TERRITORIAL MOBILITY: A MEASURING PROPOSAL

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Abstract

As a contribution to the study of intradistributional mobility, this paper introduces a family of functions whose usefulness as mobility measures is justified by different theoretical results. Indeed, as a particular case, this family includes the Bartholomew index, which is widely used in the literature devoted to the dynamic analysis of personal income distribution. The paper also contains, by way of example, an application to the study of mobility in regional per capita income distribution in the European Union between 1977 and 1999.

Key words: Mobility, income, transition matrices, regions, European Union.
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1 Introduction

In recent years, the issue of spatial disparities has been examined in numerous papers covering widely varying geographical areas[1]. Most of these studies are essentially static in their approach, since they are based on the information provided by various indicators calculated from cross sectional data of the distribution in question. However, as Quah (1993, 1996a, 1997) has repeatedly pointed out, this type of analysis fails to capture a series of potentially interesting issues relating to the distribution dynamics. In particular, the conventional static approach does not contemplate, for example, the possibility of regions modifying their relative positions over time, and thereby neglects the whole question of intradistributional mobility.

As an illustration of the relevance of issues relating to the analysis of distribution dynamics, let us consider the following example. Let us assume that we have information for a period of several years on regional incomes and populations in two given countries, A and B, each of which is in turn divided into two regions with exactly the same size of population. To eliminate the effects of population shifts, let us also suppose that there is no change over time in the distribution of the population share in either of the two countries considered. In both A and B, the per capita income of one of the two regions is exactly twice that of the other region, and this situation remains unaltered for the whole of the period considered. There is, however, one major difference between these two countries. A is characterised by a high degree of regional mobility, such that, every year, its two regions switch positions. The situation in B, by contrast, is that the relative positions of its regions remain constant year on year. As we have already mentioned, the

[1] See, for example, Theil (1989), Barro and Sala-i-Martin (1992), Ram (1992), Duro and Esteban (1998), Schultz (1998) or Sala-i-Martin (2002), among many others.

type of analysis commonly found in the literature is essentially static in its approach, since it is based on cross sectional information, and therefore reveals no appreciable difference between A and B. In fact, given that there is no change over time in the cross sectional structure of the per capita income distribution of either of the countries, any inequality index that satisfies the properties of symmetry and scale independence will give exactly the same value for A and B throughout the period considered[2].

This example highlights the need to supplement standard inequality studies with additional data relating to the mobility of the distribution under analysis. However, the few papers that have examined this issue from the spatial perspective tend to use the econometric tools proposed by Quah (1996a, 1997)[3]. However, it is not possible via this methodological approach to obtain an exact measure of changes in mobility levels over time. One possible solution to this problem would be to take into account the mobility measures used in the literature devoted to the dynamic study of personal income distribution and adapt them to the regional context. Surprisingly, this is an approach that has so far received very little attention from researchers engaged in the analysis of territorial imbalances[4]. This is no doubt due, in part, to the obvious limitations of the theoretical and empirical basis for the analysis of intradistributional mobility[5].

In order to make a deeper analysis of this issue, in this paper we present a family of functions whose usefulness as mobility measures is justified by different theoretical

[2]The properties of symmetry and scale independence do not constitute a major limitation. Indeed both are basic properties that any inequality index can be reasonably expected to fulfil (Cowell, 1995). In any event, for the purposes of our example, we can overcome the need for the inequality index to satisfy the property of scale independence by simply assuming the per capita incomes of A and B to be equal.

[3]As well as Quah (1996a, 1997), see Desdoigts (1994), Jones (1997) or Johnson (2000).

[4]As an exception, see Parker and Gardner (2002).

[5]Indeed, as stated in Fields and Ok (1999), considerably different approaches are currently taken in the study of inequality and mobility. Nevertheless, over the course of the last decade, major theoretical advances have been made in the analysis of intradistributional mobility. In particular, there have been proposed a series of measuring procedures with similar axiomatic contents to those used in the study of inequality.

results. Indeed, as a particular case, this family includes the Bartholomew index, which is widely used in the empirical literature devoted to the analysis of personal income distribution. The paper also contains, by way of example, an application to the study of mobility in regional per capita income distribution in the European Union between 1977 and 1999.

The paper is structured into four sections. After this introduction, in section 2 we present a proposal to measure intradistributional mobility using information supplied by various transition matrices. In section 3 we carry out an empirical application to the European context. The main conclusions are summarised in section 4.

2 A family of mobility measures

Let us begin by assuming that we have information for a given period of time on n groups of individuals, to which will refer henceforth as regions. In this setting, we will denote per capita income of region i in period t by x_i^t, with $x_i^t = \frac{X_i^t}{N_i^t}$, where X_i^t and N_i^t are respectively the income and population of region i, $i = 1, 2, \ldots, n$. Likewise, let p_i^t be the population share of region i in period t, $p_i^t = \frac{N_i^t}{N^t}$, with $N^t = \sum_{i=1}^{n} N_i^t$. The associated per capita income and population distributions, therefore, will be given by $x^t = (x_1^t, x_2^t, \ldots, x_n^t)$ and $p^t = (p_1^t, p_2^t, \ldots, p_n^t)$[6]. Finally, let us suppose that $x^t \in \mathbb{R}_+^n$, while $p^t \in \mathbb{R}_{++}^n$.

Our objective is to measure the intradistributional mobility associated to x^t over time. For that, one of the options most commonly used in the literature involves the construction of transition matrices to obtain information concerning shifts in the relative

[6]Obviously $\sum_{i=1}^{n} p_i^t = 1$.

positions of regions over a given period of time. Further, in order to define the concept of transition matrix, let us now suppose that we have classified the different regions in the distribution into m exhaustive and mutually exclusive classes according to their per capita income level. Let us also imagine that we have information on the distribution of interest for two moments in time, t_0 and t_1. In a case such as this, the matrix that summarises the probabilities of regions shifting from one class to another between t_0 and t_1 is known as a transition matrix. Supposing, therefore, that the probabilities can be reasonably estimated from the corresponding relative frequencies, the transition matrix associated with the transformation experienced by the distribution between t_0 and t_1 ($x^{t_0} \longrightarrow x^{t_1}$), will be the square matrix $\Pi(x^{t_0}, x^{t_1}) = \left[\pi_{jk}(x^{t_0}, x^{t_1})\right] \in \mathbb{R}_+^{m \times m}$, where $\pi_{jk}(x^{t_0}, x^{t_1})$ denotes the proportion of regions that belonged to class j in t_0 and have shifted to class k in t_1. According to this definition, we have that $\sum_{k=1}^{m} \pi_{jk}(x^{t_0}, x^{t_1}) = 1$ for any $j = 1, 2, \ldots, m$, so that $\Pi(x^{t_0}, x^{t_1})$ is a stochastic matrix.

The information provided by transition matrices can be summarised in various indicators, the most basic of these being shifts between classes either towards a higher or to a lower relative position. However, the literature usually considers synthetic measures derived from the specific characteristics of the transition matrix[7]. Within this framework, therefore, a mobility index can be defined as a continuous function $M(\Pi) : \Omega \longrightarrow \mathbb{R}$, where Ω is the set of transition matrices (Shorrocks, 1978).

Nevertheless, having reached this point, it is worth mentioning the existence of different approaches to the study of intradistributional mobility as a consequence of the multidimensional nature of the concept that concern us (Fields and Ok, 1999a). The main difference between the various approaches lies in the way in which each one defines

[7]In relation to this, see, for example, Prais (1955), Bartholomew (1973), Bibby (1975), Shorrocks (1978), Sommers and Conlisk (1978) or Conlisk (1985, 1990).

situations characterised by maximum mobility. One alternative, for example, is to relate perfect mobility to situations in which there is no dependence between t_0 and t_1 (independence of origin). Note that this would imply the assumption in this case that the probability of shifting from one class to any other is always the same, such that $\pi_{jk} = \frac{1}{m}$ for any j, k[8]. However, bearing in mind the objective of this paper, we decided to adopt an alternative approach that highlights the dimension of mobility directly related to movement *per se*[9].

Pursuing this idea, we define perfect mobility as a situation in which, all the regions in each of the m classes considered move to the class furthest away from their baseline class. Formally, this can be written in the form of the following transition matrices:

$$\Pi^* = \begin{cases} m \text{ even} \begin{cases} \text{if } j \leq \frac{m+1}{2},\ \pi_{jk} = 0 \ \ \forall k \neq m \text{ and } \pi_{jm} = 1 \\ \\ \text{if } j \geq \frac{m+1}{2},\ \pi_{jk} = 0 \ \ \forall k \neq 1 \text{ and } \pi_{j1} = 1 \end{cases} \\ \\ m \text{ odd} \begin{cases} \text{if } j < \frac{m+1}{2},\ \pi_{jk} = 0 \ \ \forall k \neq m \text{ and } \pi_{jm} = 1 \\ \\ \text{if } j = \frac{m+1}{2},\ \pi_{jk} = 0 \ \ \forall k \neq 1, m \text{ and } \pi_{j1} + \pi_{jm} = 1 \\ \\ \text{if } j > \frac{m+1}{2},\ \pi_{jk} = 0 \ \ \forall k \neq 1 \text{ and } \pi_{j1} = 1 \end{cases} \end{cases}$$

There are other alternatives, however. According to Ramos (1998), for example, the transition matrices that present perfect mobility in this context are those in which for any j, k with $k = (m - j + 1)$, $\pi_{jk} = 1$; while in any other case $\pi_{jk} = 0$. In other

[8] For further details, see Shorrocks (1978).

[9] In relation to this, see Cowell (1985), Fields and Ok (1996, 1999b) or Mitra and Ok (1998).

words, matrices of this type would have a line of ones along the diagonal joining the last element of the first row and the first element of the last row. Nevertheless, let us consider the following transition matrices with $m = 4$:

$$\Pi^1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

and

$$\Pi^2 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

It would appear to be obvious that if we identify mobility with movement, matrix Π^1 exhibits more mobility than matrix Π^2. Note, however, that the idea of perfect mobility based on the definition of Π^* allows for the presence in the corresponding transition matrix of columns in which the sum of the elements is zero. This fact imposes a series of restrictions with respect to the criterion used to classify the different groupings of individuals into the m initial classes, because it excludes the possibility of dividing the population by quantiles. This could represent a major limitation in studies based on microdata, where it is usual to examine the distribution pattern in terms of quintiles or deciles, for example. In our case, however, this definition of perfect mobility raises no practical problems, since our units of reference are geographical areas that contain a varying number of individuals, and it is therefore perfectly reasonable to define the m classes in terms of their per capita income normalised according to the average of the distribution under analysis.

We will now examine a series of basic properties that a mobility measure based on the data given by a transition matrix $\Pi \in \Omega$ can reasonably be expected to fulfil.

- *Monotonicity (MN):* If $\Pi > \Pi'$, then $M(\Pi) > M(\Pi')$, being $\Pi > \Pi'$ when $\pi_{jk} \geq \pi'_{jk}$ for any $j \neq k$ and $\pi_{jk} > \pi'_{jk}$ for some $j \neq k$.

  This property focuses on the non diagonal elements of transition matrices, that is, the ones that give the probability of moving from the original class over the time period considered. Therefore, if the non diagonal elements are greater in Π than in Π', it would seem fitting to impose that $M(\Pi)$ should be larger than $M(\Pi')$.

- *Strong immobility (SIM):* $M(\Pi) = 0$ if and only if $\Pi = \mathrm{I}$.

  In the absence of movements between classes, from t_0 to t_1, Π coincides with the identity matrix, I. On the one hand, this property brings up the advisability of attributing the lowest value of the index precisely to the identity matrix. It also excludes the possibility of there being some other matrix $\Pi' \in \Omega$ distinct from I, such that $M(\Pi') = 0$.

- *Strong maximum mobility (SMM):* $M(\Pi)$ reaches its maximum at Π. Likewise, if Π is the maximum, then $\Pi \in \Pi^*$.

All three of the above properties, with some variations, were proposed by Shorrocks (1978). In this study, we have also considered the following:

- *Independence of irrelevant classes (IIC):* Let there be $\Pi^A, \Pi^B, \Pi^C, \Pi^D \in \Omega$, such that $\Pi^A = (\Pi_1, \ldots, \Pi_h, \ldots, \Pi_m)$, $\Pi^B = (\Pi_1, \ldots, \Pi_{h-1}, \widehat{\Pi}_h, \Pi_{h+1}, \ldots, \Pi_m)$, $\Pi^C =$

$(\Pi'_1, \ldots, \Pi'_{h-1}, \Pi_h, \Pi'_{h+1}, \ldots, \Pi'_m)$ and $\Pi^D = (\Pi'_1, \ldots, \Pi'_{h-1}, \widehat{\Pi}_h, \Pi'_{h+1}, \ldots, \Pi'_m)$, where Π_j denotes row j of matrix Π. Then, $M(\Pi^A) \geq M(\Pi^B)$ if and only if $M(\Pi^C) \geq M(\Pi^D)$.

In order to clarify the implications of this property, let us consider the following example. In particular, let there be the following transitions matrices with $m = 3$:

$$\Pi^3 = \begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.5 & 0.5 & 0 \\ 0 & 0.3 & 0.7 \end{bmatrix}$$

and

$$\Pi^4 = \begin{bmatrix} 0.2 & 0.6 & 0.2 \\ 0.5 & 0.5 & 0 \\ 0 & 0.3 & 0.7 \end{bmatrix}$$

Likewise,

$$\Pi^5 = \begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and

$$\Pi^6 = \begin{bmatrix} 0.2 & 0.6 & 0.2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

As yet we have no criterion by which to classify Π^3 and Π^4 in terms of their mobility. Obviously, this reasoning can be extended to Π^5 and Π^6. In any event, by virtue of (IIC) we are able to establish that $M(\Pi^3) \geq M(\Pi^4)$ if and only if $M(\Pi^5) \geq M(\Pi^6)$.

- *Symmetry of rows along the main diagonal (SRD):* Let there be $\Pi, \Pi' \in \Omega$, such that $\Pi = (\Pi_1, \ldots, \Pi_j, \ldots, \Pi_m)$ and $\Pi' = (\Pi_1, \ldots, \Pi_{j-1}, \Pi'_j, \Pi_{j+1}, \ldots, \Pi_m)$. For row j, $j \notin \{1, m\}$, it is verified that:

  1. $\pi_{jk} = \pi'_{jk}$ for any $k \neq j - \lambda, j + \lambda$, where λ is any natural number in the interval $[1,\ Min\{j-1,\ m-j\}]$.

  2. $\pi_{j(j-\lambda)} + \pi_{j(j+\lambda)} = \pi'_{j(j-\lambda)} + \pi'_{j(j+\lambda)}$.

  Then, $M(\Pi) = M(\Pi')$.

According to this property, equal degrees of movement away from the class of origin but with a different sign should be valued equally by $M(\Pi)$ irrespective of their direction, as long as the total probability in each case remains constant. To further illustrate the implications of this property, let us consider the following example. Then, let there be two matrices with $m = 3$ such that:

$$\Pi^7 = \begin{bmatrix} 1 & 0 & 0 \\ 0.2 & 0.5 & 0.3 \\ 0 & 0.3 & 0.7 \end{bmatrix}$$

and

$$\Pi^8 = \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0.3 & 0.7 \end{bmatrix}$$

(SRD) guarantees that $M(\Pi^7) = M(\Pi^8)$.

Having reached this point, we are now in a position to introduce a family of indices that enables us to measure mobility conceived as movement, by means of the information supplied by a transition matrix $\Pi \in \Omega$. As we will see later, as a particular case, this family includes the Bartholomew index (1973), widely used in the empirical literature on the analysis of intradistributional mobility.

As stated earlier, a transition matrix provides information on movements from one class to another in a distribution over the time period defined by t_0 and t_1. In this context, lack of movement between classes implies that Π is equal to I. We can therefore consider the possibility of measuring the mobility corresponding to transformation $x^{t_0} \longrightarrow x^{t_1}$ by calculating the distance between Π and I for an appropriate distance function[10].

Taking this idea as our starting point, let us begin by considering the following family of functions:

$$MD(\Pi, \omega, v, \alpha) = \sum_{j=1}^{m} \omega_j \left[\sum_{k=1}^{m} |\pi_{jk} - i_{jk}|^{\alpha} \, v(j,k) \right]^{\frac{1}{\alpha}} \tag{1}$$

where i_{jk} is the corresponding element of the identity matrix, and $\sum_{j=1}^{m} \omega_j = 1$ with $\omega_j > 0$

[10]Dagum (1980), Shorrocks (1982), Ebert (1984), Chakravarty and Dutta (1987) and Silber and Berrebi (1988) have all used various distance functions within the context of inequality measurement. Up to the present, however, this approach has received very little attention from mobility analysis, save for a few exceptions [Cowell (1985), Fields and Ok (1996)].

for any $j = 1, 2, \ldots, m$. Likewise, $\alpha \geq 1$[11], while $v : \{1, ..., m\} \times \{1, ..., m\} \longrightarrow \mathbb{R}_+$ is a strictly increasing function on variable $|j - k|$[12].

Note that the above expression includes a double weighting. Specifically, we have considered the possibility when calculating $MD(\Pi, \omega, v, \alpha)$, of assigning a different weight, ω_j, to each of the Π rows. This is not common practice in the literature devoted to the study of intradistributional mobility using transition matrix data. However, it would appear advisable to endow the various indicators considered in the empirical analysis with some kind of instrument to deal with possible differences in population or income shares between classes. Also, by including $v(j, k)$ in expression (1), we are able to assign within each row different weightings according to the degree of movements between classes. Nevertheless, we will take up this issue in greater detail later in the paper.

We will now examine the suitability of using the family of functions described in expression (1) as mobility measures. For this, let us consider the following result.

Proposition 1: $MD(\Pi, \omega, v, \alpha)$ *satisfies (MN), (SIM), (SMM) and (IIC).*

Proof:

•*Monotonicity (MN):*

Let there be $\Pi, \Pi' \in \Omega$ such that $\Pi > \Pi'$ with $\omega_j = \omega'_j$ for any $j = 1, 2, ..., m$. Π and Π' are by definition stochastic matrices, so that they verify that $\sum\limits_{k=1}^{m} \pi_{jk} = \sum\limits_{k=1}^{m} \pi'_{jk} = 1$ for any $j = 1, 2, \ldots, m$. We can therefore write that:

$MD(\Pi, \omega, v, \alpha) - MD(\Pi', \omega, v, \alpha) = \omega_1 \{[(\pi_{12} + \ldots + \pi_{1m})^\alpha v(1, 1) + \pi_{12}^\alpha v(1, 2) + \ldots + \pi_{1m}^\alpha v(1, m)]^{\frac{1}{\alpha}} - [(\pi'_{12} + \ldots + \pi'_{1m})^\alpha v(1, 1) + \pi'^{\alpha}_{12} v(1, 2) + \ldots + \pi'^{\alpha}_{1m} v(1, m)]^{\frac{1}{\alpha}}\} + \ldots +$

[11] In the event of $\alpha < 1$, it would be possible to obtain contradictory orderings of transition matrices from $MD(\Pi, \omega, v, \alpha)$ according to the notion of mobility as movement.

[12] In other words, $|j_1 - k_1| > |j_2 - k_2|$ if and only if $v(|j_1 - k_1|) > v(|j_2 - k_2|)$.

$\omega_j\{[\pi_{j1}^\alpha v(j,1)+\ldots+\pi_{j(j-1)}^\alpha v(j,j-1)+(\pi_{j1}+\ldots+\pi_{j(j-1)}+\pi_{j(j+1)}+\ldots+\pi_{jm})^\alpha v(j,j)+\pi_{j(j+1)}^\alpha v(j,j+1)+\ldots+\pi_{jm}^\alpha v(j,m)]^{\frac{1}{\alpha}}-[\pi'^\alpha_{j1}v(j,1)+\ldots+\pi'^\alpha_{j(j-1)}v(j,j-1)+(\pi'_{j1}+\ldots+\pi'_{j(j-1)}+\pi'_{j(j+1)}+\ldots+\pi'_{jm})^\alpha v(j,j)+\pi'^\alpha_{j(j+1)}v(j,j+1)+\ldots+\pi'^\alpha_{jm}v(j,m)]^{\frac{1}{\alpha}}\}+\ldots+\omega_m\{[\pi_{m1}^\alpha v(m,1)+\ldots+\pi_{m(m-1)}^\alpha v(m,m-1)+(\pi_{m1}+\ldots+\pi_{m(m-1)})^\alpha v(m,m)]^{\frac{1}{\alpha}}-[\pi'^\alpha_{m1}v(m,1)+\ldots+\pi'^\alpha_{m(m-1)}v(m,m-1)+(\pi'_{m1}+\ldots+\pi'_{m(m-1)})^\alpha v(m,m)]^{\frac{1}{\alpha}}\}>0$

since $\Pi > \Pi'$ and $\omega_j > 0$ for any $j = 1, 2, ..., m$.

Therefore, $MD(\Pi, \omega, v, \alpha) > MD(\Pi', \omega, v, \alpha)$.

•*Strong Immobility (SIM):*

If $\Pi = \mathrm{I}$, we have that $\left[\sum_{k=1}^{m} |\pi_{jk} - i_{jk}|^\alpha v(j,k)\right]^{\frac{1}{\alpha}} = 0$ for any $j = 1, 2, ..., m$, such that $MD(\Pi, \omega, v, \alpha) = 0$.

To test sufficiency, let us consider a matrix $\Pi' \in \Omega$ with $\Pi' \neq \mathrm{I}$. If $\Pi' \neq \mathrm{I}$, there must be some $j \neq k$ such that $\pi_{jk} \neq 0$. Given that $v(j,k) > v(j,j) \geq 0$, $MD(\Pi, \omega, v, \alpha) > 0$.

•*Strong maximum mobility (SMM):*

Let there be $\Pi^0 \in \Pi^*$, with Π^0 such that $\pi^0_{jk} \in \{0, 1\}$. Then,

$$MD(\Pi^0, \omega, v, \alpha) = \sum_{j=1}^{m} \omega_j \left[\sum_{k=1}^{m} \left|\pi^0_{jk} - i_{jk}\right|^\alpha v(j,k)\right]^{\frac{1}{\alpha}} =$$

$$= \sum_{j=1}^{m} \omega_j \left[v(j,j) + Max\{v(j,1), v(j,m)\}\right]^{\frac{1}{\alpha}}$$

We will now prove that $MD(\Pi^0, \omega, v, \alpha) \geq MD(\Pi, \omega, v, \alpha)$ for any $\Pi \in \Omega$. For this we consider two separate cases:

1. Case A: $\Pi \in \Pi^*$.

   If $\pi_{jk} \in \{0, 1\}$ for any j, k, clearly, $MD(\Pi, \omega, v, \alpha) = MD(\Pi^0, \omega, v, \alpha)$.

   Otherwise there exists a pair (j, k), such that $\pi_{jk} \notin \{0, 1\}$. Since $\Pi \in \Pi^*$, m must be odd and $j = \frac{m+1}{2}$. Also, $k = 1$ or $k = m$. Therefore, for this row j:

   $$\left[\sum_{k=1}^{m} |\pi_{jk} - i_{jk}|^\alpha v(j,k)\right]^{\frac{1}{\alpha}} = \left[v(j,j) + \pi_{j1}^\alpha v(j,1) + \pi_{jm}^\alpha v(j,m)\right]^{\frac{1}{\alpha}} \leq$$

$$\leq [v(j,j) + 1^{\alpha} Max\{v(j,1), v(j,m)\}]^{\frac{1}{\alpha}}$$

due to the concavity of x^{α} with $\alpha \geq 1$[13].

2. Case B: $\Pi \notin \Pi^*$.

   If $\Pi \notin \Pi^*$, there must be a pair (j,k) such that,

$$k \neq \begin{cases} m \text{ if } j \leq \frac{m+1}{2} \\ \\ 1 \text{ if } j \geq \frac{m+1}{2} \end{cases}$$

   if m is even and

$$k \neq \begin{cases} m \text{ if } j < \frac{m+1}{2} \\ \\ m,\ 1 \text{ if } j = \frac{m+1}{2} \\ \\ 1 \text{ if } j > \frac{m+1}{2} \end{cases}$$

   if m is odd, such that $\pi_{jk} \neq 0$.

   - Let us assume that the pair in question is such that $j = k$.

     If we select row j, we have that $\pi_{jj} \neq 0$, so we can write:

$$\left[|\pi_{jj} - 1|^{\alpha} v(j,j) + \sum_{k \neq j} \pi_{jk}^{\alpha} v(j,k) \right]^{\frac{1}{\alpha}} \leq$$

$$\leq \left[1^{\alpha} v(j,j) + \sum_{k \neq j} \pi_{jk}^{\alpha} Max\{v(j,1), v(j,m)\} \right]^{\frac{1}{\alpha}} =$$

$$= \left[1^{\alpha} v(j,j) + Max\{v(j,1), v(j,m)\} \sum_{k \neq j} \pi_{jk}^{\alpha} \right]^{\frac{1}{\alpha}}$$

     Applying the concavity of x^{α} with $\alpha \geq 1$, we have that the above expression is less than or equal to:

[13]Note that if $\alpha = 1$, we have that $MD(\Pi, \omega, v, \alpha = 1) = MD(\Pi^0, \omega, v, \alpha = 1)$.

$$\left[1^{\alpha}v(j,j)+Max\{v(j,1),v(j,m)\}\left(\sum_{k\neq j}\pi_{jk}\right)^{\alpha}\right]^{\frac{1}{\alpha}}<$$

$$<\left[1^{\alpha}v(j,j)+Max\{v(j,1),v(j,m)\}1^{\alpha}\right]^{\frac{1}{\alpha}}=$$

$$=\left[v(j,j)+Max\{v(j,1),v(j,m)\}\right]^{\frac{1}{\alpha}}$$

since because $v(j,k)$ is a strictly increasing function, $Max\{v(j,1),v(j,m)\}>0$. Likewise, $\left(\sum_{k\neq j}\pi_{jk}\right)^{\alpha}<1^{\alpha}$, since $\pi_{jj}\neq 0$.

- If the pair that concerns us is such that $j\neq k$, similar reasoning can be applied:

$$\left[|\pi_{jj}-1|^{\alpha}v(j,j)+\sum_{k\neq j}\pi_{jk}^{\alpha}v(j,k)\right]^{\frac{1}{\alpha}}<$$

$$<\left[1^{\alpha}v(j,j)+\sum_{k\neq j}\pi_{jk}^{\alpha}Max\{v(j,1),v(j,m)\}\right]^{\frac{1}{\alpha}}=$$

$$=\left[1^{\alpha}v(j,j)+Max\{v(j,1),v(j,m)\}\sum_{k\neq j}\pi_{jk}^{\alpha}\right]^{\frac{1}{\alpha}}$$

since there is a $\pi_{jk}\neq 0$ with $v(j,k)>0$. Again applying the concavity of x^{α} with $\alpha\geq 1$, we get that the above expression is less than or equal to:

$$\left[1^{\alpha}v(j,j)+Max\{v(j,1),v(j,m)\}\left(\sum_{k\neq j}\pi_{jk}\right)^{\alpha}\right]^{\frac{1}{\alpha}}\leq$$

$$\leq\left[1^{\alpha}v(j,j)+Max\{v(j,1),v(j,m)\}1^{\alpha}\right]^{\frac{1}{\alpha}}=$$

$$=\left[v(j,j)+Max\{v(j,1),v(j,m)\}\right]^{\frac{1}{\alpha}}$$

since, because $v(j,k)$ is a strictly increasing function, $Max\ \{v(j,1),v(j,m)\}>0$. Also, by contrast to the previous case, $\left(\sum_{k\neq j}\pi_{jk}\right)^{\alpha}\leq 1^{\alpha}$.

Therefore, agreggating by rows in both cases, we have:

$$MD(\Pi,\omega,v,\alpha)<MD(\Pi^{0},\omega,v,\alpha)$$

We have therefore proved that $MD(\Pi,\omega,v,\alpha)$ reaches a maximum at Π. Further, if Π is the maximum it must belong to Π^{*}.

•*Independence of irrelevant classes (IIC):*

The proof of this property is straightforward, since $MD(\Pi,\omega,v,\alpha)$ can also be written as:

$$MD(\Pi,\omega,v,\alpha) = \omega_h \left[\sum_{k=1}^{m} |\pi_{hk} - i_{hk}|^{\alpha} v(h,k)\right]^{\frac{1}{\alpha}} + \sum_{j\neq h}^{m} \omega_j \left[\sum_{k=1}^{m} |\pi_{jk} - i_{jk}|^{\alpha} v(j,k)\right]^{\frac{1}{\alpha}}$$

□

$MD(\Pi,\omega,v,\alpha)$, meanwhile, presents an apparent limitation in terms of interpretation. In particular, Shorrocks (1978) recommends that any mobility index derived from a transition matrix, $M(\Pi)$, should verify for any $\Pi \in \Omega$ that $0 \leq M(\Pi) \leq 1$. However, as we are aware, the range of variation of $MD(\Pi,\omega,v,\alpha)$ is not generally limited to the interval $[0,1]$, since it is not possible a priori to establish a predefined upper bound independent of m. This does not raise a major problem, however, since it is possible to normalise $MD(\Pi,\omega,v,\alpha)$. Specifically:

$$0 \leq MD(\Pi,\omega,v,\alpha) \leq \sum_{j=1}^{m} \omega_j \left[v(j,j) + Max\{v(j,1), v(j,m)\}\right]^{\frac{1}{\alpha}} \tag{2}$$

Therefore,

$$0 \leq MD_N(\Pi,\omega,v,\alpha) \leq 1 \tag{3}$$

with

$$MD_N(\Pi,\omega,v,\alpha) = \frac{\sum_{j=1}^{m} \omega_j \left[\sum_{k=1}^{m} |\pi_{jk} - i_{jk}|^{\alpha} v(j,k)\right]^{\frac{1}{\alpha}}}{\sum_{j=1}^{m} \omega_j \left[v(j,j) + Max\{v(j,1), v(j,m)\}\right]^{\frac{1}{\alpha}}} \tag{4}$$

such that $MD_N(\Pi,\omega,v,\alpha)$ takes values between 0 (absence of any movement between classes) and 1 (maximum mobility). In fact, the above expression satisfies the same properties as $MD(\Pi,\omega,v,\alpha)$.

In addition, it is worth mentioning that the characteristics of $MD(\Pi, \omega, v, \alpha)$ will enable us to obtain a decomposition of the observed mobility based on the partition of the population used to define the m classes. Indeed, from expression (1) we can write that:

$$\sum_{j=1}^{m} C_j(\Pi, \omega, v, \alpha) = 1 \tag{5}$$

where

$$C_j(\Pi, \omega, v, \alpha) = \frac{\omega_j \left[\sum\limits_{k=1}^{m} |\pi_{jk} - i_{jk}|^{\alpha}\, v(j,k)\right]^{\frac{1}{\alpha}}}{\sum\limits_{j=1}^{m} \omega_j \left[\sum\limits_{k=1}^{m} |\pi_{jk} - i_{jk}|^{\alpha}\, v(j,k)\right]^{\frac{1}{\alpha}}} \tag{6}$$

represents the share of overall observed mobility according to $MD(\Pi, \omega, v, \alpha)$ attributed to class j. Meanwhile, $C_j(\Pi, \omega, v, \alpha)$ gives the proportion of the decrease that would take place in $MD(\Pi, \omega, v, \alpha)$ assuming there were no movements between classes originating in class j $(\pi_{jj} = 1)$[14].

The family of functions $MD(\Pi, \omega, v, \alpha)$ is of limited applicability, however, since it contains an infinite number of potential mobility measures with no prior arguments to support the choice of one over any of the others. To address this problem, we consider the following result.

Proposition 2: $MD(\Pi, \omega, v, \alpha)$ satisfies (SRD) if and only if $\alpha = 1$.

Proof:

If $\alpha = 1$ it is immediate that it also satisfies (SRD). To prove that $MD(\Pi, \omega, v, \alpha = 1)$ are the only measurements from the family $MD(\Pi, \omega, v, \alpha)$ that satisfy (SRD), let us consider two matrices $\Pi', \Pi'' \in \Omega$ such that:

[14] Obviously, this decomposition is extendable to $MD_N(\Pi, \omega, v, \alpha)$.

$$\Pi' = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

and

$$\Pi'' = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0.5 & 0 & 0.5 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

If we assume in this case that $\alpha > 1$, for row 2 we have:

$[1^\alpha v(2,1) + v(2,2)]^{\frac{1}{\alpha}} > [0,5^\alpha v(2,1) + v(2,2) + 0,5^\alpha v(2,3)]^{\frac{1}{\alpha}}$

Therefore, $MD(\Pi', \omega, v, \alpha) > MD(\Pi'', \omega, v, \alpha)$.

□

This proposition shows that there exist theoretical arguments to justify the use of $MD(\Pi, \omega, v, \alpha = 1)$ to measure mobility conceived as movement.

However, $MD(\Pi, \omega, v, \alpha = 1)$ still includes an infinite number of mobility indices, depending on the functional form of $v(j,k)$. Indeed, the mobility measure proposed by Bartholomew (1973),

$$MB(\Pi) = \sum_{j=1}^{m} \sum_{k=1}^{m} \omega_j \pi_{jk} |j - k| \tag{7}$$

is simply a particular case of $MD(\Pi, \omega, v, \alpha = 1)$. In fact, if $v^1(j,k) = |j-k|$, we have:

$$MD(\Pi, \omega, v^1, \alpha = 1) = \sum_{j=1}^{m} \sum_{k=1}^{m} \omega_j |\pi_{jk} - i_{jk}||j-k| = $$
$$= \sum_{j=1}^{m} \sum_{k=1}^{m} \omega_j \pi_{jk} |j-k| \quad (8)$$

In any event, there are no a priori theoretical arguments to justify the use of a given $v(j,k)$, beyond the requirement that it should be a strictly increasing function on the variable $|j-k|$. In particular, the ultimate choice will depend on the value assigned to the movements that take place between the different classes. Thus, as an alternative to $v^1(j,k)$ we might consider, for example, the possibility of using $v^2(j,k) = |j-k|^2$. Note that $v^2(j,k)$ assigns a greater weight than $v^1(j,k)$ to movements between non-adjacent classes, which may be considered desirable in certain cases.

In fact, it is possible to reformulate $MD(\Pi, \omega, v, \alpha = 1)$, such that the weighting scheme used for movements between classes varies as a function of the original class. In other words[15],

$$MD'(\Pi, \omega, v, \alpha = 1) = \sum_{j=1}^{m} \omega_j \sum_{k=1}^{m} |\pi_{jk} - i_{jk}| v_j(j,k) \quad (9)$$

Summing up, in this section we have introduced a family of functions to measure mobility conceived as movement, and have presented various theoretical results that justify its usefulness. The main characteristic of $MD(\Pi, \omega, v, \alpha = 1)$ is its flexibility, given that it allows the use of different weighting schemes for movements between classes, according to the desired objective. Furthermore, we have shown that $MD(\Pi, \omega, v, \alpha = 1)$ includes, as a particular case, the mobility measure proposed by Bartholomew (1973).

[15] It can be proved that $MD'(\Pi, \omega, v, \alpha = 1)$ satisfies (MN), (SIM), (SMM), (IIC) and (SRD).

3 An application: Regional mobility in the European Union

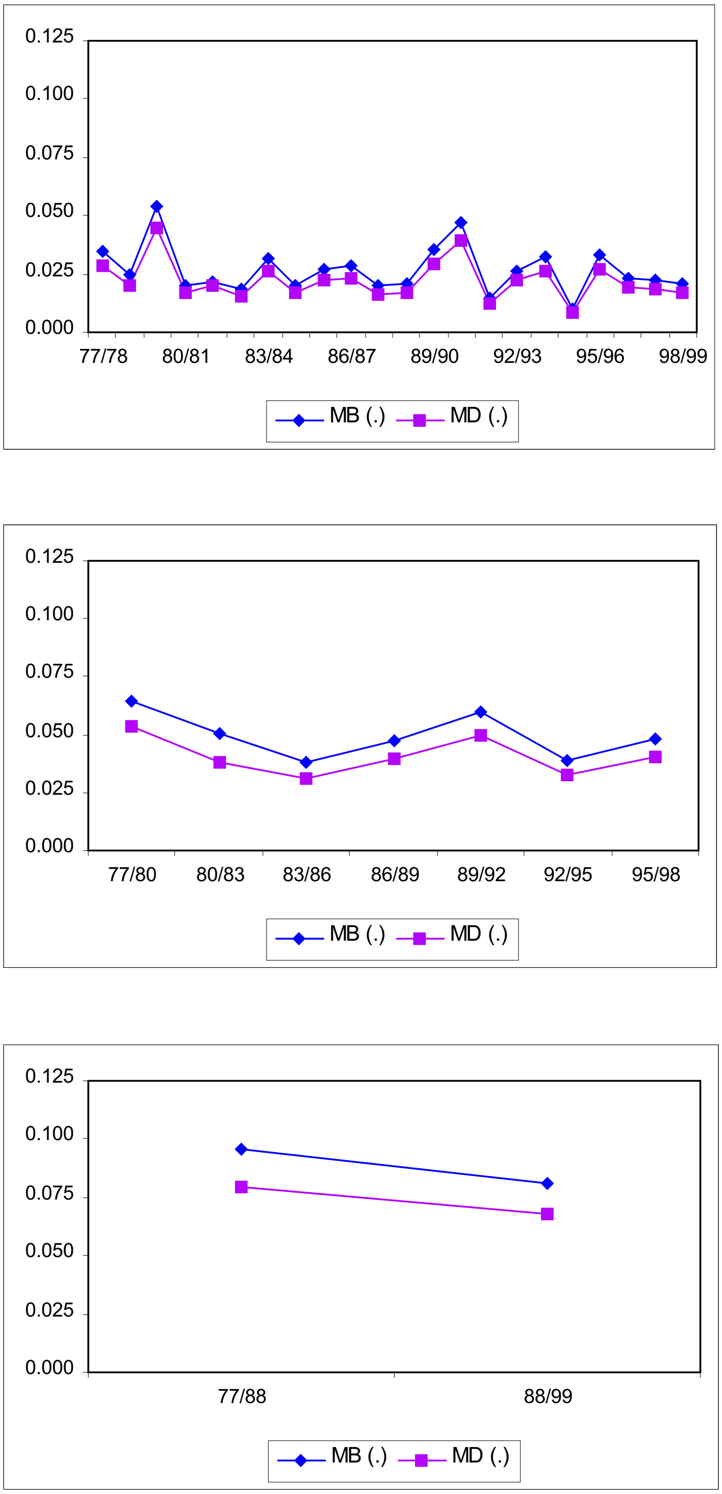
In this section, we will apply $MD(\Pi, \omega, v, \alpha = 1)$ to the study of mobility in the regional distribution of per capita income in the European Union, using statistical information supplied by Cambridge Econometrics for 197 NUTS2 regions over the period 1977-1999[16]. For this, the first step is to select an appropriate definition of the various classes. To address this problem, we opted for a solution that provides reasonably precise details of the movements of regions among a sufficient number of groups, without this making the sample any less representative. Thus, we divided the regions that make up the distribution under analysis into five exhaustive and mutually exclusive classes, according to their per capita income in relation to the European average, which was assigned a value of 100: [0,75), [75,90), [90,110), [110,125) and [125,+∞)[1].

Figure 1 shows the results obtained from calculating $MB_N(\Pi, \omega)$ and MD_N $(\Pi, \omega, v^2, \alpha = 1)$ after estimating the corresponding transition matrices, with $\omega_j = \sum_{i \in j} p_i^{t_0}$ and $v^2(j,k) = |j-k|^2$. In addition, in order to isolate the effect of transient per capita income fluctuations associated with annual changes, we decided in our analysis to use time periods of different length, thus we were also able to distinguish between short and medium term mobility.

The results obtained reveal that regional per capita income distribution exhibits greater mobility when the time interval taken as a reference is increased. Thus on average, 91% of the regions considered continued in the same class after a year. This

[16]Lack of complete series, however, has obliged us to eliminate from the analysis the member States admitted to the European Union in May 2004, the Länder of former East Germany, The French overseas departments and the Spanish territories in North Africa. Nevertheless, the appendix includes a complete list of all the regions considered in this study.

Figure 1: Regional mobility according to $MB_N(\Pi, \omega)$ and $MD_N(\Pi, \omega, v^2, \alpha = 1)$, $m = 5$.



percentage drops to 63% when the period is taken as a whole, however.

It is also worth stressing that the two mobility indices considered follow very similar trends. Given that the main difference between them lies in the distinct valuation given to shifts between classes, this result suggests a relatively low degree of intradistributional mobility[17]. Further confirmation of this is to be found in the various transition matrices estimated, which exhibit the highest values around the main diagonal[18].

Whatever index is used, the empirical evidence presented shows a reduction in the mobility of the European Union regional per capita income distribution between 1997 and 1999. Nevertheless, since mobility has not fallen at an even rate over time, it is possible to identify a series of separate stages, each with its distinguishing features. Thus, the main reduction in $MB_N(\Pi, \omega)$ and $MD_N(\Pi, \omega,\ v^2, \alpha = 1)$ took place between 1977 and the early eighties. From then onwards, however, there is a change of trend leading to an increase in regional mobility continuing until the end of that decade. During the early nineties, there was a further decrease in regional mobility, which, however, seemed to mark the beginning of a new stage, characterised by a new rise in $MB_N(\Pi, \omega)$ and $MD_N(\Pi, \omega,\ v^2, \alpha = 1)$.

In this context, however, it is necessary to stress that the above results cannot be valued normatively without taking into account the degree of inequality observed in the distribution under analysis. In this respect, a large number of studies have coincided in reporting a lack of regional convergence in per capita income in the European context from the mid-seventies onwards [Armstrong (1995), Neven and Gouyette (1995), López-Bazo *et al.* (1999), Rodríguez-Pose (1999), etc.]. The analysis performed in this section,

[17]Neven and Gouyette (1995) and López-Bazo *et al.* (1999) reach a similar conclusion for a more reduced geographical area and a shorter time period than considered in this article.

[18]The medium and full term transition matrices are included in the appendix. The rest, which are not shown for lack of space, are available from the authors upon request.

for its part, shows that this maintenance of territorial imbalances has coincided in time with a process of consolidation in the relative positions of the various regions, which stresses the need for an active regional policy at European level[19].

In light of the volatility of $MB_N(\Pi, \omega)$ and $MD_N(\Pi, \omega,\ v^2, \alpha = 1)$ in short term observations, we performed a preliminary analysis of the relationship between the economic cycle and regional mobility trends in the European context. To this end, we estimated the statistical correlation between per capita income growth rates in the European Union and annual fluctuations in the two mobility measures considered. We then repeated the exercise incorporating the assumption that economic cycle influences on regional mobility with a lag[20]. In both cases, however, the correlation coefficients, though positive, were not statistically significant[21].

Tables 1 and 2 show the average relative contribution of the classes considered to overall observed mobility in the above analysis. The last row of Table 1, for instance, indicates that, according to $MB_N(\Pi, \omega)$, about 60% of long term mobility can be attributed to classes grouping regions in which per capita income in 1977 was between 75 and 125% of the European average. In other words, if over the sample period, there had been no movement originating in the groups at either end of the distribution, the value of the index would have fallen by about 40%.

Nevertheless, to ensure correct interpretation of the figures in Tables 1 and 2, it appears advisable to relate for each class $C_j(.)$ to its population share. Thus, if we limit the analysis to include only those contributions where $C_j(.) > \omega_j$, it is possible to

[19] Note that, for a given level of inequality, high mobility would be a sign of strong cyclical variability in regional incomes. In this kind of context, regional policy should address the need to mitigate adverse cyclical effects before applying traditional convergence policies.

[20] In relation to this, see Fischer and Nijkamp (1987).

[21] Quah (1996b) obtains a similar finding for the United States.

Table 1: Average relative contribution of each class to overall mobility according to $MB_N(\Pi, \omega)$.

Mobility	$C_1(.)$	$C_2(.)$	$C_3(.)$	$C_4(.)$	$C_5(.)$
Biannual	0.12	0.26*	0.24*	0.25*	0.13
Short term (4 years)	0.10	0.27*	0.27*	0.24*	0.12
Medium term (12 years)	0.09	0.28*	0.27*	0.18*	0.21
Long term (23 years)	0.15	0.26*	0.23*	0.11*	0.25

Note: The (*) sign shows that $C_j(.) > \omega_j$.

Table 2: Average relative contribution of each class to overall moblility according to $MD_N(\Pi, \omega, v^2, \alpha = 1)$.

Mobility	$C_1(.)$	$C_2(.)$	$C_3(.)$	$C_4(.)$	$C_5(.)$
Biannual	0.12	0.25*	0.24*	0.26*	0.13
Short term(4 years)	0.11	0.27*	0.26*	0.24*	0.12
Medium term (12 years)	0.09	0.27*	0.26*	0.17*	0.21
Long term (23 years)	0.16	0.20	0.25*	0.12	0.28*

Note: The (*) sign shows that $C_j(.) > \omega_j$.

detect the existence of differentiated behaviour patterns in terms of mobility according to regional development levels. Particularly, results suggest that the main relative contributions correspond basically to groups in the middle of the distribution, with those at either end playing a minor role.

The analysis carried out so far is based on the application of the family $MD(\Pi, \omega, v, \alpha = 1)$ to examine mobility in the regional distribution of per capita income in the European Union. This has involved the use of data from various transition matrices, obtained by dividing the distribution under analysis into a series of exhaustive and mutually exclusive classes. However, since there is no procedure for finding the optimal number of classes in each case, the researcher is obliged to make an arbitrary decision in this respect[22].

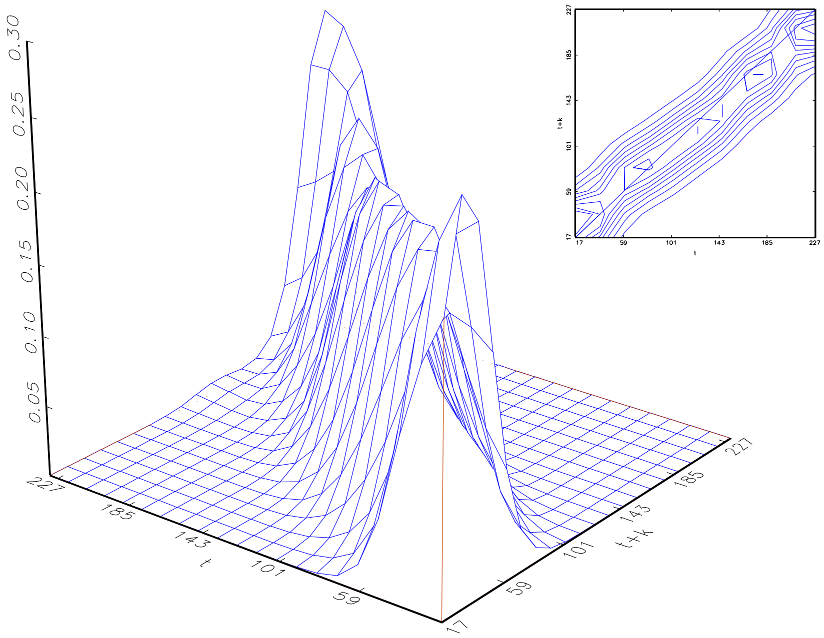
Keeping this issue in mind, we decided to test the robustness of the results obtained in the preceding pages by estimating a stochastic kernel for the entire sample period ($t = 1977$ and $t + k = 1999$)[23]. The three-dimensional graph that appears in Figure 2 can be intuitively interpreted as a transition matrix with an infinite number of classes, such that it gives the associated probability of each pair of values for the first and last years of the sample period. In other words, the stochastic kernel provides, in a way analogous to that of a discrete transition matrix, the probability distribution of 1999 per capita income for regions with a given per capita income in 1977. The peaks on the graph represent high levels of probability. Thus, if the probability mass is concentrated around the main diagonal, the intradistributional dynamics are characterised by a high level of persistence in the relative positions of the regions over time and, therefore, low mobility. If, on the other hand, the density is located mainly on the opposite diagonal to the main diagonal, this would indicate that regions at each end of the distribution switch their relative positions throughout the period. Finally, the probability mass could, in theory, accumulate parallel to the t axis. This would reflect the convergence of regional per capita incomes towards the European average. In order to aid interpretation of the graph, Figure 2 also includes a contour plot on which the lines connect points at the same height on the three-dimensional kernel.

The results obtained fully uphold the conclusions reached in the previous analysis based on the data from the discrete transition matrices. Indeed, as can be seen from Figure 2, the mass of probability is concentrated around the main diagonal. As we are

[22] In relation to this, see Kremer *et al.* (2001).

[23] Gaussian kernel functions were used in all cases, while the smoothing parameter was determined following Silverman (1986).

Figure 2: Stochastic kernel and contour plot of the regional distribution of per capita income, 1977-1999.



already aware, this indicates that there is little mobility in the distribution of regional per capita income between 1977 and 1999. There is a general tendency, therefore, for the European regions to maintain their relative positions throughout the twenty-three years considered. We can also observe how regions with a per capita income around the European average exhibit a relatively higher degree of mobility over time, while those located at either end of the distribution are characterised by a stronger persistence in their relative positions[24].

[24]In order to test the robustness of the results, we decided to repeat the above analysis using data only for the subperiods 1977-1988 and 1988-1999. The results, shown in the appendix, are very similar to those already described.

4 Conclusions

In this paper, as a contribution to the study of intradistributional mobility, we have introduced a family of functions, $MD(\Pi, \omega, v, \alpha = 1)$, whose usefulness as mobility measures has been justified by various theoretical results. This family summarises the information contained in a transition matrix into a single value, and, as a particular case, includes the Bartholomew index (1973), which is widely used in the empirical literature devoted to the dynamic analysis of personal income distribution.

The most outstanding feature of $MD(\Pi, \omega, v, \alpha = 1)$ is its flexibility, given that, depending on the desired objective, it allows the use of different weighting schemes for movements between the classes into which the distribution under analysis is divided. Furthermore, in addition to informing about the level and evolution of intradistributional mobility, the proposed family of measures can be used to examine its origin. Indeed, it is possible to decompose $MD(\Pi, \omega, v, \alpha = 1)$ according to the partition of the population considered to define the various classes, in order to determine the contribution to overall mobility that can be attributed to each class.

As an example, we have applied some of the indices derived from the prior theoretical analysis to study mobility in the regional distribution of per capita income in the European Union from 1977 to 1999. The results obtained show a decline in the mobility of the distribution in question over the time period considered. In addition, it is also worth noting that the level of intradistributional mobility is relatively low. With some exceptions, therefore, the European regions on the whole have tended to maintain their relative positions over the twenty-three years contemplated. All of this underlines the need for the European Union to reinforce its regional development policies.

The analysis carried out also reveals that regional mobility patterns vary as a function of economic development levels. In fact, the regions with a per capita income around the European average tend to register a relatively higher degree of mobility over time, while those at either end of the distribution are characeterised by a stronger persistence in their relative positions.

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Appendix

The 197 territorial units considered in this study are as follows:
Belgium: Bruxelles-Brussel, Antwerpen, Limburg, Oost-Vlaanderen, Vlaams Brabant, West-Vlaanderen, Brabant Wallon, Hainaut, Liège, Luxembourg and Namur. *Denmark*. *Germany*: Stuttgart, Karlsruhe, Freiburg, Tübingen, Oberbayern, Niederbayern, Oberpfalz, Oberfranken, Mittelfranken, Unterfranken, Schwaben, Berlin, Bremen, Hamburg, Darmstadt, Giessen, Kassel, Braunschweig, Hannover, Lüneburg, Weser-Ems, Düsseldorf, Köln, Münster, Detmold, Arnsberg, Koblenz, Trier, Rheinhessen-Pfalz, Saarland and Schleswig-Holstein. *Greece*: Anatoliki Makedonia, Kentriki Makedonia, Dytiki Makedonia, Thessalia, Ipeiros, Ionia Nisia, Dytiki Ellada, Sterea Ellada, Peloponnisos, Attiki, Voreio Aigaio, Notio Aigaio and Kriti. *Spain*: Galicia, Asturias, Cantabria, País Vasco, Navarra, La Rioja, Aragón, Madrid, Castilla-León, Castilla-la Mancha, Extremadura, Cataluña, Com. Valenciana, Baleares, Andalucía, Murcia and Canarias. *France*: Île de France, Champagne-Ardenne, Picardie, Haute-Normandie, Centre, Basse-Normandie, Bourgogne, Nord-Pas de Calais, Lorraine, Alsace, Franche-Comté, Pays de la Loire, Bretagne, Poitou-Charentes, Aquitaine, Midi-Pyrénées, Limousin, Rhône-Alpes, Auvergne, Languedoc-Rousillon, Provence-Alpes-Côte d'Azur and Corse. *Ireland*: Border-Midland and Western and Southern and Eastern. *Italy*: Piemonte, Valle d'Aosta, Liguria, Lombardia, Trentino-Alto Adige, Veneto, Friuli-Venezia Giulia, Emilia-Romagna, Toscana, Umbria, Marche, Lazio, Abruzzi, Molise, Campania, Puglia, Basilicata, Calabria, Sicilia and Sardegna. *Luxembourg*. *The Netherlands*: Groningen, Friesland, Drenthe, Overijssel, Gelderland, Flevoland, Utrecht, Noord-Holland, Zuid-Holland, Zeeland, Noord-Brabant and Limburg. *Austria*: Burgenland, Niederösterreich, Wien, Kärnten, Steiermark, Oberösterreich, Salzburg, Tirol and Vorarlberg. *Portugal*: Norte, Centro, Lisboa e Vale do Tejo, Alentejo, Algarve, Açores and Madeira. *Finland*: Itä-Suomi, Väli-Suomi, Pohjois-Suomi, Uusimaa, Etelä-Suomi and Aland. *Sweden*: Stockholm, Östra Mellansverige, Sydsverige, Norra, Mellansverige, Mellersta Norrland, Övre Norrland, Smaland med oarna and Västsverige. *United Kingdom*: Tees Valley and Durham, Northumberland *et al.*, Cumbria, Cheshire, Greater Manchester, Lancashire, Merseyside, East Riding, North Yorkshire, South Yorkshire, West Yorkshire, Derbyshire, Leicestershire, Lincolnshire, Hereford *et al.*, Shropshire, West Midlands (county), East Anglia, Bedfordshire, Essex, Inner London, Outer London, Berkshire *et al.*, Surrey, Hampshire, Kent,, Avon *et al.*, Dorset, Cornwall, Devon, West Wales, East Wales, North East Scotland, Eastern Scotland, South West Scotland, Highlands and Islands and Northern Ireland.

Figure A1: Stochastic kernel and contour plot of the regional distribution of per capita income, 1977-1988.

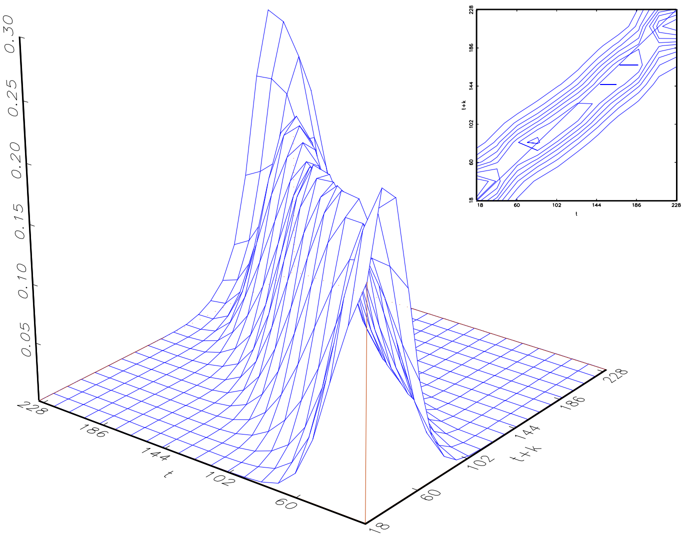

Figure A2: Stochastic kernel and contour plot of the regional distribution of per capita income, 1988-1999.

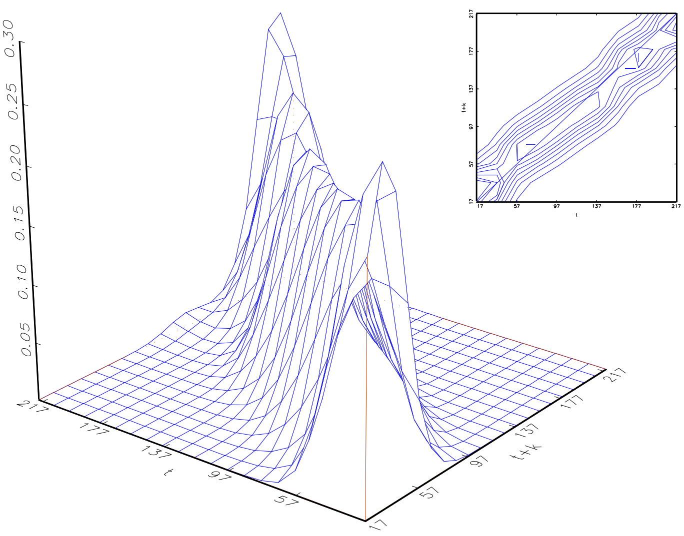

Table A1: Transition matrix, 1977-1988.

Regions	ω_j	[0,75)	[75,90)	[90,110)	[110,125)	[125,∞)
46	0.19	0.81	0.17	0.02	0.00	0.00
45	0.20	0.18	0.67	0.15	0.00	0.00
46	0.20	0.07	0.13	0.71	0.07	0.02
24	0.15	0.00	0.00	0.33	0.63	0.04
36	0.26	0.00	0.00	0.03	0.22	0.75

Table A2: Transition matrix, 1988-1999.

Regions	ω_j	[0,75)	[75,90)	[90,110)	[110,125)	[125,∞)
48	0.22	0.98	0.00	0.02	0.00	0.00
44	0.19	0.15	0.55	0.30	0.00	0.00
50	0.23	0.00	0.16	0.68	0.16	0.00
26	0.16	0.00	0.00	0.15	0.73	0.12
29	0.20	0.00	0.00	0.07	0.10	0.83

Table A3: Transition matrix, 1977-1999.

Classes	ω_j	[0,75)	[75,90)	[90,110)	[110,125)	[125,∞)
46	0.19	0.78	0.09	0.13	0.00	0.00
45	0.20	0.33	0.43	0.24	0.00	0.00
46	0.20	0.07	0.17	0.59	0.15	0.02
24	0.15	0.00	0.04	0.21	0.71	0.04
36	0.26	0.00	0.00	0.14	0.17	0.69