DYNAMIC ANALYSIS IN AN OPTIMIZING MONETARY MODEL WITH TRANSACTION COSTS AND ENDOGENOUS INVESTMENT

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Abstract

This paper analyzes the period-to-period changes that occur in an optimizing monetary model with uncertainty and sticky prices. Money is incorporate in its role as a medium of exchange through a time-cost transactions technology. Another important characteristic of the model is that both capital and investment are obtained endogenously. In this regard, adjustment costs of installing investment are incorporated to smooth and delay capital movements over the economic cycle. We will focus attention on analyzing the consumption, investment, and real money demand functions resulting from the model. These three equations gives rise to the structural IS-LM economy as part of the general equilibrium described in the paper. Nominal prices are sticky, i.e., they do not adjust instantly thereby allowing departures from general equilibrium obtained when there is absence of nominal frictions. We chose to have the Fuhrer-Moore specification for nominal contract prices. The model is calibrated on quarterly observations from United States data. Four types of exogenous shocks are included in our setup: production technology shocks, consumption preference (demand) shocks, monetary policy shocks, and shopping time shocks. Hence, variability of output, consumption, investment, etc., may result from several sources. The impact of each shock in the economic cycle will be examined by plotting impulse-response functions implied by the solutions of the model.

Keywords: money, dynamic optimizing models, business cycle.

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1 Introduction

In the recent macroeconomic literature, most of the monetary structural models incorporate money in its role as a medium of exchange.[1] Money is used to carry out transactions because it is well-accepted by all the agents in exchange for any good. This fact makes money valuable and gives rise to the standard money demand expression depending on total expenditures (consumption, output), and on the nominal interest rate. Although the authors who work with these models often refer to this role of money, it is rare to find papers in which the use of money to facilitate transactions is explicitly represented (and measured) through a transaction costs technology. Conversely, monetary models are generally built under the cash-in-advance constraint (see Dow (1995), Woodford (1994), and Yun (1996)), or the money in the utility function approach (see Benassy (1995), Chari, Kehoe and McGrattan (1996), Ireland (1997), McCallum and Nelson (1997, 1999), and Kollman (1996)).[2]

There are two ways to include transaction costs: transactions take time resources (time-cost transactions approach, described by McCallum and Goodfriend (1987), and recently used in Attanasio, Guiso, Jappelli (1998), Chadha, Haldane and Janssen (1998), and Pakko (1998)), or transactions require output usages from the budget constraint (output-cost transactions approach, as in Feenstra (1986), and Sims (1994)). In any case, real money holdings should be included in the transactions technology reflecting the advantages of possessing the medium of exchange when conducting transactions. This paper explores the economic dynamics of the time-cost transactions approach.

Another important feature of the model in this paper is that both capital and investment are obtained endogenously. As we intend to analyze economic fluctuations here, investment movements must account for much of the output variability over the business cycle. Capital

[1] Money also serves as a store of value function. But in the presence of assets that pay positive interest and are no riskier, money would, under typical circumstances, not be held at all if its medium-of-exchange role were neglected.

[2] The cash-in-advance setup treats money as the required counterpart in any transaction. It turns out that the money demand expression is very restrictive for a binding cash-in-advance constraint: the consumption elasticity is one and the nominal interest rate elasticity is zero. Money in the utility function model has real money as one of the arguments in the utility function. There can be different representations of the utility function leading to distinct money demand functions. In fact, the transactions time approach can be considered one particular case of the money in the utility function framework. However, the assumptions taken to define the transaction costs function (in particular the issue of separability) must be set to be reflected in the properties of the utility function and these issues are seldom discussed.

movements are smoothed by considering adjustment costs of installing investment.

There are three other main characteristics in our model:

i) Optimizing criteria. A neoclassical discrete-time model is constructed in the well-known optimizing framework described by Sidrauski (1967), and Brock (1975). Thus, a set of first order conditions are derived from rational choices of the households who act as consumers and producers. Decisions are made to maximize a discounted sum of expected utility values subject to resource and time constraints. The resulting structural equations have the potential to be policy invariant. A general equilibrium system will be formed by adding up market clearing conditions.

ii) Sticky prices in the supply side. The presence of sluggish mechanisms in nominal wages or prices has been broadly included in the business cycle literature to improve the performance of the simulations (for recent papers see Cho and Cooley (1995), Dotsey, King, and Wolman (1997), King and Watson (1995), Roberts (1995), Woodford (1996), and a survey by Nelson (1998)). We chose to have the price setting following the Fuhrer-Moore specification (Fuhrer-Moore (1995)). The Fuhrer-Moore equations guarantee a high degree of persistence of inflation in the model, much closer to the pattern of inflation in actual data than with any other sticky-price formulation (see Nelson (1998)). The existence of sticky prices leads to departures from the general equilibrium solution. As a result, two measures of output must be distinguished: current output and capacity (or market-clearing) output.

iii) Variety of shocks. Uncertainty surrounds the model where stochastic shocks may hit the system shaping the economic cycle. Agents utilize rational expectations to make predictions. Four types of exogenous shocks are included in the model: production technology shocks, consumption preference (demand) shocks, monetary shocks, and transactions technology shocks. Hence, variability of output, consumption, investment, etc., may come from several reasons. The impact of each shock in the economic cycle will be examined by discussing the impulse-response functions implied by the solutions of the model.

Two are the motivations for this chapter. First, derive and analyze the structural consumption, investment, and money demand functions obtained in an optimizing monetary model with transaction costs in terms of time and endogenous investment. Second, we will describe how the model predicts the responses of macroeconomic variables to changes in technology, monetary policy, or household's preferences.

The paper proceeds as follows. Section 2 is devoted to describing the elements of the model with special attention on the proposed shopping time function. In section 3, we see how the households make their optimal decisions, and then derive a general equilibrium framework. Section 4 defines endogenously current output, capacity output, and the output gap. In section 5, we complete the setup by presenting the Fuhrer-Moore price setting relation on the supply side. The model is calibrated in section 6. Further, the behavior of the main variables of the model are characterized in section 7 by means of impulse response functions that determine the effects of exogenous shocks to the system. The paper finishes in section 8 with some conclusions.

2 The elements of the model

The shopping time function.

Our economy consists of a large number N of infinitely-lived households, each of them producing a different single good that can be consumed, sold, or transformed into physical capital for producing in the next period. In this many goods economy, consumption is defined as an aggregate. In particular, we use indexes as in Dixit and Stiglitz (1977)[3] for both bundles of consumption (c_t) and the price level (P_t). Households share the same utility function, and they have access to the same technology. Symmetric equilibria is examined in which $P_t(i) = P_t(j) = P_t$ for all $i, j \in [1, ..., N]$.[4]

Let m_t represent the amount of real money balances held by the household at the end of period t

$$m_t = \frac{M_t}{P_t} \tag{2.1}$$

where M_t denotes the amount of nominal money. Households put up a shop where they sell the good that they produce to the other households. Likewise, they have to spend part of their time on shopping for the other $N-1$ goods. Money is used during the purchases

[3] The consumption and price level indexes are taken in discrete units over the N distinct goods:
$c_t = N^{\frac{1}{1-\vartheta}} \left(\sum_{i=1}^{N} c_t(i)^{\frac{\vartheta-1}{\vartheta}} \right)^{\frac{\vartheta}{\vartheta-1}}$ *with* $\vartheta > 1$
$P_t = N^{-1} \left(\sum_{i=1}^{N} P_t(i)^{1-\vartheta} \right)^{\frac{1}{1-\vartheta}}$ *with* $\vartheta > 1$.

[4] In turn, demand for each individual good will be equal to $N^{-1}c_t$, being c_t the consumption aggregate.

to facilitate the exchange and economize time. Without money, the household would have to negotiate a payment by credit or spend some time on transforming part of its income to money. These alternatives suppose a higher amount of time. On the other hand, more shopping time will be needed when the level of consumption desired is higher. Hence, we present this shopping time function whose arguments are consumption c_t, real money balances m_t, and a transactions technology shock χ_t,

$$s_t = s(\chi_t, c_t, m_t) = \left\{ \begin{array}{ll} 0 & if\ c_t = 0 \\ \exp(\chi_t)\left(a_0 + a_1 \dfrac{c_t^{a_2}}{m_t^{a_3}}\right) & if\ c_t > 0 \end{array} \right\} \tag{2.2}$$

$$with\ a_0, a_1 > 0, a_2, a_3 > 1.$$

The amount of time devoted to shopping increases with the level of consumption ($s_c > 0$) and decreases with more real money balances ($s_m < 0$). Moreover, consumption has increasing costs in terms of time ($s_{cc} > 0$) and real money has diminishing returns ($s_{mm} > 0$). Finally, the cross marginal effect is positive ($s_{cm} > 0$).

The shopping time specification proposed presents a discontinuity when consumption becomes positive. Thus, the appearance of a constant term (a_0) in (2.2) reflects the initial transaction costs due to a positive consumption level. To justify this term, we rely on the construction of the consumption bundles c_t. A positive value of this aggregate implies that transactions were conducted in order to obtain a positive amount of all the goods of the economy. And this is true even for a very small positive number of c_t. At that moment, households had to spend some time to buy at the shops of the other $N-1$ households. We assume that this searching cost (a_0) is fixed. This cost is independent from the volume of transactions. It only depends on the number of goods (or household units) in the economy N. Other transaction costs vary with the quantity of consumption and the money services (negotiation costs, transportation costs,...).

The scale parameter a_1 indicates the size of the variable time in the total shopping time. There are two other coefficients: the elasticity parameters a_2 and a_3. Let $\eta_{x,y}$ denote the elasticity between variable x and variable y. The two parameters a_2 and a_3 inform about the elasticity of the transaction costs with respect to consumption and real money respectively.[5] Actually, the elasticities are these two values multiplied by a reduction factor,

[5] Loglinearizing (2.2) when consumption is positive it yields

$\eta_{s_t,c_t} = a_2(1 - \frac{a_0}{s^{ss}})$ and $\eta_{s_t,m_t} = -a_3(1 - \frac{a_0}{s^{ss}})$. From now on, the ss superscript over one variable denotes its steady state value. The higher is the constant searching cost a_0 relative to the steady-state shopping time, the lower is the impact of consumption and money in the total shopping time.

Another argument of the shopping time function is a shopping technology shock χ_t. A random walk generates observations of this shock

$$\chi_t = \chi_{t-1} + \epsilon_t \tag{2.3}$$

where ϵ_t are draws from a normalized normal distribution, i.e. $\epsilon_t \sim N(0, \sigma_\epsilon)$.

The utility function.

Utility depends on consumption (c_t), leisure time (l_t), and a random shock reflecting consumption preferences (ζ_t). In particular, we chose a constant relative risk aversion (CRRA) instantaneous utility function

$$U(\zeta_t, c_t, l_t) = \exp(\zeta_t)\frac{c_t^{1-\sigma}}{1-\sigma} + \Upsilon\frac{l_t^{1-\gamma}}{1-\gamma} \tag{2.4}$$
$$\textit{with } \sigma, \gamma, \Upsilon > 0$$

in which the standard first and second order assumptions $U_c > 0$, $U_{cc} < 0$, $U_l > 0$, and $U_{ll} < 0$ hold. The cross marginal effect is null ($U_{cl} = 0$) being consumption and leisure separable in (2.4). The consumption preference shock follows an AR(1) process

$$\zeta_t = \phi^c \zeta_{t-1} + \iota_t, \tag{2.5}$$

with $|\phi^c| < 1$ and $\iota_t \sim N(0, \sigma_\iota)$.

The production function and the adjustment costs of investment.

Households produce output (y_t) according to a Cobb-Douglas production function depending on the amounts of labor demanded (n_t^d), employed capital (k_t), and on the state of technology represented by a technological shock (z_t)

$\log s_t = a_2(1 - \frac{a_0}{s^{ss}}) \log c_t - a_3(1 - \frac{a_0}{s^{ss}}) \log m_t + \chi_t$,
where the constant term was left out.

$$y_t = f(z_t, n_t^d, k_t) = \exp(z_t)(n_t^d)^{1-\alpha} k_t^{\alpha}, \tag{2.6}$$

where we do not include any notation referring to the i-th household because, as supposed above, all the households have access to the same $f(.)$ technology and produce the same amount of output under symmetric equilibrium. The law of motion for the productivity shock is an AR(1) without constant from a normal distribution

$$z_t = \phi^z z_{t-1} + \varepsilon_t, \tag{2.7}$$

with $|\phi^z| < 1$, and $\varepsilon_t \sim N(0, \sigma_\varepsilon)$. Households can save part of their output for the next period to be used in their production function as capital. They also need to replace the depreciated capital. Thus, investment is defined as:

$$x_t = k_{t+1} - (1-\delta)k_t \tag{2.8}$$

where δ is the depreciation rate. There exist adjustment costs to transform output into capital. The adjustment costs of investment function $C(x_t)$ used here was taken from Abel (1982)

$$C(x_t) = \psi x_t^{\eta} \tag{2.9}$$
$$with \qquad \psi > 0 \qquad and \qquad \eta > 1.$$

This functional form implies constant elasticity with respect to the amount of investment and increasing marginal adjustment costs. As a consequence of the latter, the production function net of adjustment costs $f(.) - C(x_t)$ has decreasing returns to scale so that the optimal size of the production plants is bound to be small.

Money creation and the government budget constraint.

Money is issued by the government. A monetary policy rule is designed based on the nominal money growth. Let μ_t be the rate of growth of nominal money in period t

$$\mu_t = \frac{M_t}{M_{t-1}} - 1. \tag{2.10}$$

The monetary policy rule used is

$$\mu_t = \phi_0^\mu + \phi_1^\mu \mu_{t-1} + \upsilon_t, \tag{2.11}$$

with $|\phi_1^\mu| < 1$, and $\upsilon_t \sim N(0, \sigma_\upsilon)$ is Gaussian white noise. The monetary authorities decide the values of the coefficients of (2.11) to determine the degree of persistence of the money-growth (ϕ_1) and the long-term rate of inflation ($\pi^{ss} = \mu^{ss} = \frac{\phi_0^\mu}{1-\phi_1^\mu}$).[6]

Increments in nominal money are paid to the households as lump-sum government transfers. There are also government bonds which can be purchased by the households and yield a nominal interest rate. Hence, the government (nominal) budget constraint is

$$G_t = M_t - M_{t-1} + (1 + R_t)^{-1} B_{t+1} - B_t \tag{2.12}$$

in which G_t are nominal lump-sum transfers and $(1+R_t)^{-1}B_{t+1}$ are the nominal purchases of bonds in period t that will be reimbursed for B_{t+1} nominal units in period $t+1$. Therefore, R_t is the rate of nominal interest of bonds bought in period t. Dividing both sides of (2.12) by the price level, we obtain the government budget constraint in real magnitudes

$$g_t = m_t - (1 + \pi_t)^{-1} m_{t-1} + (1 + r_t)^{-1} b_{t+1} - b_t. \tag{2.13}$$

Lower case letters in (2.13) denote real values of capital letters and the following definitions were utilized for inflation π_t, and the real return on bonds $1 + r_t$,

$$\pi_t = \frac{P_t}{P_{t-1}} - 1, \tag{2.14}$$

$$1 + r_t = \frac{1 + R_t}{E_t\left[1 + \pi_{t+1}\right]}. \tag{2.15}$$

The rational expectation operator $E_t[.]$ appearing in (2.15) represents the expectation in period t conditioned to the set of information available in that period.

[6] Since this is a no-growth model, the steady state rate of inflation is equal to the steady state rate of growth of nominal money ($\pi^{ss} = \mu^{ss}$).

3 Building a general equilibrium

3.1 The decision process

Households face the following budget and time constraints in period t

$$\begin{aligned} g_t + f(z_t, n_t^d, k_t) - C(x_t) &= c_t + k_{t+1} - (1-\delta)k_t + w_t(n_t^d - n_t^s) + \\ & \quad m_t - (1+\pi_t)^{-1} m_{t-1} + (1+r_t)^{-1} b_{t+1} - b_t, \\ T &= n_t^s + s_t + l_t. \end{aligned}$$

There are two sources of real income for the household: lump-sum real transfers from the government (g_t) and their own output production after subtracting the adjustments cost of investment ($f(z_t, n_t^d, k_t) - C(x_t)$). Income is spent on consumption (c_t), on investment ($k_{t+1} - (1-\delta)k_t$), on payments to the labor force hired in the market ($w_t(n_t^d - n_t^s)$, with w_t denoting the real wage, and on increasing the amounts of real money ($m_t - (1+\pi_t)^{-1} m_{t-1}$) or bonds ($(1+r_t)^{-1} b_{t+1} - b_t$).

As for the time constraint, planned allocation of total time T consists of an amount of worktime supplied n_t^s, some time required for shopping s_t, and the remaining leisure time l_t. Obviously, labor must be supplied by the households. As producers, they will raise a labor demand which may be different from their labor supply. If it is the case that they coincide, we would say that the labor market clears at the current prices. Otherwise, the labor demand dominates in the labor market and becomes the actual labor force.

Households make decisions according to the following program:

$$\underset{c_t, k_{t+1}, n_t^d, n_t^s, l_t, m_t, b_{t+1}}{Max} \quad E_t \sum_{j=0}^{\infty} \beta^j U(\zeta_{t+j}, c_{t+j}, l_{t+j})$$

subject to

$$\begin{gathered} E_t[g_{t+j} + f(z_{t+j}, n_{t+j}^d, k_{t+j}) - C(x_{t+j}) - c_{t+j} - k_{t+1+j} + (1-\delta)k_{t+j} - w_{t+j}(n_{t+j}^d - n_{t+j}^s) \\ - m_{t+j} + (1+\pi_{t+j})^{-1} m_{t-1+j} - (1+r_{t+j})^{-1} b_{t+1+j} + b_{t+j}] = 0 \quad \text{for all } j \geq 0 \\ E_t[T - n_{t+j}^s - s(\chi_{t+j}, c_{t+j}, m_{t+j}) - l_{t+j}] = 0 \qquad \text{for all } j \geq 0. \end{gathered}$$

The first order conditions resulting from the household rational choices in period t are[7] [8]

$$U_{c_t} - \lambda_{1t} - \lambda_{2t} s_{c_t} = 0 \tag{3.1}$$

$$-\lambda_{1t}(1 + C_{x_t}) + \beta E_t[\lambda_{1t+1}(1 + f_{k_{t+1}} - \delta + (1-\delta)C_{x_{t+1}})] = 0 \tag{3.2}$$

$$\lambda_{1t}(f_{n_t} - w_t) = 0 \tag{3.3}$$

$$\lambda_{1t} w_t - \lambda_{2t} = 0 \tag{3.4}$$

$$U_{l_t} - \lambda_{2t} = 0 \tag{3.5}$$

$$-\lambda_{1t} + \beta E_t[\lambda_{1t+1}(1 + \pi_{t+1})^{-1}] + \lambda_{2t} s_{m_t} = 0 \tag{3.6}$$

$$-\lambda_{1t}(1 + r_t)^{-1} + \beta E_t[\lambda_{1t+1}] = 0 \tag{3.7}$$

$$\begin{aligned} g_t + f(z_t, n_t^d, k_t) - C(x_t) - c_t - k_{t+1} + (1-\delta)k_t - \\ w_t(n_t^d - n_t^s) - m_t + (1+\pi_t)^{-1} m_{t-1} - (1+r_t)^{-1} b_{t+1} + b_t = 0 \end{aligned} \tag{3.8}$$

$$T - n_t^s - s(\chi_t, c_t, m_t) - l_t = 0 \tag{3.9}$$

where the discount factor is $\beta = \frac{1}{1+\rho}$ being $\rho > 0$ the rate of intertemporal preference, and λ_{1t} and λ_{2t} are the Lagrange multipliers attached to the budget and time constraints respectively.

3.2 The consumption function

We begin the analysis with the consumption rational choice. Plugging the Lagrange multiplier of the time constraint λ_{2t} from (3.4) into (3.1) and solving for the Lagrange multiplier of the budget constraint λ_{1t}, we have

[7] We employed the following notation for partial derivatives in a particular time period. If $V(x_t, y_t)$ is a generic function in period t, we have
$\frac{\partial V(x_t,y_t)}{\partial x_t} \equiv V_{x_t}$ and $\frac{\partial V(x_t,y_t)}{\partial y_t} \equiv V_{y_t}$.

[8] The following transversality conditions must also be included to rule out undesired dynamic paths for capital and bonds respectively:

$$\begin{aligned} \lim_{j\to\infty} k_{t+1+j}\beta^{t-1+j}\lambda_{1t+j} &= 0, \\ \lim_{j\to\infty} b_{t+1+j}\beta^{t-1+j}\lambda_{1t+j} &= 0. \end{aligned}$$

$$\lambda_{1t} = \frac{U_{c_t}}{1 + w_t s_{c_t}}. \tag{3.10}$$

Our shopping time functional form (2.2) and the CRRA utility function (2.4) were substituted in (3.10) to yield

$$\lambda_{1t} = \frac{\exp(\zeta_t) c_t^{-\sigma}}{1 + w_t \exp(\chi_t) a_1 a_2 c_t^{a_2-1} m_t^{-a_3}}. \tag{3.11}$$

The resulting non-linear expression (3.11) can be log-linearized so as to obtain[9]

$$\widehat{\lambda}_{1t} = -(\sigma + (a_2 - 1) ws_c)\widehat{c}_t + \zeta_t - ws_c(\widehat{w}_t + \chi_t - a_3 \widehat{m}_t) \tag{3.12}$$

where $ws_c = w^{ss} s_c(0, c^{ss}, m^{ss}) \geq 0$ is the steady-state marginal cost of consumption in output units. Here the "hat" variables denote log-approximations of percent deviations from steady state values, e.g., $\widehat{c}_t = \log\left(\frac{c_t}{c^{ss}}\right)$. Taking logarithms in (3.7), we obtain

$$\log \lambda_{1t} = \log \beta + E_t \log \lambda_{1t+1} + \log(1 + r_t). \tag{3.13}$$

We use $\log(1 + r_t) \simeq r_t$ and $\log \beta = -\log(1 + r^{ss}) \simeq -r^{ss}$ [10] to approximate fairly well (3.13) with

$$\widehat{\lambda}_{1t} = E_t \widehat{\lambda}_{1t+1} + (r_t - r^{ss}). \tag{3.14}$$

Moving equation (3.12) into period $t + 1$ and taking rational expectations, a expression for $E_t \widehat{\lambda}_{1t+1}$ is reached. If we substitute the resulting expression and (3.12) itself in equation (3.14), it will yield after some algebra

$$\widehat{c}_t = E_t \widehat{c}_{t+1} - \tfrac{1}{\sigma+(a_2-1)ws_c}(r_t - r^{ss}) - \tfrac{ws_c}{\sigma+(a_2-1)ws_c}(\widehat{w}_t - E_t \widehat{w}_{t+1}) +$$
$$\tfrac{a_3 ws_c}{\sigma+(a_2-1)ws_c}(\widehat{m}_t - E_t \widehat{m}_{t+1}) + \tfrac{1}{\sigma+(a_2-1)ws_c}\left(\zeta_t - E_t \zeta_{t+1}\right) - \tfrac{ws_c}{\sigma+(a_2-1)ws_c}\left(\chi_t - E_t \chi_{t+1}\right) \tag{3.15}$$

[9] See Uhlig (1998) for a complete description of the techniques used in the loglinearization process.

[10] Note that in a Sidrauski-type optimizing model, the steady state real interest rate r^{ss} is equal to the rate of intertemporal preference ρ. As a result, $\beta = (1 + \rho)^{-1} = (1 + r^{ss})^{-1}$.

The laws of motion for the shopping time shock χ_t and the preferences shock ζ_t were defined in (2.3) and (2.5) respectively. Using these expressions to calculate $E_t\chi_{t+1}$ and $E_t\zeta_{t+1}$, (3.15) becomes

$$\widehat{c}_t = E_t\widehat{c}_{t+1} - \tfrac{1}{\sigma+(a_2-1)ws_c}(r_t - r^{ss}) - \tfrac{ws_c}{\sigma+(a_2-1)ws_c}(\widehat{w}_t - E_t\widehat{w}_{t+1}) + \tfrac{a_3 ws_c}{\sigma+(a_2-1)ws_c}(\widehat{m}_t - E_t\widehat{m}_{t+1}) + \tfrac{(1-\phi^c)}{\sigma+(a_2-1)ws_c}\zeta_t. \qquad (3.16)$$

In turn, we may represent the consumption function through the following generic function

$$\widehat{c}_t = E_t\widehat{c}_{t+1} + \alpha_1(r_t - r^{ss}) + \alpha_2(\widehat{w}_t - E_t\widehat{w}_{t+1}) + \alpha_3(\widehat{m}_t - E_t\widehat{m}_{t+1}) + \alpha_4\zeta_t$$
$$\textit{with} \qquad \alpha_1, \alpha_2 < 0 \qquad \textit{and} \qquad \alpha_3, \alpha_4 > 0 \qquad (3.17)$$

which is a forward-looking consumption function depending on the real interest rate, a real wage effect, a monetary component, and a preference stochastic process.[11] A rise in the real interest rate induces the household to save more and consume less now that the real return on risk-free assets is greater. This behavior appears as a negative consumption semi-elasticity to the real interest rate (also called elasticity of intertemporal substitution), $\alpha_1 < 0$. The higher (in absolute value) this number is the higher is the variability of consumption over the cycle with a more sensitive response to departures of the real interest rate from its steady state value. Even though many other parameters take part in the composition of α_1 (see 3.16), its figure mostly depends quantitatively upon the coefficient of relative risk aversion $(\frac{-1}{\sigma})$.[12]

[11] Alternatively, the consumption function (3.17) can be expressed as
$\widehat{c}_t = \alpha_1 \sum\limits_{j=0}^{\infty} E_t(r_{t+j} - r^{ss}) + \alpha_2\widehat{w}_t + \alpha_3\widehat{m}_t + \alpha_4 \sum\limits_{j=0}^{\infty} E_t\zeta_{t+j}$,
from which consumption depends on current and expected future real interest rate, on the current real wage, on current real money holdings, and on current and expected future preference shocks.

[12] After calibration in Secton 6, the value of ws_c obtained will be rather close to zero ($ws_c = 0.0052$) implying a quantitatively small significance of the real money and real wage variables in the consumption decision. In the limit, $\alpha_1 = \frac{-1}{\sigma}$ when ws_c is exactly zero and then neither real money balances nor the real wage appear in the consumption function. This result is reached for money in the utility function models that present separability between consumption and real money in the utility function (see McCallum (1999a) for a more extensive discussion).

Consumption choices are neither separable from real money nor from the real wage because these two variables affect the shadow price of consumption λ_{1t}. A higher amount for real wage paid in the labor market implies a higher price (or opportunity cost) for the time spent purchasing bundles of consumption. As (3.10) shows, higher real wage makes the shopping time more costly and thus reduces the shadow price of consumption λ_{1t}, i.e., the degree of satisfaction obtained with the last bundle of goods consumed. The response from the household is consume less, $\alpha_2 < 0$. Regarding the monetary terms, more real money balances lessen the marginal cost of consumption in terms of time ($s_{cm} < 0$) so as to make the satisfaction of consuming higher. This leads to a positive value of α_3 in (3.16). Finally, the state of the consumption preference coming from the utility function ζ_t enters the consumption function affecting positively to the consumption level ($\alpha_4 > 0$).

Two particular cases of the consumption function are worthwhile to view. In both situations, the shopping time function (2.2) involves separability between consumption and real money ($s_{cm} = 0$). First, when the time allocation does not vary with the level of consumption ($a_2 = 0$ in (2.2)),[13] there are neither real money nor real wage in the resulting consumption function which resembles the IS curve commonly utilized in the recent literature:[14]

$$\widehat{c}_t = E_t\widehat{c}_{t+1} + \alpha_1'(r_t - r^{ss}) + \alpha_4'\zeta_t \tag{3.17'}$$
$$\textit{with} \qquad \alpha_1' = \frac{-1}{\sigma} \qquad \textit{and} \qquad \alpha_4' = \frac{(1-\phi^c)}{\sigma}.$$

Second, if the shopping time is real money invariant ($a_3 = 0$ in (2.2)), there still is a real wage effect but the real money balances disappear from the consumption equation:

$$\widehat{c}_t = E_t\widehat{c}_{t+1} + \alpha_1''(r_t - r^{ss}) + \alpha_2''(\widehat{w}_t - E_t\widehat{w}_{t+1}) + \alpha_4''\zeta_t \tag{3.17''}$$
$$\textit{with} \qquad \alpha_1'' = \alpha_1, \alpha_2'' = \alpha_2 \qquad \textit{and} \qquad \alpha_4'' = \alpha_4.$$

[13] Note that in this case $s_c(c^{ss}, m^{ss}, 0) = a_1 a_2 (c^{ss})^{a_2-1} (m^{ss})^{-a_3} = 0$ and then $ws_c = 0$ in expression (3.16).

[14] For examples, see Kerr and King (1996), Rotemberg and Woodford (1997), McCallum and Nelson (1999), and many others.

3.3 The investment function

We turn now to derive the investment function. The first order condition of k_{t+1} (3.2) governs the decision about the investment to take during period t. Combining (3.2) with (3.7) to eliminate the Lagrange multiplier λ_{1t}, we can reach

$$1 + r_t = \frac{1 + E_t f_{k_{t+1}} - \delta + (1-\delta) E_t C_{x_{t+1}}}{1 + C_{x_t}}. \tag{3.18}$$

The left hand side of (3.18) is the return on the financial asset (government bonds) and the right hand side is the expected return on the real asset (physical capital). As shown, the real interest rate does not equal the expected net marginal product of capital ($r_t \neq E_t f_{k_{t+1}} - \delta$). The existence of adjustment costs of investment creates a gap between the expected net return on the physical asset and the return on the financial asset. The arbitrage between physical and financial assets occurs taking into account increasing marginal adjustment costs that lead to a slower responsiveness of investment. Changes in the stock of capital are split up over time in order to avoid high installation costs. In steady state terms, the physical asset premium implied by (3.18) is positive

$$\frac{f_k(0, n^{ss}, k^{ss}) - \delta - \delta C_x(x^{ss})}{r^{ss}} = 1 + C_x(x^{ss}).$$

since $C_x(x^{ss})$ is unambiguously positive for our adjustment cost specification (2.9). The existence of a greater steady-state return can be considered as a risk-premium to the real asset over the risk-free financial asset.

Let us move back to the investment equation. By means of taking natural logarithms on both sides of (3.18) and assuming that the marginal adjustment cost, the marginal product of capital net of depreciation, and the real interest rate are small relative to zero,[15] we obtain the approximation

$$C_{x_t} = E_t f_{k_{t+1}} - \delta - r_t + (1-\delta) E_t C_{x_{t+1}}. \tag{3.19}$$

[15] Consequently, we use the approximation $\log(1+z) \simeq z$ when $z \simeq 0$. All the variables under this approximation represent fractional returns on quarterly units that typically report figures close to zero and their steady state values indeed are close to zero.

For our adjustment cost specification (2.9), equation (3.19) implies the following non-linear relationship

$$\psi\eta x_t^{\eta-1} = E_t f_{k_{t+1}} - \delta - r_t + (1-\delta)\psi\eta E_t x_{t+1}^{\eta-1}. \tag{3.20}$$

Again using log-linearization techniques, we can fairly approximate the latter with the following semi-loglinear investment equation

$$\widehat{x}_t = (1-\delta)E_t\widehat{x}_{t+1} + \frac{1}{(\eta-1)C_x(x^{ss})}(E_t f_{k_{t+1}} - \delta - r_t), \tag{3.21}$$

where $C_x(x^{ss}) = \eta\psi(x^{ss})^{\eta-1}$. This is an "expectational" investment equation in which current period's investment depends on next period's investment and on the gap between the expected return on the physical asset and the return on the financial asset. Let q_t denote this expected premium:

$$q_t = E_t f_{k_{t+1}} - \delta - r_t.$$

When we substitute sequentially expected investment, (3.21) becomes

$$\widehat{x}_t = \frac{1}{(\eta-1)C_x(x^{ss})}E_t\sum_{j=0}^{\infty}(1-\delta)^j q_{t+j}. \tag{3.21'}$$

Our investment equation (3.21') indicates now that current investment depends positively on all the expected future premiums on investing in real assets. In addition, the value of $\frac{1}{(\eta-1)C_x(x^{ss})}$ is the semi-elasticity of investment to a change in the real asset's premium. This number directly affects the variability of investment over the economic cycle. Clearly, both parameters of the adjustment cost function influence the size of this number. In this regard, Table 1 reports figures of the semi-elasticity of the physical asset premium in (3.21) for various calibrations of the parameters ψ and η from the adjustment cost function.

Table 1

Semi-elasticity of q_t in the investment equation (3.21).

	$\psi = 0.025$	$\psi = 0.05$	$\psi = 0.075$	$\psi = 0.1$
$\eta = 2$	17.93	9.61	6.80	5.37
$\eta = 3$	5.84	3.45	2.61	2.16
$\eta = 4$	3.03	1.95	1.54	1.33
$\eta = 5$	1.95	1.34	1.10	0.96

A higher η reduces the semi-elasticity of q_t in the investment equation. It should be recalled that η is the elasticity of the total adjustment cost with respect to investment. When this cost increases more rapidly with x_t, cyclical movements in investment will be more moderate. Likewise, a rise in the scale parameter ψ increases the size of the adjustment cost and thus the value of the semi-elasticity in (3.21). Notably, the absence of adjustment costs ($\psi = 0$) implies $C_x(x^{ss}) = 0$ and an "infinite response" of investment to changes in the expected marginal product of capital premium in (3.21).[16]

3.4 The money demand function

First, let us substitute the value of $\beta E_t \lambda_{1t+1}$ from equation (3.7) into the first order condition with respect to the real money balances (3.6)

$$\lambda_{1t} = \lambda_{1t} E_t[(1 + r_t)^{-1}(1 + \pi_{t+1})^{-1}] + \lambda_{2t} s_{m_t}. \tag{3.22}$$

Using the labor supply first order condition (3.4), the Lagrange multiplier of the time constraint (λ_{2t}) can be expressed as a function of the Lagrange multiplier of the budget constraint (λ_{1t}). As a result, (3.22) yields

$$\lambda_{1t} = \lambda_{1t} E_t[(1 + r_t)^{-1}(1 + \pi_{t+1})^{-1}] + \lambda_{1t} w_t s_{m_t}. \tag{3.23}$$

[16] Finite responses when $\psi = 0$ can be calculated by solving an analogous model without explicit adjustment costs from installing capital. Note that equation (3.2) and (3.7) would imply $r_t = E_t f_{k_{t+1}} - \delta$ in comparison to the case with adjustment costs. Casares and McCallum (2000) solve this model and compare it with the adjustment-cost model.

Dividing both sides of (3.23) by λ_{1t} and applying the real interest rate definition (2.9), we can obtain

$$\frac{R_t}{1+R_t} = -w_t s_{m_t} \tag{3.24}$$

where the left hand side is the marginal cost of the last unit of real money held and the right hand side is the marginal benefit of the last unit of real money held. Both sides in (3.24) are measured in output value. Inserting the value of s_{m_t} implied by our shopping time function (2.2) yields

$$\frac{R_t}{1+R_t} = -w_t \exp(\chi_t) a_3 a_1 c_t^{a_2} m_t^{-a_3-1}. \tag{3.25}$$

Equation (3.25) can be loglinearized to formulate this semi-log money demand function[17]

$$\widehat{m}_t = \frac{a_2}{1+a_3}\widehat{c}_t + \frac{1}{1+a_3}\widehat{w}_t - \frac{1}{R^{ss}(1+a_3)}(R_t - R^{ss}) + \frac{1}{1+a_3}\chi_t, \tag{3.26}$$

which is to say

$$\widehat{m}_t = \varphi_1\widehat{c}_t + \varphi_2\widehat{w}_t - \varphi_2(R^{ss})^{-1}(R_t - R^{ss}) + \varphi_2\chi_t \tag{3.27}$$
$$with \qquad \varphi_1, \varphi_2 > 0.$$

Here it is noticeable that both the real wage and the shopping time shock accompany the standard determinants of real money: consumption and the nominal interest rate. What occurs in the labor market will have an impact in the demand of real money.[18] The real wage is the opportunity cost of one unit of time. When it goes up, there will be an incentive to demand more money and save some transactions time.

By construction, a positive shock to the transactions technology χ_t would represent an increase in the shopping time. More real balances would be held ($\varphi_2 > 0$) to compensate with more monetary services the effects of such shock.

[17] For the sake of simplicity and provided that the nominal interest rate is given in fractional quarterly units (small relative to zero), we took $R_t \simeq \frac{R_t}{1+R_t}$ to derive equation (3.26).

[18] This has been recognized in the literature for example in papers by Karni (1973), and Dutton and Gramm (1973).

When the volume of transactions to carry out is larger (higher c_t) more real balances are demanded. In addition, a rise in its opportunity cost R_t leads to a decrease in the amount of real money held.

The parameters of the shopping time function a_2 and a_3 define the size of the elasticities in the money demand. Actually, they are the only parameters appearing in (3.26). A change in the consumption elasticity in the shopping time function ($\eta_{s_t,c_t} = a_2(1 - \frac{a_0}{s^{ss}})$) caused by a movement in a_2 would only affect to the elasticity of consumption in the money demand. The impact would be of the same sign as the change in a_2. However, if the shopping time function becomes more elastic with respect to real money (higher $\eta_{s_t,m_t} = -a_3(1 - \frac{a_0}{s^{ss}})$ in absolute value caused by an increase in a_3), there would be a fall on all the elasticities of the money demand function. All of them would go down when real money is more useful in the shopping process.

3.5 The IS-LM economy and the general equilibrium

The overall resource constraint implied by combining the household budget constraint (3.8) and the government budget constraint (2.13) defines final goods market equilibria where the amount of output produced net of adjustment costs is spent on consumption and investment[19]

$$y_t - C(x_t) = c_t + x_t. \tag{3.28}$$

This expression (3.28) can be transformed in percent deviations from steady state to have

$$\widehat{y}_t = j_1 \widehat{c}_t + j_2 \widehat{x}_t \tag{3.29}$$

in which $j_1 = \frac{c^{ss}}{y^{ss}}$ and $j_2 = \frac{x^{ss}+C(x^{ss})}{y^{ss}}$. Current output deviations $\widehat{y}_t$ are demand determined. In other words, changes in either consumption or investment conform the level of $\widehat{y}_t$ resulting in the economy. Hence, the demand-side behavior of the model is represented by the structural seven-equation IS-LM sector:

[19] It is also used the fact previously mentioned that the labor market is demand-dominated and in turn the "effective" labor supply coincides with the labor demand. As a result, $w_t(n_t^d - n_t^s) = 0$.

$$
\begin{aligned}
\widehat{c}_t &= E_t\widehat{c}_{t+1} + \alpha_1(r_t - r^{ss}) + \alpha_2(\widehat{w}_t - E_t\widehat{w}_{t+1}) + \alpha_3(\widehat{m}_t - E_t\widehat{m}_{t+1}) + \alpha_4\zeta_t, && \text{(3.30a)}\\
\widehat{x}_t &= (1-\delta)E_t\widehat{x}_{t+1} + \tfrac{1}{(\eta-1)C_x(x^{ss})}(E_t f_{k_{t+1}} - \delta - r_t), && \text{(3.30b)}\\
\widehat{y}_t &= j_1\widehat{c}_t + j_2\widehat{x}_t, && \text{(3.30c)}\\
\widehat{m}_t &= \varphi_1\widehat{c}_t + \varphi_2\widehat{w}_t - \varphi_2(R^{ss})^{-1}(R_t - R^{ss}) + \varphi_2\chi_t, && \text{(3.30d)}\\
\widehat{w}_t &= \widehat{y}_t - \widehat{n}_t^d, && \text{(3.30e)}\\
f_{k_t} &= \tfrac{\alpha y^{ss}}{k^{ss}}(\widehat{y}_t - \widehat{k}_t), && \text{(3.30f)}\\
\widehat{k}_{t+1} &= \delta\widehat{x}_t + (1-\delta)\widehat{k}_t. && \text{(3.30g)}
\end{aligned}
$$

The first two equations are the consumption function (3.17) and the investment function (3.21) previously derived. The third equation is the demand-determined composition of output (3.28) as consumption plus investment. Applying Fisher equation $r_t = R_t - E_t\pi_{t+1}$, both components of output are negatively related to the nominal interest rate. In turn, there exists a negative functional form between output and the nominal interest rate, similar to the traditional IS relationship appearing in the macroeconomics textbooks. Unlike the latter, the structural IS sector presented here includes next period's expected values in the consumption and investment decision rules as implied by the optimizing analysis developed above.

The fourth relation is our structural money demand LM equation (3.30d) that determines time paths for $\widehat{m}_t$ as derived in the previous subsection. Typically, the LM relationship is represented in the textbooks as a functional form relating demand for real money positively to the level of output and in a negative way to the nominal interest rate.

Remarkably, the IS and LM relationships are not separable because the consumption choice, accounting for the IS sector, is affected by the amount of money determined in the LM function.[20] In addition, the presence of the real wage in the consumption and demand for money decisions require the inclusion of (3.30e). This equation states that the marginal product of labor is equal to the real wage as derived in the household maximizing program in (3.3). The sixth equation is the marginal product of capital definition which is used in the investment equation. The last equation (3.30g) is the time path for $\widehat{k}_{t+1}$ defined as a weighted average of the (predetermined) value of $\widehat{k}_t$, and the magnitude of $\widehat{x}_t$ decided in

[20] This fact arises from the non-separability of c_t and m_t in the transactions technology function. This assumption is necessary to imply $s_{cm} < 0$ as expected due to the transactions-facilitating role of money.

(3.30b). This expression is required to have capital endogenously determined and thus to obtain the expected marginal product of capital entering the investment equation (3.30b).

A general equilibrium economy is defined when all the households are acting optimally and markets clear. Therefore, we introduce the following relations to guarantee equilibrium in the labor market:

$$\begin{aligned}
\widehat{n}_t^d &= \widehat{y}_t - \widehat{w}_t, && (3.30e)\\
\widehat{n}_t^s &= \frac{l^{ss}}{\gamma n^{ss}}\widehat{\lambda}_{1t} + \frac{l^{ss}}{\gamma n^{ss}}\widehat{w}_t - \frac{s^{ss}}{n^{ss}}\widehat{s}_t, && (3.31)\\
\widehat{\lambda}_{1t} &= -(\sigma + (a_2 - 1)ws_c)\widehat{c}_t + \zeta_t - ws_c(\widehat{w}_t + \chi_t - a_3\widehat{m}_t) && (3.32)\\
\widehat{s}_t &= a_2(1 - \frac{a_0}{s^{ss}})\widehat{c}_t - a_3(1 - \frac{a_0}{s^{ss}})\widehat{m}_t + \chi_t, && (3.33)\\
\widehat{n}_t^d &= \widehat{n}_t^s. && (3.34)
\end{aligned}$$

Note that the labor demand expression (3.30e) has already been included within the IS-LM sector in order to determine $\widehat{w}_t$. Hence, we have four new equations (3.31)-(3.34). Labor supply behavior is governed by (3.31) which can be obtained after combining and loglinearizing the first order conditions (3.4), (3.5), and (3.9). Equations (3.32) and (3.33) determine the values of the consumption shadow price $\widehat{\lambda}_{1t}$ and the shopping time $\widehat{s}_t$, both employed in making labor supply decisions in (3.31). The former is identical to (3.12) as derived above from the household's first order condition of consumption, and the latter results from loglinearizing the shopping time function (2.2). Labor market clears by holding (3.34).

The general equilibrium system includes a loglinearization of the production function (2.6),

$$\widehat{y}_t = z_t + \alpha\widehat{k}_t + (1-\alpha)\widehat{n}_t^d, \tag{3.35}$$

together with the definitions:

$$\begin{aligned}
r_t &= R_t - E_t\pi_{t+1}, && (3.36)\\
\widehat{m}_t &= \log M_t - \log P_t, && (3.37)\\
\pi_t &= \log P_t - \log P_{t-1}, && (3.38)\\
\mu_t &= \log M_t - \log M_{t-1}, && (3.39)
\end{aligned}$$

that are the Fisher equation (3.36), and log-linear expressions equivalent to (2.1), (2.10), and (2.14) respectively.[21] In order to close the general equilibrium model, we also define a monetary policy rule such as (2.11):[22]

$$\mu_t = \phi_0^\mu + \phi_1^\mu \mu_{t-1} + \upsilon_t, \tag{2.11}$$

In turn, there exists a general equilibrium consisting of seventeen equations (3.30a)-(3.30g), (3.31)-(3.39), and (2.11) that permit to solve solution paths for the seventeen endogenous variables $\widehat{y}$, $\widehat{c}$, $\widehat{x}$, $\widehat{m}$, $\widehat{w}$, $\widehat{n}^d$, $\widehat{n}^s$, $\widehat{\lambda}_1$, $\widehat{s}$, $\widehat{k}$, f_k, $\log M$, $\log P$, μ, R, r, and π.

4 The output gap

How is $\widehat{y}_t$ produced? As we saw above, the existing Cobb-Douglas technology can be expressed in a log-linear fashion as follows

$$\widehat{y}_t = z_t + \alpha \widehat{k}_t + (1-\alpha)\widehat{n}_t^d. \tag{4.1}$$

The stock of capital $\widehat{k}_t$ is a predetermined variable,[23] and the state of technology z_t is exogenous. Thus, it is the amount of labor hired $\widehat{n}_t^d$ which accommodates production to satisfy the demand in (4.1) on a period-to-period basis.

At this point, we may ask whether the workforce hired $\widehat{n}_t^d$ is equal to the corresponding desired labor supply $\widehat{n}_t^s$ to test the existence of equilibrium in the labor market. In a flexible-price scenario, prices would adjust immediately to restore equilibrium whenever an excess of demand (or supply) occurred. But if it is not the case, we may have non-equilibrium situations. Then two measures of output become relevant: current output and capacity (market-clearing) output. The latter is the amount that would be produced in the economy if there were perfect flexibility on prices, what is to say, if there were equilibrium in the

[21] Expression (2.1) can be understood both as the definition of real money balances and as the equilibrium condition in the money market.

[22] Alternatively, we could have R_t as the monetary policy instrument, and then $\log M_t$ would be the amount clearing the money market at a given R_t.

[23] However, the time sequence of the stock of capital is endogenous and linked to the investment decision as indicated by (3.30g), $\widehat{k}_{t+1} = \delta \widehat{x}_t + (1-\delta)\widehat{k}_t$.

labor market. Hence, denoting $\widehat{\overline{n}}_t$ as the market-clearing labor deviations, capacity output deviations $\widehat{\overline{y}}_t$ is

$$\widehat{\overline{y}}_t = z_t + \alpha \widehat{k}_t + (1 - \alpha)\widehat{\overline{n}}_t. \tag{4.2}$$

At this point, it is necessary to determine the value of $\widehat{\overline{n}}_t$. We proceed by using the labor supply and labor demand functions obtained in the general equilibrium described above. There, first order conditions (3.4), (3.5), and (3.9) led to the following labor supply function (after loglinearization)

$$\widehat{n}_t^s = \frac{l^{ss}}{\gamma n^{ss}} \widehat{\lambda}_{1t} + \frac{l^{ss}}{\gamma n^{ss}} \widehat{w}_t - \frac{s^{ss}}{n^{ss}} \widehat{s}_t. \tag{4.3}$$

The labor supply function presents a positive slope with respect to the real wage. The value of the elasticity of the labor supply function is set by means of the parameter γ. Willingness to work is also positively influenced by the current marginal utility of consumption $\widehat{\lambda}_{1t}$. When consumption is more satisfying, households wish to work more to reach a higher level of consumption. The third element in (4.3) is the shopping time. More shopping time reduces the labor supply. Turning to the labor demand, its behavior is governed by first order condition (3.3), or (3.30e) in the IS-LM sector, which yields

$$\widehat{n}_t^d = \widehat{y}_t - \widehat{w}_t. \tag{4.4}$$

The interpretation of (4.4) is the standard assumption to define a labor demand function: marginal product of labor equal to real wage. Now, we can define the market-clearing real wage $\widehat{\overline{w}}_t$ as the value such that the quantities of labor supplied and demanded coincide ($\widehat{n}_t^s = \widehat{n}_t^d = \widehat{\overline{n}}_t$). Hence, making (4.3) and (4.4) equal, we have

$$\widehat{\overline{w}}_t = \frac{\gamma n^{ss}}{l^{ss} + \gamma n^{ss}} \widehat{y}_t - \frac{l^{ss}}{l^{ss} + \gamma n^{ss}} \widehat{\lambda}_{1t} + \frac{\gamma s^{ss}}{l^{ss} + \gamma n^{ss}} \widehat{s}_t. \tag{4.5}$$

Finally, substituting $\widehat{\overline{w}}_t$ from (4.5) into either the labor supply (4.3) or labor demand functions (4.4) and rearranging terms, the market-clearing labor $\widehat{\overline{n}}_t$ is determined as follows

$$\widehat{\overline{n}}_t = \frac{l^{ss}}{l^{ss} + \gamma n^{ss}} \widehat{y}_t + \frac{l^{ss}}{l^{ss} + \gamma n^{ss}} \widehat{\lambda}_{1t} - \frac{\gamma s^{ss}}{l^{ss} + \gamma n^{ss}} \widehat{s}_t. \tag{4.6}$$

The value of $\widehat{\overline{n}}_t$ obtained in (4.6) enters the production function (4.2) together with z_t and $\widehat{k}_t$ to define capacity output $\widehat{\overline{y}}_t$. Once we have determined both current output $\widehat{y}_t$ and capacity output $\widehat{\overline{y}}_t$, the output gap $\widetilde{y}_t$ is the difference between the former and the latter

$$\widetilde{y}_t = \widehat{y}_t - \widehat{\overline{y}}_t. \tag{4.7}$$

The output gap says how demand is relative to capacity. Of course, if prices were not sticky, they would instantaneously adjust to remove the output gap and have $\widehat{y}_t = \widehat{\overline{y}}_t$.

5 Price stickiness

Price stickiness has become an habitual element in macro-monetary models. An enormous literature of sluggishness mechanisms erupted after the seminal works by Taylor (1979) and Calvo (1983) in an attempt to explain how monetary impulses (or 'surprises') were transmitted to generate "real effects". The presence of nominal rigidities leads to disruptions from the general equilibrium setup due to the existence of non-equilibrium situations in (at least) one of the markets. In particular, our partial equilibrium model typically presents excess demand or excess supply in the labor market and equilibrium in the goods, capital, bonds, and money markets.

A two-period contract price specification is used as in Fuhrer and Moore (1995). Our choice comes from the fact that Fuhrer-Moore price formulations guarantee a realistically high degree of persistence on inflation whereas most other models do not (see Nelson (1998)).

Nominal contract prices are set to last for two periods. Thus, half of the households decide in period t the contract price d_t at which they will sell their production in the current and in the next period. The other half of the households made this decision one period ago and their contract price d_{t-1} is still in effect until the end of period t. Thus, the price level in period t is the average between these two contract prices[24]

[24] The price level is expressed here in logarithmic form ($p_t = \log P_t$). Consequently, the contract prices, d_t and d_{t-1}, are also in logarithms.

$$p_t = 0.5(d_t + d_{t-1}). \tag{5.1}$$

Unlike Taylor's staggered nominal contracts, the value of the contract is set as follows

$$d_t - p_t = 0.5(d_{t-1} - p_{t-1}) + 0.5E_t(d_{t+1} - p_{t+1}) + \varrho\widetilde{y}_t + \varrho E_t\widetilde{y}_{t+1}. \tag{5.2}$$

Comparisons take place in determining the contracts. Hence, households include both the past and expected future period real contracts in their decision because they will be traded while d_t is in effect. Furthermore, current and expected output gaps also enter the real contract equation to represent the demand pressure during the lifetime of the contracts. We assume $\varrho > 0$ in (5.2).

6 Calibration

Some sensible figures must now be assigned to the parameters of the model. In this regard, we look at actual data and intend to replicate what is observed in reality. Particularly, quarterly US data are used to calibrate the model. Indeed, any of the parameters calibrated here might be altered under particular circumstances, and actually it would result in a sensitivity analysis of much interest.

The standard figure in the literature is given to the capital share parameter in the production function ($\alpha = 0.36$). The rate of intertemporal preference is set at 0.5% per quarter ($\rho = 0.005$), 2% per year. It implies $\beta = 0.995$. The depreciation rate for capital δ will be 2.5% per quarter ($\delta = 0.025$).

The coefficients of the monetary policy rule (2.9) have been estimated numerous times in the literature. As pointed out by Benassy (1995), the resulting estimates for the US economy are quite sensitive to the period chosen, the estimation method, and the definition of the money aggregate. Thus, Cooley and Hassen (1989) obtained $\phi_0 = 0.008$ and $\phi_1 = 0.48$. More recently, Hairault and Portier (1993) reported $\phi_0 = 0.0087$ and $\phi_1 = 0.377$, Ireland (1997) $\phi_0 = 0.0048$ and $\phi_1 = 0.68$, and Chari, Kehoe and McGrattan (1996) $\phi_0 = 0.0064$ and $\phi_1 = 0.57$. In our model, we set $\phi_0^\mu = 0.0025$ and $\phi_1^\mu = 0.5$.[25]

[25] These are realistic figures for the current low-inflation environment since they imply a steady-state inflation rate of 0.5% per quarter ($\pi^{ss} = \mu^{ss} = \frac{\phi_0^\mu}{1-\phi_1^\mu} = 0.005$), say 2% per year.

In a Sidrauski-type (neoclassical) model like the one presented here, the steady state real interest rate r^{ss} is the rate of intertemporal preference ρ. Therefore, the steady state nominal interest rate (neglecting the cross product $\rho\pi$) is $R^{ss} = \pi^{ss} + r^{ss} = 0.005 + 0.005 = 0.01$ which implies a 4% annual rate.

The utility function includes three parameters: σ, γ, and Υ. We take the common figure $\sigma = 5$ as in Chari, Kehoe and McGrattan (1996), Chari, Christiano and Kehoe (1994), and McCallum and Nelson (1997). With $\sigma = 5$, the coefficient of relative risk aversion is 0.2 and the real interest rate semi-elasticity of consumption is practically equal to -0.2 ($\alpha_1 = -0.1991$). The value of Υ is defined so that one third of the total time is spent at work in the steady state solution of the model, $\Upsilon = 116.25$. Arbitrarily, it is set $n^{ss} = 1$, and therefore $T = 3$. The response of the amount of labor supplied by the households to the real wage is governed in the labor supply function (4.3) by the magnitude of γ. Actually, the labor supply elasticity depends there on the steady state leisure-to-work time ratio and on the value assigned to γ ($\eta_{n_t^s,w_t} = \frac{l^{ss}}{\gamma n^{ss}}$). Being $\frac{l^{ss}}{n^{ss}} = 1.9688$, we set $\gamma = 13.125$ in order to have $\eta_{n_t^s,w_t} = 0.15$. This figure is taken from Johnson and Pencavel (1984) where an extensive labor supply empirical study is conducted for US data. A low number of the labor supply elasticity has been generally utilized in the literature (see Pencavel (1986) for a survey).

The figure of η in the adjustment cost of investment function (2.9) is chosen to imply a semi-elasticity of investment relative to the expected real asset premium z_t equal to 1.5 as in Casares and McCallum (2000). In addition, the size of the adjustment cost implies a ratio between the total adjustment cost and output equal to 1% in steady state, i.e., $\frac{C(x^{ss})}{y^{ss}} = 0.01$. Accordingly, we set $\eta = 4.65$ and $\psi = 0.05$.

The money demand formulation (3.26) derived above can be used to calibrate the coefficients of the shopping time function (2.2). Thus, a_3 takes a value such that the interest rate semi-elasticity in the real money demand is -10, or there is an interest rate elasticity equal to -0.1 since $R^{ss} = 0.01$. The consumption elasticity depends on both a_2 and a_3. Having set a_3 already, we chose the figure of a_2 to imply a consumption elasticity of 0.5. These figures are consistent with estimates recently reported in long-run money demand studies on US data by Ball (1997), and Meltzer and Rasche (1996). The other two parameters a_0 and a_1 are calibrated looking at the baseline steady state solution. The steady state version of the money demand function has a constant term. The value assigned to a_1 leads to an unitary consumption over real money ratio[26] in steady state ($\frac{c^{ss}}{m^{ss}} = 1$). Finally, a_0 was calibrated by considering the steady state shopping time as 1.04% of the total time. This is equivalent to say that 2% of output in steady state is spent on transaction costs for the baseline inflation economy.

The coefficient of autocorrelation of the technology shock is set to the standard figure $\phi^z = 0.95$. As for the consumption preference random process, the coefficient of autocorrelation ϕ^c is 0.323 as in McCallum and Nelson (1997).

The Fuhrer-Moore equation presented in the previous section enters the model with a coefficient on the output gap equal to 0.008. This value was utilized by Nelson (1998) for US data analysis.

All the numbers obtained in the calibration are reported in Table 2. Furthermore, some relevant steady state information for the calibration at hand is contained in Table 3.

Table 2

Calibration of the parameters of the model

$\alpha = 0.36$	$\sigma = 5$	$a_0 = 0.0300$	$\psi = 0.05$	$\phi_0^\mu = 0.005$	$\varrho = 0.008$
$\delta = 0.025$	$\gamma = 13.125$	$a_1 = 0.0684$	$\eta = 4.65$	$\phi_1^\mu = 0.5$	
$\rho = 0.005$	$\Upsilon = 116.25$	$a_2 = 5$		$\phi^c = 0.323$	
$\beta = 0.995$		$a_3 = 9$		$\phi^z = 0.95$	

Table 3

Relevant ratios in the steady state solution.

$\frac{c}{y} = 0.7365$	$\frac{x}{y} = 0.2535$	$\frac{C(x)}{y} = 0.01$	$\frac{k}{y} = 10.15$
$\frac{c}{m} = 1$	$\frac{x}{m} = 0.3445$	$\frac{y}{m} = 1.32$	$\frac{k}{m} = 13.78$
$\frac{n}{T} = 0.3333$	$\frac{l}{T} = 0.6562$	$\frac{s}{T} = 0.0104$	$\frac{l}{n} = 1.9688$

The partial-equilibrium model is solved by means of a system of linear rational expectations equations using Paul Klein's algorithm (Klein (1997)).[27] This method reaches the

[26] On quarterly US data, the ratio of real money over consumption has a sample mean equal to 1.03 in the period 1965:1-1997:4. M1 money stock seasonally adjusted (SA) was taken to represent nominal money. As for the price level, the GDP implicit deflator was chosen. Finally, consumption is given by the amount of (SA) real personal consumption expenditures. The base year was 1992 for both consumption and the price level. Source: Federal Reserve Bank of St. Louis.

[27] Particularly, we ran the routine called "solvek.m" in MatLab 4.2.

minimal state variable solution of the system as explained in McCallum (1999b). The solution is expressed as decision rule functions of the endogenous variables responding to the state variables. There are two types of state variables: predetermined and exogenous (shocks). In the next section, we analyze the effects of changes in the latter.

7 Impulse-response functions

Impulse response functions are calculated as the impact on the endogenous variables of each of the four types of shocks incorporated to the model: production function (technology) shock, monetary policy rule shock, consumption preference shock in the utility function, and shopping time function shock. These shocks are generated by assigning a value of 1 to the error term of the exogenous processes in one particular quarter and a value of 0 for all the coming quarters. Figures 1-4 show plots of the resulting impulse response functions for the variables (in order of appearance) $\widehat{y}_t$, $\widehat{\overline{y}}_t$, $\pi_t - \pi^{ss}$, $\widehat{c}_t$, $\widehat{x}_t$, $\widehat{m}_t$, $\widehat{s}_t$, $\widehat{k}_{t+1}$, $\widehat{\overline{n}}_t$, $f_{k_t} - f_{k^{ss}}$, $r_t - r^{ss}$, and $R_t - R^{ss}$. As defined above, the "hat" figures plot percent deviations of these variables from their steady-state values. The marginal product of capital, the nominal and real interest rates, and the inflation rate are plotted as level deviations from steady state in fractional units.

7.1 Technology shock

A positive shock to the production function occurs when the exogenous component z_t takes a positive value in $f(.)$. Due to the high degree of serial correlation of the process ($\phi^z = 0.95$), a one-period shock implies lasting effects in the responses of output, consumption, investment, and real money balances. Figure 1 depicts these effects during the 30 quarters coming after the shock.

Capacity output rises more and earlier than current output creating a temporary negative output gap. It is interesting to observe in Figure 1 how current output reaches its peak (68% of the shock) with a delay of eight quarters with respect to the shock. Demand components, both consumption and investment, show their maximum responses after eight quarters as well. However, investment has a rather higher peak of around three times the size of consumption. On the contrary, the stock of capital presents a very slow positive response to a technology shock. Its maximum value is 45% of the shock, reached after 32 quarters (8 years!), featuring a smooth shape as Figure 1 shows.

Inflation drops during the next quarters after the shock. The major impact on inflation occurs four periods after the shock when it falls by 11% of the shock.[28] This quarter is later than the major responses on capacity output but earlier than the peak on current output. Both the marginal product of capital and, especially, the real interest rate go up as a result of a technology shock.

The demand for real money balances reports an increase by 64% of the shock with certain delay that is caused by a larger volume of consumption and a late fall in the nominal interest rate.

The shopping time function measures the transaction costs. Although consumption is rising with a technology shock there is no significant impact in transaction costs due to a simultaneous greater demand of more monetary services.

7.2 Monetary shock

In accordance with the monetary policy rule (2.11), a monetary shock represents an unexpected increase in the nominal money growth rate μ_t. The effects of a unit shock can be observed in Figure 2.

The response of inflation to a higher money growth reaches its maximum five quarters after the shock when there is an increase of 33% of the shock.[29] This represents a bell-shaped impulse response function as reported in the third box of Figure 2..

A liquidity (decreasing) effect on the nominal interest rate together with price stickiness that results in a slow rise of inflation lead to a u-shape fall in the real interest rate by 42% of the shock. In turn, both consumption and investment go up and make current output increase by 120% of the shock[30]. Investment is much more sensitive to the shock than consumption which presents a smoother pattern. Indeed, price stickiness brings about significant short-term real effects from money-growth surprises due to the persistent fall in

[28] Therefore, a technology shock that increases capacity output by 1% gives rise to a maximum drop in annual inflation of 0.44%.

[29] In other words, a 4% unexpected increase in the annual nominal money growth rate leads to a 1.32% maximum increase in the annual rate of inflation reached 5 quarters after the shock.

[30] One may believe that this response is too large. The reason for this can be the lack of any stabilizing mechanism in the money-growth monetary policy rule. If we introduced stabilizing properties in the monetary policy rule, for example, a Taylor-type rule, the response would be quite smaller.

the real interest rate.

On the supply side, we must see how capital and market clearing labor react to determine capacity output. Capital rises slowly to reach a 25% of the shock peak 6 quarters after the shock. As for market clearing labor, there is an immediate drop by 16% of the shock due to the decrease in the labor supply. The early fall on market-clearing labor dominates in the production function over the later increase in the stock of capital, and capacity output reports a slight decrease (10% of the shock).

The amount of $\widehat{m}_t$ rises by 120% of the shock due to lower nominal interest rates (liquidity effect to clear the money market), and more consumption expenditures. This response of real money is considerably larger than the response of consumption (by approximately 3 times). As a result, the transactions (shopping) time falls dawn by near 35% of the size of the shock during the first quarters after the shock. However, there is a progressive return to the steady state value and figures go over the positive side ten quarters after the shock.

7.3 Consumption preference shock

As shown in Figure 3, a unitary positive shock to the state of consumption preference entering the utility function leads to a rise in the consumption level by 20% of the size of the shock. Since consumption is one of the elements of output, there is as well a positive effect in this variable (14% of the shock increase).

The nominal interest rate, inflation, and the real interest rate respond very little. Real money balances also remain close to the steady state figures.

The consumption preference shock resulted in a higher consumption level with no significant change in real money. Thus, the transaction costs measured by the shopping time function rise during the quarters with higher consumption. This is quite a temporary increase and 3 quarters after the shock the shopping time practically returns to the steady state value. The size of the response of the shopping time is 4% of the shock.

7.4 Transactions (shopping) technology shock

A random walk was specified to describe the process of the exogenous shock to the shopping time function. Therefore, a shock that enters the process through the innovation the random walk will stay in the process for all future periods. A positive shock represents more transaction costs for the same level of $\widehat{c}_t$ and $\widehat{m}_t$, i.e., a worsening in shopping conditions. Figure 4 shows the impact of an unit shock in the variables of the model.

The transactions technology shock appears in the model as one of the explanatory variables in the money demand function. More money will be demanded to carry out transactions when the shopping time process is more costly. Hence, there is a permanent increase in the quantity of real money held completed 12 quarters after the shock. This rise amounts to about 10% of the shock.

The nominal interest rate and the rate of inflation also vary. A higher demand of real money generates an increase in the nominal interest rate and diminishing inflation. These changes are not quantitatively important but both together make the real interest rate rise about 2.3% of the shock in its peak. Higher real interest rates result into lower consumption, investment, and output. So, there will be some (significantly small) real effects of a transactions technology shock (see Figure 4).

Neither capacity output nor its two components, capital and market-clearing labor, shows substantial deviations from the steady state when a transactions technology shock occurs.

Obviously, the quantitatively most important impact of the transactions technology shock is observed in the shopping time function (seventh box in Figure 4). Transaction costs rise directly as a consequence of the shock. This increment becomes permanent under the assumption of a random walk process for the shock. As long as time passes, the initial effect is slightly reduced because of the higher demand for real money balances. In the end, there is a permanent response of the shopping time measured as an increment by 96% of the initial shock.

8 Conclusions

This paper has presented and solved a Sidrauski-type optimizing model with money incorporated in its role as a medium of exchange. We followed the time-cost transactions approach linking the amount of real money with the time needed to carry out purchases. Thus, a shopping time function gives the transaction costs as a function of the level of consumption, the amount of real money balances, and the state of transactions technology.

Economic decisions are made by a representative household that maximize utility subject to budget and time constraints. In turn, a set of first order conditions describe the dynamics of the system. We focused attention on the resulting consumption, investment, and money demand equations.

The consumption function is forward-looking because of the inclusion of the expected next period's consumption value as one of the explanatory variables. The amount of consumption depends negatively on the real interest rate and the real wage, and positively on a consumption preference shock and a real balance term.

Unlike most of the models of the recent literature, investment is one more endogenous variable of the model. We incorporated adjustment costs associated to the transformation of output savings into capital. Optimizing investors decide the quantity of current investment depending on all the expected premiums of the return on the physical asset (capital) over the return on the financial asset (government bonds). The existence of adjustment costs imply a positive steady state premium to the real assets which is equal to the marginal cost of installing investment. The results obtained here can be derived for any kind of monetary and non-monetary specification within the Sidrauski-type models.

Our money demand function gives the optimal quantity of real balances depending positively on consumption, the real wage and the transactions technology shock, and negatively on the nominal interest rate.

Output is demand-determined and was presented in deviations from its steady state as a weighted sum of consumption and investment. The demand side of the economy can be summarized in a set of seven equations that we named structural IS-LM economy. Furthermore, some definitions, market-clearing conditions, and a monetary policy rule on money growth are added to the IS-LM sector so as to obtain a general equilibrium.

Sticky prices were introduced following the Fuhrer-Moore contract prices setting. The existence of nominal rigidities gives rise to two definitions of output: current output and capacity (market-clearing) output. The setup described in the paper permits to have capacity output as another endogenous variable. Thus, using a Cobb-Douglas technology, capacity output is determined depending on capital movements, market-clearing labor, and the state of production technology.

Impulse response representations were calculated from shocks to the production function, to the monetary policy rule, to consumption preferences, and to the shopping time function. Current output featured sizeable responses to the production technology shock and to the monetary shock. Either positive shock had a significant positive effect on output: 70% of the size of the shock for the technology shock and 120% for the monetary shock. Investment behaved much more sensitive to the shocks than consumption. In addition, the technology shock featured much more persistent effects than the money growth shock.

Inflation shows a positive impact from a monetary shock and a negative effect from a technology shock. These responses reached peaks of 33% (increase) and 11% (decrease), respectively, of the shock. Two patterns were common in the responses of inflation: high persistence and delayed maximum.

The existence of adjustment costs made the responses of capital to technology or monetary shocks be reasonably small and well-constrained.

A consumption preference shock led to a rise in consumption and subsequently in output. The response of inflation was positive but not substantial. The last kind of shock analyzed was a shopping technology shock. The most significant response was a 95% permanent increase in transaction costs. There was also a permanent effect of a 10% increase on the amount of real money balances and some negative real effect on current output and its two components.

APPENDIX

Deriving and calibrating the IS curve.

The structural IS-LM set of equations in the main text includes the consumption function (3.30a), the investment function (3.30b), and the overall resources constraint (3.30c)

$$\begin{aligned} \widehat{c}_t &= E_t\widehat{c}_{t+1} + \alpha_1(r_t - r^{ss}) + \alpha_2(\widehat{w}_t - E_t\widehat{w}_{t+1}) + \alpha_3(\widehat{m}_t - E_t\widehat{m}_{t+1}) + \alpha_4\zeta_t & \text{(3.30a)} \\ \widehat{x}_t &= (1-\delta)E_t\widehat{x}_{t+1} + \tfrac{1}{(\eta-1)C_x(x^{ss})}(E_t f_{k_{t+1}} - \delta - r_t) & \text{(3.30b)} \\ \widehat{y}_t &= j_1\widehat{c}_t + j_2\widehat{x}_t & \text{(3.30c)} \end{aligned}$$

Inserting both consumption and investment from the first two equations in the third one

$$\begin{aligned} \widehat{y}_t = {} & j_1E_t\widehat{c}_{t+1} + j_1\alpha_1(r_t - r^{ss}) + j_1\alpha_2(\widehat{w}_t - E_t\widehat{w}_{t+1}) + j_1\alpha_3(\widehat{m}_t - E_t\widehat{m}_{t+1}) + \\ & j_1\alpha_4\zeta_t + j_2(1-\delta)E_t\widehat{x}_{t+1} + j_2\tfrac{1}{(\eta-1)C_x(x^{ss})}(E_t f_{k_{t+1}} - \delta - r_t). \end{aligned} \quad \text{(A.1)}$$

From equation (3.30c) one period ahead, we have

$$E_t\widehat{y}_{t+1} = j_1E_t\widehat{c}_{t+1} + j_2E_t\widehat{x}_{t+1}. \quad \text{(A.2)}$$

Combining (A.1) and (A.2), it yields the following expectational IS function

$$\begin{aligned} \widehat{y}_t = {} & E_t\widehat{y}_{t+1} + j_1\alpha_1(r_t - r^{ss}) + j_1\alpha_2(\widehat{w}_t - E_t\widehat{w}_{t+1}) + j_1\alpha_3(\widehat{m}_t - E_t\widehat{m}_{t+1}) \quad \text{(A.3)} \\ & + j_1\alpha_4\zeta_t - j_2\delta E_t\widehat{x}_{t+1} + j_2\tfrac{1}{(\eta-1)C_x(x^{ss})}(E_t f_{k_{t+1}} - \delta - r_t). \end{aligned}$$

Using the calibration of the parameters described in section 6 and calculating steady-state figures, one obtains the following numbers:

$j1 = \frac{c^{ss}}{y^{ss}} = 0.7364,$

$j2 = \frac{x^{ss}+C(x^{ss})}{y^{ss}} = 0.2636,$

$\alpha_1 = \frac{-1}{\sigma+(a_2-1)ws_c} = \frac{-1}{5+(5-1)(2.35)(0.0023)} = -0.2,$

$\alpha_2 = \frac{-ws_c}{\sigma+(a_2-1)ws_c} = \frac{-(2.35)(0.0023)}{5+(5-1)(2.35)(0.0023)} = -0.0011,$

$\alpha_3 = \frac{a_3ws_c}{\sigma+(a_2-1)ws_c} = \frac{(9)(2.35)(0.0023)}{5+(5-1)(2.35)(0.0023)} = -0.01,$

$\alpha_4 = \frac{1-\phi^c}{\sigma+(a_2-1)ws_c} = \frac{1-0.323}{5+(5-1)(2.35)(0.0023)} = 0.135$,
$\frac{1}{(\eta-1)C_x(x^{ss})} = 1.5$,
and $\delta = 0.025$.

Substituting them in (A.3) we have

$$\widehat{y}_t = E_t\widehat{y}_{t+1} - 0.2(r_t - r^{ss}) + 0.1\zeta_t - 0.0008(\widehat{w}_t - E_t\widehat{w}_{t+1}) \\ + 0.0073(\widehat{m}_t - E_t\widehat{m}_{t+1}) - 0.0066E_t\widehat{x}_{t+1} + 0.395(E_t f_{k_{t+1}} - \delta - r_t). \quad \text{(A.4)}$$

Interestingly, the real money balances and real wage components present a very small quantitative significance. Neglecting the small numbers,i.e., those smaller than 0.01, the expectational IS equation (A.4) could be approximated by this simpler equation

$$\widehat{y}_t = E_t\widehat{y}_{t+1} - 0.2(r_t - r^{ss}) + 0.1\zeta_t + 0.395(E_t f_{k_{t+1}} - \delta - r_t). \quad \text{(A.5)}$$

The appearance of (A.5) reminds the type of expectational IS function frequently used to simulate macroeconomic dynamics (for examples see Kerr and King (1996), Rotemberg and Woodford (1997), and McCallum and Nelson (1997)) with the inclusion of the expected premium to the return on the real asset $\left(E_t f_{k_{t+1}} - \delta - r_t\right)$ to explain investment behavior.

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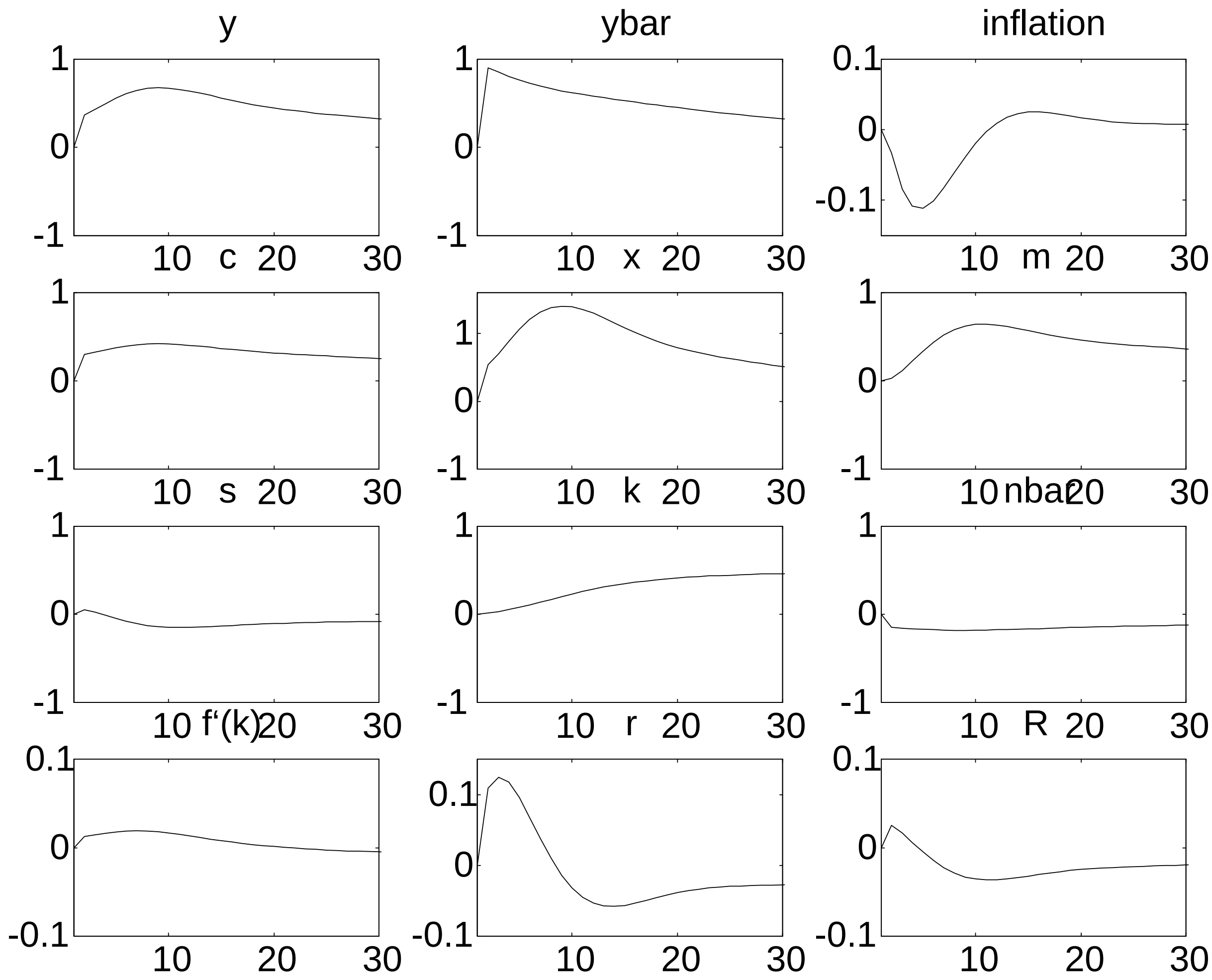

Figure 1: Technology unit shock to the production function (2.6). Impulse response functions.

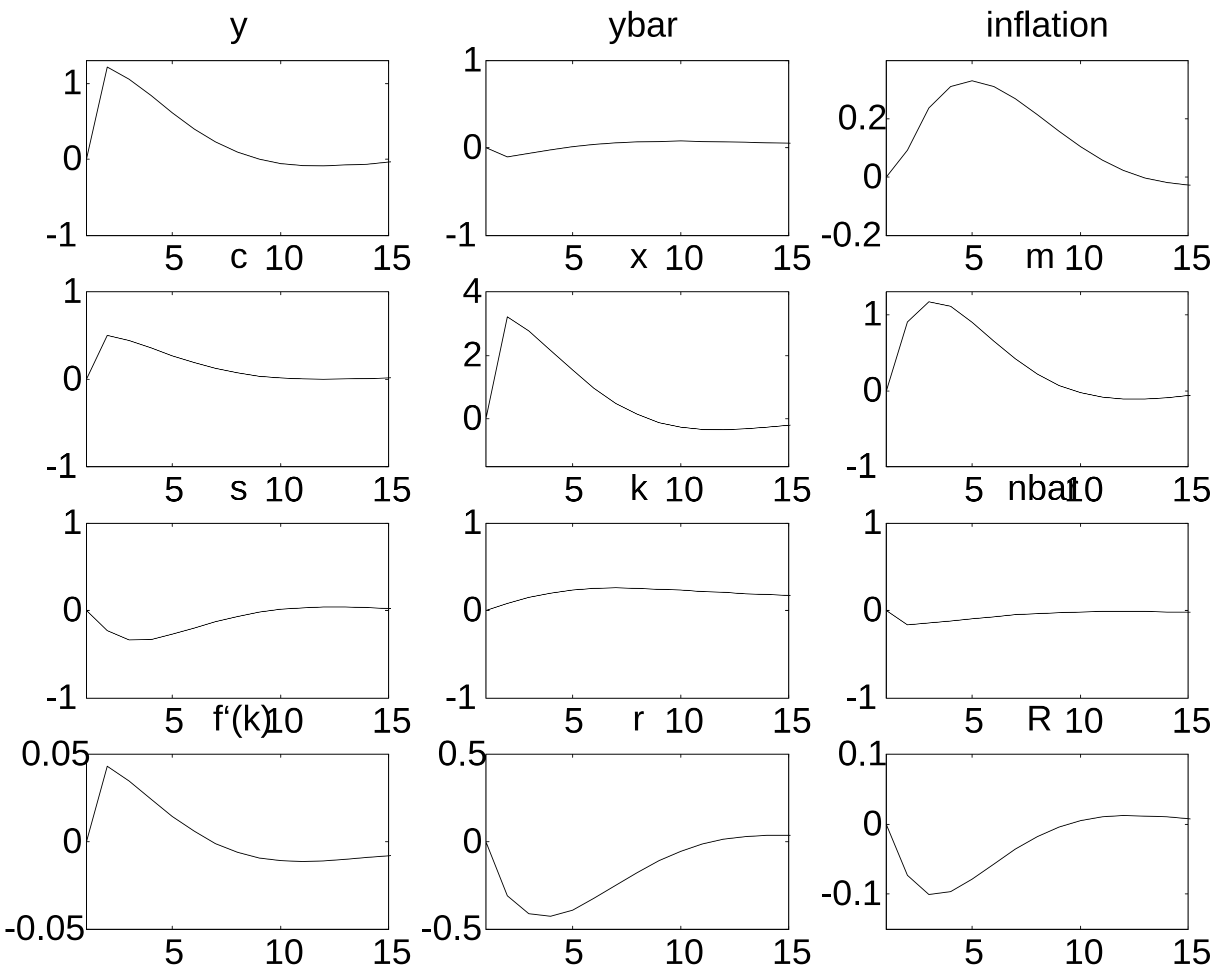

Figure 2: Money-growth unit shock to the monetary policy rule (2.11). Impulse response functions.

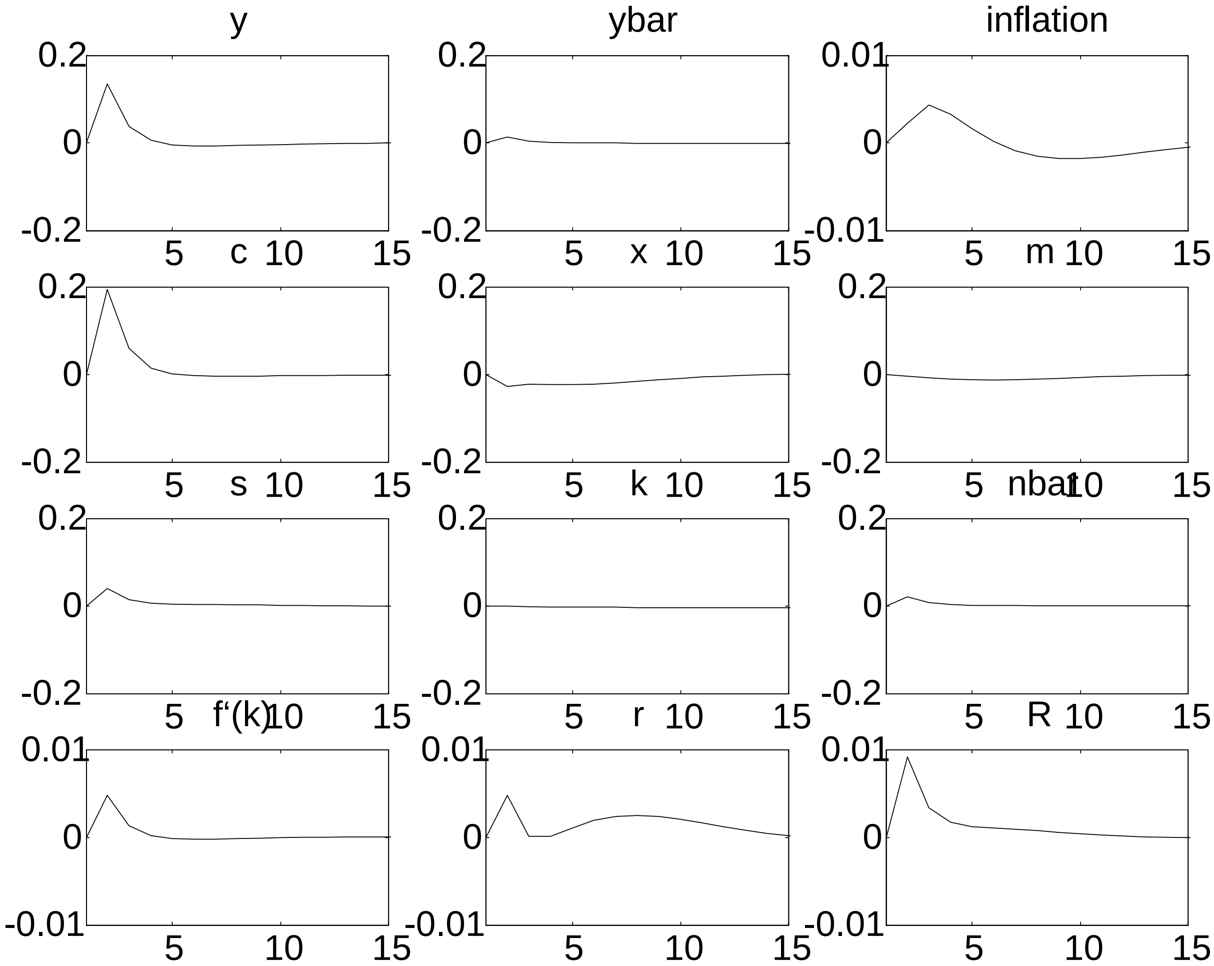

Figure 3: Consumption preference unit shock to the utility function (2.4). Impulse response functions.

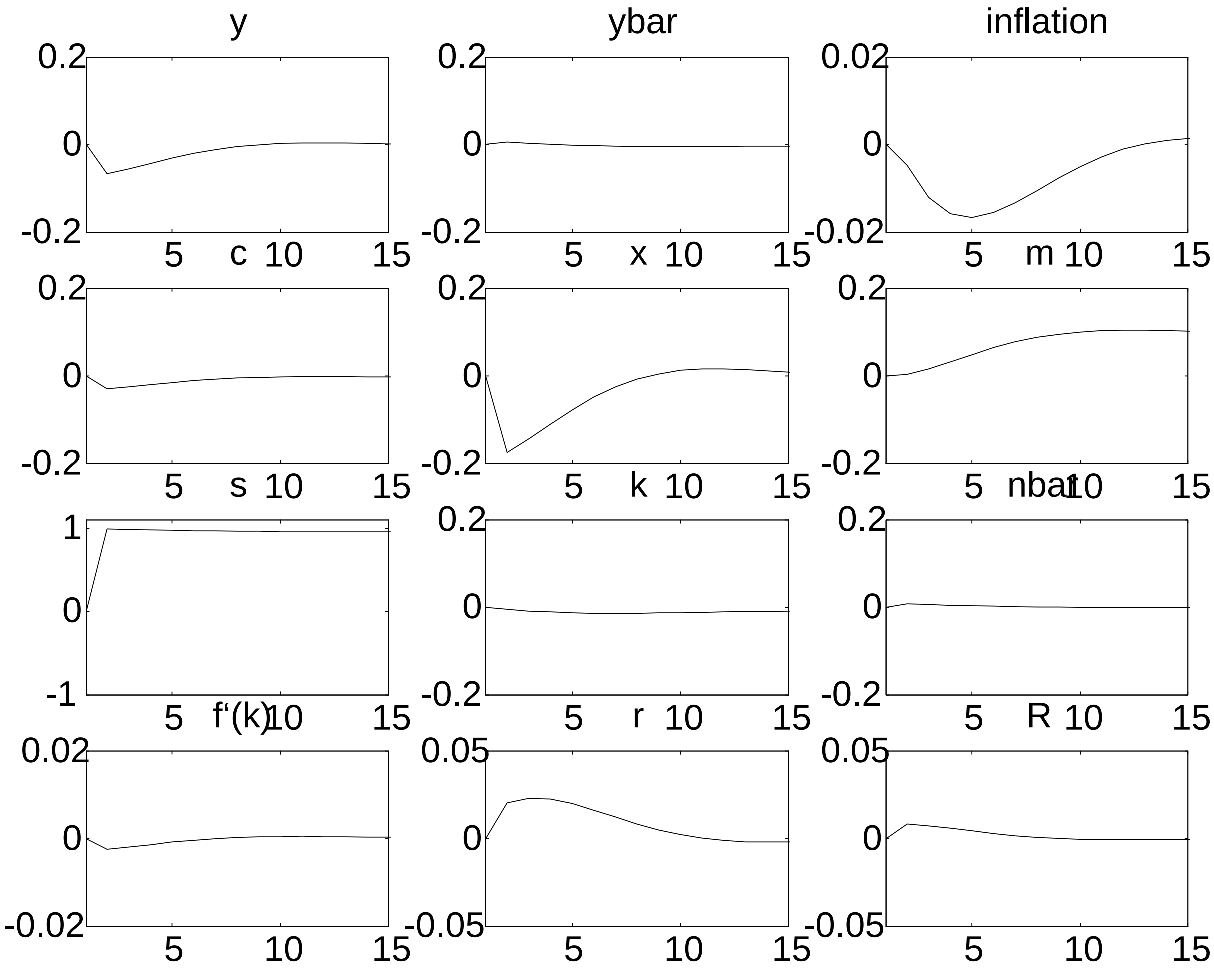

Figure 4: Transactions technology unit shock to the shopping time function (2.2). Impulse response functions.