Documentos de Trabajo

Lan Gaiak

THE GREAT MODERATION OF INFLATION: A STRUCTURAL ANALYSIS OF RECENT U.S. MONETARY BUSINESS CYCLES

Miguel Casares
Jesús Vázquez
D.T. 1215

Departamento de Economía

Ekonomia Saila

upna

Universidad
Pública de Navarra
Nafarroako
Unibertsitate Publikoa

The Great Moderation of Inflation: a structural analysis of recent U.S. monetary business cycles[*]

Miguel Casares[†]and Jesús Vázquez[‡]

December 22, 2012

Abstract

U.S. inflation has experienced a great moderation in the last two decades. This paper examines the factors behind this and other stylized facts, such as the weaker correlation of inflation and nominal interest rate (Gibson paradox). Our findings point at lower exogenous variability of supply-side shocks and, to a lower extent, structural changes in money demand, monetary policy, and firms' sticky pricing behavior as the main driving forces of the changes observed in recent U.S. business cycles.

Keywords: DSGE monetary model, inflation moderation, structural changes.

JEL codes: E32, E47.

1

1   Introduction

Since 1995, there has been a great moderation of U.S. inflation, characterized by a low average rate and mild fluctuations around it (see Figure 1). This is particular striking when compared to high average inflation and strong volatility observed in the three decades before 1995. The inflation moderation might be connected to changes in other economic variables. Indeed, the Fed funds rate also displays low levels and low variability after 1995. The decline in volatility is also found in the rate of growth of some monetary aggregates.[1]

Apart from changes in volatilities, other statistics measuring cyclical correlation and persistence have also shifted in the post-1995 period. To illustrate these changes, Figure 2 shows dynamic correlation functions of inflation and the nominal interest rate computed from an empirical VAR using U.S. quarterly data.[2] A comparison across different samples helps us to highlight three additional important changes of inflation and the nominal interest rate dynamics across periods: (i) inflation persistence is much lower since 1995, (ii) the positive correlation between inflation and the nominal interest rate observed in the first period almost vanishes (i.e. Gibson paradox) in the most recent period,[3] (iii) the Fed funds rate seems to anticipate inflation movements by 10 quarters in the most recent period but not before 1995. The first two additional stylized facts were recently uncovered by Cogley, Sargent and Surico (2012), CSS (2012) from now on.

This paper considers an extended version of the canonical DSGE model (Smets and Wouters,

---

[1] This stylized fact holds for some definitions of money as *money with zero maturity* (MZM) and Divisia recently used in the monetary economics literature (Altig, Christiano, Eichenbaum, Linde, 2011; McCallum and Nelson, 2010), but does not hold for others as the M1.

[2] A comprehensive analysis of the actual vector autocorrelations of the four variables included in the VAR (inflation, nominal interest rate, nominal money growth and output growth rates) is provided below. See Hamilton (1994, pp. 264-266) for a derivation of the analytical expressions of the vector autocorrelation functions. Fuhrer and Moore (1995) and Ireland (2003) are two prominent papers in this literature using unconstrained VAR to summarize data features in this way. In contrast to these papers, we consider output growth instead of a measure of detrended output to characterize the dynamic comovement between economic activity and the three nominal variables included in the VAR.

[3] The term Gibson paradox was coined by Keynes (1930) in honor of A. H. Gibson who detected historical episodes during the Gold standard period where nominal interest rates were positively correlated with the aggregate price level, but a weak/null correlation with inflation, which contradicts conventional monetary theory. **More recently, many papers (Friedman and Schwartz, 1982; Barsky, 1987; Barsky and Summers, 1988; Cogley, Sargent and Surico, 2012; among others) have revisited the Gibson paradox.**
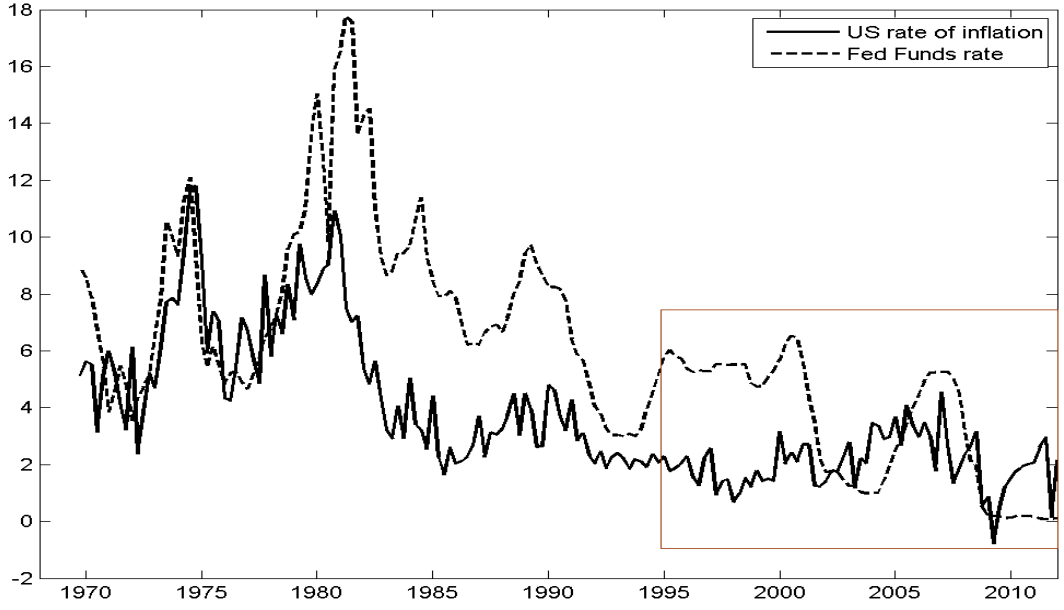
Figure 1: U.S. inflation and nominal interest rates, 1968-2011.

2007) to understand the underlying forces explaining these changes in US business cycles.[4] The extended version of the model incorporates a transactions-facilitating role of money, and a variant of the Taylor rule, which allows for nominal interest rate responses to nominal money growth.

Following Ireland (2003) and CSS (2012), the extended model is estimated for two sub-sample periods, which allows to identify the sources of the changing patterns. Thus, we use a DSGE model to explaining the business cycle dynamics of output growth, inflation, nominal interest rate and nominal money growth. CSS (2012) do it with a highly stylized four-equation model. However, both our model and that of Ireland (2003) estimate medium-scale DSGE models, which in particular incorporate adjustment costs for changes in the stock of capital. This feature improves the fitting of sticky-price models to regularities found in actual business cycle fluctuations (see Kimball, 1995; King and Watson, 1996; Casares and McCallum, 2000; among others). Moreover, Ireland (2003) has shown that using data on both consumption and investment in addition to output, as opposed to only using output in CSS (2012), helps to estimate adjustment cost parameters for both capital

---

[4]As discussed below, this extension is mainly motivated because the cash-less model of Smets and Wouters (2007) fails to reproduce some observed dynamic shifts such as the Gibson paradox.

3

and prices. This medium-scale modelling approach further explains some of the different results found in this paper compared to the ones reported by CSS (2012). In addition, CSS (2012) take the great inflation period (1968-1983) as the first subsample, whereas ours covers the period 1968-1994. The reason is that by estimating a longer subsample that also includes the disinflation period (1984-1994) the parameter estimates do not change significantly. We also extend our second subsample in order to cover the most recent period after the subprime mortgage crisis, which is ignored in CSS (2012). Ireland's (2003) sample runs from 1959 to 2000. He studied the pre-1979 and post-1979 periods.[5]

The estimation results show that the structural parameter estimates are fairly stable across the two sample periods studied. However, many parameters describing the shock processes have changed significantly across periods. We assess model's performance by reporting key second-moment statistics obtained from both actual US data and synthetic data. The model does a good job in capturing the volatility reduction of inflation, the nominal interest rate and the growth of nominal money. The model also reproduces the mild increase of the inertia in both nominal interest rates and the nominal money growth and the lower persistence of inflation since 1995. Moreover, the model performs well in replicating the comovement of inflation with the nominal interest rate, nominal money growth and output growth rates. In particular, the model explains the re-emergence of the Gibson Paradox in the post-1995 period. The impulse-response functions and the counterfactual experiments carried out in the paper show that the change in the parameters characterizing wage mark-up shocks is the main determinant explaining the changes in U.S. business cycle fluctuations after 1995.

The rest of the paper is organized as follows. Section 2 extends the medium-scale DSGE model suggested by Smets and Wouters (2007) with both money demand and monetary policy behavior. Section 3 describes the estimation strategy and discusses the empirical results. Section 4 analyzes model's performance by comparing actual and simulated second-moment statistics. Based on the estimates of the structural parameters obtained from the two sub-samples, Section 5 illustrates the changes in the structural equations of the model from 1995 onwards. In addition, this section runs a large set of counterfactual exercises to assess the relative importance of key structural elements of

---

[5]For obvious data sample reasons, he could not analyze the changes observed after 1995, which are studied in this paper and CSS (2012).
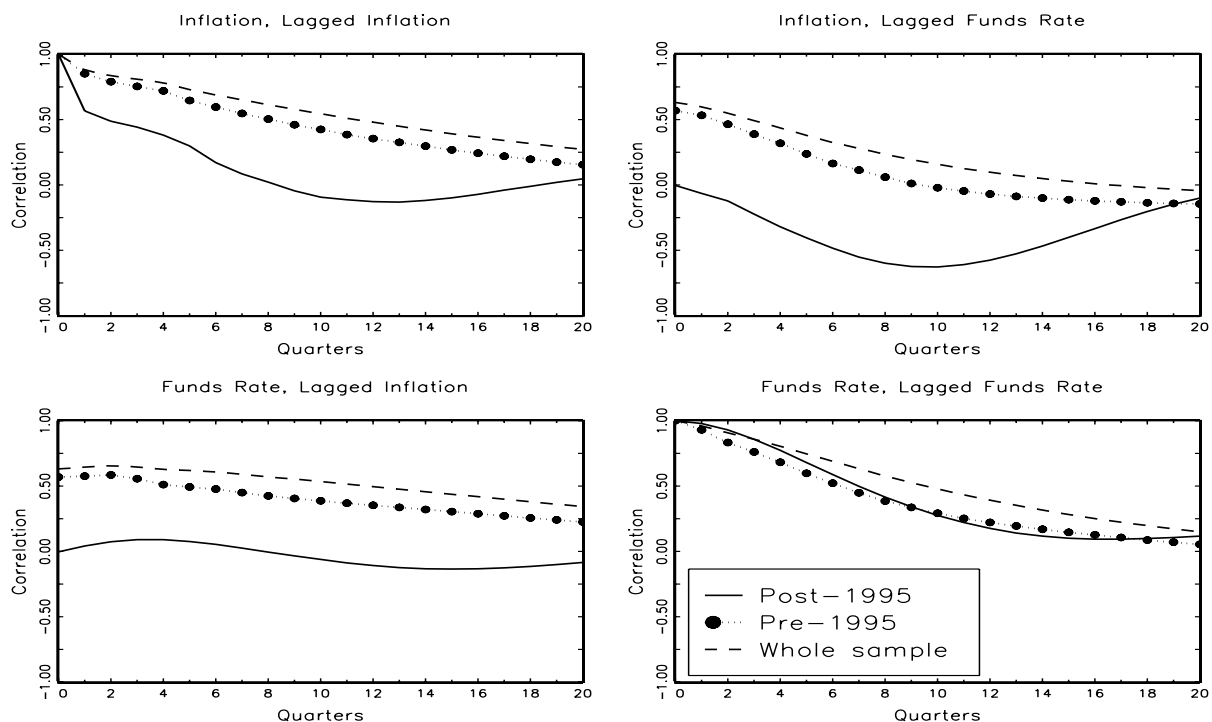
Figure 2: Vector autocorrelation and cross-correlation functions from US inflation and nominal interest rates.

the model when it comes to explaining the swing of cyclical patterns. Finally, Section 6 concludes.

2   A DSGE model with money

This paper considers a modified version of the Smets and Wouters (2007) model to introduce money. The role of money is defined by its specific function: being the medium-of-exchange to carry out transactions. Thus, a transactions technology is presented where the stock of real money can be used to save transaction costs.[6] Any increase in real money holdings has a negative impact on transactions costs, with decreasing marginal returns. Meanwhile, money is supplied by the central bank to support the implementation of a Taylor (1993)-style stabilizing monetary policy rule.

Households maximize intertemporal utility. Recalling the specification of Smets and Wouters

---

[6]The introduction of money through a transaction cost technology instead of a money-in-the-utility function specification is empirically motivated. The former approach is somewhat more flexible than the latter to accommodate the observed dynamic shifts.

(2007), and adding a consumption preference shock ε_t^b, the instantaneous utility of the representative j-th household is

$$\left[\exp\left(\varepsilon_t^b\right)\frac{1}{1-\sigma_c}\left(C_t(j)-\lambda C_{t-1}\right)^{1-\sigma_c}\right]\exp\left(\frac{\sigma_c-1}{1+\sigma_l}\left(L_t(j)\right)^{1+\sigma_l}\right),$$

where $\sigma_c, \sigma_l > 0$, $C_t(j)$ is current consumption of the j-indexed representative household, C_{t-1} is lagged aggregate consumption, and $L_t(j)$ is the supply of household-specific labor. In addition, the budget constraint incorporates (real) spending on transaction costs, $H_t(j)$, and the possibility of a net increase in real money balances, $\frac{M_t(j)}{P_t}-\frac{M_{t-1}(j)}{P_t}$. For period t, the budget constraint is written as follows

$$\frac{W_t(j)L_t(j)}{P_t}+\frac{R_t^k Z_t(j)K_{t-1}(j)}{P_t}-a\left(Z_t(j)\right)K_{t-1}(j)+\frac{Div_t}{P_t}-T_t =$$
$$C_t(j)+I_t(j)+\frac{B_t(j)}{(1+R_t)P_t}-\frac{B_{t-1}(j)}{P_t}+\frac{M_t(j)}{P_t}-\frac{M_{t-1}(j)}{P_t}+H_t(j),$$

which is equivalent to

$$\frac{W_t(j)L_t(j)}{P_t}+\frac{R_t^k Z_t(j)K_{t-1}(j)}{P_t}-a\left(Z_t(j)\right)K_{t-1}(j)+\frac{Div_t}{P_t}-T_t =$$
$$C_t(j)+I_t(j)+\frac{B_t(j)}{(1+R_t)P_t}-\frac{B_{t-1}(j)}{P_{t-1}}\left(1+\pi_t\right)^{-1}+\frac{M_t(j)}{P_t}-\frac{M_{t-1}(j)}{(1+\pi_t)P_{t-1}}+H_t(j),$$

where $\pi_t = (P_t/P_{t-1}) - 1$ is the rate of inflation between periods $t-1$ and t.

Based on the monetary model of Casares (2007), let us consider the following transactions technology

$$H_t(j) = a_0 + a_1 C_t(j)\left(\frac{C_t(j)}{\exp\left(\varepsilon_t^\chi\right)\left(\frac{M_t(j)}{P_t}-\lambda_m\frac{M_{t-1}}{P_{t-1}}\right)}\right)^{\frac{a_2}{1-a_2}},$$

where $a_0, a_1 > 0$, $0 < a_2 < 1$, $0 < \lambda_m < 1$ and ε_t^χ is a money-augmenting AR(1) shock.[7] As a distinctive characteristic from Casares (2007), there is (external) monetary habits that measure endogenous inertia on the demand for real money. The first order conditions for consumption, real

---

[7] The partial derivatives are

$$H_{C_t(j)} = \frac{a_1}{1-a_2}\left(\frac{C_t(j)}{\exp\left(\varepsilon_t^\chi\right)\left(\frac{M_t(j)}{P_t}-\lambda_m\frac{M_{t-1}}{P_{t-1}}\right)}\right)^{\frac{a_2}{1-a_2}},$$

$$H_{\frac{M_t(j)}{P_t}} = -\frac{a_1 a_2}{1-a_2}\exp\left(\varepsilon_t^\chi\right)\left(\frac{C_t(j)}{\exp\left(\varepsilon_t^\chi\right)\left(\frac{M_t(j)}{P_t}-\lambda_m\frac{M_{t-1}}{P_{t-1}}\right)}\right)^{\frac{1}{1-a_2}},$$

which satisfy the desirable properties: $H_{C_t(j)} > 0$, $H_{C_t(j)C_t(j)} > 0$, $H_{\frac{M_t(j)}{P_t}} < 0$, $H_{\frac{M_t(j)}{P_t}\frac{M_t(j)}{P_t}} > 0$ and $H_{C_t(j)\frac{M_t(j)}{P_t}} < 0$.

money, labor supply, and bonds that result from the household optimizing program are

$$\exp\left(\varepsilon_t^b\right)\left(C_t\left(j\right)-\lambda C_{t-1}\right)^{-\sigma_c}\exp\left(\frac{\sigma_c-1}{1+\sigma_l}\left(L_t^s(j)\right)^{1+\sigma_l}\right)-\Xi_t\left(1+H_{C_t(j)}\right)=0, \qquad (C_t^{foc}(j))$$

$$-\Xi_t\left(1+H_{\frac{M_t(j)}{P_t}}\right)+\beta E_t\Xi_{t+1}\left(1+\pi_{t+1}\right)^{-1}=0, \qquad \left(\left(\tfrac{M_t(j)}{P_t}\right)^{foc}\right)$$

$$\exp\left(\varepsilon_t^b\right)\frac{(C_t(j)-\lambda C_{t-1})^{1-\sigma_c}}{1-\sigma_c}\exp\left(\tfrac{\sigma_c-1}{1+\sigma_l}\left(L_t(j)\right)^{1+\sigma_l}\right)\left(\sigma_c-1\right)\left(L_t(j)\right)^{\sigma_l}+\Xi_t\frac{W_t(j)}{P_t}=0, \qquad (L_t(j))$$

$$-\Xi_t\left(1+R_t\right)^{-1}+\beta E_t\Xi_{t+1}\left(1+\pi_{t+1}\right)^{-1}=0, \qquad \left(\left(\tfrac{B_t(j)}{P_t}\right)^{foc}\right)$$

where Ξ_t is the Lagrange multiplier of the budget constraint in period t, and $H_{C_t(j)}$ and $H_{\frac{M_t(j)}{P_t}}$ are the partial derivatives of the transaction costs function with respect to consumption and real money balances, respectively. Plugging the expression for $\beta E_t\Xi_{t+1}\left(1+\pi_{t+1}\right)^{-1}$ obtained from $\left(\frac{B_t(j)}{P_t}\right)^{foc}$ in $\left(\frac{M_t(j)}{P_t}\right)^{foc}$ yields the money demand equation

$$-H_{\frac{M_t(j)}{P_t}}=\frac{R_t}{1+R_t},$$

that presents a standard microeconomic optimality condition that equates the marginal return of monetary services $(-H_{\frac{M_t(j)}{P_t}})$ to the marginal (opportunity) cost of money holdings $(\frac{R_t}{1+R_t})$. After loglinearizing, it gives

$$\log H_{\frac{M_t}{P_t}}-\log H_{\frac{M}{P}}=-\frac{1}{R^{ss}}R_t, \qquad (1)$$

where R^{ss} is the steady-state nominal interest rate. In equilibrium, the representative household assumption implies that aggregate and household-level amounts of consumption or money demand are identical. Using this result when calculating the log deviations of the marginal transactions-facilitating service of money, we have

$$\log H_{\frac{M_t}{P_t}}-\log H_{\frac{M}{P}}=-\tfrac{1}{1-a_2}c_t+\tfrac{1}{1-a_2}\left(\tfrac{1}{1-\lambda_m/\gamma}m_t-\tfrac{\lambda_m/\gamma}{1-\lambda_m/\gamma}m_{t-1}\right)+\tfrac{a_2}{1-a_2}\varepsilon_t^\chi, \qquad (2)$$

where the standard notation is used to denote the log deviations from steady state of consumption $c_t=\log\left(\frac{C_t}{C^{ss}}\right)$ and real money $m_t=\log\left(\frac{M_t/P_t}{M^{ss}/P^{ss}}\right)$. The fluctuations of the marginal service of transactions-facilitating money can be taken from equation (2) and substituted in the optimality condition (1) to yield the semi-log real money demand equation

$$m_t=\left(\lambda_m/\gamma\right)m_{t-1}+\left(1-\lambda_m/\gamma\right)c_t-\tfrac{(1-\lambda_m/\gamma)(1-a_2)}{R^{ss}}R_t-a_2\left(1-\lambda_m/\gamma\right)\varepsilon_t^\chi, \qquad (3)$$

where γ is the steady state output growth. The loglinear version of $\left(\frac{B_t(j)}{P_t}\right)^{foc}$ is

$$\log\Xi_t=E_t\log\Xi_{t+1}+\left(R_t-E_t\pi_{t+1}\right). \qquad (4)$$

We need one expression for $\log \Xi_t$ that can be found by loglinearizing $C_t^{foc}(j)$. It leads to (for simplicity, the constant terms were dropped)

$$\log \Xi_t = -\sigma_c \left(\frac{1}{1-\lambda/\gamma} c_t - \frac{\lambda/\gamma}{1-\lambda/\gamma} c_{t-1} \right) + \frac{(\sigma_c-1)w^{ss}}{\phi_w(1-\lambda/\gamma)c^{ss}} \log L_t(j) - H_{C(j)} \log H_{C_t(j)} + \varepsilon_t^b, \qquad (5)$$

that uses the standard notation denoting $c_t = \log\left(\frac{C_t}{C^{ss}}\right)$ and $c_{t-1} = \log\left(\frac{C_{t-1}}{C^{ss}}\right)$. Taking both $\log \Xi_t$ and the corresponding expression for $\log \Xi_{t+1}$ from (5), and inserting them both in (4) result in

$$-\sigma_c \left(\frac{1}{1-\lambda/\gamma} c_t - \frac{\lambda/\gamma}{1-\lambda/\gamma} c_{t-1} \right) + \frac{(\sigma_c-1)w^{ss}}{\phi_w(1-\lambda/\gamma)c^{ss}} \log L_t(j) - H_C^{ss} \log H_{C_t(j)} =$$

$$E_t \left(-\sigma_c \left(\frac{1}{1-\lambda/\gamma} c_{t+1} - \frac{\lambda/\gamma}{1-\lambda/\gamma} c_t \right) + \frac{(\sigma_c-1)w^{ss}}{\phi_w(1-\lambda/\gamma)c^{ss}} \log L_{t+1}(j) - H_C^{ss} \log H_{C_{t+1}(j)} \right)$$

$$+ (R_t - E_t \pi_{t+1}) - (1-\rho_b)\varepsilon_t^b,$$

where terms can be rearranged for the IS-type curve

$$c_t = \frac{\lambda/\gamma}{1+\lambda/\gamma} c_{t-1} + \frac{1}{1+\lambda/\gamma} E_t c_{t+1} + \frac{(\sigma_c-1)w^{ss}}{\sigma_c \phi_w(1+\lambda/\gamma)c^{ss}} (l_t - E_t l_{t+1})$$

$$- \frac{(1-\lambda/\gamma)H_C^{ss}}{\sigma_c(1+\lambda/\gamma)} \left(\log H_{C_t} - E_t \log H_{C_{t+1}} \right) - \frac{1-\lambda/\gamma}{\sigma_c(1+\lambda/\gamma)} (R_t - E_t \pi_{t+1}) + \frac{(1-\rho_b)(1-\lambda/\gamma)}{\sigma_c(1+\lambda/\gamma)} \varepsilon_t^b. \qquad (6)$$

The consumption marginal transaction costs in loglinear terms is

$$\log H_{C_t} - \log H_C = \frac{a_2}{1-a_2} c_t - \frac{a_2}{1-a_2} \left(\frac{1}{1-\lambda_m/\gamma} m_t - \frac{\lambda_m/\gamma}{1-\lambda_m/\gamma} m_{t-1} \right) - \frac{a_2}{1-a_2} \varepsilon_t^\chi. \qquad (7)$$

Inserting (7) and the corresponding expression for period $t+1$ into equation (6) gives rise to the IS curve with real-money balance effects

$$(1+c_4)c_t = c_1 c_{t-1} + c_2 E_t c_{t+1} + c_3 (l_t - E_t l_{t+1})$$

$$+ c_4 \left(\frac{1}{1-\lambda_m/\gamma} m_t - \frac{\lambda_m/\gamma}{1-\lambda_m/\gamma} m_{t-1} \right) - c_4 \left(\frac{1}{1-\lambda_m/\gamma} E_t m_{t+1} - \frac{\lambda_m/\gamma}{1-\lambda_m/\gamma} m_t \right)$$

$$- c_5 (R_t - E_t \pi_{t+1}) + c_4 (1-\rho_\chi) \varepsilon_t^\chi + c_5 (1-\rho_b) \varepsilon_t^b, \qquad (8)$$

where $c_1 = \frac{\lambda/\gamma}{1+\lambda/\gamma}$, $c_2 = \left(\frac{1}{1+\lambda/\gamma} + c_4 \right)$, $c_3 = \frac{(\sigma_c-1)w^{ss}}{\sigma_c(1+\lambda/\gamma)\phi_w c^{ss}}$, $c_4 = \frac{(1-\lambda/\gamma)H_C a_2}{\sigma_c(1+\lambda/\gamma)(1-a_2)}$, and $c_5 = \frac{1-\lambda/\gamma}{\sigma_c(1+\lambda/\gamma)}$.

The marginal rate of substitution between consumption and hours worked, $\frac{-U_{L_t(j)}}{U_{C_t(j)}/\left(1+H_{C_t(j)}\right)}$, is also affected by the introduction of money in the utility function. It yields

$$-\frac{U_{L_t(j)}}{U_{C_t(j)}/\left(1+H_{C_t(j)}\right)} = (L_t(j))^{\sigma_l} (C_t(j) - \lambda C_{t-1}) \left(1 + H_{C_t(j)} \right),$$

which in a log-linear approximation and aggregating across households becomes

$$mrs_t = \sigma_l l_t + \left(\frac{1}{1-\lambda/\gamma} + \frac{H_C a_2}{1-a_2}\right) c_t - \frac{\lambda/\gamma}{1-\lambda/\gamma} c_{t-1} - \frac{H_C a_2}{1-a_2}\left(\frac{1}{1-\lambda_m/\gamma} m_t - \frac{\lambda_m/\gamma}{1-\lambda_m/\gamma} m_{t-1}\right) - \frac{H_C a_2}{1-a_2} \varepsilon_t^\chi. \quad (9)$$

Unlike Smets and Wouters (2007), log fluctuations of transaction costs, h_t, appear in the log-linearized aggregate resource constraint:

$$y_t = c_y c_t + i_y i_t + z_y z_t + h_y h_t + \varepsilon_t^g, \quad (10)$$

where $c_y = \frac{C}{Y} = 1 - g_y - i_y$, $i_y = \frac{I}{Y} = (\gamma - 1 + \delta)\frac{K}{Y}$, $z_y = r^k \frac{K}{Y}$ and $h_y = \frac{H}{Y}$ are steady-state ratios. The value of h_t is provided by the loglinearized transactions technology function

$$h_t = \frac{1-(a_0/H)}{1-a_2}\left(c_t - a_2\left(\frac{1}{1-\lambda_m/\gamma} m_t - \frac{\lambda_m/\gamma}{1-\lambda_m/\gamma} m_{t-1}\right) - a_2 \varepsilon_t^\chi\right). \quad (11)$$

Regarding the central-bank behavior, systematic monetary policy actions are governed by a Taylor (1983)-type rule extended with a stabilizing response to changes in the growth of nominal money, $\mu_t = \log M_t - \log M_{t-1}$. In addition, there is a smoothing component, $0 < \rho < 1$, that brings a partial adjustment between the previous nominal interest rate and the Taylor-style targeting as follows:[8]

$$R_t = \rho R_{t-1} + (1-\rho)[r_\pi \pi_t + r_y(y_t - y_t^p) + r_\mu \mu_t] + \varepsilon_t^R, \quad (12)$$

where y_t and y_t^p denote output and potential output, respectively; and ε_t^R is a random disturbance following an AR(1) process with persistence parameter denoted by ρ_R and the standard deviation of the innovations of the AR(1) process denoted by σ_R.[9]

3   Estimation results

The DSGE model with money has been estimated for two sub-samples of quarterly U.S. data, 1968:1 to 1994:4 and 1995:1 to 2011:3, taking the first quarter of 1995 as the starting period of the

---

[8] We have experimented with additional formulations for the policy rule. The qualitative results of the paper are robust to all policy specifications considered. The rationale for this empirical finding is simple: equation (12) can be viewed as an observational equivalent reduced-form of a wide range of alternative monetary rules where either the nominal interest rate or the nominal money growth is the instrument monitored by the central bank. Of course, the interpretation of the monetary policy shock changes accordingly.

[9] The Appendix collects all the dynamic equations of the extended DSGE model and the set of structural parameters.

great moderation of inflation with low average rates and little variability.[10] As a monetary model, one of the observable series is the log difference of the monetary aggregate called "Money with zero maturity" (MZM) calculated by the Federal Reserve Bank of St. Louis.[11] The rest of the variables corresponds to the list used in Smets and Wouters (2007): the inflation rate, the Federal funds rate, the log of hours worked and the log differences of the real GDP, real consumption, real investment and the real wage.

As has become rather standard in the literature, our DSGE model is estimated using a two-step Bayesian procedure. In the first step, the log posterior function is maximized in a way that combines the prior information of the parameters with the empirical likelihood of the data. In a second step, we perform the Metropolis-Hastings algorithm to compute the posterior distribution of the parameter set.[12]

In terms of the priors, we select the same prior distributions as Smets and Wouters (2007) for the estimation of the model (see the first three columns in Tables 1A and 1B), and we also borrow their notation for the structural parameters. In our extended model to incorporate money, we have a few additional parameters. The prior distribution of money habit parameter, λ_m, is identical to the one associated with consumption habit, λ, with the exception of the standard deviation, which is twice larger for λ_m reflecting our rather diffuse prior knowledge of this parameter. The prior distribution of the elasticity parameter in the transaction costs technology, a_2, is described by a beta distribution with mean 0.5 and standard deviation 0.2. The prior distribution of the policy parameter r_μ is a normal distribution with mean 0.5 and standard deviation 0.2. Finally, the prior distributions of the two parameters describing the money demand shock, ρ_χ and σ_χ are identical to the corresponding parameters describing the other shocks of the

---

[10] The choice of the sample split can be found by simple inspection of the U.S. inflation time series plot. Interestingly, a more sophisticated method suggested in CSS (2012), based on a VAR with drifting coefficients and stochastic volatility, points out 1995 as the year when the Gibson paradox re-emerged and inflation persistence fell.

[11] As discussed in McCallum and Nelson (2010) and Altig, Christiano, Eichenbaum and Linde (2011), there are hybrid definitions of money such as MZM or Divisia more adequate for providing a representation of the medium-of-exchange role of money than more conventional aggregates such as the Monetary Base, M1 or M2.

[12] All estimation exercises are performed with DYNARE free routine software, which can be downloaded from http://www.dynare.org. A sample of 250,000 draws was used (ignoring the first 20% of draws). A step size of 0.30 resulted in an average acceptation rate of roughly 31% (27%) in the estimation procedure of the first (second) sub-sample period.

model. In addition to

Table 1A. Priors and estimated posteriors of the structural parameters

		Priors			Posteriors					
					Pre-1995 model			Post-1995 model		
					Log density $= -954.56$			Log density $= -481.65$		
	Distr	Mean	Std D.		Mean	5%	95%	Mean	5%	95%
φ	Normal	4.00	1.50		6.54	4.73	8.37	7.38	5.50	9.34
λ	Beta	0.70	0.10		0.73*	0.66	0.81	0.82*	0.74	0.90
λ_m	Beta	0.70	0.20		0.52	0.39	0.64	0.46	0.28	0.64
σ_c	Normal	1.50	0.37		1.75*	1.41	2.09	1.30*	0.98	1.61
σ_l	Normal	2.00	0.75		1.78	0.66	2.89	0.86	-0.15	1.90
a_2	Beta	0.50	0.20		0.77	0.70	0.85	0.75	0.68	0.82
ξ_p	Beta	0.50	0.10		0.62*	0.51	0.72	0.75*	0.66	0.84
ξ_w	Beta	0.50	0.10		0.73	0.62	0.84	0.74	0.64	0.83
ι_p	Beta	0.50	0.15		0.34	0.14	0.53	0.28	0.11	0.45
ι_w	Beta	0.50	0.15		0.61	0.43	0.82	0.44	0.19	0.66
ψ	Beta	0.50	0.15		0.37*	0.19	0.56	0.69*	0.52	0.86
Φ	Normal	1.25	0.12		1.57*	1.44	1.70	1.41*	1.27	1.55
ρ	Beta	0.75	0.10		0.66*	0.57	0.75	0.79*	0.69	0.89
r_π	Normal	1.50	0.25		1.87	1.58	2.16	1.57*	1.19	1.95
r_y	Normal	0.12	0.05		0.16	0.09	0.22	0.09*	0.01	0.17
r_μ	Normal	0.5	0.20		0.41	0.28	0.54	0.41	0.16	0.66
π^{ss}	Gamma	0.62	0.50		1.10**	0.81	1.40	0.53**	0.30	0.76
$100(\beta^{-1}-1)$	Gamma	0.25	0.10		0.18	0.08	0.29	0.15	0.06	0.24
α	Normal	0.30	0.05		0.20*	0.17	0.24	0.15*	0.11	0.18

Notes to Table 1: Table A in the appendix shows the definition for each estimated parameter. A double asterisk, **, means that the two confidence intervals associated with a particular parameter do not overlap across sub-samples. Meanwhile, a single asterisk, *, means that the estimated parameter does not lie inside the confidence interval obtained from the other sub-sample.

Table 1B. Priors and estimated posteriors of the shock processes

| | Priors | | | Posteriors | | | | | |
| | | | | Pre-1995 model | | | Post-1995 model | | |
	Distr	Mean	Std D.	Mean	5%	95%	Mean	5%	95%
σ_a	Invgamma	0.10	2.00	0.46	0.41	0.51	0.48	0.41	0.56
σ_b	Invgamma	0.10	2.00	3.93	3.07	4.81	5.22*	3.35	7.06
σ_i	Invgamma	0.10	2.00	0.59**	0.46	0.70	0.32**	0.24	0.41
σ_g	Invgamma	0.10	2.00	0.54**	0.47	0.60	0.38**	0.32	0.43
σ_p	Invgamma	0.10	2.00	0.15*	0.12	0.19	0.11*	0.08	0.14
σ_w	Invgamma	0.10	2.00	0.18**	0.13	0.22	0.40**	0.33	0.48
σ_R	Invgamma	0.10	2.00	0.43**	0.34	0.52	0.18**	0.10	0.26
σ_χ	Invgamma	0.10	2.00	5.48*	3.60	7.20	2.70*	1.74	3.75
ρ_a	Beta	0.50	0.20	0.92	0.88	0.96	0.94	0.89	0.99
ρ_b	Beta	0.50	0.20	0.14**	0.03	0.24	0.70**	0.55	0.85
ρ_i	Beta	0.50	0.20	0.64**	0.54	0.76	0.88**	0.81	0.96
ρ_g	Beta	0.50	0.20	0.90	0.81	0.98	0.87	0.78	0.96
ρ_p	Beta	0.50	0.20	0.88	0.79	0.98	0.81	0.64	0.98
ρ_w	Beta	0.50	0.20	0.87**	0.78	0.97	0.44**	0.18	0.71
ρ_R	Beta	0.50	0.20	0.27**	0.14	0.40	0.60**	0.49	0.73
ρ_χ	Beta	0.50	0.20	0.93	0.87	0.99	0.96	0.92	0.99
μ_p	Beta	0.50	0.20	0.68	0.48	0.88	0.61	0.36	0.86
μ_w	Beta	0.50	0.20	0.59	0.37	0.82	0.47	0.22	0.72
ρ_{ga}	Beta	0.50	0.20	0.58*	0.40	0.75	0.41	0.25	0.57

the parameters fixed in Smets and Wouters (2007), the steady-state growth parameter, γ, was also fixed to the estimated value reported by them. The reason is that our empirical strategy splits up the period into two subsamples, which makes it harder to identify parameters characterizing long-run dynamics. Moreover, the scale parameter of transaction-cost technology, a_1, is calibrated to match the steady-state money velocity with the average ratio of nominal GDP/MZM over the whole sample period (1968-2011). The fixed transaction cost, a_0, is also predetermined at the value

that implies that the ratio of total transaction costs over consumption is equal to 0.01 in steady state.

Table 1 shows the estimation results by reporting the posterior mean estimates together with the 5% and 95% quantiles of the posterior distribution for the two sub-samples studied. Many parameter estimates look rather stable across sub-samples. In particular, indexation parameters are rather similar in the two sub-samples whereas policy parameters ρ and r_π have only changed marginally. However, there are several noticeable differences. Thus, the consumption habit formation parameter, λ, and the price rigidity probability parameter, ξ_p, are higher in the after-1995 subsample. Meanwhile, the steady-state rate of inflation, π^{ss}, is much lower. Moreover, many of the estimates of shock processes change significantly across sub-samples. Thus, the persistence of preference, investment, and monetary policy shocks (ρ_b, ρ_i, and ρ_R) has increased over the whole sample period, whereas the opposite occurs for the persistence of wage indexation shocks (ρ_w). Furthermore, the standard deviation of the innovations associated with most shocks have changed across sub-samples. Those corresponding to money demand, monetary policy, investment, government spending and price indexation shocks are higher in the first sub-sample than in the second whereas the opposite is true for consumption preference and wage indexation shocks.

4   Model performance

As described in the Introduction, business cycle dynamics have shifted in many ways during the great moderation of inflation period. What of these changes can be captured by the estimated DSGE model extended with money? Table 2 reports changes in the standard deviations of key aggregate variables obtained from both actual U.S. data and synthetic data. The model does a great job in matching the standard deviations of inflation, nominal interest rate and nominal money growth in each period, which implies that the model explains very well the fall of volatility observed in the three nominal variables. Moreover, the model captures the lower volatility of output growth since 1995, though it generates larger output growth volatility than what it is observed in actual data.

By using the vector autocorrelation function method, Figures 3 and 4 show a comprehensive analysis of model's performance based on dynamic auto-correlations and cross-correlations obtained

from actual and synthetic data across subsamples. A comparison of these two figures shows that in general the model fits well the two subsamples. More precisely, the model does a very good job in matching the shape of the serial correlation of output and nominal money growth. However, it falls short to characterize the persistence of inflation and the nominal interest rate in the pre-1995 period. The model also performs well in capturing the comovement of inflation with the nominal interest rate, nominal money growth and output growth. In particular, the model does a good job in explaining the fall of the contemporaneous correlation between inflation and the nominal interest rate, which results in a weak correlation between the two variables (i.e. Gibson paradox) during the great moderation of inflation. Moreover, the model does fairly well when describing the low correlations of nominal money growth with the other three variables in the pre-1995 period. However, the model fails to capture the positive, although weak, correlation of the nominal interest rate with both output and money growth rates since 1995.

Table 2. The moderation of macroeconomic volatility.

	Pre-1995		Post-1995	
	US data	Model	US data	Model
Standard deviation, %				
$\sigma(\pi_t)$	0.61	0.65	0.23	0.38
		$(0.51, 0.76)$		$(0.26, 0.47)$
$\sigma(R_t)$	0.82	0.74	0.55	0.40
		$(0.62, 0.84)$		$(0.31, 0.46)$
$\sigma(\mu_t)$	2.17	2.30	1.41	1.55
		$(1.97, 2.56)$		$(1.32, 1.80)$
$\sigma(\Delta y_t)$	0.95	1.14	0.72	0.96
		$(1.02, 1.23)$		$(0.84, 1.06)$

Note: the posterior 5%-95% confidence interval for each estimated second-moment statistic is reported on parenthesis.
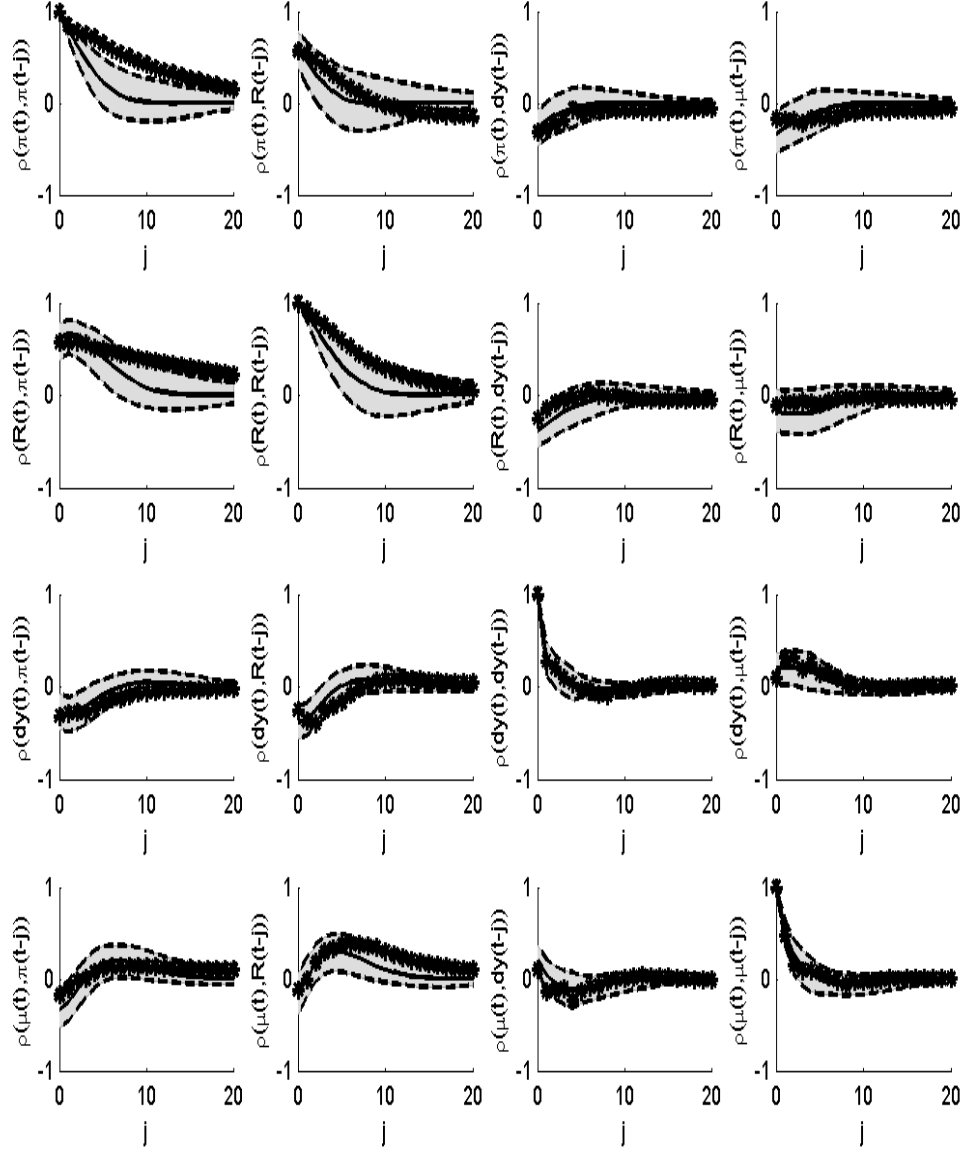
Figure 3: Dynamic correlations (1968-1994). US data (*) and estimated model (solid lines with shaded area showing the 5%-95% confidence interval).
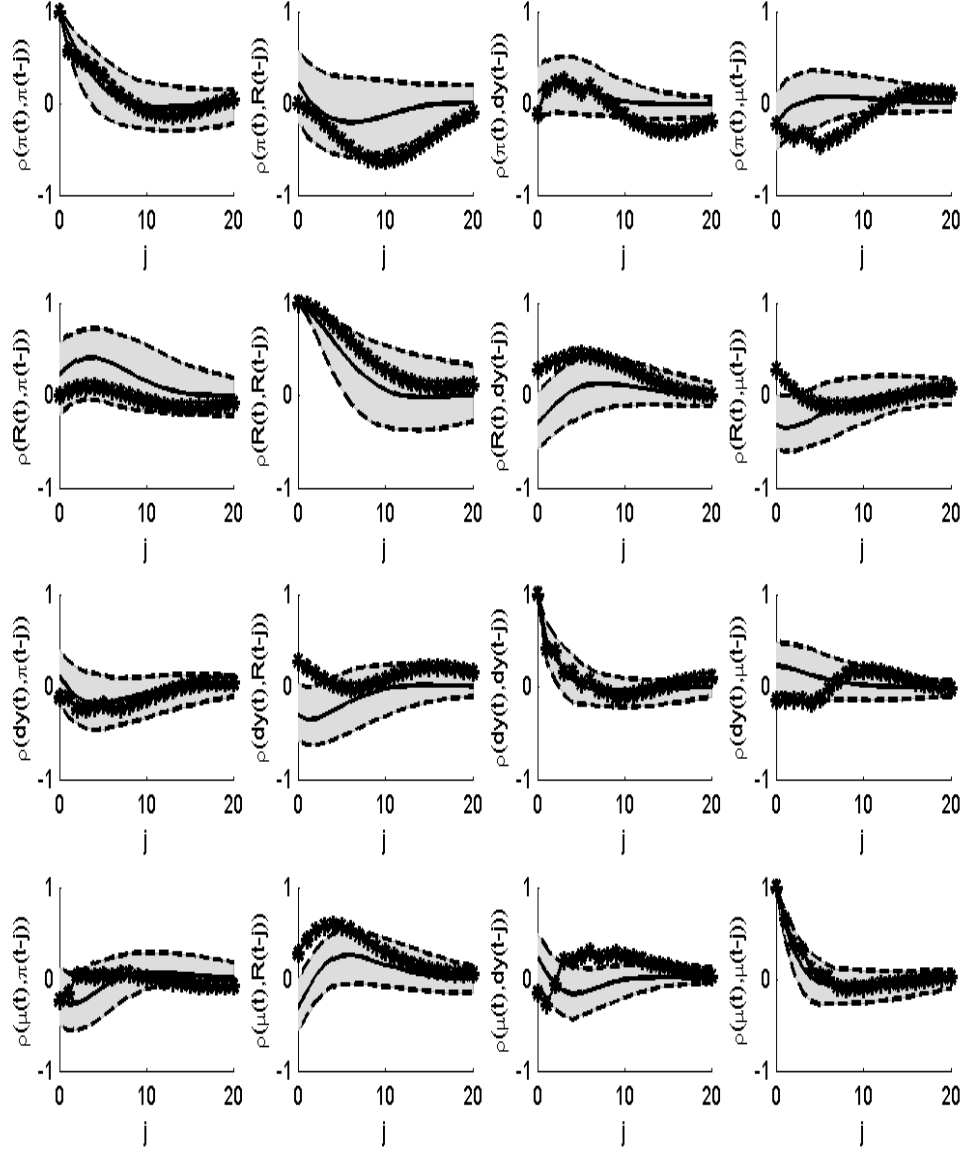
Figure 4: Dynamic correlations (1995-2011). US data (*) and estimated model (solid lines with shaded area showing the 5%-95% confidence interval).

5 What did it change from 1995 onwards?

5.1 Inflation dynamics

Price inflation dynamics are governed in the model by the New Keynesian Phillips Curve (NKPC):

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} + \pi_3 mc_t + \varepsilon_t^p,$$

where $\pi_1 = \frac{\iota_p}{1 + \overline{\beta} \iota_p}$, $\pi_2 = \frac{\overline{\beta}}{1 + \overline{\beta} \iota_p}$, and $\pi_3 = \frac{1}{1 + \overline{\beta} \iota_p} \left[\frac{\left(1 - \overline{\beta} \xi_p\right)\left(1 - \xi_p\right)}{\xi_p \left(\left(\phi_p - 1\right) \varepsilon_p + 1\right)} \right]$ and mc_t denotes the log deviation of the real marginal cost with respect to the steady-state level. The structural analysis of inflation dynamics can be illustrated by examining the NKPC across both sub-samples. Hence, the estimates of the structural parameters give the following NKPC over the first sub-sample (1968:1-1994:4)

$$\pi_t = 0.25 \pi_{t-1} + 0.74 E_t \pi_{t+1} + 0.0270 mc_t + \varepsilon_t^p,$$

$$\varepsilon_t^p = 0.88 \varepsilon_{t-1}^p - 0.68 \eta_{t-1}^p + \eta_t^p, \quad std(\eta^p) = 0.15\%,$$

whereas for the second sub-sample (1995:1-2011:4) the estimated NKPC is

$$\pi_t = 0.22 \pi_{t-1} + 0.78 E_t \pi_{t+1} + 0.0125 mc_t + \varepsilon_t^p,$$

$$\varepsilon_t^p = 0.81 \varepsilon_{t-1}^p - 0.61 \eta_{t-1}^p + \eta_t^p, \quad std(\eta^p) = 0.11\%.$$

The backward-looking component of inflation is slightly lower after 1995, which indicates that the endogenous inflation inertia has diminished a bit after 1995. This result is based on the decrease of nominal inertia described by the price indexation parameter (i.e., ι_p falls from 0.34 to 0.28 as reported in Table 1). Remarkably, the backward-looking dynamics of price inflation are much weaker in our estimated NKPC than in the estimation of CSS (2012).

The estimate of the slope coefficient falls substantially in the second period, as it comes down to less than half of the value found in the first sub-sample. This result is consistent with the increase in the estimate of the Calvo sticky-price probability. As shown in Table 1, $\xi_p = 0.62$ in the period before 1995, and $\xi_p = 0.75$ in the period after 1995. So, the average number of months without optimal pricing increases from 7.9 months to 12 months. As argued by Smets and Wouters (2007), the price stability period of the Great Moderation (of real variables) may explain the increase in price stickiness associated with lower menu costs.

As for the exogenous variability of inflation, the comparison of the estimated NKPCs shows that price mark-up shocks are less volatile and less persistent in the second sub-sample. The

autoregressive coefficient falls by around 8% while the standard deviation of the innovations is 27% lower.

Summarizing, the decline in inflation volatility after 1995 can be explained by two factors: i) stickier prices turn inflation less sensitive to real marginal cost fluctuations and ii) firms receive lower and, more important, less persistent mark-up pricing shocks.[13]

5.2 Money market

Money demand

The extension of the DSGE model with transactions-facilitating model can shed some light on the possible influence of variations of money demand behavior to explaining the changes of inflation and interest rate dynamics. Making the first difference on the money demand equation (3) gives

$$\mu_t - \pi_t = (\lambda_m/\gamma)\left(\mu_{t-1} - \pi_{t-1}\right) + (1 - \lambda_m/\gamma)\,\Delta c_t - \frac{(1-\lambda_m/\gamma)(1-a_2)}{R^{ss}}\Delta R_t - a_2\left(1 - \lambda_m/\gamma\right)\Delta\varepsilon_t^{\chi}.$$

Using the estimates reported in Table 1A, the pre-1995 subsample is characterized by the following money demand behavior

$$\begin{aligned}
\mu_t - \pi_t &= 0.52\left(\mu_{t-1} - \pi_{t-1}\right) + 0.48\Delta c_t - 8.63\Delta R_t - 0.37\Delta\varepsilon_t^{\chi},\\
\varepsilon_t^{\chi} &= 0.93\varepsilon_{t-1}^{\chi} + \eta_t^{\chi},\quad std(\eta^{\chi}) = 5.48\%,
\end{aligned}$$

whereas in the post-1995 subsample

$$\begin{aligned}
\mu_t - \pi_t &= 0.46\left(\mu_{t-1} - \pi_{t-1}\right) + 0.54\Delta c_t - 19.85\Delta R_t - 0.41\Delta\varepsilon_t^{\chi},\\
\varepsilon_t^{\chi} &= 0.96\varepsilon_{t-1}^{\chi} + \eta_t^{\chi},\quad std(\eta^{\chi}) = 2.70\%.
\end{aligned}$$

The semi-elasticity of real money with respect to the nominal interest rate increases dramatically in the second period due to the large fall of (inflation and) the nominal interest rate in the steady state. A highly elastic money demand implies that nominal interest rates are quite insensitive to changes in either consumption, inflation, nominal money growth or exogenous perturbations. For example, after a cost-push shock that raises inflation, the required increase in the nominal interest to adjust down real money demand would be quantitatively much smaller. Hence, the increase in

---

[13]The qualification "more important", associated with less persistent mark-up shocks, made in this sentence is rather relevant because intertemporal rational agents decisions are severely affected by persistent shocks.

interest-rate semi-elasticity of money demand can serve to explain both the low volatility of the nominal interest rate and the reduction in its cyclical correlation with inflation, observed after 1995.

Money supply (monetary policy)

Regarding money supply behavior, the Taylor-style rule (12) incorporates responses of the nominal interest rate to nominal money growth as part of a stabilizing systematic monetary policy. Before 1995, the estimated monetary policy rule is

$$R_t = 0.66R_{t-1} + 0.64\pi_t + 0.05(y_t - y_t^p) + 0.14\mu_t + \varepsilon_t^R,$$

$$\varepsilon_t^R = 0.27\varepsilon_{t-1}^R + \eta_t^R, \ \ std(\eta^R) = 0.43\%.$$

Meanwhile, in the second sub-sample that begins in 1995 the estimated monetary policy is

$$R_t = 0.79R_{t-1} + 0.33\pi_t + 0.02(y_t - y_t^p) + 0.09\mu_t + \varepsilon_t^R,$$

$$\varepsilon_t^R = 0.60\varepsilon_{t-1}^R + \eta_t^R, \ \ std(\eta^R) = 0.18\%.$$

The comparison highlights some relevant changes. First, nominal interest rates adjust more gradually during the period of the great inflation moderation, as the smoothing (inertia) coefficient increases from 0.66 to 0.79. Second, the response coefficients to inflation, the output gap, and money growth are lower after 1995, which might somewhat reflect a sense of a loose central-bank policy during the period of inflation moderation. In other words, monetary policy becomes more discretional and less rule-oriented, as the systematic behavior weakens.[14] By contrast, CSS (2012) find a more anti-inflationary monetary-policy rule as one of the factors behind the return of the Gibson paradox. These different results might be explained by the different sample periods used in the two papers.[15]

5.3 Real wage dynamics

Wage setting behavior and nominal rigidities *à la* Calvo (1983) lead to the following expression for the dynamic evolution of real wages

$$w_t = w_1 w_{t-1} + (1 - w_1)(E_t w_{t+1} + E_t \pi_{t+1}) - w_2 \pi_t + w_3 \pi_{t-1} - w_4(w_t - mrs_t) + \varepsilon_t^w,$$

[14]Contributing to this line of argument, Taylor (2012) claims that, from 2003, Fed's monetary policy deviated significantly from a Taylor (1983)-type rule to become quite discretional.

[15]In particular, our paper considers observations after 2007, ignored by CSS (2012), which capture the rather loose monetary policy implemented by the Fed since the subprime mortgage crisis.

where $w_1 = \frac{1}{1+\overline{\beta}}$, $w_2 = \frac{1+\overline{\beta}\iota_w}{1+\overline{\beta}}$, $w_3 = \frac{\iota_w}{1+\overline{\beta}}$, and $w_4 = \frac{1}{1+\overline{\beta}}\left[\frac{\left(1-\overline{\beta}\xi_w\right)\left(1-\xi_w\right)}{\xi_w\left(\left(\phi_w-1\right)\varepsilon_w+1\right)}\right]$. In the estimation, the curvature of the Kimball labor aggregator is fixed at $\varepsilon_w = 10.0$ and the steady-state wage mark-up is $\phi_w = 1.5$, following Smets and Wouters (2007). The wage mark-up, $w_t - mrs_t$, measured as the log difference between the real wage and the marginal rate of substitution between working and consuming is the key determinant of real wage fluctuations. The estimates before 1995 imply

$$w_t = 0.50w_{t-1} + 0.50\left(E_t w_{t+1} + E_t \pi_{t+1}\right) - 0.80\pi_t + 0.30\pi_{t-1} - 0.0094\left(w_t - mrs_t\right) + \varepsilon_t^w,$$

$$\varepsilon_t^w = 0.87\varepsilon_{t-1}^w - 0.59\eta_{t-1}^w + \eta_t^w, \quad std(\eta^w) = 0.18\%.$$

Meanwhile, in the sample after 1995 the estimated real wage equation is

$$w_t = 0.50w_{t-1} + 0.50\left(E_t w_{t+1} + E_t \pi_{t+1}\right) - 0.72\pi_t + 0.22\pi_{t-1} - 0.0079\left(w_t - mrs_t\right) + \varepsilon_t^w,$$

$$\varepsilon_t^w = 0.44\varepsilon_{t-1}^w - 0.47\eta_{t-1}^w + \eta_t^w, \quad std(\eta^w) = 0.40\%.$$

The structural components of real wage dynamics (backward/forward looking coefficients, slope coefficient) show slight shifts after 1995. The estimates of wage-stickiness, ξ_w, barely change across samples, while wage indexation on lagged inflation, ι_w, falls in the second subsample (see Table 2 for the numbers). Nevertheless, significant differences are observed in the exogenous process that collects wage mark-up shocks. The coefficient of autocorrelation falls from 0.87 to 0.44. The moving-average coefficient is also lower, while the innovations have a higher standard deviation after 1995. The sizable reduction of persistence in wage mark-up shocks has dramatic effects on the sources of business cycle variability after 1995, as documented next.

5.4   Sources of variability

Both the impulse-response functions and the variance decomposition provide information about how the exogenous sources of variability shape the business cycle fluctuations. Figures 5 and 6 display the responses of output, inflation and the nominal interest rate to the eight exogenous shocks in the pre-1995 and post-1995 periods, respectively. The size of the shocks has been normalized at the estimated standard deviation of their innovations.

The monetary policy shock to the extended Taylor (1993)-type rule (12) drives a negative comovement between inflation and the nominal interest rate. A positive ε_t^R raises the nominal interest rate that increases productivity and cuts real marginal costs through the demand-side
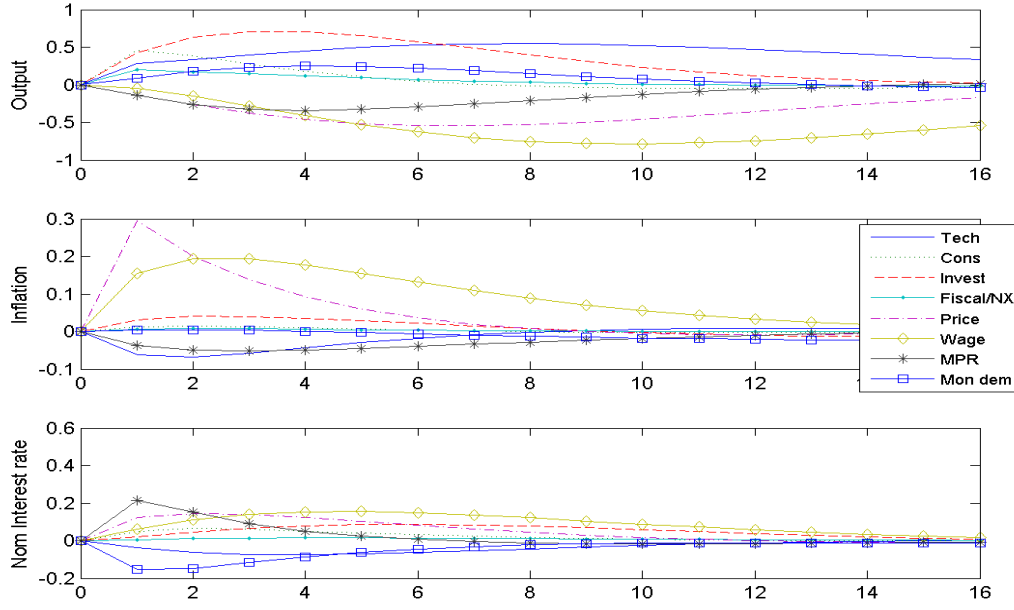
Figure 5: Impulse-response functions. Pre-1995 subsample.

contraction. In turn, the rate of inflation falls at higher nominal interest rates. By contrast, all the other seven shocks display a positive comovement between inflation and the nominal interest rates (see Figures 5 and 6). In the pre-1995 subsample, the negative comovement induced by monetary policy shocks is weak. As a result, cyclical fluctuations of inflation and the nominal interest rate are dominated by the remaining shocks and give a moderately high coefficient of correlation (higher than 0.5 as shown in Figures 2 and 3). In particular, supply-side shocks on both price mark-up and wage mark-up are very influential on inflation and output. After 1995, the impulse-response functions show that the effects of wage mark-up shocks are clearly mitigated (compare diamond-marked lines in Figures 5 and 6). The loss of influence of wage mark-up shocks, can explain both the reduction of volatility on interest rates and inflation as well as the presence of the Gibson paradox. By contrast, demand-side shocks (such as those on consumption spending) induce much stronger responses of output and the nominal interest rate during this recent period.

21
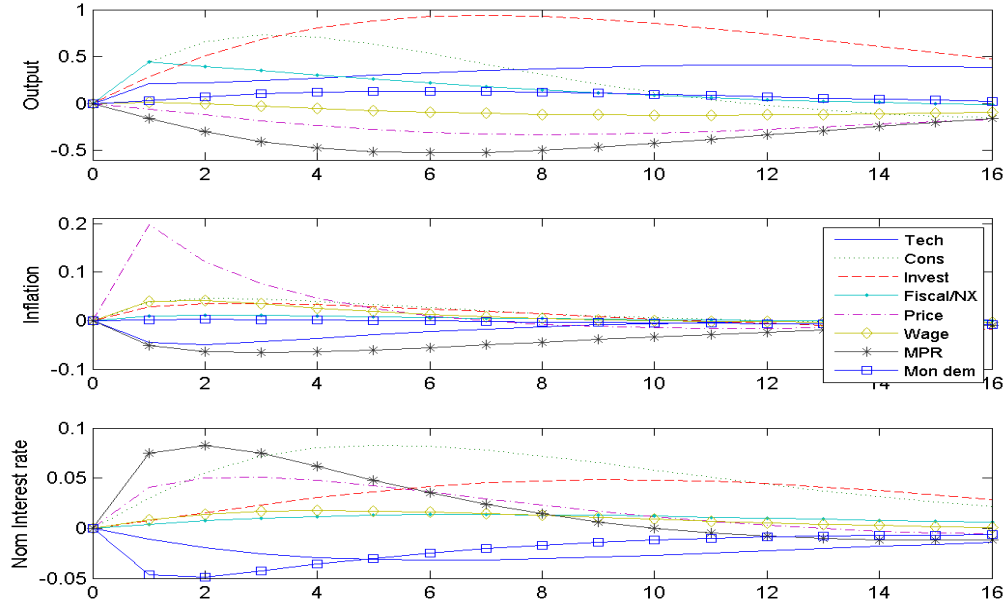
Figure 6: Impulse-response functions. Post-1995 subsample.

Table 3. Variance decomposition, %

Innovations	Pre-1995 model					Post-1995 model				
	y	Δy	R	π	μ	y	Δy	R	π	μ
Technology, η^a	21.3	8.7	6.5	4.4	2.8	20.2	6.9	7.5	6.8	2.4
Consumption pref., η^b	2.4	18.5	3.2	0.2	0.5	11.6	31.3	36.8	9.1	3.6
Investment, η^i	14.8	21.3	11.2	2.9	2.0	42.0	25.4	19.2	9.1	2.7
Fiscal/Net exports, η^g	5.7	29.8	2.7	0.7	0.6	3.0	20.0	1.3	0.4	0.2
Price-push, η^p	12.9	5.7	15.7	36.6	9.4	8.6	3.8	9.0	46.5	8.4
Wage-push, η^w	37.3	8.8	31.2	49.2	12.5	1.0	0.6	1.6	5.0	1.2
MP rule, η^R	4.0	5.3	15.4	4.3	40.3	12.9	11.4	17.4	21.8	65.8
Money demand, η^χ	1.7	1.8	14.3	1.8	31.9	0.6	0.6	7.2	1.2	15.7

In the variance decomposition of the estimated model (Table 3), the percentages of monetary policy shocks explaining inflation fluctuations increase in the second sub-sample (from 4% to 22%). As discussed above, monetary policy shocks increases the negative comovement variability, which

help to explaining some part of the Gibson paradox. However, the most significant change in the sources of variability is the dramatic decline in the participation of wage-push shocks. As Table 3 reports, these shocks were responsible for 49.2% of the variability of inflation and 31.2% of the variability of the nominal interest rate before 1995. The percentages fall below 5% from 1995 onwards. In addition, demand-side shocks determine a much greater portion of business cycle fluctuations after 1995 when consumption and investment shocks explain more than 50% of output and the nominal interest rate variability. Hence, the estimated variance decomposition across samples confirms the findings of the impulse-response analysis.

6   Counterfactual experiments

The previous section has discussed the different driving forces explaining the monetary dynamic changes since 1995. In this section, we carry out a large number of counterfactual experiments in order to assess the relative importance of these sources. In all these experiments we consider the model parameter estimates obtained from the post-1995 period as the benchmark parameter values and we recalculate the variance/covariance matrix by considering alternative sets of parameters values obtained from the pre-1995 estimated model. Eighteen counterfactual experiments were conducted. The following are the parameters changing in each of them: (i) r_π, r_y, r_μ and ρ, which describe the systematic part of monetary policy, (ii) monetary policy shock parameters (ρ_R, σ_R), (iii) monetary policy rule parameters (r_π, r_y, r_μ, ρ, ρ_R, σ_R), (iv) money demand technology parameters (λ_m, a_2, R^{ss}), (v) money demand shock parameters (ρ_χ, σ_χ), (vi) money demand parameters (λ_m, a_2, R^{ss}, ρ_χ, σ_χ), (vii) price setting parameters (ξ_p, ι_p, $\overline{\beta}$), (viii) price mark-up shock parameters (σ_p, ρ_p, μ_p), (ix) new Keynesian Phillips curve parameters (ξ_p, ι_p, $\overline{\beta}$, σ_p, ρ_p, μ_p), (x) wage setting parameters (ξ_w, ι_w, $\overline{\beta}$), (xi) wage mark-up shock parameters (σ_w, ρ_w, μ_w), (xii) real wage dynamic parameters (ξ_w, ι_w, $\overline{\beta}$, σ_w, ρ_w, μ_w), (xiii) consumption preference parameters (λ, σ_c), (xiv) consumption preference shock parameters (ρ_b, σ_b), (xv) consumption dynamic parameters (λ, σ_c, ρ_b, σ_b), (xvi) investment technology parameters (φ, ψ), (xvii) investment shock parameters (ρ_i, σ_i), and (xviii) investment dynamic parameters (φ, ψ, ρ_i, σ_i).

Table 4. Change in second-moment statistics, Post-95 minus Pre-95

	US data	Baseline model	MP Rule [i,ii,iii]'	Money demand [iv,v,vi]'	Prices [vii,viii,ix]'	Wages [x,xi,xii]'
Standard deviations:						
			-0.32	-0.27	-0.20	-0.26
$\sigma(\pi_t)$	-0.38	-0.27	-0.25	-0.27	$\mathbf{-0.12}$	$\mathbf{-0.13}$
			-0.32	-0.27	$\mathbf{-0.13}$	$\mathbf{-0.12}$
			-0.29	-0.25	-0.32	-0.33
$\sigma(R_t)$	-0.27	-0.34	-0.26	-0.30	-0.29	-0.27
			-0.25	$\mathbf{-0.19}$	-0.30	-0.26
			-0.83	-1.03	-0.77	-0.77
$\sigma(\mu_t)$	-0.76	-0.75	$\mathbf{+0.49}$	-0.45	-0.62	-0.62
			$\mathbf{-0.01}$	-0.51	-0.70	-0.64
Autocorrelations:						
			-0.10	-0.05	$+0.00$	-0.04
$\rho(\pi_t, \pi_{t-1})$	-0.34	-0.05	-0.03	-0.04	-0.07	$\mathbf{+0.03}$
			-0.10	-0.04	-0.05	$\mathbf{+0.03}$
			$+0.07$	$+0.06$	$+0.06$	$+0.06$
$\rho(R_t, R_{t-1})$	$+0.05$	$+0.07$	-0.01	$+0.05$	$+0.06$	$+0.07$
			$+0.05$	$+0.05$	$+0.06$	$+0.07$
			$+0.16$	$+0.08$	$+0.09$	$+0.09$
$\rho(\mu_t, \mu_{t-1})$	$+0.17$	$+0.10$	$\mathbf{-0.12}$	$\mathbf{-0.01}$	$+0.13$	$+0.14$
			$\mathbf{-0.10}$	$\mathbf{-0.05}$	$+0.11$	$+0.14$
Correlations:						
			-0.24	-0.33	-0.36	-0.39
$\rho(\pi_t, R_t)$	-0.45	-0.39	-0.59	-0.40	-0.25	$\mathbf{-0.18}$
			-0.32	-0.32	-0.31	$\mathbf{-0.18}$
			$+0.27$	$+0.35$	$+0.44$	$+0.38$
$\rho(\pi_t, \Delta y_t)$	$+0.25$	$+0.34$	$+0.41$	$+0.39$	$+0.20$	$+0.21$
			$+0.28$	$+0.35$	$+0.35$	$+0.21$

Table 4 shows the changes in some selected second-moment statistics between sub-samples for actual data, model estimates and each of the counterfactual experiments.[16] Bold characters highlight the main sources for these second-moment statistics changes. Counterfactual experiment results show that the fall of volatility associated with each nominal variable is due to a different source. Thus, the fall of inflation volatility, $\sigma(\pi_t)$, is mainly explained by changes in both the parameters describing wage mark-up shocks (σ_w, ρ_w, μ_w) and price mark-up shocks (σ_p, ρ_p, μ_p). Meanwhile, the milder fluctuations of the nominal interest rate and the nominal money growth observed in the most recent period are due to changes in money demand parameters (λ_m, a_2, R^{ss}, ρ_χ, σ_χ) and changes in monetary policy shock parameters (ρ_R, σ_R), respectively.

The fall of inflation first order autocorrelation is not quantitatively replicated by the model, but only qualitatively. Experiment (xi) suggests that the changes in the wage mark-up shock parameters are partially responsible for the fall of inflation persistence. Moreover, the changes in monetary policy shock parameters and money demand parameters help to explain the small increase in autocorrelation of nominal money growth.

Finally, the changes in the wage mark-up shock parameters largely explain the Gibson paradox besides the fall of both inflation volatility and inflation persistence. These results are in contrast with the results found by CSS (2012) in a small-scale DSGE models. Our results suggest that the fall of inflation persistence and the re-emergence of the Gibson paradox share common sources as in CSS (2012). However, the sources are different in the two papers. CSS (2012) suggest that a more anti-inflationary policy rule and a decline of price indexation are the two sources for the re-emergence of the Gibson paradox and the fall of inflation persistence. Our counterfactual experiments show that, by considering a medium-scale DSGE model that incorporates an imperfect-monopolistic labor market together with a more detailed description of consumption and investment demands, the Gibson paradox, the fall of inflation persistence and many of the changes observed since 1995 are mostly explained by the low persistence exhibited by wage markup shocks in the last fifteen years. These findings are consistent with the variance decomposition analysis described above.

---

[16]The latest six experiments involving parameters describing consumption and investment demands do not imply significant changes in second-moments statistics. For the sake of brevity, these experiment results are excluded from Table 4. They are available from the authors upon request.

7 Conclusions

This paper builds a full-fledged estimated DSGE model with money to study the driving forces of the recent U.S. inflation moderation period and other stylized facts, such as the fall of inflation persistence and the weakening of inflation and nominal interest rate correlation. Compared to standard DSGE models, the model incorporates a transaction-facilitating demand for money, a real-balance effect on consumption, transaction costs as a small percentage of the overall household spending, and a Taylor-type monetary policy rule that reacts to changes in nominal money growth.

Our findings suggest a combination of lower exogenous variability and, to a lower extent, structural changes in money demand, monetary policy and firms' pricing behavior as the main driving forces of the changes observed in the U.S. business cycles since 1995. Regarding the exogenous variability, the supply-side shocks on both prices and wages, that were dominant in the cyclical variability in the 70's and 80's, loose much of their significance after 1995. Their explanatory power for business cycle fluctuations has been mostly replaced in the recent period by either investment spending or interest-rate shocks. The structural analysis of the money market shows that the estimated interest-rate elasticity of money demand more than doubles in the recent period the value estimated for the earlier sample period. The lower average rates of return explains this higher responsiveness of money demand to changes in the nominal interest rates. Other non-modeled factors could be the greater accessibility of households to a variety of money-like assets, and the financial innovation of the period. We have also found a swing in monetary policy towards a more conservative and gradual strategy after 1995. The Fed's response coefficients to inflation, the output gap and money growth have been considerably lower after 1995 than what they had been before. Finally, the estimates of private sector decision making show little differences across periods. The most remarkable one is that the level of price stickiness rises in the post-95 sample period. It helps to characterize the observed lower inflation volatility as firms moderate the reaction of prices to changes in the marginal costs. This factor and the substantial decline of persistence in wage-push shocks are key elements explaining US business cycles in the last two decades.

References

Altig, David, Lawrence J. Christiano, Martin Eichenbaum, and Jesper Linde. 2011. "Firm-Specific Capital, Nominal Rigidities and the Business Cycle". *Review of Economic Dynamics* 14, 225-247.

Barsky Robert B. 1987. "The Fisher Hypothesis and the Forecastability and Persistence of Inflation". *Journal of Monetary Economics* 19, 3-24.

Barsky Robert B., and Lawrence H. Summers. 1988. "Gibson's Paradox and the Gold Standard". *Journal of Political Economy* 96, 528-550.

Calvo, Guillermo A. 1983. "Staggered Pricing in a Utility-Maximizing Framework." *Journal of Monetary Economics* 12, 383-396.

Casares, Miguel. 2007. "The New Keynesian Model and the Euro Area Business Cycle." *Oxford Bulletin of Economics and Statistics* 69, 209-244.

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans. 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." *Journal of Political Economy* 113, 1-45.

Cogley, Timothy, Thomas J. Sargent, and Paolo Surico. 2012. "The Return of the Gibson Paradox." working paper.

Friedman, Milton, and Anna J. Schwartz. 1982. *Monetary Trends in the United States and the United Kingdom: Their Relation to Income, Prices and Interest Rates, 1867-1975.* University of Chicago Press: Chicago, IL.

Fuhrer, Jeff and George Moore. 1995. "Inflation Persistence." *Quarterly Journal of Economics* 440, 127-159.

Hamilton, James D. 1994. *Time Series Analysis.* Princeton University Press: Princeton, New Jersey.

Ireland, Peter. 2003. "Endogenous Money or Sticky Prices?" *Journal of Monetary Economics* 50, 1623-1648.

Keynes, John M. 1930. *A Treatise on Money.* Mcmillan: London.

Kimball, Miles S. 1995. "The Quantitative Analytics of the Basic Neomonetarist Model." *Journal of Money, Credit, and Banking*, 27: 1241-1277.

McCallum, Bennett T., and Edward Nelson. 2010. "Money and inflation: Some critical issues." *Handbook of Monetary Economics*, Volume 3, (Chapter 3), pp. 97-153.

Smets, Frank R., and Rafael Wouters. 2007. "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach." *American Economic Review* 97, 586-606.

Taylor, John B. 1993. "Discretion Versus Policy Rules in Practice." *Carnegie-Rochester Conference Series on Public Policy*, 39, 195-214.

Taylor, John B. 2012. "Monetary Policy Rules Work and Discretion Doesn't: A Tale of Two Eras", *Journal of Money Credit and Banking* 44, 1017-1032.

Appendix

I. Set of model parameters:

Table A. Model parameter description

φ	Elasticity of the cost of adjusting capital
λ	Consumption habits
σ_c	Inverse of the elasticity of intertemporal substitution in utility function
σ_l	Inverse of the elasticity of labor supply with respect to the real wage
ξ_p	Calvo probability of price stickiness
ξ_w	Calvo probability of wage stickiness
ι_w	Wage indexation to lagged wage inflation
ι_p	Price indexation to lagged price inflation
ψ	Elasticity of capital utilization adjustment cost
Φ	One plus steady-state fixed cost to total cost ratio (price mark-up)
ρ	Smoothing coefficient in monetary policy rule
r_π	Inflation coefficient in monetary policy rule
r_Y	Output gap coefficient in monetary policy rule
r_μ	Money-growth coefficient in monetary policy rule
π	Steady-state rate of inflation
$100(\beta^{-1}-1)$	Steady-state rate of discount
l	Steady-state labor
$100(\gamma - 1)$	One plus steady-state rate of output growth
α	Capital share in production function
λ_m	Monetary habits
a_0	Fixed transaction costs
a_1	Scale parameter of variable transaction costs
a_2	Elasticity parameter of transaction costs function

Table A. (*Continued*)

σ_a	Standard deviation of productivity innovation
σ_b	Standard deviation of risk premium innovation
σ_g	Standard deviation of exogenous spending innovation
σ_i	Standard deviation of investment-specific innovation
σ_R	Standard deviation of monetary policy rule innovation
σ_p	Standard deviation of price mark-up innovation
σ_w	Standard deviation of wage mark-up innovation
σ_χ	Standard deviation of money demand innovation
ρ_a	Autoregressive coefficient of productivity shock
ρ_b	Autoregressive coefficient of risk premium shock
ρ_g	Autoregressive coefficient of exogenous spending shock
ρ_i	Autoregressive coefficient of investment-specific shock
ρ_R	Autoregressive coefficient of policy rule shock
ρ_p	Autoregressive coefficient of price mark-up shock
μ_p	Moving-average coefficient of price mark-up shock
ρ_w	Autoregressive coefficient of wage mark-up shock
μ_w	Moving-average coefficient of wage mark-up shock
ρ_p	Autoregressive coefficient of money demand shock

II. Set of log-linearized dynamic equations:

Real money demand equation:

$$m_t = (\lambda_m/\gamma)\, m_{t-1} + (1 - \lambda_m/\gamma)\, c_t - \frac{(1-\lambda_m/\gamma)(1-a_2)}{R^{ss}} R_t - a_2 \left(1 - \lambda_m/\gamma\right) \varepsilon_t^\chi. \qquad (A1)$$

Transaction costs equation:

$$h_t = \frac{1-(a_0/H)}{1-a_2} \left(c_t - a_2 \left(\frac{1}{1-\lambda_m/\gamma} m_t - \frac{\lambda_m/\gamma}{1-\lambda_m/\gamma} m_{t-1} \right) - a_2 \varepsilon_t^\chi \right). \qquad (A2)$$

Aggregate resource constraint:

$$y_t = c_y c_t + i_y i_t + z_y z_t + h_y h_t + \varepsilon_t^g, \qquad (A3)$$

where $c_y = \frac{C}{Y} = 1 - g_y - i_y$, $i_y = \frac{I}{Y} = (\gamma - 1 + \delta)\frac{K}{Y}$, $z_y = r^k \frac{K}{Y}$, and $h_y = \frac{H}{Y}$ are steady-state ratios.

As in Smets and Wouters (2007), the depreciation rate and the exogenous spending-GDP ratio are fixed in the estimation procedure at $\delta = 0.025$ and $g_y = 0.18$.

Consumption equation:

$$(1 + c_4) c_t = c_1 c_{t-1} + c_2 E_t c_{t+1} + c_3 (l_t - E_t l_{t+1})$$
$$+ c_4 \left(\frac{1}{1-\lambda_m/\gamma} m_t - \frac{\lambda_m/\gamma}{1-\lambda_m/\gamma} m_{t-1} \right) - c_4 \left(\frac{1}{1-\lambda_m/\gamma} E_t m_{t+1} - \frac{\lambda_m/\gamma}{1-\lambda_m/\gamma} m_t \right)$$
$$- c_5 (R_t - E_t \pi_{t+1}) + c_4 (1 - \rho_\chi) \varepsilon_t^\chi + c_5 (1 - \rho_b) \varepsilon_t^b, \quad \text{(A4)}$$

where $c_1 = \frac{\lambda/\gamma}{1+\lambda/\gamma}$, $c_2 = \left(\frac{1}{1+\lambda/\gamma} + c_4 \right)$, $c_3 = \frac{(\sigma_c-1)w^{ss}}{\sigma_c(1+\lambda/\gamma)\phi_w c^{ss}}$, $c_4 = \frac{(1-\lambda/\gamma)H_C a_2}{\sigma_c(1+\lambda/\gamma)(1-a_2)}$, and $c_5 = \frac{1-\lambda/\gamma}{\sigma_c(1+\lambda/\gamma)}$

Investment equation:

$$i_t = i_1 i_{t-1} + (1 - i_1) E_t i_{t+1} + i_2 q_t + \varepsilon_t^i, \quad \text{(A5)}$$

where $i_1 = \frac{1}{1+\overline{\beta}}$, and $i_2 = \frac{1}{(1+\overline{\beta})\gamma^2 \varphi}$ with $\overline{\beta} = \beta \gamma^{(1-\sigma_c)}$.

Arbitrage condition (value of capital, q_t):

$$q_t = q_1 E_t q_{t+1} + (1 - q_1) E_t r_{t+1}^k - (R_t - E_t \pi_{t+1}) + c_3^{-1} \varepsilon_t^b, \quad \text{(A6)}$$

where $q_1 = \overline{\beta}\gamma^{-1}(1 - \delta) = \frac{(1-\delta)}{(r^k+1-\delta)}$.

Log-linearized aggregate production function:

$$y_t = \phi_p \left(\alpha k_t^s + (1 - \alpha)l_t + \varepsilon_t^a \right), \quad \text{(A7)}$$

where $\phi_p = 1 + \frac{\phi}{Y} = 1 + \frac{\text{Steady-state fixed cost}}{Y}$ and α is the capital-share in the production function.[17]

Effective capital (with one period time-to-build):

$$k_t^s = k_{t-1} + z_t. \quad \text{(A8)}$$

Capital utilization:

$$z_t = z_1 r_t^k, \quad \text{(A9)}$$

where $z_1 = \frac{1-\psi}{\psi}$.

Capital accumulation equation:

$$k_t = k_1 k_{t-1} + (1 - k_1)i_t + k_2 \varepsilon_t^i, \quad \text{(A10)}$$

---

[17] From the zero profit condition in steady-state, it should be noticed that ϕ_p also represents the value of the steady-state price mark-up.

where $k_1 = \frac{1-\delta}{\gamma}$ and $k_2 = \left(1 - \frac{1-\delta}{\gamma}\right)\left(1 + \overline{\beta}\right)\gamma^2\varphi$.

Log fluctuations of the real marginal cost:

$$mc_t = w_t - \alpha\left(k_t^s - l_t\right) - \varepsilon_t^a. \tag{A11}$$

New-Keynesian Phillips curve (price inflation dynamics):

$$\pi_t = \pi_1\pi_{t-1} + \pi_2 E_t\pi_{t+1} + \pi_3 mc_t + \varepsilon_t^p, \tag{A12}$$

where $\pi_1 = \frac{\iota_p}{1+\overline{\beta}\iota_p}$, $\pi_2 = \frac{\overline{\beta}}{1+\overline{\beta}\iota_p}$, and $\pi_3 = \frac{1}{1+\overline{\beta}\iota_p}\left[\frac{\left(1-\overline{\beta}\xi_p\right)\left(1-\xi_p\right)}{\xi_p\left(\left(\phi_p-1\right)\varepsilon_p+1\right)}\right]$. The coefficient of the curvature of the Kimball goods market aggregator, included in the definition of A, is fixed in the estimation procedure at $\varepsilon_p = 10$ as in Smets and Wouters (2007).

Optimal demand for capital by firms:

$$-\left(k_t^s - l_t\right) + w_t = r_t^k. \tag{A13}$$

Wage markup equation:

$$w_t - mrs_t = w_t - \left(\sigma_l l_t + \left(\frac{1}{1-\lambda/\gamma} + \frac{H_C a_2}{1-a_2}\right)c_t - \frac{\lambda/\gamma}{1-\lambda/\gamma}c_{t-1} - \frac{H_C a_2}{1-a_2}\left(\frac{1}{1-\lambda_m/\gamma}m_t - \frac{\lambda_m/\gamma}{1-\lambda_m/\gamma}m_{t-1}\right) - \frac{H_C a_2}{1-a_2}\varepsilon_t^\chi \cdot\right). \tag{A14}$$

Real wage dynamic equation:

$$w_t = w_1 w_{t-1} + \left(1 - w_1\right)\left(E_t w_{t+1} + E_t\pi_{t+1}\right) - w_2\pi_t + w_3\pi_{t-1} - w_4\left(w_t - mrs_t\right) + \varepsilon_t^w, \tag{A15}$$

where $w_1 = \frac{1}{1+\overline{\beta}}$, $w_2 = \frac{1+\overline{\beta}\iota_w}{1+\overline{\beta}}$, $w_3 = \frac{\iota_w}{1+\overline{\beta}}$, and $w_4 = \frac{1}{1+\overline{\beta}}\left[\frac{\left(1-\overline{\beta}\xi_w\right)\left(1-\xi_w\right)}{\xi_w\left(\left(\phi_w-1\right)\varepsilon_w+1\right)}\right]$ with the curvature of the Kimball labor aggregator fixed at $\varepsilon_w = 10.0$ and a steady-state wage mark-up fixed at $\phi_w = 1.5$ as in Smets and Wouters (2007).

Monetary policy rule, a Taylor-type rule including responses to the rate of nominal money growth:

$$R_t = \rho R_{t-1} + \left(1 - \rho\right)\left[r_\pi\pi_t + r_y\left(y_t - y_t^p\right) + r_\mu\mu_t\right] + \varepsilon_t^R. \tag{A16}$$

Relationship between nominal money growth, inflation and real money dynamics

$$\mu_t - \pi_t = m_t - m_{t-1}. \tag{A17}$$

Block of potential variables (with ^{p} superscript), obtained when assuming flexible prices, flexible wages and shutting down price mark-up and wage indexation shocks as well as revision shocks.

Flexible-price condition (no price mark-up fluctuations, $mpl_t^p = w_t^p$):

$$\alpha \left(k_t^{s,p} - l_t^p \right) + \varepsilon_t^a = w_t^p. \tag{A18}$$

Flexible-wage condition (no wage mark-up fluctuations, $w_t^p = mrs_t^p$):

$$w_t^p = \sigma_l l_t^p + \left(\frac{1}{1-\lambda/\gamma} + \frac{H_C a_2}{1-a_2} \right) c_t^p - \frac{\lambda/\gamma}{1-\lambda/\gamma} c_{t-1}^p - \frac{H_C a_2}{1-a_2} \left(\frac{1}{1-\lambda_m/\gamma} m_t^p - \frac{\lambda_m/\gamma}{1-\lambda_m/\gamma} m_{t-1}^p \right) - \frac{H_C a_2}{1-a_2} \varepsilon_t^\chi \tag{A19}$$

Potential real money equation

$$m_t^p = \left(\lambda_m/\gamma \right) m_{t-1}^p + \left(1 - \lambda_m/\gamma \right) c_t^p - \frac{(1-\lambda_m/\gamma)(1-a_2)}{R^{ss}} R_t^p - a_2 \left(1 - \lambda_m/\gamma \right) \varepsilon_t^\chi. \tag{A20}$$

Potential transaction costs equation:

$$h_t^p = \frac{1-(a_0/H)}{1-a_2} \left(c_t^p - a_2 \left(\frac{1}{1-\lambda_m/\gamma} m_t^p - \frac{\lambda_m/\gamma}{1-\lambda_m/\gamma} m_{t-1}^p \right) - a_2 \varepsilon_t^\chi \right). \tag{A21}$$

Potential aggregate resource constraint:

$$y_t^p = c_y c_t^p + i_y i_t^p + z_y z_t^p + h_y h_t^p + \varepsilon_t^g \tag{A22}$$

Potential consumption equation:

$$\begin{aligned}
(1 + c_4) c_t^p = {} & c_1 c_{t-1}^p + c_2 E_t c_{t+1}^p + c_3 \left(l_t^p - E_t l_{t+1}^p \right) \\
& + c_4 \left(\frac{1}{1-\lambda_m/\gamma} m_t^p - \frac{\lambda_m/\gamma}{1-\lambda_m/\gamma} m_{t-1}^p \right) - c_4 \left(\frac{1}{1-\lambda_m/\gamma} E_t m_{t+1}^p - \frac{\lambda_m/\gamma}{1-\lambda_m/\gamma} m_t^p \right) \\
& - c_5 \left(R_t^p - E_t \pi_{t+1}^p \right) + c_4 \left(1 - \rho_\chi \right) \varepsilon_t^\chi + c_5 \left(1 - \rho_b \right) \varepsilon_t^b,
\end{aligned} \tag{A23}$$

Potential investment equation:

$$i_t^p = i_1 i_{t-1}^p + (1 - i_1) E_t i_{t+1}^p + i_2 q_t^p + \varepsilon_t^i. \tag{A24}$$

Arbitrage condition (value of potential capital, q_t^p):

$$q_t^p = q_1 E_t q_{t+1}^p + (1-q_1) E_t r_{t+1}^{k,p} - \left(R_t^p - E_t \pi_{t+1}^p \right) + c_3^{-1} \varepsilon_t^b. \tag{A25}$$

Log-linearized potential aggregate production function:

$$y_t^p = \phi_p \left(\alpha k_t^{s,p} + (1-\alpha) l_t^p + \varepsilon_t^a \right). \tag{A26}$$

Potential capital (with one period time-to-build):

$$k_t^{s,p} = k_{t-1}^p + z_t^p. \tag{A27}$$

33

Potential capital utilization:

$$z_t^p = z_1 r_t^{k,p}. \tag{A28}$$

Potential capital accumulation equation:

$$k_t^p = k_1 k_{t-1}^p + (1 - k_1)i_t^p + k_2 \varepsilon_t^i. \tag{A29}$$

Potential demand for capital by firms ($r_t^{k,p}$ is the potential log of the rental rate of capital):

$$-(k_t^{s,p} - l_t^p) + w_t^p = r_t^{k,p}. \tag{A30}$$

Monetary policy rule (under flexible prices and flexible wages):

$$R_t^p = \rho R_{t-1}^p + (1 - \rho)[r_\pi \pi_t^p + r_\mu \mu_t^p] + \varepsilon_t^R. \tag{A31}$$

Potential nominal money growth, inflation and real money dynamics:

$$\mu_t^p - \pi_t^p = m_t^p - m_{t-1}^p. \tag{A32}$$

III. Equations-and-variables summary

- Set of equations:

Equations (A1)-(A32) determine solution paths for 32 endogenous variables.

- Set of variables:

Endogenous variables (32): y_t, c_t, i_t, z_t, l_t, R_t, π_t, mc_t, mrs_t, q_t, r_t^k, k_t^s, k_t, μ_t, m_t, h_t, w_t, y_t^p, c_t^p, i_t^p, z_t^p, l_t^p, R_t^p, π_t^p, q_t^p, $r_t^{k,p}$, $k_t^{s,p}$, k_t^p, μ_t^p, m_t^p, h_t^p, and w_t^p.

Predetermined variables (13): c_{t-1}, i_{t-1}, k_{t-1}, π_{t-1}, w_{t-1}, R_{t-1}, m_{t-1}, y_{t-1}, c_{t-1}^p, i_{t-1}^p, m_{t-1}^p, k_{t-1}^p, and R_{t-1}^p.

Exogenous variables (8): AR(1) technology shock $\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \eta_t^a$, AR(1) risk premium shock $\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b$, AR(1) exogenous spending shock cross-correlated to technology innovations $\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \eta_t^g + \rho_{ga} \eta_t^a$, AR(1) investment shock $\varepsilon_t^i = \rho_i \varepsilon_{t-1}^i + \eta_t^i$, AR(1) monetary policy shock $\varepsilon_t^R = \rho_R \varepsilon_{t-1}^R + \eta_t^R$, ARMA(1,1) price mark-up shock $\varepsilon_t^p = \rho_p \varepsilon_{t-1}^p + \eta_t^p - \mu_p \eta_{t-1}^p$, ARMA(1,1) wage mark-up shock $\varepsilon_t^w = \rho_w \varepsilon_{t-1}^w + \eta_t^w - \mu_w \eta_{t-1}^w$, and AR(1) money demand shock $\varepsilon_t^\chi = \rho_\chi \varepsilon_{t-1}^\chi + \eta_t^\chi$.

Table 2. Second-moment statistics

	Pre-1995			Post-1995		
	US data	Model		US data	Model	
Standard deviations						
$\sigma(\pi_t)$	0.61	0.65	(0.51,0.76)	0.23	0.38	(0.26,0.47)
$\sigma(R_t)$	0.82	0.74	(0.62,0.84)	0.55	0.40	(0.31,0.46)
$\sigma(\mu_t)$	2.17	2.30	(1.97,2.56)	1.41	1.55	(1.32,1.80)
$\sigma(\Delta y_t)$	0.95	1.14	(1.02,1.23)	0.72	0.96	(0.84,1.06)
Autocorrelations						
$\rho(\pi_t, \pi_{t-1})$	0.85	0.83	(0.77,0.88)	0.51	0.78	(0.68,0.86)
$\rho(R_t, R_{t-1})$	0.93	0.89	(0.86,0.92)	0.98	0.96	(0.94,0.97)
$\rho(\mu_t, \mu_{t-1})$	0.47	0.52	(0.44,0.60)	0.64	0.62	(0.53,0.71)
Correlations						
$\rho(\pi_t, R_t)$	0.54	0.62	(0.49,0.72)	0.09	0.23	(0.02,0.44)
$\rho(\pi_t, \mu_t)$	-0.21	-0.26	(-0.38,-0.14)	-0.17	-0.15	(-0.33,0.02)
$\rho(\pi_t, \Delta y_t)$	-0.33	-0.26	(-0.34,-0.18)	-0.08	0.08	(-0.05,0.20)
$\rho(R_t, \mu_t)$	-0.13	-0.12	(-0.22,-0.00)	0.16	-0.23	(-0.33,-0.13)
$\rho(R_t, \Delta y_t)$	-0.27	-0.36	(-0.42,-0.31)	0.25	-0.26	(-0.33,-0.18)
$\rho(\mu_t, \Delta y_t)$	0.09	0.24	(0.17,0.33)	-0.15	0.30	(0.18,0.42)

Table . Second-moment statistics

	Pre-1984			1995-2007		
	US data	Model		US data	Model	
Standard deviations						
$\sigma(\pi_t)$	0.55	0.69	(0.50,0.83)	0.22	0.41	(0.28,0.51)
$\sigma(R_t)$	0.90	0.75	(0.61,0.87)	0.45	0.36	(0.28,0.43)
$\sigma(\mu_t)$	2.55	2.82	(2.37,3.21)	1.32	1.56	(1.24,1.82)
$\sigma(\Delta y_t)$	1.15	1.24	(1.10,1.38)	0.56	0.83	(0.72,0.93)
Autocorrelations						
$\rho(\pi_t, \pi_{t-1})$	0.74	0.83	(0.77,0.88)	0.48	0.81	(0.73,0.88)
$\rho(R_t, R_{t-1})$	0.91	0.89	(0.86,0.92)	0.97	0.95	(0.93,0.97)
$\rho(\mu_t, \mu_{t-1})$	0.43	0.50	(0.42,0.58)	0.65	0.65	(0.55,0.76)
$\rho(\Delta y_t, \Delta y_{t-1})$	0.24	0.25	(0.15,0.35)	-0.05	0.40	(0.32,0.50)
Correlations						
$\rho(\pi_t, R_t)$	0.49	0.58	(0.44,0.73)	-0.20	0.27	(0.05,0.49)
$\rho(\pi_t, \mu_t)$	-0.28	-0.29	(-0.47,-0.13)	-0.39	-0.10	(-0.33,0.14)
$\rho(\pi_t, \Delta y_t)$	-0.35	-0.22	(-0.33,-0.12)	-0.36	-0.03	(-0.19,0.11)
$\rho(R_t, \mu_t)$	-0.14	-0.24	(-0.45,-0.07)	0.14	-0.30	(-0.43,-0.21)
$\rho(R_t, \Delta y_t)$	-0.30	-0.32	(-0.40,-0.25)	0.07	-0.33	(-0.42,-0.26)
$\rho(\mu_t, \Delta y_t)$	0.12	0.23	(0.14,0.33)	0.01	0.35	(0.23,0.46)

Table Second-moment statistics (SW model)

	Pre-1995			Post-1995		
	US data	Model		US data	Model	
Standard deviations						
$\sigma(\pi_t)$	0.61	0.69	(0.53,0.83)	0.23	0.37	(0.24,0.46)
$\sigma(R_t)$	0.82	0.78	(0.62,0.91)	0.55	0.46	(0.31,0.55)
$\sigma(\Delta y_t)$	0.95	1.14	(1.01,1.23)	0.72	0.92	(0.80,1.03)
Autocorrelations						
$\rho(\pi_t, \pi_{t-1})$	0.85	0.85	(0.80,0.92)	0.51	0.77	(0.66,0.89)
$\rho(R_t, R_{t-1})$	0.93	0.91	(0.88,0.94)	0.98	0.96	(0.95,0.98)
$\rho(\Delta y_t, \Delta y_{t-1})$	0.28	0.31	(0.23,0.41)	0.40	0.42	(0.36,0.50)
Correlations						
$\rho(\pi_t, R_t)$	0.54	0.65	(0.54,0.77)	0.09	0.45	(0.27,0.65)
$\rho(\pi_t, \Delta y_t)$	−0.33	−0.29	(-0.38,-0.19)	−0.08	−0.07	(-0.22,0.03)
$\rho(R_t, \Delta y_t)$	−0.27	−0.18	(-0.27,-0.11)	0.25	−0.07	(-0.14,-0.01)