \underline{t}, \bar{t}) = \frac{c_{n-1} \cdot s}{1-s}$. If $j = 2$, this implies that $x_1^2(N, C, \underline{t}, \bar{t}) = \frac{c_1 \cdot s}{1-s}$. If $j \geq 3$ and $s = 0$, we have that $x_j^j(N, C, \underline{t}, \bar{t}) = c_j$. If $j \geq 3$ and $s > 0$, we have by CR that $x_i^n(N, C, \underline{t}, \bar{t}) \cdot x_k^{n-1}(N, C, \underline{t}, \bar{t}) = x_k^n(N, C, \underline{t}, \bar{t}) \cdot x_i^{n-1}(N, C, \underline{t}, \bar{t})$ for all $i, k \in \{1, \ldots, n-1\}$. Or, equivalently, we have that $x_i^n(N, C, \underline{t}, \bar{t}) \cdot \sum_{i=1}^{n-1} x_i^{n-1}(N, C, \underline{t}, \bar{t}) = x_i^{n-1}(N, C, \underline{t}, \bar{t}) \cdot \sum_{i=1}^{n-1} x_i^n(N, C, \underline{t}, \bar{t})$ for all $i \in \{1, \ldots, n-1\}$.

  Given that $\sum_{i=1}^{n-1} x_i^{n-1}(N, C, \underline{t}, \bar{t}) = c_{n-1}$ and that we also know from step $j-1$ that $x_i^{n-1}(N, C, \underline{t}, \bar{t}) = c_i \cdot s^{n-1-i} - c_{i-1} \cdot s^{n-i}$, we have that for all $i \in \{1, \ldots, n-1\}$,

$$x_i^n(N, C, \underline{t}, \bar{t}) = \frac{c_i \cdot s^{n-i-1} - c_{i-1} \cdot s^{n-i}}{c_{n-1}} \cdot \frac{c_{n-1} \cdot s}{1-s}.$$

Therefore, for all $i \in \{1, \ldots, n-1\}$,

$$x_i^n(N, C, \underline{t}, \bar{t}) = \frac{c_i \cdot s^{n-i} - c_{i-1} \cdot s^{n-i+1}}{1-s}.$$

Proof of Proposition 6

The following examples prove that the axioms are independent.

Limits of Responsibility: The UES rule, β satisfies NDR, CR and MIT. However, it does not satisfy LR as we have shown in Example 2.

No Downstream Responsibility: Let ω be the following rule:

$$\omega_i^j(N, C, \underline{t}, \bar{t}) = \begin{cases} c_i - c_{i-1} \cdot s & \text{if } i = j < n, \\ c_i - \frac{c_{i-1} \cdot s}{1-s} & \text{if } i = j = n, \\ c_{i-2} \cdot s & \text{if } i = j+1, \\ \frac{c_i \cdot s}{1-s} & \text{if } i+1 = j = n, \\ 0 & \text{otherwise,} \end{cases}$$

where $s = \frac{t + \bar{t}^*(\bar{t}, C)}{2}$ and c_0 is set to 0.

It is easy to see that this rule ω satisfies MIT, LR and CR. However, the following example shows that it does not satisfy NDR. Let $N = \{1, 2, 3\}$, $C = \{10, 10, 10\}$, $\underline{t} = 0$ and $\bar{t} = 1$ be a cost allocation problem. We have that $\omega_3^2(N, C, \underline{t}, \bar{t}) = \frac{5}{2}$, while NDR states that $\omega_3^2(N, C, \underline{t}, \bar{t}) = 0$.

Consistent Responsibility: Let φ be the following rule:

$$\varphi_i^j(N,C,\underline{t},\overline{t}) = \begin{cases} 0 & \text{if } i > j, \\ c_i - c_{i-1} \cdot s & \text{if } i = j < n, \\ c_i - \frac{c_{i-1} \cdot s}{1-s} & \text{if } i = j = n, \\ \frac{c_{j-1} \cdot s}{j-1} & \text{if } i < j < n, \\ \frac{c_{j-1}}{j-1} \cdot \frac{s}{1-s} & \text{if } i < j = n, \end{cases}$$

where $s = \frac{\underline{t} + \overline{t}^*(\overline{t}, C)}{2}$ and c_0 is set to 0.

It is easy to see that φ satisfies LR, NDR and MIT. However, the following example shows that it does not satisfy CR. Let $N = \{1,2,3\}$, $C = \{10, 10, 5\}$, $\underline{t} = 0$ and $\overline{t} = 1$ be a cost allocation problem. We have that $\varphi_2^2(\cdot) = \frac{25}{3}$, $\varphi_1^3(\cdot) = 1$, $\varphi_2^3(\cdot) = 1$ and $\varphi_1^2(\cdot) = \frac{5}{3}$. Then, $\varphi_2^2(\cdot) \cdot \varphi_1^3(\cdot) = \frac{25}{3} \neq \frac{5}{3} = \varphi_1^2(\cdot) \cdot \varphi_2^3(\cdot)$, while CR would imply that $\varphi_2^2(\cdot) \cdot \varphi_1^3(\cdot) = \varphi_1^2(\cdot) \cdot \varphi_2^3(\cdot)$.

Monotonicity with respect to Information on the transfer rate: Let ρ be the following rule:

$$\rho_i^j(N,C,\underline{t},\overline{t}) = \begin{cases} 0 & \text{if } i > j, \\ c_i \cdot \underline{t}^{j-i} - c_{i-1} \cdot \underline{t}^{j+1-i} & \text{if } i \leq j < n, \\ c_i - \frac{c_{i-1} \cdot \underline{t}}{1-\underline{t}} & \text{if } i = j = n, \\ \frac{c_i \cdot \underline{t}^{j-i} - c_{i-1} \cdot \underline{t}^{j-i+1}}{1-\underline{t}} & \text{if } i < j = n, \end{cases}$$

where c_0 is set to 0 and the indeterminate form 0^0 is set to 1.

It is easy to see that ρ satisfies LR, NDR and CR. However, the following example shows that it does not satisfy MIT. Let $(N,C,\underline{t},\overline{t})$ and $(N,C,\underline{u},\overline{u})$ be two cost allocation problems, with $N = \{1,2\}$, $C = \{10, 20\}$, $\underline{t} = 0$, $\overline{t} = 1$ and $\underline{u} = \overline{u} = \frac{1}{4}$. We have that $\rho_1^2(N,C,\underline{t},\overline{t}) = 0$ and $\rho_1^2(N,C,\underline{u},\overline{u}) = \frac{10}{3}$, although MIT would imply that $\rho_1^2(N,C,\underline{u},\overline{u}) \leq \rho_1^2(N,C,\underline{t},\overline{t})$.

References

[1] Ambec, S., Ehlers, L., 2008. Sharing a river among satiable agents. *Game Econ. Behav.* 64, 35-50.

[2] Ambec, S., Sprumont, Y., 2002. Sharing a river. *J. Econ. Theory* 107, 453-462.

[3] Barrett, S., 1994. Conflict and cooperation in managing international water resources. Working paper 1303. World Bank, Washington.

[4] Béal S., Ghintran, A., Rémila, E., Solal, P., 2013. The river sharing problem: a survey. *Int. Game Theory Rev.* 15, 1-19.

[5] Beard, R., 2011. The river sharing problem: a review of the technical literature for policy economists. MPRA Paper No. 34382, University Library of Munich, Germany.

[6] Dong, B., Ni, D., Wang, Y., 2012. Sharing a polluted river network. *Environ. Resour. Econ.* 53, 367-387.

[7] Gengenbach, M.F., Weikard, H.P., Ansink, E., 2010. Cleaning a river: an analysis of voluntary joint action. *Nat. Resour. Model.* 23, 565-589.

[8] Godana, B., 1985. Africa's shared water resources. France Printer, London.

[9] Gómez-Rúa, M., 2013. Sharing a polluted river through environmental taxes. *SERIEs* 4, 137-153.

[10] Khmelnitskaya, A.B., 2010. Values for rooted-tree and sink-tree digraphs games and sharing a river. *Theor. Decis.* 69, 657-669.

[11] Kilgour, M., Dinar A., 1996. Are stable agreements for sharing international river waters now possible? Working paper 1474. World Bank, Washington.

[12] Littlechild, S.C., Owen, G. 1973. A simple expression for the Shapley Value in a special case. *Manage. Sci.* 20, 370-372.

[13] Ni, D., Wang, Y., 2007. Sharing a polluted river. *Game. Econ. Behav.* 60, 176-186.

[14] OECD, 2008. Freshwater. OECD Environmental Outlook to 2030, Paris.

[15] Parrachino, I., Dinar, A., Patrone, F., 2006. Cooperative game theory and its application to natural, environmental and water resource issues: 3. Application to water resources. World Bank Policy Research Working Paper 4074.

[16] Rébille, Y., Richefort, L., 2012. Sharing water from many rivers. LEMNA Working paper EA 4272.

[17] Segerson, K., 1988. Uncertainty and incentives for nonpoint pollution control. *J. Environ. Econ. Manag.*, 15, 87-98.

[18] van den Brink, R., van der Laan, G., 2008. Comment on "Sharing a polluted river". Mimeo.

[19] van den Brink, R., van der Laan, G. and Moes, N., 2012. Fair agreements for sharing international rivers with multiple springs and externalities. *J. Environ. Econ. Manag.* 3, 388-403.

[20] van der Laan, G., Moes, N., 2012. Transboundary externalities and property rights: an international river pollution model. Tinbergen Discussion Paper 12/006-1, Tinbergen Institute and VU University, Amsterdam.

[21] Wang, Y., 2011. Trading water along a river. *Math. Soc. Sci.* 61, 124-130.