Millimeter-wave phase resonances in compound reflection gratings with subwavelength grooves

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Abstract: Experimental evidence of phase resonances in a dual-period reflection structure comprising three subwavelength grooves in each period is provided in the millimeter-wave regime. We have analyzed and measured the response of these structures and show that phase resonances are characterized by a minimum in the reflected response, as predicted by numerical calculations. It is also shown that under oblique incidence these structures exhibit additional phase resonances not present for normal illumination because of the potentially permitted odd field distribution. A satisfactory agreement between the experimental and numerical reflectance curves is obtained. These results confirm the recent theoretical predictions of phase resonances in reflection gratings in the millimeter-wave regime, and encourage research in this subject due to the multiple potential applications, such as frequency selective surfaces, backscattering reduction and complex-surface-wave-based sensing. In addition, it is underlined here that the response becomes much more complex than the mere infinite analysis when one considers finite periodic structures as in the real experiment.

OCIS codes: (240.5445) Polarization selective devices; (290.5820) Scattering measurements; (290.5880) Scattering, rough surfaces; (350.4010) Microwaves.

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1. Introduction

It is well known that when an infinite metallic grating is illuminated by *p*-polarized light, a surface plasmon polariton (SPP) can be excited along the surface [1]. This excitation is accompanied by a significant power absorption [2, 3], and consequently it produces a sudden change in the efficiency curves of the reflected orders. This phenomenon is particularly important when the corrugations are shallow. However, as the depth of the grooves is increased, another type of resonance can take place: the eigenmodes of each cavity can be excited, producing interesting resonant effects, such as field enhancement inside the corrugations [4, 5]. Both effects manifest themselves as dips in the reflected power curves and absorption peaks, and they can even merge into one another forming hybrid resonances [6, 7]. Based on the results for infinite periodic structures, resonances occuring in finite gratings comprising rectangular grooves have also been investigated [8–12].

The study of finite compound arrays has been historically connected to the superdirective property exhibited by structures formed by an array of passive elements such as slotted cylinders [13] or rectangular cavities [14]. Phase resonances were first reported in such structures, and were later observed in compound infinitely periodic gratings [15–17]. Le Perchec *et al.* analyzed the excitation of phase resonances in a two-slit system [18], and recently Barbara *et al.* investigated a new kind of resonances that arise in commensurate arrangements of deep metallic sub-wavelength grooves [19].

Phase resonances take place under p-polarized incidence for subwavelength grooves and they are only permitted when the period is formed by several slits or cavities. They are usually excited within a waveguide mode resonance, and are characterized by a phase reversal of the magnetic field in adjacent slits within each period [14]. Phase resonances manifest themselves as sharp features in the reflected and transmitted responses. This kind of resonances has recently been reported for s polarization, but in this case a material with a large dielectric constant is needed to be placed within the grooves to increase their effective width [20].

There are a few recent works in the literature that report experimental demonstrations of phase resonances, most of them in slit transmission structures [21–23]. For reflection gratings with dual-period, reflectance measurements have been reported in [24], where the authors analyze the excitation of surface plasmons and the absorption they produce at resonance. Recently, Barbara *et al.* gave experimental evidence of phase resonance excitation in the infrared region [19, 25].

In this work we report experimental demonstration of phase resonances in compound reflection structures comprising three grooves within each period, and highlight the complexity physics arising from finiteness issues on such gratings. Copper samples have been fabricated, and their reflectance has been measured for normal and oblique incidence in the millimeter wave regime. Strictly speaking, SPPs do not excite in this regime. Nevertheless, they are mimicked by a leaky wave [26, 27]. The experimental results are compared with those obtained by numerical calculations.

2. Configuration and experimental setup

We fabricated a copper sample by wire-cut electrical discharge machining in which a total of 51 rulings of rectangular cross section were practised. The grooves are distributed in 17 groups of three grooves each, so that the structure can be regarded as a finite compound grating comprising three grooves in each period (see Fig. 1). The grooves' width and the separation between nearest grooves is $c = a = 0.4$mm, their nominal depth is $h = 1.1$mm, and the period is $d = 2.8$mm. The copper wafer has a total diameter of 62.4 mm. The structure is illuminated by a linearly polarized Gaussian beam of wavelength λ and frequency f, coming with an angle θ_0 with the y-axis.

The measurements of the reflected intensity were performed with an ABmm^{TM} Quasi Optical (QO) Vector Network Analyzer. In this solid-state instrument, the millimeter-submillimeter wave frequencies are obtained by the frequency multiplication of centimeter wave frequencies generated by an internal tunable source. Afterwards, the detection is done by harmonic mixing heterodyne down-conversion. The QO set-up consists of a corrugated horn antenna that

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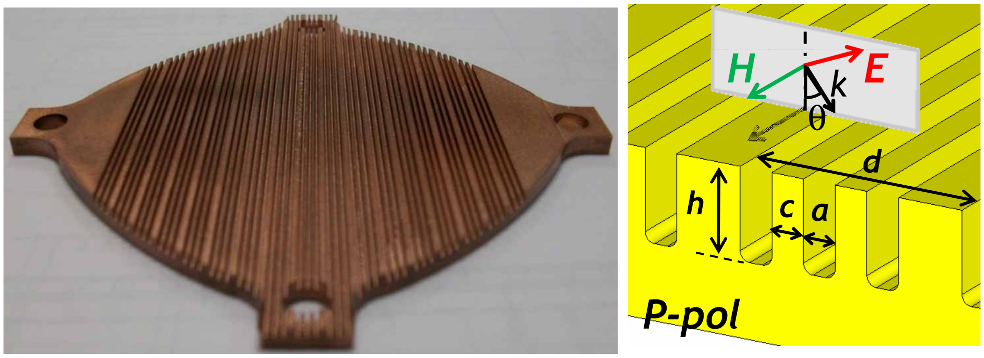

Fig. 1. Fabricated sample and scheme of the compound reflection grating with subwavelength rectangular grooves.

generates a very well linearly polarized Gaussian beam which, after two ellipsoidal mirrors, is focused over the sample under test with a beamwaist of 28 mm for the frequency range of this work. The reflected beam covers the same path inversely, and is steered towards the harmonic mixer by a directional coupler placed after the corrugated horn antenna. This set-up is optimized for normal incidence. When oblique incidence angles are considered, the reflected beam is misaligned with respect to the ellipsoidal mirrors, and thus, a penalty on the recorded reflected power may be suffered. For close-to-normal angles of incidence the penalty may be minimal, and this is the reason to restrict the analysis just to $\theta_0 = 5^\circ$ [28, 29].

3. Results

The experimental results are compared with the simulations obtained with CST Microwave Studio^{TM}, a completely numerical approach based upon finite integration time domain method. To analyze the response of finitely periodic structures, many authors [19,22,30] compare the experimental results with the simulated ones obtained by applying periodic boundary conditions, i.e., those corresponding to an infinitely periodic structure. However, for analyzing the generation of phase resonances in dual-period reflection structures in the millimeter-wave regime, it is crucial to use the actual structure to simulate the response, i.e., an infinite plane with a finite number of grooves. In what follows, we call this structure “finite structure”. To illustrate this fact, in Fig. 2 we compare the reflection coefficient for an infinitely periodic compound structure and that corresponding to a finite structure with 17 groups of three grooves each, which represents more accurately the fabricated sample. The infinite case is simulated using the solver defined “unit cell” boundary conditions, reducing the infinite problem to a single cell, and the illumination is done with the zeroth-order mode, which is equivalent to a normally incident plane wave. The finite case, which is actually semi-infinite since the grooves’ length is infinite in order to simplify the simulation model, is illuminated with a plane wave (thus, illumination is uniform) and the reflected component is sensed with a magnetic field probe placed at 100 mm from the surface. It was found that the best matching between the experimental and the simulated results was obtained introducing a thickness $h = 1.01$mm in the numerical calculation, instead of the actual thickness of the fabricated sample. This implies a fabrication error of approximately 8% in the thickness, which is completely reasonable for the fabrication method of this kind of samples. This thickness value was used in all the results presented. The blue thick curve in Fig. 2, which corresponds to the infinite structure, exhibits a remarkable minimum at

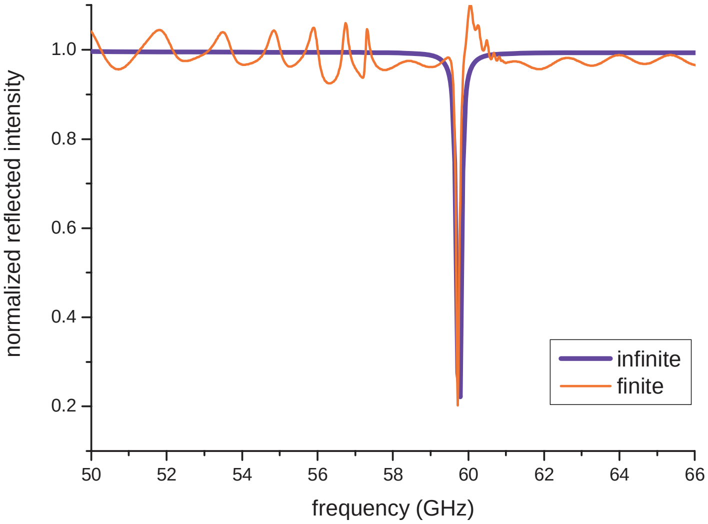

Fig. 2. Comparison of the simulated reflection coefficient as a function of the frequency calculated for an infinitely periodic structure and for a finite structure comprising 17 groups of three grooves for normal incidence. The parameters of the structure are: $c = a = 0.4$ mm, $h = 1.01$ mm, $d = 2.8$ mm.

a frequency $f \approx 60$ GHz on an otherwise planar reflectance of unit value. The red thin curve, which corresponds to the finite structure, has a similar dip at the same frequency, but the rest of the curve exhibits ripples in the frequency range considered. These undulations are generated by the multiple resonances supported by the structure. When the structure is infinitely periodic, only two phase resonances are allowed, i.e., the (+ + +) mode and the (+ - +) mode (the first groove is arbitrarily assigned a plus sign; for the subsequent grooves, if the phase difference is 0 (π), the sign will be the same as (opposite to) that of the preceding one) [14]. In the latter case this means that in every group of three grooves, the phase of the field within the external grooves is opposite to that of the central one. However, for a finite structure, the number of phase resonances increases with the number of grooves, producing sudden changes in the reflected response. This effect becomes even more pronounced as the groups of grooves become closer to each other, i.e., as the period of the grating is reduced while keeping the other lengths fixed. In this case, field coupling between the fields at adjacent groups of grooves is allowed, and this produces new resonant modes. Taking into account that the fabricated structure has 51 grooves, a very oscillating response must be expected, as shown in Fig. 2. Consequently, in what follows we compare the experimental results with those simulated for the actual finite structure.

In Fig. 3 we compare the experimental and the numerical results for a copper structure comprising 17 groups of three grooves each, illuminated by a Gaussian beam at normal incidence [Fig. 3(a)] and at $\theta_0 = 5^\circ$ [Fig. 3(b)]. The thick blue curves correspond to the measurements and the thin red ones, to the simulations.

The most striking characteristic of the reflectance curves in Fig. 3(a) is the minimum at $f \approx 60$ GHz. It corresponds to a π resonance mode, which is characterized by a phase reversal of the field within adjacent grooves, as it was predicted in [14]. The π resonance is the highest quality resonance among the so called phase resonances, which are characterized by phase differences of 0 or π radians between the fields at adjacent grooves [15]. One of the requirements for a phase resonance to occur is to have at least one phase reversal of the magnetic field

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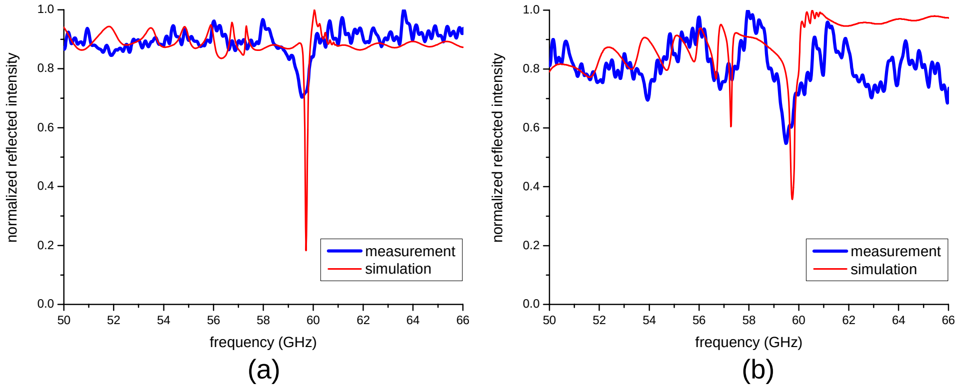

Fig. 3. Comparison of the measured and the simulated reflection coefficient as a function of the frequency. The simulated results correspond to a finite structure comprising 17 groups of three grooves each, and the parameters of the structure are: $c = a = 0.4$ mm, $h = 1.01$ mm, $d = 2.8$ mm. (a) Normal incidence; (b) oblique incidence (5°).

in adjacent grooves. The number of possible configurations with at least one phase reversal in adjacent grooves is determined by the total number of grooves in the structure. The symmetry imposed by the normal illumination also restricts the number of allowed modes. In Fig. 3(a) the measured reflectance exhibits a marked minimum at the phase resonance, and the rest of the curve is rather oscillating, with no other significant features. When the structure is obliquely illuminated, new possibilities of resonant configurations open up: the symmetry condition imposed by the normal illumination is removed, thus allowing new phase configurations inside the grooves. Notice that for $\theta_0 = 5^\circ$ [Fig. 3(b)], the π resonance at $f \approx 60$ GHz is still present in the measured as well as in the simulated reflectance, and there is also another minimum at $f \approx 57$ GHz, which corresponds to the excitation of another, non-symmetric phase resonance [31]. As observed, there is a good agreement between the experimental and the simulated results, and these results are in qualitative agreement with previously reported experimental data in the infrared region [19, 25]. However, it is important to remark that in all cases the measured dips are wider than the simulated ones, as it is to expect due to a variety of reasons: small period changes along the fabricated structure, experimental Gaussian beam illumination, which is non-uniform, reception of power employing a horn antenna, instead of the point magnetic field probe used in the simulation. Also, out of the main resonances, the agreement between simulation and experiment is only moderately good. In both cases a strong ripple is observed out of band, but the amplitude as well as frequency peak location is different in each case. As explained above, these oscillations arise from the multiple resonances that can be excited in finite dual-period gratings. Other cause that cannot be neglected is Fabry-Perot interference, inherent and inevitable in the reflection characterization of strongly reflecting structures. Both phenomena can combine to produce the observed ripple, but again the mentioned fundamental differences between simulation and measurement set-ups can lead to different curves in each case. This probably explains the wide experimental dip at $f \approx 57$ GHz, which can be the result of the overlap of two resonant dips at $f \approx 56.6$ GHz and $f \approx 57.3$ GHz. The excitation of phase resonances is extremely sensitive to changes in the incidence angle, and it was observed that even for $\theta_0 = 1^\circ$ (not shown) this new dip already appears in the reflected response, as it was also found for transmission structures [31]. This implies that a very small misalignment in the

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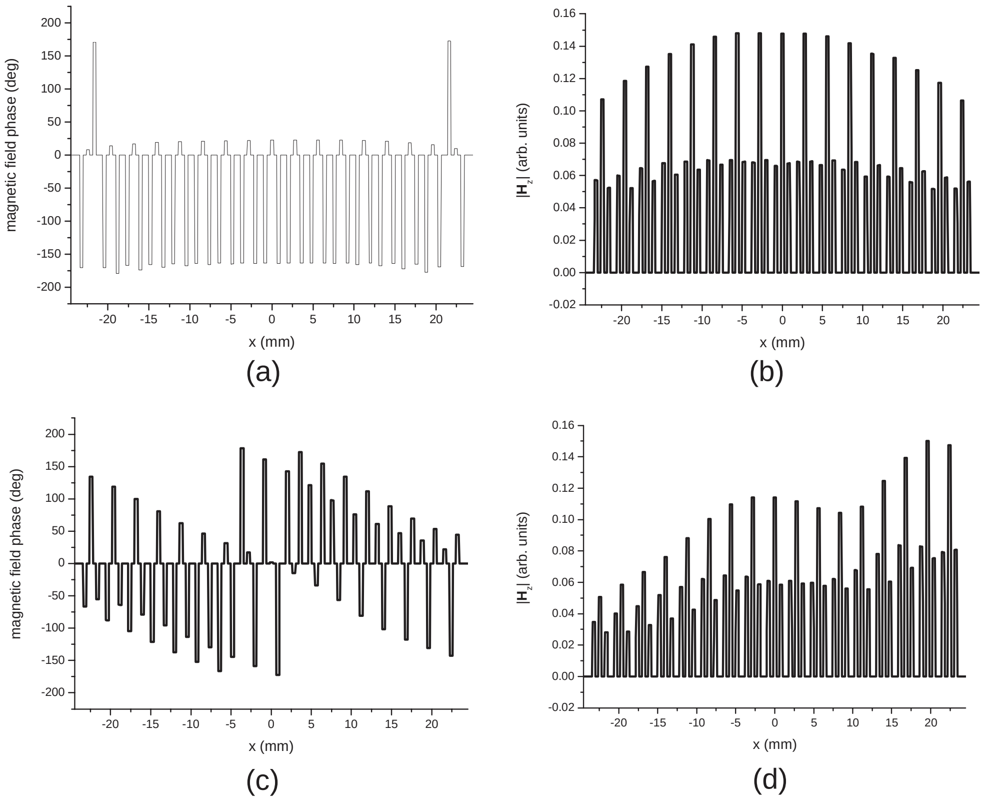

Fig. 4. Magnetic field (phase and modulus) at the bottom of the grooves for two resonant situations: (a) Phase and (b) modulus for $\theta_0 = 0^\circ$ and $f = 59.67$ GHz; (c) phase and (d) modulus for $\theta_0 = 5^\circ$ and $f = 59.78$ GHz.

measurement system can produce additional resonant features in the reflectance, that should not appear under normal illumination. It is important to recall that in the reflection measurement the dynamic range and the sensitivity of the detection is much smaller than in the transmission measurement [23], due to the directional coupler used to direct the reflected Gaussian beam. Besides, as the incidence angle is increased, the experimental configuration might be misaligned and this implies an additional penalty in sensitivity of the measurement.

The results in Fig. 4 confirm the nature of the resonances reported in this paper. We plot the calculated magnetic field at the bottom of the grooves for the π resonance under normal incidence [$f = 59.67$ GHz, Figs. 4(a) and 4(b)], and for $\theta_0 = 5^\circ$ [$f = 59.78$ GHz, Figs. 4(c) and 4(d)]. For normal illumination, the magnetic field pattern is symmetric, as imposed by the incidence conditions. The phase exhibits the typical behaviour of the π-mode: opposite phases in adjacent grooves of the same period. This mode can be identified as (+ - +) [15]. At the same time, the internal field is intensified, mostly in the central grooves of each period. It is important to mention that in this resonant case the magnetic field within the grooves is an order of magnitude stronger than for other non-resonant frequencies (not shown) [14,32]. For oblique incidence, the internal field is no longer symmetric and different phases could be found in adjacent slits, which eventually may generate a phase resonance [31]. In particular for $f = 59.78$ GHz, the phase distribution within the grooves is nearly of the type (+ - +), which confirms that also for $\theta_0 = 5^\circ$ the π resonance is excited. The internal field is also enhanced in this case, especially in the central groove of each period. Different phase configurations are expected to appear for the other resonant frequencies, which, in turn, generate other phase resonances different from the π one, which has the highest quality factor among phase resonances. The quality of those other resonances is not as high as that of the π resonance, and the magnetic field within the grooves is not as strong as in the π-mode (not shown) [15]. The results reported here in the millimeter-wave regime are in qualitative agreement with previous measurements of phase resonances in the infrared region [19,25]. The π resonance (called symmetric and pseudo-symmetric mode in [19] and [25], respectively) emerges both at normal incidence and at 5 deg and has the strongest field enhancement inside the grooves, as stated in [19] and [25]. Also, the anti-symmetric mode only exists under oblique incidence, as it was already demonstrated in those works.

4. Conclusion

In conclusion, we have demonstrated experimentally the recent theoretical predictions of phase resonances in a dual-period reflection structure comprising three subwavelength grooves of rectangular cross section, in the millimeter wave regime. A copper sample was fabricated and its reflectance has been measured under normal and oblique incidence. It was shown that the reflected response exhibits minima that are associated with the excitation of phase resonances in the structure. In particular, for oblique incidence there appear additional resonant modes that cannot be excited under normal illumination. The experimental curves have been compared with numerical results obtained by a commercial software based upon finite integration time domain method, and a satisfactory agreement was obtained. In addition, aided by the software, we have been able to catch more details arising from the finiteness of the structure than those manifested on previously reported infinite compound grating analysis. We have shown that a groove width of roughly $\lambda/10$ is sufficiently subwavelength to excite phase resonances. This is an attractive fact if we think of applying this property for the design of selective devices. The results here presented confirm the common nature of π resonances of reflection gratings in frequency regimes where metals have very different behavior such as nearly perfect conductors as is the case in this paper or plasmonic models as in Refs. [19] and [25].

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