OWA Operators Based on Admissible Permutations

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***Abstract*—In this work we propose a new OWA operator defined on bounded convex posets of a vector-lattice. In order to overcome the non-existence of a total order, which is necessary to obtain a non-decreasing arrangement of the input data, we use the concept of admissible permutation. Based on it, our proposal calculates the different ways in which the input vector could be arranged, always respecting the partial order. For each admissible arrangement, we calculate an intermediate value which is finally collected and averaged by means of the arithmetic mean. We analyze several properties of this operator and we give some counterexamples of those properties of aggregation functions which are not satisfied.**

***Index Terms*—Aggregation functions, admissible order, admissible permutation, OWA operator.**

I. Introduction

Information fusion is a very important step in almost every real-world application, from computer vision [1], [2] to machine learning [3], [4], among others. In this type of applications, one must combine several sources of information (inputs), aggregate experts' opinions or merge different outputs in order to give a single response.

From a theoretical point of view, aggregation functions (see [5]–[8]) are the most import mathematical tool to deal with information fusion. However, nowadays aggregation functions are being generalized by new functions satisfying only certain properties of aggregation functions (see, for example pre-aggregation functions [3]) or extended to deal with information coming from more complex structures than the usual unit interval $[0, 1]$ (see, for example [9]).

In this work we investigate the definition of a new operator defined on some specific lattices where there does not necessarily exist a total order. Specifically, we focus on the definition of OWA operators, a family of aggregation functions that weights the set of inputs by means of their magnitude. Therefore, the problem arises in determining which is the the largest input, the second largest, and so on, in a structure where a total order may not exist.

In the literature, we can find several approaches that deal with the same idea but with different approximations to the problem. For example, in [10], OWA operators are defined in lattices endowed with a t-norm and a t-conorm. This approach transforms the input vector into a new input vector that forms a chain, so it can be arranged. These operators were generalized in [11] and have been also studied in [12], [13]. Another approach, which is the inspiration of our work, is the one given in [14]. In this new definition, OWA operators are defined associated with an admissible order (see also [15]), a linear order that refines the given partial order.

Unlike the proposal given in [14], which is based on the election of a specific admissible order, in this work we propose to analyze, for each input vector (fixed-size sample of elements from the set), in how many different ways we are able to arrange its elements with only one restriction: the partial order must be always kept. Based on each possible way of arranging the input vector, that we will call admissible permutation, we will generate an intermediate OWA result. Finally, the set of intermediate outputs, each one generated by means of a specific admissible permutation, will be collected into a single fused value by means of the arithmetic mean.

In this paper, we analyze the main properties of the OWA operator based on admissible permutations, such as monotonicity, boundedness, among others, and we analyze whether we recover special cases of OWA operators such as the maximum or the minimum when we consider certain weighting vectors. Moreover, we also investigate some transformations of the input vector that allow us to better understand the mechanisms of our operator.

The structure of this work is as follows: in Section 2 we

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recall some preliminary definitions. In Section 3 we propose the OWA operator based on admissible permutations and we study its main properties. We finish this work with some conclusions, remarks and future lines of research.

II. Preliminaries

We start recalling the concept of an aggregation function defined on a partially ordered set.

Definition 1: [16] Let $(L, \leq_L)$ be a bounded partially ordered set with a least element 0_L and a greatest element 1_L. A mapping $M : L^n \to L$ is an n-ary aggregation function if it satisfies the properties:

(i) $M(0_L, \ldots, 0_L) = 0_L$ and $M(1_L, \ldots, 1_L) = 1_L$;
(ii) it is increasing in each argument, i.e., for all $(x_1, \ldots, x_n), (y_1, \ldots, y_n) \in L^n$, $M(x_1, \ldots, x_n) \leq_L M(y_1, \ldots, y_n)$ whenever $x_1 \leq_L y_1, \ldots, x_n \leq_L y_n$.

In this work we will focus on bounded convex sublattices of vector-lattices (also called Riesz spaces [17]). Therefore, we have that any convex combination of elements of the set belongs to the set. Namely, for any $(x_1, \ldots, x_n) \in L^n$ and any $(w_1, \ldots, w_n) \in [0, 1]^n$ with $w_1 + \cdots + w_n = 1$, we have that $w_1 x_1 + \cdots + w_n x_n \in L$. This is due to the existence and properties of the operations inherited from vector-lattices

A. Admissible orders

We recall the concept of admissible order, a concept which is related to the concept of admissible permutation used in this work.

Definition 2: [15] Consider the partially order set $(L, \leq_L)$. The order $\preceq$ on L is called an admissible order if

(i) $\preceq$ is a linear order on L;
(ii) for all $x, y \in L$, $x \preceq y$ whenever $x \leq_L y$.

In [14] admissible orders are used to define interval-valued OWA operators where $L = L([0, 1])$ is the set of all closed subintervals of the unit interval $[0, 1]$.

Definition 3: Let $\preceq$ be an admissible order on $L([0, 1])$, and $\mathbf{w} = (w_1, \ldots, w_n) \in [0, 1]^n$, $w_1 + \cdots + w_n = 1$, a weighting vector. An Interval-Valued OWA (IVOWA) operator associated with $\preceq$, and $\mathbf{w}$ is a mapping $IVOWA^{\preceq}_{\mathbf{w}} : (L([0, 1]))^n \to L([0, 1])$ defined by

$$IVOWA^{\preceq}_{\mathbf{w}}(x_1, \ldots, x_n) = \sum_{i=1}^{n} w_i x_{(i)}$$

where $x_{(i)}$, $i = 1, \ldots, n$, denotes the ith greatest interval of the input intervals with respect to $\preceq$.

Observe that, even if $(L([0, 1]), \leq_L)$ is a poset and $\leq_L$ is a partial order, once we define an admissible order, we are able to arrange the input vector according to that specific admissible order.

III. OWA operators based on admissible permutations

In this section we propose the definition of OWA operators based on the idea of admissible permutation. As we have stated in the introduction, given an input vector of fixed (and finite) length, an admissible permutation represents a possible arrangement of the input vector in such a way that the partial order is respected. We formalize this concept in the following definition (see also [18]).

Definition 4: Let $(L, \leq_L)$ be a poset and let $x_1, \ldots, x_n \in L$. A permutation $\sigma : \{1, \ldots, n\} \to \{1, \ldots, n\}$ of $x_1, \ldots, x_n$ is said to be an admissible permutation with respect to the partial order $\leq_L$ if

(i) for every $x_i <_L x_j$, we have that $\sigma^{-1}(i) < \sigma^{-1}(j)$ and
(ii) for each x_i, the set $\{\sigma^{-1}(j) | j \in \{1, \ldots, n\}$ with $x_i = x_j\}$ is an interval in $\mathbb{N}$.

Remark: Notice that the concept of admissible permutation is similar to the concept of labeling of a poset: if $(L, \leq_L)$, $|L| = n$ is a poset, a labeling is a mapping $u : (L, \leq_L) \to \{1, \ldots, n\}$ such that for any $x, y \in L$, $x <_L y$ it holds that $u(x) < u(y)$ (see, for example [19]).

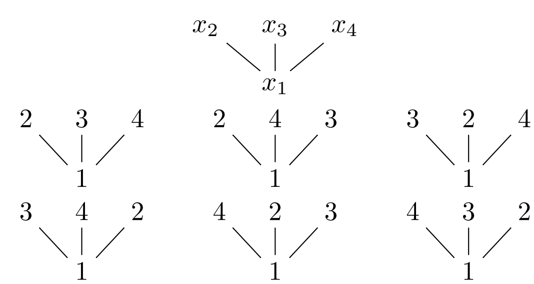
Example 1: Let $L = [-1, 1]^2$ and $\leq_L$ be the lexicographic order defined in the following way: for any $x_1 = (x_{11}, x_{12}), x_2 = (x_{21}, x_{22}) \in L$, $x_1 \leq_L x_2$ if and only if

$$x_{11} \leq x_{21} \text{ and } x_{12} \leq x_{22}.$$

Let $n = 4$ and $x_1 = (-0.2, -0.2), x_2 = (-0.1, 0.3), x_3 = (0.1, 0.1), x_4 = (0.2, 0.0)$. Observe that $x_1 <_L x_i, i = 2, 3, 4$ but no other relation can be established among the rest of elements. Then, there exist six admissible permutations of $x_1, \ldots, x_4$, namely $\sigma_1, \ldots, \sigma_6 : \{1, \ldots, 4\} \to \{1, \ldots, 4\}$ that generate the following arrangements:

$$\begin{array}{ll} \sigma_1: & x_1 \prec x_2 \prec x_3 \prec x_4 \\ \sigma_2: & x_1 \prec x_2 \prec x_4 \prec x_3 \\ \sigma_3: & x_1 \prec x_3 \prec x_2 \prec x_4 \\ \sigma_4: & x_1 \prec x_3 \prec x_4 \prec x_2 \\ \sigma_5: & x_1 \prec x_4 \prec x_2 \prec x_3 \\ \sigma_6: & x_1 \prec x_4 \prec x_3 \prec x_2 \end{array}$$

We next show the Hasse diagram of the elements and the labelings of each element on each admissible permutation:



Using the concept of admissible permutation, the OWA operator that we propose is based on the following steps:

- Given a weighting vector $\mathbf{w}$ and an input vector $\mathbf{x} = (x_1, \ldots, x_n) \in L^n$, we calculate the set of all admissible permutations of $\mathbf{x}$. Recall that the number of admissible permutations may vary from 1 to $n!$. We denote this set as $\Sigma = \{\sigma_1, \ldots, \sigma_p : \{1, \ldots, n\} \to \{1, \ldots, n\}\}$, being $1 \leq p \leq n!$;

- For each admissible permutation σ_j, $j = 1, \ldots, p$, we calculate an intermediate result $OWA_{\mathbf{w}}^{\sigma_j}$ which is based on the specific arrangement generated by σ_j:

$$OWA_{\mathbf{w}}^{\sigma_j}(x_1, \ldots, x_n) = \sum_{i=1}^{n} w_i x_{\sigma_j(n-i+1)},$$

where $x_{\sigma_j(1)} \preceq \cdots \preceq x_{\sigma_j(n)}$. Notice that this intermediate OWA is very similar to the one given in Definition 3, unlike the fact that the arrangement is induced by an admissible permutation and not by an admissible order.
- Finally, we fuse each intermediate value $OWA_{\mathbf{w}}^{\sigma_j}(x_1, \ldots, x_n)$ via the arithmetic mean to obtain the final result $OWA_{\mathbf{w}}^{\Sigma}$.

The formalization of these ideas is given in the following definition.

Definition 5: Let $(L, \leq_L)$ be a bounded convex poset of a vector-lattice and $\mathbf{w} = (w_1, \ldots, w_n) \in [0,1]^n$, $w_1 + \cdots + w_n = 1$, a weighting vector. For each $\mathbf{x} = (x_1, \ldots, x_n) \in L^n$, let $\Sigma = \{\sigma_1, \ldots, \sigma_p : \{1, \ldots, n\} \to \{1, \ldots, n\}\}$ be the set of admissible permutations of $\mathbf{x}$. The OWA operator based on admissible permutations $OWA_{\mathbf{w}}^{\Sigma}$ is a mapping $OWA_{\mathbf{w}}^{\Sigma} : L^n \to L$ given by

$$OWA_{\mathbf{w}}^{\Sigma}(x_1, \ldots, x_p) = \frac{1}{p} \sum_{j=1}^{p} OWA_{\mathbf{w}}^{\sigma_j}(x_1, \ldots, x_n)$$

where

$$OWA_{\mathbf{w}}^{\sigma_j}(x_1, \ldots, x_n) = \sum_{i=1}^{n} w_i x_{\sigma_j(n-i+1)}.$$

Notice that Definition 5 recovers the classical definition of OWA operator if $L = [0,1]$ and $\leq_L$ is the usual total order between real numbers. Recall that, even if L is a chain, the cardinality of the set of admissible permutations of any input vector $\mathbf{x}$ need not be one. This can be easily seen when $x_1 = \cdots = x_n$, since we have $n!$ different arrangements, although all of them yield the same result. Definition 5 is well defined even if there exist indistinguishable elements in the input vector.

Moreover, our definition of OWA operator satisfies idempotence and, as a consequence, it satisfies boundary conditions of aggregation functions. However, we will see that it fails in satisfying the monotonicity property with respect to the partial order $\leq_L$.

Proposition 1: Let $(L, \leq_L)$ be a bounded convex poset of a vector-lattice and let $OWA_{\mathbf{w}}^{\Sigma} : L^n \to L$ be the OWA operator based on admissible permutations. Then,

$$OWA_{\mathbf{w}}^{\Sigma}(x, \ldots, x) = x$$

holds for every $x \in L$.

Proof: Given the input vector $\mathbf{x} = (x, \ldots, x)$ for some $x \in L$, observe that $n!$ admissible permutations exist. Since $\mathbf{w}$ is a weighting vector, we have that $OWA_{\mathbf{w}}^{\sigma_j}(x, \ldots, x) = x$ for every $j = 1, \ldots, n!$ and due to the idempotence of the arithmetic mean the result follows. ■

Remark: Evidently, we have that $OWA_{\mathbf{w}}^{\Sigma}(0_L, \ldots, 0_L) = 0_L$ and $OWA_{\mathbf{w}}^{\Sigma}(1_L, \ldots, 1_L) = 1_L$ for any weighting vector $\mathbf{w}$.

Now, we show an example where the monotonicity with respect to $\leq_L$ of the OWA operator fails.

Example 2: Let $L = L([0,1])$ be the set of all closed subintervals of the unit interval, i.e.,

$$L([0,1]) = \{[\underline{x}, \overline{x}] | 0 \leq \underline{x} \leq \overline{x} \leq 1]\}$$

and $\leq_L$ be the partial given in the following way: for any $x = [\underline{x}, \overline{x}], y = [\underline{y}, \overline{y}] \in L$, $x \leq_L y$ if and only if

$$\underline{x} \leq \underline{y} \text{ and } \overline{x} \leq \overline{y}.$$

Let $\mathbf{x} = ([0.1, 0.8], [0.2, 0.7])$ and $\mathbf{w} = (0, 1)$. Now, let $\mathbf{x}' = ([0.1, 0.8], [0.2, 0.9])$. We have that

$$OWA_{\mathbf{w}}^{\Sigma}(\mathbf{x}) = [0.15, 0.75],$$

$$OWA_{\mathbf{w}}^{\Sigma}(\mathbf{x}') = [0.1, 0.8],$$

so $OWA_{\mathbf{w}}^{\Sigma}(\mathbf{x}) \not\leq_L OWA_{\mathbf{w}}^{\Sigma}(\mathbf{x}')$ although $\mathbf{x} <_L \mathbf{x}'$, so monotonicity fails.

Finally, we are also interested in analyzing whether OWA operators based on admissible permutations recover classical aggregation functions, such as the minimum or maximum operator when we consider some specific weighting vectors.

Proposition 2: Let $(L, \leq_L)$ be a bounded convex poset of a vector-lattice and $\mathbf{w}^* = (1, \ldots, 0)$, $\mathbf{w}_* = (0, \ldots, 1)$ be weighting vector. Given $\mathbf{x} \in L^n$ and Σ its set of admissible permutations, if $x_{\sigma_j(1)} \leq_L x_{\sigma_j(k)} \leq_L x_{\sigma_j(n)}$ holds for every $k \in \{2, \ldots, n-1\}$ and every $\sigma_j \in \Sigma$, then

$$OWA_{\mathbf{w}^*}^{\Sigma}(\mathbf{x}) = \sup\{\mathbf{x}\},$$

$$OWA_{\mathbf{w}_*}^{\Sigma}(\mathbf{x}) = \inf\{\mathbf{x}\}.$$

Corollary 1: If $\mathbf{x} \in L^n$ forms a chain, then

$$OWA_{\mathbf{w}^*}^{\Sigma}(\mathbf{x}) = \sup\{\mathbf{x}\},$$

$$OWA_{\mathbf{w}_*}^{\Sigma}(\mathbf{x}) = \inf\{\mathbf{x}\}$$

hold.

Example 3: let $L = L([0,1])$ with $\leq_L$ as in Example 2, and let $x_1 = [0, 0]$, $x_2 = [0.2, 0.8]$ $x_3 = [0.3, 0.5]$. It is clear that $\{x_1, x_2, x_3\}$ does not form a chain and there exist two admissible permutations satisfying $x_{\sigma_1(1)} = x_{\sigma_2(1)}$, so we have that

$$OWA_{\mathbf{w}_*}^{\Sigma}(x_1, x_2, x_3) = x_1 = \inf\{x_1, x_2, x_3\},$$

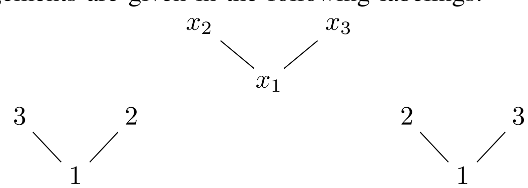
while

$$OWA_{\mathbf{w}^*}^{\Sigma}(x_1, x_2, x_3) = \frac{1}{2} x_2 + \frac{1}{2} x_3 \neq \sup\{x_1, x_2, x_3\}.$$

This example, which illustrates Proposition 2, shows that it is not evident the use of the orness function (see [20], [21]) to determine how close (or far) the OWA operator based on admissible permutations is from the maximum operator. In fact, we can see that the orness cannot be derived only from the weighting vector, since if we take w_* there is no equivalence between the corresponding OWA operator and the minimum operator.

One of the main disadvantages of our proposal is the complexity for calculating the set of admissible permutations Σ of a given input vector. In fact, as long as incomparable elements appear in the input vector, the cardinality of Σ increases rapidly. Therefore, we are interested in studying ways of reducing such complexity. Although in this work we do not reach any improvement in the reduction of complexity, we have analyzed a transformation of the input vector $\mathbf{x}$ in a new input vector $\mathbf{y}$ such that $OWA^{\Sigma}_{\mathbf{w}}(\mathbf{x}) = OWA^{\Sigma}_{\mathbf{w}}(\mathbf{y})$.

Example 4: Let $L([0,1]), \leq_L$ be given as in Example 2 and let $x_1 = [0,0]$, $x_2 = [0,1]$ and $x_3 = [0.5, 0.5]$. There exist two admissible permutations σ_1, σ_2, whose corresponding arrangements are given in the following labelings:



The OWA operator based on admissible permutations is given by

$$OWA^{\Sigma}_{\mathbf{w}}(\mathbf{x}) = \frac{1}{2}\sum_{j=1}^{2}\left(\sum_{i=1}^{3} w_i x_{\sigma_j(4-i)}\right) =$$
$$\frac{1}{2}(w_1x_2 + w_2x_3 + w_3x_1 + w_1x_3 + w_2x_2 + w_3x_3) =$$
$$\frac{1}{2}((w_1+w_2)x_2 + (w_1+w_2)x_3 + 2w_3x_1)$$

Observe that we can rewrite this expression as

$$OWA^{\Sigma}_{\mathbf{w}}(\mathbf{x}) = \frac{1}{2}\left(w_1(x_2+x_3) + w_2(x_2+x_3) + w_3 2x_1\right). \tag{1}$$

Since $x_1 <_L \frac{x_2+x_3}{2}$, we have that

$$OWA^{\Sigma}_{\mathbf{w}}(\mathbf{x}) = OWA^{\Sigma}_{\mathbf{w}}\left(\frac{x_2+x_3}{2}, \frac{x_2+x_3}{2}, x_1\right). \tag{2}$$

This result is studied in the following propositions.

Proposition 3: Let $(L, \leq_L)$ be a bounded convex poset of a vector-lattice. For every $\mathbf{x} = (x_1, \ldots, x_n) \in L^n$, calculate the set of admissible permutations $\Sigma = \{\sigma_1, \ldots, \sigma_p\}$ and calculate

$$y_i = \frac{1}{p}\sum_{j=1}^{p} x_{\sigma_j(i)}, \quad i = 1, \ldots, n.$$

Under these conditions, $y_i \leq_L y_j$ whenever $i \leq j$.

Proof: Suppose that, for each $i, j \in \{1, \ldots, n\}$, $i < j$ and every admissible permutation $\sigma_k \in \Sigma$ we have $x_{\sigma_k(i)} \leq_L x_{\sigma_k(j)}$. Then, the result hold immediately. If the condition does not hold, then for each permutation k' where $x_{\sigma_{k'}(i)} \prec x_{\sigma_{k'}(j)}$ there exits another permutation k'' where $x_{\sigma_{k''}(j)} \prec x_{\sigma_{k''}(i)}$, and the result follows. ■

Proposition 4: Let $(L, \leq_L)$ be a bounded convex poset of a vector-lattice and $\mathbf{w}$ a weighting vector. Then, for each input vector $\mathbf{x} \in L^n$, we have that

$$OWA^{\Sigma}_{\mathbf{w}}(\mathbf{x}) = OWA^{\Sigma}_{\mathbf{w}}(\mathbf{y})$$

where $\mathbf{y} \in L^n$ is obtained from $\mathbf{x}$ as in Proposition 3.

Example 5: Following with Example 4, we have that

$$\mathbf{x} = ([0.0, 0.0], [0.0, 1.0], [0.5, 0.5])$$

and the transformed vector is given by

$$\mathbf{y} = ([0.0, 0.0], [0.25, 0.75], [0.25, 0.75]).$$

Under this condition, $OWA^{\Sigma}_{\mathbf{w}}(\mathbf{x}) = OWA^{\Sigma}_{\mathbf{w}}(\mathbf{y})$ for every weighting vector $\mathbf{w}$.

From these propositions we conclude that although the transformed vector $\mathbf{y}$ is more simple than $\mathbf{x}$ in terms of order structure (the elements of $\mathbf{y}$ always form a chain), one needs to calculate the set of admissible permutations of $\mathbf{x}$ in order to obtain $\mathbf{y}$, so the complexity remains. However, if we are able to find some similar transformation that does not require the calculation of Σ, we can reduce the complexity of OWA operators based on admissible permutations.

IV. Conclusions

In this work we have presented the definition of a new OWA operator on bounded convex posets of vector-lattices. The proposed operator calculates the set of admissible permutation of a given input vector and aggregates intermediate results based on each possible arrangement. Although the operator satisfies some usual properties of aggregation functions, it fails in the satisfaction of monotonicity, so it cannot be properly considered as an aggregation functions.

Although the complexity is one of the main drawbacks when calculating the set of admissible permutations of a given input vector, our idea is to continue studying some simplifications that allow us to reduce the time complexity. In this sense, this paper does not propose any full idea but it opens the possibility of studying transformations of the input vector to new vectors whose ordinal structure is more simple.

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