NIFS 1 (1995) 2, 104-115

Concept lattices associated with interval-valued L-Fuzzy contexts.

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Abstract.

In 1982, Wille published the paper titled *Restructuring lattice theory: an approach based on hierarchies of concepts*([5]), where he developed a new theory called formal concept theory. In that paper, he analyzes 0/1 relations between the object and attribute sets.

Aplying the L-Fuzzy set theory to the results of Wille, we have developed the L-Fuzzy concept theory that deals with L-Fuzzy relations inside of 0/1 relations. The final purpose of our theory is the knowledge acquisition and classification.

Taking as departure point the L-Fuzzy concept theory ([2]), we study in this paper contexts defined by interval-valued L-Fuzzy relations between the object and attribute sets. We give a new definition of L-Fuzzy concept valid for this case and analyze the structure of the L-Fuzzy concept set. We aply these results to a medical example.

Keywords. Knowledge Acquisition and Learning, Data Analysis Methods, Interval-valued L-Fuzzy Concepts.

1 Introduction.

Starting from the formal concept analysis of Wille ([5]) and the L-Fuzzy set theory, we have developed in [2] a new model from L-Fuzzy relations between the objects and the attributes. We are going to see the main aspects:

We defined an *L-Fuzzy context* as a tuple $(L, X, Y, \overset{\backsim}{R})$ where L was a complete lattice, X and Y were the object and attribute sets respectively, and $\overset{\backsim}{R}$ was an L-Fuzzy relation.

At this point, we gave in [2] the new definitions of *derivation operators weighted by a complementation* noted by the subindexes 1 and 2:

For every $\overset{\backsim}{A} \in L^X$, we associated with it the L-Fuzzy set $\overset{\backsim}{A}_1$ of L^Y such that

$$\overset{\backsim}{A}_1(y) = \inf_{x \in X} (\overset{\backsim}{A}'(x) \top \overset{\backsim}{R}(x, y))$$

In the same way, for every $\overset{\backsim}{B} \in L^Y$ we associated with it $\overset{\backsim}{B}_2 \in L^X$ such that

$$\overset{\backsim}{B}_2(x) = \inf_{y \in Y} (\overset{\backsim}{B}'(y) \top \overset{\backsim}{R}(x, y))$$

It can be proved, in a simple way, the relationship between this definitions of derivation operators and the composition sup $-\star$ of the Fuzzy theory.

$$(\forall \overset{\backsim}{A} \in L^X), \overset{\backsim}{A}_1 = (\overset{\backsim}{A} \star \overset{\backsim}{R}')'$$
$$(\forall \overset{\backsim}{B} \in L^Y), \overset{\backsim}{B}_2 = (\overset{\backsim}{B} \star \overset{\backsim}{R}')'.$$

where $\star$ is the t-norm in L defined by $\alpha \star \beta = (\alpha' \top \beta')', \forall \alpha, \beta \in L$.

We also defined the operators φ and ψ:

$$\varphi : L^X \to L^X \,/\, \varphi(\overset{\backsim}{A}) = \overset{\backsim}{A}_{12}$$

$$\psi : L^Y \to L^Y \,/\, \psi(\overset{\backsim}{B}) = \overset{\backsim}{B}_{21}$$

where $\overset{\backsim}{A}_{12}$ and $\overset{\backsim}{B}_{21}$ represented the L-Fuzzy sets $(\overset{\backsim}{A}_1)_2$ and $(\overset{\backsim}{B}_2)_1$ respectively. The operators φ and ψ are said to be *constructor operators*.

These operators preserve the order and we used them to give the following definition:

If $\overset{\backsim}{M} \in fix(\varphi)$, then the pair $(\overset{\backsim}{M}, \overset{\backsim}{M}_1)$ is said to be *L-Fuzzy concept* of the L-Fuzzy context $(L, X, Y, \overset{\backsim}{R})$.

We used the set $fix(\varphi)$ to define the L-Fuzzy concepts, but as we saw in ([2]), we could do it with $fix(\psi)$.

We also proved in [2], using the theorem of Tarski ([4]), that the L-Fuzzy concept set $\breve{\mathcal{L}}(L, X, Y, \breve{R})$ with the order relation $\preceq$ defined by:

$$(\forall(\breve{A}, \breve{B}), (\breve{C}, \breve{D}) \in \breve{\mathcal{L}}(L, X, Y, \breve{R}))\ ((\breve{A}, \breve{B}) \preceq (\breve{C}, \breve{D}) \iff \breve{A} \leq \breve{C})$$

is a complete lattice.

The pair $(\breve{\mathcal{L}}(L, X, Y, \breve{R}), \preceq)$ is said to be *L-Fuzzy concept lattice* of the L-Fuzzy context $(L, X, Y, \breve{R})$.

We will use $(\breve{\mathcal{L}}(L, X, Y, \breve{R}), \preceq)$ or $\breve{\mathcal{L}}$ to denote the L-Fuzzy concept lattice.

On the other hand, we realize that in same cases it is very common to have relations between the object and the attribute sets with interval-valued observations:

1.1 Example. We want to study the influence of a great quantity of alcohol and cholesterol in the evolution of two illnesses: the leukemia and the diabetes. To do that, we ask his opinion to a doctor and the answer is

$\breve{R}$	leukemia	diabetes
alcohol	[0.3,0.5]	[0.5,0.5]
cholesterol	[0.2,0.6]	[0.5,0.8]

In general, the doctor preferes to give an interval-valued answer inside of an exact answer because it is very difficult to determinate the exact value of the relationship between the factors and the illnesses.

In this paper, we are going to work with this kind of relations between the objects and the attributes aplying the L-Fuzzy concept theory.

2 Preliminaries.

In 1975, Zadeh ([6]) defines an *interval-valued fuzzy set* as an application $\breve{A}$ given by

$$\breve{A} : X \longrightarrow I([0,1])$$

where $X \neq \emptyset$ and $I([0,1])$ is the set of closed subintervals of the interval $[0,1]$.

We can extend the definition to any complete lattice L and say that an *interval-valued L-Fuzzy set* is an application

$$\breve{A} : X \longrightarrow I(L)$$

The order relation defined by us is a generalization of that defined in [0,1]:

$$(\forall \breve{A}, \breve{B} \in I(L)^X) \quad (\breve{A} \leq \breve{B} \Longleftrightarrow (\forall x \in X)(\breve{A}_l(x) \leq \breve{B}_l(x) \;\&\; \breve{A}_u(x) \leq \breve{B}_u(x)))$$

where $\breve{A}_l(x)$ and $\breve{A}_u(x)$ are the lower bound and the upper bound of the $\breve{A}(x)$ interval, and the same for $\breve{B}(x)$.

The set of interval-valued L-Fuzzy sets with this order relation is a complete lattice where the SUP and INF operators, and the complementation are defined by: $(\forall x \in X)$

$$\underset{i \in I}{\mathrm{SUP}}(\breve{A}_i)(x) = [\bigvee_{i \in I} \breve{A}_{i_l}(x), \bigvee_{i \in I} \breve{A}_{i_u}(x)]$$

$$\underset{i \in I}{\mathrm{INF}}(\breve{A}_i)(x) = [\bigwedge_{i \in I} \breve{A}_{i_l}(x), \bigwedge_{i \in I} \breve{A}_{i_u}(x)]$$

$$\breve{A}'(x) = [\breve{A}'_u(x), \breve{A}'_l(x)]$$

Moreover, the minimum and maximum elements will be $\breve{0}(x) = [0,0]$ and $\breve{1}(x) = [1,1]$, $\forall x \in X$, with 0 and 1 the minimum and maximum elements of L.

3 Derivation and constructor operators.

The derivation operator definitions gived in [2] are not valid in the case of interval-valued observations, so we have to adapt them to this situation.

Let X,Y be finite and not empty sets. For every $\widetilde{A} \in I(L)^X$, we are going to define the interval-valued fuzzy set $\widetilde{A}_1 \in I(L)^Y$ such that:

$$\widetilde{A}_1(y) = \underset{x \in X}{\mathrm{INF}}(\mathrm{SUP}(\widetilde{A}'(x), \widetilde{R}(x,y)))$$

In the same way, for every $\widetilde{B} \in I(L)^Y$ we can define $\widetilde{B}_2 \in I(L)^X$ such that:

$$\widetilde{B}_2(x) = \underset{y \in Y}{\mathrm{INF}}(\mathrm{SUP}(\widetilde{B}'(y), \widetilde{R}(x,y)))$$

3.1 Definition. The operators noted by 1 and 2 are said to be <u>derivation operators weighted by a complementation.</u>

From these definitions we can prove the following properties:

3.2 Proposición.

The previous operators verify:

i) $(\forall \widetilde{A}, \widetilde{B} \in I(L)^X)\ (\widetilde{A} \leq \widetilde{B} \Longrightarrow \widetilde{A}_1 \geq \widetilde{B}_1)$.

i') $(\forall \widetilde{C}, \widetilde{D} \in I(L)^Y)\ (\widetilde{C} \leq \widetilde{D} \Longrightarrow \widetilde{C}_2 \geq \widetilde{D}_2)$.

<u>Proof:</u> i) Let $\widetilde{A}, \widetilde{B} \in I(L)^X, \widetilde{R} \in I(L)^{X \times Y}$.

$\widetilde{A} \leq \widetilde{B} \Longrightarrow (\forall x \in X)(\widetilde{A}(x) \leq \widetilde{B}(x)) \Longrightarrow (\forall x \in X)(\widetilde{A}_l(x) \leq \widetilde{B}_l(x)\ \&\ \widetilde{A}_u(x) \leq \widetilde{B}_u(x)) \Longrightarrow (\forall x \in X)(\widetilde{A}'_l(x) \geq \widetilde{B}'_l(x)\ \&\ \widetilde{A}'_u(x) \geq \widetilde{B}'_u(x)) \Longrightarrow (\forall x \in X)([\widetilde{A}'_u(x), \widetilde{A}'_l(x)] \geq [\widetilde{B}'_u(x), \widetilde{B}'_l(x)]) \Longrightarrow (\forall x \in X)(\widetilde{A}'(x) \geq \widetilde{B}'(x))$.

So, we have that

$(\forall x \in X)(\forall y \in Y)\ (\mathrm{SUP}(\widetilde{A}'(x), \widetilde{R}(x,y)) \geq \mathrm{SUP}(\widetilde{B}'(x), \widetilde{R}(x,y)))$,

And finally, taking infimum,

$$(\forall y \in Y)(\underset{x \in X}{\mathrm{INF}}(\mathrm{SUP}(\widetilde{A}'(x), \widetilde{R}(x,y))) \geq \underset{x \in X}{\mathrm{INF}}(\mathrm{SUP}(\widetilde{B}'(x), \widetilde{R}(x,y))))$$

$$\Longrightarrow (\forall y \in Y)\ (\widetilde{A}_1(y) \geq \widetilde{B}_1(y) \Longrightarrow \widetilde{A}_1 \geq \widetilde{B}_1).$$

The proof of i') is similar to i).■

If we write $\breve{A}_{12}$ and $\breve{B}_{21}$ to represent the interval-valued fuzzy sets $(\breve{A}_1)_2$ and $(\breve{B}_2)_1$ respectively, we can define the operators $\breve{\varphi}$ and $\breve{\psi}$:

$$\breve{\varphi} : I(L)^X \rightarrow I(L)^X \ / \ \breve{\varphi}(\breve{A}) = \breve{A}_{12}$$

$$\breve{\psi} : I(L)^Y \rightarrow I(L)^Y \ / \ \breve{\psi}(\breve{B}) = \breve{B}_{21}$$

that we will use later.

3.3 Definition. The operators $\breve{\varphi}$ and $\breve{\psi}$ are said to be constructor operators.

3.4 Proposición. The operators $\breve{\varphi}$ and $\breve{\psi}$ preserve the order:

$$(\forall \breve{A}, \breve{B} \in I(L)^X) \ (\breve{A} \leq \breve{B} \Longrightarrow \breve{\varphi}(\breve{A}) \leq \breve{\varphi}(\breve{B}))$$

$$(\forall \breve{C}, \breve{D} \in I(L)^Y) \ (\breve{C} \leq \breve{D} \Longrightarrow \breve{\psi}(\breve{C}) \leq \breve{\psi}(\breve{D})).$$

Proof: We only have to aply the previous proposition two times.■

4 Interval-valued L-Fuzzy concept lattices.

At this point we can introduce some very important definitions that allow us to work with interval-valued L-Fuzzy relations.

4.1 Definición. An interval-valued L-Fuzzy context is a tuple $(I(L), X, Y, \breve{R})$ where $X \neq \emptyset$ and $Y \neq \emptyset$ are the object and attribute sets respectively, and $\breve{R} \in I(L)^{X \times Y}$ is a interval-valued L-Fuzzy relation.

As we see from the constructor operator definitions, any valid proposition for $\breve{\varphi}$ is also valid for $\breve{\psi}$. Then, we will choose one of them to work, for example the first one.

Let $fix(\breve{\varphi}) = \{\breve{A} \in L^X \ / \ \breve{A} = \breve{\varphi}(\breve{A})\}$ be the fixed points set of $\breve{\varphi}$.

4.2 Definition. If $\breve{M} \in fix(\breve{\varphi})$, then the pair $(\breve{M}, \breve{M}_1)$ is said to be interval-valued L-Fuzzy concept of the interval-valued L-Fuzzy context $(I(L), X, Y, \breve{R})$.

In order to study the set of these new concepts, we show the following theorem of Tarski ([4]) for fixed point sets:

4.3 Theorem. If L is a complete lattice and $\breve{\varphi} : L \to L$ preserves the order, then the fixed points set of $\breve{\varphi}$ is a complete lattice.

In our case, $I(L)^X$ y $I(L)^Y$ are complete lattices, and the constructor operators $\breve{\varphi}$ and $\breve{\psi}$ preserve the order; therefore, by the Tarski theorem, $\Omega = (fix(\breve{\varphi}), \leq, \breve{0}_\Omega, \breve{1}_\Omega, \vee_\Omega, \wedge_\Omega)$ and $\Sigma = (fix(\breve{\psi}), \geq, \breve{0}_\Sigma, \breve{1}_\Sigma, \vee_\Sigma, \wedge_\Sigma)$ are complete lattices.

To calculate the minimum and maximum elements of these lattices, and the supremum and infimum of a family, we will look at the work developed by P. Cousot and R. Cousot ([3]), that provides a constructive version of the theorem of Tarski ([4]).

Therefore, we can calculate

$$\breve{0}_\Omega = luis(\breve{\varphi})(\breve{0}) \qquad \breve{0}_\Sigma = luis(\breve{\psi})(\breve{0}) \tag{1}$$

$$\breve{1}_\Omega = llis(\breve{\varphi})(\breve{1}) \qquad \breve{1}_\Sigma = llis(\breve{\psi})(\breve{1}) \tag{2}$$

$$\forall \{\breve{A}_i, i \in I\} \in \Omega \qquad \forall \{\breve{B}_i, i \in I\} \in \Sigma$$

$$\bigvee_\Omega \breve{A}_i = luis(\breve{\varphi})\left(\bigvee \breve{A}_i\right) \qquad \bigvee_\Sigma \breve{B}_i = luis(\breve{\psi})\left(\bigvee \breve{B}_i\right) \tag{3}$$

$$\bigwedge_\Omega \breve{A}_i = llis(\breve{\varphi})\left(\bigwedge \breve{A}_i\right) \qquad \bigwedge_\Sigma \breve{B}_i = llis(\breve{\psi})\left(\bigwedge \breve{B}_i\right) \tag{4}$$

where in general, given a function f that preserves the order, $luis(f)(\underset{\smile}{A})$ is the limit of a stationary upper iteration sequence for f starting with $\underset{\smile}{A}$, and $llis(f)(\underset{\smile}{B})$ is the limit of a stationary lower iteration sequence for f starting with $\underset{\smile}{B}$.

As we did in [2], we can prove that the sets $\mathcal{L}_1 = \{(\breve{A}, \breve{A}_1) \,/\, \breve{A} \in fix(\breve{\varphi})\}$ and $\mathcal{L}_2 = \{(\breve{B}_2, \breve{B}) \,/\, \breve{B} \in fix(\breve{\psi})\}$ are equal; therefore we will use $\breve{\mathcal{L}}(I(L), X, Y, \breve{R})$ to denote these sets.

4.4 Definition. We define the order relation $\preceq$ in $\widetilde{\mathcal{L}}(I(L), X, Y, \widetilde{R})$ as follows:

$$(\forall(\widetilde{A}, \widetilde{B}), (\widetilde{C}, \widetilde{D}) \in \widetilde{\mathcal{L}}(I(L), X, Y, \widetilde{R})) \quad ((\widetilde{A}, \widetilde{B}) \preceq (\widetilde{C}, \widetilde{D}) \Longleftrightarrow \widetilde{A} \leq \widetilde{C})$$

It is immediate to prove that $\widetilde{A} \leq \widetilde{C}$ is equivalent to $\widetilde{B} \geq \widetilde{D}$ taking into account the proposition 3.2 and the interval-valued L-Fuzzy concept definition.

We can observe that the order definition $\preceq$ is induced by the order relation $\leq$ in Ω and its opposite $\geq$ in Σ.

4.5 Theorem. $(\widetilde{\mathcal{L}}(I(L), X, Y, \widetilde{R}), \preceq)$ is a complete lattice.

<u>Proof:</u> To prove this theorem you only have to aply the proof of the theorem 1 of [2] to the interval-valued case. ■

4.6 Definición. The complete lattice $(\widetilde{\mathcal{L}}(I(L), X, Y, \widetilde{R}), \preceq)$ is said to be <u>interval-valued L-Fuzzy concept lattice</u> of the context $(I(L), X, Y, \widetilde{R})$.

From this point we will use $(\widetilde{\mathcal{L}}(I(L), X, Y, \widetilde{R}), \preceq)$ or $\widetilde{\mathcal{L}}$ to call the interval-valued L-Fuzzy concept lattice.

It is very easy to prove the next proposition as we did in the L-Fuzzy case ([2]):

4.7 Proposition. The maximum and minimum elements of $(\widetilde{\mathcal{L}}(I(L), X, Y, \widetilde{R}), \preceq)$ are respectively:

$$\widetilde{0}_{\widetilde{\mathcal{L}}} = (\widetilde{0}_{\Omega}, \widetilde{1}_{\Sigma}) \text{ y } \widetilde{1}_{\widetilde{\mathcal{L}}} = (\widetilde{1}_{\Omega}, \widetilde{0}_{\Sigma})$$

Moreover, since the SUP operator is an upper semicontinous t-conorm

$$(\forall \alpha, \beta_i \in L), SUP(\alpha, \bigwedge_{i \in I}(\beta_i)) = \bigwedge_{i \in I}(SUP(\alpha, \beta_i)).$$

hence:

4.8 Theorem. For every interval-valued L-Fuzzy concept family

$$\mathcal{F} = \{(\widetilde{A}_i, (\widetilde{A}_i)_1), \ \widetilde{A}_i \in \Omega\} = \{((\widetilde{B}_i)_2, \widetilde{B}_i), \ \widetilde{B}_i \in \Sigma)\} \subseteq \widetilde{\mathcal{L}}$$

we can express the supremum and the infimum of $\mathcal{F}$ by:

$$\bigvee_{\widetilde{\mathcal{L}}} \left(\widetilde{A}_i, (\widetilde{A}_i)_1\right) = \left(\bigvee_{\Omega} \widetilde{A}_i, \bigwedge_{\Sigma} (\widetilde{A}_i)_1\right)$$

$$\bigwedge_{\widetilde{\mathcal{L}}} \left((\widetilde{B}_i)_2, \widetilde{B}_i\right) = \left(\bigwedge_{\Omega} (\widetilde{B}_i)_2, \bigvee_{\Sigma} (\widetilde{B}_i)\right)$$

where the supremum and infimum in Σ and Ω are calculed with the expressions (3) and (4).

5 Example.

We can come back to the example shown up at the introduction. In this case, we will take the interval-valued L-Fuzzy context $(I(L), X, Y, \widetilde{R})$ where:

L={0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1}

X={alcohol, cholesterol} is the factor set

Y={leukemia, diabetes} is the illness set

$\widetilde{R}$ shows the influence of the factors into the illnesses evolution

$\widetilde{R}$	leukemia	diabetes
alcohol	[0.3,0.5]	[0.5,0.5]
cholesterol	[0.2,0.6]	[0.5,0.8]

This is an small example, but you can aply the same technique to others.

We have constructed the interval-valued L-Fuzzy concept lattice and the result is the following:

$$1\begin{cases} \tilde{A}1 = \{\text{alcohol}/[0.3, 0.6], \text{cholesterol}/[0.3, 0.6]\} \\ \tilde{B}1 = \{\text{leukemia}/[0.4, 0.7], \text{diabetes}/[0.4, 0.7]\} \end{cases}$$

$$2\begin{cases} \tilde{A}2 = \{\text{alcohol}/[0.3, 0.6], \text{cholesterol}/[0.3, 0.7]\} \\ \tilde{B}2 = \{\text{leukemia}/[0.3, 0.7], \text{diabetes}/[0.4, 0.7]\} \end{cases}$$

$$3\begin{cases} \tilde{A}2 = \{\text{alcohol}/[0.3, 0.6], \text{cholesterol}/[0.3, 0.8]\} \\ \tilde{B}2 = \{\text{leukemia}/[0.2, 0.7], \text{diabetes}/[0.4, 0.7]\} \end{cases}$$

$$4\begin{cases} \tilde{A}2 = \{\text{alcohol}/[0.4, 0.6], \text{cholesterol}/[0.4, 0.6]\} \\ \tilde{B}2 = \{\text{leukemia}/[0.4, 0.6], \text{diabetes}/[0.4, 0.6]\} \end{cases}$$

$$5\begin{cases} \tilde{A}2 = \{\text{alcohol}/[0.4, 0.6], \text{cholesterol}/[0.4, 0.7]\} \\ \tilde{B}2 = \{\text{leukemia}/[0.3, 0.6], \text{diabetes}/[0.4, 0.6]\} \end{cases}$$

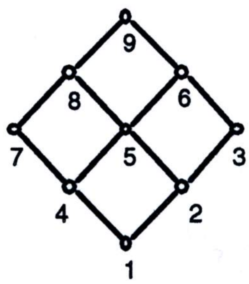
$$6\begin{cases} \tilde{A}2 = \{\text{alcohol}/[0.4, 0.6], \text{cholesterol}/[0.4, 0.8]\} \\ \tilde{B}2 = \{\text{leukemia}/[0.2, 0.6], \text{diabetes}/[0.4, 0.6]\} \end{cases}$$

$$7\begin{cases} \tilde{A}2 = \{\text{alcohol}/[0.5, 0.6], \text{cholesterol}/[0.5, 0.6]\} \\ \tilde{B}2 = \{\text{leukemia}/[0.4, 0.5], \text{diabetes}/[0.4, 0.5]\} \end{cases}$$

$$8\begin{cases} \tilde{A}2 = \{\text{alcohol}/[0.5, 0.6], \text{cholesterol}/[0.5, 0.7]\} \\ \tilde{B}2 = \{\text{leukemia}/[0.3, 0.5], \text{diabetes}/[0.4, 0.5]\} \end{cases}$$

$$9\begin{cases} \tilde{A}3 = \{\text{alcohol}/[0.5, 0.6], \text{cholesterol}/[0.5, 0.8]\} \\ \tilde{B}3 = \{\text{leukemia}/[0.2, 0.5], \text{diabetes}/[0.4, 0.5]\} \end{cases}$$

We can represent this lattice as follows:



From a very quick view of the interval-valued L-Fuzzy concepts we can observe that all the intervals have at least amplitude equal to 0.1; this means that the results must be interpreted with a certain degree of ambiguity. This ambiguity will be bigger if the amplitude of the interval is bigger.

Moreover, we can interpret each concept alone. For example, we can say, looking at concept number 6, that the factor cholesterol is worse than the alcohol for the evolution of these illnesses, mainly for the diabetes.

Finally we can compare two concepts; for example, if we compare number 1 and number 3, we can say that the only difference is the degree of membership of the cholesterol and the leukemia (the first one goes up and the second one goes down). Once more we can say that the cholesterol is more injorious to the diabetes than the leukemia.

This interpretation is not yet very formal, but we are now studying the possibility of introduce a relation that allows classificate these interval-valued L-Fuzzy concepts.

Conclusions.

The theory developed in this paper allows us obtain information from an interval-valued L-Fuzzy relation by not stadistic methods.

In same cases, the cardinality of the interval-valued L-Fuzzy concept lattice is very big and we will have problems to interpret these concepts.

On the other hand, due to the map defined by Atanassov and Gargov [1], that associates an intuitionistic Fuzzy set to every intervalo-valued Fuzzy set, we will be able to translate these results to the intuitionistic case.

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