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A method for inference in approximate reasoning based on normal intuitionistic fuzzy sets

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Abstract: *This paper introduces a method of approximate inference which operates with normal intuitionistic fuzzy. We give a definition of degree of compatibility between intuitionistic fuzzy sets and we present a method for the construction of these sets. Lastly we present the method of inference in approximate reasoning with normal intuitionistic fuzzy sets and we study its most immediate properties.*

Keywords: *Fuzzy set; intuitionistic fuzzy set; approximate reasoning; fuzzy inference; inference methods.*

1. Introduction

Approximate reasoning is, imformally speaking, as I. B. Turksen says in ([3]), the process or processes by which a possible imprecise conclusion is deduced from a collection of imprecise premises.

Our objetive in this paper is, parting from a collection of imprecise premises, represented by intuitionistic fuzzy sets, and from their corresponding imprecise conclusions, also represented by intuitionistic fuzzy sets, to provide a method of inference by which it is possible to obtain from any one imprecise premise its corresponding imprecise conclusion, being both represented by intuitionistic fuzzy sets.

That is, the problem (1) of approximate inference in our particular case, will be represented in the following way:

Let's consider the following sequence

If A_1 then B_1 or
If A_2 then B_2 or
...
If A_n then B_n

where A_i are intuitionistic fuzzy sets on a non-empty set X, B_i are intuitionistic fuzzy sets on a non-empty set Y and $i = 1, 2, \cdots, n$. For an arbitrary intuitionistic fuzzy set on X, A, a method is provided that generates its corresponding intuitionistic fuzzy set B on Y. We shall represent this method symbolically as follows: MAI-IFSs(A)=B.

It is clear that the method we present operates with intuitionistic fuzzy sets, for this reason we start the paper offering the definition of an intuitionistic fuzzy set and a brief summary of its most important properties.

2. Some definitions on Intuitionistic fuzzy sets

An *intuitionistic fuzzy set* ([1]) in X is an expression A given by

$$A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}, \quad \text{where}$$

$$\mu_A : X \longrightarrow [0,1]$$

$$\nu_A : X \longrightarrow [0,1]$$

with the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for all $x \in X$

Numbers $\mu_A(x)$ and $\nu_A(x)$ denote respectively the degree of membership and the degree of nonmembership of element x to set A. We will denote with $IFSs(X)$ the set of all the intuitionistic fuzzy sets on X. We shall say that an intuitionistic fuzzy set A is *normal* if there is at least one $x \in X$ such that $\mu_A(x) = 1$. In this paper we shall use exclusively normal intuitionistic fuzzy sets.

The following expressions are defined in ([1]),([2]) for all $A, B \in X$

1. $A \leq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ & $\nu_A(x) \geq \nu_B(x)$ $\forall x \in X$
2. $A \preceq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ & $\nu_A(x) \leq \nu_B(x)$ $\forall x \in X$
3. $A = B$ if and only if $A \leq B$ & $B \leq A$
4. $A_c = \{< x, \nu_A(x), \mu_A(x) > | x \in X\}$
5. $A \vee B = \{< x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x)) > | x \in X\}$
6. $A \wedge B = \{< x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x)) > | x \in X\}$

The operations 5 and 6 are easily generalizable to the case of n intuitionistic fuzzy sets.

Let $A \in IFSs(X)$ and $B \in IFSs(Y)$, we shall call the intuitionistic fuzzy set given by

$$A \times B = \{< (x,y), \mu_{A \times B}(x,y), \nu_{A \times B}(x,y) > | (x,y) \in X \times Y\} \qquad \text{where}$$

$$\mu_{A\times B}(x,y) = \min(\mu_A(x), \mu_B(y))$$
$$\nu_{A\times B}(x,y) = \max(\nu_A(x), \nu_B(y))$$

cartesian product of A and B and we will denote it with $A \times B$.

Let $A, B \in IFSs(X)$ and $C \in IFSs(Y)$, set $A \circ (B \times C) \in IFSs(Y)$ is given as follows

$$\mu_{A\circ(B\times C)}(y) = \max_{x\in X}\left[\min\Big(\mu_A(x), \min(\mu_B(x), \mu_C(y))\Big)\right]$$
$$\nu_{A\circ(B\times C)}(y) = \min_{x\in X}\left[\max\Big(\nu_A(x), \max(\nu_B(x), \nu_C(y))\Big)\right]$$

Definition 1. Let's consider normal $A, B \in IFSs(X)$. We shall call the following element of $[0,1] \times [0,1]$ the *degree of compatibility* between A and B

$$\Gamma(A,B) = \Big(\max_{x\in X}(\min(\mu_A(x), \mu_B(x))), \min_{x\in X}(\max(\nu_A(x), \nu_B(x)))\Big)$$

The most important properties of this measure are the following:

1- $\Gamma(A,A) = (1,0)$, for all normal $A \in IFSs(X)$.

2- For all normal $A, B \in IFSs(X)$, $\Gamma(A,B) = \Gamma(B,A)$.

3- $\Gamma(A,B) = (0,1) \Leftrightarrow \min(\mu_A(x), \mu_B(x)) = 0$ and $\max(\nu_A(x), \nu_B(x)) = 1$ for all $x \in X$.

We will now present a method of construction of intuitionistic fuzzy sets on the referential Y from two intuitionistic fuzzy sets on the referential X.

For each $A_i, A_j \in IFSs(X)$ the set $\Phi^{i,j} = \{< y, \mu_{\Phi^{i,j}}(y), \nu_{\Phi^{i,j}}(y) > | y \in Y\}$ with

$$\Gamma(A_i, A_j) = (\mu_{\Phi^{i,j}}(y), \nu_{\Phi^{i,j}}(y)) \qquad \text{that is}$$
$$\mu_{\Phi^{i,j}}(y) = \max_{x\in X}(\min(\mu_{A_i}(x), \mu_{A_j}(x)))$$
$$\nu_{\Phi^{i,j}}(y) = \min_{x\in X}(\max(\nu_{A_i}(x), \nu_{A_i}(x)))$$

is an intuitionistic fuzzy set on Y. We can point out that set $\Phi^{i,j}$ constructed in this way is such that $\mu_{\Phi^{i,j}}(y) = \text{const}$ and $\nu_{\Phi^{i,j}}(y) = \text{const}$, for all $y \in Y$.

Let $\Phi^{i,j} \in IFSs(Y)$ be the intuitionistic fuzzy set constructed above and let B be any intuitionistic fuzzy set on Y we define the operation $*$ between both as follows:

Definition 2. $\Phi^{i,j} * B = \{< y, \min\Big(\mu_{\Phi^{i,j}}(y), \mu_B(y)\Big), \max\Big(\nu_{\Phi^{i,j}}(y), \nu_B(y)\Big) > | y \in Y\}$. Evidently $\Phi^{i,j} * B \in IFSs(Y)$.

3. Method for inference in approximate reasoning based on normal IFSs

In these conditions, in the problem (1) stated in the introduction set $B \in IFSs(Y)$ such that $MAI - IFSs(A) = B$, is obtained by means of the two following steps:

1.- The degree of compatibility of A with each of the $A_i \in IFSs(X)$ is determined in a consecutive way. We shall denote $\Gamma(A_i, A)$ with Γ_i, $i = 1, 2, \cdots, n$. Next for each Γ_i the set $\Phi^i = \{< y, \mu_{\Phi^i}(y), \nu_{\Phi^i}(y) > |y \in Y\}$ is constructed similarly to how $\Phi^{i,j}$ was calculated. After, for each $i = 1, 2, \cdots, n$ the sets $\Phi^i * B_i$ are calculated.

2.- B is generated in the following way $B = \bigvee_{i=1}^{n} (\Phi^i * B_i)$.

4. First characteristics of MAI-IFSs

a) If $A \in IFSs(X)$ coincides with $A_i \in IFSs(X)$, then $\Gamma_i = (1, 0)$ therefore $\Phi^i * B_i = B_i$

b) If for any A_i, $\Gamma_i = (0, 1)$ then $B_i = \{< y, 0, 1 > |y \in Y\}$

c) If $A_i \wedge A_j = \{< y, 0, 1 > |y \in Y\}$, for all $i, j = 1, 2, \cdots, n$ with $i \neq j$ then $MAI - IFSs(A_i) = B_i$ for all i.

d) **Theorem 1.**

$$B = A \circ \left[\bigvee_{i=1}^{n} (A_i \times B_i) \right]$$

Proof. We just need to use the following property of the t-conorm max and of the t-norm min ([2])

$$\bigvee_{i=1,2,\cdots,n} \left(\wedge \left(\bigvee_{x} \left(\wedge(\mu_{A_i}(x), \mu_A(x)), \mu_{B_i}(y) \right) \right) \right) =$$

$$= \bigvee_{x} \left(\wedge \left(\mu_A(x), \bigvee_{i=1,2,\cdots,n} \left(\wedge(\mu_{A_i}(x), \mu_{B_i}(y)) \right) \right) \right)$$

and the same for the non-membership function. □

This property is not verified when the intuitionistic fuzzy sets are not normal, in this case it is necessary to use another degree of compatibility.

e) If in the method presented we use fuzzy sets instead of intuitionistic fuzzy sets, set B is obtained by $B = A \circ \left[\bigvee_{i=1}^{n} (A_i \times B_i) \right]$, therefore, MAI with fuzzy sets is a special case of MAI-IFSs for ordinary fuzzy sets under the additional assumption of normality of fuzzy sets $A_i, B_i, i = 1, 2, \cdots, n$ operates in the same way as Zadeh's compositional rule of inference when cartesian product is defined by min-operator ([4])

5. Future research

In this paper we present a very specific method of inference, in the one-dimensional case and for normal intuitionistic fuzzy sets. Nevertheless, it is only a first step in the generalization of approximate reasoning for general $IFSs$, as well as in the construction of other methods that allow us to compare some to others. We also intend to study the possible measures of compatibility in the setting of the relations between fuzzy and intuitionistic fuzzy, and their application to the methods of approximate inference for the multi-dimensional case with $IFSs$. We will study the effectiveness of the computer representation of these methods. At this moment we have numerical examples with this method.

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