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Numerical evaluation of Airy-type integrals arising in uniform asymptotic analysis.

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We consider the numerical evaluation of integrals of the form

$$F(z) = \frac{1}{2\pi i} \int_{\mathcal{C}} e^{\frac{1}{3}r^3 - zr} f(r)\, dr, \quad z \in \mathbb{R}.$$

The contour $\mathcal{C}$ runs from $\infty e^{-\frac{1}{3}\pi i}$ to $\infty e^{+\frac{1}{3}\pi i}$. The function $f(r)$ is assumed to be analytic in a neighborhood of the contour $\mathcal{C}$. When we take $f(r) = 1$ the integral becomes the Airy function: $F(z) = \mathrm{Ai}(z)$.

The considered integral arises in asymptotic analysis when two saddle points are close together, or coalesce. These integrals occur frequently in problems from physics, but also in the asymptotic analysis of special functions. Examples are Bessel functions and the classical orthogonal polynomials.

In a recent paper [1], the approach is based on Gauss quadrature on the contour $\mathcal{C}$ with polynomials that are orthogonal on this contour. The Gauss quadrature requires the computation of the zeros of special polynomials and of their moments. In this talk we describe a simpler quadrature method, in fact, the trapezoidal rule, which appears to be very efficient on the saddle point contour, and on a slightly shifted one.

References

[1] Daan Huybrechs, Arno B.J. Kuijlaars and Nele Lejon, A numerical method for oscillatory integrals with coalescing saddle points, preprint, arXiv: 1806.06549 (2018).

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