Progress In Electromagnetics Research, Vol. 142, 799–823, 2013

CIRCUIT AND MULTIPOLAR APPROACHES TO INVESTIGATE THE BALANCE OF POWERS IN 2D SCATTERING PROBLEMS

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Abstract—Circuit and multipolar approaches are presented to investigate the correlation between absorption and scattering processes in 2D problems. This investigation was inspired by earlier works of R. E. Collin, which pointed out deficiencies of the Thévenin/Norton circuit models to evaluate the scattered and absorbed powers associated with receiving antennas and, thus, encouraged research on new analytical tools to address these problems. Power balance results are obtained with both circuit and multipolar approaches that are fully consistent. This analysis serves to illustrate how the correlation between absorption and scattering processes results in upper bounds for their power magnitudes, as well as stringent design trade-offs in both far-field and near-field source and scattering technologies.

1. INTRODUCTION

The manner by which receiving antennas not only absorb but also scatter electromagnetic fields has attracted the interest of the antenna community from its very foundation. Aside from the desire to maximize the power received by an antenna, an antenna designer must also be aware of its scattering properties. In many practical applications, an understanding of an antenna's scattering characteristics is essential, for example, to reduce its visibility [1]

Received 14 August 2013, Accepted 13 October 2013, Scheduled 18 October 2013

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Invited paper dedicated to the memory of Robert E. Collin.

(e.g., cloaking devices), to mitigate its influence on neighboring systems [2] (e.g., EMI/EMC behaviors), and/or to avoid an unwanted leakage of power to other channels in RFID and near-field wireless power transfer (WPT) systems [3] (e.g., energy harvesting).

The pioneering work of Dicke [4] was one of the first to point out the importance of controlling the scattering by a receiving antenna. Inspired by this work, the topic would be investigated in the 60's under the label of minimum scattering antennas [5–8], mostly based on a scattering matrix approach. It was explored further in conjunction with antenna arrays and frequency selective surfaces, and their radar cross-section properties. Much of this work has been summarized nicely, for instance, in [9]. More recently, the discussion was intensified when, in [10], the power extracted by a receiving antenna was associated to the power dissipated within its Thévenin/Norton equivalent circuit models. This paper started a series of articles debating the matter: [10–16]. It is perhaps worth noting that this extended use of Thévenin/Norton circuit models is also included in a number of basic antenna textbooks [17, 18].

R. E. Collin also participated in this discussion [12, 14]. In our view, he made two critical contributions to the resolution of the receiving antenna problem. First, he pointed out the limitations of the Thévenin/Norton circuit models to retrieve the power scattered by a receiving antenna. In particular, his detailed analysis of the Thévenin/Norton circuit models revealed that such models only retrieve the component of the scattered field originated by the re-radiation from the load of the receiving antenna. Second, he paraphrased Aharoni [19] to note that when two antennas are coupled, the scattered power does not exist as a separate quantity. Rather, he emphasized that it is a component of the total power radiated away towards infinity which also contains interaction terms describing interference phenomena between the incident and scattered waves. As will be elucidated in this paper, this comment is of paramount importance to understand the differences between the far-field (FF) and near-field (NF) interactions between a source of the incident wave and the scatterer, e.g., a receiving antenna.

In our opinion, the work of R. E. Collin greatly influenced later investigations on the receiving antenna problem. For example, his cautionary note on the inaccuracy of the Thévenin/Norton circuit models has stimulated research on new analytical tools to address this problem. In particular, these efforts include using the optical theorem [20] and spherical harmonic decompositions [3, 21, 22], to investigate the intimate correlation between the associated absorption and scattering processes. In addition, his work has motivated the development of new circuit models that overcome the difficulties of the Thévenin/Norton models, and provide an accurate description of the scattered power [22]. Complementary efforts include the completion of the analytical description of a receiving dipole antenna, including all components of the scattered field, while ensuring a proper energy balance [23].

This paper presents both circuit and multipolar approaches to investigate the correlation between absorption and scattering processes in 2D source-scattering problems. It also serves as a review of recent advances in the receiving antenna problem, particularizing them to 2D geometries. We emphasize that while physical bodies are naturally 3D, a large number of scattering problems are better described by 2D geometries. The latter include problems in which there are no or little field variations along one direction (e.g., parallel plate waveguides or infinitely long cylinders) and/or problems in which the objects are electrically large only along one particular direction (e.g., very thin rectangular waveguides or high aspect ratio, long cylinders). In such cases, the scattering problem can be more easily addressed by using 2D geometries. Moreover, the upper bounds in the system performance are more accurately described in terms of cylindrical harmonics, as they adjust better to the volume efficiently occupied by the object. Therefore, the study of 2D geometries is relevant from both fundamental and applied points of view. In addition, the discussion will illustrate how the work of R. E. Collin advanced a better understanding of this long-studied problem, which continues to have significant implications for many practical applications.

This article is organized as follows. Section 2 first introduces the basic definitions of the power quantities involved in a generic 2D scattering problem. Section 3 then summarizes the cylindrical harmonic representation of the related field and power quantities in a 2D space. Next, an equivalent circuit model for an electrically small scatterer/receiving antenna that correctly determines the absorbed and scatterer powers is derived in Section 4. This model thus helps to extricate the correlations between both the scattered and absorbed powers and their fundamental limits. Section 5 then presents a more general analysis of the problem based on a multipolar decomposition into cylindrical harmonics. This approach helps to extrapolate the results of Section 4 to quite generic scatterers, as well as to clarify the nuances of both the far-field and near-field scenarios. Finally, conclusions of the presented results are drawn in Section 6.

2. GEOMETRY AND DEFINITIONS

Let us consider the generic scattering problem depicted in Fig. 1: a given distribution of sources ($\mathbf{J}_i$, $\mathbf{K}_i$), enclosed within a surface S_i, produces an incident electromagnetic field ($\mathbf{E}^i$, $\mathbf{H}^i$) that illuminates a scatterer/receiving antenna enclosed within a surface S. All of these surfaces are assumed to have outward pointing normals: $\hat{n}_i$, $\hat{n}_S$, and $\hat{n}_\infty$. In response to such an incident field, the currents excited inside/on the scatterer/receiving antenna produce a certain scattered field ($\mathbf{E}^s$, $\mathbf{H}^s$). The total field ($\mathbf{E}^t$, $\mathbf{H}^t$) is equal to the combination of the incident plus scattered fields, i.e., ($\mathbf{E}^t = \mathbf{E}^i + \mathbf{E}^s$, $\mathbf{H}^t = \mathbf{H}^i + \mathbf{H}^s$).

This formulation emphasizes the complete parallelism between generic scatterers and receiving antennas. Without any loss of generality, a receiving antenna can be considered as an ordinary obstacle in which part of the absorbed power has been abstracted and written in terms of circuit quantities. Consequently, receiving antennas must obey the same physics that is associated with generic scatterers.

For the sake of simplicity, let us assume that both the sources and scatterer are immersed in free-space. As noted by Collin [12], the first point for the resolution of the problem is to establish a proper energy conservation statement. According to Fig. 1, power conservation implies that all of the power supplied by the sources, P_{sup}, is either radiated away from the system, P_{rad}, or absorbed inside

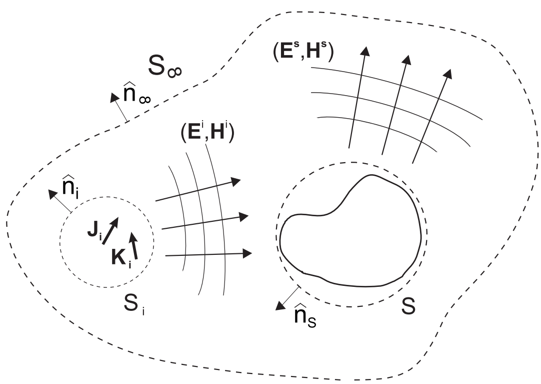

Figure 1. Sketch of an arbitrary scattering problem. A given distribution of free electric and magnetic sources ($\mathbf{J}_i$, $\mathbf{K}_i$), enclosed within a surface S_i, illuminates an arbitrary scatterer enclosed within a surface S. The surface S_∞ includes both the source and scatterer regions.

the scatterer, P_{abs}, i.e.,

$$P_{\mathrm{sup}} = P_{\mathrm{rad}} + P_{\mathrm{abs}} \tag{1}$$

Both P_{rad} and P_{abs} can be determined in terms of the flux of the Poynting vector field through the appropriate surface. Note that since all source and field quantities will be represented in their time-harmonic form (with an $\exp(j\omega t)$ time dependence), all power quantities are assumed to be their time-averaged (real) values, unless otherwise noted, to simplify the terminology used in this discussion. On the one hand, P_{rad} represents the power propagating away from the entire system; and thus it is found as the outward flux of the total Poynting vector field through the surface S_∞, which encloses both the source and scatterer regions:

$$P_{\mathrm{rad}} = P_{S_\infty} = \frac{1}{2}\oiint_{S_\infty} \mathrm{Re}\left\{\mathbf{E}^t \times \left(\mathbf{H}^t\right)^*\right\} \cdot \hat{\mathbf{n}}_\infty dS \tag{2}$$

On the other hand, P_{abs} represents the total absorbed power, i.e., the power dissipated within the scatterer; and thus it is found as the inward flux of the total Poynting vector field over a surface S, which encloses only the scatterer region:

$$P_{\mathrm{abs}} = -\frac{1}{2}\oiint_{S} \mathrm{Re}\left\{\mathbf{E}^t \times \left(\mathbf{H}^t\right)^*\right\} \cdot \hat{\mathbf{n}}_S dS \tag{3}$$

It is worth investigating the individual contributions of the incident, scattered and cross-terms to the total radiated and absorbed powers. To begin, P_{rad} can be decomposed as

$$P_{\mathrm{rad}} = \frac{1}{2}\oiint_{S_\infty} \mathrm{Re}\left\{\mathbf{E}^i \times \left(\mathbf{H}^i\right)^* + \mathbf{E}^s \times (\mathbf{H}^s)^* + \mathbf{E}^i \times (\mathbf{H}^s)^* + \mathbf{E}^s \times \left(\mathbf{H}^i\right)^*\right\} \cdot \hat{\mathbf{n}}_\infty dS \tag{4}$$

Next, the contribution from only the incident field is identified with the power radiated by the free currents, i.e., the power supplied by the sources *in the absence of the scatterer*:

$$P_0 = \frac{1}{2}\oiint_{S_i} \mathrm{Re}\left\{\mathbf{E}^i \times \left(\mathbf{H}^i\right)^*\right\} \cdot \hat{\mathbf{n}}_i dS = \frac{1}{2}\oiint_{S_\infty} \mathrm{Re}\left\{\mathbf{E}^i \times \left(\mathbf{H}^i\right)^*\right\} \cdot \hat{\mathbf{n}}_\infty dS \tag{5}$$

Then the contribution from the scattered field is identified with the power radiated *only by the scatterer*, i.e., the scattered power:

$$P_{\mathrm{scat}} = \frac{1}{2}\oiint_{S} \mathrm{Re}\{\mathbf{E}^s \times (\mathbf{H}^s)^*\} \cdot \hat{\mathbf{n}}_S dS = \frac{1}{2}\oiint_{S_\infty} \mathrm{Re}\{\mathbf{E}^s \times (\mathbf{H}^s)^*\} \cdot \hat{\mathbf{n}}_\infty dS \tag{6}$$

Finally, the contributions from the cross-terms are identified with interference phenomena between the incident and scattered fields. In particular,

$$P_{\text{cross-terms}} = \frac{1}{2}\oiint_{S_\infty} \mathrm{Re}\left\{\mathbf{E}^i \times (\mathbf{H}^s)^* + \mathbf{E}^s \times \left(\mathbf{H}^i\right)^*\right\} \cdot \hat{\mathbf{n}}_\infty dS \tag{7}$$

which means the total radiated power can be composed as

$$P_{\text{rad}} = P_0 + P_{\text{scat}} + P_{\text{cross-terms}} \tag{8}$$

As noted by Collin [12], the scattered power does not exist on its own, and it is not a measurable quantity in this coupled scenario. It is just part of the power radiated away from the system. This is explicitly revealed by (8). This result is an outcome of the fact that the scattered field does not exist on its own and it is not a measurable quantity, but only is just a part of the total field. It is of particular relevance in the analysis of NF wireless power transfer systems, in which the total radiated power P_{rad} represents a leakage of power that restricts the transfer efficiency, i.e., the percentage of the power supplied by the sources that is absorbed by the scatterer/receiving antenna.

The situation can be treated differently for FF interactions, in which the sources, e.g., plane wave sources, are asymptotically placed at infinity. In that case the sources and scatterer/receiving antenna are effectively decoupled, and the fields generated by the latter do not affect the power supplied by the sources, i.e., $P_{\text{sup}} \simeq P_0$. In fact, the power absorbed by the scatterer is then only a very small fraction of the power supplied by the sources $P_{\text{abs}} \ll P_{\text{sup}}$. From a physical standpoint, the electromagnetic fields produced by the sources at infinity decouple from them and propagate away to the far zone, where they are then intercepted by the scatterer. In virtue of the optical theorem [20, 24], the power intercepted by the scatterer is usually associated with the extracted power, defined as the addition of the scattered and absorbed powers and called the extinction power, i.e.,

$$P_{\text{ext}} = P_{\text{scat}} + P_{\text{abs}} \tag{9}$$

Within the FF approximation, P_{ext} can be considered as a physically sound quantity since it corresponds to the power depleted from the incident field. In fact, P_{ext} is a measurable quantity (see, e.g., the experiments carried out in [25]).

To relate the extracted power to the forward scattering behavior in this FF scenario, let the incident field be of the form of a plane-wave with an electric field magnitude E_0, which is propagating in free-space along the $\hat{\mathbf{k}}_i$ direction and is polarized along the $\hat{\mathbf{p}}_0$ direction. In addition, let $\mathbf{F}(\hat{\mathbf{r}})$ represent the amplitude of the scattered field in the far zone along the direction $\hat{\mathbf{r}}$. Then the FF scattered electric field takes the form:

$$\mathbf{E}_s = \frac{e^{-jk_0 r}}{k_0\, r}\, \mathbf{F}\left(\hat{\mathbf{r}}\right) \tag{10}$$

where $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$ and $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$ are the free-space propagation constant and wave impedance, respectively. The associated magnetic field is $\mathbf{H}_s = \hat{\mathbf{r}} \times \mathbf{E}_s/\eta_0$. According to the optical theorem [20, 24], P_{ext} is related to the forward amplitude of the scattered field as:

$$P_{\text{ext}} = P_{\text{scat}} + P_{\text{abs}} = \frac{8\pi\eta_0}{k_0^2 |E_0|^2} \operatorname{Im}\left[\hat{\mathbf{p}}_0 \cdot \mathbf{F}\left(\hat{\mathbf{k}}_i\right)\right] \tag{11}$$

Equation (11) is the classical formulation of the optical theorem. More recent formulations of the optical theorem generalize this result for a wide range of incident fields [26] and background media [27].

Here we note that although Collin [14] did not directly refer to the optical theorem as a tool to elucidate the power balance in FF interactions, he followed many times a parallel thought process, e.g., when explaining the shadow of a parabolic-reflector antenna. This is most clearly evident in the following paragraph extracted from [14]: "The power interaction between the incident field and the forward-scattered field occurs over a vanishingly small solid angle, centered on the axis in the forward direction, as the observation point moves towards infinity. This interaction represents the removal of power from the incident field, and balances the absorbed and scattered powers".

3. DECOMPOSITION IN CYLINDRICAL HARMONICS

Exterior to the free source and scatterer regions, the fields are solutions to the homogeneous Maxwell Equations. Therefore, for a 2D problem, they can be decomposed as a series of cylindrical harmonics [28]. As will be demonstrated, this decomposition is particularly useful to compute the power quantities of interest, as well as to elucidate the correlations between the absorbed and scattered powers and their fundamental limits. In particular, assuming that the origin of the coordinate system is centered within S, the fields can be written as [28]

$$\mathbf{E}^i = \sum_{n=-\infty}^{\infty} e^{-jn\phi}\left[\hat{\mathbf{z}} A_n^{TM\lessgtr} M_n^{\lessgtr}(k_0 r) - A_n^{TE\lessgtr}\left(\hat{\boldsymbol{\phi}} M_n^{\lessgtr\prime}(k_0 r) - \hat{\mathbf{r}} jn \frac{M_n^{\lessgtr}(k_0 r)}{k_0 r}\right)\right] \tag{12}$$

$$\mathbf{H}^i = -\frac{j}{\eta_0}\sum_{n=-\infty}^{\infty} e^{-jn\phi}\left[\hat{\mathbf{z}} A_n^{TE\lessgtr} M_n^{\lessgtr}(k_0 r) + A_n^{TM\lessgtr}\left(\hat{\boldsymbol{\phi}} M_n^{\lessgtr\prime}(k_0 r) - \hat{\mathbf{r}} jn \frac{M_n^{\lessgtr}(k_0 r)}{k_0 r}\right)\right] \tag{13}$$

$$\mathbf{E}^s = \sum_{n=-\infty}^{\infty} e^{-jn\phi}\left[\hat{\mathbf{z}} B_n^{\text{TM}} H_n^{(2)}(k_0 r) - B_n^{TE}\left(\hat{\boldsymbol{\phi}} H_n^{(2)\prime}(k_0 r) - \hat{\mathbf{r}} jn \frac{H_n^{(2)}(k_0 r)}{k_0 r}\right)\right] \tag{14}$$

$$\mathbf{H}^s = -\frac{j}{\eta_0}\sum_{n=-\infty}^{\infty} e^{-jn\phi}\left[\hat{\mathbf{z}} B_n^{TE} H_n^{(2)}(k_0 r) + B_n^{\text{TM}}\left(\hat{\boldsymbol{\phi}} H_n^{(2)\prime}(k_0 r) - \hat{\mathbf{r}} jn \frac{H_n^{(2)}(k_0 r)}{k_0 r}\right)\right] \tag{15}$$

The terms: $A_n^{TZ\lessgtr}$ and B_n^{TZ}, $Z = E, M$, are the incident and scattered field coefficients, respectively, with electric field units. The $A_n^{TZ\lessgtr}$

coefficients are defined by the properties of the sources of the incident field, and the B_n^{TZ} coefficients are functions of the geometrical and electromagnetic properties of the scatterer. In general, the B_n^{TZ} coefficients are found by solving the boundary value problem defined relative to the surface of the scatterer. The $M_n^{\lessgtr}(z)$ functions represent Bessel functions, with $M_n^{<}(z) = J_n(z)$ being the Bessel function of the first kind and order n, and $M_n^{>}(z) = H_n^{(2)}(z)$ being the Hankel function of the second kind and order n. The $\lessgtr$ index indicates the representation for the $r < r'$ and $r > r'$ regions, with r' being the distance from the origin of the coordinates to a point in the source region.

We emphasize that, being a 2D problem, all the aforementioned power quantities will correspond to power per unit length magnitudes, with W/m units. Hereafter, this fact is emphasized by using a superscript L. Furthermore, due to the orthogonality of the cylindrical harmonics, a total power quantity is equal to the sum of the same power quantity associated with each mode [28]. Specifically, after a substantial number of mathematical details which are readily reproduced, the powers: P_{abs}^L, P_{scat}^L, P_{ext}^L, P_{rad}^L, can be rewritten explicitly as the multipole sums:

$$P_{\mathrm{abs}}^L = -\frac{2}{\eta_0 k_0} \sum_{n=-\infty}^{\infty} \sum_{Z=E,M} \left\{ \mathrm{Re}\left[\left(A_n^{TZ<}\right)^* B_n^{TZ} \right] + \left|B_n^{TZ}\right|^2 \right\} \tag{16}$$

$$P_{\mathrm{scat}}^L = \frac{2}{\eta_0 k_0} \sum_{n=-\infty}^{\infty} \sum_{Z=E,M} \left|B_n^{TZ}\right|^2 \tag{17}$$

$$P_{\mathrm{ext}}^L = -\frac{2}{\eta_0 k_0} \sum_{n=-\infty}^{\infty} \sum_{Z=E,M} \mathrm{Re}\left[\left(A_n^{TZ<}\right)^* B_n^{TZ} \right] \tag{18}$$

$$P_{\mathrm{rad}}^L = \frac{2}{\eta_0 k_0} \sum_{n=-\infty}^{\infty} \sum_{Z=E,M} \left|B_n^{TZ} + A_n^{TZ>}\right|^2 \tag{19}$$

4. CIRCUIT MODEL APPROACH

Inspired by [12] in which the deficiencies of the Thévenin/Norton circuit models in determining the scattered power were pointed out, we attempted to find circuit models that provide an accurate description of the balance of powers in scattering problems. For example, circuit models to describe the scattering of spherical bodies of arbitrary size were presented in [22]. To illustrate this concept further, this section presents a circuit model to describe the scattering of electrically small

2D bodies. While a similar circuit model was introduced in [32–34] to explain the peculiarities in the scattering of ferromagnetic wires, this section provides a step forward by illustrating how this circuit model actually holds for objects with arbitrary constitutive parameters, and how it can help to describe the balance of powers in 2D problems.

4.1. Derivation of the Circuit Model

Let us consider then an electrically small 2D body illuminated by a plane wave of magnitude E_0 with the electric field polarized along $\hat{\mathbf{z}}$. For this electrically small obstacle, the scattered field is dominated by the $n = 0$ TM mode, so that it can be simply written as

$$\mathbf{E}^s\big|_{n=0} = \hat{\mathbf{z}} B_0^{\mathrm{TM}} H_0^{(2)}(k_0 r) \tag{20}$$

Additional insight can be obtained by examining the equivalent sources which can produce such a field. In particular, the electric field given by (20) can be identified with the electric field produced by an electric line source $\hat{\mathbf{z}} I_{\mathrm{eq}}$, which is given by the expression [29]:

$$\mathbf{E}^{line} = -\hat{\mathbf{z}} \frac{\eta_0 k_0}{4} I_{\mathrm{eq}} H_0^{(2)}(k_0 r) \tag{21}$$

The magnitude and phase of the equivalent current, I_{eq}, are defined by the geometrical and electromagnetic properties of the scatterer. This simple field equivalence allows us to derive a equivalent circuit model. To this end, note that the scattered field produced by the obstacle is equal to that of any structure supporting the same current distribution. For example, the electric field produced by an impedance-loaded perfect electric conductor (PEC) wire is given by [30, 31]:

$$\mathbf{E}^{lw} = -\hat{\mathbf{z}} \frac{\eta_0 k_0}{4} \frac{E_0}{\alpha_0^{-1} + Z} H_0^{(2)}(k_0 r) \tag{22}$$

where

$$\alpha_0^{-1} = \frac{\eta_0 k_0}{4} \left\{ 1 + j \frac{2}{\pi} \left[\ln\left(\frac{2}{k_0 a} \right) - \gamma \right] \right\} \tag{23}$$

is the susceptibility of this PEC wire, E_0 the field acting on the wire, $\gamma \simeq 0.5772$ the Euler constant, and Z the effective distributed impedance associated with this scatterer. By comparing (20) and (22) it is found that the fields produced by our generic electrically small obstacle are equivalent to those of a PEC wire with an equivalent distributed impedance given by

$$Z = -\frac{\eta_0 k_0}{4} \frac{E_0}{B_0^{\mathrm{TM}}} - \alpha_0^{-1} \tag{24}$$

In this manner, we can describe the scattering problem as the excitation of a line source current whose magnitude and phase are found from the equivalent circuit represented in Fig. 2. The equivalent circuit corresponds to a voltage source connected to a load. On the one hand, the voltage source is defined by the incident electric field. This source representation is convenient to describe the far-field interactions in which the sources and the scatterer/receiving antenna are decoupled. On the other hand, the load is described in terms of several components connected in series. These include a generic distributed impedance, Z, which describes the physical phenomena taking place within the scatterer/receiving antenna, and an impedance consisting of a resistance and an inductance, i.e., $R_{\text{scat}} + j\omega L = \alpha_0^{-1}$. The latter describe, respectively, the radiation and stored magnetic energy produced by the equivalent electric line source, i.e., the dominant physical phenomena outside the scatterer/receiving antenna.

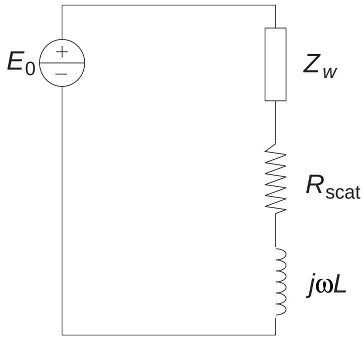

Figure 2. Equivalent circuit model of the scattering by an electrically small 2D body.

Since the equivalent model constructs the same fields outside the receiving antenna/scatterer, any magnitude associated with such external fields can be also determined through the circuit model. For example, (3)–(6) reveal that the absorbed and scattered powers can be computed through the incident and scattered fields. Therefore, it can be readily shown that the absorbed and scattered powers can also be computed in terms of the equivalent circuit model as follows:

$$P_{\text{abs}}^{L} = \frac{1}{2} \text{Re}\,[Z]\, |I_{\text{eq}}|^2 \tag{25}$$

$$P_{\text{scat}}^{L} = \frac{1}{2} R_{\text{scat}}\, |I_{\text{eq}}|^2 \tag{26}$$

$$P_{\mathrm{ext}}^{L} = \frac{1}{2} \left(R_{\mathrm{scat}} + \mathrm{Re}\left[Z \right] \right) \left| I_{\mathrm{eq}} \right|^2 \tag{27}$$

This type of circuit model can be applied to a large number of scattering/receiving structures. As a matter of fact, the model can be applied to any electrically small 2D body in which the $n = 0$ TM mode is dominant, provided that the B_1^{TM} scattering coefficient is known. There are analytical formulations of this coefficient for a number of canonical problems, and it can be obtained numerically for a wider range of objects. For example, Fig. 3 depicts the equivalent impedance for a set of circular cylinders of radius a. By the definition of the equivalent circuit, the distributed impedance of a PEC cylinder corresponds to a short-circuit, while it is non-zero for finite conductivity cylinders. Furthermore, if the cylinder radius is much smaller than the penetration depth, the distributed impedance reduces to its DC resistance. Contrarily, the field is constrained to the surface of the cylinder if its radius is much larger than the penetration depth, and the distributed impedance is composed of a resistance and a reactance, accounting for the losses and magnetic flux associated with this surface effect.

As a another example, if the PEC cylinder is covered by a lossless magnetic layer, the distributed impedance is given by an inductance

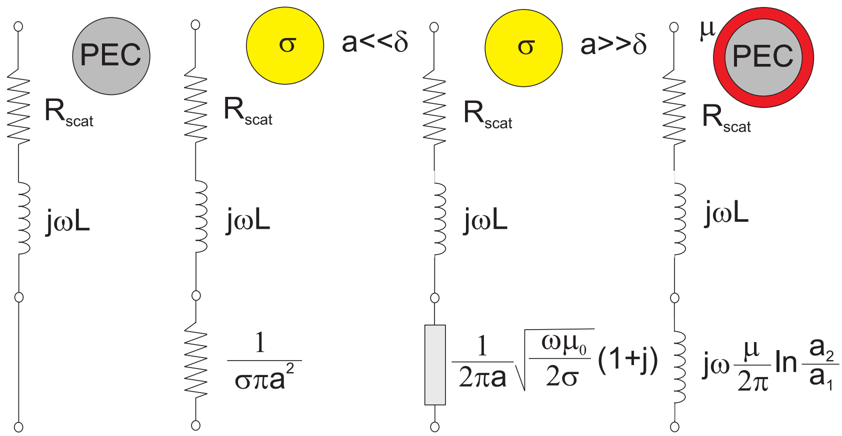

Figure 3. Canonical examples for which an equivalent circuit model is readily obtained: PEC cylinder, conductive cylinder of radius much smaller than the penetration depth, conductive cylinder of radius much larger than the penetration depth, and PEC cylinder coated by a magnetic layer. The terms: a stands for the radius of the cylinder, σ for conductivity, and δ for the penetration depth.

proportional to the magnetic layer permeability. Finally, note that it can also be applied to an ideal PEC wire periodically loaded with lumped elements of impedance Z_{lumped}, in which case the distributed impedance would be given by Z_{lumped}/p, with p being the periodicity of the load [30]. Since lumped elements could perfectly represent the port of a receiving antenna, this example illustrates that there is no physical difference between generic scatterers and receiving antennas.

4.2. Implications of the Circuit Model Representation

Formulating power quantities in circuital terms, as in (25)–(27), not only provides a straightforward way to compute the absorbed, scattered and extracted powers, but also helps to extricate the correlations between the absorbed and scattered powers, as well as their fundamental limits. Moreover, it helps an antenna engineer understand the practical implications of a design.

The scattering resistance in Fig. 2 for the 2D plane wave excitation problem, $R_{\text{scat}} = \eta_0 k_0/4$, is real, positive and non-vanishing. More strikingly, it is independent of the wire and incident field properties. Therefore, it can be concluded that there is an inherent radiative component associated with the equivalent current distribution that forms the scattered field. Consistently, there cannot be absorption without scattering. In connection with the optical theorem, the reason for this fact is that the power carried by the incident field outside the scatterer must be reduced by the amount of absorbed power, which can only be done through a destructive interference produced by the scattered field. In this regard, the necessity of the scattered field to produce this destructive interference in the forward direction was illustrated by Collin [14] in a number of scattering problems, including typical obstacles such as reflectors and disks.

In circuital terms, if there is a certain amount of current I_{eq} flowing on the circuit, then there must also be scattered power given by (26): $\frac{1}{2}R_{\text{scat}}\left|I_{\text{eq}}\right|^2$. Since R_{scat} cannot be controlled either by the wire geometry or its electromagnetic properties, the amount of scattered power can only be manipulated through the amount of current. This fact leads to limitations and inter-dependencies between the absorbed and scattered powers.

Since R_{scat} cannot be zero, the absorbed power is maximized when the effective distributed impedance Z is the complex conjugate of the scattering impedance, i.e., when $Z = R_{\text{scat}} - j\omega L$. In this maximal case, the absorbed power per unit length (25) is equal to

$$P_{\text{abs}}^{L,MAX} = \frac{1}{2}\frac{\left|E_0\right|^2}{4R_{\text{scat}}} = \frac{\left|E_0\right|^2}{2\eta_0 k_0} \tag{28}$$

Moreover, with $R_{\mathrm{scat}} = R_{\mathrm{abs}}$, it then directly follows that the scattered and absorbed powers are equal when the absorbed power is maximized, i.e.,

$$\text{if} \quad P_{\mathrm{abs}} = P_{\mathrm{abs}}^{L,MAX} \quad \text{then} \quad P_{\mathrm{scat}} = P_{\mathrm{abs}} \tag{29}$$

This necessary condition for maximizing the absorbed power has far-ranging practical implications beyond simply receiving antennas.

It is worth remarking that the extracted power, which is identified as the total power dissipated within the equivalent circuit, is not a constant; but it changes as a function of the scatterer properties, i.e., it depends on the distributed impedance Z. Specifically, the power dissipated within the circuit is maximized when there is no reactance, and the resistance is made as low as possible, i.e., when $Z = -j\omega L$. Strikingly, the extracted power is maximized for the lossless, zero absorption case. In such a particular case, the extracted power can be written as

$$P_{\mathrm{ext}}^{L,MAX} = \frac{1}{2}\frac{|E_0|^2}{R_{\mathrm{scat}}} = \frac{2\,|E_0|^2}{\eta_0 k_0} \tag{30}$$

Moreover, since the extracted power is maximized in the lossless case, the extracted and scattered powers are equal in this idealized limit, i.e., $P_{\mathrm{ext}}^L = P_{\mathrm{scat}}^L$. Consequently, the maximal scattered power is also given by (30), i.e., in the lossless limit $P_{\mathrm{ext}}^{L,MAX} = P_{\mathrm{scat}}^{L,MAX}$.

Since the maximal extracted power is obtained for the lossless case, it must be concluded that the presence of absorption actually limits the amount of power that can be extracted from the incident field. Specifically, it is found by comparing (30) and (28) that the maximum absorbed power is a quarter of the maximum extracted power. This means that no scatterer can absorb more than 25% of the maximum amount of power that can be extracted from the same incident field, i.e., from (28) and (30), one finds:

$$P_{\mathrm{abs}}^{L,MAX} = \frac{1}{4}\,P_{\mathrm{ext}}^{L,MAX} \tag{31}$$

To finalize this discussion, this fact should not lead one to the wrong conclusion that the absorbed power can never exceed 50% of the power extracted from the incident field. On the contrary, absorption efficiencies larger than 50% are perfectly possible. This was recognized by Collin [14]. Green's antenna [6] is but one of the many examples in which the absorbed power is larger than a 50% of the extracted power. This possibility is also illustrated by the circuit model presented herein, in which the ratio between the absorbed and scattered powers is equal to the ratio between the distributed and scattering resistances, i.e.,

$$\frac{P_{\mathrm{abs}}^L}{P_{\mathrm{scat}}^L} = \frac{\mathrm{Re}\,[Z]}{R_{\mathrm{scat}}} = \frac{P_{\mathrm{abs}}^L}{P_{\mathrm{ext}}^L - P_{\mathrm{abs}}^L} \tag{32}$$

As the losses are increased so that $P_{\text{abs}}^L \to P_{\text{ext}}^L$, this ratio can be made as large as desired. However, when constrained by (30) and its lossless limit, this outcome comes at the cost of reducing P_{abs}^L. Consequently, there is a compromise between the absorbed power (i.e., effective area) and the visibility of a receiving antenna. Cloaked [1] and forward-scattering [20] sensors are examples of strategies to efficiently solve this design trade-off.

5. MULTIPOLAR APPROACH

One may wonder up to which point the conclusions extracted by means of the equivalent circuit model can be extrapolated to other 2D structures. Admittedly, the equivalent circuit model has been derived for electrically small 2D structures. However, because the limits seem so natural when a circuit model representation is considered, one might anticipate that similar conclusions could be drawn for the scattering by a large number of, if not all, 2D objects.

As indicated earlier, the work of R. E. Collin motivated the use of tools, such as the optical theorem and multipolar approaches, as alternatives to the circuit models. As a matter of fact, a more generic, albeit more mathematical, derivation of the demonstrated limits on the absorbed and scattered powers can be accomplished by following a purely multipolar approach. This section presents the 2D derivation in terms of the cylindrical harmonic decomposition, which is the most convenient for 2D geometries. The analogous 3D analysis based on spherical harmonics was presented in [3]. The following 2D analysis allow us to emphasize the differences between the 2D and 3D problems.

5.1. Far-field Interactions

Let us address the limits of the absorbed, scattered and extracted powers in FF interactions on the basis of their multipolar sums (16)–(19). Let us first inspect the multipolar representation of the absorbed power (16). To find the maximum absorbed power for a generic source, defined by the coefficients $A_n^{TZ\lessgtr}$, one can take derivatives of (16) with respect to the terms $\mathrm{Re}[B_n^{TZ}]$ and $\mathrm{Im}[B_n^{TZ}]$. This approach leads to the conclusion that the absorbed power P_{abs}^L is maximized for the following condition between the source and scattering coefficients:

$$B_n^{TZ} = -\frac{1}{2} A_n^{TZ<} \tag{33}$$

which means the maximum absorbed power is given explicitly by the expression:

$$P_{\text{abs}}^{L,MAX} = \frac{1}{2\eta_0 k_0} \sum_{n=-\infty}^{\infty} \sum_{Z=E,M} \left|A_n^{TZ<}\right|^2 \tag{34}$$

In consistency with the circuit model representation, it is found by introducing (33) into (17) that the absorbed and scattered powers are equal when the absorbed power is maximized, which means the absorbed power is 50% of the extracted power. On the other hand, as noted earlier, in many cases the absorbed power can be much larger than 50% of the extracted power. This fact can also be checked in the multipolar formulation by taking the limit: $B_n^{TZ} \to 0$, and noting that the scattered power (17) decreases faster than the absorbed power (16). Therefore, arbitrarily large ratios between the absorbed and scattered powers are indeed feasible, though at the cost of a similar decrease in the absorbed power. In fact, it is also apparent from (17) that a zero scattered power can only be achieved in the exact limit: $B_n^{TZ} = 0$. However, it is found from (16) that $B_n^{TZ} = 0$ also implies a zero absorbed power. Consequently, the cylindrical harmonic representation ratifies the principle that there can not be absorption without scattering.

Summarizing, the considerations on the absorbed power derived through the circuit model are fully consistent with those derived by means of the multipolar approach. However, the latter results are much more general. They have been derived for 2D scatterers of arbitrary size, shape and constitutive parameters, as well as for arbitrary incident fields. Thus, the multipolar approach allows us to generalize the results intuitively obtained with the circuit model representation to general 2D scatterers.

As a particular example, consider a uniform plane wave propagating along the x-axis with an electric field magnitude: E_0, and polarized along the wire axis (z-axis). It has the expansion coefficients: $A_n^{TM<} = j^n E_0$, $A_n^{TE<} = 0$ [29]. (Note that the $A_n^{TZ>}$ coefficients do not exist because the sources of the plane wave are located at infinity.) Introducing these coefficients into (34), the maximum (limit) of the absorbed power for this particular excitation is given explicitly by

$$\left.P_{\text{abs}}^{L,MAX}\right|_{\text{pw}} = \frac{1}{2\eta_0 k_0} \sum_{n=-\infty}^{\infty} |E_0|^2 \tag{35}$$

It can be immediately verified that for electrically small structures, i.e., scatterers for which only the $n = 0$ TM cylindrical harmonic is non-trivial, (35) reduces to (28), i.e., that

$$P_{\text{abs,elecsmall}}^{L,MAX}\Big|_{\text{pw}} = \frac{1}{2\eta_0 k_0} |E_0|^2 \tag{36}$$

In other words, the limit derived based on the multipolar approach is fully consistent with the limit derived on the basis of the circuit model. Consider also a finite-size scatterer able to efficiently couple with a number of multipoles up to order $n = N$. The limit of absorbed power is then given by

$$P_{\text{abs,N-multipoles}}^{L,MAX}\Big|_{\text{pw}} = \frac{|E_0|^2}{2\eta_0 k_0} (2N + 1) \tag{37}$$

Similarly, for electrically large scatterers, a rule of thumb employed for the truncation of this series is $N = k_0 a$ [28]. With this truncation rule, the absorbed power limit becomes:

$$P_{\text{abs,eleclarge}}^{L,MAX}\Big|_{\text{pw}} = \left\{ \frac{|E_0|^2}{2\eta_0} \right\} 2a \tag{38}$$

which corresponds to the integration of the density of the incident power over the diameter of the circumference which circumscribes the scatterer. In other words, the derived limit consistently recovers the geometrical optics limit for electrically large structures.

Note that the maximal absorbed power (35) is infinite, i.e., (37) diverges as $N \to \infty$. This is an artifact that is due to the infinite amount of energy artificially carried by an ideal plane wave. In practice, as the effective area of the scatterer grows, the assumption of uniform illumination over the effective area of the scatterer no longer holds; and the plane-wave model cannot be employed. As a matter of fact, within the range of applicability of the plane-wave excitation, the scatterer is typically extracting only a small fraction of the power produced by the plane wave sources.

To emphasize further, this cylindrical harmonic formulation allows us to accentuate the differences between the 2D and 3D geometries and to illustrate why the upper bounds of objects that are electrically large only along one particular direction are more accurately described with it. Specifically, one finds from (37) that the upper bound of absorbed power for 2D objects increases as $2N + 1$ along with the number of harmonics N, while it was found to increase as $N^2 + 2N$ for 3D geometries [22]. Thus, it can be concluded that the growth rate per harmonic is much smaller in 2D geometries. This behavior also illustrates the difficulty of examining structures having a high aspect ratio with the more general 3D spherical harmonics decomposition. To this end, let us consider an arbitrary cylinder with electrically large length L, but much smaller cross-section, whose size suggests the use of N_0 harmonics, i.e., whose electrical length $k_0L \gg N_0$. According to the 3D spherical decomposition, the upper bound of absorbed power would be as high as $\{[|E_0|^2/(2\eta_0)] \cdot L/k_0\} \cdot \pi(k_0L + 1)$, while the decomposition into cylindrical harmonics would lead to the tighter bound $\{[|E_0|^2/(2\eta_0)] \cdot L/k_0\} \cdot (2N_0 + 1)$.

Let us now focus on the limits of the scattered and extracted powers. Inspecting (17) reveals that P^L_{scat} grows along with the coefficients B^{TZ}_n, and thus it cannot be maximized through the same derivation approach that was used for the absorbed power P^L_{abs}. In fact, the generality of the scatterer will be restricted in our analysis from now on to enable the derivation of the limits for the scattered and extracted powers. Specifically, let us assume that the scatterer is passive, linear, and that its surface allows the cylindrical harmonics to interact independently. In such a case, the scattered field coefficients are proportional to the incident field coefficients in the region of the scatterer, i.e.,

$$B^{TZ}_n = b^{TZ}_n A^{TZ}_n \tag{39}$$

and passivity holds independently for each multipole, i.e.,

$$P^{L,TZ}_{\text{abs},n} = -\left|A^{TZ}_n\right|^2 \left\{\text{Re}\left[b^{TZ}_n\right] + \left|b^{TZ}_n\right|^2\right\} > 0 \tag{40}$$

Thus, positive definiteness of the absorbed power: $P^{L,TZ}_{\text{abs},n} > 0$, imposes the conditions: $\text{Re}[b^{TZ}_n] < 0$ and $|\text{Re}[b^{TZ}_n]| \geq |b^{TZ}_n|^2$. Furthermore, since $|\text{Re}[b^{TZ}_n]| \leq |b^{TZ}_n|$, it also requires that $|b^{TZ}_n| \leq 1$. Therefore, the condition on the b^{TZ}_n coefficients to achieve the maximum scattered power can be written as

$$\left|b^{TZ}_n\right|^2 = 1 \longrightarrow b^{TZ}_n = -1 \tag{41}$$

Therefore, the maximal scattered power per unit length is given by

$$P^{L,MAX}_{\text{scat}} = \frac{2}{\eta_0 k_0} \sum_{n=-\infty}^{\infty} \sum_{Z=E,M} \left|A^{TZ}_n\right|^2 \tag{42}$$

Again, the maximum scattered power is found to be four times larger than the maximum absorbed power, i.e.,

$$P^{L,MAX}_{\text{scat}} = 4\, P^{L,MAX}_{\text{abs}} \tag{43}$$

Moreover, the assumption (39) also allows us to write the extracted power as

$$P^{L,TZ}_{\text{ext},n} = -\frac{2}{\eta_0 k_0} \sum_{n=-\infty}^{\infty} \sum_{Z=E,M} \left|A^{TZ}_n\right|^2 \text{Re}\left[b^{TZ}_n\right] \tag{44}$$

Equation (44) reveals that extracted power increases along with $\mathrm{Re}[b_n^{TZ}]$. Therefore, its upper limit is reached when $b_n^{TZ} = -1$. In this manner the multipolar approach generalizes the result that the extracted and scattered powers share the same upper bound. Thus, it can be concluded that the extracted power per unit length is maximized for the ideal lossless case; and, therefore, the presence of losses limits the amount of power that can be extracted from the incident field.

Throughout the multipolar discussion of the scattered and extracted powers, the balance of powers for each multipole has been considered independently, assuming (39). This condition is rigorously satisfied for cylindrical objects, and it is approximately satisfied by 2D objects with soft surfaces. However, the limit of absorbed power (34) has been derived for completely arbitrary scatterers. It was found, according to (33), that the absorbed power is maximized under a condition in which the balance of powers for each multipole is considered independently. This result encourages us to believe that the results in terms of the scattered and extracted powers can indeed be generalized to arbitrary scatterers.

5.2. Near-field Interactions

In contrast with FF interactions, the source and scatterer/receiving antenna are coupled in NF interactions. Therefore, the scatterer affects the power supplied by the sources, P_{sup}^L. Consequently, the magnitude of interest for an efficient transmission of energy is typically the power transfer efficiency, which is defined as the fraction of the power supplied by the sources that is absorbed by the scatterer:

$$PTE = \frac{P_{\mathrm{abs}}^L}{P_{\mathrm{sup}}^L} \tag{45}$$

In FF interactions this ratio is very small, and the PTE is maximized by increasing the absorbed power, no matter how much power is scattered. On the other hand, maximizing P_{abs}^L does not necessarily lead to the highest PTE in NF interactions. In particular, an uncontrolled leakage of power into the radiated field (possibly produced by the scattered power which is inevitably associated to the absorbed power) could decrease the overall PTE.

Following Collin's description of the process [12], power conservation implies that all the supplied power is either absorbed by the scatterer or radiated away from the system, i.e.,

$$P_{\mathrm{sup}}^L = P_{\mathrm{abs}}^L + P_{\mathrm{rad}}^L \tag{46}$$

This simple statement suggests two main strategies to asymptotically get a 100% power transfer. First, one would want to suppress P_{rad}^L by destructive interference. Second, one would want to obtain an absorbed power much larger than the power radiated away from the system, i.e., $P^L_{\text{abs}}/P^L_{\text{rad}} \gg 1$.

Let us analyze both possibilities through the cylindrical harmonic representation of the power magnitudes (16)–(19). In view of (19), we begin with the condition that the multipolar coefficients must satisfy to achieve zero radiated power, $P^L_{\text{rad}} = 0$, which is:

$$B_n^{TZ} = -A_n^{TZ>} \tag{47}$$

This condition (47) means that the contributions from each of the multipoles associated with the sources and the scatterer must be equal in magnitude, but out-of-phase. This results in a net radiated field that is equal to zero due to destructive interference. Due to the degrees of freedom provided by the coefficients of the incident field, $A_n^{TZ>}$, and the scatterer, B_n^{TZ}, in their exterior regions, the condition (47) is compatible with the net absorbed power result. Therefore, there are configurations in which all of the power supplied by the sources is absorbed by the scatterer. Moreover, in theory, the condition (47) is indeed compatible with the absorbed power being maximized. Consequently, there are configurations leading to a 100% PTE, while keeping the absorbed power at a maximum. In particular, this is achieved when the following combined condition is satisfied

$$B_n^{TZ} = -A_n^{TZ>} = -\frac{1}{2}A_n^{TZ<} \tag{48}$$

Unfortunately, the combined condition (48), though possible, imposes very stringent limits on the sources of the incident field. When the sources are close to the scatterer (i.e., only NF interactions exist between the sources and the scatterer), the absolute values of the source coefficients for the $r < r'$ region are much larger than the corresponding absolute values of the source coefficients in the $r > r'$ region, i.e., $|A_{nm}^{lTZ<}| \gg |A_{nm}^{lTZ>}|$. Therefore, it can be concluded that (48) cannot be satisfied in near-field interactions. In other words, it is not possible to simultaneously suppress the radiated power and keep the absorbed power to be at its maximum in NF interactions. Moreover, the multipolar approach applied to the 3D case concluded that the combined condition (48) cannot be satisfied when a Hertzian dipole excites a finite scatterer [3]. Therefore, it can be inferred that (48) can not be satisfied with small devices.

This conclusion led us to the second strategy to asymptotically get a 100% PTE. Comparing (16) and (19), one finds that while the coefficient $A_n^{TZ<}$ is present in the multipolar formulation of the absorbed power, this coefficient is not present in the multipolar formulation of the radiated power. Therefore, if $A_n^{TZ<} \gg A_n^{TZ>}$

and $A_n^{TZ<} \gg B_n^{TZ}$, it is possible to asymptotically approach the $PTE = 100\%$ limit. On the one hand, the condition $A_n^{TZ<} \gg A_n^{TZ>}$ also means that the incident field in the region of the scatterer is much larger than the field radiated by the sources into the exterior region. This condition can be satisfied by simply placing a poor radiator (e.g., one with a large reactive field) in the vicinity of the scatterer. On the other hand, the condition $A_n^{TZ<} \gg B_n^{TZ}$ means that the scattered field is significantly weaker than the incident field on its surface, and that the absorbed power is well below its maximum value (33). Thus, the PTE can approach the desired 100% limit, but the actual total amount of absorbed power must be much lower than what the scatterer could actually handle.

This result exemplifies how the correlations between absorbed and scattered powers also influence the transfer of power in NF interactions. In essence, unless there is a perfect destructive interference configuration, a large absorbed to scattered power ratio is needed to avoid a leakage of power in the form of radiated power. However, as was found in the FF interactions, it was demonstrated that such large ratios can only be achieved with absorbed powers much smaller than the upper bound. Therefore, the optimal receiver to realize a large PTE in the very NF case features a poor performance in FF interactions. Nevertheless, note that reducing the possible absorbed power would also make the system more vulnerable against undesired parasitic losses.

6. CONCLUSIONS

This work has introduced circuit and multipolar approaches to investigate the correlations between absorption and scattering processes in 2D scattering problems. Both formulations were described with the intent to illustrate the tools that can be used to straightforwardly compute the set of powers involved in any scattering process. They also provide a means to investigate the associated balance of powers and the fundamental limits on each contribution. This work therefore completes previous studies based on 3D geometries [3, 21, 22]. Specifically, we have presented upper bounds of the absorbed, scattered and extracted power for 2D geometries, thus revealing the main similarities and differences between the 2D and 3D scenarios, as well as the difficulties of analyzing 3D objects with high aspect ratio by using a vector spherical harmonic decomposition.

We have also intended with this analysis to illustrate how the seminal work of R. E. Collin has led to an improvement in the understanding of the long-standing problem of the powers associated with a receiving antenna, and how it stimulated research seeking out new and more adequate tools to address it. In this manner, this article also serves to review recent advances in this receiving antenna problem. In summary, we believe there is a consensus that, due to energy conservation issues exemplified by the optical theorem, there cannot be absorption without scattering. In addition, this correlation between the absorption and scattering processes leads to the fact that, when the absorbed power is maximized, the absorbed and scattered powers must be equal. Despite this fact, it also was demonstrated that the ratio between the absorbed and scattered powers can be arbitrarily large, although at the cost of decreasing the actual amount of power absorbed. Cloaked and forward scattering sensors are examples of how obtain such large ratios, while minimizing the sacrifice in terms of the absorbed power. As discussed, it can also be demonstrated that the maximal extracted and scattered powers are equal, which means that the maximum extracted power is four times larger than the maximal absorbed power. Consequently, despite its seemingly contradiction, the presence of losses actually limits the amount of power that can be extracted by any scatterer/receiving antenna.

Aside from being of fundamental interest, the correlation between the absorption and scattering processes in a scatterer/receiving antenna scenario has far-reaching technological implications in both FF and NF scenarios. In FF interactions, such correlations impose a compromise between the effective area and the visibility of a receiving antenna. In NF interactions, the leakage of radiation from a coupled system can, in theory, be totally suppressed by means of destructive interference or by emphasizing reactive effects. However, the correlations between the absorption and scattering processes also impose practical trade-offs in NF systems. Specifically, it is found that a total suppression of the radiated power can only be achieved by sacrificing both the PTE for larger distances and the robustness of the system against undesired dissipation for smaller distances. Recognizing these FF and NF tradeoffs, an antenna engineer has a better perspective from which to design an optimal receiving antenna system for a specific application.

Finally, it was shown that all of the aforementioned conclusions hold for both 2D and 3D geometries. Nonetheless, it was also demonstrated that the dissimilarities between both the 2D and 3D geometries lead to quantitative differences on the upper bounds of the absorbed, scattered and extracted powers. As a consequence, it also was demonstrated how important it is to select the appropriate approach to describe the problem, e.g., choosing either the cylindrical or spherical harmonics approach to analyze the problem as a function of the aspect ratio of the object under consideration.

ACKNOWLEDGMENT

This work was supported in part by the Spanish Ministry of Science and Innovation, Projects No. TEC2009-11995 and No. CSD2008-00066 and by the NSF Contract No. ECCS-1126572. Prof. R. W. Ziolkowski would like to give special thanks to Prof. I. M. Besieris for his invitation to contribute to this special issue in honor of Prof. R. E. Collin and to Prof. W. Chew for helping make this issue a reality. He recalls his original impressions in awe of Prof. Collin while learning waveguide theory from his textbook [35] as a graduate student at the University of Illinois at Urbana-Champaign in the late 1970's. He was introduced to Prof. Collin by Prof. Besieris and interacted with him at several IEEE International Symposium on Antennas and Propagation and U. R. S. I. Meetings in the 1980's and 1990's. It was his great pleasure to find that Prof. Collin was a wonderful person in addition to being an exceptionally talented electromagnetic theorist. While at the Lawrence Livermore National Laboratory in the 1980's, he was on the receiving end of Prof. Collin's many insightful comments on his aperture coupling work and his localized wave work with Prof. Besieris. He later used Prof. Collin's chapter on "Artificial Dielectrics" in [35] during his initial efforts on artificial atoms and molecules in the 1990's and on metamaterials in this century. He is delighted to acknowledge the continuing impact of Prof. Collin's numerous books and publications on his students at the University of Arizona and their enduring impact on students worldwide.

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