Does a small cost share reflect a negligible role for energy in economic production? Testing for aggregate production functions including capital, labor, and useful exergy through a cointegration-based method

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Abstract

Neoclassical models disregard the role of energy in production, equating a factor's output elasticity with its cost share, but failing to explain growth without a residual term. In contrast, ecological economics acknowledges energy's importance in production, regardless of its cost share. The aggregate production function (APF) concept, central to neoclassical theory, is also disputed.

We apply cointegration analysis to test for APFs between output, capital, and labor. We investigate the inclusion of energy inputs, measuring energy's capacity to generate productive work (useful exergy). Plausible APFs must verify cointegration and Granger-causality between output and inputs; and non-negative output elasticities. This method recognizes cases where: a) plausible APFs don't exist; b) energy impacts growth directly; c) energy impacts growth indirectly, through other inputs. We apply the method to Portugal (1960-2009), considering standard and quality-corrected capital and labor measures.

Plausible APFs are rarely obtained for capital-labor models. When they are, the residual growth component is large, and output elasticities disagree with historical cost shares. However, the residual is virtually eliminated for capital-labor-energy models with two cointegration relationships: a) a capital-labor APF, with output elasticities matching historical cost shares; b) a function estimating capital from useful exergy. These models reconcile energy's significance in production with cost-share neoclassical assumptions.

Keywords: *Cointegration, Aggregate production function, Cost shares, Solow residual, Useful exergy*

Highlights:

- We apply a cointegration-based method to test for aggregate production functions;
- Plausible production functions are subject to empirically-defined criteria;
- We test for the inclusion of energy inputs, measured through a useful exergy metric;
- Cointegration reveals indirect impact of useful exergy on growth, through capital;
- In this way, cost share theorem and energy's importance to production are reconciled;

1. Introduction

The true nature of the relationship between economic growth and energy use has sparked an ongoing debate between two seemingly contradictory approaches in the literature: neoclassical and ecological economics.

On the one hand, neoclassical theory acknowledges two (freely substitutable) factors of production – capital and labor – while ignoring the role of energy inputs. On the other hand, ecological economics argues that energy is essential to economic production, and real-world economic processes cannot be fully understood without accounting for energy use. In the next sections we will review some of the major contrasting features regarding energy inputs from both these approaches, and propose an econometric methodology to conciliate the two.

1.1. Neoclassical growth theory

The absence of energy from neoclassical growth models can be traced to (Ayres et al., 2013): a) an accounting identity; b) a historically observed stylized fact; c) an equilibrium condition arising from a simplifying income allocation theorem (i.e. cost share theorem).

The identity, commonly adopted in national accounts, equates GDP to the sum of payments to capital (interests, rents, dividends, royalties) and labor (wages, salaries). Historically, a stylized fact observed across countries verifies stable (average) cost shares for these factors, with labor receiving 70% of payments, and capital the remaining 30%[1]. Then, for a state of market equilibrium – maximizing profits without technological constraints on factor combinations – and for a simple economy consisting of small price-taking firms, a factor's productive power (i.e. output elasticity) can be equated with its respective cost share in total income (Gans et al., 2012).

Unlike capital and labor, payments to energy are generally not explicitly represented in national accounts. Even when these payments are roughly equated with revenues from energy industries (e.g. coal mining, electricity generation and distribution), they correspond to no more than 10% of total income (Denison, 1979; US EIA, 2011; Platchkov & Pollitt, 2011). Hence, by the cost share theorem, energy's productive power will be correspondingly small, thus justifying its exclusion from the principal neoclassical growth models, in which energy is neither a constraint nor an enabler of growth (Aghion & Howitt, 2009).

Ironically, while acknowledging only capital and labor as independent factors of production, neoclassical theory is open to a major criticism: its inability to account for the majority of economic growth with just these two factors. The pioneering work of Solow (1957) found that, after acknowledging the contributions from capital and labor in a growth accounting framework, an exogenous residual term is necessary in order to explain most (>85%) of economic growth in the US (1909-1949). This residual term (a.k.a. Solow residual, or total factor productivity – TFP), corresponding by definition to growth not explicable by measurable changes in either capital nor labor, is often found to be the major driver of growth within most industrialized economies (Easterly & Levine, 2001). And, while commonly regarded as a place holder for disembodied exogenous technological progress, there is no established theory regarding the factors that influence TFP. Hence, neoclassical theory rests on a tautological explanation of economic growth that, paradoxically, leaves most of growth unaccounted by measurable factors.

Since Solow's original results, attempts have been made to reduce the exogenous residual, by breaking it down into its components.

Jorgenson & Griliches (1967) proposed to eliminate TFP by carefully measuring capital and labor inputs according to their actual contribution to production, thereby avoiding inflation in the residual term from absorption of measurement errors in these inputs. Based on a strict application of neoclassical theory, the authors replace, in turn, traditional measures for capital and labor inputs by *quality-adjusted* measures that take into account not only the quantitative magnitude of these inputs, but also qualitative differences from each inputs in their contribution to production. Hence, the authors introduce capital measures based on utilization, rather than stocks[2]; and intensity-corrected labor measures[3].

The seminal work of Jorgenson & Griliches (1967)[4] forms the basis for current growth accounting exercises undertaken, among others, by the EU KLEMS database (Timmer et al., 2010). The importance of acknowledging the heterogeneity of capital and labor inputs in order to accurately account for economic growth is nowadays widely recognized. Labor measured in total hours worked is adjusted for the "skill" level of different workers, through a human capital index[5] (Whalley & Zhao, 2013; Manuelli & Seshadri, 2014), while differences in capital productivity are accounted by estimating the services provided by different capital assets, rather than their stocks (Schreyer et al., 2003; Wallis, 2009).

Since quality-adjusted measures for traditional factors of production generally grow at a faster rate than their unadjusted counterparts, adoption of these measures has a significant effect in reducing TFP in growth accounting (Groth et al. 2004). However, it's important to note that a considerable portion of TFP that cannot be "explained away" even when accounting for quality-adjusted inputs (van Ark, 2014).

1.2. Aggregate production function critique

Besides the need for an exogenous residual to account for economic growth, the very concept of an aggregate production function, which sits at the heart of neoclassical theory, has also been the subject of critique, mostly due to the "aggregation problem": the conditions under which different heterogeneous outputs and factors of production can be aggregated and summed across different micro-production functions to give an aggregate production function are so stringent that it is hard to believe in the existence of such a function (Felipe & McCombie, 2005; Felipe & McCombie, 2013).The most famous of the debates surrounding the aggregation problem – the Cambridge "capital controversy", sparked by the work of Pierro Sraffa and Joan Robinson (Sraffa, 1975 ;Robinson, 1953) – pointed out that the aggregate measurement of the amount of capital inputs involves adding up incomparable heterogeneous physical assets.

The arguments concerning this aggregation issue state that the aggregate production function is merely capturing an underlying accounting identity, which accounts for the good statistical fits commonly obtained, despite the "capital controversy". However, this means that no reliance should be placed on estimates for elasticity of substitution as reflecting technological parameters (Felipe & McCombie, 2005).

Despite the damaging implications for neoclassical macroeconomic theory, these fundamental issues surrounding the aggregate production function concept have been overlooked by neoclassical economists. Growth accounting exercises presume the existence of an aggregate homogeneous of degree one production function between factors of production and output. Most neoclassical models simply assume that such a relationship can always be written between the variables.

Because studies have shown that an aggregate Cobb-Douglas production function (or other, more flexible formulation) can have a good fit to a given dataset – even though the aggregation conditions are violated, or there is no neoclassical production function – results of regressions purporting to estimate such functions should be treated with caution (Felipe & McCombie, 2005; Felipe & McCombie, 2013). Hence, any approach to the estimation of aggregate production functions between time series for output and factors of production should take into account that such an aggregate relationship may not exist. In our work we take this into consideration.

1.3. Energy’s role in economic production

Ecological economics distinguishes itself from neoclassical economics by arguing that the economic system is embedded within a larger, environmental system with which it performs energy/matter transactions. The proponents of ecological economics argue that economic thinking should therefore be grounded in physical reality, namely the laws of thermodynamics.

The ecological economics literature generally posits a central role for energy in driving growth, going as far as to construct biophysical models that consider energy the only primary factor of production, while capital and labor are intermediates created and maintained by energy (Cleveland, 1991).

However, it can also be argued that an active use of energy is only possible through information and accumulated knowledge (Stern, 2011), which must be incorporated into machines/workers to be made productive, thus justifying the treatment of capital and labor as factors of production in their own right. These factors are also easier to measure than either information of knowledge, though their measurement is still very imperfect when compared to that of energy inputs (Stern, 2011).

In the field of ecological economics there have been several attempts to endogenize and better account for economic growth by treating energy as an independent factor of production, incorporating it alongside capital and labor in standard production function approaches (Tintner et al., 1977; Kümmel et al., 2000; Lindenberger & Kummel, 2002). By computing output elasticities through statistical fitting methods, the implications of the cost share theorem are generally invalidated, since the estimated economic weight associated with each factor may be higher/lower than suggested by its cost share.

Furthermore, most energy-augmented growth models do not include realistic constraints on substitution possibilities between energy, capital, and labor. In fact, thermodynamic considerations would suggest that production of a given level of output has minimum energy requirements (Stern, 1997), and energy scarcity will be a limiting factor to future economic growth (d’Arge & Kogiku, 1973; Gross & Veendorp, 1990; van den Bergh & Nijkamp, 1994; Lindenberger & Kümmel, 2011).

Besides being a constraint on economic production and growth, recent evidence (Giraud & Kahraman, 2014) suggests that tremendous increases in energy consumption – and efficiency – that accompanied the post WWII years in fact account for a large fraction of growth (generally ascribed to technological change) in the Western countries during the 30 Glorious Years[6].

Among studies that tested the inclusion of energy as an independent factor in aggregate production functions, the work of Ayres & Warr (2005) stands out in the ecological economics literature. The authors focus on 100 years of US economic growth, and find that by including primary energy in either a Cobb-Douglas or a more complex function (i.e. linear exponential, or LINEX[7]), alongside capital and labor, growth cannot be fully explained without a time-dependent residual (TFP).

However, Ayres & Warr (2005) defend that appropriate measuring and aggregation of energy inputs affect estimated TFP, and proceed to quantify energy not in terms of primary energy[8], but in terms of *useful exergy* (i.e. useful work)[9].

In thermodynamics, exergy is an energy flow's capacity to produce work. By the 2nd law of thermodynamics, work can be completely converted into heat, but the converse is not true (entropy). Accounting for exergy instead of energy takes this into consideration, changing the way heat and work are added up, by giving work a mark of quality. Useful exergy measures exergy flows at their useful stage, i.e. after all transformation and conversion losses, just before becoming energy services in the economy. Useful exergy accounts, for example, for heat delivered by an electric heater to provide thermal comfort, or mechanical work delivered by a car engine, through the driveshaft to the tires, to provide transport. It is a *quality-adjusted* measure for the energy used productively.

When useful exergy is included in a LINEX production function, Ayres & Warr (2005) obtain a remarkable result, successfully accounting for US economic growth without resorting to an exogenous residual.

Still, for all its importance in ecological economics literature, a twofold criticism falls on the work of Ayres & Warr (2005): 1) they adopt a non-standard production function (LINEX), which has found little acceptance within the economics community – Saunders (2008) states that the LINEX's thermodynamic considerations come at the price of not satisfying standard production function concavity conditions; 2) while quality-adjusting energy inputs through useful exergy measures, the authors fail to recognize the need to account for quality-adjusted measures for capital and labor.

1.4. Reconciling the neoclassical and ecological approaches?

Can the neoclassical and ecological strands of economic growth theory be reconciled? Namely, can an essential role for energy in production be compatible with neoclassical assumptions, such as the cost share theorem?

A pair of recent papers – Stern & Kander (2010), Kander & Stern (2014) – attempt to answer question. The authors link the omission of energy from mainstream growth models to the relative abundance of energy inputs in recent decades, compared with past constraints on growth imposed by energy availability. They show that expansion in the supply of energy services[10] over the last 200 years in Sweden has reduced the apparent importance of energy in economic growth, despite energy being essential to production[11].

Furthermore, while for recent decades the cost share for energy is small when compared with that of capital or labor – and mainstream growth models can yield good results while disregarding energy's contribution – Kander et al. (2014) show there is evidence for a declining cost share of energy in the very long-run, for Sweden[12] and other countries[13]. Such a decline is incompatible with a (Cobb-Douglas) unitary elasticity of substitution, since as the price of one factor rises, a movement along the isoquant implies that less of that factor is used, as to keep its cost share constant.

This led Stern & Kander (2010), Kander & Stern (2014) and Kander et al. (2014) to expand from the Cobb-Douglas formulation, allowing for restrictions to the substitution between energy, capital, and labor. By considering a capital-labor Cobb-Douglas function, embedded within a nested Constant Elasticity of Substitution (CES) production function with energy inputs[14], the authors successfully include energy as an additional factor of production to capital and labor, while still preserving, within the nested CES, the capital-labor neoclassical Cobb-Douglas production function[15]. Under this approach, energy can be shown to be essential to production, while the cost share theorem is preserved.

Sharing the motivation behind Stern & Kander (2010) and Kander & Stern (2014), we propose an alternative approach: to identify long-run economic relationships through joint statistical properties of economic data. Namely, we apply cointegration analysis to the estimation of aggregate Cobb-Douglas production functions and corresponding output elasticities for capital, labor, and energy inputs. Similar approaches can be found in Schröer & Stahlecker (1996) – that discussed whether the aggregate Cobb-Douglas production function represents a cointegration relationship between time series for output, capital, and labor – and Stern (2000) and Cleveland et al. (2000), which also included energy inputs.

Besides Stern (2000) and Cleveland et al. (2000), Ghali & El-Sakka (2004), Stresing et al. (2008), and Warr & Ayres (2010) are among the most recent studies testing multivariate cointegration between output, capital, labor, and energy. We also adopt a multivariate approach to test for cointegration, which reduces potentially omitted-variable biases[16]. Of the selected studies, only Stern (2000) and Warr & Ayres (2010) adopt a quality-adjusted measure for energy (a Divisia index in the former; useful exergy in the latter), and none adopts quality-adjusted measures for either capital or labor. In our work we will consider quality-adjusted energy inputs – i.e. useful exergy, as in Warr & Ayres (2010) –, and both unadjusted and quality-adjusted measures for capital and labor.

Stern (2000) presents evidence of cointegration between GDP, capital, labor, and energy for the US (1947-94), showing that energy cannot be excluded from the cointegration space[17]. Similar results are obtained for Canada by Cleveland et al. (2000) and Ghali & El-Sakka (2004). Stern (2000) also interprets observed cointegration relationships as production functions, and possibly as labor supply and/or capital acceleration functions. However, when restricting the analysis to Cobb-Douglas production functions without a time trend, and imposing that output elasticities for capital and labor must sum to unity, cointegration is no longer observed.

More recently, Stresing et al. (2008) tested for cointegration between output, capital, labor, and energy in Germany, Japan, and the US (1960-2008). The authors make a direct correspondence between cointegration coefficients and output elasticities of aggregate, energy-dependent, Cobb-Douglas production functions. These coefficients are required to meet certain restrictions (non-negativity, summing to unity) in order to be economically meaningful. Stresing et al. (2008) find that the hypothesis of cointegration cannot be rejected for any country, and that estimated output elasticities for labor (energy) are much smaller (larger) than corresponding historical cost shares.

Similarly to Stresing et al. (2008), in our analysis we will work within a sub-space of economically meaningful cointegration vectors, which we will interpret as aggregate Cobb-Douglas production functions if certain criteria are met. We define these criteria to be: 1) existence of cointegration relationships; 2) non-negative and distinguishable from zero normalized cointegration coefficients, corresponding to output elasticities for capital, labor, and energy; 3) long-run Granger causality between factor inputs and economic output. Estimated models are also compared with historical data in terms of goodness-of-fit and magnitude of the growth accounting residual.

The remainder of this paper is organized as follows: Section 2 presents the methodology in detail, including cointegration analysis, Granger causality, criteria for economically meaningful neoclassical aggregate production functions, and an application to past economic growth in Portugal. Results are presented and discussed in Section 3. Section 4 concludes and provides suggestions for future work.

2. Methodology

In this section we propose a detailed methodology to test for production functions through cointegration analysis on a selected dataset, comprised of time series for economic output, unadjusted and quality-adjusted capital and labor, and quality-adjusted energy (i.e. useful exergy).

Our methodology is built around a set of criteria, corresponding to conditions that must be met by our econometric models, in order to allow for their interpretation as economically meaningful neoclassical aggregate Cobb-Douglas production functions. The criteria – listed below and detailed throughout this section – are divided between those focused specifically on the econometric analysis, and those that focus on comparison with historical data.

The first set of criteria correspond to conditions imposed on the cointegration subspace and causal dynamics between variables. Each of these three criteria constitutes a stage of acceptance or rejection of empirically tested models. The first two criteria are related to the outcome of cointegration tests on each of the econometric models. The third criterion concerns the causal dynamics between variables.

- **There must be at least one estimated cointegration vector between output and inputs**, in order for a statistically significant relationship between the variables to be inferred. To test for cointegration, time series corresponding to each variable must be non-stationary and integrated of the same order[18]. Hence, we begin by testing each time series individually for the presence of unit roots. Working variables are grouped in vector autoregressive models (VAR), and tested for cointegration relationships following the Johansen procedure.
- **Estimated values for multiplicative coefficients in cointegration vectors observed between economic output and factor inputs – after normalizing to output – must be non-negative and distinguishable from zero.** When interpreting the cointegration relationships as aggregate Cobb-Douglas production functions, these normalized coefficients will correspond to output elasticities.
- **“Strong” long-run Granger causality must be observed between factor inputs and economic output.** By formulating a corresponding vector error-correction model (VECM) for combinations of variables for which cointegration is observed, tests for short-run and long-run causal dynamics between endogenous variables can be conducted.

Besides the econometric-driven criteria described above, comparative criteria, regarding the empirical application of accepted models to a case-study, are also defined:

- **Lowest discrepancy between estimated level of output and historical observations**, evaluated through goodness-of-fit statistics;
- **Lowest portion of growth attributed to an unexplained residual**, evaluated through growth accounting;
- **Closest correspondence between estimated output elasticities for capital and labor, and respective historical cost shares for these factors;**

The motivation for these criteria is to select among estimated production functions those that are better able to account for past economic growth in a case-study country – for this work Portugal, for the period 1960-2009.

The following sections describe the methodology adopted concerning each of the defined production function criteria.

2.1. Cointegration analysis

Testing for the existence of a cointegrated combination of two or more series is a way of testing the hypothesis that there is a statistically significant connection between them. For our models, a cointegration relationship must be established between output and input variables, or else we cannot interpret the resulting cointegration vector as a Cobb-Douglas production function. Failure to detect any cointegration vectors in a given VAR model results in the immediate rejection of that model from the remaining analysis.

2.1.1. Non-stationary tests and definition of working variables

The non-stationarity of time series quantifying each of the selected variables is tested through the Augmented Dickey-Fuller (ADF) and Philips-Perron (PP) tests for the presence of unit roots. These tests use different approaches to cope with serial correlation in the data. For time series in levels, a time trend and a constant term are included. First-differenced series represent growth rates, so there is no need to include a time trend. The number of lags is chosen according to the minimum observed value of the Schwarz Information Criteria (SIC)[19].

While output, labor, and energy variables are generally integrated of order one – I(1), capital measures are constructed so that the first difference of these measures – investment – is an I(1) series. Hence, capital measures tend to have a double unit root, i.e. they are integrated of second order – I(2). One way to deal with this situation is to define a set of working variables for our cointegration analysis. These will correspond to time series for the ratios of output, capital, and useful exergy per labor inputs (i.e. $Q/L, K/L, U/L$). We take the logarithms of these ratios, so that parameters may be interpreted as output elasticities (i.e. $q = log(Q/K), k = log(K/L), u = log(U/L)$). These will be our working variables for the rest of the analysis.

2.1.2. VAR models and testing for cointegration

Working variables are grouped in VAR models in the following way: the output variable (q) is included in all VAR models, alongside one or two input variables (k and/or u)[20].

Diagnostic tests are applied on the VAR models to assess if they are well-specified statistically. We observe available information criteria[21] and determine the adequate lag length (p) according to a majority rule. The choice is confirmed through serial correlation and normality tests on the VAR model residuals. If the lag length chosen is inadequate, the number of lags is increased until all tests are statistically sound.

Tests for cointegration are conducted on well-specified VAR models following the Johansen procedure (Johansen, 1988), a multivariate approach preferable over bivariate settings (e.g. the Engle-Granger two-step method), which do not account for additional channels of direct and indirect causality. This feature is especially relevant in our analysis, in order to assess not only the effect of useful exergy consumption on output directly, but also its effect in combination with capital and labor. The Johansen test is applied here under the assumption of an unrestricted constant term in the VAR model but no linear trend in the cointegration vector. This is the empirically most common specification, and corresponds to case 3 in Juselius (2006).

The Johansen procedure is composed of two separate tests: trace and eigenvalue – see Appendix B.2. For VAR models with at most three endogenous variables, there are three possible outcomes from cointegration analysis: a) no cointegration; b) at most one cointegration vector between all variables – which can be interpreted as a three-factor Cobb-Douglas production function, if it satisfies the necessary criteria (defined in Section 2.3); c) at most two cointegration vectors between combinations of variables – which can be simultaneously interpreted (again, if the necessary criteria are satisfied) as a two-factor Cobb-Douglas production function (output as a dependent variable), and a Cobb-Douglas type function (but not a production function) linking all variable inputs. For models with at most two endogenous variables, at most one cointegration vector (interpretable as a production function) can be observed.

Occasionally, the two separate tests in the Johansen procedure may provide contradictory information on the number of cointegration vectors observed for a given VAR model. In this case, we adopt the following criterion: as long as at least one of the Johansen tests rejects the null hypothesis (of no cointegration or at most one cointegration vector), we proceed by actively rejecting that hypothesis. It may also occur that the number of cointegration vectors suggested by tests is identical to the number of variables (n) included in a given VAR model. This suggests that the model can be specified as a VAR in levels of stationary time series. Since we only perform tests for cointegration with integrated variables, these situations contradict our univariate unit root tests. This may be due to low power of the cointegration tests, small sample size, or indication of a specification error. In our analysis, we deal with such cases by giving more weight to the unit root tests than to cointegration tests, and assuming for each model at most $n-1$ cointegration vectors.

2.2. Vector error-correction models and cointegration coefficients

Multiplicative coefficients obtained through cointegration analysis are unconstrained, i.e. they can assume positive or negative values. However, after normalizing to output , these coefficients will correspond to output elasticities in our interpretation of cointegration vectors as Cobb-Douglas aggregate production functions (e.g. β_2/β_1 in Equation 3 below). Therefore, the estimated coefficients must assume non-negative, bounded between 0 and 1, values. The sum of all entrepreneurial decisions will always lead to a state of the economy where the increase of an input never results in a decrease of output. The estimated values for the output elasticities must also be statistically significant. If any model fails to satisfy these conditions, it is immediately rejected from the remaining analysis.

If the n time series included in any given VAR model are found to be cointegrated, the corresponding VECM is given by:

$$\Delta\mathbf{Y}_t = \mathbf{c} + \alpha(\beta\mathbf{Y}_{t-1} + \mu) + \sum_{j=1}^{p}\Gamma_j\Delta\mathbf{Y}_{t-j} + \varepsilon_j \tag{1}$$

Where $\mathbf{Y}_t$ is a $(n \times 1)$ vector of each model's time series, $\mathbf{c}$ is a vector of constant terms, Γ_j represents matrices of short-run dynamics coefficients (lags), and ε_j is a vector of random disturbances. The term in parentheses in Equation 1 is the error-correction term (ECT), with β a $(n \times r)$ matrix of cointegration vectors (r being the number of cointegration vectors) and μ a vector of coefficients representing a constant in the cointegration space; α is a $(n \times r)$ matrix of adjustment coefficients.

The ECT represents a statistically significant long-term relationship between the variables in $\mathbf{Y}_t$, which under certain conditions can be interpreted as economically meaningful neoclassical Cobb-Douglas production functions. For a two-variable model (q, k) with at most one cointegration vector between its variables, the corresponding VECM is

$$\begin{aligned} \Delta q_t &= c_1 + \alpha_1 ECT_{t-1} + \textstyle\sum_{j=1}^{p} \phi_{1j} \Delta q_{t-j} + \sum_{j=1}^{p} \theta_{1j} \Delta k_{t-j} \\ \Delta k_t &= c_2 + \alpha_2 ECT_{t-1} + \textstyle\sum_{j=1}^{p} \phi_{2j} \Delta q_{t-j} + \sum_{j=1}^{p} \theta_{2j} \Delta k_{t-j} \end{aligned} \quad (2)$$

Where the cointegration vector ECT_{t-1} normalized to output is

$$ECT_{t-1} = q_{t-1} + \left(\frac{\beta_2}{\beta_1}\right) k_{t-1} + \mu_1 \quad (3)$$

Setting Equation 3 equal to zero, moving the normalized variable to the l.h.s., and multiplying both sides by the labor inputs yields (time subscripts are removed in the next step):

$$Q = e^{\mu_1} K^{\alpha_K} L^{\alpha_L} \qquad (4)$$

With $\alpha_K = (-\beta_2/\beta_1)$ and $\alpha_L = 1 + (\beta_2/\beta_1)$ necessarily summing to one: $\alpha_K + \alpha_L = 1$.

If no cointegration is observed between the variables in $\mathbf{Y}_t$, Equation 1 is written as a VAR, without the ECT, and depending only on a constant term and lagged terms.

For models where at most two simultaneous cointegration vectors are observed between pairs of endogenous variables, we opt to algebraically manipulate both vectors to form a single long-term relationship between output, capital, and labor (i.e. a two-factor Cobb-Douglas production function), and a second long-term relationship linking all factors of production. The normalized ECT, for these models, are written as

$$\begin{cases} ECT_{1,t-1} = q_{t-1} + \left(\frac{\beta_2}{\beta_1}\right) k_{t-1} + \mu_1 \\ ECT_{2,t-1} = k_{t-1} + \left(\frac{\beta_4}{\beta_3}\right) u_{t-1} + \mu_2 \end{cases} \quad (5)$$

Where the first ECT is normalized with respect to q, and the second ECT normalized to k. Setting both ECT in Equation 5 equal to zero, and reformulating as in Equation 3, yields

$$\begin{cases} Q = e^{\mu_1} K^{\alpha_K} L^{\alpha_L} \\ K = e^{\mu_2} U^{\gamma} L^{\delta} \end{cases} \qquad (6)$$

The first equation in Equation 6 is similar to Equation 4. In the second equation in Equation 6, the coefficients correspond to $\gamma = (-\beta_4/\beta_3)$ and $\delta = 1 + (\beta_4/\beta_3)$. These coefficients sum to unity, but they are not interpreted as output elasticities or subjected to our production function criteria defined below since, although Cobb-Douglas in form, the second equation in Equation 6 is not a production function[22].

Our reasoning for estimating capital (and not labor or useful exergy) from other inputs using the second cointegration vector is as follows: when compared with both labor and energy inputs, which are measured in physical units of hours worked and Joules, respectively; adopted monetary measures for capital inputs are less accurate in representing actual productive uses of these inputs in the economy. Even approaches to the measurement of service flows of capital assets such as the ones adopted in our analysis (see Section 2.5 and Appendix A.2) rely on numerous assumptions on rates of return, depreciation of assets (in value and efficiency), initial benchmarks of stocks of assets, and price variations. By estimating capital inputs as a function of useful exergy and labor inputs, we hope to obtain a better estimate of the utilization of capital in the economy, depending uniquely on physical measures.

2.3. Granger causality

Economically meaningful neoclassical Cobb-Douglas production functions should exhibit long-term Granger causality between factor inputs and economic output, in any direction[23]. In effect, this means that for any model we should observe long-run (unidirectional or bidirectional) causality running from k (and/or u) to the output variable q, thus indicating that the considered inputs to economic production are having a long term causal influence on production output. Models that fail to exhibit such a map of causality relationships will be discarded from analysis.

In order to test for causal dynamics between variables in the VAR models, we adopt the Engle and Granger (1987) approach. Cointegration analysis precedes testing for Granger causality since the presence of cointegration vectors between variables has implications for the way in which short and long-run causality is carried out.

There are two sources of causation in Equation 2: coefficients associated with the ECT – translating an error-correction mechanism driving the variables towards a long-run relationship; and coefficients on lagged terms, indicating short-run dynamics. Analogously, we can write Equation 2 for a three-variable VAR and interpret the results in the same way.

Granger non-causality tests are applied by testing the statistical significance of coefficients. Short-run Granger non-causality tests are applied to lagged coefficients (e.g. testing the null hypothesis $H_0\colon \theta_{1j} = 0, \forall j$ in Equation 2 using a Wald test), while long-run tests are applied to the ECT adjustment coefficients. "Strong" long-run Granger non-causality is tested by examining whether the two sources of causation are jointly significant (e.g. testing the null hypothesis $H_0\colon \alpha_1 = \theta_{1j} = 0, \forall j$ in Equation 2). In our analysis we consider only short-run and "strong" long-run causality relationships between variables.

2.4. Goodness-of-fit and Solow residual

Econometric models that satisfy all econometric criteria defined above can be further compared in terms of their adjustment to historical values for real output in a given country, as to infer which estimated production function is better suited to account for past trends. The comparison is made through: 1) goodness-of-fit to historical output; 2) magnitude of unexplained residual in growth accounting.

Goodness-of-fit is evaluated through computation and comparison of the root mean squared error (RMSE) in both levels and growth rates, and coefficient of determination R^2. The RMSE is a good measure of accuracy for the estimated production functions, regarding past output trends, while R^2 is a widely used statistic to indicate how well data fits a particular model. In order to account for the phenomenon by which R^2 automatically increases when extra explanatory variables are added to the model, a modification of this statistic (the adjusted R^2) is used.

In order to weigh, for each estimated production function, the contribution from each factor of production – and indirectly estimate the residual component, TFP – to historical output growth, we conduct growth accounting exercises. The fundamental equation for growth accounting, valid for any production function homogeneous of degree one satisfying constant returns to scale (such as the Cobb-Douglas), can be expressed for two factors of production (capital and labor) as:

$$g_Q = \alpha_K g_K + \alpha_L g_L + g_{TFP} \qquad (7)$$

In Equation 7 g_Q, g_K, g_L stand, respectively, for the growth rates of output, capital, and labor. The coefficients α_K and α_L represent the marginal productivities of capital and labor, respectively, and correspond to observed annual shares of payments to these factors (obtained from national accounts). The growth rate of total factor productivity, g_{TFP}, is computed as a residual, by subtracting the capital and labor contributions to growth from total observed output growth. Functions with smaller average growth of TFP are better suited to account for economic growth only with the contributions from inputs to production.

For two-variable models with at most one cointegration vector, goodness-of-fit statistics and growth accounting is conducted on the estimated Cobb-Douglas production function – Equation 4.

For three-variable models with at most two cointegration vectors – normalized as in Equation 6 – are tested for goodness-of-fit and growth accounting considering two alternatives. First, using observed time series for capital and labor inputs as dependent variables on the first equation in Equation 6. Second, estimating, using the second equation in Equation 6, a time series for capital inputs as a function of observed series for useful exergy and labor. This estimated series for capital is then substituted in the two-factor Cobb-Douglas production function given by the first equation in Equation 6, alongside observed labor inputs. Hence, we compare between capital inputs measures by estimating alternative fits to real output and growth accounting for three-variable models with at most two cointegration vectors: 1) using observed capital inputs; 2) using estimated capital inputs.

2.5. Case-study: Portugal 1960-2009

We apply our proposed methodology to a single-country case-study: economic output in Portugal in recent decades.

Annual data is collected for 50 years, starting in 1960 and ending in 2009. We measure economic output (Q) as gross value added (GVA), corresponding to the sum of payments to the traditional inputs to production: capital (K) and labor (L). We will consider both unadjusted and quality-adjusted measures for both these factors. By performing econometric analysis with both unadjusted and quality-adjusted capital and labor measures, we will be able to observe the effects of quality-adjusting for these inputs in our statistical estimates. We also consider quality-adjusted energy inputs to production, measured as aggregate consumption of useful exergy (U).

We will be working with two measures for labor inputs, so we will produce two sets of working variables, depending on whether these are defined by the ratio of output, capital, and useful exergy per unadjusted labor inputs (indicated by a label L), or per quality-adjusted labor inputs (label hL). All models are then defined in terms of both unadjusted and quality-adjusted labor inputs[24].

Overall, our empirical analysis begins by considering two separate sets of 5 models each: three two-variable models and 2 three-variable models. Table 1 specifies the models, as well as the measure, units, and source of data concerning each input to production. Figure 1 shows collected data for each variable, normalized to base year (1960).

In the next section we present and discuss results obtained from our analysis.

Table 1 - List of models used in statistical analysis. For all models, the output variable q corresponds to GVA in Mrd 2006 €, obtained from Pinheiro (1997) and INE database. The 5 models presented are defined for both unadjusted (subscript L) and quality-adjusted (hL) labor inputs, obtained from Amaral (2009) (L) and PWT8.1 (h).

2 variables	Capital inputs (k)			Energy inputs (u)	
	Measure	Units	Source	Units	Source
$(q, u)_{L,hL}$	-	-	-	TJ	Serrenho et al. (2016)
$(q, k^{AMECO}_{Stock})_{L,hL}$	Stock	Mrd 2006 €	AMECO	-	-
$(q, k^{S\&L}_{Services})_{L,hL}$	Services	VICS	da Silva & Lains (2013)	-	-
3 variables	**Capital inputs (k)**			**Energy inputs (u)**	
	Measure	Units	Source	Units	Source
$(q, k^{AMECO}_{Stock}, u)_{L,hL}$	Stock	Mrd 2006 €	AMECO	TJ	Serrenho et al. (2016)
$(q, k^{S\&L}_{Services}, u)_{L,hL}$	Services	VICS	da Silva & Lains (2013)	TJ	Serrenho et al. (2016)

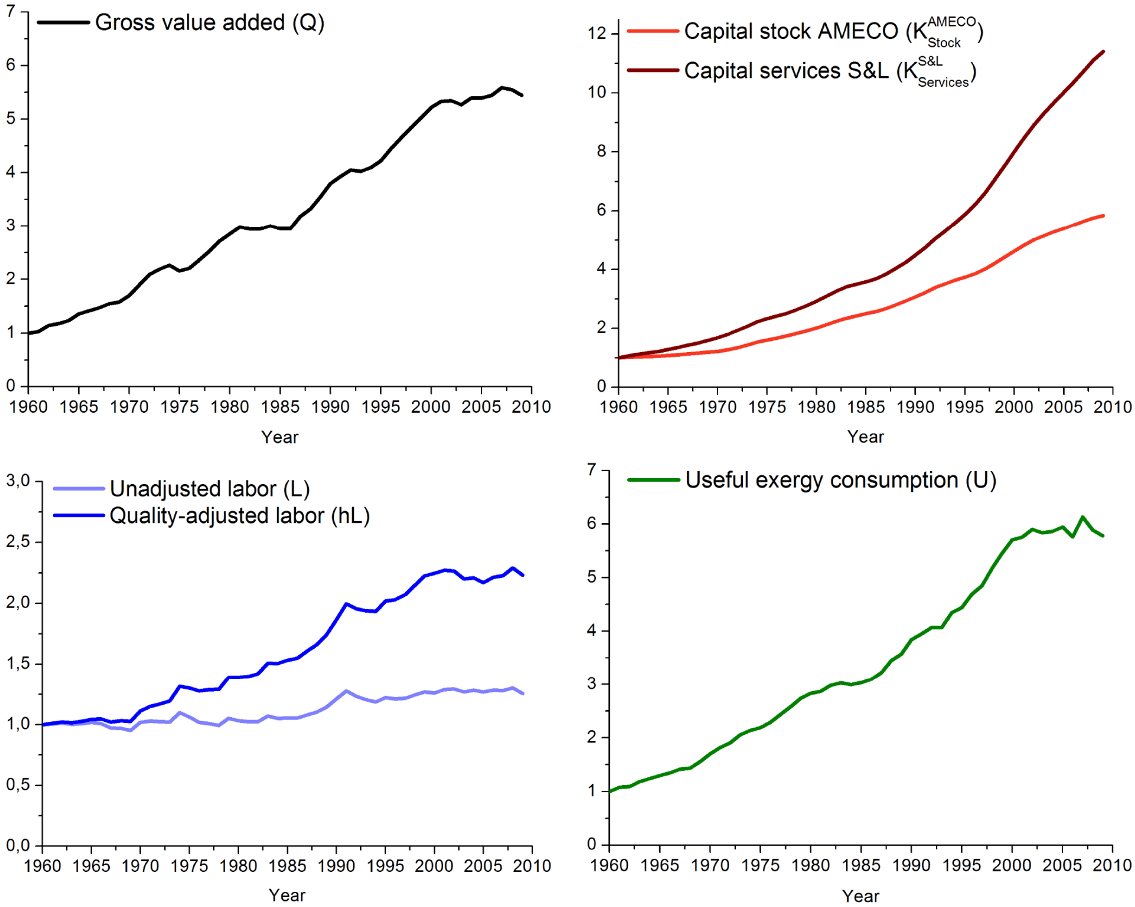

Figure 1 - Normalized collected data for output and inputs to production. Top left: output, measured as gross value added. Top right: capital inputs, measured as stocks of assets (light red) and service flows (dark red). Bottom left: labor inputs, measured as unadjusted total number of hours worked (light blue) and total hours worked adjusted using a human capital index (dark blue).

3. Results and discussion

3.1. Cointegration analysis

3.1.1. Non-stationarity tests

Explicit results for the ADF and PP tests are presented in Appendix C.1 and summarized here.

We find, for both tests, that the time series quantifying the variables for output (Q), capital (stock, K_{Stock}^{AMECO}, and services, $K_{Services}^{S\&L}$), labor (L and hL), and useful exergy (U) all fail to reject the hypothesis of non-stationarity, i.e. they all have unit roots.

However, while level variables Q, L, hL, and U are found to be integrated of order one (first differences are stationary), both capital measures are integrated of order two (only second differences are stationary). Hence, we cannot conduct tests for cointegration between these variables, following the Johansen procedure.

Defining the variables per labor inputs, as proposed in our methods section, produces time series corresponding to the variables q, k, and u. The ADF and PP unit root tests fail to reject the hypothesis of non-stationarity for all these time series in levels (considering both unadjusted and quality-adjusted labor).

Concerning the first differences of the time series defined per labor inputs, the non-stationarity hypothesis is always rejected by both tests at the 1% significance level. We therefore conclude that all time series defined per labor inputs are integrated of first order, and hence we can conduct tests for cointegration between these variables.

3.1.2. Cointegration and Granger causality

Results corresponding to the cointegration and Granger causality tests conducted in our analysis are presented in Table 2. These results concern the above-defined econometric criteria for economically meaningful neoclassical Cobb-Douglas production functions. Explicit numerical results for each of these models, and additional results concerning models that include alternative measures for capital stock are presented in Appendices C.2 and C.3.

For each model we indicate the number and normalized formulation (to output) of estimated cointegration vectors, the direction of short and long-run Granger causality between variables, and which of the defined econometric criteria are satisfied: 1) existence of cointegration; 2) economically meaningful normalized coefficients in cointegration vectors linking inputs to output; 3) long-run causality between factor inputs and output.

Of the 10 models considered at the beginning of our analysis, 7 satisfy all econometric criteria. These are highlighted in Table 2. Of the original 10, one model – $(q, k^{S\&L}_{Services})_L$ – is rejected due to no cointegration vectors being detected using the Johansen procedure. Of the remaining 9 models, for which at least one cointegration vector is observed, 2 more models – $(q, k^{S\&L}_{Services})_{hL}$ and $(q, k^{AMECO}_{Stock}, u)_{hL}$ – are rejected due to the normalized estimated coefficients falling outside the defined boundaries for economically meaningful neoclassical output elasticities in a Cobb-Douglas production function (i.e. they assume negative values).

We observe that none of the considered models is rejected based solely on the econometric criteria of long-run Granger causality between inputs and output. All models show capital and useful exergy inputs (per labor) having a significant long-run causal effect on output (per labor). The one exception – $(q, k^{S\&L}_{Services})_{hL}$ –, for which no significant Granger causality relations are observed, also fails to satisfy the criterion of economically meaningful neoclassical normalized cointegration coefficients.

Granger causality relationships concerning causal effects of inputs or output variables on themselves (e.g. $q \rightarrow q$) are not represented in Table 2, since these results are not directly linked with the production function criteria defined in our analysis. They can, however, be consulted in Appendix C.3, as well as all numerical results concerning Granger causality tests.

For three or our models – $(q, k^{AMECO}_{Stock}, u)_L$, $(q, k^{S\&L}_{Services}, u)_L$ and $(q, k^{S\&L}_{Services}, u)_{hL}$ – at most two cointegration vectors are observed between endogenous variables. In this cases, as suggested in Section 2.2, we normalize the first cointegration vector to output, and the second cointegration vector to capital inputs. Then, manipulating both vectors algebraically, we obtain: 1) a Cobb-Douglas production function formulation with output (Q) depending on capital (K) and labor (L, or hL) inputs; 2) a Cobb-Douglas function where capital (K) is estimated from independent variables useful exergy (U) and labor (L or hL) inputs. For each of these models, both formulations are represented in Table 2.

The models presented in Table 2 that satisfy all econometric production function criteria are further tested according to our defined comparative criteria concerning how well estimated production functions are adjusted to the Portuguese economy in the past 50 years.

3.2. Goodness-of-fit and Solow residual

Estimated aggregate Cobb-Douglas production functions obtained from the 7 models in Table 2 that satisfy all econometric production function criteria are compared in terms of goodness-of-fit to real output (GVA) for the Portuguese economy in the past 50 years, as well as in terms of the magnitude of the estimated TFP component in long-run output growth. In the former case, models are evaluated in terms of RMSE and (adjusted) R^2 statistics, while for the latter comparison is made through growth accounting.

All relevant results are shown in Table 3. For each model we show the estimated production function formulation, and whether (or not) capital inputs correspond to observed historical time series or are instead estimated as a function of both useful exergy and labor inputs. The remaining columns in Table 3 compare RMSE and (adjusted) R^2 statistics, as well as the components of average output growth, for all models. Graphical representations of fits to output levels, obtained with each of the 7 models in Table 3 are shown in Figure 2 (two variables) and Figure 3 (three variables).

In terms of fits to historical levels and growth rates of output for the Portuguese economy, the lowest RMSE (and correspondingly highest adjusted R^2) are obtained with estimated production functions that either: 1) have a useful exergy-labor form (no capital inputs); 2) have a capital-labor form, but capital is estimated as a function of both useful exergy and labor, through a second cointegration vector. These results are not unexpected, given that useful exergy by itself tracks Portuguese economic output very closely (see Figure 2).

Model	No. CV	Normalized long-run relationship	Causality		Absolute production function criteria		
			Short-run	Long-run	Cointegration	Output elasticities	Granger causality
$(q,u)_L$	≤ 1	$Q = e^{-7.77} \cdot U^{0.84} \cdot L^{0.16}$	$u \rightarrow q$	$u \rightarrow q$	Yes	Yes	Yes
$(q,k_{Stock}^{AMECO})_L$	≤ 1	$Q = e^{-5.19} \cdot K^{0.60} \cdot L^{0.40}$	-	$k \rightarrow q$	Yes	Yes	Yes
$(q,k_{Services}^{S\&L})_L$	0	-	-	-	No	-	-
$(q,k_{Stock}^{AMECO},u)_L$	≤ 2	$\begin{cases} Q = e^{-4.79} \cdot K^{0.64} \cdot L^{0.36} \\ K = e^{-4.68} \cdot U^{1.31} \cdot L^{-0.31} \end{cases}$	$u \rightarrow q$ $u \rightarrow k$	$k \rightarrow q$ $u \leftrightarrow q$	Yes	Yes	Yes
$(q,k_{Services}^{S\&L},u)_L$	≤ 2	$\begin{cases} Q = e^{-8.65} \cdot K^{0.37} \cdot L^{0.63} \\ K = e^{2.78} \cdot U^{2.34} \cdot L^{-1.34} \end{cases}$	$u \leftrightarrow q$	$k \rightarrow q$ $u \leftrightarrow q$	Yes	Yes	Yes
$(q,u)_{hL}$	≤ 1	$Q = e^{-8.19} \cdot U^{0.78} \cdot L^{0.22}$	-	$u \rightarrow q$	Yes	Yes	Yes
$(q,k_{Stock}^{AMECO})_{hL}$	≤ 1	$Q = e^{-7.12} \cdot K^{0.45} \cdot L^{0.55}$	-	$k \rightarrow q$	Yes	Yes	Yes
$(q,k_{Services}^{S\&L})_{hL}$	≤ 1	$Q = e^{-15.00} \cdot K^{-0.34} \cdot L^{1.34}$	-	-	Yes	No	No
$(q,k_{Stock}^{AMECO},u)_{hL}$	≤ 1	$Q = e^{-7.98} \cdot K^{-0.52} \cdot L^{-0.04} \cdot U^{1.57}$	$u \leftrightarrow q$ $u \rightarrow k$	$k \rightarrow q$ $u \leftrightarrow q$ $u \leftrightarrow k$	Yes	No	Yes
$(q,k_{Services}^{S\&L},u)_{hL}$	≤ 2	$\begin{cases} Q = e^{-9.65} \cdot K^{0.31} \cdot L^{0.69} \\ K = e^{7.69} \cdot U^{3.11} \cdot L^{-2.11} \end{cases}$	$u \rightarrow q$	$k \rightarrow q$ $u \leftrightarrow q$	Yes	Yes	Yes

Table 2 – **Production function criteria – results from cointegration analysis and Granger causality tests.** Top half models with variables defined per unadjusted labor inputs (L). Bottom half models with variables defined per quality-adjusted labor inputs (hL). Column (1) specifies the model; columns (2) and (3) present the number and normalized formulation of observed cointegration vectors; columns (4) and (5) present short and long-run Granger causality (arrows represent direction of causality); columns (6-8) indicate whether the model satisfies the econometric production function criteria of: existence of cointegration vector (column 6), plausible output elasticities (column 7), and causal effects between inputs and output (column 8). Models that satisfy every econometric criteria are highlighted.

The TFP components in long-run output growth for the $(q, k_{Services}^{S\&L}, u)_L$ and $(q, k_{Services}^{S\&L}, u)_{hL}$ models, when capital is estimated through the second cointegration vector, are the smallest among all estimated production functions. However, the residual term obtained with the $(q, k_{Services}^{S\&L}, u)_{hL}$ model is particularly striking – corresponding approximately to only 1% of overall output growth for the period 1960-2009, and approximately 4% of the value for the next smallest residual i.e. model $(q, k_{Services}^{S\&L}, u)_L$. In comparison, the largest obtained TFP component – model $(q, k_{Stock}^{AMECO})_L$, accounts for more than 60% of overall output growth.

Within models with at most two cointegration vectors, there is a significant difference in terms of goodness-of-fit between the case where the capital-labor production function is computed with historical capital series, and when capital series are estimated as a function of useful exergy and labor (i.e. a 70-80% smaller RMSE in levels and a 2-10% higher adjusted R^2). In comparison, adoption of quality-adjusted measures for labor inputs (in the models where it is comparable[25]) has a positive but less significant effect on goodness-of-fit (a 5-10% smaller RMSE in levels and at most a 1% higher adjusted R^2). The effect of estimating actual utilization of capital in production as a function of useful exergy and labor inputs, instead of using historical observed values, seems to have a much more significant impact on goodness-of-fit than introducing quality-adjusted measures for either of the traditional factors of production.

While it can be argued that introducing additional variables will *per se* result in better fits to historical data, it should be noted that two-factor useful exergy-labor production functions already produce very good fits[26]. The inclusion of a capital variable in the models then serves the purpose of having a production function formulation with the traditional factors of production, as used in the neoclassical approach, and hence obtain information on payments to these factors, assuming that the cost-share theorem holds.

The results in Table 3 suggest that, for those models satisfying all econometric production function criteria and for which useful exergy is included in the estimated production function (either directly, or indirectly through estimation of capital inputs), output growth can be mostly accounted for only with the inputs to production, through the ECT in Equation 1 (i.e. observed cointegration vectors). In contrast, models with worse fits rely more on the constant and lagged terms than the ECT in their respective VECM formulation to account for economic growth. Moreover, models for which no cointegration vector is observed (but for which growth accounting can still be performed – see Table 3) are not written in VECM but VAR form, and all of their explanatory power lies in the constant and lagged terms.

Model/Data	Estimated production function	Estimated factor	RMSE (level)	Adj. R^2 (%)	Output growth (%)	$\alpha_K g_K$ (%)	$\alpha_L g_L$ (%)	Residual (%)
$(q,u)_L$	$Q = e^{-7.77} \cdot U^{0.84} \cdot L^{0.16}$	None	0.051	99.54	-	-	-	-
$(q,k_{Stock}^{AMECO})_L$	$Q = e^{-5.19} \cdot K^{0.60} \cdot L^{0.40}$	None	0.167	95.39	3.45	1.04	0.32	2.08
$(q,k_{Stock}^{AMECO},u)_L$	$Q = e^{-4.79} \cdot K^{0.64} \cdot L^{0.36}$	None	0.152	96.52	3.45	1.04	0.32	2.08
		$K(U,L)$	0.052	99.50	3.45	1.33	0.32	1.80
$(q,k_{Services}^{S\&L},u)_L$	$Q = e^{-8.65} \cdot K^{0.37} \cdot L^{0.63}$	None	0.206	89.31	3.45	1.45	0.32	1.67
		$K(U,L)$	0.040	99.70	3.45	2.30	0.32	0.82
$(q,u)_{hL}$	$Q = e^{-8.19} \cdot U^{0.78} \cdot L^{0.22}$	None	0.048	99.69	-	-	-	-
$(q,k_{Stock}^{AMECO})_{hL}$	$Q = e^{-7.12} \cdot K^{0.45} \cdot L^{0.55}$	None	0.150	96.45	3.45	1.04	1.04	1.36
$(q,k_{Services}^{S\&L},u)_{hL}$	$Q = e^{-9.65} \cdot K^{0.31} \cdot L^{0.69}$	None	0.144	96.26	3.45	1.45	1.04	0.95
		$K(U,L)$	0.040	98.89	3.45	2.37	1.04	0.03

Table 3 – **Fits to historical output for the Portuguese economy – results from Root Mean Squared Error, adjusted R^2 and growth accounting.** Only models that satisfy the econometric production function criteria defined earlier are represented. Top half shows models with variables defined per unadjusted labor inputs (L). Bottom half shows models with variables defined per quality-adjusted labor inputs (hL). Column (1) specifies the model; columns (2) and (3) present the production function formulation and whether any of the factor inputs is estimated from the other inputs (i.e. K as a function of U and L); columns (4) and (5) show values for RMSE and adjusted R^2, respectively; columns (6-9) refer to growth accounting, subtracting from average output growth (column 6) the contributions from capital (column 7) and labor (column 8), to obtain a residual component (column 9).

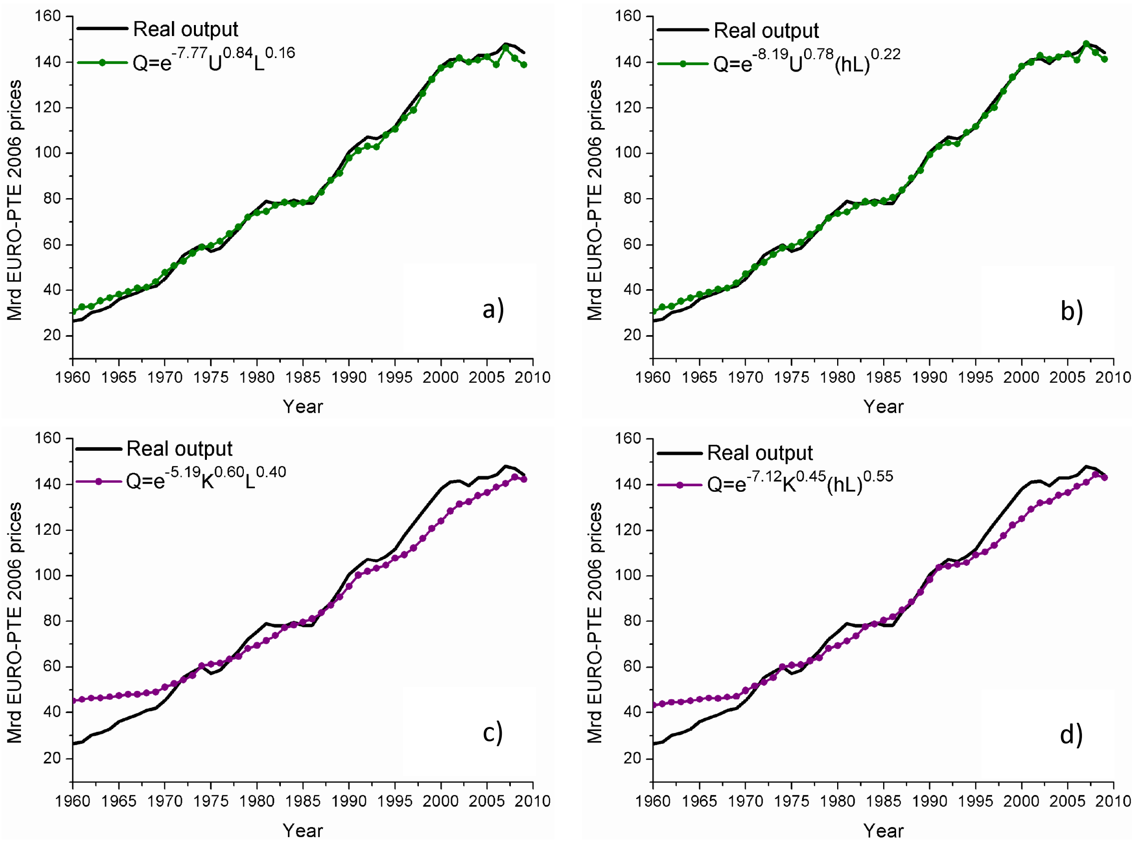

*Figure 2 – **Fits to historical output for estimated production functions obtained from two-variable models satisfying all econometric production function criteria.** Left: Estimated production functions with unadjusted labor inputs (L). Right: Estimated production functions with quality-adjusted labor inputs (hL). a)-b) Estimated production functions for models $(q,u)_L$ and $(q,u)_{hL}$, respectively. c)-d) Estimated production functions for models $(q,k_{Stock}^{AMECO})_L$ and $(q,k_{Stock}^{AMECO})_{hL}$, respectively.*

The most interesting results are those obtained with model $(q,k_{Services}^{S\&L},u)_{hL}$, when capital is estimated through the second cointegration vector. This is the only model that combines: 1) inclusion of capital services and useful exergy in the cointegration space; b) working variables defined per quality-adjusted labor inputs; 3) actual utilization of capital in production estimated as a function of both useful exergy and labor. The estimated cointegration coefficients for this model, corresponding to output elasticities ($\hat{\alpha}_K = 30.80\%$ and $\hat{\alpha}_L = 69.20\%$) can be compared to average historical cost shares associated with capital and labor inputs in national accounts – Figure 4. In this graph it can be seen that, while the first decades are marked by strong variation of cost shares, overall they assume average values of $\bar{\alpha}_K \cong 29.54\%$ for capital and $\bar{\alpha}_L \cong 70.46\%$ for labor.

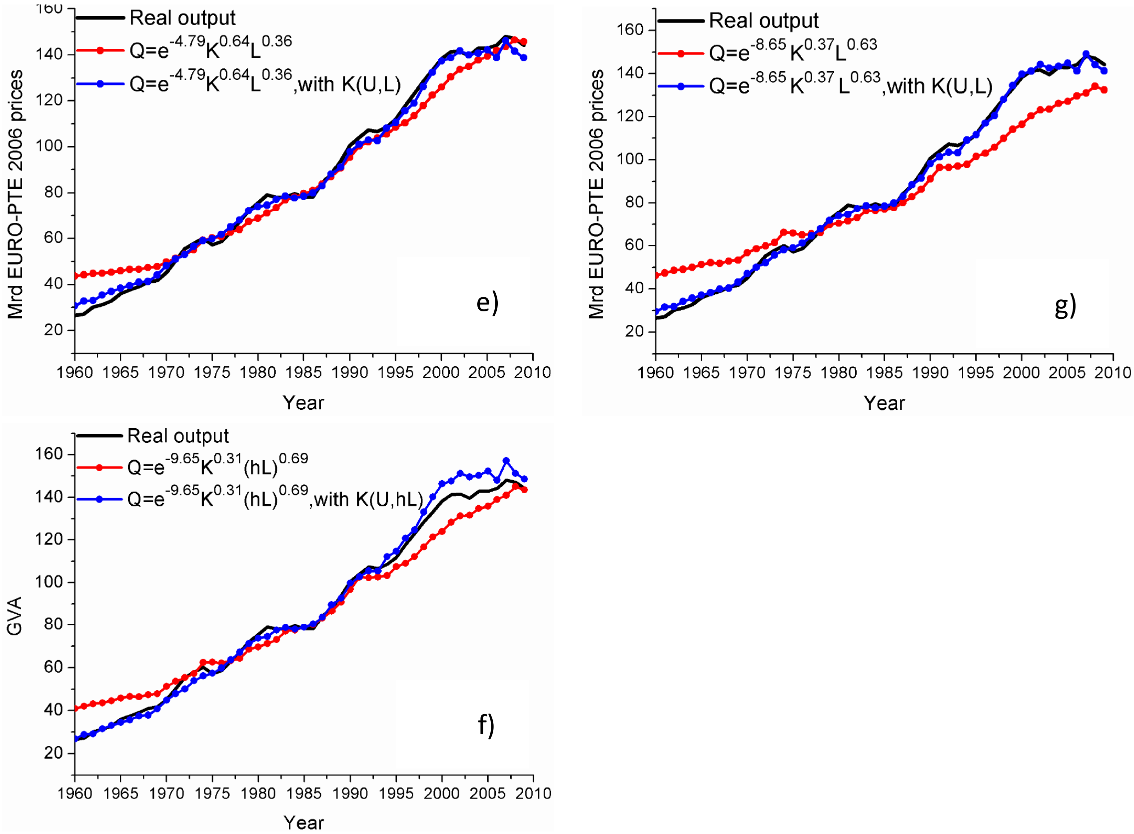

Figure 3 - ***Fits to historical output for estimated production functions obtained from three-variable models satisfying all econometric production function criteria.*** *Left: Estimated production functions with unadjusted labor inputs (L). Right: Estimated production functions with quality-adjusted labor inputs (hL). e) Estimated production function for model* $(q, k_{Stock}^{AMECO}, u)_L$*. f)-g) Estimated production functions for models* $(q, k_{Services}^{S\&L}, u)_L$ *and* $(q, k_{Services}^{S\&L}, u)_{hL}$*, respectively. Estimated production functions are represented assuming historically observed capital inputs (red) and estimated series for capital inputs (blue).*

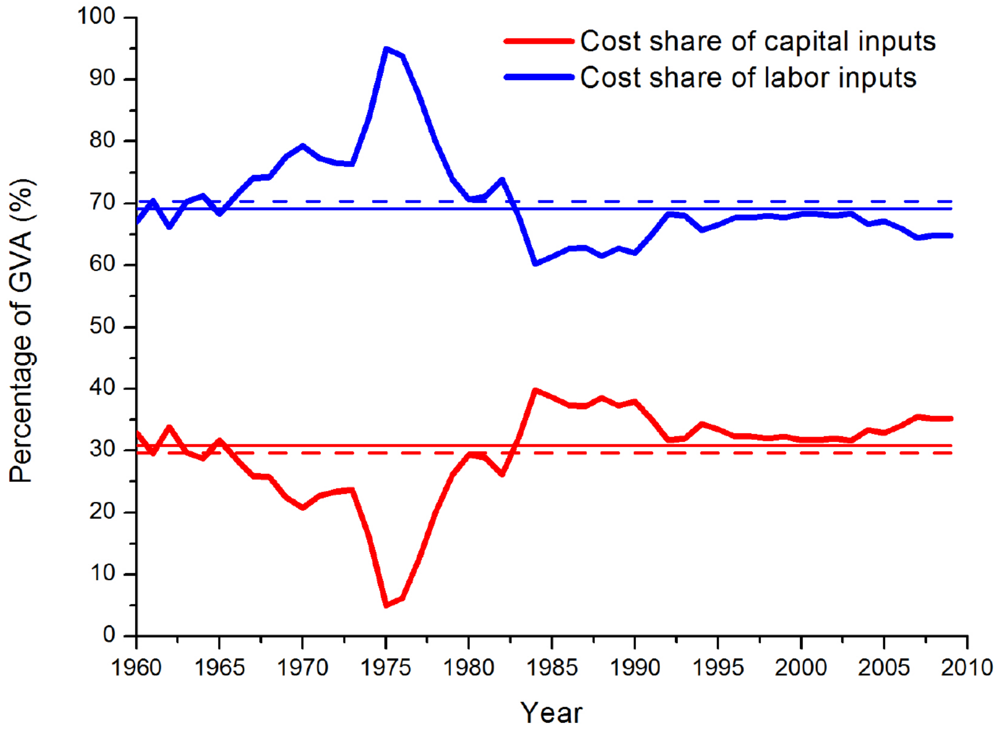

*Figure 4 – **Historical cost shares of traditional factors of production (capital and labor) for the Portuguese economy.** Shares are adjusted for payments to self-employed individuals. Average values (straight dashed lines): for capital ($\bar{\alpha}_K \cong 29.54\%$); for labor ($\bar{\alpha}_L \cong 70.46\%$). Estimated values with model $(q, k^{S\&L}_{Services}, u)_{hL}$ (straight full lines): for capital ($\hat{\alpha}_K \cong 30.80\%$); for labor ($\hat{\alpha}_L \cong 69.20\%$).*

The output elasticities for the estimated production function obtained with model $(q, k^{S\&L}_{Services}, u)_{hL}$ are remarkably similar to average historical cost shares for capital and labor in the Portuguese economy. Hence, one additional advantage of this model is that the estimated production function has the form of a mainstream two-factor Cobb-Douglas function with output elasticities corresponding to average historical cost shares, thus resembling in almost every way the traditional production functions used in the neoclassical approach to account for economic output. The difference, of course, is in the fact that for our model there is a second function, through which a more accurate measure for the real utilization of capital in production can be estimated from useful exergy and labor inputs – $K[U, L]$. Within the estimated $(q, k^{S\&L}_{Services}, u)_{hL}$ model, we can observe both the essentiality of energy inputs to production, through the estimation of real utilization of capital; and neoclassical assumptions, through a correspondence with the cost-share theorem.

4. Conclusions

We have examined the cointegration relationships between combinations of output, capital, labor, and energy for Portugal, in the past 50 years. Our methodology suggests that, under the appropriate criteria, these relationships can be interpreted as economically meaningful neoclassical Cobb-Douglas production functions.

The main conclusion is that, for our case-study, and in contrast with the literature, the argument of energy essentiality in production from ecological economics is not *a priori* incompatible with the neoclassical assumptions of the cost share theorem.

Overall, the best fits to past Portuguese economic output are obtained when capital inputs are either excluded from the cointegration space – (q, u); or are estimated as a function of both historically observed useful exergy and labor inputs. In contrast, the worst fits obtained in our analysis all refer to production functions estimated from models where energy is absent from the cointegration space – (q, k). These functions are the ones which most resemble the neoclassical Cobb-Douglas approach.

Specifically, we find that the best estimated fits to past economic trends (and lowest TFP component in growth accounting) for Portugal are obtained with econometric models for which two simultaneous cointegration vectors are predicted by the econometric approach. These are: 1) one cointegration vector linking all factor inputs, through which the actual utilization of capital in production can be estimated as a function of historically observed time series of useful exergy and labor inputs; 2) one cointegration vector linking output, capital, and labor inputs, forming a two-factor Cobb-Douglas production function in which labor inputs are observed but capital inputs are estimated from the first cointegration vector.

By estimating such a model with quality-adjusted capital (services), labor (human-capital adjusted), and energy (useful exergy) inputs in the cointegration space, alongside output, we are able to obtain, from the first cointegration vector, an estimate for the actual utilization of capital in production which is better than the stocks of assets or the flow of services provided by capital. The second cointegration vector provides estimated values of (constant) output elasticities for capital and labor, which are very similar to the average values for historically observed cost shares associated to these factors, in our case-study. This function fits historical output very well, while reporting a remarkably small TFP component in growth accounting (less than 1% of overall output growth).

In this model, useful exergy inputs do not appear explicitly in the estimated production function, but they assume a major role in explaining economic output growth: they form an accurate proxy for the utilization of capital in production, supported by the fact that capital assets are useless without being activated by useful exergy (and labor). Payments to useful exergy inputs must therefore be implicit in the share of payments to capital.

Comparing between econometric models, results show that adjusting for qualitative differences in capital and labor, *ceteris paribus*, has a marginal effect on the goodness-of-fit to past economic growth for estimated production functions, and some positive effect in reducing TFP. However, when capital inputs are estimated as a function of historically observed useful exergy and labor inputs, goodness-of-fit increases significantly. This suggests that despite adjusting for the quality of capital inputs by accounting for the services provided by the stocks of assets, it is only by accounting for the exergy consumed in devices and machines – and used productively in the economy – that one can accurately estimate the real utilization of capital in production.

Since energy inputs do not appear explicitly in the estimated Cobb-Douglas function, their cost-share is zero, as in the neoclassical two-factor approach. However, as argued in the field of ecological economics, energy is an essential input to the production of economic output. This can be seen as an ecological economics growth model that does not reject the cost share assumptions associated with neoclassical theory; or, alternatively, a neoclassical model of economic growth that acknowledges the importance of energy to economic production.

Future work should focus on expanding this analysis to: 1) other individual countries, and groups of countries; 2) more complex production functions, such as the generalized formulation of the Cobb-Douglas – the Constant Elasticity of Substitution (CES) production function.

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Supplementary information

Appendix A. Measures for output and inputs to production

This section details the adopted measures and sources of data concerning economic output and factors of production, namely capital, labor, and useful exergy.

A.1. Output

Both gross domestic product (GDP) and gross value added (GVA) are widespread measures for economic output. In this analysis, we opt for the latter. Our choice is based on the fact that GVA corresponds to GDP without components relating to taxes and subsidies on production and imports. It accounts just for the total of income, traditionally divided between payments to capital and labor. This is especially relevant for growth accounting exercises performed in our analysis.

A.2. Capital

As reported in the main paper, two distinct measures for capital inputs are adopted in the empirical analysis. The first corresponds to the stock of capital assets, as accounted by AMECO database (AMECO, 2015). The second corresponds to a measure of services provided by capital assets, as estimated by da Silva & Lains (2013). The differences between both these measures are highlighted in the main text, and detailed below in this Appendix.

Besides the capital stock measure obtained from AMECO database and featured in the main paper, analysis was conducted with two other measures for capital stock, obtained from different sources. These correspond to capital stocks as accounted by the Penn World Tables (PWT8.1, 2015), and capital stocks as estimated in da Silva & Lains (2013).

Details concerning capital stocks as accounted by the Penn World Tables can be found in Inklaar & Timmer (2013) and Feenstra, Inklaar & Timmer (2015).

Details concerning the capital stock measures estimated by da Silva & Lains (2013) are synthesized below, along with the authors estimated of capital services.

A.2.1. Capital stocks and services estimation (da Silva & Lains, 2013)

Integrated time series for net capital stock and the volume index of capital services (VICS) for Portugal are obtained from da Silva & Lains (2013). The step-by-step methodology adopted by these authors begins with the crucial task of constructing a fully integrated investment (i.e. gross fixed capital formation, of GFCF) series from published GFCF and corresponding price indices in

Pinheiro et al. (1997) (1953-1995), and INE (2015) (1977-2011). Both series are integrated by da Silva & Lains (2013), applying backwards the growth rates implicit in the earlier temporal series.

After performing consistency checks on the GFCF time series, da Silva & Lains (2013) proceed to estimate net capital stocks following the Perpetual Inventory Method, which requires assumptions on the depreciation in value of each asset type, and an initial benchmark for the respective stocks of capital, besides GFCF and price indices series.

Concerning depreciation rates, da Silva & Lains (2013) adopt the method of declining balances suggested in Hulten & Wykoff (1996), under which the depreciation rate (in value) of an asset i is computed as $\delta^i = R/\bar{T}^i$, with R as an estimated declining balance rate[27] and $\bar{T}^i$ the average service life of the asset[28].

Initial benchmarks for capital stocks at the beginning of the period (1910) are constructed by da Silva & Lains (2013) following the steady-state approach, widely used in the literature (e.g. Ohanian & Wright, 2010; de la Escosura & Rosés, 2009; Kamps, 2006).

After computation of capital stocks for each asset type, the volume index of capital services (VICS) is derived. The method followed by da Silva & Lains (2013) is the one pioneered by the Bureau of Labor Statistics (BLS): capital stocks for each type of asset are aggregated to form overall measures of capital services, taking the user costs of capital as appropriate weights. The user costs reflect the marginal productivity of different assets under the usual assumptions of competitive markets.

Specifically, the user costs ($\mu_{i,t}$) measure the cost of financing the asset as the sum of depreciation ($d_{i,t}$) plus the nominal cost of financial capital ($r_{i,t}$), minus the nominal capital gain (or loss) from holding the asset for each accounting period ($p_{i,t} - p_{i,t-1}$):

$$\mu_{i,t} = r_{i,t} \cdot p_{i,t-1} + d_{i,t} \cdot p_{i,t} - (p_{i,t} - p_{i,t-1}) \qquad \text{(A.1)}$$

After user costs have been derived, da Silva & Lains (2013) combine the stocks of each asset type to obtain VICS, using a Törnqvist index:

$$\ln\left[\frac{K_t}{K_{t-1}}\right] = \sum_i \bar{v}_i \ln\left[\frac{S_{i,t}}{S_{i,t-1}}\right] \quad \text{(A.2)}$$

Where $S_{i,t}$ represent the stocks of capital for each asset type i at time t, and $\bar{v}_i = 0.5(v_{i,t} + v_{i,t-1})$, with:

$$v_{i,t} = \frac{\mu_{i,t} S_{i,t}}{\sum_i \mu_{i,t} S_{i,t}} \qquad \text{(A.3)}$$

A.3. Labor

The simplest way to account for labor inputs to production is to account for the number of workers, or persons engaged, in the economy. However, this measure weighs all workers equally, regardless of whether they work part-time or full-time.

Alternatively, a standard measure for labor inputs is to account for total work hours, recognizing the fact that the number of hours worked differs from individual to individual. However, this measure does not account for differences in the productive capacity of different individuals. The aim of quality-adjusting labor inputs is thus to assess the impact of growth in labor services to economic growth (Jorgenson et al., 1987).

Adjusting labor inputs for qualitative differences, by measuring the skill level (quality) of workers, is difficult, because “skill” is a loose term that cannot be directly observed: individual’s skill are inherently subjective, and embodied in a variety of forms[29]. To capture the quality of labor, it is necessary to resort to proxies.

A production-oriented definition of “human capital” – defined as an amalgam of factors such as education, experience, training, etc. – can be approximated by a limited number of observable characteristics, primarily the amount of formal schooling[30].

In this work we use both unadjusted and quality-adjusted measures to account for labor inputs. Unadjusted labor is measured as average annual hours worked by engaged individuals, and it is obtained from the work of Amaral (2009).

The Penn World Table database (PWT8.1) provides annual time series for a dimensionless human capital proxy (based on average returns to years of education), which we multiply by our unadjusted measures for labor inputs in order to obtain a quality-adjusted measure for labor inputs.

This proxy is estimated assuming that perfect competition in factor and goods markets implies that the average wage of a worker with s years of education is proportional to his human capital h. It also considers a log-linear relation between h and s, based on the wage-schooling relationships (Caselli, 2005):

$$h = e^{\varphi(s)} \quad \text{(A.4)}$$

Where $\varphi(s)$ expresses average returns as function of the years of schooling s. The specific form for this function is based on evidence that earlier years of education have a higher return than later years (Caselli, 2005; Psacharopoulos & Patrinos, 2004). This finding is based on

Mincerian cross-country wage regressions. The function $\varphi(s)$ is chosen to be piece-wise linear with slope defined according to a range of average years of schooling[31].

$$\varphi(s)=\begin{cases}0{,}134\cdot s, & if\ s\leq 4,\\ 0{,}134\cdot 4+0{,}101\cdot s, & if\ 4\leq s\leq 8,\\ 0{,}134\cdot 4+0{,}101\cdot 4+0{,}068\cdot(s-8), & if\ s\geq 8\end{cases} \tag{A.5}$$

The rates of return are based on Psacharopoulos & Patrinos (2004). International data on education-wage profiles suggests that in Sub-Saharan Africa – which has the lowest levels of education – the returns to one extra year of schooling are in the order of 13,4%, while the World average is in the order of 10,1%, and the OECD average is in the order of 6,8%.

A.4. Useful exergy

We obtain time series for useful exergy consumption in the Portuguese economy from Serrenho et al. (2016), whose useful exergy accounting study covers a 154-year period from 1856 to 2009, focusing on final-to-useful processes.

The step-by-step methodology applied by Serrenho et al. (2016) for each year and energy carrier improves on the basic approach developed in Warr et al. (2010):

1. Conversion of existing final energy data[32] to final exergy values;
2. Allocation of final exergy consumption from each final use sector to useful exergy categories;
3. Estimation of second-law efficiencies for each final-to-useful transformation;
4. Calculation of aggregate useful exergy values by summing total values obtained for each useful exergy category;

Serrenho et al. (2016) consider the following categories for energy (exergy) end-uses: heat (high, medium, and low temperature); mechanical drive; light; electricity; and muscle work. As a refinement of the basic methodology by Warr et al. (2010), electricity is treated separately, since it can be used either for heating, lighting, mechanical drive, or other electrical uses, depending on the sector where it is used.

Besides the typical energy carriers (coal & coal products, oil & oil products, natural gas, combustible renewables, electricity & CHP heat), Serrenho et al. (2016) also take into account energy (exergy) inputs that go beyond conventional energy accounting statistics: food for humans; feed for working animals; and non-conventional sources[33]. The dataset is compiled from different sources, for final energy consumption: IEA energy balances (for typical energy carriers, 1960 onwards); Henriques (2011) (for typical energy carriers and food/feed before

1960, as well as corrected combustible renewables data prior to 1990 and non-conventional carriers for the entire period 1856-2009); FAO database (food/feed, 1960 onward).

For each year t and each combination of energy carrier i, economic sector j, and energy end-uses category k, useful exergy consumption is calculated as follows:

$$U_{t,ijk} = \varepsilon_{t,k}\varphi_i E_{F\ t,ijk} \qquad \text{(A.6)}$$

The process requires a mapping for energy end-uses, estimation of thermodynamics second-law efficiencies for each end-use category $\varepsilon_{t,k}$, and the definition of an exergy factor[34] for each energy carrier φ_i. The mapping depends on the level of disaggregation of th energy data for final energy consumption $E_{F\ t,ijk}$. For details on the estimation of second-law efficiencies and exergy factors in this study, consult Serrenho et al. (2016).

Appendix B. Detailed statistical methods

B.1. Non-stationary unit root tests

The augmented Dickey-Fuller (ADF) and Philips-Perron (PP) tests are two alternative unit root tests used in statistics to determine if a given time series is non-stationary (i.e. if it has a unit root). The ADF test is an extension of the Dickey-Fuller test for larger sets of time series models. The procedure of the ADF test is applied to the model:

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \delta_1 \Delta y_{t-1} + \cdots + \delta_{p-1}\Delta y_{t-p+1} + \epsilon_t \qquad \text{(B.1)}$$

In Equation B.1 the relevant time series is represented by y_t, α is a constant, β is a coefficient in a time trend, and p is the lag order of the autoregressive process. The lag order p is chosen before applying the test, by examining relevant information criteria. The unit root test is then carried out under the null hypothesis $\gamma = 0$ against the alternative hypothesis of $\gamma < 0$. A value is determined for the test statistic:

$$DF_\tau = \hat{\gamma}/SE(\hat{\gamma}) \qquad \text{(B.2)}$$

This value is compared with a critical value for the Dickey-Fuller test. If the test statistic is smaller[35] than the critical value, the null hypothesis is rejected and no unit root is detected in the time series sample.

The PP test builds on the Dickey-Fuller test of the null hypothesis $\delta = 0$ in a model:

$$\Delta y_t = \delta y_{t-1} + v_t \quad \text{(B.3)}$$

Like the ADf test, the PP test addresses the issue that the process generating data for y_t might have a higher order of autocorrelation than what is admitted in the test equation.

However, while the ADF test introduces lags of Δy_t as regressors in the test equation, the PP test makes a non-parametric correction to the t-test statistic.

When the ADF or PP statistic is smaller than the tabulated critical value, the null non-stationarity hypothesis is rejected, and the time series has a unit root. For the ADF tests, the final number of lags to be included was chosen according to the minimum observed value for the Schwarz Information Criteria (SIC) statistic. For the PP tests, the bandwidth parameter for the kernel-based estimator of the residual spectrum at frequency zero was obtained by the Newey-West method using Bartlett kernel (Newey & West, 1994).

B.2. Cointegration

For this analysis, the models defined in the paper are written as autoregressive models (VAR) and tested for cointegration, following the multivariate cointegration procedure developed in Johansen (1988; 1991). We test all defined models in this way. The procedure is as follows:

Any given $(n \times 1)$ vector of time series $\mathbf{Y}_t$ is represented by a VAR model such as:

$$\mathbf{Y}_t = \mathbf{c} + \sum_{j=1}^{p} \Gamma_j \mathbf{Y}_{t-j} + \boldsymbol{\epsilon}_t \quad \text{(B.4)}$$

Where $\mathbf{c}$ is a vector of constants, Γ_j represents matrices of short-run dynamics coefficients, and $\boldsymbol{\epsilon}_t$ is a vector of random disturbances.

If the time series in $\mathbf{Y}_t$ are all integrated of order one, I(1), the VAR in equation B.4 is non-stationary. If there is no cointegration, statistical inference is not possible using the usual tests and *p-values*, as statistics will not have standard tabulated distributions. In this case, it is appropriate to first-difference the series in $\mathbf{Y}_t$ and to estimate the first-differences VAR of the form:

$$\Delta\mathbf{Y}_t = \mathbf{c} + \sum_{j=1}^{p} \Gamma_j \Delta\mathbf{Y}_{t-j} + \boldsymbol{\epsilon}_t \qquad \text{(B.5)}$$

When cointegration is verified between the variables, there is at least one linear combination of $\mathbf{Y}_t$ (cointegration vector) that is stationary. In this case, we can write equation B.5 as a vector error-correction model (VECM):

$$\Delta\mathbf{Y}_t = \mathbf{c} + \sum_{j=1}^{p} \Gamma_j \Delta\mathbf{Y}_{t-j} + \Pi\mathbf{Y}_{t-1} + \boldsymbol{\epsilon}_t \qquad \text{(B.6)}$$

Where Π is a rank r matrix that can be decomposed as

$$\Pi = \alpha\beta' \qquad \text{(B.7)}$$

With α being a $n \times r$ loading matrix and β a $n \times r$ matrix of cointegration vectors. The number of cointegration vectors (CV) in (B.4) is r, and it is tested following the procedure in

Johansen (1988). If no cointegration vectors are estimated, the analysis proceeds by taking first-differences VAR. If one or more cointegration vectors are found, these are estimated and a VECM formulation is considered, as in (B.6).

For all models, the maximum number of lags admitted was 10, and the lag order was set to that suggested by the majority of all available information criteria (Likelihood Ratio, Final Prediction Error, Akaike, Schwarz, and Hannan-Quinn). In order to check whether the obtained VAR is well-defined, we have tested for serial independence in the residuals, applying a Lagrange Multiplier autocorrelation test. Autocorrelation issues have been resolved by increasing the lag order p.

Carrying out the cointegration tests, a choice was also made regarding the underlying trend in the data. A linear deterministic trend was allowed in the level data, but the cointegration vector was chosen to have only intercepts, since it was assumed that all trends in the data are stochastic.

There are two separate tests in the Johansen procedure: the trace test and the maximum eigenvalue test. Both are a test of the null hypothesis of no cointegration against the alternative of cointegration. They are based on eigenvalues of transformations of the data (λ) and represent linear combinations of the data that have maximum correlation (canonical correlations). The tests differ in terms of alternative hypothesis.

The trace test is a test of whether the rank of matrix Π is zero. The null hypothesis is that $rank(\Pi) = r_0$. The alternative hypothesis is that $r_0 < rank(\Pi) \leq n-1$, where $n-1$ is the maximum number of possible cointegration vectors. Succeding tests have the null hypothesis $rank(\Pi) = r_0 + 1$ against the alternative $r_0 + 1 < rank(\Pi) \leq n-1$. This test is a likelihood ratio test:

$$LR(r_0, n-1) = -T\sum_{i=r_0+1}^{n-1}(1-\lambda_i) \qquad \text{(B.8)}$$

Where $LR(r_0, n-1)$ is the likelihood ratio statistics for testing whether $rank(\Pi) = r$ versus the alternative hypothesis that $rank(\Pi) \leq n-1$.

The maximum eigenvalue test begins by testing whether the rank of matrix Π is zero against the alternative that $rank(\Pi) = 1$. It then proceeds to test the null hypothesis that $rank(\Pi) = 1,2, \ldots$ against the alternative hypothesis that $rank(\Pi) = 2,3, \ldots$. This test is also a likelihood ratio test:

$$LR(r_0, r_0+1) = -T\ln(1-\lambda_{r_0+1}) \text{ (B.9)}$$

Where $LR(r_0, r_0 + 1)$ is the likelihood ratio test statistics for testing whether $rank(\Pi) = r_0$ versus the alternative hypothesis that $rank(\Pi) = r_0 + 1$. This likelihood ratio statistic does not have the usual asymptotic chi-square distribution.

B.3. Granger non-causality tests

The causality tests performed in this analysis are preceded by the cointegration tests, since the presence of cointegrated relationships between the variables has implications for the way in which short-run and long-run causality is carried out. If cointegration is detected among the variables in any of the models, then tests for causality are conducted by employing the methodology by Engle & Granger (1987).

According to this approach, cointegrated variables must have an error correction representation in which an error correction term (ECT) is incorporated into the model. Therefore, a VECM is formulated in order to reintroduce the information lost in the differencing process – equation B.5 – thereby allowing for long-run equilibrium as well as short-run dynamics.

For a three-variable model case (y_t, x_t, z_t) with one cointegration relationship between the variables, the relevant VECM can be written as:

$$\begin{aligned}
\Delta y_t &= a_1 + \alpha_{11} ECT_{t-1} + \textstyle\sum_{j=1}^{p-1} \phi_{1j} \Delta y_{t-j} + \sum_{j=1}^{p-1} \theta_{1j} \Delta x_{t-j} + \sum_{j=1}^{p-1} \psi_{1j} \Delta z_{t-j} \\
\Delta x_t &= a_2 + \alpha_{21} ECT_{t-1} + \textstyle\sum_{j=1}^{p-1} \phi_{2j} \Delta y_{t-j} + \sum_{j=1}^{p-1} \theta_{2j} \Delta x_{t-j} + \sum_{j=1}^{p-1} \psi_{2j} \Delta z_{t-j} \\
\Delta z_t &= a_3 + \alpha_{31} ECT_{t-1} + \textstyle\sum_{j=1}^{p-1} \phi_{3j} \Delta y_{t-j} + \sum_{j=1}^{p-1} \theta_{3j} \Delta x_{t-j} + \sum_{j=1}^{p-1} \psi_{3j} \Delta z_{t-j}
\end{aligned} \qquad \text{(B.10)}$$

Where $ECT_{t-1} = y_{t-1} + (\beta_{21}/\beta_{11}) x_{t-1} + (\beta_{31}/\beta_{11}) z_{t-1} + c$ is the normalized cointegration vector, with β_k representing the coefficients on lagged terms of the conditional VECm and c a constant term. There are two sources of causation: through the ECT (if $\alpha \neq 0$); and through the lagged dynamics terms. The ECT measures the long-run equilibrium relationship while the coefficients on lagged difference terms indicate the short-run dynamics. The statistical significance of the coefficients associated with ECT provides evidence of an error correction mechanism that drives the variables back to their long-run relationship.

Given the two sources of causality, there are distinct causality tests that can be performed on such a model. Short-run Granger non-causality tests are applied to the lagged coefficients. Long-run causality is indicated by the significance of the one period lagged error-correction term. Lastly, “strong” Granger non-causality can be tested by examining whether the two sources of causation are jointly significant. For example, for the first equation in B.10, in order to test whether Δx_t Granger-causes Δy_t in the short-run, the significance of the lagged dynamic terms is examined by testing the null hypothesis $H_0: \theta_{1j} = 0, \forall j$ using the Wald test. Rejection of the null hypothesis implies that Δx_t Granger-causes Δy_t in the short-run.

Testing the significance of the adjustment coefficient $H_0: \alpha_{11} = 0$ allows for the investigation of how fast the dependent variable (in this case, Δy_t) responds to deviations from the long-run equilibrium.

Finally, the "strong" long-run Granger non-causality can be tested by performing a joint significance *F-test* on both the ECT and the lagged dynamic terms. For example, in system (B.10), testing the null hypothesis $H_0: \alpha_{11} = \theta_{1j} = 0$ is equivalent to testing whether "strong" long-run non-causality runs from Δx_t to Δy_t. Rejection of the null hypothesis implies that Δx_t Granger-causes Δy_t in the long-run.

Appendix C. Additional econometric results

In this section additional results from the analysis proposed and conducted in the main paper are disclosed. These results help clarify the statistical interpretation of the main paper's results. All additional results are presented for analysis conducted using the output and input measures adopted in the main text, and also the alternative capital inputs measures discusses in Appendix A.

C.1. Non-stationarity tests

Unit root test results concerning the time series adopted for output, and capital, labor and useful exergy inputs are presented in Table C.1.1. Results are divided between non-stationarity tests for the variables in levels, first-differences, and second-differences. Values correspond to ADF and PP test statistics, which are compared with tabulated values (not shown).

It can be observed that all time series corresponding to output, labor inputs, and useful exergy inputs have unit roots in levels, but are stationary in first-differences. Time series corresponding to measures for capital inputs (stocks or services), however, have unit roots in both levels and first-differences, and only become stationary when taking second differences.

Tables C.1.2. and C.1.3. show the ADF and PP test results for the working variables defined in our analysis, per unadjusted and quality-adjusted labor inputs, respectively. Here, we test for the presence of unit roots in the logarithms of the ratio between time series for output, capital, and useful exergy; and time series for unadjusted and quality-adjusted labor inputs.

It can be observed, in both cases (variables defined per unadjusted and quality-adjusted labor), that all time series in levels accuse the presence of unit roots, but are stationary in first-differences. By defining our working variables in this way, we obtain time series that are all integrated of order one, and the Johansen procedure to test for cointegration can thus be applied.

*Table C.1.1 – **Unit root tests. Historical annual time series for output, and capital, labor, and useful exergy inputs.** * Rejection of the null hypothesis that the series has a unit root (i.e. is non-stationary), at the 1% level.*

Variable	Measure / Source	Lags	ADF	PP	Unit root
	Level variables				
Q	GVA / INE and Pinheiro (1997)	1	-2.483	-2.142	Yes
K	Stock / AMECO	1	-2.611	-2.475	Yes
	Stock / PWT8.1	2	-0.856	0.229	Yes
	Stock / da Silva & Lains (2013)	2	-1.973	-1.147	Yes
	Services / da Silva & Lains (2013)	2	-1.383	0.233	Yes
L	Hours worked / Amaral (2009)	0	-2.514	-2.519	Yes
hL	Human-capital adjusted hours / PWT8.1 and Amaral (2009)	0	-2.456	-2.534	Yes
U	Useful exergy / Serrenho et al. (2016)	2	-1.900	-2.014	Yes
	First-differences variables				
Variable	Measure / Source	Lags	ADF	PP	Unit root
Q	GVA / INE and Pinheiro (1997)	0	-3.900*	-3.958*	No
K	Stock / AMECO	0	-1.673	-1.673	Yes
	Stock / PWT8.1	1	-2.174	-1.476	Yes
	Stock / da Silva & Lains (2013)	1	-2.045	-1.532	Yes
	Services / da Silva & Lains (2013)	1	-1.689	-1.176	Yes
L	Hours worked / Amaral (2009)	0	-6.667*	-6.835*	No
hL	Human-capital adjusted hours / PWT8.1 and Amaral (2009)	0	-6.108*	-6.019*	No
U	Useful exergy / Serrenho et al. (2016)	1	-2.220	-6.687*	No
	Second-differences variables				
Variable	Measure / Source	Lags	ADF	PP	Unit root
K	Stock / AMECO	0	-5.010*	-5.068*	No
	Stock / PWT8.1	1	-3.631*	-3.477*	No
	Stock / da Silva & Lains (2013)	1	-4.628*	-3.885*	No
	Services / da Silva & Lains (2013)	0	-3.305*	-3.184*	No

*Table C.1.2 - **Unit root tests. Variables defined per unadjusted labor inputs (L).** * Rejection of the null hypothesis that the series has a unit root (i.e. is non-stationary), at the 1% level.*

Level variables					
Variable	Measure / Source	Lags	ADF	PP	Unit root
q	GVA / INE and Pinheiro (1997)	0	-1.586	-1.551	Yes
k	Stock / AMECO	0	-1.950	-2.071	Yes
	Stock / PWT8.1	0	-2.007	-2.003	Yes
	Stock / da Silva & Lains (2013)	0	-1.795	-1.793	Yes
	Services / da Silva & Lains (2013)	0	-1.974	-1.995	Yes
u	Useful exergy / Serrenho et al. (2016)	0	-1.104	-0.914	Yes

First-differences variables					
Variable	Measure / Source	Lags	ADF	PP	Unit root
q	GVA / INE and Pinheiro (1997)	0	-5.139*	-5.129*	No
k	Stock / AMECO	0	-6.262*	-6.233*	No
	Stock / PWT8.1	0	-5.744*	-5.734*	No
	Stock / da Silva & Lains (2013)	0	-5.973*	-5.966*	No
	Services / da Silva & Lains (2013)	0	-6.620*	-6.619*	No
u	Useful exergy / Serrenho et al. (2016)	1	-2.790	-8.093*	No

*Table C.1.3 - **Unit root tests. Variables defined per quality-adjusted labor inputs (hL).** * Rejection of the null hypothesis that the series has a unit root (i.e. is non-stationary), at the 1% level.*

Level variables					
Variable	Measure / Source	Lags	ADF	PP	Unit root
q	GVA / INE and Pinheiro (1997)	0	-2.840	-2.231	Yes
k	Stock / AMECO	0	-2.997	-3.059	Yes
	Stock / PWT8.1	0	-2.276	-2.267	Yes
	Stock / da Silva & Lains (2013)	0	-2.327	-2.327	Yes
	Services / da Silva & Lains (2013)	0	-1.867	-1.867	Yes
u	Useful exergy / Serrenho et al. (2016)	0	-1.744	-1.599	Yes

First-differences variables					
Variable	Measure / Source	Lags	ADF	PP	Unit root
q	GVA / INE and Pinheiro (1997)	0	-5.156*	-5.141*	No
k	Stock / AMECO	0	-6.380*	-6.595*	No
	Stock / PWT8.1	0	-5.660*	-5.660*	No
	Stock / da Silva & Lains (2013)	0	-5.985*	-5.984*	No
	Services / da Silva & Lains (2013)	0	-6.256*	-6.243*	No
u	Useful exergy / Serrenho et al. (2016)	1	-2.961	-8.158*	No

C.2. Cointegration tests

All results obtained for the trace and max eigenvalue tests constituting the Johansen procedure are presented in Tables C.2.1 and C.2.2, for models with variables defined per unadjusted (L) and quality-adjusted (hL) labor inputs, respectively. The number of lags considered is indicated for each model.

Models for which no cointegration relationships are observed between variables are the ones that fail to reject the null hypothesis of “None” cointegration vectors in Tables C.2.1 and C.2.2. Models with at least two cointegration vectors between variables are the ones that reject the null hypothesis of “At most 1” cointegration vectors.

*Table C.2.1 - **Johansen cointegration test results for models with working variables defined per unadjusted labor inputs (L).** Columns (1) and (2) indicate model and number of lags considered, respectively. Columns (3-8) indicate results for Johansen cointegration tests when the null hypothesis is that there is no cointegration relationship (3-4), at most 1 cointegration relationship (5-6), or at most 2 cointegration relationships (7-8). Variables in levels, assuming only an intercept (no trend) in the cointegration vector. Values represented correspond to p-values for the trace and max-eigenvalue tests. Critical values in parenthesis. *Rejection of the null hypothesis at 1% significance level.*

Model	Lags	None		At most 1		At most 2	
		Trace	Max	Trace	Max	Trace	Max
2 variables		(19.937)	(18.520)	(6.635)	(6.635)	-	-
$(q,u)_L$	4	24.429*	15.227	9.202*	9.202*	-	-
$\left(q,k_{Stock}^{AMECO}\right)_L$	4	28.720*	24.635*	4.086	4.086	-	-
$\left(q,k_{Stock}^{PWT8.1}\right)_L$	5	14.739	13.209	1.531	1.531	-	-
$\left(q,k_{Stock}^{S\&L}\right)_L$	2	16.488	11.546	4.941	4.941	-	-
$\left(q,k_{Services}^{S\&L}\right)_L$	5	18.010	16.917	1.093	1.093	-	-
3 variables		(35.458)	(25.861)	(19.937)	(18.520)	(6.635)	(6.635)
$\left(q,k_{Stock}^{AMECO},u\right)_L$	4	44.501*	23.139	21.362*	17.678	3.687	3.687
$\left(q,k_{Stock}^{PWT8.1},u\right)_L$	3	36.695*	21.762	14.934	14.902	0.032	0.032
$\left(q,k_{Stock}^{S\&L},u\right)_L$	3	40.209*	21.951	18.259	12.335	5.924	5.924
$\left(q,k_{Services}^{S\&L},u\right)_L$	4	38.362*	18.138	20.223*	14.123	6.101	6.101

*Table C.2.2 - **Johansen cointegration test results for models with working variables defined per quality-adjusted labor inputs (L).** Columns (1) and (2) indicate model and number of lags considered, respectively. Columns (3-8) indicate results for Johansen cointegration tests when the null hypothesis is that there is no cointegration relationship (3-4), at most 1 cointegration relationship (5-6), or at most 2 cointegration relationships (7-8). Variables in levels, assuming only an intercept (no trend) in the cointegration vector. Values represented correspond to p-values for the trace and max-eigenvalue tests. Critical values in parenthesis. *Rejection of the null hypothesis at 1% significance level.*

Model	Lags	None		At most 1		At most 2	
		Trace	Max	Trace	Max	Trace	Max
2 variables		(19.937)	(18.520)	(6.635)	(6.635)	-	-
$(q,u)_{hL}$	4	22.952*	16.968	5.984	5.984	-	-
$(q,k_{Stock}^{AMECO})_{hL}$	4	26.288*	25.610*	0.678	0.678	-	-
$(q,k_{Stock}^{PWT8.1})_{hL}$	5	13.563	11.320	2.243	2.243	-	-
$(q,k_{Stpck}^{S\&L})_{hL}$	4	11.080	10.947	0.132	0.132	-	-
$(q,k_{Services}^{S\&L})_{hL}$	5	20.314*	18.071	2.243	2.243	-	-
3 variables		(35.458)	(25.861)	(19.937)	(18.520)	(6.635)	(6.635)
$(q,k_{Stock}^{AMECO},u)_{hL}$	4	39.142*	23.024	16.118	16.063	0.055	0.055
$(q,k_{Stock}^{PWT8.1},u)_{hL}$	3	32.890	20.745	12.145	12.130	0.015	0.015
$(q,k_{Stock}^{S\&L},u)_{hL}$	4	27.470	16.814	10.656	10.286	0.370	0.370
$(q,k_{Services}^{S\&L},u)_{hL}$	4	39.404*	18.139	21.265*	15.121	6.144	6.144

Tables C.2.3 and C.2.4 represent the cointegration coefficients, normalized to the output variable, obtained for each of the 11 models for which at least one cointegration vectors were predicted by the Johansen procedure. Table C.2.3 refers to models with variables defined per unadjusted labor inputs (L), while Table C.2.4 refers to models with variables defined per quality-adjusted labor inputs (hL).

In each case, the normalized coefficients corresponding to capital and useful exergy inputs, retrieved directly from the software output, are represented. The normalized coefficient corresponding to labor inputs – estimated assuming constant returns to scale $\alpha_K + \alpha_L = 1$ ($\alpha_K + \alpha_L + \alpha_U = 1$ for three-variable models) – is also represented. Both Table C.2.3 and C.2.4 also show a constant term (written as the exponent of a natural exponential function e^{cst}) obtained for each model.

Table C.2.3 - ***Normalized cointegration coefficients for models with cointegrated variables defined per unadjusted labor inputs (L).*** *Columns (1) and (2) indicate model and number of cointegration vectors, respectively. Coefficients for capital (K) and useful exergy (U) obtained directly from EViews® output. Labor (L) coefficients estimated assuming constant returns to scale. Error terms are represented in parenthesis. Constants CST presented as exponentials.*

Model	Nr. CV	Coefficients		
2 variables		$Q = G(U, L)$		
		α_U	α_L	CST
$(q, u)_L$	1	0.840 (0.026)	0.160	-7.772

Model	Nr. CV	Coefficients		
2 variables		$Q = F(K, L)$		
		α_K	α_L	CST
$\left(q, k_{Stock}^{AMECO}\right)_L$	1	0.600 (0.057)	0.400	-5.188

Model	Nr. CV	Coefficients			
3 variables		$Q = F(K, L, U)$			
		α_K	α_L	α_U	CST
$\left(q, k_{Stock}^{PWT8.1}, u\right)_L$	1	0.073 (0.209)	0.741	0.186 (0.267)	-7.977
$\left(q, k_{Stock}^{S\&L2014}, u\right)_L$	1	-0.094 (0.412)	0.902	0.192 (0.431)	-8.449

Model	Nr. CV	$Q = G(U, L)$			$K = H(U, L)$			$Q = F[K = H(U, L), L]$		
3 variables		α_U	α_L	CST	α_U	α_L	CST	α_K	α_L	CST
$\left(q, k_{Stock}^{AMECO}, u\right)_L$	2	0.838 (0.024)	0.162	-7.781	1.313 (0.076)	-0.313	-4.682	0.638	0.362	-4.794
$\left(q, k_{Services}^{S\&L2014}, u\right)_L$	2	0.876 (0.021)	0.124	-7.614	2.344 (0.273)	-1.344	2.778	0.374	0.626	-8.652

*Table C.2.4 - **Normalized cointegration coefficients for models with cointegrated variables defined per unadjusted labor inputs (L).** Columns (1) and (2) indicate model and number of cointegration vectors, respectively. Coefficients for capital (K) and useful exergy (U) obtained directly from EViews® output. Labor (L) coefficients estimated assuming constant returns to scale. Error terms are represented in parenthesis. Constants CST presented as exponentials.*

Model	Nr. CV	Coefficients								
2 variables		$Q = G(U, hL)$								
		α_U			α_{hL}			CST		
$(q, u)_{hL}$	1	0.780 (0.030)			0.220			-8.194		
2 variables		$Q = F(K, hL)$								
		α_K			α_{hL}			CST		
$(q, k_{Stock}^{AMECO})_{hL}$	1	0.454 (0.065)			0.546			-7.115		
$(q, k_{Services}^{S\&L})_{hL}$	1	-0.341 (0.213)			1.341			-15.003		
3 variables		$Q = F(K, hL, U)$								
		α_K		α_{hL}		α_U		CST		
$(q, k_{Stock}^{AMECO}, u)_{hL}$	1	-0.522 (0.113)		-0.043		1.565 (0.136)		-7.977		
3 variables		$Q = G(U, hL)$			$K = H(U, hL)$			$Q = F[K = H(U, hL), hL]$		
		α_U	α_{hL}	CST	α_U	α_{hL}	CST	α_K	α_{hL}	CST
$(q, k_{Services}^{S\&L}, u)_{hL}$	2	0.958 (0.040)	0.042	-7.282	3.111 (0.431)	-2.111	7.694	0.308	0.692	-9.651

For models with more than one cointegration vector, both vectors are represented (coefficients normalized to output and capital inputs, respectively), as well as the algebraicaly manipulated vector corresponding to a Cobb-Douglas type aggregate production function.

C.3. Granger causality tests

Tables C.3.1 and C.3.2 show the numerical results for the Granger non-causality tests performed on models that evidence at least one cointegration vector between its variables. Rejection of the null hypothesis of non-causality between two variables, at a 1% significance level, is indicated by an asterisk. Table C.3.1 refers to models with variables defined per unadjusted labor inputs (L), while Table C.3.2 refers to models with variables defined per quality-adjusted labor inputs (hL).

*Table C.3.1 - **Short-run and long-run Granger non-causality test results, for models with working variables defined per unadjusted labor inputs (L).** Columns (1) and (2) indicate the model and independent variable, respectively. Columns (3-8) indicate dependent variables in short-run (3-5) and long-run (6-8). Statistics are chi-square. *Rejection of null hypothesis at 1% level.*

Model	Independent variable	Dependent variables					
Bivariate models		Short run			Long run		
		$\Delta\ln q$	$\Delta\ln k$	$\Delta\ln u$	$\Delta\ln q$	$\Delta\ln k$	$\Delta\ln u$
(q, u)	$\Delta\ln q$	13.629*		10.228	29.433*		10.845
	$\Delta\ln u$	13.911*		8.470	35.117*		10.874
		$\Delta\ln q$	$\Delta\ln k$	$\Delta\ln u$	$\Delta\ln q$	$\Delta\ln k$	$\Delta\ln u$
(q, k^{AMECO}_{Stock})	$\Delta\ln q$	3.101	8.142		17.812*	8.175	
	$\Delta\ln k$	6.733	0.973		17.013*	1.421	
Multivariate models		Short run			Long run		
		$\Delta\ln q$	$\Delta\ln k$	$\Delta\ln u$	$\Delta\ln q$	$\Delta\ln k$	$\Delta\ln u$
$(q, k^{AMECO}_{Stock}, u)$	$\Delta\ln q$	3.243	6.955	12.327	13.713	9.571	18.209*
	$\Delta\ln k$	9.294	10.922	4.989	22.171*	11.649	14.725
	$\Delta\ln u$	16.451*	13.723*	20.134*	38.533*	14.454	25.382*
		$\Delta\ln q$	$\Delta\ln k$	$\Delta\ln u$	$\Delta\ln q$	$\Delta\ln k$	$\Delta\ln u$
$(q, k^{PWT8.1}_{Stock}, u)$	$\Delta\ln q$	3.704	0.777	4.669	23.437*	7.014	10.603
	$\Delta\ln k$	13.613*	7.860	7.725	33.102*	12.880	10.791
	$\Delta\ln u$	25.712*	16.355*	19.971*	51.834*	18.438*	22.178*
		$\Delta\ln q$	$\Delta\ln k$	$\Delta\ln u$	$\Delta\ln q$	$\Delta\ln k$	$\Delta\ln u$
$(q, k^{S\&L2014}_{Stock}, u)$	$\Delta\ln q$	6.599	2.056	6.773	26.272*	9.769	15.332*
	$\Delta\ln k$	11.042	9.512	8.784	34.017*	14.796*	14.789*
	$\Delta\ln u$	23.100*	19.373*	23.264*	49.278*	22.165*	27.521*
		$\Delta\ln q$	$\Delta\ln k$	$\Delta\ln u$	$\Delta\ln q$	$\Delta\ln k$	$\Delta\ln u$
$(q, k^{S\&L}_{Services}, u)$	$\Delta\ln q$	6.537	6.405	14.097*	16.408	8.166	20.049*
	$\Delta\ln k$	7.747	6.885	5.737	21.241*	7.429	16.591
	$\Delta\ln u$	18.404*	10.197	17.133*	37.925*	10.594	26.576*

*Table C.3.2 - **Short-run and long-run Granger non-causality test results, for models with working variables defined per quality-adjusted labor inputs (hL).** Columns (1) and (2) indicate the model and independent variable, respectively. Columns (3-8) indicate dependent variables in short-run (3-5) and long-run (6-8). Statistics are chi-square. *Rejection of null hypothesis at 1% level.*

Model	Ind. variable	Dependent variables					
2 variables		Short run			Long run		
		$\Delta \ln q$	$\Delta \ln k$	$\Delta \ln u$	$\Delta \ln q$	$\Delta \ln k$	$\Delta \ln u$
(q, u)	$\Delta \ln q$	12.465		9.042	29.325*		9.448
	$\Delta \ln u$	12.844		7.266	34.436*		9.227
		$\Delta \ln q$	$\Delta \ln k$	$\Delta \ln u$	$\Delta \ln q$	$\Delta \ln k$	$\Delta \ln u$
(q, k_{Stock}^{AMECO})	$\Delta \ln q$	2.785	8.102		17.418*	8.361	
	$\Delta \ln k$	5.793	1.935		16.850*	2.137	
		$\Delta \ln q$	$\Delta \ln k$	$\Delta \ln u$	$\Delta \ln q$	$\Delta \ln k$	$\Delta \ln u$
$\left(q, k_{Services}^{S\&L(2014)}\right)$	$\Delta \ln q$	3.344	6.845		10.136	7.319	
	$\Delta \ln k$	5.052	0.544		9.635	0.843	
3 variables		Short run			Long run		
		$\Delta \ln q$	$\Delta \ln k$	$\Delta \ln u$	$\Delta \ln q$	$\Delta \ln k$	$\Delta \ln u$
$(q, k_{Stock}^{AMECO}, u)$	$\Delta \ln q$	10.471	9.204	16.279*	10.527	10.306	19.562*
	$\Delta \ln k$	10.795	14.455*	6.814	17.719*	14.171	17.287*
	$\Delta \ln u$	32.318*	15.292*	22.134*	32.393*	15.477*	27.289*
		$\Delta \ln q$	$\Delta \ln k$	$\Delta \ln u$	$\Delta \ln q$	$\Delta \ln k$	$\Delta \ln u$
$(q, k_{Services}^{S\&L}, u)$	$\Delta \ln q$	7.565	6.612	12.231	17.766*	9.641	18.191*
	$\Delta \ln k$	7.238	10.539	8.864	21.029*	11.063	16.343
	$\Delta \ln u$	20.179*	12.802	16.243*	37.714*	13.458	25.124*

[1] The stylized fact was originally proposed by Kaldor (1961) for the US, and other studies support the long-term stability of cost shares for this country (Denison, 1974; Jorgenson et al., 1987). Young (1995) reports reasonably stable cost shares for 4 East Asian countries – Hong Kong, Singapore, South Korea, and Taiwan – between 1960 and 1990. Studies for 7 developed countries – Canada, France, Germany, Italy, Japan, the Netherlands, and the United Kingdom – indicate cost shares similar to those in the US (Christensen et al., 1980; Dougherty, 1991).

[2] It should be noted that Solow (1957) also acknowledged the distinction between capital in use and capital in place, correcting the latter by the fraction of labor force unemployment, in lack of a more reliable measure.

[3] i.e. the relative proportion of labor used in a process.

[4] Along with Jorgenson et al. (1987), Diewert (1976), and Caves et al. (1982).

[5] Skill is difficult to measure objectively, so human capital indexes generally adopt the sum of years of education, or the returns to each additional year of education (or both), as proxies.

[6] For Portugal, technological change (or total factor productivity) accounts for approximately half of GDP growth during the period 1960-1974 (da Silva & Lains, 2013).

[7] The LINEX is a particular form for the production function, derived by Kümmel et al. (1985), obtained by choosing simple mathematical forms for the output elasticities (based on plausible assumptions on asymptotic behavior), and performing partial integrations. Unlike the Cobb-Douglas, the LINEX does not imply that factors of production are strict substitutes of each other, and hence provides a more complex and realistic substitution-complement relationship among its variables.

[8] i.e. the heat content of fuels, in Joule or BTU.

[9] Useful exergy and useful work are interchangeable terms defining the same concept. Throughout the rest of our work only the nomenclature “useful exergy” will be adopted.

[10] The authors define energy services as the product of the level of energy supply by energy quality (measured using Divisia indexation), and the level of (exogenous) energy-augmenting technology.

[11] Energy services expansion is the major factor explaining growth in Sweden, until the second half of the 20th century. After that, labor-augmenting technical change becomes the major explanatory factor.

[12] From 90% of GDP in 1800 to close to 10% of GDP today (Kander et al., 2014).

[13] See Jones (2002) for the US, and Smulders & De Nooij (2003) for Japan, France, UK, and West Germany.

[14] For three factors of production (capital, labor, and energy), it is common to write a capital-labor two-factor CES production function as a factor of production, alongside energy, within another CES two-factor production function (i.e. nested CES functions). For details, see for example Kemfert (1998).

[15] Economic output (GDP) is adjusted by adding the value of primary energy, thereby creating a gross output measures, as opposed to a value-added measure.

[16] A multivariate approach makes sense in our analysis, since we are expecting to find statistically significant relationships between economic output and *all* relevant inputs factors, rather than between output and only *some* of the factors.

[17] His results show that capital, on the other hand, can be excluded from the cointegration space.

[18] For example, if all variables are quantified by time series which are non-stationary in levels but stationary when taking first differences, they are all integrated of order one – I(1).

[19] The maximum lag order is defined according to the formula proposed by Schwert (2002): $p_{max} = \left[12\left(\frac{T}{100}\right)^{1/4}\right]$, where T is the number of observations.

[20] We want to be able to interpret observed cointegration vectors (CV) as production functions, so the output variable must be present in all models.

[21] i.e. Likelihood Ratio (LR), Final Prediction Error (FPE), Akaike (AIC), Schwarz (SIC), and Hannan-Quinn (HQ).

[22] i.e. output is not a dependent variable.

[23] Changes in factor inputs should have a causal effect on output in a production function formulation. However, it can also occur that variation of output, resulting from variation of demand, can have a causal effect on factor inputs, and the production function formulation would remain valid.

[24] e.g. a two-variable model grouping output and useful exergy defined per unadjusted labor inputs $(q,u)_L$, or per quality-adjusted labor inputs $(q,u)_{hL}$.

[25] i.e. $(q,u)_L$ and $(q,u)_L$, or $(q,k^{AMECO}_{Stock})_L$ and $(q,k^{AMECO}_{Stock})_{hL}$.

[26] Also, values for adjusted R^2 only increase when the increase in R^2 (due to the inclusion of a extra explanatory variable) is more than one would expect to see by chance alone.

[27] da Silva & Lains (2013) set a fixed declining balance rate of 1.65 for machinery & equipment (including the asset types of transport equipment, machinery & equipment, and other investment), and 0.91 for structures (asset types of dwellings, and other buildings & structures).

[28] Service life assumptions are based on previous historical studies on capital formation, along with recent evidence on the Portuguese case (da Silva, 2010). Different service lives are assumed in different sub-periods, considering shorter assets' lives in the more recent decades (1960 onwards).

[29] E.g. innovation and creativity, work experience, education, etc.

[30] Given its broad coverage of countries and years, the average years of schooling remains the most useful proxy for human capital. However, the quality of education, as reflected in internationally comparable test scores, is also increasingly flagged as an important dimension of human capital (Hanushek & Woessmann, 2012).

[31] Data on average years of schooling for population aged 15+ is obtained from Barro & Lee (2011).

[32] Serrenho et al. (2016) define final energy consumption as the total effective consumption, i.e. standard final energy consumption as commonly defined in official energy statistics plus energy sector own energy uses.

[33] E.g. wind and water streams for mechanical drive uses in boats, mills and wells.

[34] Defined as the ratio of exergy to energy.

[35] The test is non-symmetrical, so the absolute value is not considered.