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Department of Statistics, Computer Science, and Mathematics

PhD Thesis

Univariate and multivariate spatio-temporal areal models to study crimes against women

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Introduction

Spatio-temporal models for areal data have been extensively applied in epidemiology and public health to study geographical and temporal patterns of incidence or mortality of several diseases, mainly cancer. The utility of these models has become crucial in public health, and methodological research has evolved in line with the necessity of analyzing the increasingly more complex data registers. However, these techniques have not been used to study crimes against women, a complex and intricate problem where risk factors are not clearly identified. This dissertation is aimed at improving and developing methodology to disentangle the phenomenon of crimes against women in general and in India in particular. The dissertation pursues the following main goals.

The first objective is to put the focus on the terrible problem of crimes against women in India and to revise statistical methods that could be useful to understand the dynamics of the phenomenon. In Chapter 1, a general introduction about crimes against women in general and about particular forms of crimes in India is provided to understand the relevance of the problem. In this chapter, we also review some spatio-temporal models used in disease mapping that can be a valuable tool to study violence on women.

The second objective of this dissertation is to compare some of the existing spatial priors, analyzing their induced correlation structure and their impact on the final risk estimates. A thourough inspection is carried out in Chapter 2. The models will be used to study the spatio-temporal patterns of dowry deaths, a form of crime against women very specific to India, in the state of Uttar Pradesh, the most populated Indian state. Additionally, some spatio-temporal covariates have been included in the model to assess their relationship with dowry deaths.

The third objective of this dissertation is to extend a class of multivariate spatial models, known as M-based models (Botella-Rocamora et al., 2015) to the spatio-temporal setting. The novel proposals presented in Chapter 3 take into account the correlations between the spatial and temporal patterns of the phenomena under study, which may suggest connections between the different crimes that will certainly benefit a comprehensive understanding of the problem. A key objective of this chapter is the implementation of these models in a fully Bayesian context using integrated nested Laplace approximations, a procedure known as INLA in short, (Rue et al., 2009). Additionally, a comparison of the results obtained using INLA and Markov chain Monte Carlo (MCMC) techniques is performed through the joint analysis of dowry deaths and rapes in Uttar Pradesh.

The fourth objective of this dissertation is to broaden the class of existing multivariate spatio-temporal models, beyond multivariate CAR priors. In particular, in Chapter 4, we propose multivariate P-spline models to discover the spatio-temporal evolution of different crimes. P-spline models have been used in univariate disease mapping (Ugarte et al., 2010, 2017) with promising results, but they have not been extended to the multivariate setting yet. The implementation of these models in INLA is also a central goal of this chapter. The methodology is used to analyze four different crimes against women in the Indian state of Maharashtra. Namely, rape, assault or criminal force to woman with intent to outrage her modesty, cruelty by husband or husband's relatives, and kidnapping and abduction.

The fifth objective is transversal to all chapters. We have a strong commitment with reproducibility, and the code developed in this dissertation is publicly available at the GitHub of our research group. (`https://github.com/spatialstatisticsupna`)

The dissertation is closed with the main conclusions and further work.

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1

Gender-based violence and spatio-temporal areal models

1.1 Introduction

Women are one of the most vulnerable groups in terms of violence (Powell and Wahidin, 2007), and violence against them is considered at present, as one of the most widespread violations of human rights, trascending the boundaries of culture, race, age and religion. Such violence is an almost universal phenomenon and a public health problem (Rose, 2012), which can develop in the public or private sphere, and includes physical and psychological abuse with direct consequences for physical, sexual and emotional health. However, years ago violence against women was not perceived as a matter of international concern and research on gender violence was scarce, to such an extent that Mukherjee et al. (2001) claimed that social scientists had neglected the study of crimes against women. Fortunately, change is actually taking place, mainly due to the emergence of organized women's groups both locally and internationally to demand attention to the physical and psychological abuse of women, and to ensure that gender issues are included in the political agenda (Heise et al., 1994; Niaz, 2003; Ellsberg and Heise, 2005). The news that we receive practically every day about crimes committed against women, together with the work of these groupings, began to raise society's awareness of this terrible problem.

United Nations General Assembly (1993) defined the term *violence against women* (VAW) as any act of *gender violence* that results in physical, sexual or psychological harm to women. The same year, the World Conference on Human Rights in Vienna recognized for the first time *gender violence* as a violation of human rights. In 1996, the World Health Assembly declared VAW as a serious public health problem, and as a result, the World Health Organization in 2002 published the first World Report on Violence and Health. The WHO subsequently stated that violence against women is a *"a public health problem of epidemic proportions that requires immediate action"* (Ellsberg and Heise, 2005) that takes the lives of more than 1.6 million women due to domestic violence. Years later, the WHO (2013) also estimated that the global lifetime prevalence of intimate partner violence among ever-partnered women is 30.0% (with a 95% confidence interval of 27.8% to 32.2%), but this rate is higher in South-East Asia with an estimate of 37.7% (with a 95% confidence interval of 32.8% to 42.6%).

Though crimes against women are found all over the world, there are countries where this issue is particularly worrying for three main reasons: the large number of affected women, the nature of certain forms of crimes, and the social acceptability of violence against women. This is the case of India, one of the most populated countries in the world (around 1380 million people in 2020 according to world population prospects made by United Nations (2019) of which 663 million are women approximately), where gender violence is deeply entrenched in society with very specific forms of crimes against women.

1.2 Crimes against women in India

Crimes against women (CAW) vary according to the country's culture (Kishor and Johnson, 2005) and there are places where this problem is alarming. In South Asian countries, and particularly in India, gender-based violence is deeply institutionalized and it develops under the mantle of religious, cultural or social practices (Johnson et al., 2007). In India the long standing power imbalance between men and women contributes to legitimizing violence against them, especially sexual violence (see Gupta et al., 2004; Rahman and Rao, 2004; Russo and Pirlott, 2006; Kaur, 2011; Kohli, 2012; Solotaroff and Pande, 2014), and this kind of violence is considered necessary for the preservation of the patriarchal structure of society (Watts and Zimmerman, 2002). Indian sex-gender system operates exposing girls and young women to different forms of violence including selective abortion and infanticide practices, harassment, rape, kidnapping, dowry homicide, and murder, that preclude them from having a dignified life (Patel, 2015).

Violence against women begins early in life in India through gender bias. South Asia has the highest levels of excess female infant mortality in the world with India being the country with the highest female infant mortality among all countries for which data are available, (Solotaroff and Pande, 2014). Numerous studies agree that the high infant mortality rate is a manifestation of a patriarchal environment with a strong preference for boys, see for example Drèze and Khera (2000); Ghansham (2002); Watts and Zimmerman (2002); Banerjee (2014). High rates of female mortality are the result of selective abortion by sex, female infanticide and systematic neglect of girls' health and nutrition needs, (Das Gupta and Mari Bhat, 1997; Arnold et al.,

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1998; Watts and Zimmerman, 2002; Miller et al., 2011). According to Ghansham (2002); Amin and Khondoker (2004); and Singh (2012), in patriarchal societies, girls are prevented from receiving immunizations and treatment for childhood diseases. In addition, as Banerjee (2014) states, they are denied the right to adequate nutrition. This gender bias has decimated the female population in a worrying way, to such an extent that India occupies the position 189 (of 201) in terms of the number of women per 100 men, (United Nations, 2019). The dangers women face in such an unbalanced society are terrifying.

Although women can be victims of any of the general crimes, the Indian Penal Code (IPC) only characterizes gender-specific crimes as "crimes against women (CAW)". Namely, *Cruelty by husband or relatives of husband, Rape, Kidnapping and Abduction of Women, Dowry Deaths, Assault or Criminal Force to Woman with Intent to Outrage Her Modesty, Insult to the Modesty of Women, Importation of Girl from Foreign Country* and *Abetment of Suicide of Women*. In this dissertation some of these crimes are analyzed and they are detailed below briefly [1]. Data on the number of crimes against women used in this dissertation have been obtained from the Indian National Crime Records Bureau, (`https://ncrb.gov.in/crime-in-india`), the governmental agency responsible for collecting data on crimes defined by the IPC and Special & Local Laws.

The crime *Kidnapping and abduction* is defined in sections 364, 364A and 366 of the IPC, and comprises kidnapping in order to murder, kidnapping for ransom, or kidnapping to compel a woman to marry a person against her will.

Violence in the marital sphere is framed, in the IPC, within *Cruelty by husband or relatives of husband*, and refers to "*any conduct to drive the woman to commit suicide or to cause grave injury or danger to life, limb or health (whether mental or physical) of the woman*". It also includes "*harassment with the intention of coercing her or any person related to her to meet any unlawful demand for any property or valuable security...*". There are numerous studies in different countries that state that the abuse of women by their intimate partner is the most endemic form of CAW, and at the same time has one of the highest rates of underreporting, (see Jejeebhoy, 1998; Visaria, 1999; Heise et al., 1994, and references therein).

Sexual violence affects 27.5 million women, and the reported number of rapes is increasing every year becoming a major issue (Raj and McDougal, 2014). The promotion of patriarchal ideologies by political, community and religious leaders has led to rape culture in India, (Kohli, 2012). According to the IPC, *Rape* is committed if a "*man penetrates his penis, to any extent, into the vagina, mouth, urethra or anus of a woman or makes her to do so with him or any other person*" (see the IPC

[1]For more information and details about the definition of these crimes, the reader is referred to the Indian Penal Code sections 304B for dowry deaths, 375 for rape, 354 (A, B, C, D) for assault, 498A for cruelty, and sections 364, 364A, and 366 for kidnapping and abduction (`http://legislative.gov.in/sites/default/files/A1860-45.pdf`).

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for a more complete definition of Rape), and *Assault or criminal force to women to outrage her modesty* includes sexual harassment, use of criminal force to a woman with intent to disrobe, voyeurism, and stalking.

Studies have shown that most rape and sexual violence are perpetrated by individuals known to the victim, such as the couple, male family members and known individuals in positions of authority (see Heise et al., 1994; Watts and Zimmerman, 2002; Kaur, 2011). Moreover, according to Jewkes et al. (2013), rapes and sexual aggressions are much more frequent within marriage, at the hands of her husband, than outside of it. Around the world, sexual assaults by strangers are recognized as crimes, however, rape and sexual assault in marriage are socially tolerated in many countries. The almost universal stigma surrounding sexual assault, rape and other sexual crimes, coupled with the fact that marital sexual violence is not seen as a crime in India, causes sexual crimes to be noticeably less denounced.

Besides traditional forms of violence, in India women are exposed to crimes associated with dowry [2]. Haveripeth (2013) defines dowry as any form of wealth (property, money, or ornaments) that a man or his family receives from his wife or family at the time of marriage. Different authors consider that due to social heritage and traditional mentality, the practice of dowry has become so deeply rooted in Indian culture that the legal provisions that prohibit this practice have failed in their attempt to eliminate or diminish it (see for example Parmar, 2014; Jeyaseelan et al., 2015; Haroon, 2017). In many cases, if the dowry is considered insufficient, the bride is harassed, abused and tortured. When this violence leads to death, it is called *dowry death*. The practice of dowry is universally widespread in India and it is present in all religions, socio-economic status, and educational levels despite de existence of laws that prohibit it.

In this dissertion we aim at identifying spatio-temporal patterns of CAW in the districts of some Indian states as well as at pinpointing high-risk districts within a given state. As far as we know, it is the first time researchers analyse CAW in India at this administrative level. Uncover spatial patterns and temporal trends can be crucial to identify underlying risk factors that help to understand the dynamic of this phenomenon, and to explore to what extent cases of CAW vary in different communities over time. That is why we are convinced that spatio-temporal models for areal data are an invaluable tool for studying and understanding crimes against women.

1.3 Spatio-temporal models for areal data

Areal data appear when the study region is divided into a subset of smaller domains or small areas in which the variable of interest is aggregated. A common example

[2] Dowry and dowry death are defined in detail in Chapter 2.

arises when studying the number of mortality or incidence cases of certain diseases. The statistical techniques for dealing with areal data have experienced a tremendous evolution since the nineties, and they have been mainly used to analyze chronic diseases like cancer. One of the reasons is that these techniques allow to estimate spatial and temporal patterns, something crucial to deep into the study of new plausible risk factors related to cancer, as known risk factors underlying most cancer types only explain a small percentage of cases.

The statistical methodology to study spatio-temporal patterns of a disease has been commonly known as spatio-temporal epidemiology or disease mapping (as the estimated risks/rates are usually represented in maps). In this dissertation we will use and also derive novel methodology to analyze areal data with a particular focus on studying crimes against women in India, a phenomenon where risk factors are not well stablished and understood as it is the case of certain cancer locations. The main goal of these techniques is smoothing standardized incidence/mortality ratios or crude rates to uncover geographical patterns and temporal trends of the phenomena under study. Traditional measures such as standardized mortality/incidence ratio or crude rates can be extremely variable for low populated areas or rare phenomena, and models are required to smooth the risk or rates reducing variability. These models *borrow information* from neighboring areas, nearby moments of time, or both, and generate estimates in which the risk variation in space and time describes smooth transitions. The research in spatial and spatio-temporal models for areal data is now rich and abundant. Here we make a revision of univariate models to analyze one single response, and multivariate models that allow the study of several outcomes establishing connections between them.

1.3.1 Univariate areal models

One of the most influential papers in the field of spatial areal models, and possibly the starting point of modern disease mapping, is that of Besag et al. (1991). These authors propose the so call convolution model for the area effect. Specifically, this model (named BYM hereafter in this dissertation) incorporates two spatial random effects to model spatially structured and unstructured heterogeneity. The first of these random effects accounts for spatial dependence through an intrinsic autoregressive conditional distribution, iCAR, (Besag, 1974). In contrast, the second random effect deals with spatially unstructured variability through an exchangeable Gaussian prior. However, the data can only inform on the sum of the two components, not about the two effects individually, and an identifiability issue arises (see, for example, Gelfand and Sahu 1999; MacNab 2011, or Banerjee et al. 2015, Chap. 6). Hence, other alternatives have been proposed to model both spatially structured and unstructured heterogeneity. Two of these proposals are the Leroux et al. (1999) model (LCAR) which is being widely used (e.g. Ugarte et al., 2014, 2016; Goicoa et al., 2016) and the model proposed by Dean et al. (2001) (DCAR), that has been revisited recently to construct the so-called PC priors (Riebler et al., 2016). The main characteristic of these proposals is that they consider only one spatial random effect and it is the covariance (or precision) matrix that separates the type of dependence. While the LCAR prior splits the precision matrix into two terms, one dealing with spatial dependence and the other one with unstructured heterogeneity, the DCAR prior splits the covariance matrix into the structured and unstructured parts.

The increasing availability of data over the years has caused the merely spatial approach to be limited on many occasions, giving way to a much richer class of spatio-temporal models. Spatio-temporal models *borrow information* from both neighboring areas and nearby time points. A first extension of the spatial BYM model was given by Bernardinelli et al. (1995) including linear temporal trends specific for each small area. However, this model could be inadequate and very restrictive in many cases, since temporal trends are not usually linear. A huge leap forward was made by Knorr-Held (2000). This author proposes a non-parametric modeling approach where conditional autoregressive (CAR) priors and random walks of first and second order are considered for the spatial and temporal random effects, respectively. In addition, to capture the specific temporal evolution of the incidence risk in each area, different types of interactions between the spatial and temporal terms are introduced. Namely, Type I interaction (independent effects), Type II interaction (effects structured in time and unstructured in space), Type III interaction (effects structured in space and unstructured in time), and Type IV interaction (effects structured in both space and time). These four types of interaction will be considered along this dissertation and will be explained in detail in the next chapter.

The work by Knorr-Held (2000) has been fundamental for the spatio-temporal modeling, but new methodological contributions have also been made. For example, Martínez-Beneito et al. (2008) combine a CAR model for space and an autoregressive model for time. A novel and different form of modelling spatio-temporal areal count data without relying on CAR prior is the use of P-splines for smoothing risks in space and time. MacNab (2007) proposes one dimensional P-splines to smooth temporal trends and space-time interactions allowing different spline coefficients for each area. Later, Ugarte et al. (2010) consider a pure interaction three-dimensional P-spline model, and Ugarte et al. (2012) extended this model including main effects and different smoothing parameters for the interaction. From then on, the research on P-spline models has been rich and fruitful. A recent paper by Goicoa et al. (2019a) considers age-space-time models models based on one and two-dimensional P-splines to study breast cancer mortality data in Spain during the period 1985-2010.

Although these models have experienced a great success to study incidence and mortality of different diseases, their use to study gender-based violence is still very scarce. Gracia et al. (2015) and Kelling et al. (2020) use spatial models to study intimate and domestic and sexual violence, respectively. As far as we now, the first attempts to unveil spatio-temporal patterns of crimes against women using spatio-temporal models for areal count data has been carried out by Vicente et al. (2018, 2020) and Goicoa et al. (2019b).

1.3.2 Multivariate areal models

Univariate models are powerful tools to unveil spatial and temporal patterns and to look into the underlying factors related to the crime under study. However modelling different crimes jointly can broaden the goals of the study as not only is precision improved, but also connections among the different crimes may be established. This is relevant in the research of crimes against women, an extraordinary complex problem in which the factors that may be related to the phenomenon are unclear. This problem is even more arduous in India, a country with a huge diverse of people, traditions, religious and social practices, where unravelling potential factors exerting some influence on violence against women is a challenge. A multivariate modelling of several crimes may be useful to provide correlations between the spatial and temporal patterns of the crimes that may reflect the association with common factors and the connections between them. This would help to narrow down the range of social, religious or economic characteristics that may be related to the crimes.

There is a considerable amount of theoretical research about multivariate spatial models for count data. However, their use is not widespread due to computational burden and a lack of ready to use software. Joint modelling has mainly relied on multivariate conditional autoregressive models (MCAR) within a fully Bayesian perspective. The seminal work by Mardia (1988) establishes a theoretical framework for multivariate CAR models (MCAR), extending the work of Besag (1974), and develops a theoretical framework based on multivariate conditionals. This multivariate approach considers spatial models with a separable covariance structure specified as the Kronecker product of two matrices: one to capture spatial dependence and the other to account for potential associations between the different responses. In this way, the same spatial smoothing is considered for all responses, which makes these models somewhat restrictive. Zhang et al. (2006) also use a separable spatial dependence structure, and propose an MCAR model whose precision matrix is the Kronecker product of three components: one that models the covariance structure of some demographic factors such as age group and sex, another that models the spatial correlation, and the third one modelling the temporal correlation. A non-separable multivariate proper CAR (pCAR) model with different smoothing parameters for each univariate response is given by Gelfand and Vounatsou (2003). This separable structure adds flexibility to the model as it allows different spatial smoothing for the responses. Later, Jin et al. (2005) propose a generalized MCAR (GMCAR) model for areal data, in which dependence between responses is induced conditionally. This

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conditional specification has the inconvenience of imposing a potentially arbitrary order on the responses, since different marginal distributions arise according to the order of responses when the sequence of conditional probabilities is defined. The same authors (Jin et al., 2007) solve this problem by formulating a linear model of coregionalization that avoids undesired dependence on the responses order. Generally, this proposal does not allow the incorporation of univariate spatial dependence structures beyond conditional autoregressive distributions, which turns out to be a limitation of this methodology. A multivariate generalization of spatial structures beyond conditional autoregressive distributions is proposed by MacNab (2011).

A general coregionalization framework for multivariate areal models that covers many of the proposals in the literature is derived by Martínez-Beneito (2013). This author provides an attractive procedure in which the spatial and the between-response dependence are including by pre and post multiplication of independent Gaussian random effects by appropriate matrices. However, like most multivariate areal models, this approach may seriously increase computational burden, which makes simultaneous modeling of a moderate to large number of responses unapproachable. Botella-Rocamora et al. (2015) present an interesting alternative to overcome this problem, the so-called M-based models. This approach is a reformulation of the Martínez-Beneito framework developing a simpler and computationally efficient technique that achieves a tradeoff between computational tractability and model's identification. Again, within the framework of linear coregionalization models, MacNab (2016a,b) presents a class of coregionalized multivariate conditional autoregressive models that allow flexible modeling of multivariate spatial interactions. A thorough review of the topic can be found in MacNab (2018), where the three main lines on the construction of multivariate proposals are discussed. Namely, the approach based on multivariate conditionals (Mardia, 1988), an approach based on univariate conditionals (Sain et al., 2011), and a coregionalization framework (Jin et al., 2007).

Most of the research on multivariate models for areal count data is focused in the spatial setting. In this dissertation, we model different crimes against women at the same time identifying correlations between the spatial and temporal patterns of the crimes. In particular, we will consider two approaches: an spatio-temporal extension of the M-based models (Chapter 3) and a multivariate approach based on spatial and temporal P-splines (Chapter 4).

1.3.3 Model fitting and inference

Model fitting and inference for spatial and spatio-temporal models for count data have been carried out within the general Bayesian framework following two main approaches: the empirical and the full Bayes approach. The empirical Bayes or classical approach provides point estimates and it relies on the penalized quasi-

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likelihood technique (see Breslow and Clayton, 1993). Though its use has been extensive in univariate modelling (see for instance Dean et al., 2004), it has not been popular in the multivariate framework. The full Bayes approach provides the posterior distribution of the quantities of interest, and it has traditionally based on Markov chain Monte Carlo (MCMC) techniques. This is the approach traditionally followed in multivariate spatial models for count data. However, MCMC can be computationally very demanding, particularly in multivariate settings, if the number of responses, areas, and time periods increase.

Recently, a new fitting technique for approximate Bayesian inference known as INLA in short, has been proposed (Rue et al., 2009). The method does not rely on simulations, but on an integrated nested Laplace approximations and numerical integration to compute the posterior marginals of the target parameters. This technique provides accurate inference and reduces computing time substantially in comparison to MCMC techniques. For theoretical details and nice applications of spatio-temporal models fitted with INLA, the reader is referred to the book by Blangiardo and Cameletti (2015). The paper by Ugarte et al. (2014) is the first reference on how to fit spatio-temporal models with a LCAR prior for the spatial effect and the four types of spatio-temporal interactions in INLA. One of the advantages of the INLA technique is that it is implemented in R through the R-INLA package (see Lindgren and Rue, 2015). The package allows to fit a wide range of models than can cope with many applications in practice. In this dissertation we will develop some functions to fit multivariate models in INLA that will be publicly available through the GitHub of our research group.

2

Unveiling spatial patterns and temporal trends of dowry deaths in the districts of Uttar Pradesh

2.1 Introduction

This chapter focuses on dowry death, a form of crime against women which is very specific to India. Even though this form of violence is included in the Indian Penal Code, and the Dowry Prohibition Act (1961) proscribes any form of dowry, it remains a widespread practice in India, a country with the largest number of dowry deaths in the world (8,455 deaths in 2014 according to NCRB, 2015). Dowry is defined in the Dowry Prohibition Act (1961) as *any property or valuable security given or agreed to be given either directly or indirectly by one party to a marriage to the other party to the marriage, or by the parents of either party to a marriage or by any other person to either party to the marriage or to any other person; at or before or any time after the marriage in connection with the marriage of said parities.*

Whereas modern adoption of a dowry as the preferred marriage payment or agreement is a significant issue to be addressed, a different but subsequent issue revolves around its inflation (Banerjee, 1999). Dowries replaced bride-price in the South (Bloch and Rao, 2002) but also became a cultural common practice transferred from high castes to middle and low castes throughout the country (Shenk, 2007). From a structural perspective, dowry system relates to kin and property as a constructed and deployed rule to preserve social and economic advantage of socially powerful groups (Banerjee, 1999). Some authors (see Shenk, 2007, and the references therein) explain that during the British Raj (1858-1947), weddings represented a large expenditure for peasants in Northwest India, leading to female infanticide in that region. In addition to that, others claim that dowry-related violence can be used as an instrument to redistribute resources (Bloch and Rao, 2002). Awarding a large dowry to the groom's family is a symbol of wealth and social status and historically, was devised as a form of inheritance for daughters in a culture where women were not allowed to own immovable property (see, for example, Banerjee, 2014). Hence a dowry's first purpose was to protect women from unfair traditions. However, it is unclear why it has become a means of extortion and an instrument of female exploitation.

Social Anthropologists might agree that marriage in India is a virtually universal institution, mainly monogamous and rather restrictive. While India is a secular state, 2011 Census showed nearly an 80% of Hindu population where social classification perpetuates through caste system. Assuming caste as a colonial translation for Hindu term varna (order, type, colour or class), marriage in India reproduces class segmentation safeguarding caste endogamy. Indian prevalent kinship system follows patrilocality, meaning a residence pattern where bride is required to migrate to the groom's clan location. Bride is consequently separated from her natal family at a very early age to enter a completely new scenario that eventually may lead to a greater subordination to her husband and in-laws. Indian Census 2001 shows female migrants representing 218 million against 91 million for male (Khan, 2015).

Marriage in India is ruled by caste endogamy but also requires clan or lineage exogamy, especially in North India where marriage becomes a complex arrangement since matches are not to be found within the surrounding community but far away from natal village where inner clan resides (Shenk, 2007; Prasad, 2016). For the case of Northwest India, where our study is located, it is common that bride's family pursues a groom of higher status, rank or age. This hypergamy, as anthropologists name it, might be considered either as cause or consequence of an even more restrictive marriage system. Some economists would speak on that in terms of marriage squeeze, as main cause for dowry inflation, subsequent extortion, violence against young married women and deaths (Dang et al., 2018; South et al., 2014). Inflation can be also related to dowry dynamics tending to what some authors have called the groom-price, meaning that when desirable eligible men are scarce the competition among brides becomes fierce (Bloch and Rao, 2002).

Shenk (2007) views dowry prohibition as problematic and maintains that a major cause in effectiveness of the Indian dowry policy may be found in the lack of understanding between those who view a dowry as a social evil and those who view it as a form of positive investment. According to Srinivasan and Lee (2004) "(...) the Indian dowry system should not be viewed simply as a traditional practice that will eventually be eliminated by processes of social change, but rather as an important component of a marriage system that is changing in response to a progressively more materialistic culture". Non-governmental organizations (NGO's) and academics also point out that the dowry system is related to discrimination against women leading to female infanticide and sex-selective abortion preventing female births (see Banerjee, 2014, and the references therein). Dowries are also related to violence

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against women by husbands and husbands' relatives, and this is a way of blackmailing the bride's family if the dowry seems unsatisfactory. Unfortunately, this form of violence against brides does not stop once the marriage has taken place but continues over time, leading to what is called a dowry death. 304B Indian Penal Code considers Dowry Death those young women deceased within seven years of marriage, either murdered or driven to suicide by the husband and in-laws to force an amplified dowry (Mohanty et al., 2013). A suicide committed by a woman who has suffered mental or physical violence in relation to her dowry is also regarded as a dowry death. The most common forms of dowry deaths are burning, drowning, and poisoning a bride as they might easily be considered as accidents (see Banerjee, 2014, and the references therein for more information about dowry and dowry violence).

Some studies have attempted to identify factors related to dowry deaths (see for example Jeyaseelan et al., 2015) but their conclusions are rather limited. Regarding sex ratio, female education, and their participation to the workforce, results are even contradictory (Dang et al., 2018; Belur et al., 2014; Sharma et al., 2002).

The chapter aims to help reveal spatial patterns for dowry death and how these patterns change over time, which in turn may be very helpful to identify unknown potential risk factors and to assess the effects of political or legal actions in time. Some district-level covariates have been considered to assess their relationship with dowry deaths. We also look into different spatial priors and their effects on final risk estimates. To our knowledge, no spatio-temporal analyses have been performed on dowry deaths at district level in India yet. Here, we focus specifically on Uttar Pradesh, the most populated Indian state, with the highest rate of death by dowry.

The rest of the chapter is organised as follows: Section 2.2 presents the data, provides descriptive figures, and discusses some covariates that may be associated to dowry deaths. Section 2.3 reviews the methodology to be used in the real data analysis. As different spatial priors are possible, we study carefully their induced correlation structures and their impact on the final risk estimates. Various priors for the hyperparameters are also revised. In Section 2.4, we provide the results of the real data analysis. A discussion is given in Section 2.5.

2.2 Dowry deaths in Uttar Pradesh

Dowry deaths are a deep-rooted form of violence against women in India. Here we focus on Uttar Pradesh, the most populous state in India, located in the north of the country. It borders Nepal and Uttarakhand to the North, Madhya Pradesh and Chhattisgarh to the South, Bihar and Jharkhand to the East, and Haryana, Delhi & NCR, and Rajasthan to the West. Nowadays, Uttar Pradesh is divided into 75 districts, but at the beginning of the study period there were 70 districts. We maintain the original 70 districts because data is not available for the current 75 districts during the years of the study. Figure 2.1 displays a map of India with Uttar Pradesh shaded (top right corner), and a map of Uttar Pradesh with the 70 districts (main central map). The corresponding names of the districts can be found in Table A.1 in Appendix A.

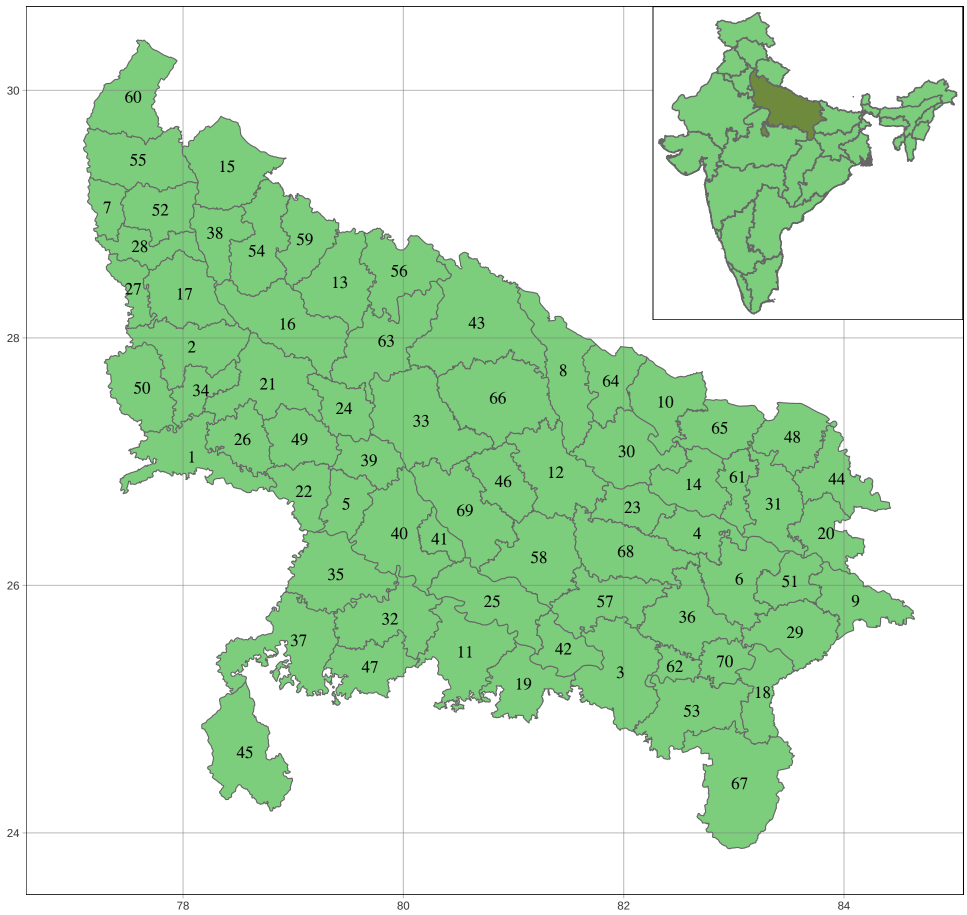

Figure 2.1: Map of India with Uttar Pradesh shaded (top right corner), and districts of Uttar Pradesh (main central map).

The total female population of Uttar Pradesh is 95,331,831 (data from 2011 census), but here we focus on 47,282,080 women between 15 and 49 years of age. In Uttar Pradesh, the male literacy rate is 77.28%, whereas for females the figure is notably lower, 57.18%. Dowry deaths figures for districts 24 (Farrukhabad) and 40 (Kanpur Dehat) are not available and consequently, they have been imputed from neighbouring areas. A total of 8,455 dowry deaths were registered during

2014 in India. This represents 2.50% of all crimes against women (CAW) in the country. The percentage of dowry deaths over the total CAW is three times higher in Uttar Pradesh, 6.42%, with 2,469 registered cases. Furthermore, the total number of dowry deaths in Uttar Pradesh in 2014 represents 29.20% of all dowry deaths in India. According to various studies, a serious problem here is incorrect reporting and classification of dowry deaths, from a criminal and forensic perspective, when dealing with unnatural deaths specially related to fire-related injuries and kitchen accidents. In popular discourse, “bride burning” and dowry death are synonymous (Sharma et al., 2002; Belur et al., 2014). Dang et al. (2018) argue that the police act as “death brokers” when using culturally appropriated scripts to classify unnatural death of female victims within seven years of marriage. Health care providers and police officers construct dowry deaths based on the perception of the victim and the need to protect themselves from accusations of incompetence and corruption (Belur et al., 2014). Other authors also highlight parents’ attitude towards dowry deaths, as on many occasions bride’s parents may be reluctant to report cases and to demand prosecution to avoid social discredit that would ruin the possibilities of marriage for other daughters (see Verma et al., 2015).

Figure 2.2 shows the temporal evolution of crude rates (per 100,000 women between 15 and 49 years old) in India (brown), Uttar Pradesh (green), and the districts of Uttar Pradesh (grey). Two districts are highlighted: Aligarth (orange) and Kheri (blue) showing increasing and decreasing trends respectively. The crude rates in Uttar Pradesh nearly double the crude rates in India, whereas the rates in Aligarth are about four times the crude rates in India from 2008 onward. Clearly, most of the districts present crude rates higher than global rates for the whole India.

Table 2.1 shows the minimum, first quartile, mean, standard deviation, third quartile, and maximum of the number of dowry deaths in the districts of Uttar Pradesh by year. The names of the districts with the minimum and maximum number of dowry deaths in each year are displayed in brackets. The number of registered deaths varies widely with minimum values between 1 and 7 and maximum values ranging from 55 to 98. In general the mean value increases from 2003 to 2008, and then it remains fairly stable until the end of the period when it shows a slight increase.

Clearly, crude rates (see Figure 2.2) are highly variable and models to smooth them are required. As the temporal evolution of rates by district is different, some trends increase (see for example Aligarh) whereas some others decrease (see for instance Kheri), models including space time interactions are considered.

The phenomenon of dowry deaths in India is complex and multifaceted, and identifying potential risk factors could help to understand and combat this form of violence against women. However, it is unclear which socio-economic indicators, or some other covariates may be associated with dowry deaths. South et al. (2014)

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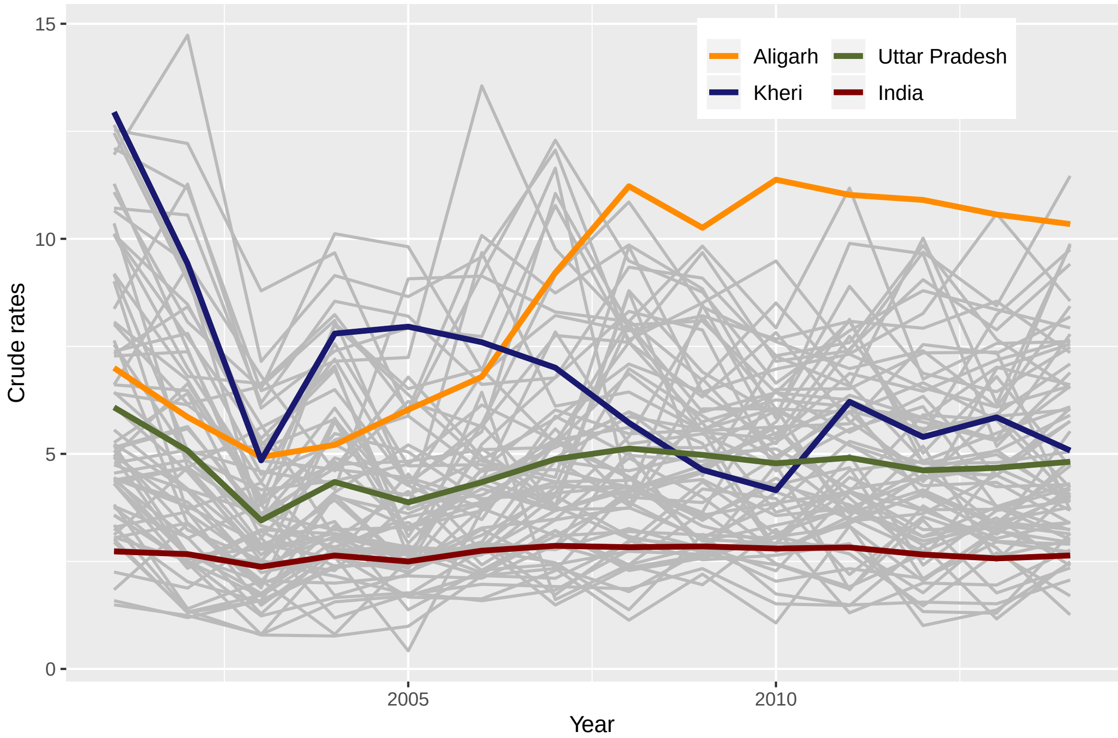

Figure 2.2: Evolution of crude rates in India (brown), Uttar Pradesh (green) and the districts Aligarh (orange) and Kheri (blue). The lines in grey represent the crude rates for the remaining districts of Uttar Pradesh.

find a surplus of male offenders correlated to gender violence. In the same way, and according to Mukherjee et al. (2001), overall crime is expected to be associated with crimes against women. Interestingly, these authors also find that the only crime that seems to be clearly associated with sex ratio (defined as female to male ratio) is dowry death, and the relationship is negative, that is, the lower the sex ratio, the higher the rate of dowry deaths. South et al. (2014); Barakade (2012) and More et al. (2012) also hypothesize a negative relationship between sex ratio with dowry extorsion and violence against women. Yet, Dang et al. (2018) find the opposite relationship when they approach marriage squeeze as one of the main factors that might be driving to dowry inflation in India, along with population growth and hypergamy. Literature also shows contradictory results on other potential risk factors, like female education and their participation on the workforce (Mukherjee et al., 2001; Dang et al., 2018; Belur et al., 2014; Sharma et al., 2002). As dowry deaths are usually registered as counts at area level (district, state), most of the covariates that have been contemplated to study this phenomenon are thought to operate at area level. In general, they are population-based variables available from the Indian

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Table 2.1: Descriptive statistics. Minimum, first quartile ($q_{.25}$), mean, standard deviation (sd), third quartile ($q_{.75}$), and maximum number of dowry deaths in the districts of Uttar Pradesh by year. The names of the districts with the minimum and maximum number of dowry deaths in each year are displayed in brackets.

Year	min	$q_{.25}$	mean	sd	$q_{.75}$	max
2001	4 (Shrawasti)	18.00	31.57	19.00	43.75	88 (Kheri)
2002	3 (Shrawasti)	14.25	27.04	18.08	34.75	83 (Sitapur)
2003	3 (Balrampur)	10.25	18.89	11.49	24.00	55 (Agra)
2004	3 (Balrampur)	14.25	24.40	15.34	29.00	71 (Sitapur)
2005	1 (Lalitpur)	12.25	22.33	13.93	26.75	70 (Sitapur)
2006	7 (Sant Kabir Nagar)	14.25	25.69	14.37	34.75	67 (Kanpur Nagar)
2007	4 (Shrawasti)	16.00	29.63	17.28	36.75	78 (Agra)
2008	5 (Balrampur)	17.25	31.96	18.73	38.75	88 (Aligarh)
2009	8 (Lalitpur)	19.25	31.87	17.98	40.75	83 (Agra, Aligarh)
2010	5 (Balrampur)	18.25	31.44	19.67	40.00	95 (Aligarh)
2011	6 (Sant Kabir Nagar)	17.00	33.16	18.69	41.75	95 (Aligarh)
2012	5 (Balrampur)	19.00	32.03	17.88	40.75	97 (Aligarh)
2013	5 (Sant Kabir Nagar)	19.00	33.31	19.52	41.25	98 (Agra)
2014	6 (Sonbhadra)	23.25	35.26	18.39	46.75	98 (Aligarh)

Census Bureau, or other variables collecting information about some form of crime or deprivation, measured at area level and usually available from official registers.

In this chapter we consider several district-level covariates to evaluate their association with dowry deaths (see Appendix A for a description). We consider sex ratio (x_1), population density (x_2), and female literacy rate (x_3) as population-based variables obtained from the Indian Census Bureau. These variables are only available for 2001 and 2011 and we have imputed the values for the remaining years using linear interpolation. These population-based variables are spatio-temporal, but the variation in time is not very pronounced, so they mainly capture spatial variability. Although sex ratio is usually defined in other settings as the number of males per 1000 females, in this chapter, sex ratio is defined as the number of females per 1000 males as this is the way it is registered by the Indian Census Bureau. Sex ratio and female literacy rate have been previously studied at state level (see for example Mukherjee et al., 2001) and the aim here is to study the relationship at district level in Uttar Pradesh.

In addition to these covariates we also consider a dummy variable indicating the political party of the Uttar Pradesh Chief Minister during the study period (x_0): the Bharatiya Janata Party (BJP), a center right wing party (2001), the Bahujan Samaj Party (BSP), a party that defends equality and social justice (2002-2003

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Table 2.2: Correlations between log crude rates of dowry deaths and the covariates in the years 2001 and 2011.

Year	Sex ratio (x_1)	Population density (x_2)	Female literacy rate (x_3)	Per capita income (x_4)	Murders (x_5)	Burglaries (x_6)
2001	-0.568	-0.289	0.274	0.206	0.608	0.278
2011	-0.721	-0.161	0.030	0.248	0.511	0.232

and 2007-2011), and Samajwadi Party (SP), of socialist ideology (2004-2006 and 2012-2014). Three more spatio-temporal variables are also considered: per capita income (x_4), number of murders per 100000 people (x_5), and number of burglaries per 100000 people (x_6). Here burglary means entering a building or residence with the intention to commit a theft or other crime. These two last covariates have been included to assess relationships of dowry deaths with any other form of crime (Mukherjee et al., 2001). Recently, sociologists and women's organizations associate an increase of dowry deaths with increase of consumerism in India (Verma et al., 2015), and per capita income is included as a proxy.

As an initial descriptive approach, correlations between log crude rates of dowry deaths and the standardized covariates are provided in Table 2.2 in the census years 2001 and 2011. At first glance, sex ratio (x_1) and number of murders per 100000 inhabitants (x_5) exhibit the strongest correlations, suggesting a potential negative association between sex ratio and dowry deaths, and a positive association between murders and dowry deaths (the spatial patterns of SMRs -standardized mortality ratios- of dowry deaths, sex ratio, and murders is shown in Figure A.1 in Appendix A for the years 2001 and 2011). The correlation between female literacy rate and dowry deaths in 2011 is practically negligible. These preliminary results are in line with those in Mukherjee et al. (2001). Moderate to low correlations are observed for the rest of variables. Summary statistics of correlations in all the years of the study period (not shown here to save space) reveal moderate to high correlations between sex ratio and dowry deaths and between murders and dowry deaths.

2.3 Spatio-temporal models

Spatial and spatio-temporal models for count data have been widely applied to study geographical and temporal patterns of incidence and mortality risks of many diseases, particularly cancer (see for example Aragonés et al. 2013; Marí-Dell'Olmo et al. 2014; Goicoa et al. 2016 or Ugarte et al. 2012). However, and to the best of our knowledge, it has been only one attempt to reveal spatio-temporal patterns of crimes against women in India. More precisely, Vicente et al. (2018) and Goicoa et al. (2019b) study

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rape incidence in Uttar Pradesh. Nowadays, the social awakening of concern for this problem and institutional support for victims have encouraged reporting of these crimes and hence the availability of records with data that can be further explored and investigated.

Models incorporating fixed effects, the main spatial and temporal random effects, and spatio-temporal interactions are valuable tools to reveal geographical patterns of risk, global temporal trends, and region-specific temporal trends. Geographic patterns may indicate the risk intrinsically related to a region, so that specific traditions or cultural practices, for example, may be exerting some influence on gender-based violence. Global temporal trends indicate how risk evolves over time and they might be valuable to assess the effect of political actions, prevention or intervention measures, as well as specific national laws to protect women. The space-time interaction term describes the specific temporal evolution of the risk in individual areas, and it may capture the effect of regional programmes, or the effect of region-specific cultural practices over time. In general, random effects may be proxies of unobserved or unknown spatial, temporal or spatio-temporal covariates associated with dowry deaths.

In this chapter we consider spatio-temporal models with spatial and temporal main effects, and spatio-temporal interactions. We review the LCAR and DCAR and evaluate their impact on the final risk estimates. We also study the scaled version of the DCAR model proposed by Riebler et al. (2016). These authors proposed this scaled version because the interpretation of the hyperparameter distribution does not depend on the graph. Moreover, the authors derive the so-called PC-priors (Simpson et al., 2017) for the hyperparameters. Here, we also study the impact of using different priors for the hyperparameters on the final inference that will be carried out using INLA.

2.3.1 Model description

In this subsection we consider spatio-temporal models to study dowry death risks including fixed effects, main spatial and temporal effects, and space-time interactions. This class of models is flexible enough to assess the effects of covariates, to describe spatial relationships between regions, to capture temporal trends that may be or may not be linear, and to account for the idiosyncrasy of regions.

Let us assume that the study region, the Indian state of Uttar Pradesh in this chapter, comprises S small areas or districts ($S = 70$ here) labelled as $i = 1, \ldots, S$. For each district, data on dowry deaths is available for $T = 14$ years denoted by $t = 1, \ldots, T$. In this setting, conditional on the rate p_{it}, the number of dowry deaths O_{it} in district i and time period t is assumed to follow a Poisson distribution with mean $\mu_{it} = n_{it} \cdot p_{it}$, where n_{it} is the population at risk in area i and time period t.

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That is,

$$O_{it}|p_{it} \sim Poisson(\mu_{it}),$$
$$\log \mu_{it} = \log(n_{it}) + \log p_{it}. \quad (2.1)$$

Note that in this case, the specific region-time rate p_{it} and the relative risk, denoted as R_{it}, are related through the global rate $R = \sum_i \sum_t O_{it} / \sum_i \sum_t n_{it}$, that is $p_{it} = R \times R_{it}$. Then, posterior samples for R_{it} can be easily obtained. Different proposals to model $\log p_{it}$ can be found in the literature, the simplest ones being models with temporal linear trends (see Bernardinelli et al., 1995). Nevertheless when temporal trends are not linear these models may be very restrictive. Hence, we consider non-parametric models with different space-time interactions (Knorr-Held, 2000). The log rate is modelled as

$$\log(p_{it}) = \alpha + \boldsymbol{x}'_{it}\boldsymbol{\beta} + \xi_i + \gamma_t + \delta_{it}, \quad (2.2)$$

where α is the overall rate, $\boldsymbol{x}'_{it} = (x_{0_{it}}, x_{1_{it}}, x_{2_{it}}, x_{3_{it}}, x_{4_{it}}, x_{5_{it}}, x_{6_{it}})$ is the vector of covariates outlined in Section 2, $\boldsymbol{\beta}$ is the coefficients of the covariates or fixed effects, ξ_i and γ_t are the main spatial and temporal random effects capturing global spatial and temporal patterns that may be associated with unobserved and unknown covariates, and δ_{it} is the spatio-temporal interaction random effect dealing with specific temporal trends in each district or changes in the global spatial pattern with time.

In this chapter we consider different conditional autoregressive (CAR) type priors for the vector of spatial random effects $\boldsymbol{\xi} = (\xi_1, \ldots, \xi_S)'$. Namely, the Leroux et al. (1999) prior (LCAR) and the Dean et al. (2001) prior (DCAR).

LCAR prior: The LCAR prior assumes the following multivariate normal distribution for the vector of spatial random effects $\boldsymbol{\xi}$

$$\boldsymbol{\xi} \sim N\left(\mathbf{0}, \sigma^2_\xi \mathbf{D}^{-1}\right),$$

where $\sigma^2_\xi \boldsymbol{D}^{-1}$ is the covariance matrix, $\mathbf{D} = \lambda_\xi \mathbf{Q}_\xi + (1 - \lambda_\xi)\mathbf{I}_S$, $\mathbf{I}_S$ is the $S \times S$ identity matrix, and $\mathbf{Q}_\xi$ is the neighbourhood matrix defined by contiguity, that is, two districts are neighbours if they share a common border. The diagonal elements are equal to the number of neighbours of each district, and non-diagonal elements $(\mathbf{Q}_\xi)_{ij} = -1$ if districts i and j are neighbours and $(\mathbf{Q}_\xi)_{ij} = 0$ otherwise. The spatial smoothing parameter λ_ξ takes values between 0 and 1.

DCAR prior: The DCAR prior assumes the following multivariate normal distribution for the vector of spatial random effects $\boldsymbol{\xi}$

$$\boldsymbol{\xi} \sim N\left(\mathbf{0}, \sigma^2_\xi \mathbf{M}\right),$$

where $\sigma_\xi^2\mathbf{M}$ is the covariance matrix, $\mathbf{M} = \lambda_\xi \mathbf{Q}_\xi^- + (1-\lambda_\xi)\mathbf{I}_S$, and the symbol $^-$ denotes the Moore-Penrose generalized inverse. Riebler et al. (2016) propose a modified version of this prior with a scaled version of the covariance matrix. These authors consider a scaled matrix $\mathbf{Q}_{\xi_\star}^-$ such that the geometric mean of its diagonal elements is equal to one. Using this scaled matrix, the priors for the hyperparameters have the same interpretation irrespective of the graph. This prior is implemented in INLA as BYM2 and this is the acronym we adopt hereafter in the chapter. The three priors, LCAR, DCAR, and BYM2 lead to the well known intrinsic CAR (iCAR) prior when $\lambda_\xi = 1$, and to an exchangeable prior when $\lambda_\xi = 0$. The key difference between the LCAR and the DCAR and BYM2 priors is that in the LCAR, λ_ξ enters in the precision matrix (and hence it is related to the conditional correlations), whereas in the DCAR and BYM2, λ_ξ enters in the covariance matrix (and it controls the marginal correlations). When all the variability is spatially structured, the iCAR would be a suitable prior, however if this is unknown the other priors are more general.

For the vector of temporal random effects $\boldsymbol{\gamma} = (\gamma_1, \ldots, \gamma_T)'$, random walks of first or second order are considered. That is, $\boldsymbol{\gamma}$ is assumed to follow a multivariate normal distribution

$$\boldsymbol{\gamma} \sim N\left(\mathbf{0}, \sigma_\gamma^2 \mathbf{R}_\gamma^-\right),$$

where $\mathbf{R}_\gamma$ is the structure matrix (see for example Rue and Held, 2005, pp. 95 and 110).

Finally, the interaction random effect $\boldsymbol{\delta} = (\delta_{11}, \ldots \delta_{S1}, \ldots, \delta_{1T}, \ldots \delta_{ST})'$ is assumed to follow a multivariate normal distribution, $N\left(\mathbf{0}, \sigma_\delta^2 \mathbf{R}_\delta^-\right)$, with structure matrix $\mathbf{R}_\delta$ defined as the Kronecker product of the spatial and temporal structure matrices. Depending on the definition of the precision (or covariance matrix), four different types of interaction can be defined (Knorr-Held, 2000): Type I (independent interactions), $\mathbf{R}_\delta = \mathbf{I}_T \otimes \mathbf{I}_S$; Type II (structured in time and unstructured in space), $\mathbf{R}_\delta = \mathbf{R}_\gamma \otimes \mathbf{I}_S$; Type III (structured in space and unstructured in time), $\mathbf{R}_\delta = \mathbf{I}_T \otimes \mathbf{Q}_\xi$ and Type IV (structured in space and time), $\mathbf{R}_\delta = \mathbf{R}_\gamma \otimes \mathbf{Q}_\xi$. It is also worth noting that these spatio-temporal models are not identifiable. The spatial and temporal random effects have implicitly defined an intercept giving rise to an identifiability issue with the model intercept. In addition, some of the interaction effects are confounded with the main effects. In this chapter, we fit the models using appropriate constraints following Goicoa et al. (2018). These authors provide the set of constraints for these spatio-temporal models with the different types of interactions.

2.3.2 A closer look into spatial priors

In this subsection we look into the the LCAR and DCAR spatial priors. The LCAR has gained popularity in disease mapping as it copes with both spatially structured and unstructured variability in one single random effect. In addition, Ugarte et al. (2014) show how to fit this model with INLA, something that has encouraged practitioners to use it. The LCAR prior considers a precision matrix that is a weighted combination of a matrix capturing the spatial relationships and an identity matrix to deal with spatially unstructured heterogeneity. The DCAR prior considers the covariance matrix as a weighted combination of a matrix including the spatial information and the identity matrix for the unstructured variability. This prior is gaining popularity because PC-priors can be obtained preventing practitioners from choosing priors for the hyperparameters. The DCAR prior has a modified version where the spatial matrix is scaled (Riebler et al., 2016), so that the interpretation of the priors for the hyperparameters is the same irrespective of the spatial graph.

Though CAR-type priors have been and still are widely used in disease mapping, there are few studies about some of their non-intuitive and perhaps impractical effects. For example, Wall (2004) states that there is no systematic structure in the covariance implied by the CAR model and hence no way to examine the spatial structure. Assunção and Krainski (2009) conclude that the entire neighbourhood ensemble affects the correlation of two neighbouring areas, and the more densely connected a graph is, the smaller the contribution of distant areas to this correlation. They determine that the second eigenvalue of an adjacency matrix determines how far away areas affect the correlation between adjacent areas. They conclude that in maps with many areas, correlation between adjacent domains are more influenced by far away areas than in maps with fewer regions. MacNab (2014) also shows that in the LCAR the correlation between areas tends to zero with distance. These authors use a series expansion of the adjacency matrix to approximate the covariance matrix and study its properties. However, the DCAR model includes the covariance matrix of an iCAR, which is rank deficient, and it does not seem possible to apply the same series expansion to look into its properties. MacNab (2011) describes a convolution prior with same covariance structure than the DCAR model and she finds that regions located further apart can be negatively correlated. Botella-Rocamora et al. (2013) also points out this unappealing property with the iCAR prior. These non intuitive and undesired properties appear in analogous priors for time effects. For example, a random walk of first order is a conditional autoregressive prior where one time point (except the first and the last one) has two "neighbours", the preceding and the subsequent time points. Here we review the LCAR and DCAR prior and look into their induced correlations using the graph of Uttar Pradesh, India (see the map with the number of the districts in Figure 2.1, and the corresponding names of the districts in Table A.1 in Appendix A).

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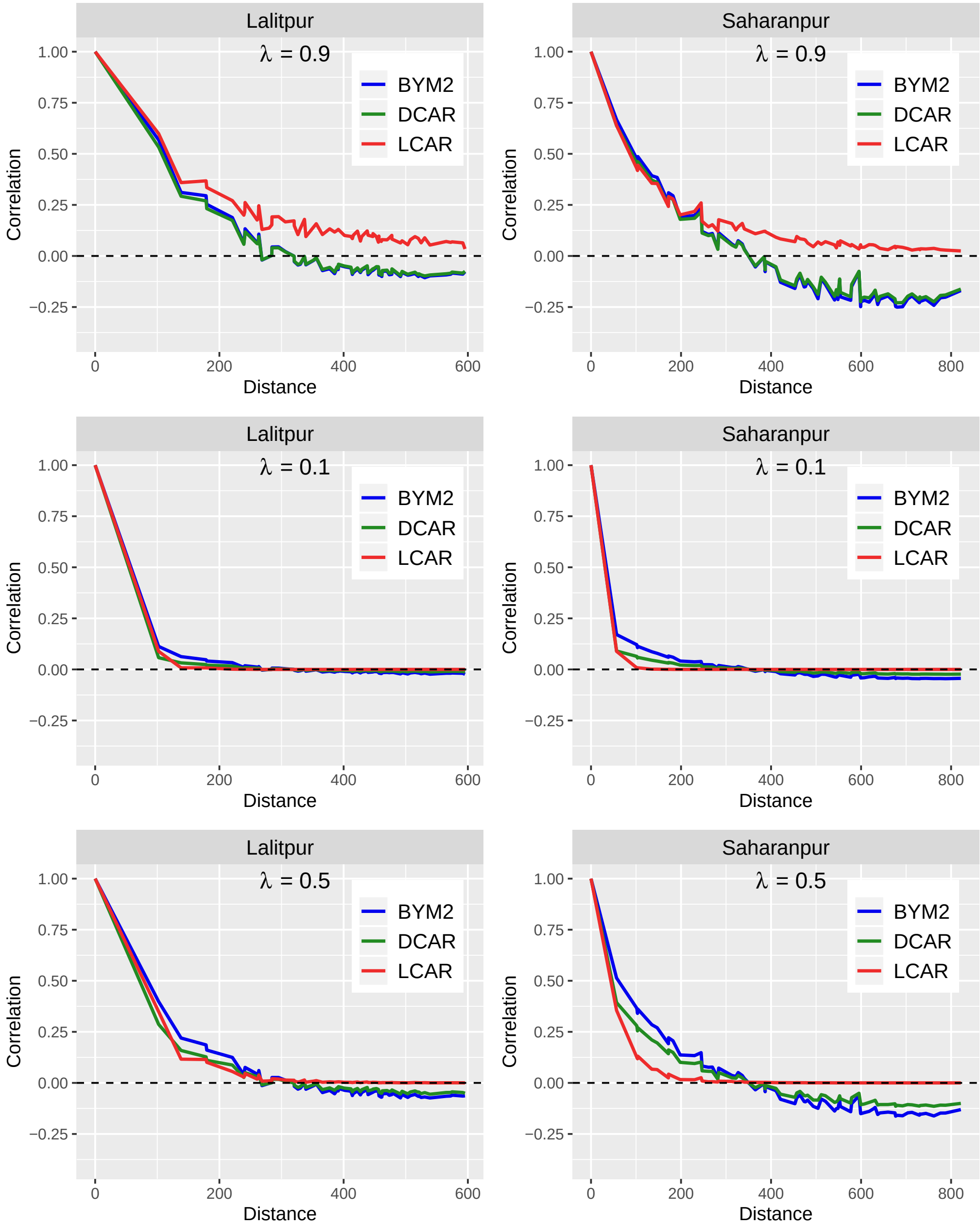

Figure 2.3: Induced correlations of the LCAR, DCAR, and BYM2 models for districts Lalitpur (45) and Saharanpur (60) which have only one neighbour. Top row corresponds to strong spatial correlation ($\lambda = 0.9$), middle row corresponds to weak spatial correlation ($\lambda = 0.1$), and bottom row corresponds to ($\lambda = 0.5$).

Figure 2.3 displays the correlations of districts Lalitpur (45) and Saharanpur (60),

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the only districts with one neighbour, with the rest of districts sorted by distance (in km) induced by the LCAR (red line), DCAR (green line) and BYM2 (blue line) models. Top row corresponds to strong spatial correlation ($\lambda = 0.9$), middle row relates to weak spatial correlation ($\lambda = 0.1$), and bottom row displays a situation where the spatially structured and the unstructured variability are equally weighted ($\lambda = 0.5$). It can be observed that locally, the three models behave similarly as the correlation with the neighbouring district is similar. However, we see some interesting differences. When the spatial correlation is high ($\lambda = 0.9$), we observe similar correlations of districts 45 (Lalitpur) and 60 (Saharanpur) with their single neighbour but the correlation decays with distance at a higher speed in the DCAR and BYM2 models. The correlations seems to be more stable in the LCAR model indicating that the correlations with areas located at similar distances are more alike. A main difference between the LCAR and both the DCAR and BYM2 priors is that the latter induce negative correlation between far away regions, whereas the correlations induced by the LCAR model tend to zero with distance, something which seems more sensible. When the spatial correlation is weak ($\lambda = 0.1$) (middle row) the local behaviour of the three models is similar, but again we observe negative correlations induced by the DCAR and BYM2 models. Moreover, in this case, the decay of the correlation with distance is higher in the LCAR, being practically zero for second order neighbours. Finally, when the spatially structured and the unstructured variability are equally weighted, the LCAR correlations decay at a faster speed than in the other models, which again produce negative correlations with regions located far apart.

Figure 2.4 displays the correlations of district Sultanpur (68), a fully connected area with 7 neighbours, with the rest of districts for $\lambda = 0.9$ (top left), $\lambda = 0.1$ (top right), and $\lambda = 0.5$ (bottom). When $\lambda = 0.9$, the LCAR induces a correlation around 0.6 between neighbouring areas and it decays slowly with distance. However, the most interesting point here is that the DCAR and BYM2 models induce negative correlations between -0.3 and -0.4 for very distant areas. When $\lambda = 0.1$, the LCAR gives rise to correlations around 0.1 between region 68 and its neighbours, whereas the DCAR and BYM2 induce practically null correlations. Finally, if $\lambda = 0.5$ we again observe rather striking negative correlations of district 68 with far apart areas induced by the DCAR and BYM2 models. MacNab (2011), following Assunção and Krainski (2009), explains the dependence of regions located farther apart as the combination of subsequent nearest-neighbours interactions in a proper CAR prior. However the same reasoning, based on a series expansion of the covariance matrix, does not apply to the DCAR and BYM2 due to the impropriety of matrix $\mathbf{Q}_{\xi}$, and the reason why these models lead to negative correlation remains unknown.

Figure 2.5 displays the correlations between neighbouring areas vs. the number of neighbours for the three models LCAR (red triangles), DCAR (green crosses), and

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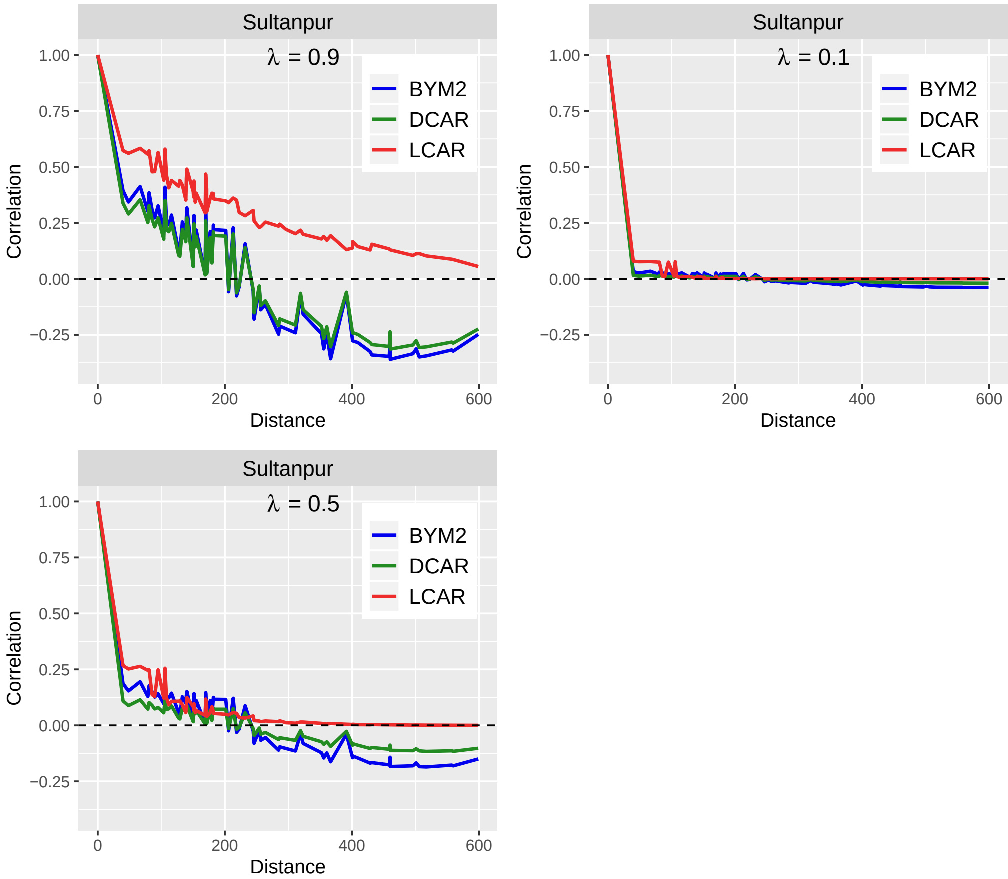

Figure 2.4: Induced correlations of the LCAR, DCAR, and BYM2 models for district Sultanpur (68), a fully connected district with seven neighbours. Top left picture corresponds to strong spatial correlation ($\lambda = 0.9$), top right plot corresponds to weak spatial correlation ($\lambda = 0.1$), and bottom picture relates to ($\lambda = 0.5$).

BYM2 (blue dots). For example, the two correlations for one neighbour indicate the correlations of district Lalitpur (45) and Saharanpur (60) with their single neighbours. The two correlations for two neighbours indicate the correlation of district 67 (the only one with two neighbours) with district 53 and 18, and so on. Clearly the variability in the correlations is higher in the DCAR and BYM2 models than in the LCAR model. This is not a desired property as the correlations of one area with their neighbouring areas should be similar. This variability seems to increase with λ. The higher the weight of the spatial matrix, the higher the variability in the correlations between neighboring regions.

To better understand the behavior of these priors we consider a LCAR and a DCAR type priors for a vector of temporal random effects of length 100 (100

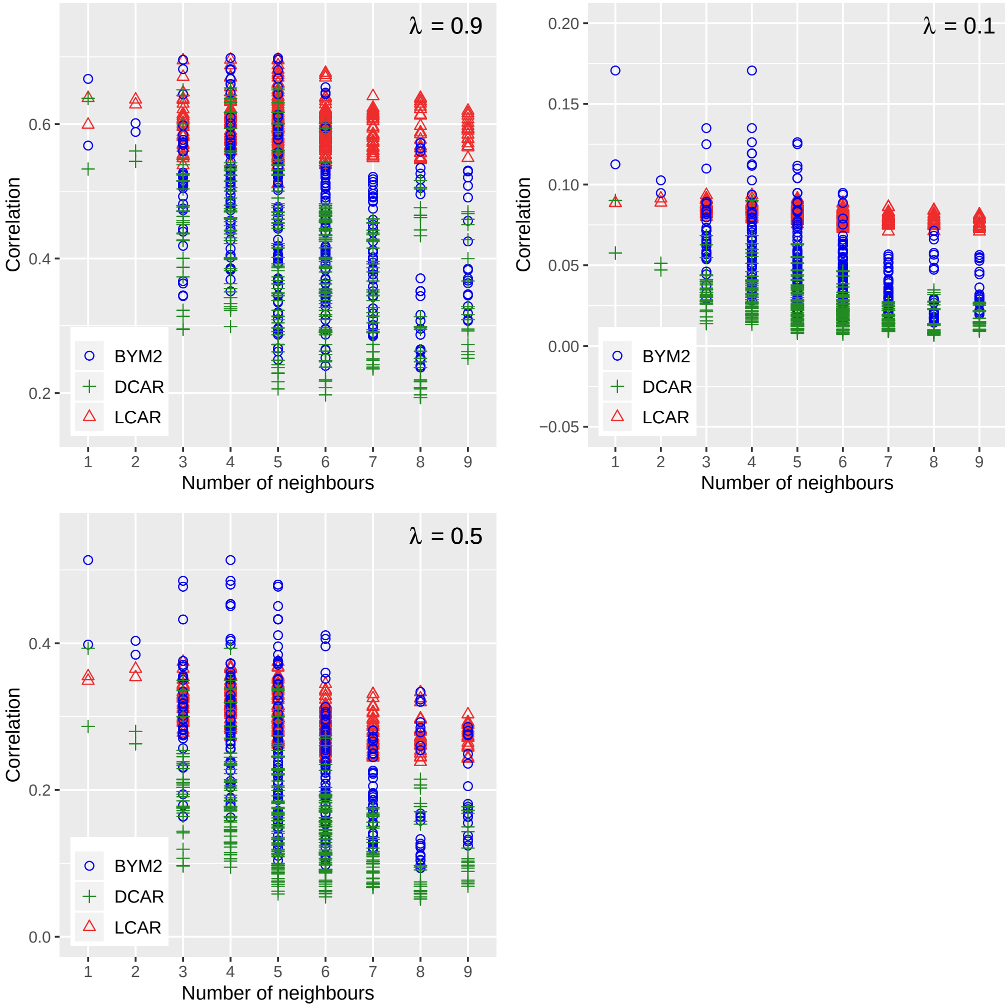

Figure 2.5: Pairwise correlation induced by the LCAR, DCAR, and BYM2 models vs. the number of neighbours for $\lambda = 0.9$ (left), $\lambda = 0.1$ (middle), $\lambda = 0.5$ (right).

time points). Figure 2.6 displays the correlation of time point 50 with the rest for the LCAR and DCAR priors with different degrees of correlation: strong (top row with $\lambda = 0.9$), weak (middle row with $\lambda = 0.1$), and moderate (bottom row with $\lambda = 0.5$). The temporal structure is more regular than the spatial one because all the points have two neighbours, the previous and the subsequent time points (except the first and last point that only have one neighbour). Here we can observe that the correlations of point 50 with points 49 and 51 are practically identical with both the LCAR and DCAR. This is expected given the symmetric relationship (but this is not what we observed in space). The pairwise correlations decay faster in the LCAR type prior than in the DCAR. This is particularly noticeable with low and moderate

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weight of the temporally structure covariance matrix ($\lambda = 0.1$ and $\lambda = 0.5$). Again we observe that the DCAR type prior leads to negative correlations between distant time points, something undesirable.

Though the behavior of these priors in time seems more appealing as we have similar correlations between neighbouring time points, we still observe the undesired property of negative correlations with the DCAR prior.

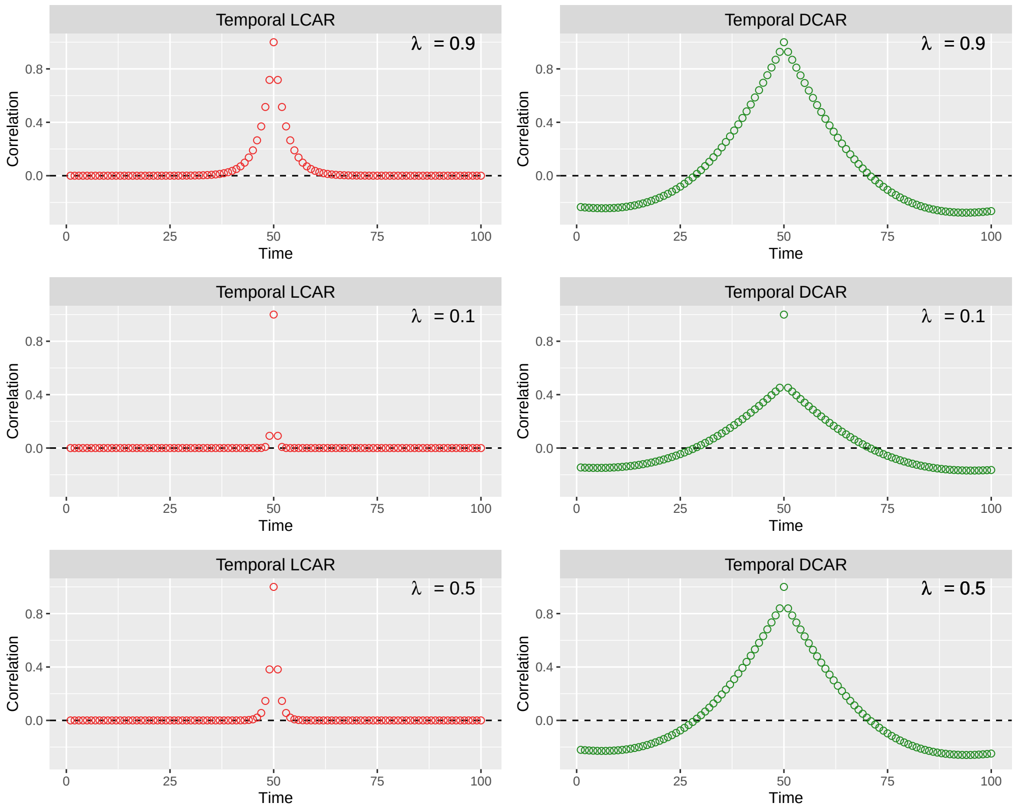

Figure 2.6: Pairwise temporal correlation induced by LCAR and DCAR type priors for the temporal random effects. Top row corresponds to strong temporal correlation ($\lambda = 0.9$), middle row represents weak temporal correlation ($\lambda = 0.1$), and bottom row displays moderate temporal correlation ($\lambda = 0.5$).

2.3.3 Hyperpriors

Given that models constitute a valuable tool to gain knowledge about the incidence of crime against women, it seems sensible to evaluate these models appropriately. This includes assessing the effects of different priors and hyperpriors on the final risk estimates. Here we consider different sets of hyperpriors for the hyperparameters

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λ, $\tau_\xi = 1/\sigma_\xi^2$, and $\tau_\gamma = 1/\sigma_\gamma^2$. On one hand, we consider a $Unif(0,1)$ prior for the spatial parameter λ and logGamma$(1, 0.00005)$ priors on the log-precisions. However, as the log-Gamma priors may produce wrong results (Carroll et al., 2015), a uniform prior on the positive real line is considered for the standard deviation (Ugarte et al., 2017; Goicoa et al., 2019a). This agrees with the recommendation from Gelman et al. (2006) about using non-informative uniform priors on the standard deviations.

Very recently, Simpson et al. (2017) proposed what they call PC-priors. These priors cannot be derived for the LCAR model, but they can be obtained for the BYM2 model. Riebler et al. (2016) explain that the PC-prior for the standard deviation is exponential with rate θ and they choose the value of θ according to $P(\sigma_\xi > U) = \alpha$, that is $\theta = -log(\alpha)/U$. If the risks are assumed to be smaller than 2 with probability 0.99, then $U = 1$ and $\alpha = 0.01$. These are the default values in INLA. The authors also provide a PC-prior for λ in the BYM2 models, but this prior has not closed form. The authors propose the rule $P(\lambda < 0.5) = 2/3$ favouring a simpler model without spatial variability unless the data supports the more complex model.

2.4 Statistical Analysis: results

The spatio-temporal Model (2.2) has been fitted considering the LCAR, the DCAR, the BYM2, and the iCAR ($\lambda_\xi = 1$) priors for space, RW1 and RW2 priors for time, and the four types of interactions. Different model selection criteria have been proposed in the literature, for example DIC (Spiegelhalter et al., 2002) or WAIC (Watanabe, 2010). Here the objective is local smoothing rather than goodness of fit, and hence, these model selection criteria may not be the most appropriate. For comparative purposes we also consider additive models, that is Model (2.2) without the spatio-temporal interaction random effects ($\log(p_{it}) = \alpha + \boldsymbol{x}'_{it}\boldsymbol{\beta} + \xi_i + \gamma_t$) to show that a lack of fit may lead to wrong results. Additionally we also compute cross validation measures. We use the logarithmic score (LS) (Gneiting and Raftery, 2007), which is a cross validation measure based on the conditional predictive ordinate or CPO in short (Pettit, 1990). The mean log score is defined as $-\frac{1}{ST}\sum_{i,t}\log CPO_{it}$, where $CPO_{it} = p(O_{it}|\mathbf{O}_{-it})$, $\mathbf{O}_{-it}$ is the set of observations excluded observation -*it*-, and $p(\cdot|\mathbf{O}_{-it})$ is the predictive distribution of a new observation given $\mathbf{O}_{-it}$. So this is a criterion to select a model in terms of its predictive ability.

Originally, we fitted Model (2.2) with the seven covariates introduced in Section 2.2. Uniform distributions on the positive real line have been considered for the standard deviations in the iCAR and LCAR priors whereas default PC-priors have been chosen for DCAR and BYM2 priors. (Results with loggamma priors on the log-precisions are not shown here as they are nearly identical). Model selection criteria points toward a Type II interaction model with a RW1 prior for time (not shown here). Table A.2 in Appendix A displays the posterior means for the fixed effects, their standard deviations, the medians and 95% credible intervals obtained from a Type II interaction model with a RW1 prior for time and an iCAR for space. Only sex ratio (x_1), murders (x_5), and burglaries (x_6) have a significant effect. Then, we fitted Model (2.2) with these three variables. Model selection criteria (DIC, WAIC, and LS) for the complete set of models are displayed in Table A.3 in Appendix A. Clearly, Type II interaction models with a RW1 prior for time are selected. Although all model selection criteria undoubtedly point to Type II interaction models with a RW1 prior for time, they do not present differences among the different spatial priors. The DIC for the Type II interaction model and RW1 for time is around 6171 for all spatial priors. Something similar happens with the WAIC (around 6180) and the LS (about 3.17).

In addition to model selection criteria, the probability integral transform (PIT) (Dawid, 1984) and its adjusted version for discrete data (Czado et al., 2009) can be used as a diagnostic tool. The PIT is simply the value that the cumulative predictive distribution function attains at the observation. Deviations from uniformity in a PIT histogram point towards model deficiencies. U-shaped histograms mean underdispersed predictive distributions and hump-shaped distributions indicate overdispersed predictive distributions. Figure 2.7 displays the PIT histograms for the additive model (left) and a model with Type II interaction (right). The models include the three significant covariates and iCAR and RW1 priors for space and time, respectively. Clearly, the additive model shows a U-shaped histogram indicating an underdispersed predictive distribution. On the other hand, the Type II interaction model seems to behave reasonably well.

Figure 2.8 shows the estimated log relative risks with the additive model (left) and with the Type II interaction model (right) for six districts in Uttar Pradesh: Aligarth, Kanpur Dehat, Kheri, Shrawasti, Sitapur, and Varanasi. The estimated risk trends with the additive model are all parallel (in fact this is the global trend shifted according to the estimated spatial effect ξ_i). On the other hand, the Type II interaction model is more flexible and it allows different estimated risk trends for each district, something that seems more plausible in this study. (See also Figure A.2 in Appendix A and the corresponding comments there).

In the following, and given that the iCAR spatial prior is the simplest model and there is no difference with the other spatial priors, we will display the results for the Type II interaction model with an iCAR spatial prior, a RW1 prior for the temporal effect, and the three significant covariates. For comparison purposes, Figure A.3 in Appendix A displays the estimated relative risks with a Type II interaction model with an iCAR spatial prior vs. the same Type II interaction model with LCAR, DCAR, and BYM2 spatial priors. The estimated relative risks are identical.

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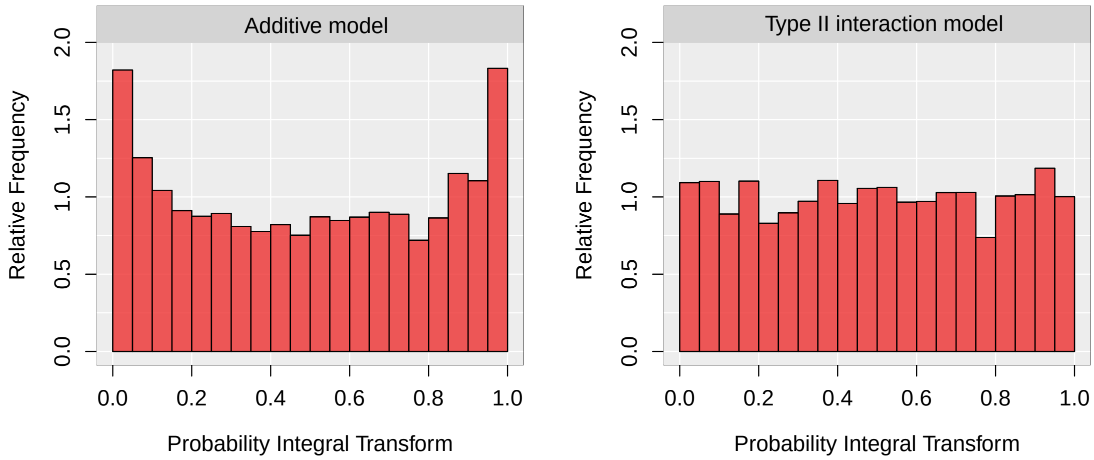

Figure 2.7: PIT histograms for the additive model (left) and the Type II interaction model (right).

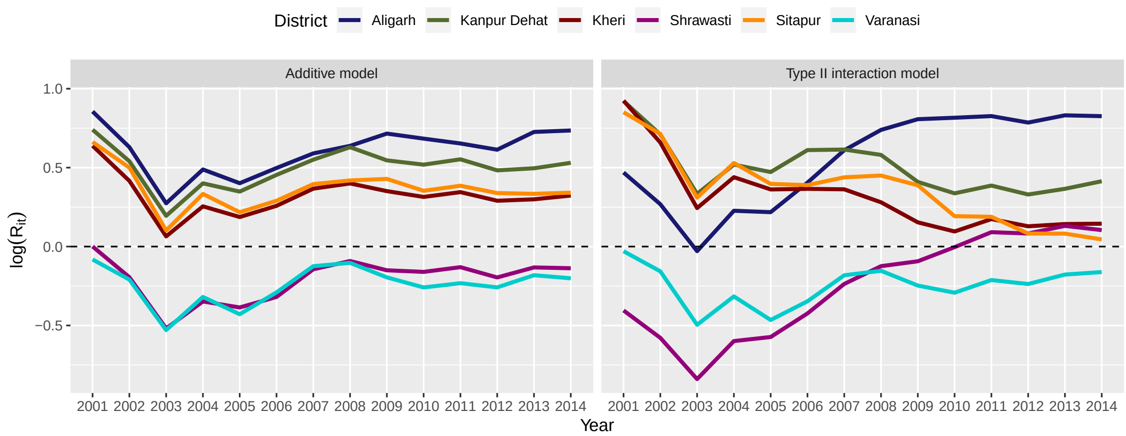

Figure 2.8: Relative risk temporal trends for six selected districts in Uttar Pradesh: Aligarth, Kanpur Dehat, Kheri, Shrawasti, Sitapur, and Varanasi. The additive model (left) and Type II interaction model (right).

Table 2.3 displays the posterior means, standard deviation, and medians of the fixed effects together with 95% credible intervals. A negative association of dowry deaths with sex ratio and a positive association with murders and burglary is obtained. Consequently, areas with low sex ratio tend to have a high risk of dowry deaths and areas with high number of murders and burglaries also have high risk of dowry

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Table 2.3: Posterior means, standard deviations and medians of the fixed effects together with a 95% credible interval.

	mean	sd	median	95% C.I.
α	-10.026	0.007	-10.026	(-10.040,-10.013)
x_1 (sex ratio)	-0.094	0.046	-0.094	(-0.183 , -0.004)
x_5 (murders)	0.089	0.020	0.090	(0.050 , 0.129)
x_6 (burglaries)	0.054	0.016	0.054	(0.023 , 0.084)

deaths. These results agree with those in Mukherjee et al. (2001).

The spatio-temporal models described in Section 2.3.1, and consequently the fitted model, are very useful and interesting because once the covariate effects are accounted for, they allow to split the final risk into different components with a valuable interpretation. Namely, an overall risk level given by $R^{-1} \cdot \exp(\alpha)$ (or an overall rate level if we do not divide by R), an spatial component $\zeta_i = \exp(\xi_i)$ indicating the risk associated to each district (this may be capturing unknown or unobserved spatial covariates), a temporal component $\exp(\gamma_t)$ that may capture the effect of unobserved temporal covariates such as long-term policies, intervention or prevention programmes over time, and finally, a spatio-temporal component $\exp(\delta_{it})$ related to each district idiosyncrasy. In our case, after the effect of sex ratio, murders, and burglaries has been discounted, most of the variability is explained by the spatial effect, 78% of the variability. The temporal effect accounts for 13% of the variability, and the remaining 9% is captured by the space-time interaction effect.

Figure 2.9 displays posterior means of the district-specific relative risks $\zeta_i = \exp(\xi_i)$ (left), and posterior probabilities that these district-specific risks are greater than 1, $P(\zeta_i > 1|\mathbf{O})$ (right). The spatial pattern (ζ_i) can be interpreted as the basic risk associated with a district, and the strong spatial pattern is evident. Midwestern and some northern districts present a greater risk than the other districts. In addition, most of the eastern districts present a small district-specific risk. There is also a group of districts with low spatial risk located in the north-west of the state. The map of posterior probabilities is clearly divided into two groups: one with districts whose posterior probabilities are greater than 0.9, and another group of districts with posterior probabilities smaller than 0.1. This indicates that the districts with posterior probabilities over 0.9 are classified as high-risk districts (see Richardson et al., 2004; Ugarte et al., 2009a,b). The spatial pattern and posterior probabilities obtained with the LCAR, DCAR, and BYM2 and with PC or loggamma priors are practically identical indicating robustness with regard to the choice of spatial priors and hyperpriors in this particular data analysis.

Figure 2.10 presents the overall temporal trend common to all districts, that is the posterior mean of $\exp(\gamma_t)$. This common temporal pattern may capture the

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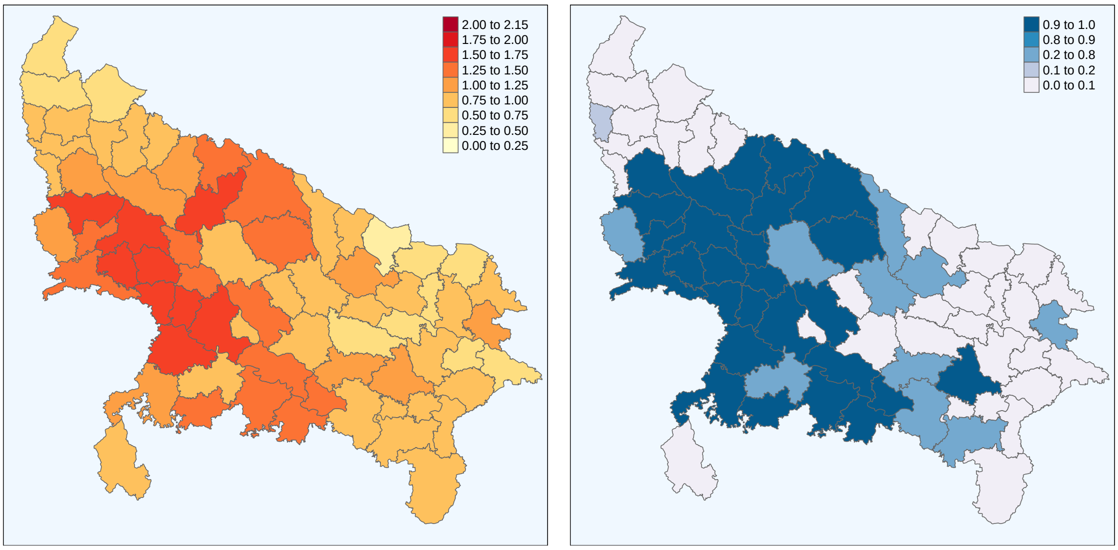

Figure 2.9: Posterior mean of the distric-specific relative risk, $\zeta_i = \exp(\xi_i)$ (left), and posterior probabilities $P(\zeta_i > 1|\mathbf{O})$ that the relative risks are greater than one (right).

effect of public policies or laws against CAW affecting the whole state. An abrupt fall is observed from 2001 to 2003. From then on, the "temporal" risk increases until 2008 and then it stabilizes around 1.1. Though the effect of the ruling party in Uttar Pradesh during the studied period was not significant, we could hypothesize that these variations may be due to past governmental actions, as typically their effects are usually noticeable some years later. The flat temporal trend around 1.1 from 2008 onwards indicates that the temporal component tend to slightly increase the final risks in this part of the period.

Area-specific temporal trends, i.e., the posterior means of $\exp(\delta_{it})$, (with 95% credible intervals) are shown in Figure 2.11 for Aligarh (district 2), Kheri (district 43), Shrawasti (district 64), and Varanasi (district 70). The specific temporal evolution in each area is clearly different indicating that in some districts (Aligarh and Shrawasti) this trend increases, whereas in other districts the area-specific temporal trend decreases (Kheri) or is flat (Varanasi). This may indicate that perhaps district-specific policies are affecting the incidence of dowry deaths over time.

The geographical patterns of dowry deaths incidence (posterior mean of the relative risk $\exp(R_{it})$) in the study period are shown in Figure 2.12. The posterior probabilities that these relative risks are greater than 1, $P(R_{it} > 1|\mathbf{O})$, are shown in Figure 2.13. From 2001 to 2003 the maps become lighter (Figure 2.12) indicating a decrease in risk. However, from 2003 to 2008 the maps start to get darker and from 2008 onward the geographical pattern seems to stabilize. Looking at Figure 2.13,

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Figure 2.10: Temporal pattern (posterior mean of $\exp(\gamma_t)$) of dowry death risks in Uttar Pradesh.

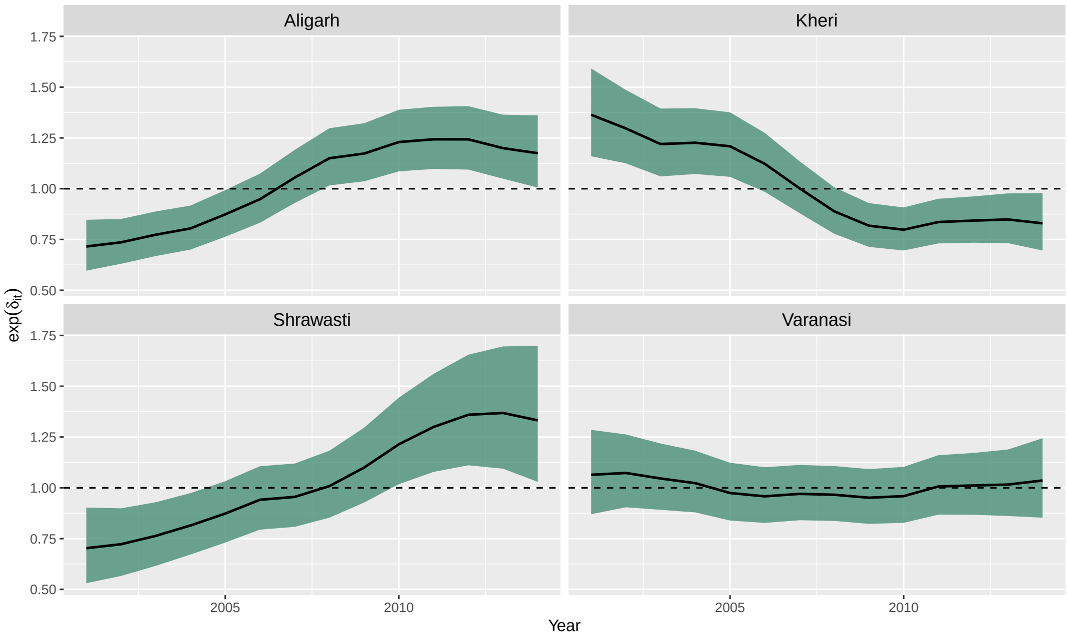

Figure 2.11: Specific temporal trends (posterior mean of $\exp(\delta_{it})$) for four selected districts: Aligarh (district 2), Kheri (district 43), Saharanpur (district 64), and Varanasi (district 70).

most of the low-risk districts are in the eastern part of Uttar Pradesh. There are also a few low-risk districts in the north-western corner. High-risk districts are in the western-central part of the state.

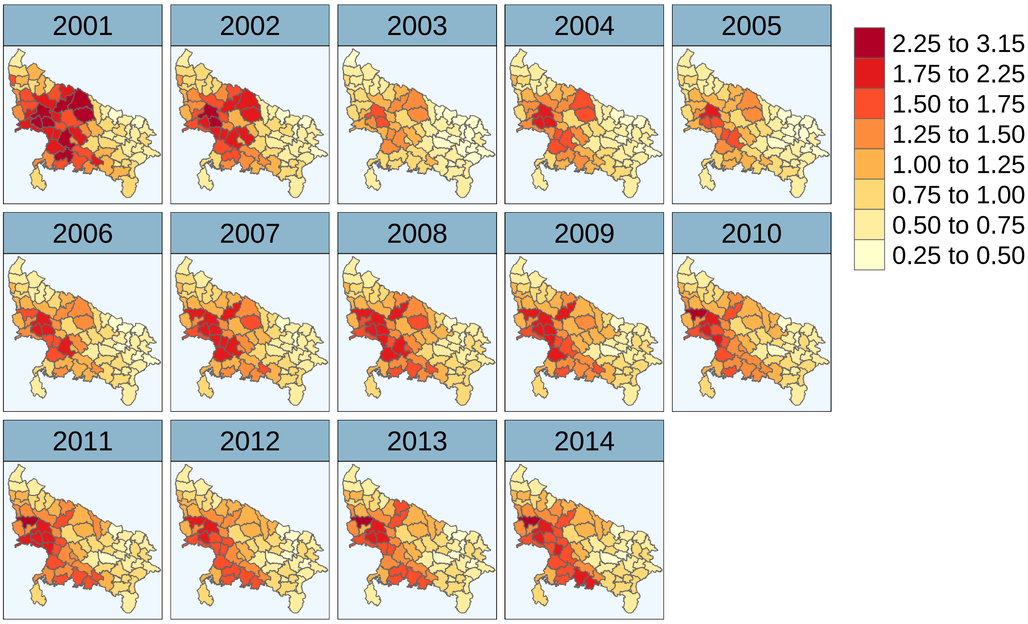

Figure 2.12: Map of estimated risks for dowry deaths in Uttar Pradesh (posterior means of R_{it}).

Finally, Figure 2.14 displays the temporal evolution of the final risk (posterior means of $\exp(R_{it})$) and 95% credible intervals for several districts: Aligarh, Kanpur Dehat, Kheri, Shrawasti, Sitapur, and Varanasi. The colours in the credible bands correspond to the posterior probabilities in Figure 2.13 and indicate whether the risk is significantly high. Clearly, Aligarh is a high-risk district and the risk increased from 2003 to 2008. Then it stabilized around 2, indicating a risk of dowry deaths about twice as high as the risk of whole Uttar Pradesh. Other districts such as Kheri or Sitapur show a decrease in risk with similar behaviour to Uttar Pradesh as a whole at the end of the period (a risk around one). These differences in the risk evolution suggest a close inspection of the districts to hypothesize about potential risk factors and possible protection factors that might be related to the districts with increasing and decreasing risks trends respectively. To have an idea about the magnitude of dowry deaths rates, the estimated rates (per 100,000 women aged between 15 and 49) for these 6 districts are displayed in Table A.4 in Appendix A.

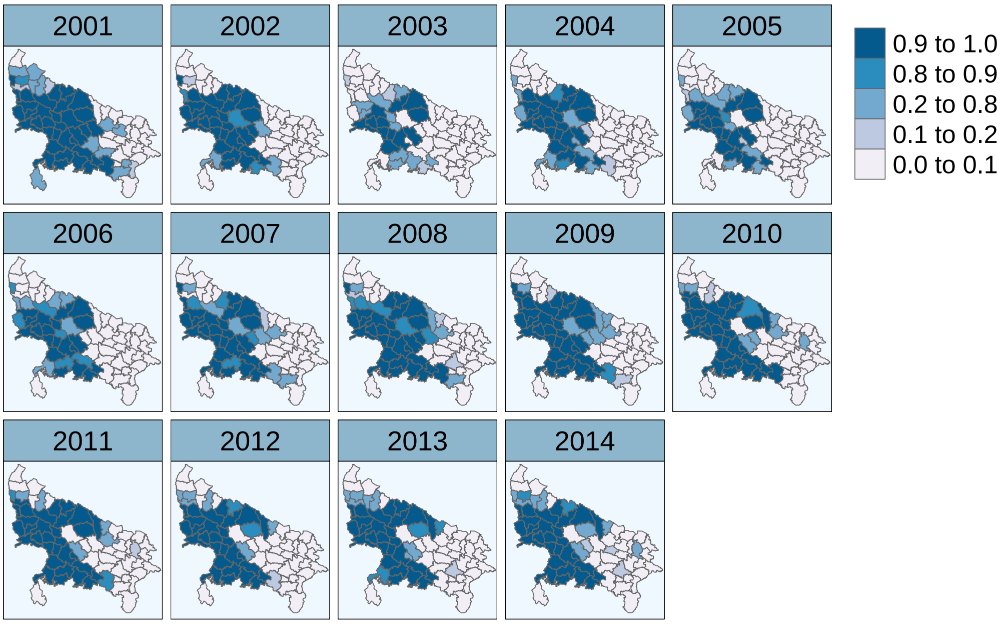

Figure 2.13: Map of posterior probabilities that the relative risks are greater than one, $P(R_{it} > 1|\mathbf{O})$.

2.5 Discussion

Statistical techniques other than simple descriptive statistics are necessary to reveal patterns of CAW in general, and dowry deaths in particular, and to calibrate to what extent this is a problem of epidemic proportions. Spatio-temporal disease mapping models are powerful and useful techniques to shed light on this problem, to localise hot spots, and to help discover the underlying risk factors. In this chapter, we fit spatio-temporal models with different spatial CAR priors to assess their effects on the final risk estimates. Although CAR priors have been and still are widely used to deal with spatial heterogeneity, they also have some counterintuitive and undesired effects. Here we study the LCAR, DCAR and BYM2 priors. They seem to be rather similar but the induced correlations are different. In particular, the DCAR and its scaled version BYM2 priors lead to non-intuitive and difficult to interpret negative correlations between regions located farther apart. On the other hand, the LCAR correlations tend to zero with distance, something more appealing. The interpretation of these priors is clearer in one dimension (time) where the correlation of one point with the preceding and subsequent points are practically identical,

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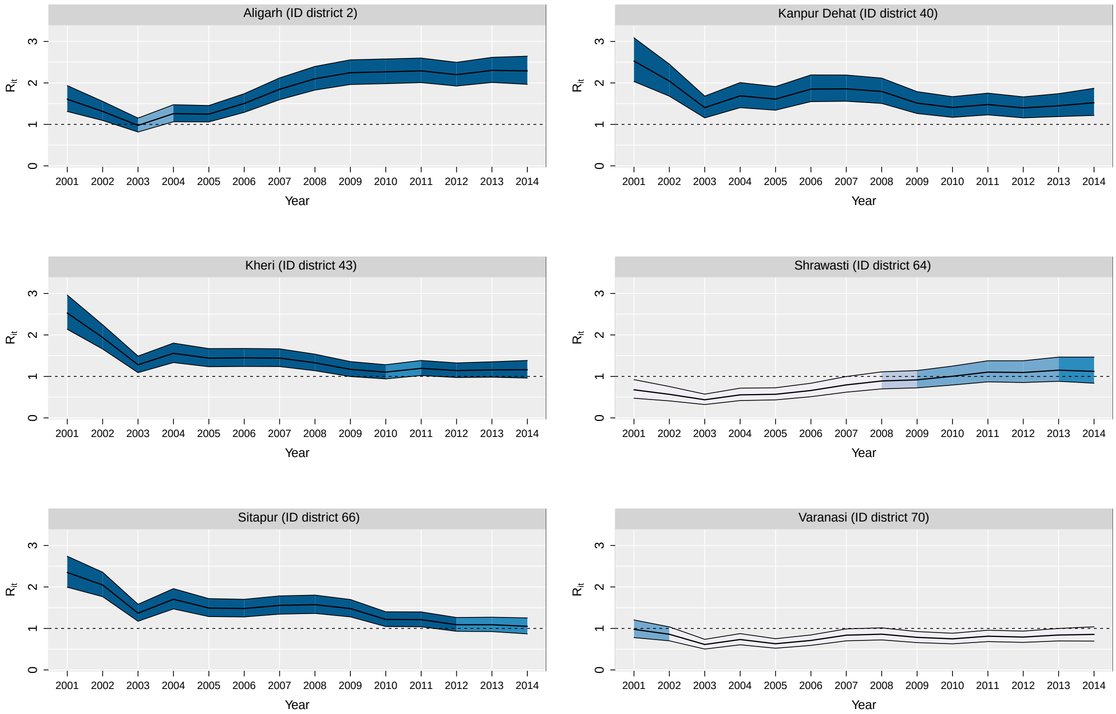

Figure 2.14: Temporal evolution of final risk estimates for some districts in Uttar Pradesh: Aligarh, Kanpur Dehat, Kheri, Shrawasti, Sitapur, and Varanasi. The range of blue colors in the credible intervals is the same as in Figure 2.13, and indicates the posterior probability that the relative risk is greater one ($P(R_{it} > 1|\mathbf{O})$). Dark blue colour means that this posterior probability is greater than 0.9, light blue colour means that the posterior probability is between 0.1 and 0.9, and grey colour means that the posterior probability is less than 0.1.

something expected as both points play the same “neighbouring” role. In the spatial case, this “symmetry” is not so evident as the correlation of one area with their neighbours is not the same. Here, the LCAR prior seems to be preferable as the variability among the correlations is much smaller than with the DCAR and BYM2 priors. The other issue related to the spatial prior is the effect of the hyperpriors. The DCAR and BYM2 are attractive as PC-prior can be derived preventing non-expert practitioner of subjective choices. Deriving PC-priors for the LCAR does not seem possible. In any case, and according to the analysis of dowry deaths in Uttar Pradesh, neither the undesired effects of the induced correlation matrix of the DCAR and BYM2 nor the different set of hyperpriors change the final risk estimates. That is, the relative risk estimates are robust to the choice of spatial priors and hyperpriors.

These priors are more general than the iCAR prior as they can cope with both

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spatially structured and unstructured variability, so they would be preferred in general. However, if the parameter λ_ξ is close to 1, all the spatial priors lead essentially to the iCAR prior. In the dowry death data analysis, the posterior mean of λ_ξ is practically equal to one if covariates are not included in the model. When covariates are included in the model, the posterior mean of λ_ξ is around 0.7. However, despite of this decrease in the value of the spatial parameter λ_ξ, the iCAR prior was selected as results with the different spatial priors were nearly identical.

Speculating on potential causes or risk factors related to the district-specific risks is very difficult in India, a country with hugely diverse traditions, a complex social structure, and high population density. Establishing mechanisms other than existing literature to link certain covariates with dowry deaths is indeed a challenge. Moreover, covariates that may be related to dowry demand (and perhaps with dowry deaths), such as man or woman's age at marriage or some characteristics related to the mother in law (Jeyaseelan et al., 2015), have been studied at individual or family level but cannot be obtained at area level. In general it is expected that crimes against women are associated with overall crime (Mukherjee et al., 2001), but, as far as we know, there are no specific studies suggesting that certain offenders are more prone to commit dowry deaths, though some literature points toward higher rates of offending in unmarried men than in married men (King et al., 2007; Sampson et al., 2006). In general, the covariates analysed in this chapter have been contemplated according to two criteria: availability and previous research. Consequently, and according to some existing literature, we have considered the following covariates: political party of the Uttar Pradesh's Chief Minister during the study period, sex ratio, population density, female literacy rate, per capita income, number of murders per 100000 inhabitants, and number of burglaries per 100000 inhabitants.

In our analysis, only sex ratio and number of murders and burglaries are statistically significant. This is consistent with the reference literature relating low child sex ratio to discrimination against daughters and sex-selective abortion (Shenk, 2007), and with literature relating low sex ratio to dowry and violence against women in the state of Maharashtra (Barakade, 2012; More et al., 2012). Moreover, Mukherjee et al. (2001) indicate that dowry deaths is the only crime against women that seems to be associated with sex ratio, and this association is negative. These authors also conclude that crimes against women are also expected to be positively associated with overall crime and that there is not a relationship with female literacy rate. Other studies investigate the higher rates of murder in western districts of Uttar Pradesh and its correlation with low sex ratio (see Oldenburg, 1992). Strong correlations of murder rate and low female to male sex ratio have been also reported (Drèze and Khera, 2000). Our analysis goes in this direction as female literacy rate is not significant whereas murders and burglaries are (with higher rates in western districts). In any case, more research is needed to disentangle social dynamics in areas with

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higher rates of crime that may have an impact on the dowry deaths phenomenon. The other two covariates in our analysis, population density and per capita income, were not found to be significant. The latter was included as Verma et al. (2015) reveal an association between dowry deaths and a rising in consumerism in India, dowry being a way to obtain certain goods still unaffordable for many families. With respect to the estimated global temporal trend, it is suspected that the observed variations could be due to changes in policies caused by alternations in power between political parties. Regarding the state political regime, Dang et al. (2018) find that where BJP party or its coalition rule, a lack of evidence on the process of implementation of laws protecting women from cruelty is revealed. In our analysis, the covariate related to the political party ruling Uttar Pradesh was not found to be significant. However, we think that it may be problematic to assess the effect of the political party in a short term. In general, policies are usually oriented to a change of mind in the society, meaning a lengthy process whose effects in practice may appear only several years after their implementation. The BJD party that was governing the state in 2001 (in fact since 1998) could have shaped a significant underreporting of cases thereafter. That could certainly explain the sudden fall of the temporal trend observed between 2001 and 2003.

Our findings should help authorities and social researchers to plan surveys to collect information in those districts with higher risks in order to look into other covariates (risk factors) that are not available at district level, such as the role of the mother in law or the age of marriage for brides and grooms from distant cohorts associated to marriage squeeze, as stated in Dang et al. (2018). Other covariates derived from low sex ratio might also be explored such as ratio of unmarried males, under the "supply of offenders" hypothesis (South et al., 2014), female migration rate (Khan, 2015), or figures on female dry thermal burn death (Pandey et al., 2014). All this research and our own analysis reveal the complexity of the problem and the difficulty in finding the underlying risk factors. A different attempt would be to use more general indicators comprising different covariates. This is the approach followed by Tanwar et al. (2016) and Kumar et al. (2018) who construct composite indexes of agricultural, social, and industrial development for 28 eastern and 25 western districts of Uttar Pradesh respectively. Their findings could give an explanation to the persistent spatial pattern that we have found in our data analysis, where districts in the east show low risks of dowry deaths whereas western districts have high risks. In general, eastern districts have better social and industrial development indexes than western districts. As an example, Sant Ravidas Nagar Bhadohi, one of the worst eastern districts in the social index (25th position), has a better index than Hathras, the best ranked western district in the same index. Similarly, concerning the industrial index, the third best district in the west, Meerut, has a value of this index worse than Sant Kabir Nagar, that is ranked 25 within the eastern districts.

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Comparisons regarding the agricultural development index are not so clear, with some districts in the west showing better scores than some districts in the east. In any case, there are eleven districts in the west with less agricultural development than 25 out of 28 districts in the east.

In summary, our study is a first attempt to unveil spatio-temporal patterns of dowry deaths in the districts of Uttar Pradesh, the most populous state in India, and to figure out underlying risk factors. We have currently detected significant associations between dowry deaths and sex ratio, murders, and burglaries. The positive association between dowry deaths and murders and burglaries confirms that in general, areas with high overall crime rates also have high rates of crimes against women.

The utility and benefit of our research is clear though the study may have some limitations. The first one is the problem of possible underreported cases. Some studies attest underreported cases of crimes against women (see for example Ellsberg et al. 2001; McNally et al. 1998, and Mukherjee et al. 2001). The social stigma related to sexual crimes makes underreporting noticeably high. Also, when violence involves family members, it is not reported because the victims do not consider such incidents as crimes (Koss, 1992). Regarding dowry deaths, Verma et al. (2015) suggest that some underreporting could be related to natal relatives fear to scandal and discredit that would ruin the possibilities of marriage for other daughters. However, Mukherjee et al. (2001) states that while some crimes such as rape are little reported, others such as dowry death rarely go unreported. Even in the case of underreporting, if we accept that regions highlighted as high-risk regions in official records are likely to be high risk regions, the discovered spatio-temporal patterns may be very useful. This debate about whether dowry deaths are underreported or not, lead us to raise some questions about how the National Bureau gets local crime statistics; if there are multiple police agencies within the state acting in the districts, or to what extent the perceptions of health care providers, forensic doctors, police or witnesses affects the classification of a dowry death (Dang et al., 2018; Belur et al., 2014). Answer to these questions could shed light on the problem.

The second limitation is that data is not available for different age groups, preventing us from studying age-specific spatio-temporal patterns. Some age-dependent differences could be expected as some authors (Jeyaseelan et al., 2015) state that as the bride's age at marriage increases, the risk of dowry demand also increases, and also that marriage squeeze depends on cohort imbalance and age gap between brides and grooms (Dang et al., 2018).

Finally, the third limitation is the difficulty to assess the statistical significance of the covariates in spatio-temporal models with spatial, temporal, and spatio-temporal random effects, as confounding problems may be present (Reich et al., 2006; Hodges and Reich, 2010; Adin et al., 2020).

The contents of this chapter have been published in *Journal of the Royal Statistical Society, A.*

The R code, data and shapefiles can be found in the GitHub of our research group, Spatial Statistics UPNA. https://github.com/spatialstatisticsupna/Dowry_JRSSA_article

A Appendix

District identifiers of Uttar Pradesh

Table A.1: District identifiers (ID) of Uttar Pradesh

ID	Dist	ID	Dist	ID	Dist
1	Agra	25	Fatehpur	49	Mainpuri
2	Aligarh	26	Firozabad	50	Mathura
3	Allahabad	27	Gautam Buddha Nagar	51	Mau
4	Ambedkar Nagar	28	Ghaziabad	52	Meerut
5	Auraiya	29	Ghazipur	53	Mirzapur
6	Azamgarh	30	Gonda	54	Moradabad
7	Baghpat	31	Gorakhpur	55	Muzaffarnagar
8	Bahraich	32	Hamirpur	56	Pilibhit
9	Ballia	33	Hardoi	57	Pratapgarh
10	Balrampur	34	Hathras	58	Rae Bareli
11	Banda	35	Jalaun	59	Rampur
12	Barabanki	36	Jaunpur	60	Saharanpur
13	Bareilly	37	Jhansi	61	Sant Kabir Nagar
14	Basti	38	Jyotiba Phule Nagar	62	Sant Ravidas Nagar Bhadohi
15	Bijnor	39	Kannauj	63	Shahjahanpur
16	Budaun	40	Kanpur Dehat	64	Shrawasti
17	Bulandshahr	41	Kanpur Nagar	65	Siddharthnagar
18	Chandauli	42	Kaushambi	66	Sitapur
19	Chitrakoot	43	Kheri	67	Sonbhadra
20	Deoria	44	Kushinagar	68	Sultanpur
21	Etah	45	Lalitpur	69	Unnao
22	Etawah	46	Lucknow	70	Varanasi
23	Faizabad	47	Mahoba		
24	Farrukhabad	48	Mahrajganj		

Description of the covariates

- x_0: Political party of the Chief Minister ruling Uttar Pradesh during the study period: Bharatiya Janata Party (BJP) during 2001; Bahujan Samaj Party (BSP) during 2002-2003 and 2007-2011; Samajwadi Party (SP) during 2004-2006 and 2012-2014 (Source: https://www.mapsofindia.com/uttar-pradesh/chief-ministers.html
or https://en.wikipedia.org/wiki/List_of_chief_ministers_of_Uttar_Pradesh)

- x_1: sex ratio. Number of females per 1000 males (Source: Office of the Registrar General and Census Commissioner, India. http://censusindia.gov.in)
- x_2: population density (People/Km^2) (Source: Office of the Registrar General and Census Commissioner, India. http://censusindia.gov.in)
- x_3: female literacy rate (Source: Office of the Registrar General and Census Commissioner, India. http://censusindia.gov.in)
- x_4: per capita income referenced to year 2004 (Source: Directorate of Economics And Statistics Government Of Uttar Pradesh. http://updes.up.nic.in)
- x_5: number of murders per 100000 inhabitants (Source: Open Government Data Platform India. https://data.gov.in)
- x_6: number of burglaries per 100000 inhabitants (Source: Open Government Data Platform India. https://data.gov.in)

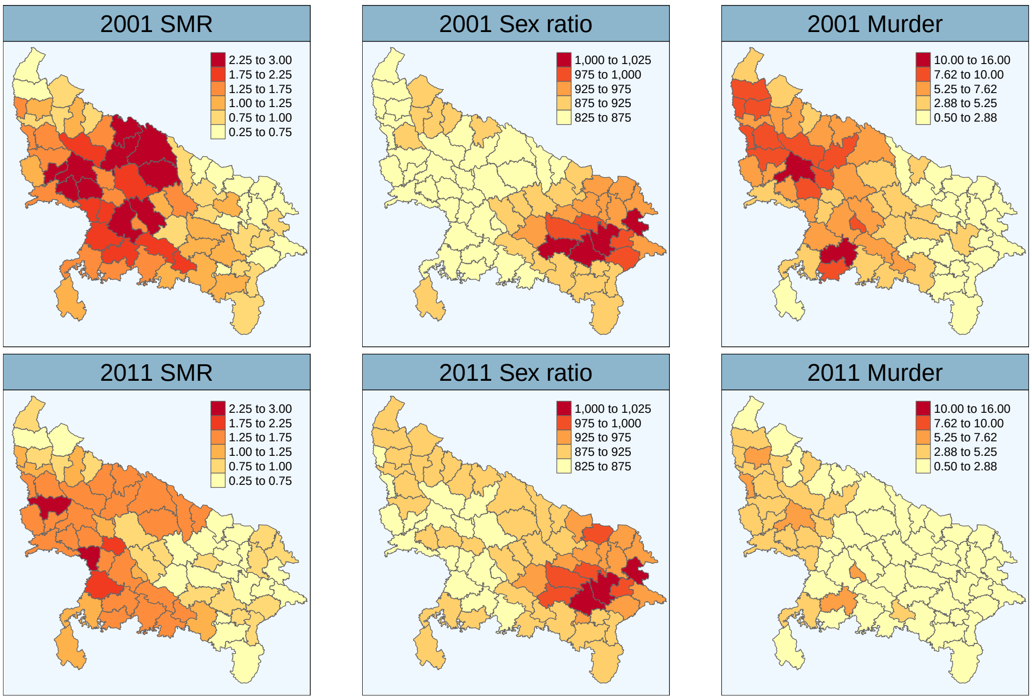

Figure A.1: Geographical pattern of SMRs of dowry deaths (left column) for years 2001 and 2011 and spatial pattern of sex ratio (center column), and murders (right column) in the same years.

Fitted models with seven covariates

We fitted Model (2.2) with the seven covariates introduced in Section 2.2. Uniform distributions on the positive real line have been considered for the standard deviations in the iCAR and LCAR priors whereas default PC-priors have been chosen for DCAR and BYM2 priors. Results with loggamma priors on the log-precisions were almost identical. Table A.2 displays the posterior means for the fixed effects, their standard deviations, the medians and 95% credible intervals obtained from a Type II interaction model with a RW1 prior for time and an iCAR for space. Only sex ratio (x_1), murders (x_5), and burglaries (x_6) have a significant effect.

Table A.2: Posterior means, standard deviations and medians of the complete set of fixed effects together with a 95% credible interval. Results correspond to a type II interaction model with a RW1 for time and an iCAR prior for space.

	mean	sd	median	95% C.I.
α (x_0 BJP)	-9.918	0.134	-9.917	(-10.189,-9.653)
x_0 (BSP)	-0.115	0.139	-0.115	(-0.392, 0.166)
x_0 (SP)	-0.118	0.161	-0.120	(-0.436, 0.208)
x_1 (sex ratio)	-0.098	0.046	-0.098	(-0.188, -0.007)
x_2 (population density)	-0.027	0.032	-0.027	(-0.088, 0.036)
x_3 (female literacy)	-0.012	0.047	-0.012	(-0.106, 0.080)
x_4 (per capita income)	-0.021	0.030	-0.022	(-0.080, 0.037)
x_5 (murders)	0.086	0.020	0.086	(0.046, 0.126)
x_6 (burglaries)	0.059	0.016	0.059	(0.027, 0.091)

Fitted models with sex ratio, murders, and burglaries

We fitted Model (2.2) with sex ratio (x_1), murders (x_5), and burglaries (x_6) covariables. Model selection criteria (DIC, WAIC, and LS) for the complete set of models are displayed in Table A.3.

In Figure A.2 risk trends, standardized mortality ratios (SMRs) and credible intervals are displayed for three different districts in Uttar Pradesh: Aligarth, Kheri, and Varanasi. On the left, we show the relative risk estimates with the additive model (solid black line) and the credible intervals (grey band). We also display the SMRs (solid orange line) and the estimated relative risk estimate with the Type II interaction model (purple line). On the right, we show the relative risk estimates with the type II interaction model (solid purple line) and the credible intervals (grey band). We also display the SMRs (solid orange line) and the estimated relative risk estimate with the additive model (black line). The credible intervals for the additive model are too narrow (indicating underdispersion of the predictive distribution as

Table A.3: Model selection criteria for different models that include covariates x_1 (sex ratio), x_5 (murders), and x_6 (burglaries). Posterior deviance ($\overline{D}$), effective number of parameters (p_D), DIC, $WAIC$, and logarithmic score (LS)

δ	ξ	γ	$\overline{D}$	p_D	DIC	$WAIC$	LS
Additive	LCAR	RW1	6395.549	80.765	6476.314	6526.625	3.331
		RW2	6397.755	80.733	6478.488	6529.162	3.332
	BYM2	RW1	6395.637	80.563	6476.200	6526.445	3.331
		RW2	6398.594	80.379	6478.972	6529.561	3.333
	DCAR	RW1	6395.615	80.575	6476.190	6526.441	3.331
		RW2	6398.130	80.481	6478.612	6529.220	3.332
	iCAR	RW1	6395.962	80.443	6476.405	6526.574	3.331
		RW2	6398.189	80.403	6478.592	6529.097	3.332
Type I	LCAR	RW1	5910.991	373.334	6284.325	6282.101	3.265
		RW2	5910.260	375.708	6285.969	6283.226	3.267
	BYM2	RW1	5911.928	372.315	6284.242	6282.566	3.265
		RW2	5911.361	374.862	6286.223	6284.099	3.267
	DCAR	RW1	5911.653	372.582	6284.235	6282.377	3.265
		RW2	5911.592	374.436	6286.028	6284.086	3.267
	iCAR	RW1	5909.702	374.435	6284.137	6281.185	3.265
		RW2	5909.508	376.289	6285.796	6282.682	3.267
Type II	LCAR	RW1	5920.320	250.875	6171.195	6180.267	3.173
		RW2	6024.192	188.442	6212.634	6239.570	3.197
	BYM2	RW1	5920.723	250.321	6171.045	6180.284	3.173
		RW2	6014.395	175.743	6190.137	6217.171	3.186
	DCAR	RW1	5920.367	250.689	6171.056	6180.169	3.173
		RW2	6018.060	180.010	6198.069	6224.701	3.190
	iCAR	RW1	5919.992	251.036	6171.028	6179.839	3.173
		RW2	6024.570	187.736	6212.306	6239.424	3.197
Type III	LCAR	RW1	6041.688	296.785	6338.473	6389.560	3.295
		RW2	6040.566	299.232	6339.798	6390.939	3.297
	BYM2	RW1	6043.307	295.461	6338.767	6390.310	3.295
		RW2	6044.678	296.366	6341.044	6393.427	3.297
	DCAR	RW1	6044.377	294.595	6338.972	6390.827	3.295
		RW2	6045.448	295.481	6340.929	6393.511	3.297
	iCAR	RW1	6041.037	296.713	6337.751	6388.626	3.295
		RW2	6041.550	297.841	6339.391	6390.788	3.296
Type IV	LCAR	RW1	5961.428	239.282	6200.710	6220.315	3.191
		RW2	6035.421	181.923	6217.344	6246.072	3.199
	BYM2	RW1	5962.441	238.325	6200.766	6220.394	3.191
		RW2	6036.816	180.974	6217.790	6246.766	3.199
	DCAR	RW1	5962.394	238.202	6200.596	6220.472	3.191
		RW2	6036.147	181.321	6217.469	6246.336	3.199
	iCAR	RW1	5961.758	238.775	6200.532	6220.062	3.191
		RW2	6036.122	181.656	6217.778	6246.202	3.199

suggested by the PIT histogram) and the estimated relative risks does not track the SMRs very well for Alligarth and Kheri. It is observed that the estimated relative risks with the type II interaction model track the SMRs much better and the credible intervals are wider. The estimated relative risk for the district of Varanasi are nearly identical with both models. In general, for those districts with rather flat trends, both models produce similar estimates. However, for those districts with increasing or decreasing trends, the Type II interaction model leads to more satisfactory results than the additive model.

Figure A.3 displays the estimated relative risks with a Type II interaction model with an iCAR spatial prior vs. the same Type II interaction model with LCAR, DCAR, and BYM2 spatial priors. The estimated relative risks are identical.

Estimated dowry deaths rates (per 100,000 women aged between 15 and 49) for districts of Aligarh, Kanpur Dehat, Kheri, Shrawasti, Sitapur, and Varanasise are displayed in Table A.4.

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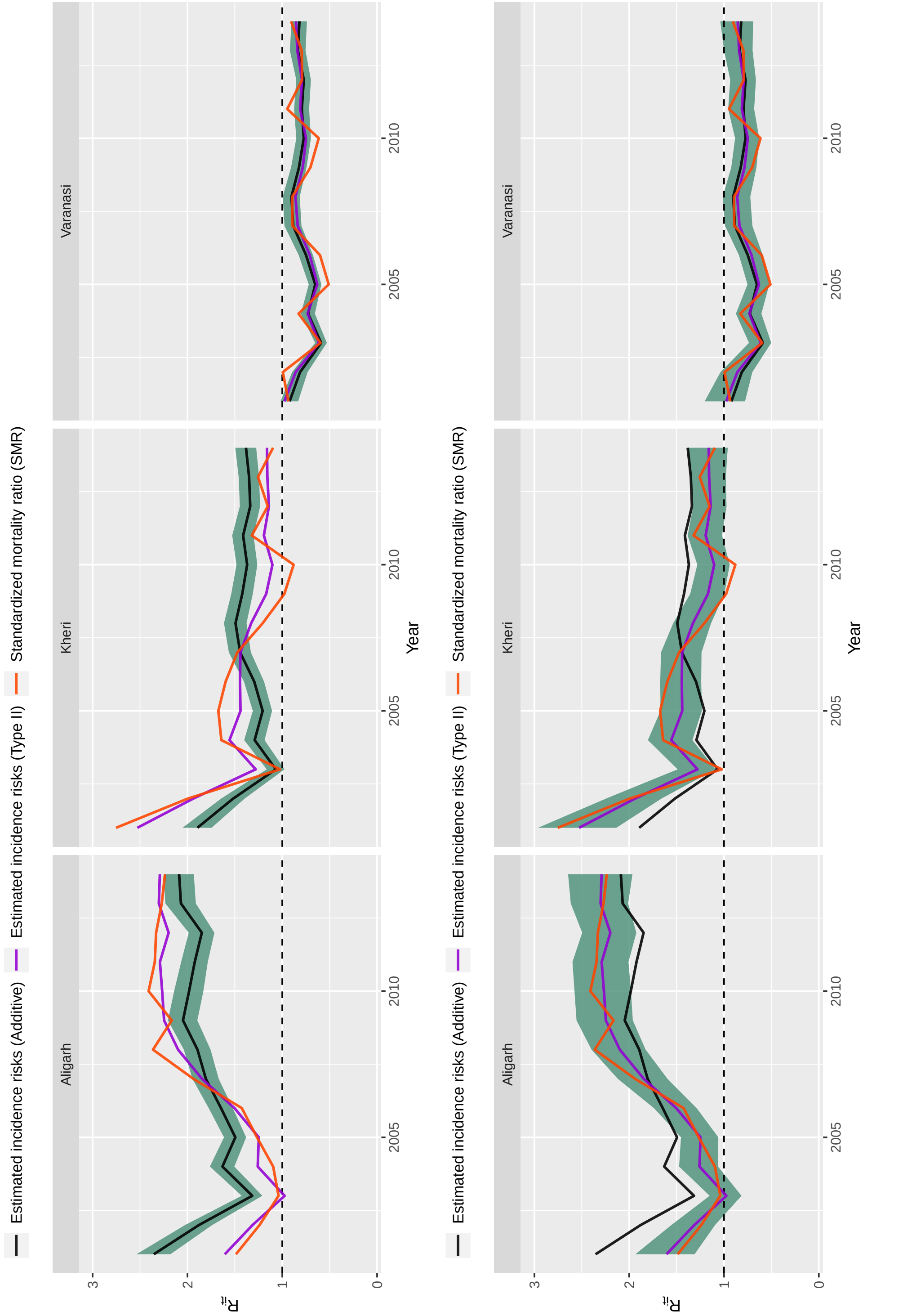

Figure A.2: Relative risk temporal trends for three different districts in Uttar Pradesh: Aligarth, Kheri, and Varanasi. On the top, estimated relative risk for the additive model (solid black line) with corresponding credible bands in grey together with the SMRs (solid orange line) and the estimated relative risk with the Type II interaction model (solid purple line). On the bottom, estimated relative risk for the Type II interaction model (solid purple line) with corresponding credible bands in grey together with the SMRs (solid orange line) and the estimated relative risk with the additive model (solid black line).

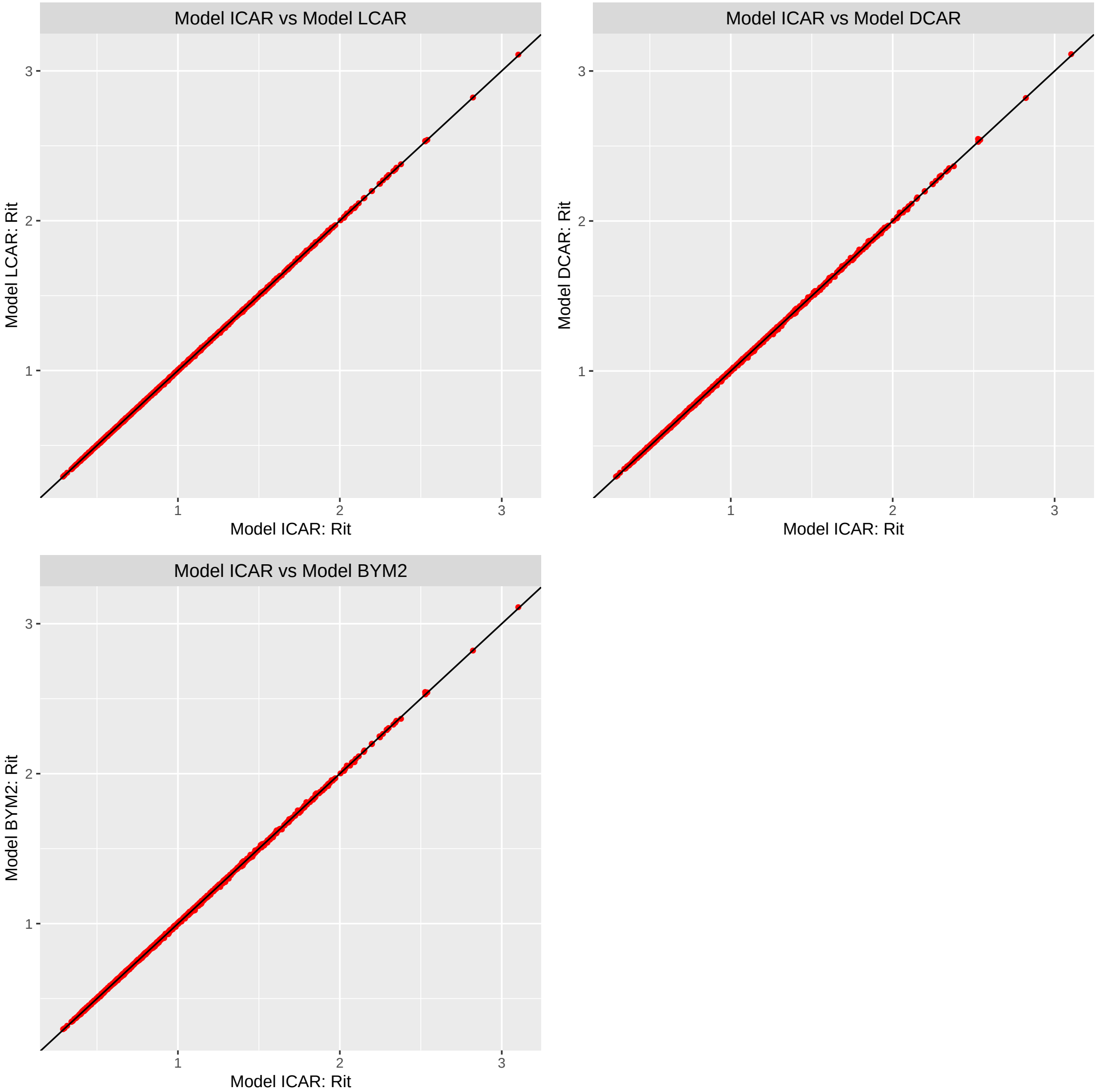

Figure A.3: Dispersion plots of the final relative risks obtained with the Type II interaction model with an iCAR spatial prior vs. Type II interaction models with LCAR (top left), DCAR (top right), and BYM2 (bottom).

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Table A.4: Estimated incidence rate of dowry deaths by year per 100,000 women aged between 15 and 49 in districts Aligarh, Kanpur Dehat, Kheri, Shrawasti, Sitapur, and Varanasi

District	2001	2002	2003	2004	2005	2006	2007
Aligarh	7.552	6.173	4.584	5.918	5.865	7.070	8.678
Kanpur Dehat	11.882	9.608	6.596	7.934	7.568	8.698	8.725
Kheri	11.888	9.112	6.018	7.318	6.771	6.796	6.775
Shrawasti	3.182	2.669	2.052	2.608	2.674	3.102	3.737
Sitapur	11.046	9.625	6.428	8.005	7.011	6.957	7.309
Varanasi	4.593	4.038	2.878	3.443	2.965	3.337	3.935
District	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**
Aligarh	9.868	10.560	10.656	10.768	10.336	10.822	10.771
Kanpur Dehat	8.435	7.109	6.613	6.947	6.567	6.811	7.155
Kheri	6.241	5.498	5.188	5.613	5.362	5.441	5.458
Shrawasti	4.181	4.312	4.715	5.186	5.145	5.404	5.273
Sitapur	7.391	6.948	5.715	5.693	5.121	5.119	4.941
Varanasi	4.047	3.686	3.523	3.817	3.722	3.953	4.022

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3

Bayesian inference in multivariate spatio-temporal areal models

3.1 Introduction

The complex and multifaceted nature of crimes against women makes it difficult to establish relationships between certain crimes, something crucial to understand the phenomenon and to develop prevention or intervention policies. To gain knowledge about crimes against women, establishing relationships between different forms of crimes can set the way forward. These relationships may be expressed in terms of similar or completely different spatial and temporal patterns, that is, in terms of correlations between spatial and temporal patterns of different crimes. This would indicate whether or not the high incidence of a particular type of crime in one specific region goes in hand with another one, or if the temporal trends of two different crimes increase or decrease in parallel. The joint analysis of different forms of crimes can be carried out using multivariate spatio-temporal models. Not only could multivariate models account for the correlations between crimes, but they would also improve estimates by borrowing information from nearby areas and time points related to the different crimes or phenomena under study.

In this chapter we extend the M-based proposal of Botella-Rocamora et al. (2015) to the spatio-temporal setting, and besides the correlation between spatial patterns of different responses, correlation between temporal trends are also included. M-based models have been derived to overcome some computational difficulties of the multivariate models propsed by Martínez-Beneito (2013). The M-based approach reduces computing time at the cost of not being able to identify some terms. Additionally, a space-time interaction term with different variance parameters for each crime is considered. Though multivariate models have been traditionally fitted using MCMC, here we also show how to fit these models using INLA with the aim of investigating if computational cost improves with respect to Gibbs sampling (the MCMC technique used by Botella-Rocamora et al. 2015).

The rest of the chapter is structured as follows: Section 3.2 reviews the M-model proposals and presents a spatio-temporal extension. Identifiability issues as well as prior specifications are also discussed in this section. Section 3.3 brings to light the problem of rapes and dowry deaths in India and gives the results of a joint analysis of rapes and dowry deaths during the period 2001-2014 in the districts of Uttar Pradesh. A comparison of the results obtained when implementing alternative models using WinBUGS (Lunn et al., 2000) and INLA is also discussed in this section. The chapter closes with a discussion.

3.2 M-models for multivariate spatio-temporal modelling

Let O_{itj} and E_{itj} be the number of observed and expected cases, respectively, in the i-th geographic unit ($i = 1, \ldots, I$), the t-th period ($t = 1, \ldots, T$), and the j-th crime ($j = 1, \ldots, J$). We assume that conditional on the relative risk, R_{itj}, the number of observed cases in each area-time-crime stratum follows a Poisson distribution

$$O_{itj}|R_{itj} \sim Poisson(\mu_{itj} = E_{itj} \cdot R_{itj}), \quad \log \mu_{itj} = \log E_{itj} + \log R_{itj},$$

where, the log-risk is modelled as

$$\log(R_{itj}) = \alpha_j + \theta_{ij} + \gamma_{tj} + \delta_{itj}.$$

Here α_j is an intercept for the j-th crime, θ_{ij} and γ_{tj} are the spatial and temporal main effects for the j-th crime, and δ_{itj} is the spatio-temporal interaction within the j-th crime. Denoting by $\mathbf{\Theta} = \{\theta_{ij} : i = 1, \ldots, I; j = 1, \ldots, J\}$ and $\mathbf{\Gamma} = \{\gamma_{tj} : t = 1, \ldots, T; j = 1, \ldots, J\}$ two matrices whose columns are the spatial and temporal random effects respectively, and by $\mathbf{\Delta}_j = \{\delta_{itj} : i = 1, \ldots, I; t = 1, \ldots, T\}$ a matrix capturing the spatio-temporal interaction within each crime, the advantage of multivariate modelling is that dependency between the spatial and temporal patterns of the different crimes can be included in the model so that a latent association between crimes can help to improve the estimates and to discover risk factors related to the phenomena being studied.

Below, we address how to incorporate into the model spatial and temporal dependencies within crimes and correlation between the spatial and temporal patterns of the crimes. Firstly, dependence between spatial patterns of the crimes is addressed through the use of M-models (Botella-Rocamora et al., 2015), and the same idea is used to deal with temporal dependence between crimes. Secondly, a disease-specific

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spatio-temporal interaction is included, and finally, some identifiability issues are raised.

3.2.1 Inducing spatial and temporal dependence within and between crimes

To understand how dependence between the spatial risks and between the global temporal trends of the different crimes are included in the model, let us express the matrices $\mathbf{\Theta}$ and $\mathbf{\Gamma}$ as

$$\begin{aligned} \mathbf{\Theta} &= \mathbf{\Phi}_\theta \mathbf{M}_\theta, \\ \mathbf{\Gamma} &= \mathbf{\Phi}_\gamma \mathbf{M}_\gamma, \end{aligned} \tag{3.1}$$

where $\mathbf{\Phi}_\theta$ and $\mathbf{\Phi}_\gamma$ are random effects matrices of order $I \times K_\theta$ and $T \times K_\gamma$ whose columns are distributed independently following a spatially correlated distribution and a temporally correlated distribution respectively. Usually K_θ and K_γ are considered equal to J, i.e., as many spatial/temporal effects as crimes, although they may be different. For example, $K_\theta = 2J$ for the multivariate formulation of the Besag et al. (1991) model, BYM, that includes two random effects to incorporate spatially structured and unstructured variability respectively. On the other hand, the dimension of the model can be reduced ($K_\theta < J$, $K_\gamma < J$) in situations where it is believed that several crimes share a common spatial/temporal pattern, obtaining computationally more efficient models (see Corpas-Burgos et al., 2019, for a discussion). The matrices $\mathbf{M}_\theta$ and $\mathbf{M}_\gamma$, of orders $K_\theta \times J$ and $K_\gamma \times J$, are responsible for inducing dependence between the different columns of $\mathbf{\Theta}$ and $\mathbf{\Gamma}$. More precisely, dependence between the columns of $\mathbf{\Theta}$ means correlation between spatial patterns of the crimes under study, whereas the dependence between their rows indicates spatial correlation within crimes. Similarly, dependence between columns of $\mathbf{\Gamma}$ means correlation between the temporal patterns of the crimes, and dependence between rows leads to temporal correlation within crimes. We refer to (3.1) as the M-model where $\mathbf{M}_\theta$ and $\mathbf{M}_\gamma$ are nonsingular but arbitrary matrices.

Different spatial priors have been considered in the literature to deal with spatial dependence. In the field of multivariate models, Botella-Rocamora et al. (2015) use a proper conditional autoregressive (pCAR) prior and Corpas-Burgos et al. (2019) consider an M-based version of the BYM. In this chapter we take into consideration both the pCAR and the BYM models. In addition, we also examine the intrinsic conditional autoregressive prior (iCAR) and the Leroux et al. (1999) prior (LCAR) for the columns of $\mathbf{\Phi}_\theta$. In the Corpas-Burgos et al.'s proposal they consider $\mathbf{\Phi}_\theta = [\mathbf{\Phi}_s : \mathbf{\Phi}_h]$, where $\mathbf{\Phi}_s$ is the $(I \times J)$ matrix of spatially correlated random effects following an iCAR distribution, and $\mathbf{\Phi}_h$ is the $(I \times J)$ matrix of spatially unstructured terms. As previously mentioned $K_\theta = 2J$ with this formulation. In

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synthesis, the columns of $\mathbf{\Phi}_\theta$ follow a multivariate normal distribution with mean $\mathbf{0}$ and covariance matrix $\mathbf{\Omega}$ whose expression depends on the spatial prior. Namely,

- iCAR:

$$\mathbf{\Omega}_{iCAR} = \sigma_s^2(\mathbf{D}_w - \mathbf{W})^- = \sigma_s^2\mathbf{Q}_\theta^-,$$

  where $\mathbf{W} = (w_{il})$ is the spatial proximity matrix defined as $w_{ii} = 0$, $w_{il} = 1$ if the i-th and the l-th areas are neighbours and 0 otherwise, $\mathbf{D}_w = diag(w_{1+}, \cdots, w_{I+})$, with the diagonal elements w_{i+} being the number of neighbours of the i-th area, and σ_s^2 is the variance parameter. The symbol $^-$ refers to the Moore-Penrose generalized inverse.

- pCAR:

$$\mathbf{\Omega}_{pCAR} = \sigma_s^2(\mathbf{D}_w - \rho\mathbf{W})^{-1},$$

  which defines a proper distribution if and only if $1/d_{min} < \rho < 1/d_{max}$ (see for example Jin et al., 2007), where d_{min} and d_{max} are the minimum and maximum eigenvalues of $\mathbf{D}_w^{-1/2}\mathbf{W}\mathbf{D}_w^{-1/2}$.

- LCAR:

$$\mathbf{\Omega}_{LCAR} = \sigma_s^2\left[\lambda(\mathbf{D}_w - \mathbf{W}) + (1-\lambda)\mathbf{I}_I\right]^{-1} = \sigma_s^2\left[\lambda\mathbf{Q}_\theta + (1-\lambda)\mathbf{I}_I\right]^{-1},$$

  where $\mathbf{I}_I$ is the $I \times I$ identity matrix. The covariance matrix $\mathbf{\Omega}_{LCAR}$ is of full rank if $\lambda \in [0, 1)$.

Note that the pCAR and the LCAR priors become the iCAR prior if $\rho = 1$ and $\lambda = 1$ respectively.

Regarding the temporal component, random walk priors of first order (RW1) are assumed for the columns of $\mathbf{\Phi}_\gamma$ i.e., each column follows a multivariate normal distribution with mean $\mathbf{0}$ and covariance matrix given by $\sigma_t^2\mathbf{Q}_\gamma^-$, where $\mathbf{Q}_\gamma$ is the structure matrix (see Rue and Held, 2005, pp. 95). This matrix is similarly defined as the spatial structure matrix $\mathbf{Q}_\theta$ but in time, that is two contiguous time points are neighbours. The variance parameters for the columns of $\mathbf{\Phi}_\theta$ and $\mathbf{\Phi}_\gamma$ are fixed at one, so the degree of spatial and temporal smoothing relies on the matrices $\mathbf{M}_\theta$ and $\mathbf{M}_\gamma$. Otherwise, these variance parameters and the cells of the $\mathbf{M}$-matrices would not be identifiable (Martínez-Beneito, 2013).

The multivariate approach allows the estimation of the correlation between the spatial patterns of the crimes, an interesting and useful feature, as a high positive correlation would support the hypotheses of common risk factors. As shown in

Botella-Rocamora et al. (2015), for models with a separable structure, this covariance matrix between the spatial patterns can be estimated as $\mathbf{M}'_\theta \mathbf{M}_\theta$. However, for BYM M-models this condition is not satisfied, as the spatial component is split into two terms with two M-matrices, so it is not reasonable to use $\mathbf{M}'_\theta \mathbf{M}_\theta$ to estimate the covariance matrix between spatial patterns of the different crimes. For this reason, Corpas-Burgos et al. (2019) recommend using the covariance matrix of the $\log(\boldsymbol{\Theta})$ columns as the covariance matrix between the spatial patterns. On the other hand, a high positive correlation between the temporal patterns would indicate that risk factors intrinsically related to the time dimension, such as certain policies, affect both crimes rather similarly and hence provide valuable information to deal with the phenomenon being studied. Employing RW1 prior distributions ensures that the $\boldsymbol{\Phi}_\gamma$ columns share a common distribution which guarantees that the covariance matrix between the temporal patterns can be estimated using $\mathbf{M}'_\gamma \mathbf{M}_\gamma$. The temporal trend could be modelled as the sum of a fixed linear term and a non linear term (random effect), similar to the work by Lombardo et al. (2018) in a different context. In such a case, one could assess if there is a significant slope. However, the final temporal trend would be the sum of the linear and the non-linear part and a positive slope might not result in a clear increase or decrease in the trend. Moreover, the matrix $\mathbf{M}'_\gamma \mathbf{M}_\gamma$ would no longer represent the covariance matrix of the temporal trends, but the covariance matrix of the non linear part. An alternative proposal would be to consider a random walk prior of second order (RW2) for time, which implicitly includes a linear term. However, in the real data analysed in this chapter, DIC and other selection criteria point towards a RW1.

3.2.2 Spatio-temporal interaction

Multivariate spatio-temporal models including the effects of area and time additively can be very restrictive in practice as the same temporal evolution is assumed for all areas within the same crime. The incorporation of a random effect for the spatio-temporal interaction models the specific behaviour of a geographical unit at a given year, thus allowing each area to have its own specific temporal evolution. Consequently, the assumption of equal time evolution for all areas is relaxed, obtaining more flexible models. Martínez-Beneito et al. (2017) propose a multidimensional framework where different dependence structures can be considered for multiple factors (space, time, and crime here). However, this procedure is computationally expensive and it is not clear how to approach this situation using M models. Given that our model already includes crime-specific spatial and temporal patterns with induced dependence between crimes, the spatio-temporal interaction within crimes is a residual term and simpler models capturing the space-time dependence can be more convenient. Here we contemplate independent spatio-temporal interactions for each crime. These spatio-temporal interactions only consider dependence in space

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and time and may have the same or different amount of smoothing for each crime.

Recalling that $\mathbf{\Delta}_j$, $j = 1, \ldots, J$, is a $(I \times T)$ matrix with the interaction random effects for the j-th crime, it is assumed that its vectorization follows a multivariate normal distribution with mean $\mathbf{0}$ and covariance matrix $\mathbf{\Sigma}_{\delta_j} = \sigma^2_{\delta_j}\mathbf{Q}^{-}_{\delta}$, i.e.,

$$\text{vec}(\mathbf{\Delta}_j) \sim N(\mathbf{0}, \sigma^2_{\delta_j}\mathbf{Q}^{-}_{\delta}).$$

Here we consider the four types of interactions defined by Knorr-Held (2000) and explained in Chapter 2. The interactions considered here are separable as they are defined in terms of Kronecker products of covariance/precision matrices. The difference between them is whether or not the elements of the interaction terms have any correlation structure in space, time or both. Regardless the correlation structure, the interaction term allows different temporal trends for each area (or different spatial pattern for each year). Other non-separable models, such as P-spline interaction models, have been proposed in the literature. Adin et al. (2017) compare the Type IV interaction with P-spline models and show that the area-specific trends are similar.

3.2.3 Identifiability issues and hyperprior specification

Univariate spatio-temporal models present some identifiability issues that can be overcome for example using constraints. These problems also arise in the multivariate setting, and to achieve identifiability between the crime-specific intercept and the corresponding main spatial and temporal random effects, sum to zero constraints are considered over these components of the model. In addition, because the main spatial and temporal effects are also included in the spatio-temporal interaction random effects, sum to zero constraints are also considered for this latter term. For more details about the required constraints for the different type of interactions (Type I, II, III, and IV), see Goicoa et al. (2018). In the multivariate setting, additional identifiability concerns emerge. As pointed out in Botella-Rocamora et al. (2015), any orthogonal transformation of the columns of $\mathbf{\Phi}_\theta$ (and $\mathbf{\Phi}_\gamma$) and the equivalent orthogonal transformation of the rows of $\mathbf{M}_\theta$ (and $\mathbf{M}_\gamma$), causes an alternative decomposition of $\mathbf{\Theta}$ (and $\mathbf{\Gamma}$), and therefore these quantities are not identifiable. However, $\mathbf{\Theta}$, $\mathbf{\Gamma}$, and the covariance matrices $\mathbf{M}_\theta{}'\mathbf{M}_\theta$ and $\mathbf{M}_\gamma{}'\mathbf{M}_\gamma$ are perfectly identifiable. Consequently, inference is confined to those quantities.

The cells of the $\mathbf{M}$-matrices act as coefficients (weights) in the decomposition of $\mathbf{\Theta}$ and $\mathbf{\Gamma}$ in Equation (3.1), so they can be seen as regression coefficients and treated as fixed effects with a normal prior with mean 0 and a large fixed variance leading to what is call fixed effects M-models (FE). Note that, assigning $N(0, \sigma^2)$ priors to the cells of the $\mathbf{M}$-matrices is equivalent to assigning a Wishart prior to $\mathbf{M}'\mathbf{M}$, i.e., $\mathbf{M}_\theta{}'\mathbf{M}_\theta \sim Wishart(J, \sigma^2_\theta\mathbf{I}_J)$ and $\mathbf{M}_\gamma{}'\mathbf{M}_\gamma \sim Wishart(J, \sigma^2_\gamma\mathbf{I}_J)$ (see Botella-Rocamora

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et al., 2015, for further details). Alternatively, random effects M-models (RE) can be considered in which the entries of the **M**-matrices are treated as independent normal random variables with mean 0 and standard deviation σ. In this case, a uniform prior between 0 and a large number is considered for σ. In our analysis, for RE M-models, Gaussian distributions with mean 0 and standard deviations σ_{θ_s} (for the spatially structured part), σ_{θ_h} (for the spatially unstructured part in the BYM model), and σ_γ (for the temporally structured part) are considered for the cells of the **M**-matrices with uniform priors between 0 and 100 for the standard deviations. The same vague uniform priors are considered for the standard deviation σ_{δ_j} of the spatio-temporal interaction. For FE M-models, and following Corpas-Burgos et al. (2019), improper $M_{ij} \propto 1$ distributions (this means that σ is set to ∞) are used for the cells of the **M**-matrices with WinBUGS. When fitting the models using INLA, a Wishart prior for $\mathbf{M}'\mathbf{M}$ is considered.

3.3 Joint analysis of crimes against women in Uttar Pradesh

3.3.1 Descriptive analysis

Uttar Pradesh accounts for the highest percentage of overall crimes against women in India, which has been increasing in the last years (11.4% in 2014; 10.9% in 2015; 14.5% in 2016 and 15.6% in 2017 according to National Crime Records Bureau (2015, 2016, 2017, 2019).

During this period, the number of rapes increased by 77% in Uttar Pradesh (1,956 in 2001, 3,462 in 2014), and this growth was even higher in the country as a whole, 138%. The increase is particularly remarkable in the last two years of the period, probably due to an improvement in the victim support system (Raj and McDougal, 2014). According to the NCRB, India is the country with the highest number of dowry deaths in the world. Some descriptive statistics about the number of rapes and dowry deaths in the districts of Uttar Pradesh by year are provided in Table 3.1. The number of rapes registered per district is highly variable, with minimum values ranging from 0 to 5 cases and maximum values between 51 and 164 cases. Figures for dowry deaths are somewhat more stable, but still coefficients of variation per year are very high. Crude rates (per 100000 women) of rapes and dowry deaths in Uttar Pradesh during the studied period are shown in Figure 3.1. An increase in rates, particularly noticeable for rapes, is observed from 2003 onward. In 2008, dowry deaths rates seem to stabilize.

The similarities between the temporal rate trends of rapes and dowry deaths during the study period leads us to hypothesize the existence of a relationship between the risk of rapes and dowry deaths. This apparent relationship may indicate

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Table 3.1: Descriptive statistics. Minimum (min), first quartile ($q_{.25}$), mean, third quartile ($q_{.75}$), maximum (max), standard desviation (sd), and coefficient of variation (cv) of the number of rapes and dowry deaths in the districts of Uttar Pradesh per year.

Year	Rapes							Dowry deaths						
	min	$q_{.25}$	mean	$q_{.75}$	max	sd	*cv*	min	$q_{.25}$	mean	$q_{.75}$	max	sd	*cv*
2001	1	13.0	27.9	41.0	93	21.5	0.8	4	18.0	31.6	43.8	88	19.0	0.6
2002	0	9.0	20.2	30.8	73	14.7	0.7	3	14.2	27.0	34.8	83	18.1	0.7
2003	0	5.0	13.0	19.5	47	11.1	0.9	3	10.2	18.9	24.0	55	11.5	0.6
2004	3	9.2	19.9	25.8	72	15.0	0.8	3	14.2	24.4	29.0	71	15.3	0.6
2005	1	7.0	17.3	24.0	61	14.2	0.8	1	12.2	22.3	26.8	70	13.9	0.6
2006	2	9.0	18.8	26.0	51	12.1	0.6	7	14.2	25.7	34.8	67	14.4	0.6
2007	1	10.0	23.5	32.5	82	16.6	0.7	4	16.0	29.6	36.8	78	17.3	0.6
2008	2	12.0	26.7	35.8	82	19.0	0.7	5	17.2	32.0	38.8	88	18.7	0.6
2009	3	13.0	25.1	35.2	77	17.5	0.7	8	19.2	31.9	40.8	83	18.0	0.6
2010	1	10.2	21.9	26.0	75	17.4	0.8	5	18.2	31.4	40.0	95	19.7	0.6
2011	2	14.2	29.1	39.0	89	20.6	0.7	6	17.0	33.2	41.8	95	18.7	0.6
2012	4	15.0	28.0	35.8	86	17.4	0.6	5	19.0	32.0	40.8	97	17.9	0.6
2013	5	23.2	43.5	53.8	119	28.5	0.7	5	19.0	33.3	41.2	98	19.5	0.6
2014	5	23.0	49.5	69.0	164	31.7	0.6	6	23.2	35.3	46.8	98	18.4	0.5

that certain facts in time (public policies, intervention programs, laws to protect women) may be exerting some influence on these phenomena. For this reason we have calculated the correlation between the standardized incidence ratio (SIR) of rapes and dowry deaths. On one hand, the SIR for rapes and dowry deaths in all districts has been obtained for each year of the period, and the correlation between the SIR vectors for rapes and dowry deaths (correlation between crude spatial patterns) has been computed. On the other hand, for each district, we have obtained the SIR vector of rapes and dowry deaths between 2001 and 2014, and the correlation between crude temporal trends has been calculated. Some summary statistics are displayed in Table 3.2. The correlations between the crude spatial patterns range between 0.32 and 0.62. We have also computed the global crude spatial patterns of rapes and dowry deaths in the whole period and the correlation is 0.53. This would indicate that certain districts are more prone to the occurrence of both crimes. The correlations between the crude temporal patterns range between -0.37 and 0.87 indicating that, depending on the district, both crimes evolve in the same or the opposite direction. We have also calculated the crude temporal trends of rapes and dowry deaths in all of Uttar Pradesh and the correlation between them is 0.59, indicating that the correlation between overall temporal patterns may be high. The correlations observed between both crimes indicate that it might be advantageous to analyse these crimes jointly.

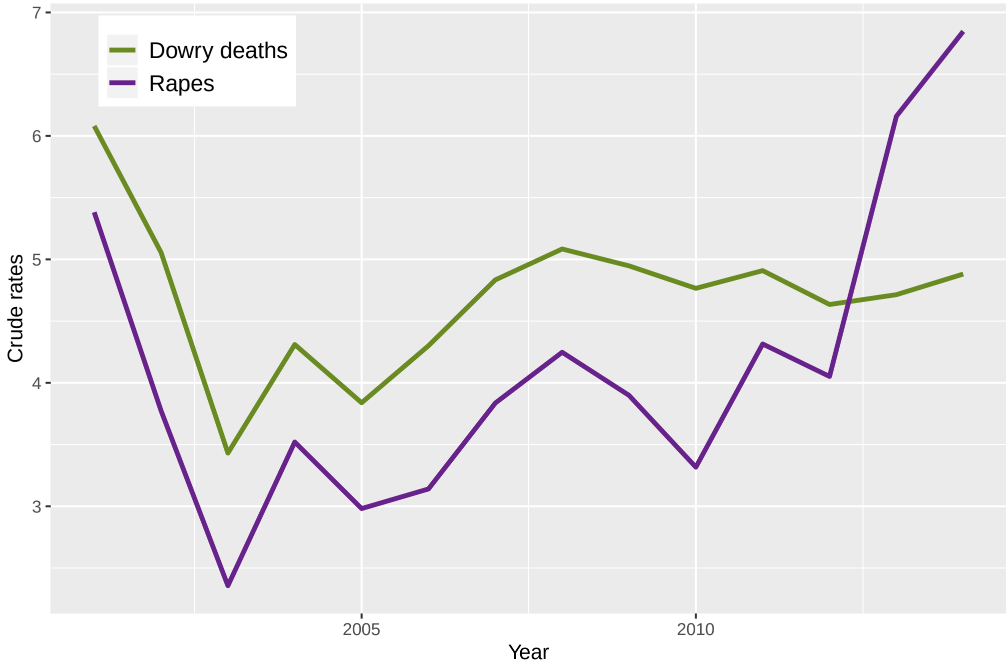

Figure 3.1: Evolution of the crude rates (per 100000 women) of rapes and dowry deaths in Uttar Pradesh in the period 2001-2014.

Table 3.2: Correlations between spatial (by year) and temporal patterns (by district) of rapes and dowry deaths based on crude standardized incidence ratios (SIR).

Correlation	min	$q_{.25}$	median	mean	$q_{.75}$	max	sd	cv
Spatial patterns	0.319	0.371	0.449	0.449	0.538	0.621	0.099	0.220
Temporal trends	-0.369	0.142	0.396	0.378	0.630	0.865	0.300	0.793

3.3.2 Model fitting using WinBUGS and INLA

Model fitting

The multivariate M-based proposal presented in Section 3.2 has been implemented to study the joint spatio-temporal distribution of rapes and dowry deaths in Uttar Pradesh between 2001 and 2014. Both specifications of M models are contemplated for the spatial and temporal effects, the fixed effects (FE) and the random effects (RE) M-models. We use BYM, iCAR, LCAR, and pCAR priors to model the spatial patterns and a RW1 prior to model the temporal effects. The four types of interactions have been considered for the spatio-temporal interaction random effect. A vague normal distribution with a precision close to zero was used for the intercepts (α_j), and uniform vague prior distributions for the standard deviations.

Initially, the models were implemented in WinBUGS. Three chains were run for each model with 30000 iterations each and a burn-in period of 5000 iterations. One out of every 75 iterations has been saved, leading to a final sample size of 1002 iterations. The Brooks-Gelman-Rubin statistic, the effective sample size, and an examination of the simulated chains were used to evaluate the convergence of the identifiable variables in the model. Convergence was checked for the standard deviations, the crime-specific intercepts, and the elements of matrices $\mathbf{\Theta}$, $\mathbf{\Gamma}$ and $\mathbf{\Delta}$. We require that the Brooks-Gelman-Rubin statistic is less than 1.1, and that the effective sample size is at least 100 for each variable. The simulated chains produced practically independent posterior draws with first order autocorrelations close to 0. Corpas-Burgos et al. (2019) present the R-code to implement spatial FE M-models and RE M-models in WinBUGS, when BYM is used to model the spatial pattern. We have extended this code to spatio-temporal M-models, and we have also considered the pCAR, LCAR, and iCAR distributions for the spatial effects.

As it is widely acknowledged that MCMC techniques can be computationally very demanding in certain cases, particularly in multivariate spatio-temporal models when the number of areas and time periods increase, the well-known INLA technique has been also considered here (Rue et al., 2009). Recently, Palmí-Perales et al. (2019) have developed the R package 'INLAMSM' (`https://CRAN.R-project.org/package=INLAMSM`) to implement multivariate spatial models for lattice data using INLA. In particular these authors consider two versions of an improper multivariate CAR and a proper multivariate CAR priors: the first version assumes a diagonal matrix for the covariance between diseases (that would indicate independence between diseases), and the second version considers more general multivariate priors with a dense symmetric matrix to model the covariance between diseases. In addition, this package includes the FE M-model (Botella-Rocamora et al., 2015) with different proper CAR priors for each disease. In this work, we have modified the INLA function for the pCAR, so that FE M-models and RE M-models with BYM, iCAR and LCAR priors for the spatial effects can be conveniently fitted. So most of the spatial priors used in the literature are extended to the multivariate setting and can be conveniently used within INLA. Moreover, these authors use a Wishart distribution for $\mathbf{M}'\mathbf{M}$ and here we also consider a $N(0, \sigma^2)$ distribution for each cell of the $\mathbf{M}$-matrices. While both alternatives are equivalent, the assignment of normal priors to each cell of the $\mathbf{M}$-matrices allows to fit more flexible models, such as those specified in Corpas-Burgos et al. (2019), relaxing the assumption of a common scale parameter for the cells of the $\mathbf{M}$-matrices. Though the advantages of INLA are clear, it may have some inconveniences in this particular setting. The computational convenience of M-models is based on the reformulation of Kronecker products of the covariance matrices as simple matrix products. However, to implement the FE M-models in INLA, INLAMSM uses a class of generic models that define the latent

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component moving away from the original philosophy of M-models as they do not replace Kronecker products by simple matrix products. In our case, with two crimes, the computational time is substantially reduced with certain spatial priors.

In what follows, a succinct comparison of the results obtained in the joint analysis of rapes and dowry deaths in Uttar Pradesh using INLA and WinBUGS is presented.

Comparing results using WinBUGS and INLA

We begin by comparing the estimated relative risks (posterior means) obtained with INLA (simplified Laplace strategy) and WinBUGS using the library pbugs to run the models in parallel (Martínez-Beneito and Vergara-Hernández, 2019). Figure 3.2 displays dispersion plots of posterior means of rapes and dowry deaths relative risks obtained with INLA vs. those obtained with WinBUGS. The estimated relative risks correspond to a FE M-model with an iCAR prior for space, a RW1 prior for time, and a Type II spatio-temporal interaction. Clearly, the relative risk estimates obtained with INLA and WinBUGS are identical. As it will be detailed later, models with a Type II spatio-temporal interaction are the most suitable candidates in terms of model selection criteria. Similar findings were obtained for the spatial ($\exp(\theta_{ij})$), temporal ($\exp(\gamma_{tj})$), and spatio-temporal pattern estimates ($\exp(\delta_{itj})$). Identical fits with INLA and WinBUGS were also observed for additive models and models with Type I, Type III, and Type IV interactions.

Regarding computing times for the models presented in Section 3.2, models with Type II and Type IV interactions are the slowest regardless the fitting technique, INLA or WinBUGS. One reason for this may be that the number of constraints is much higher for Type II and Type IV spatio-temporal interactions than for the Type I and Type III counterparts. The number of constraints on the spatio-temporal interaction random effect for Type II and Type IV are 70 (number of regions) and 84 (number of regions+number of time periods) respectively, whereas for Type I and Type III interactions the number of constraints are 1 and 14 respectively (see Goicoa et al., 2018). Given that adding restrictions entails computational cost, models with Type II and Type IV interactions are expected to run more slowly. In general, models in INLA run faster, particularly with pCAR, LCAR, and BYM priors. Computations were run on a twin superserver with two processors, Intel Xeon 4108 and 96GB RAM. For these models, the computing time ranges between 15 minutes (additive models) and 69 minutes (Type IV interaction) with INLA and between 400 minutes (additive) and 620 minutes (Type IV interaction) with WinBUGS. This indicates that the fit with INLA is between 9 and about 25 times faster than the fit with WinBUGS. Here, we would like to clarify that the pCAR and LCAR spatial priors are proper and hence WinBUGS does not place sum-to-zero constraints. However, as pointed out by Goicoa et al. (2018) a milder confounding issue still remains between the intercept and the spatial term. Consequently, sum-to-zero constraints are required.

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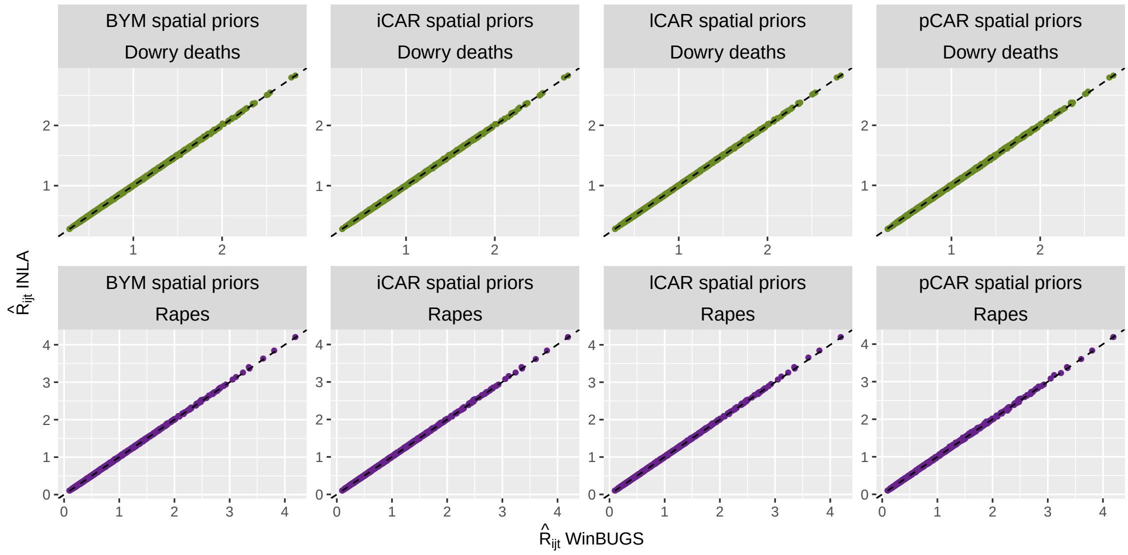

Figure 3.2: Dispersion plots of the final relative risks for rapes and dowry deaths obtained with the Type II interaction RE M-model with in INLA (y-axis) vs. WinBUGS (x-axis), using the BYM (first column), iCAR (second column), LCAR (third column) and the pCAR (last column) spatial priors.

Though this is rather simple in INLA, it is not so straightforward in WinBUGS, and we have centered the spatial random effects in each iteration of the MCMC algorithm, which in turn produces an increase in computing time. This does not happen with the iCAR (where in general WinBUGS is slightly faster than INLA) because WinBUGS internally places sum-to-zero constraints in this prior. The reason why INLA seems to be slightly slower in this case may be that constraints in this case are well handled in WinBUGS and INLA uses Kronecker instead of simple matrix products. The exception is the Type IV interaction, where the constraints slow down the computations in WinBUGS as they have to be defined manually. In summary, INLA seems to be a more efficient tool regarding computing time for the implementation of M-models.

Posterior means and 95% credible intervals for the crime-specific intercepts have been obtained and are displayed in Table 3.3. Pretty similar results are obtained with all the models and fitting techniques. We also fitted the models with the LCAR and BYM priors in WinBUGS without centering, and the final relative risk estimates were identical to the ones obtained with INLA, but differences were observed in the crime specific intercepts and the spatial patterns. Regarding the hyperparameters of the models with a spatio-temporal Type II interaction term, Table 3.4 provides the posterior mean, the posterior standard deviation, and 95% credible intervals. It is very clear that crime-specific standard deviations of the interaction term (σ_{δ_j}) do not

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Table 3.3: Posterior means, standard deviations, and 95% credible intervals for the crime-specific intercepts (α_j, $j = 1, 2$) of the models with a spatio-temporal Type II interaction term.

			FE M-models				RE M-models			
			mean	sd	$q_{.025}$	$q_{.975}$	mean	sd	$q_{.025}$	$q_{.975}$
iCAR	Rapes	MCMC	-0.187	0.008	-0.203	-0.170	-0.186	0.008	-0.201	-0.169
		INLA	-0.186	0.008	-0.203	-0.170	-0.185	0.008	-0.202	-0.169
	Dowry deaths	MCMC	-0.061	0.007	-0.075	-0.047	-0.061	0.007	-0.076	-0.046
		INLA	-0.061	0.007	-0.075	-0.047	-0.061	0.007	-0.074	-0.047
pCAR	Rapes	MCMC	-0.186	0.008	-0.203	-0.171	-0.186	0.009	-0.204	-0.169
		INLA	-0.187	0.008	-0.203	-0.170	-0.185	0.008	-0.202	-0.169
	Dowry deaths	MCMC	-0.061	0.007	-0.074	-0.048	-0.061	0.007	-0.075	-0.048
		INLA	-0.061	0.007	-0.075	-0.048	-0.062	0.007	-0.076	-0.049
LCAR	Rapes	MCMC	-0.187	0.008	-0.203	-0.170	-0.186	0.009	-0.203	-0.169
		INLA	-0.185	0.008	-0.201	-0.169	-0.186	0.008	-0.203	-0.170
	Dowry deaths	MCMC	-0.061	0.007	-0.074	-0.048	-0.061	0.007	-0.075	-0.047
		INLA	-0.060	0.007	-0.074	-0.047	-0.061	0.007	-0.075	-0.048
BYM	Rapes	MCMC	-0.187	0.008	-0.203	-0.172	-0.186	0.008	-0.203	-0.170
		INLA	-0.187	0.008	-0.204	-0.171	-0.185	0.008	-0.201	-0.169
	Dowry deaths	MCMC	-0.061	0.007	-0.074	-0.048	-0.061	0.007	-0.075	-0.048
		INLA	-0.062	0.007	-0.075	-0.048	-0.061	0.007	-0.075	-0.048

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practically change when using INLA and WinBUGS. Small differences are observed in the estimates of σ_θ and σ_γ.

Table 3.4: Posterior means, standard deviations, and 95% credible intervals for the hyperparameters of the models with a spatio-temporal Type II interaction term.

	Model	Parameter	INLA			MCMC		
			mean	$q_{.025}$	$q_{.975}$	mean	$q_{.025}$	$q_{.975}$
iCAR	FE M-models	σ_{δ_1}	0.212	0.187	0.233	0.210	0.190	0.232
		σ_{δ_2}	0.093	0.080	0.107	0.093	0.080	0.108
	RE M-models	σ_θ	0.605	0.268	1.343	0.669	0.266	1.748
		σ_γ	0.222	0.095	0.488	0.242	0.094	0.626
		σ_{δ_1}	0.210	0.190	0.237	0.210	0.190	0.233
		σ_{δ_2}	0.092	0.080	0.109	0.094	0.080	0.108
pCAR	FE M-models	σ_{δ_1}	0.210	0.191	0.234	0.209	0.189	0.231
		σ_{δ_2}	0.092	0.081	0.104	0.093	0.080	0.108
		ρ_1	0.928	0.773	0.992	0.965	0.856	0.999
		ρ_2	0.985	0.948	0.999	0.968	0.833	0.999
	RE M-models	σ_θ	0.756	0.346	1.570	0.674	0.280	1.822
		σ_γ	0.292	0.198	0.397	0.256	0.089	0.713
		σ_{δ_1}	0.214	0.189	0.252	0.210	0.188	0.232
		σ_{δ_2}	0.094	0.079	0.117	0.093	0.080	0.107
		ρ_1	0.911	0.731	0.990	0.961	0.827	0.999
		ρ_2	0.921	0.734	0.984	0.967	0.841	0.999
LCAR	FE M-models	σ_{δ_1}	0.206	0.189	0.230	0.209	0.188	0.232
		σ_{δ_2}	0.092	0.080	0.107	0.093	0.080	0.106
		λ_1	0.830	0.558	0.980	0.860	0.568	0.996
		λ_2	0.920	0.746	0.992	0.868	0.588	0.996
	RE M-models	σ_θ	0.528	0.231	1.108	0.647	0.258	1.782
		σ_γ	0.184	0.142	0.261	0.258	0.094	0.741
		σ_{δ_1}	0.212	0.199	0.226	0.209	0.187	0.230
		σ_{δ_2}	0.093	0.082	0.104	0.093	0.080	0.108
		λ_1	0.730	0.409	0.942	0.844	0.560	0.995
		λ_2	0.945	0.808	0.996	0.853	0.548	0.996
BYM	FE M-models	σ_{δ_1}	0.210	0.200	0.219	0.210	0.189	0.232
		σ_{δ_2}	0.094	0.087	0.101	0.093	0.081	0.107
	RE M-models	σ_{θ_s}	0.591	0.261	1.285	0.681	0.265	1.831
		σ_{θ_h}	0.042	0.017	0.079	0.053	0.002	0.199
		σ_γ	0.209	0.113	0.372	0.246	0.089	0.629
		σ_{δ_1}	0.211	0.190	0.233	0.210	0.189	0.231
		σ_{δ_2}	0.093	0.080	0.109	0.093	0.080	0.107

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In summary, results obtained with INLA and WinBUGS are practically identical, and given that INLA is, in general, much faster than WinBUGS, and constraints are easily handled in INLA, we consider that fitting multivariate models using INLA is an interesting alternative to WinBUGS. In the next section, we provide all the results of the real data analysis using INLA.

3.3.3 Joint analysis of rapes and dowry deaths using M-Models in INLA

Multivariate models presented in Section 3.2, including the different spatial priors and space-time interaction types, have been fitted to study rapes and dowry deaths in Uttar Pradesh during the period 2001-2014. The models are compared in terms of DIC, WAIC, and LS. The values are displayed in Table 3.5.

The same multivariate models with the same standard deviation for the spatio-temporal interaction of both crimes have been fitted, but poorer results were obtained and results have been omitted. Additive models exhibit the highest values of all the criteria, indicating that they are not flexible enough to model the data. Models with Type II spatio-temporal interaction are the most suitable candidates with notable differences in terms of DIC, WAIC, and LS with the rest of models including other interaction types. Overall, and according to all criteria, the differences between distinct models with the same Type II interaction are not very large, and it is very difficult to select the best one in terms of goodness of fit (DIC and WAIC) or prediction ability (LS). However, we notice that estimates of spatial parameters ρ_j $(j = 1, 2)$ within crimes in models with a pCAR prior, and estimates of the spatial tuning parameter within crimes λ_j $(j = 1, 2)$ with a LCAR prior, are close to 1 (see Table 3.4). This means that the differences between these models and the model with an iCAR prior tend to vanish completely. That is, the pCAR and the LCAR are essentially the iCAR, but the latter is a simpler model with a substantial reduction in computing time (about two times faster). On the other hand, estimated incidence risks using all models with Type II interaction are practically identical. Then, M-models (FE and RE) with an iCAR prior for the spatial random effect present the best tradeoff between complexity and goodness of fit. Moreover, FE M-models are in general faster than RE M-models, and consequently, we have finally selected a FE-M model with an iCAR spatial prior and a Type II spatio-temporal interaction to display the results.

The spatio-temporal multivariate model proposed in this chapter also permits to split the final risk for each crime into the spatial, temporal, and spatio-temporal component, each providing information that may be related to different issues. The crime-specific intercepts $\exp(\alpha_j)$ can be interpreted as an overall risk for each crime; the district-specific spatial risk for each crime, $\exp(\theta_{ij})$, can be related to the

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Table 3.5: Model selection criteria, DIC, WAIC and LS, for the proposed models. Within each class, iCAR, pCAR, LCAR, and BYM, the best model according to the different criteria are highlighted in bold.

$\boldsymbol{\Theta}$		Type	DIC	WAIC	LS
iCAR	FE M-models	Additive	14160.929	14413.494	7210.957
		Type I	12608.167	12522.138	6608.222
		Type II	**12355.856**	**12379.212**	**6338.481**
		Type III	12663.282	12707.279	6624.992
		Type IV	12405.457	12479.757	6370.050
	RE M-models	Additive	14161.084	14413.314	7210.853
		Type I	12607.161	12521.936	6607.393
		Type II	12356.652	12387.969	6338.562
		Type III	12661.840	12710.519	6623.163
		Type IV	12403.473	12472.729	6369.541
pCAR	FE M-models	Additive	14161.376	14415.178	7211.860
		Type I	12606.321	12507.739	6607.746
		Type II	12356.132	12373.483	6338.431
		Type III	12660.066	12693.335	6622.705
		Type IV	12403.443	12476.556	6369.834
	RE M-models	Additive	14161.125	14414.223	7211.355
		Type I	12607.587	12522.777	6608.033
		Type II	12362.337	12399.167	6342.436
		Type III	12660.802	12699.595	6622.373
		Type IV	12393.019	12441.440	6365.348
LCAR	FE M-models	Additive	14160.912	14413.994	7211.237
		Type I	12608.498	12529.463	6609.148
		Type II	12358.354	12392.021	6339.574
		Type III	12663.113	12715.302	6623.690
		Type IV	12396.433	12455.568	6366.125
	RE M-models	Additive	14159.901	14412.886	7210.686
		Type I	12609.721	12522.181	6609.431
		Type II	12354.983	12374.041	6337.927
		Type III	12657.593	12696.315	6621.781
		Type IV	12404.071	12479.757	6369.211
BYM	FE M-models	Additive	14160.500	14413.932	7211.210
		Type I	12608.295	12541.938	6609.303
		Type II	12353.168	12375.668	6337.332
		Type III	12664.510	12722.878	6623.983
		Type IV	12400.066	12463.970	6368.017
	RE M-models	Additive	14161.078	14413.632	7211.020
		Type I	12607.490	12522.248	6607.727
		Type II	12354.795	12380.401	6337.246
		Type III	12663.906	12707.357	6625.187
		Type IV	12402.443	12473.332	6368.702

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idiosyncrasy of the districts and may be reflecting the effect of certain traditions, demographic and socio-economic characteristics, or religious practices. The crime-specific temporal component $\exp(\gamma_{tj})$ indicates a global evolution of the crime in the state and may reflect the effect of factors that change over time such as policies, women supportive plans, or laws to protect women. Finally, the spatio-temporal risk $\exp(\delta_{itj})$ is a residual term that may be modelling heterogeneity related to differences in the effect of certain actions in time in each area. In general, similar spatial and temporal patterns would indicate a relationship between the crimes being studied.

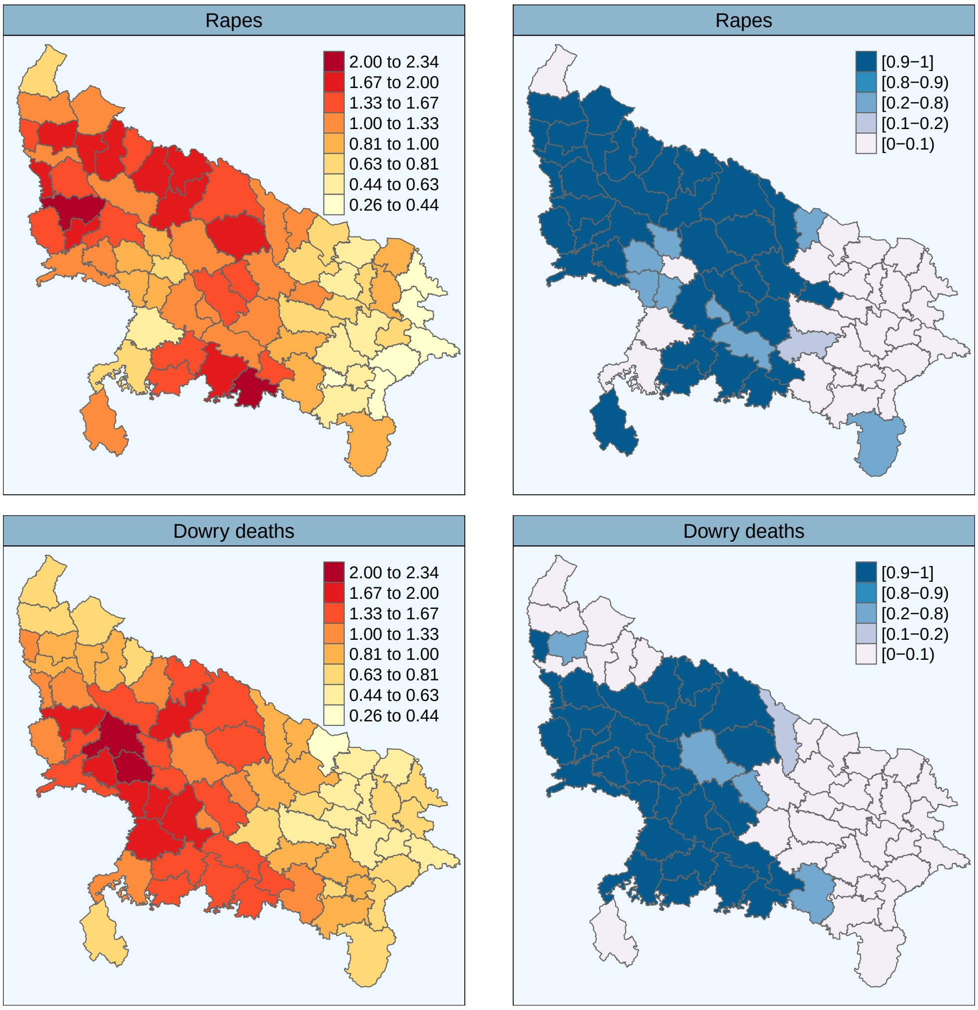

Figure 3.3: Posterior mean of the district-specific spatial risk, $\exp(\theta_{ij})$ (left column), and the exceedence probabilities, i.e., $P(\exp(\theta_{ij}) > 1|\mathbf{O})$ (right column), for rapes (top) and dowry deaths (bottom).

Figure 3.3 displays the posterior mean of the district-specific spatial risk, $\exp(\theta_{ij})$

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(left column), and the exceedence probabilities, i.e., $P(\exp(\theta_{ij}) > 1|\mathbf{O})$ (right column), for rapes (top) and dowry deaths (bottom). A clear Northwest-Southeast gradient (the largest diagonal axis of the map) is observed in the relative risk estimates for both crimes, although the spatial patterns present some differences. Whereas most of the areas in the Northwest part of the state exhibit a high risk of rapes, districts with high risk of dowry deaths are mainly located in the central part of the map. In fact, a Southwest-Northeast gradient is observed for dowry deaths in the central part of the map, something that is not clear for rapes. However, the maps reveal an interesting fact: most eastern districts present a small district-specific risk for both crimes, and this would require further insight to understand why the risk of both crimes is lower in these districts than in Uttar Pradesh as a whole. INLA allows to produce samples from the approximated joint posterior for the hyperparameters. From them, we have been able to obtain samples of the estimated correlation matrices (between spatial and between temporal patterns). The estimated posterior mean of the correlation between the spatial patterns is 0.30, with a 95% credible interval (0.08, 0.50). Similar results were obtained using WinBUGS. This positive correlation would indicate that certain districts are more prone to the occurrence of both crimes. However, finding common spatial risk factors is a challenge.

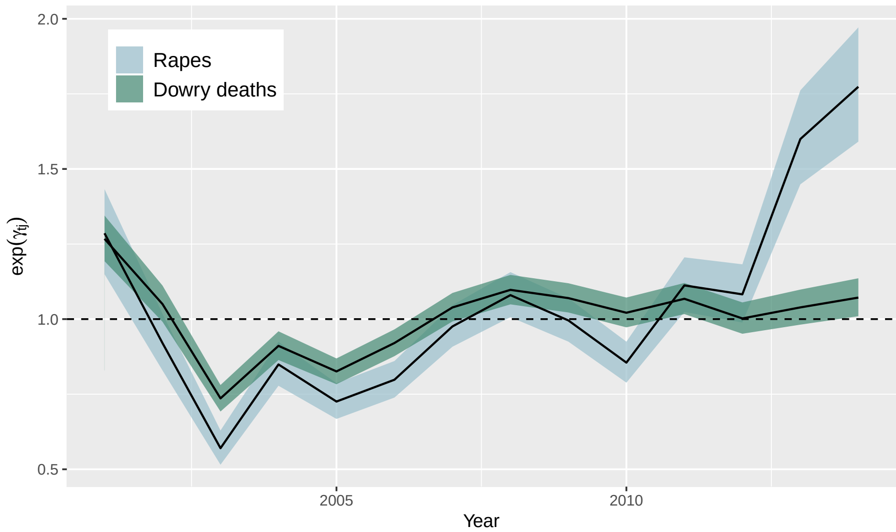

Figure 3.4: Temporal pattern of incidence risks (posterior means of $\exp(\gamma_{tj})$) for rapes and dowry deaths in Uttar Pradesh.

Figure 3.4 displays the global temporal trends common to all districts (posterior means of $\exp(\gamma_{tj})$) for each crime. Both trends exhibit a marked decrease from 2001 to 2003, and a constant increase until 2008. From then on, a remarkable increase is observed for rapes, whereas the trend remains stable for dowry deaths. The positive

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correlation here is evident, when one crime increases (decreases), the other one also increases (decreases). This is confirmed with the estimated posterior mean of the correlation between the temporal trends: 0.82 with a 95% credible interval (0.38, 1.00). This estimated high correlation indicates that rape risks keep pace with dowry death risks, indicating that some events in time may have affected the two crimes similarly. It is suspected that changes in government (and consequently in policies) may have had some influence on both crimes. During the study period, three different parties held government in India, and another three different parties ruled the state of Uttar Pradesh. A tentative hypothesis is that female protection policies (Protection of Women from Domestic Violence Act 2005) may have encouraged women to report rapes, a well known underreported crime (Vogelman and Eagle, 1991; Koss, 1992), and hence led to an increase in rape risk in the last years of the study period. It may also be responsible for the stabilization of dowry deaths, a crime where underreported cases are not expected (Mukherjee et al., 2001). However, these are mere hypotheses as evaluating the effects of certain policies requires a longer time period, and it is even more complicated to include such information in the model unless covariates about investment on plans to protect women and give them support are available.

Figure 3.5 shows the geographical risk patterns (posterior mean of the relative risk) of rapes (top) and posterior probabilities of risk exceedance, $P(R_{itj} > 1|\mathbf{O})$ (bottom) in the study period. The same information for dowry deaths is displayed in Figure 3.6. The increase in risk in rapes is clearly observed in the maps, which become darker from 2003 to 2014. The increase is particularly remarkable from 2010 onwards. The maps for dowry deaths also reveal a stable pattern in the last years of the period. Both figures show that most eastern districts exhibit a low risk for both crimes. The pattern of high risk areas (those with $P(R_{itj} > 1|\mathbf{O}) > 0.9$) of rapes is more irregular. In some years of the period (2003 and 2010 mainly), most of the areas do not exhibit high risk. However, at the end of the period, nearly all the areas do have a high risk with the exception of some districts in the eastern part of Uttar Pradesh. Regarding dowry deaths, most of the high risk areas are located in the central-western part of the state and the pattern remains fairly stable during the study period.

Finally, the temporal evolution of the final risk (posterior means of R_{itj}) and 95% credible intervals for several districts, Aligarh, Ghazlabad, Kheri, Mainpuri, Sant Kabir Nagar, and Varanasi are shown in Figure 3.7. These districts are interesting because the risk evolution is very different. Aligarh exhibits high relative risks for both crimes. In particular, the risk of rapes does not stabilize and continues growing, standing about three times higher than the overall risk in Uttar Pradesh at the end of the period. Regarding dowry deaths, the risk is significantly high, but it stabilizes over time around twice the risk of whole Uttar Pradesh. Kheri shows a decreasing evolution of risks for both crimes that stabilizes around one at the end of the period.

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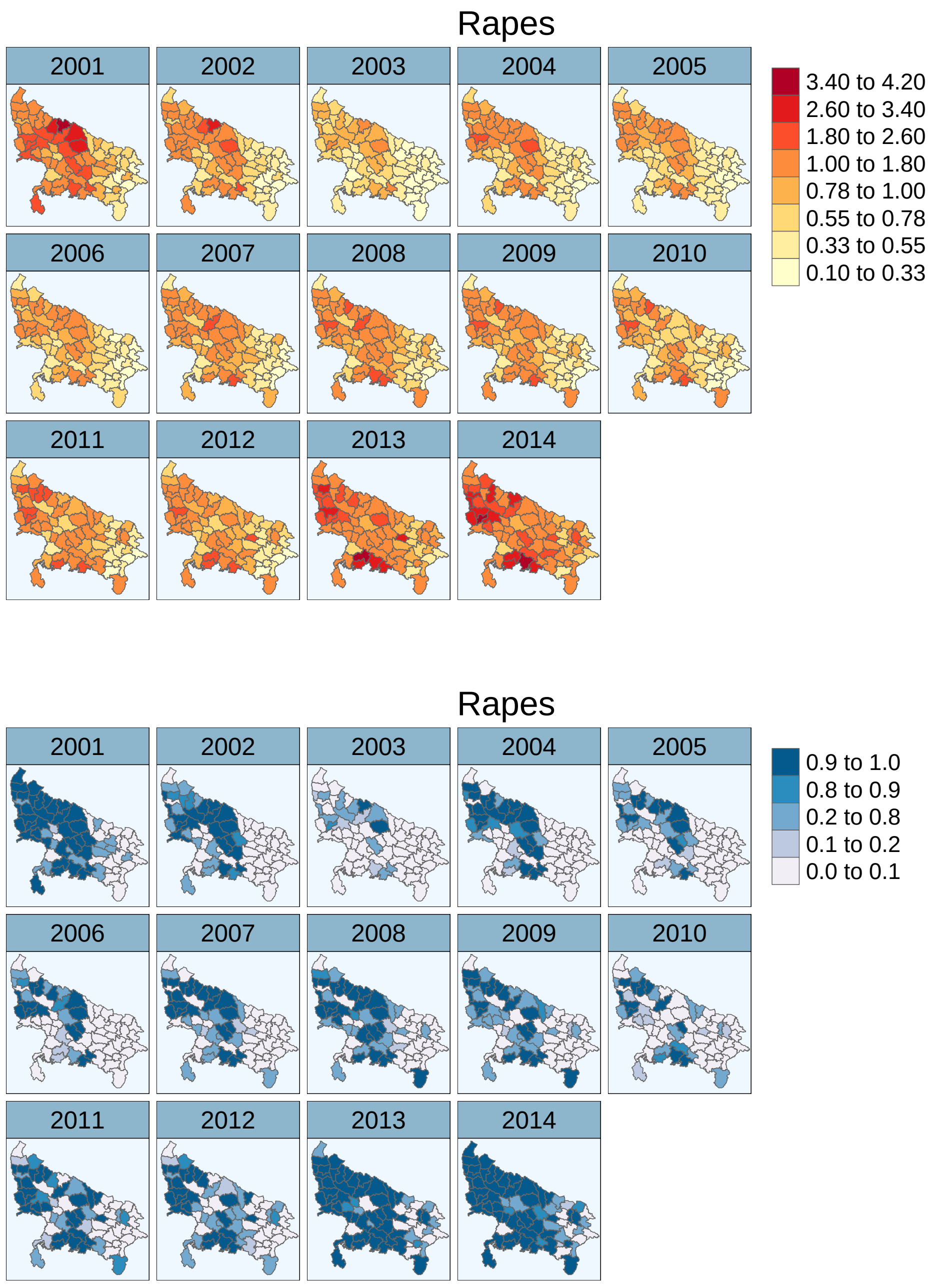

Figure 3.5: Map of estimated incidence risks for rapes (top) and posterior probabilities that the relative risk is greater than one ($P(R_{itj} > 1|\mathbf{O})$) (bottom) in Uttar Pradesh.

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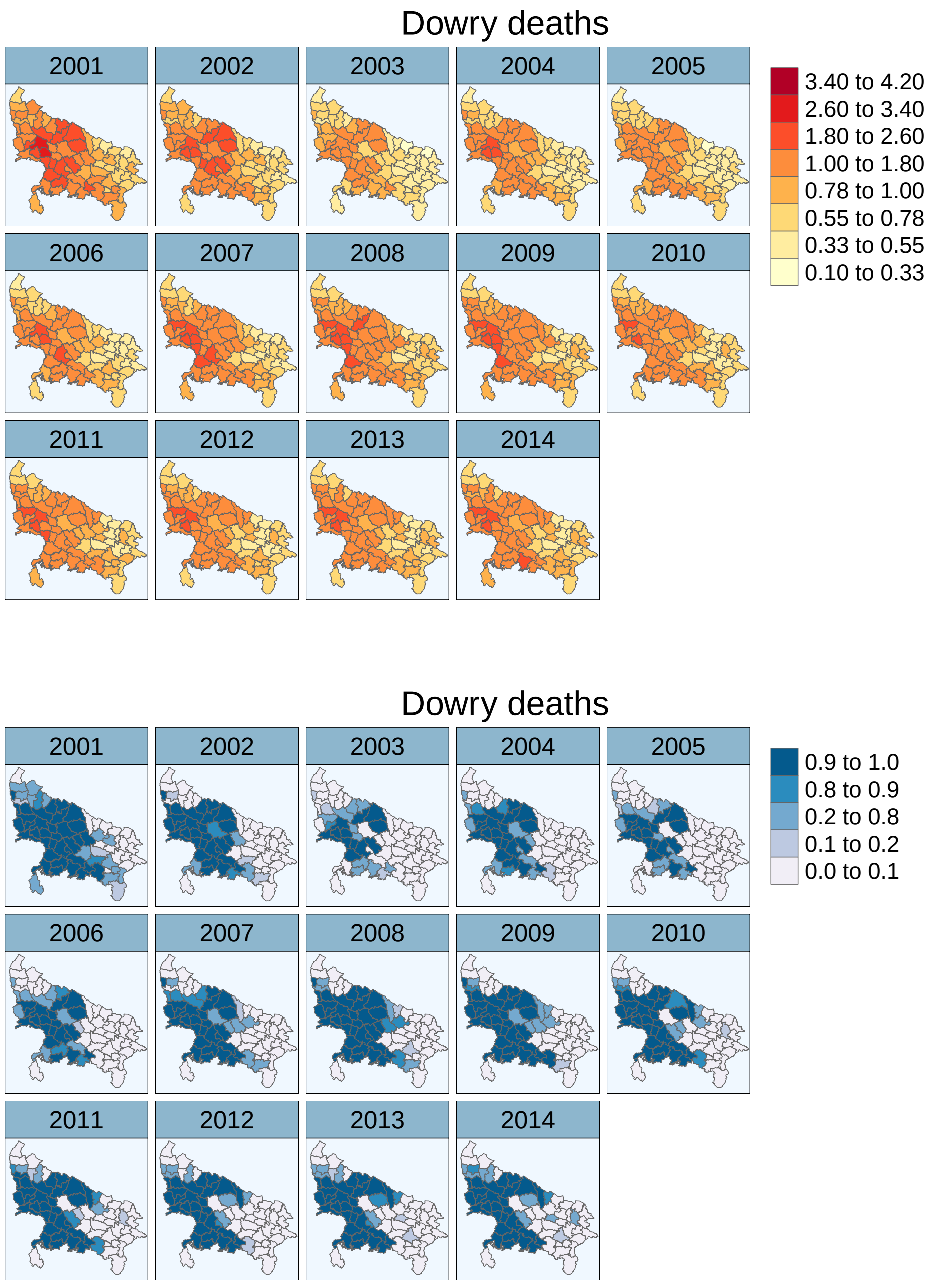

Figure 3.6: Map of estimated incidence risks for dowry deaths (top) and posterior probabilities that the relative risk is greater than one ($P(R_{itj} > 1|\mathbf{O})$) (bottom) in Uttar Pradesh.

In Mainpuri, the risk of dowry deaths is significantly high during the whole period in contrast to rapes. Sant Kabir Nagar has a significant low risk of both crimes until 2009 approximately. From then on, the trends start to diverge due to a significant increase of the risk of rapes. Varanasi has significant low risks with a fairly stable evolution for both crimes, though they tend to one at the end of the period.

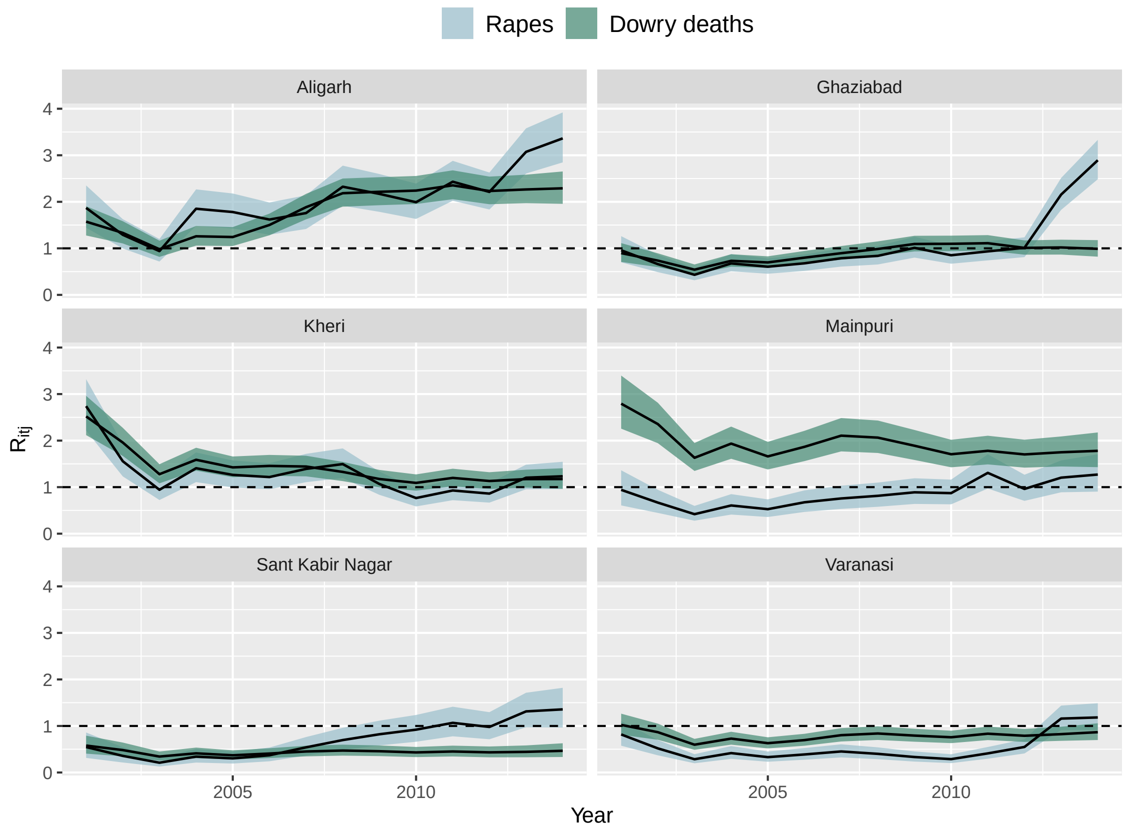

Figure 3.7: Temporal evolution of final risk estimates for rapes and dowry deaths in some districts in Uttar Pradesh: Ghazlabad, Kheri, Mainpuri, Sant Kabir Nagar, and Varanasi.

3.4 Discussion

The use of spatio-temporal areal models to analyze crimes against women has been the exception rather than the rule. Multivariate models are powerful techniques that provide valuable information to locate hot spots and may help social researchers to make hypotheses about potential risk factors related to certain forms of violence against women. Given the multifaceted dimension of crimes against women and the difficulty to determine relationships between crimes and socio-economic, demographic, religious factors, and other transitory or circumstantial elements, a multivariate approach may help to reveal relationships between different crimes that can shed light on this complex phenomenon. Moreover, if it is believed that different crimes against women could share risk factors, a rather sensible approach, the use of multivariate spatio-temporal models will make it possible to estimate these dependencies and improve understanding of the problem.

In this chapter, we extend the spatial M-models proposed by Botella-Rocamora et al. (2015) to a spatio-temporal setting. In addition to the spatial M-model, we introduce a temporal M-model and a spatio-temporal interaction. The model makes it possible to estimate correlations between spatial and temporal patterns which would respectively indicate potential geographical factors and transitory events related to both crimes. As the interaction term is a residual term, we do not consider inter-crime dependence for this term because variability is mainly captured by the main effects. Instead, we use different variance parameters for both crimes leading to a different amount of spatio-temporal smoothing. This model provides better results than a model with the same variance parameter. This seems sensible as the standard deviation of the spatio-temporal random effects for rapes is about twice the standard deviation for dowry deaths. Different models have been considered to analyse the data, but those that achieve the best tradeoff between complexity and goodness of fit (measured in terms of DIC and WAIC), and prediction ability (measured with the LS) are the so called M-models with an iCAR prior for space, a RW1 prior for time, and a Type II interaction. In fact, the crime-specific spatial parameters of the pCAR and LCAR model are very close to one pointing towards the iCAR prior for space.

The analysis of rapes and dowry deaths in Uttar Pradesh reveals interesting findings. On one hand, the correlation between the estimated spatial patterns is positive and significant, though not very strong (0.30). This indicates that certain districts tend to present high risks of both crimes, but the underlying spatial patterns are not similar. The estimated pattern reveals that the risks of rapes and dowry deaths in the most eastern districts of Uttar Pradesh are significantly low, and consequently further insight is needed to study the characteristics of these regions which could bring light to the understanding of the phenomena being studied. On the other hand, the estimated correlation between temporal patterns is 0.82, indicating a strong, positive association and that the two crimes evolve in line. We could hypothesize that certain policies or laws, such as the Protection of Women from Domestic Violence Act 2005, has had some influence on both crimes, but it is rather complex to validate such hypothesis.

Finally, model fitting has been implemented using WinBUGS and INLA. In particular, we have implemented the LCAR and BYM M-models in INLA. The implementation of our multivariate proposal using R-INLA allows non-expert users to fit these models without difficulty, as they are integrated in the usual package syntax. Our study indicates that there are practically no differences between WinBUGS and INLA in the data analysis considered in this chapter in terms of relative risk estimates, and the derived spatial and temporal patterns. Small differences were only observed in the model hyperparameter estimates. In addition, we have seen that in the cases analyzed here, INLA is, in general, a computationally more efficient alternative than WinBUGS. However, further research is needed when the number of areas, time periods, and crimes increases as INLA does not replace Kronecker products by simple matrix products. We are currently investigating this issue.

The contents of this chapter have been accepted for publication in *Stochastic Environmental Research and Risk Assessment*.

The WinBUGS and INLA code to fit all models presented in this chapter is available at https://github.com/spatialstatisticsupna/Mmodels_SERRA_article.

The univariate analysis of rapes in the districts of Uttar Pradesh has been published in *Statistics and Aplications*. See https://www.ssca.org.in/journalvolumes/1/.

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4 Multivariate spatio-temporal splines

4.1 Introduction

The statistical toolkit for analyzing spatial and spatio-temporal areal count data has been enriched during the last years with relevant advances in model proposals, algorithms for inference and the realm of applications (see for example Lawson et al., 2016; Martínez-Beneito and Botella-Rocamora, 2019). The research on multivariate models for spatial count data, and in particular for disease mapping, is now rich, though the use of multivariate models is still limited due to the computational burden and a lack of software that can be adopted by practitioners without advanced computer skills. Most of the research extends univariate CAR models to the multivariate setting.

Research on multivariate splines models for spatio-temporal count data is not so abundant, and it focuses on multivariate structures to deal with spatial and temporal dependence for one response. MacNab (2007) and MacNab and Gustafson (2007) consider B-splines to model temporal trends. In particular, they consider different B splines for the different small areas and they introduce spatial correlation in the coefficients of the B splines. Ugarte et al. (2017) consider spatial P-splines to smooth geographical patterns, and they introduce temporal dependence through the coefficients of the P-splines, such that smooth surfaces in close time points are similar. Analogously, they consider temporal P-splines with spatial dependence on the coefficients so that temporal trends from neighbouring areas are similar. As far as we know, there is no research about multivariate P-spline models for areal count data in which different responses are analysed at the same time.

In this chapter, we propose multivariate P-spline models to analyse different forms of violence on women jointly. We consider spatial and temporal P-splines to model the geographical and temporal patterns respectively. Correlations among the coefficients of the P-splines for the different crimes are introduced to look into spatial

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and temporal associations between the crimes. In particular, we illustrate the results analysing four distinct crimes against women in the Indian state of Maharashtra during the period 2001-2013. Namely, *rape*, *assault*, *cruelty by husband or relatives*, and *kidnapping and abduction*. In this chapter, we also include the implementation of these models in R-INLA using the "rgeneric" fuction to build our multivariate proposal.

The rest of the chapter has the following structure: Section 4.2 proposes multivariate spatial and temporal P-splines accounting for correlations between the different responses of interest (crimes against women here). Section 4.3 explains some identifiability issues, prior specifications and implementation in R-INLA. Section 4.4 presents the real data analysis. We close the chapter with a discussion.

4.2 Multivariate P-spline models

Denoting by O_{itj} and by E_{itj} the observed and expected number of cases respectively in area $i = 1, \ldots, I$, time $t = 1, \ldots, T$, and crime $j = 1, \ldots J$, and assuming a Poisson distribution for the number of observed cases conditional on the relative risks R_{itj}, we have

$$O_{itj} | R_{itj} \sim Poisson(\mu_{itj} = E_{itj} \cdot R_{itj}).$$

Usually, in multivariate spatial count data, the log risk is modelled using CAR models. Here we use P-spline models to smooth the spatial and temporal patterns. That is

$$\log(R_{itj}) = \alpha_j + f_j(x_{1i}, x_{2i}) + f_j(x_t) + \delta_{itj}, \tag{4.1}$$

where α_j is the intercept for the jth crime, x_{1i} and x_{2i} are the longitude and latitude of the centroid of the ith area; x_t indicates the time point, and $f_j(x_{1i}, x_{2i})$ and $f_j(x_t)$ are a smooth spatial surface and a smooth temporal trend respectively for the jth crime that are well approximated using P-splines. Finally, δ_{itj} is the spatio-temporal interaction for crime j. The smooth surface for each crime is specified as $f_j(x_1, x_2) = \mathbf{B}_s \boldsymbol{\psi}^{(j)}$, where $\mathbf{B}_s = \mathbf{B}_2 \Box \mathbf{B}_1$ is a two-dimensional B-spline basis of dimension $I \times k_1 k_2$ arising from the row-wise Kronecker product (Eilers et al., 2006) of the marginal B-splines basis for longitud, $\mathbf{B}_1$, and latitud $\mathbf{B}_2$, and $\boldsymbol{\psi}^{(j)} = (\psi_1^j, \ldots, \psi_{k_1 k_2}^j)'$, $j = 1, \ldots, J$. Note that k_1 and k_2 are the number of columns of the marginal basis $\mathbf{B}_1$ and $\mathbf{B}_2$ respectively, and depend on the number of knots and the degree of the polynomials to construct these bases. To achieve smoothness, the following prior distribution is considered for the coefficients of the two-dimensional B-splines

$$p(\boldsymbol{\psi}^{(j)}) \propto \exp\left(-\frac{1}{2} \boldsymbol{\psi}^{(j)\prime} \mathbf{P}_s \boldsymbol{\psi}^{(j)}\right). \tag{4.2}$$

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Here $\mathbf{P}_s = \lambda_1(\mathbf{I}_{k_2} \otimes \mathbf{D}_1'\mathbf{D}_1) + \lambda_2(\mathbf{D}_2'\mathbf{D}_2 \otimes \mathbf{I}_{k_1})$, $\mathbf{I}_{k_1}$ and $\mathbf{I}_{k_2}$ are $k_1 \times k_1$ and $k_2 \times k_2$ identity matrices, λ_1 and λ_2 control the amount of smoothing in longitude and latitude, and $\mathbf{D}_1$ and $\mathbf{D}_2$ are difference matrices (of order 1 or 2) of dimension $k_1 \times k_1$ and $k_2 \times k_2$ respectively. Similarly, the smooth temporal trend for each crime is specified as $f_j(x_t) = \mathbf{B}_t\boldsymbol{\gamma}^{(j)}$, where now $\mathbf{B}_t$ is the temporal B-spline basis of dimension $T \times k_t$, where k_t depends on the number of knots and the degree of the polynomials in the basis, and $\boldsymbol{\gamma}^{(j)} = (\gamma_1^j, \ldots, \gamma_{k_t}^j)'$, $j = 1, \ldots, J$. As in the spatial case, smoothing is achieved through the prior distribution for the coefficients of the B-splines. Here the following prior distribution with Gaussian kernel is considered

$$p(\boldsymbol{\gamma}^{(j)}) \propto \exp\left(-\frac{1}{2}\boldsymbol{\gamma}^{(j)'}\mathbf{P}_t\boldsymbol{\gamma}^{(j)}\right), \tag{4.3}$$

where $\mathbf{P}_t = \lambda_t\mathbf{D}_t'\mathbf{D}_t$, λ_t controls the temporal smoothing and $\mathbf{D}_t$ is a difference matrix (of order 1 or 2) of dimension $k_t \times k_t$. Note that the prior distributions in Equations (4.2) and (4.3) are the Bayesian analogues to the penalty on the coefficients proposed by Eilers and Marx (1996) to achieve smoothness. This is clear as the matrices $\mathbf{D}_l'\mathbf{D}_l$, $l = 1, 2, t$ are the precision matrices of a random walk of first or second order (see Rue and Held, 2005, pp. 95 and 110). For more details about bases and penalties in a disease mapping context, see for example Ugarte et al. (2010, 2017).

To take account of the potential relationships between the different crimes, correlations among the coefficients of the P-splines (spatial or temporal) of the different crimes are introduced in the model. That is, we rearrange the coefficients of spatial and temporal P-splines for each crime in the matrices $\boldsymbol{\Psi} = (\boldsymbol{\psi}^{(1)}, \ldots, \boldsymbol{\psi}^{(J)})$ and $\boldsymbol{\Gamma} = (\boldsymbol{\gamma}^{(1)}, \ldots, \boldsymbol{\gamma}^{(J)})$, so that each column represents the set of P-spline coefficients (spatial and temporal respectively) for each crime. Then, the following $J \times J$ covariance matrices between the sets of spatial and temporal P-splines coefficients are assumed

$$\begin{aligned} \mathrm{Cov}(\boldsymbol{\Psi}) &= \boldsymbol{\Sigma}_\psi \\ \mathrm{Cov}(\boldsymbol{\Gamma}) &= \boldsymbol{\Sigma}_\gamma. \end{aligned}$$

The diagonal elements of $\boldsymbol{\Sigma}_\psi$ are $(\boldsymbol{\Sigma}_\psi)_{jj} = \sigma_{\psi j}^2$, $j = 1, \ldots, J$, and the off-diagonal elements are given by $(\boldsymbol{\Sigma}_\psi)_{jk} = \rho_{\psi jk}\sigma_{\psi j}\sigma_{\psi k}$, where $\rho_{\psi jk}$ is the correlation between the coefficients of the spatial P-splines for crimes j and k. Similarly, the diagonal elements of $\boldsymbol{\Sigma}_\gamma$ are $(\boldsymbol{\Sigma}_\gamma)_{jj} = \sigma_{\gamma j}^2$, $j = 1, \ldots, J$, and the off-diagonal elements are expressed as $(\boldsymbol{\Sigma}_\gamma)_{jk} = \rho_{\gamma jk}\sigma_{\gamma j}\sigma_{\gamma k}$, where analogously to the spatial case, $\rho_{\gamma jk}$ represents the correlation between the coefficients of the temporal P-splines for crimes j and k.

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In matrix form, the joint multivariate P-spline model can be expressed as

$$\begin{aligned}\log(\mathbf{r}) &= (\mathbf{I}_J \otimes \mathbf{1}_T \otimes \mathbf{1}_I)\boldsymbol{\alpha} + (\mathbf{I}_J \otimes \mathbf{1}_T \otimes \mathbf{B}_s)\,\mathrm{vec}(\boldsymbol{\Psi}) + (\mathbf{I}_J \otimes \mathbf{B}_t \otimes \mathbf{1}_I)\,\mathrm{vec}(\boldsymbol{\Gamma}) \\ &\quad + (\mathbf{I}_J \otimes \mathbf{I}_T \otimes \mathbf{I}_I)\,\mathrm{vec}(\boldsymbol{\Delta}) \qquad (4.4)\end{aligned}$$

where $\mathbf{r} = (\mathbf{r}'_1, \ldots, \mathbf{r}'_J)'$, $\mathbf{r}'_j = (r_{11,j}, \ldots, r_{S1,j}, \ldots, r_{1T,j}, \ldots, r_{ST,j})$, $j = 1, \ldots, J$, $\mathbf{I}_J$ is a $J \times J$ identity matrix, and $\mathbf{1}_I$ and $\mathbf{1}_T$ are column vectors of ones of length I and T respectively. The vec operator stacks the columns of a matrix one under the other, hence $\mathrm{vec}(\boldsymbol{\Psi}) = (\boldsymbol{\psi}^{(1)'}, \ldots, \boldsymbol{\psi}^{(J)'})'$ and $\mathrm{vec}(\boldsymbol{\Gamma}) = (\boldsymbol{\gamma}^{(1)'}, \ldots, \boldsymbol{\gamma}^{(J)'})'$. The precision matrix for $\mathrm{vec}(\Psi)$ is given by $\boldsymbol{\Sigma}_\psi^{-1} \otimes \mathbf{P}_s$, and the precision matrix for $\mathrm{vec}(\boldsymbol{\Gamma})$ is given by $\boldsymbol{\Sigma}_\gamma^{-1} \otimes \mathbf{P}_t$. Here, $\boldsymbol{\Delta} = (\boldsymbol{\delta}^{(1)}, \ldots, \boldsymbol{\delta}^{(J)})$, where $\boldsymbol{\delta}^{(j)} = (\delta_{11}^{(j)}, \ldots, \delta_{S1}^{(j)}, \ldots, \delta_{1T}^{(j)}, \ldots, \delta_{ST}^{(j)})'$ is the spatio-temporal interaction for the jth crime, $j = 1 \ldots, J$, and $\mathrm{vec}(\boldsymbol{\Delta}) = (\boldsymbol{\delta}^{(1)'}, \ldots, \boldsymbol{\delta}^{(J)'})'$. For the interaction term, the following prior distribution is assumed

$$p(\boldsymbol{\delta}^{(j)}) \propto \exp\left(-\frac{1}{2}\boldsymbol{\delta}^{(j)'}\mathbf{R}\delta^{(j)}\right),$$

where $\mathbf{R}$ is a precision matrix that can take different forms according to the four types of interactions defined by Knorr-Held (2000) and already explained in Chapter 2.

4.3 INLA fit, prior distributions, and identifiability

In this chapter, the models are fitted using INLA. Here we put random walks priors of first or second order (RW1 or RW2 respectively) on the coefficients of the B-splines to achieve smoothness. The models are fitted using the "rgeneric" function in R-INLA. In particular, a modification of the "inla.rgeneric.IMCAR.model" from the R package "INLAMSM" (`https://CRAN.R-project.org/package=INLAMSM`) developed by Palmí-Perales et al. (2019) and designed for fitting multivariate extensions of intrinsic conditional autoregressive models (Besag, 1974) has been used. This function has been modified to fit P-splines. The function requires to parameterize the covariance matrix between crimes in terms of internal parameters, and the variances and correlations can be conveniently recovered through appropriate transformations. More precisely, the rgeneric function defined for the multivariate setting requires internal generic parameters $\boldsymbol{\theta}$. In particular, for the covariance matrix between the set of P-splines coefficients (spatial or temporal), $J(J+1)/2$ parameters are needed, where J is the number of crimes, that is, $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_{J(J+1)/2})$. Then to recover the standard deviations σ's and the correlations ρ's between the crimes, the following transformations are used

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$$\begin{aligned}\sigma &= g(\theta) = \exp(-0.5 * \theta)\\ \rho &= g(\theta) = 2\exp(\theta)/(1+\exp(\theta)) - 1,\end{aligned}$$

where θ is the corresponding element of the internal parameters $\boldsymbol{\theta}$. These transformations guarantee that the standard deviations are positive and the correlation parameters take values between -1 and 1. The code used to fit the models is available in the GitHub of our research team (https://github.com/spatialstatisticsupna). Users only have to download the rgeneric function created to fit the multivariate P-splines and then to write INLA formulas as usual.

4.3.1 Prior distributions and identifiability issues

Prior distributions for the hyperparamerters are always a controversial issue in Bayesian statistics due to sensitivity of result to the prior choice. Regarding the covariance matrix of the coefficients of the spatial and temporal P-splines for the different crimes, a Wishart distribution has been considered, that is $\boldsymbol{\Sigma}_{\psi}, \boldsymbol{\Sigma}_{\gamma} \sim Wishart(\upsilon, \mathbf{I}_J)$, where J indicates the number of crimes, and υ the degrees of freedom. Hence, the prior mean is $\upsilon\mathbf{I}_J$. Here, the degrees of freedom are set equal to $\upsilon = 2J+1$ (see Palmí-Perales et al., 2019). Regarding the smoothing/precision parameters λ in the precision matrices of the P-splines coefficients, uniform distributions in the interval $(0, 100)$ are considered for $1/\sqrt{\lambda_1}$, $1/\sqrt{\lambda_2}$, and $1/\sqrt{\lambda_t}$. Also improper uniform distributions in $(0, \infty)$ are considered for the standard deviations $\sigma_{\delta_j} = 1/\sqrt{\tau_j}$ of the spatio-temporal interaction (Ugarte et al., 2017).

Our models incorporate crime-specific intercepts, P-splines for space, P-splines for time, and spatio-temporal interactions of the type proposed by Knorr-Held (2000). As the crime-specific spatial surfaces (spatial P-splines) and the crime-specific temporal trends (temporal P-splines) also include an intercept, identifiability issues arise. To overcome this problem, constraints on the coefficients of the crime-specific spatial and temporal P-splines are considered. In particular, and regardless the prior for the coefficients is a RW1 or a RW2, the following constraints are considered

$$\begin{aligned}\sum_{m=1}^{k_1 k_2} \psi_m^{(j)} &= 0, \quad \forall j = 1, \ldots, J,\\ \sum_{m=1}^{k_t} \gamma_m^{(j)} &= 0, \quad \forall j = 1, \ldots, J.\end{aligned}$$

Also, the interaction effects overlap with the main spatial and temporal P-splines terms and additional constraints are required. The constraints for the interaction terms are based on the work by Goicoa et al. (2018).

Additionally, the λ_t parameter in the precision matrix of the temporal P-splines is not identifiable as it can be subsumed in the matrix $\mathbf{\Sigma}_\gamma^{-1}$ (or $\sigma_t^2 = 1/\lambda_t$ is subsumed in $\mathbf{\Sigma}_\gamma$). Similarly, the parameters $\sigma_{\gamma j}$, $j = 1, \ldots, J$ are not identifiable, so inferences on these quantities should be precluded. On the contrary, the correlation parameters $\rho_{\gamma jk}$ are identifiable. This is clear as $\rho_{\gamma jk} = \sigma_{\gamma jk}/(\sigma_{\gamma j}\sigma_{\gamma j})$, where $\sigma_{\gamma jk}$ is the covariance between the coefficients of the temporal P-splines of crimes j and k. Suppose that $\sigma_t^2 = 1/\lambda_t$ is subsumed in the between-crimes covariance matrix, denoted now by $\mathbf{\Sigma}_\gamma^*$. Then, the correlation parameters are identifiable as

$$\rho_{\gamma jk}^* = \sigma_{\gamma jk}^*/(\sigma_{\gamma j}^*\sigma_{\gamma j}^*) = (\sigma_{\gamma jk}\sigma_t^2)/(\sigma_t\sigma_{\gamma j}\sigma_t\sigma_{\gamma j}) = \rho_{\gamma jk}.$$

4.4 Case study: rape, assault, cruelty by husband and relatives, and kidnapping and abduction in Maharashtra, India

In this section, data on crimes against women in the Indian state of Maharashtra over the period 2001-2013 are analyzed using the methodology developed in the previous sections. Data consist of the observed number of cases in each of the 34 districts of Maharashtra for four crimes described in the Indian Penal Code (IPC). Namely, *Rape*, *Assault or criminal force to woman with intent to outrage her modesty*, *Cruelty by husband or relatives of husband*, and *Kidnapping and abduction*.

4.4.1 Descriptive analysis

Maharashtra is located in the middle west of the Indian peninsula, see Figure 4.1, and according to the 2011 census (see https://www.census2011.co.in), it is the second most highly populated state in the country (112,374,333 inhabitants), surpassed only by Uttar Pradesh. It is also the third largest state of India with a total of 307,713 km^2. The overall literacy rate is 82.34%, about 8 percentage points over the overall literacy rate in India (74.04%). Similar to all Indian states, male literacy rate (88.38%) is greater than female literacy rate (75.87%), though these figures are well above the Indian male (82.14%) and female (65.46%) literacy rates. Sex ratio (number of females per 1000 males) is 922, slightly lower than the sex ratio in whole India (933). The majority religion (79.83%) is Hindu.

In the last years, the number of crimes against women in Maharashtra has grown horrifyingly. According to National Crime Record Bureau (NCRB), in the study period the incidence of crimes against women has doubled, from 12,524 in 2001 to

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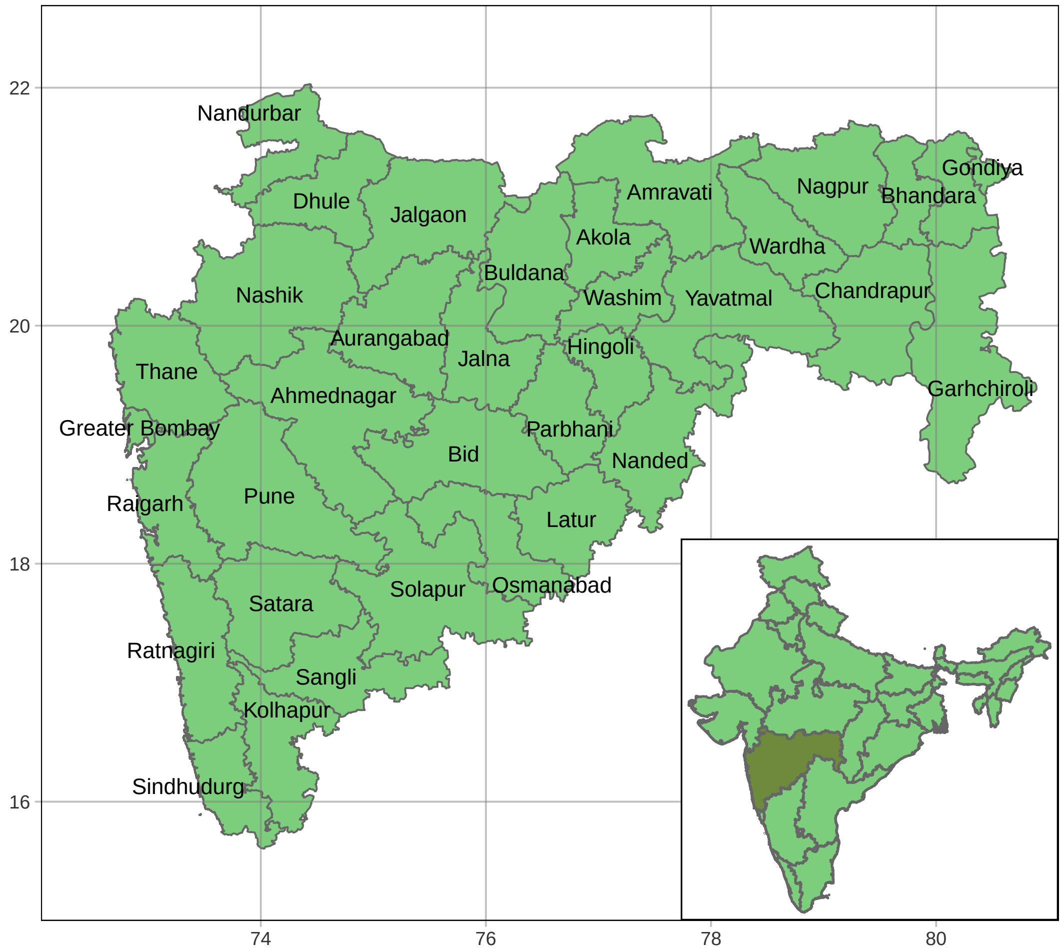

Figure 4.1: Map of the administrative division of Maharashtra into districts and its location in the west middle of India (bottom right corner).

24,895 in 2013. In terms of rates, the increase is even worse as the overall rate of crimes against women in 2013 (44.9) is about 3.5 higher than in 2001 (12.9).

One of the most endemic form of violence against women in different countries of the world is the abuse of women by their intimate partner. The problem is even more serious as it has one of the highest rates of underreporting, (see Jejeebhoy, 1998; Visaria, 1999; Heise et al., 1994, and the references therein). Unfortunately, the state of Maharashtra is not the exception. Figure 4.2 displays a circular barplot with the crude rates (per 100000 women between the age of 15 and 49 years) of the four different crimes considered in this chapter by district. Clearly, cruelty by husband or relatives is the crime against women with the highest incidence, followed by assault on women. During the thirteen years of the study period, a total of 21,049 rapes, 47,351 assaults, 12,727 kidnappings and 88,905 cases of cruelty by the husband were committed in this state. Interestingly, differences are found among the districts. In general, the lowest rates of crimes correspond to the most western districts with the exception of Great Bombay, whereas districts in the central and northeastern part of the state have the highest rates of crimes. Some descriptive statistics regarding the number of cases of the four crimes in the first and last year of the study period are shown in Table 4.1.

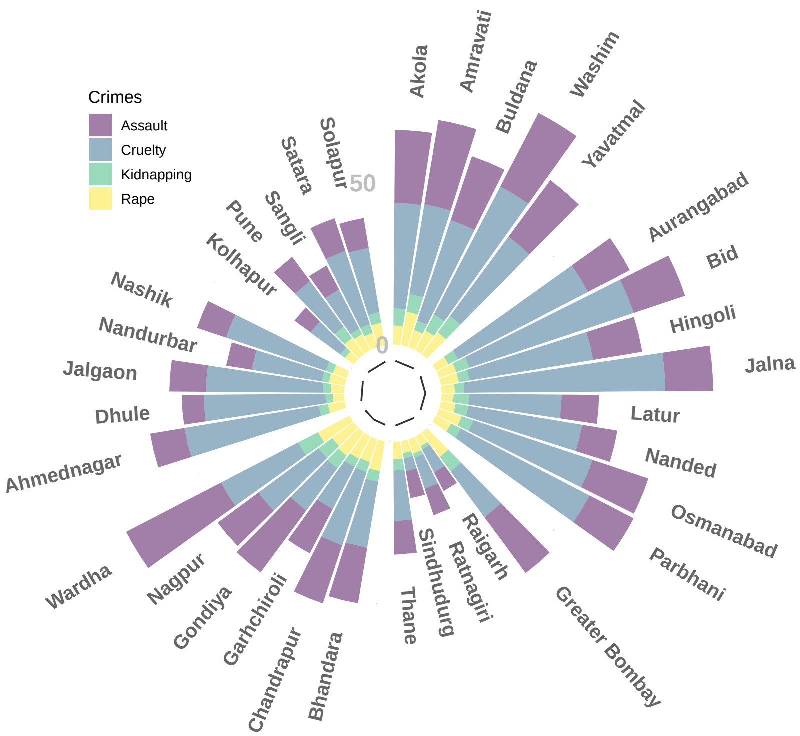

Figure 4.2: Incidence rates of crimes against women in Maharashtra between 2001 and 2013.

Figure 4.3 displays the evolution of the standardized incidence ratio (SIR) for the four crimes. A SIR over one indicates that the number of cases for one crime in a particular year is greater than expected in comparison with the whole study period. A close inspection to this figure reveals rather flat trends for rapes, assault

Table 4.1: Minimum (min), first quartile ($q_{.25}$), mean, third quartile ($q_{.75}$), maximum (max), standard desviation (sd), and coefficient of variation (cv) of the number of crimes against women in the districts of Maharastra in 2001 and 2013.

Crime	Year	min	mean	$q_{.25}$	median	$q_{.75}$	max	sd	cv
Rape	2001	7	25.50	16.75	38.29	43.75	146	33.15	1.30
	2013	11	58.00	40.00	90.09	101.00	466	96.51	1.66
Assault on women	2001	17	72.50	50.50	83.03	101.00	302	56.39	0.78
	2013	44	166.00	99.50	239.18	283.50	1342	247.74	1.49
Cruelty by husband or relatives	2001	8	148.50	90.75	179.12	217.75	496	119.58	0.81
	2013	12	202.50	117.25	251.24	373.50	741	187.74	0.93
Kidnapping and abduction	2001	1	13.50	7.00	17.97	21.50	67	15.71	1.16
	2013	5	37.50	21.00	55.12	52.00	292	64.66	1.72

and cruelty until 2012. Then, a strong rise is observed in 2013 for rape and assault. Regarding kidnapping and abduction, a steady growth in SIR is observed along the period, with a pronounced increase in the last year. According to some authors, this increase may be attributed to an improvement in the victim support system (Raj and McDougal, 2014) or studies trying to identify and localize the problem (see, for example Jain et al., 2004).

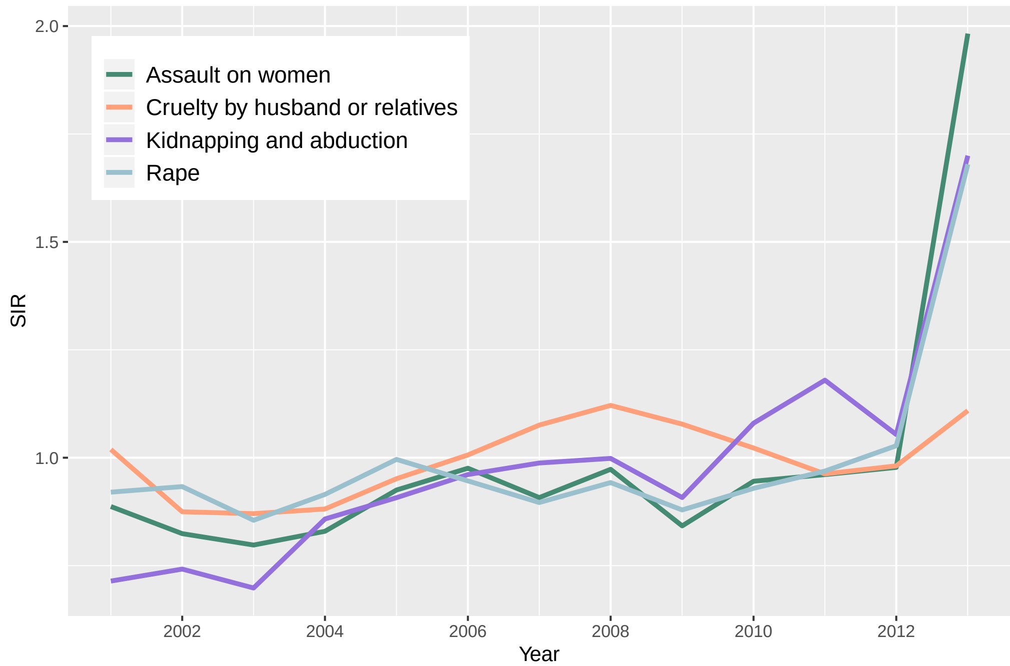

Figure 4.3: Standardized incidence rate (SIR) by crime.

To identify any potential relationship between the crimes, we have computed Pearson correlations between the overall spatial patterns of each crime. That is, we have computed the SIR for each district and crime in the study period and calculated the corresponding correlations. The greatest correlations were found between rapes and assaults (0.79), rapes and kidnapping (0.72) and assaults and kidnapping (0.81). The correlation of these three crimes with cruelty were low. Similarly, we have computed Pearson correlations between the overall temporal trends shown in Figure 4.3. Again, the highest correlations are found between rapes and assaults (0.99), rapes and kidnapping (0.87), and assaults and kidnapping (0.90). The high correlations between these three crimes may be explained because all of them are related in a greater or less extent to sexual offences. However, the statistical analysis has to confirm this exploratory analysis and then social researchers should look for potential factors that might be related to these crimes.

4.4.2 Model fitting using INLA

The spatial and temporal P-splines proposed in Section 4.2 are implemented for the joint spatio-temporal analysis of the incidence of four crimes against women in Maharashtra during the years 2001 and 2013. To fit the P-spline model, cubic B-splines with first and second order penalties have been used for the spatial and temporal dimensions. For longitude and latitude, 10 equidistant internal knots have been considered, and for the temporal dimension, 5 internal knots have been chosen. The final dimensions of the spatial and temporal B-spline bases are 34×144 and 13×7 respectively. Previous to fit the multivariate P-spline models, a battery of univariate P-spline models for each crime has been run. In particular, different combinations of first and second order random walks prior for the spatial and temporal coefficients have been considered. The four types of interactions defined by Knorr-Held (2000) and commented in previous chapters have also been examined. According to the model selection criteria, DIC, WAIC, and LS, first order penalties for space and time together with Type II interactions for the spatio-temporal term are the best option for all crimes. Consequently, and on the basis of these results, the joint multivariate P-spline model given in Equation (4.4) is fitted considering a Type II spatio-temporal interaction for all crimes. We also examine RW1 and RW2 prior distributions for the coefficients. To account for potential relationships between the different crimes, between-crime correlations among the coefficients of the P-splines (spatial and temporal) are introduced in the model. Additionally, and given that the temporal trends for some crimes are rather flat, we also consider models without correlation between the coefficients of the temporal P-splines, that is, $\mathbf{\Sigma}_{\gamma} = \text{diag}(\sigma^2_{\gamma 1}, \cdots, \sigma^2_{\gamma J})$. Table 4.2 displays model selection criteria for the multivariate P-spline models fitted with different combinations of RW1 and RW2 priors for the coefficients of the spatial and temporal P-splines. According to DIC, WAIC, and LS, the best candidate is a

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model accounting for correlation between the coefficients of the spatial and temporal B-splines, and RW1 prior distribution for the coefficients. This is the model we finally select to analyse the four crimes.

Table 4.2: Model selection criteria, DIC, WAIC and LS, for multivariate models.

vec($\mathbf{\Delta}$)	Temporal correlations	Prior Spat.	Prior Temp.	DIC	WAIC	LS
Type II	TRUE	RW1	RW1	**13081.023**	**13094.365**	**3.881**
			RW2	13292.101	13514.950	3.965
		RW2	RW1	13091.986	13104.106	3.887
			RW2	13290.992	13508.344	3.965
	FALSE	RW1	RW1	13090.897	13105.697	3.888
			RW2	13265.264	13472.533	3.952
		RW2	RW1	13103.418	13118.459	3.894
			RW2	13263.815	13470.391	3.951

Model selection criteria in Table 4.2 point towards multivariate models, as models with independent temporal trends clearly perform worse. Besides these criteria, multivariate models provide posterior correlations between crimes and hence it is possible to establish relationships between them. That is, those crimes with high posterior correlations could share some risk factors. There is one more advantage of the multivariate models over the univariate counterparts. The joint modelling of several crimes allows borrowing information not only from neighbouring areas and time points, but also from other crimes. In general, posterior medians of the quantities of interest (relative risks, spatial, temporal and spatio-temporal patterns) are rather similar using the multivariate proposal and the univariate models (not shown here). However, estimates are more precise with the multivariate proposal.

We have computed 95% credible intervals for the relative risks, the spatial pattern, the temporal trends, and the spatio-temporal interaction term for each of the four crimes analysed in Maharashtra (not shown here). On average, the selected multivariate P-spline model provides narrower credible intervals for all quantities of interest. This is specially noticeable for the temporal pattern and for rapes and kidnapping and abduction, which have the lowest incidence rate of the four crimes examined here. Figure 4.4 displays violin plots of the widths for the 95% credible intervals of the temporal patterns obtained with the selected multivariate P-spline model and the univariate counterparts (univariate P-splines models with RW1 prior distributions and a Type II interaction). From this picture, it is clear that 95% credible intervals for the temporal component obtained with the multivariate model are narrower than the ones obtained with the univariate counterparts, particularly for rapes and kidnapping and abduction.

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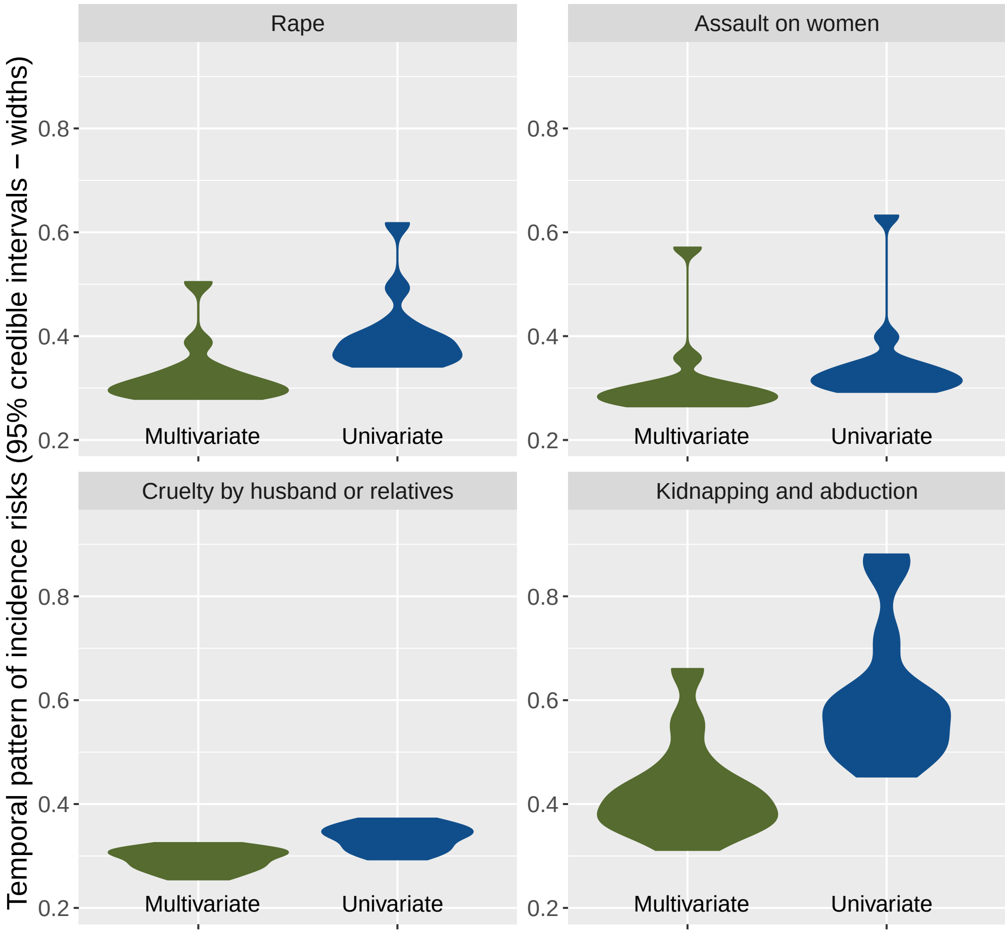

Figure 4.4: Violin plots representing the widths of the 95% credible intervals for the temporal pattern of incidence risks obtained with the multivariate and univariate P-splines.

Computations were run on a twin superserver with four processors, Intel Xeon 6C and 96GB RAM, using the R-INLA (stable) version 19.09.03. All models in the chapter have been fitted using a simplified Laplace strategy, which provides a tradeoff between computing time and accuracy. Computing times for the multivariate models range between 100 and 115 minutes for models with independent temporal P-splines and between 194 and 210 minutes for models including correlations in both the spatial and the temporal P-splines. In particular, the computing time for the selected model in this chapter is 195.979 minutes. Models have also been fitted using a Gaussian strategy, which is faster though less precise. Computing times are

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significantly reduced ranging between 5 and 11 minutes. Some small differences are found in model selection criteria, but the same model is selected and results are pretty similar. Along the chapter, results correspond to a simplified Laplace strategy.

4.4.3 Joint analysis of four crimes against a women in Maharashtra using multivariate P-spline model

In this subsection, the spatio-temporal patterns of the four crimes in the state of Maharashtra are examined using the selected multivariate P-spline model.

The underlying spatial patterns and the global temporal trends may be very informative as similarities between crimes could be detected. The district-specific spatial risk for each crime, $\exp(f_j(x_{1i}, x_{2i}))$, is related to the idiosyncrasy of the districts. It captures the risk associated to a spatial location, and it may reflect the effect of potential spatial risk factors such as certain traditions or demographic and socio-economic characteristics specific to certain districts or regions in the state of Maharashtra. Posterior medians of the district-specific spatial risk for each crime are displayed in Figure 4.5. The maps with the exceedance probability, i.e., $P(\exp(f_j(x_{1i}, x_{2i})) > 1|\mathbf{O})$ are shown in Figure B.1 in Appendix B. A Northeast-Southwest gradient (following the largest diagonal axis of the map) can be observed very clearly for rape, assault and kidnapping and abduction, the pattern being smoother for kidnapping. Though some differences exists, the spatial patterns of these three crimes are rather similar. On the other hand, the spatial pattern of cruelty by husband and relatives is different. For this crime, districts with high risk are mainly located in the central part of the map, and a Northwest-Southeast gradient can be envisaged.

The estimated posterior medians of the correlations between the coefficients of the spatial P-splines confirm these findings. Table 4.3 displays these posterior correlations below the main diagonal. Significant correlations are highlighted in bold. The posterior correlation between rape and assault is particularly high with a posterior median of 0.782. This may point towards underlying spatial factors affecting both crimes. The posterior correlations between rape and kidnapping, and between assault and kidnapping are weaker though significant, with estimated posterior medians of 0.384 and 0.411, respectively. The spatial pattern for cruelty is different. Whereas most of the districts in the Northeast part of the state exhibit a high risk of rape, assault, and kidnapping, districts with high risk of cruelty by husband are mainly located in the central part of the map. In fact, as we move away from the center, the map is becoming increasingly lighter. The posterior correlations between the coefficients of the spatial P-splines of cruelty by husband and the rest of crimes are not significant.

The global temporal evolution of each crime in Maharashtra is revealed by the

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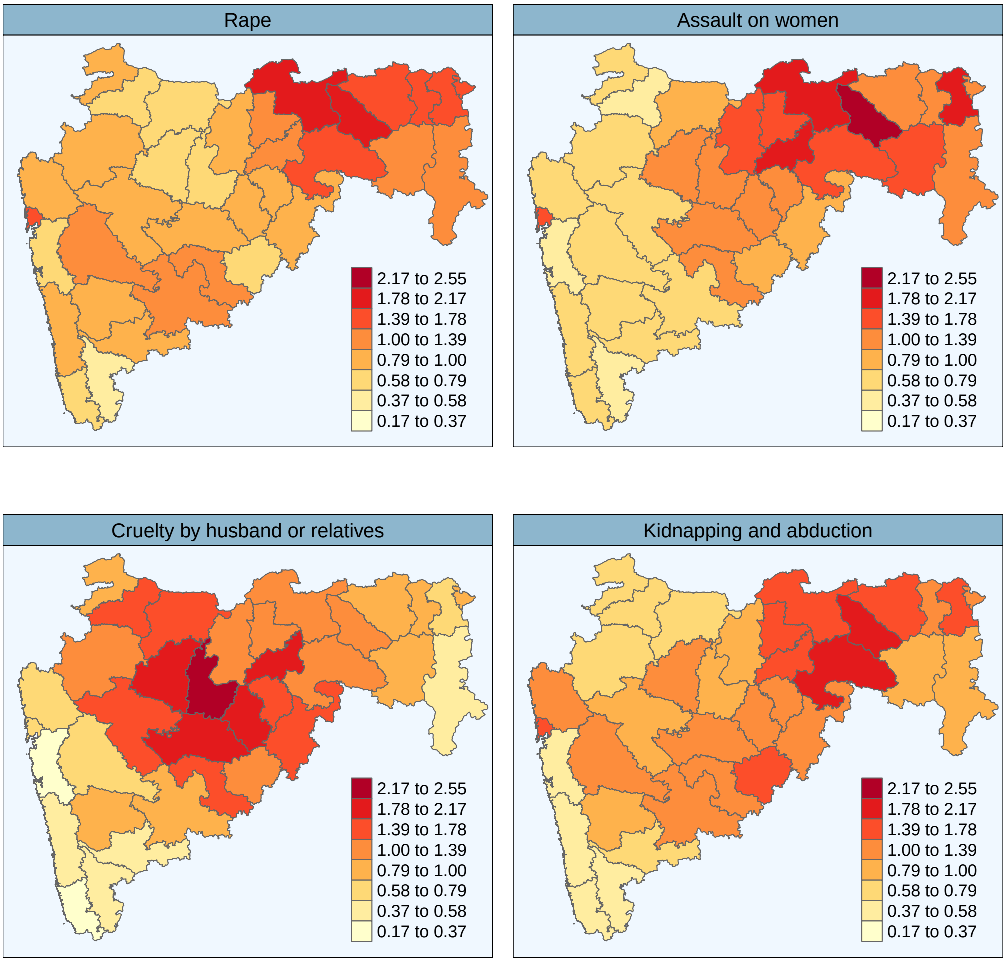

Figure 4.5: Posterior median of the district-specific spatial risk for rapes (top left), assault (top right), cruelty (bottom left), and kidnapping (bottom right).

crime-specific temporal component, $\exp(f_j(x_t))$, and may reflect if time-referenced events, such as certain policies, change in Government, or general social changes affect the incidence of the crimes in different ways. Figure 4.6 displays the posterior medians of $\exp(f_j(x_t))$, for each crime. The shaded regions represent the 95% credible intervals. Trends for rapes and assaults are fairly flat from 2001 to 2012 with a marked growth in the last year. The temporal trend for cruelty is also rather flat, but it shows a wave-shape, something that is not observed in rapes and assaults. An slight upturn is observed at the end of the period. A different behaviour is observed for kidnapping, which shows a steady growth throughout the period.

The posterior correlation between the coefficients of the temporal P-splines

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Table 4.3: Estimated correlations (posterior medians and 95% credible intervals) between the spatial P-spline coefficients (below main diagonal) and between the temporal P-spline coefficients (above main diagonal). Significant correlations are highlighted in bold.

Crime	Rapes	Assaults	Cruelty	Kidnapping
Rapes	$\ddots$	**0.609** (0.445, 0.676)	**0.344** (0.112, 0.714)	**0.354** (0.322, 0.372)
Assaults	**0.782** (0.604, 0.890)	$\ddots$	**0.370** (0.304, 0.407)	0.420 (-0.124, 0.602)
Cruelty	0.037 (-0.278, 0.335)	0.233 (-0.078, 0.489)	$\ddots$	**0.157** (0.132, 0.206)
Kidnapping	**0.384** (0.049, 0.645)	**0.411** (0.099, 0.653)	0.238 (-0.071 0.509)	$\ddots$

are displayed above the main diagonal in Table 4.3. Significant correlations are highlighted in bold. The posterior correlation between the coefficients of the temporal P-splines for rapes and assaults is moderate-high (0.609). Lower correlations are observed between rape and cruelty (0.344) and between cruelty and assault (0.370). Regarding kidnapping, the posterior correlation with assault is not significant, and it is moderate or mild with rape (0.354) and cruelty (0.157).

The interaction term δ_{ijt} allows a different time evolution for each area and disease. Note that although the same type of interaction (Type II) is considered for the four crimes, different precision/variance parameters are allowed for each crime. More precisely, the posterior medians of the standard deviations with a 95% credible interval are 0.109 (0.093, 0.131) for rapes, 0.132 (0.116, 0.151) for assault, 0.150 (0.136, 0.166) for cruelty, and 0.181 (0.154, 0.213) for kidnapping. This indicates a different amount of smoothing for each crime. Area-specific temporal trends, i.e. the posterior medians of $\exp(\delta_{ijt})$, (with 95% credible intervals) are shown in Figure B.2 (Appendix B) for three districts located in different areas of Maharashtra: Aurangabad (central part of the state), Garhchiroli (in the northeast corner) and Greater Bombay (in the middle western coast). The specific temporal evolution in each area is clearly different indicating that in some districts (Greater Bombay) this trend increases, whereas in other districts (Aurangabad and Garhchiroli) the area-specific temporal trend decreases or it is flat.

To save space, the evolution of the geographical distribution of the relative risk is provided in Appendix B. Figures B.3, B.4, B.5, and B.6 show the posterior medians of the relative risks, R_{itj}, (top) and posterior probabilities of risk exceedance, $P(R_{itj} > 1|\mathbf{O})$ (bottom) in the study period, for rapes, assaults, cruelty and kidnapping

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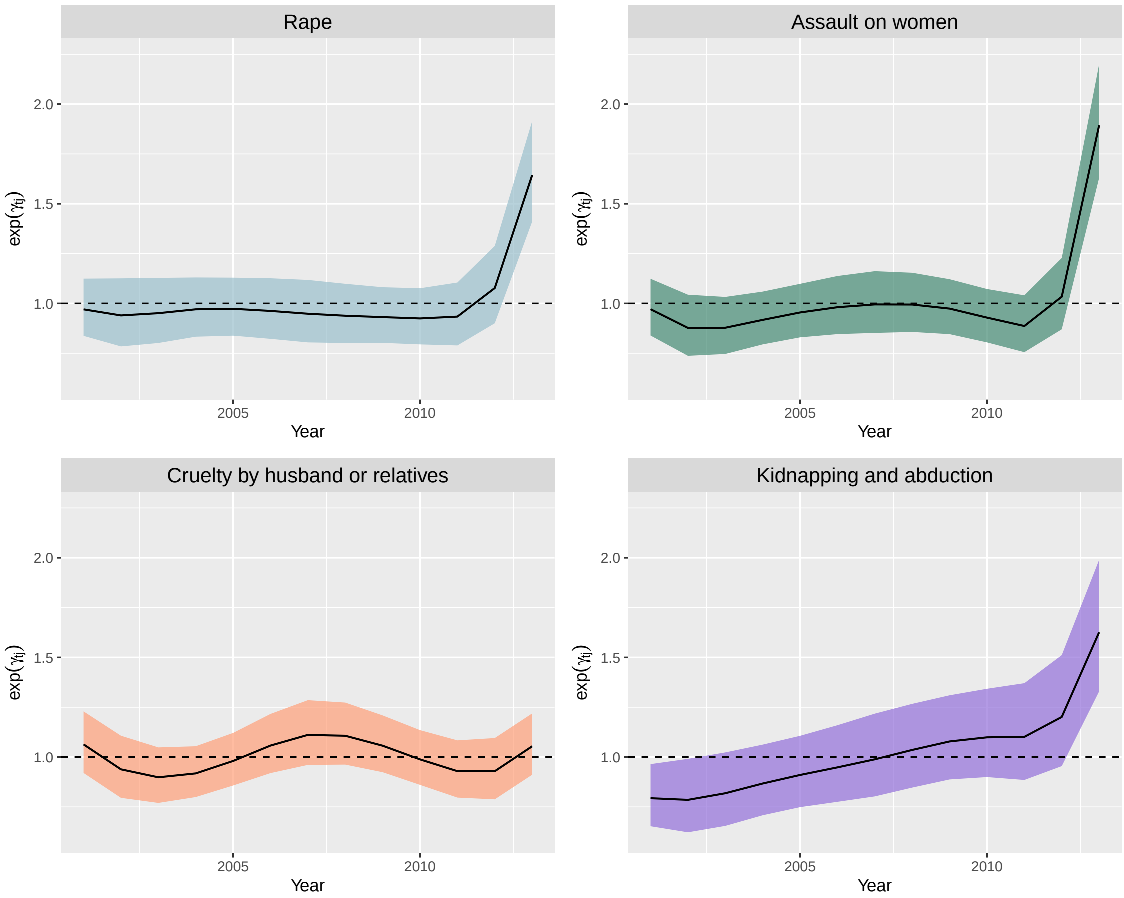

Figure 4.6: Temporal pattern of incidence risks for rapes, assaults, cruelty by husband or relatives, and kidnappings.

respectively. In general, the risk distribution for rapes, assaults and cruelty remains stable during the period with an increase in the last year. The pattern for kidnapping is different with a steady increase of the risk along the period. In summary, these risk patterns confirm the Northeast-Southwest gradient for rapes, assault and kidnapping, with most of the high risk area in the northeast of the state, and a Northwest-Southeast gradient for cruelty with most of the high risk areas concentrated in the central part of the map.

Figure 4.7 displays the relative risk evolution of the four crimes in Aurangabad, Garhchiroli and Greater Bombay. The relative risk of cruelty in Araungabad remains nearly constant and is about twice the risk of whole Maharashtra. Garhchiroli has low relative risks for all crimes, though an upturn is observed for rapes and assaults at the end of the period. Finally, Greater Bombay shows increasing and significantly high risk for rapes, assaults and kidnapping. The risk for cruelty remains low during

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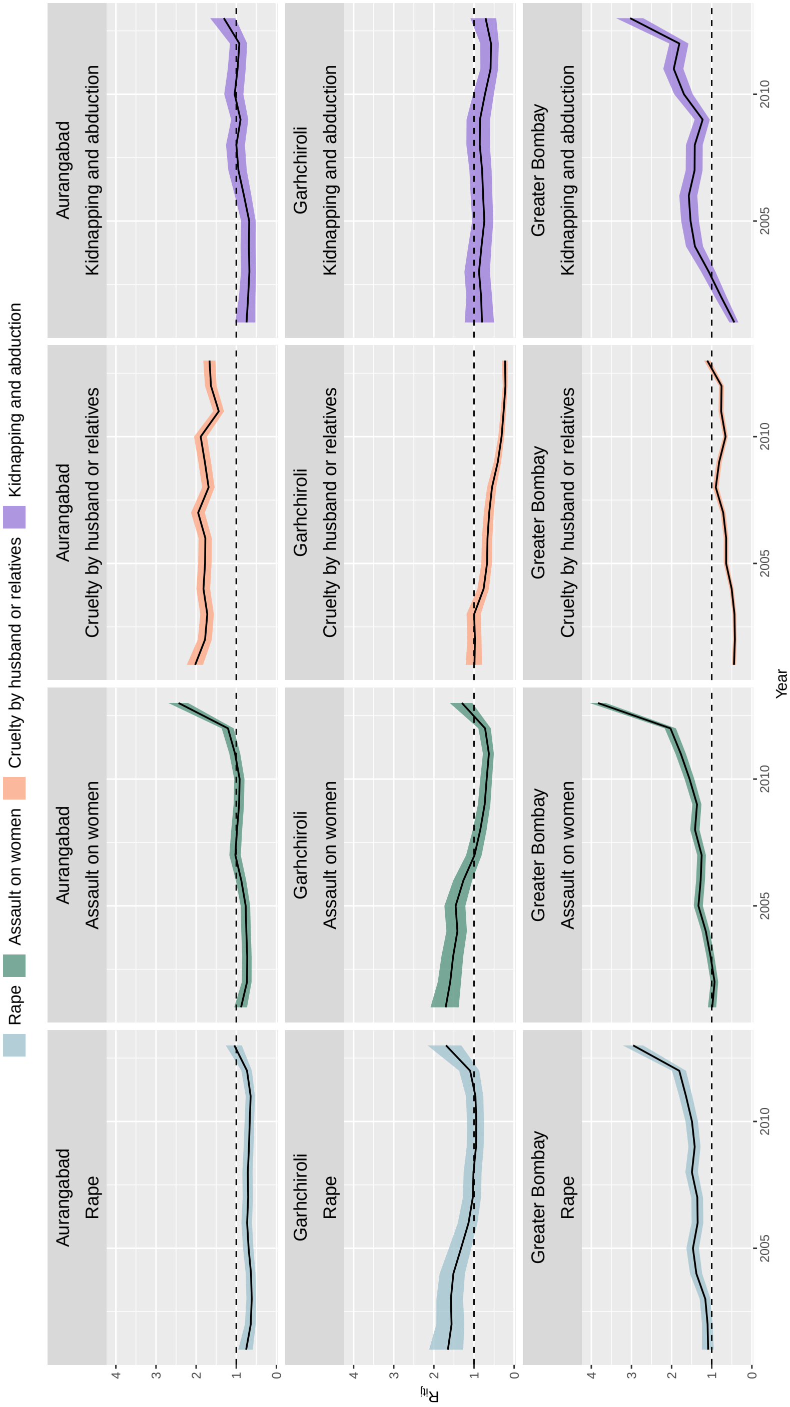

Figure 4.7: Relative risk evolution (posterior median of R_{ijt}) for three selected districts: Aurangabad, Garhchiroli, and Greater Bombay.

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the study period.

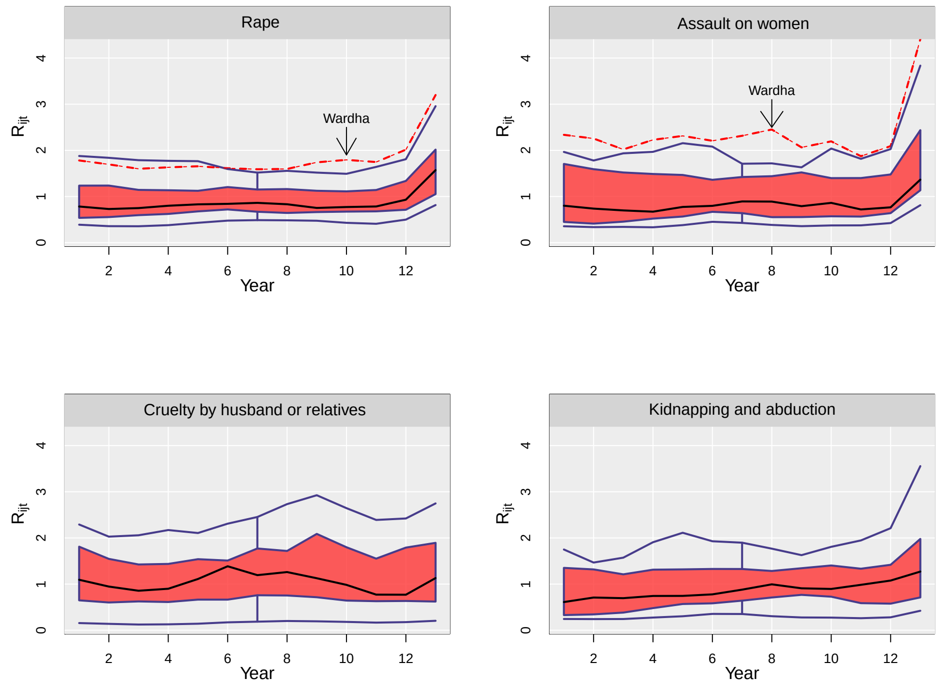

Figure 4.8: Functional boxplots of the final relative risk trends for rape (top left), assault (top right), cruelty (bottom left), and kidnapping (bottom right).

To conclude the analysis, it would be helpful to identify outlying districts regarding their relative risk temporal evolution. Sun and Genton (2011) propose a functional boxplot to visualize functional data and to detect outlying functions. If a function is outside the 1.5 times the 50% central region, it is considered an outlier. Figure 4.8 displays the functional boxplots of the final relative risk trends for rape (top left), assault (top right), cruelty (bottom left), and kidnapping (bottom right). According to the functional boxplot, the district of Wardha, in the northeastern corner of Maharashtra, is an outlying district with regard to rape and assault on women, with a risk greater than the rest of districts. Interestingly, any outlying district is found in relation to cruelty by husband and kidnapping and abduction. It remains unknown why this particular district is an outlier regarding rapes and assaults and it is a matter of investigation for social researchers and anthropologists. Some infrastructure indicators (Directorate of Economics & Statistics, 2017) in the period 2013-2014 (2013 is the last year of our study period) reveal that Wardha was one of the districts with the lowest number of total road kilometers, and also with less post offices. This could indicate some form of isolation that could favour a sense of impunity about sexual crimes. However this is mere speculation and further insight into this district is needed.

4.5 Discussion

In this chapter we consider multivariate spatio-temporal P-spline models to analyse jointly four crimes against women in the Indian state of Maharashtra. More precisely, we propose two-dimensional spatial and one-dimensional temporal P-splines to model the geographical patterns and the temporal trends respectively. Correlations among the coefficients of the P-splines for the different crimes are introduced to look into spatial and temporal associations between the crimes that could point towards connections between the crimes.

The models are fitted using integrated nested Laplace approximations in R-INLA. As far as we know, this is the first attempt to fit multivariate P-spline models with R-INLA. The model has been built using the `rgeneric` function. A modification of the function `inla.rgeneric.IMCAR.model` from the package `INLAMSM` has been written and it is at user disposal. In our data analysis, cubic B-splines with 10 inner knots have been considered for longitude and latitude whereas cubic B-splines with 5 internal knots have been chosen for time. RW1 and RW2 priors for the coefficients have been examined (this is equivalent to first or second order penalties in a frequentist setting). The models have been fitted using a simplified Laplace strategy, but a Gaussian approximation reduces computing time significantly and at least in this application, results are pretty similar. DIC, WAIC and LS criteria points towards RW1 priors. To make the models more flexible, an interaction term structured in time (Type II) has also been introduced to allow for different trends in each district. The models are complex and some issues deserve some comments. First, the smoothing parameter of the temporal P-spline is subsumed in the covariance matrix between the coefficients and hence this parameter and the variance parameters of this matrix are not identifiable. It could be possible to fix that parameter to 1, but this is arbitrary and we obtain a better fit including it in the model. Consequently inference about the non-identifiable parameters is avoided. However, the more relevant correlation parameters are identifiable. Second, the spatial and temporal effects include an intercept, and sum-to-zero constraints are needed. Additional sum-to-zero constraints are required to identify the interaction terms.

Regarding the real case study in Maharashtra, the results are very interesting. The analysis reveals similarities between the spatial patterns of rapes, assaults and kidnapping with a Northeast-Southwest gradient, whereas the spatial pattern for cruelty is different and a Northwest-Southeast gradient is observed. The temporal evolution reveals similarities between rapes and assaults. Again, cruelty seems to

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differ from the other crimes. Our study identifies districts with high risk for some or all crimes examined here. Morover, functional boxplots discover Wardha as an outliying district with a risk of rape and assault greater than the rest of districts. We firmly believe that our findings will be useful for social researchers and anthropologists to disentangle the complex phenomenon of violence on women. Additional research in those districts could bring light to identify potential risk factors that may be related with the crimes. So far, we could only make hypotheses based on our results and existing literature. For example, it has been documented that in urban slums in Bombay, the risk of cruelty increases if the man in the household consumes alcohol (see, for example Begum et al., 2015). Other descriptive studies in rural villages (see Jain et al., 2004) points towards the predominant role of men over women, economic stress, many people living in one room, or complaints of the mother in law as reasons for the abuse.

Finally, and despite the undoubted value of this research, it suffers from the same limitations commented in previous chapters. Namely, underreporting of crimes, and the difficulty to find covariates and to evaluate their effect in multivariate models. Further research is needed in this direction.

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B Appendix

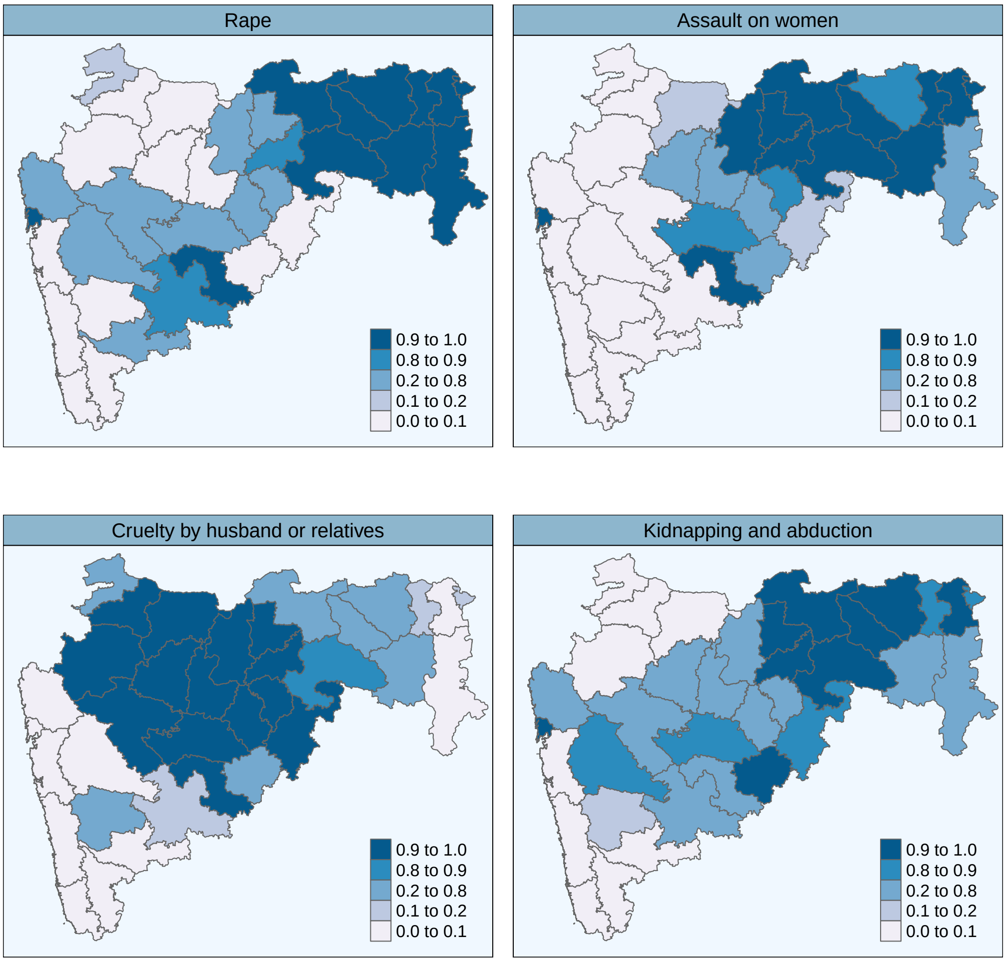

Figure B.1: Exceedance probabilities for rapes (top left), assault (top right), cruelty (bottom left), and kidnapping (bottom right).

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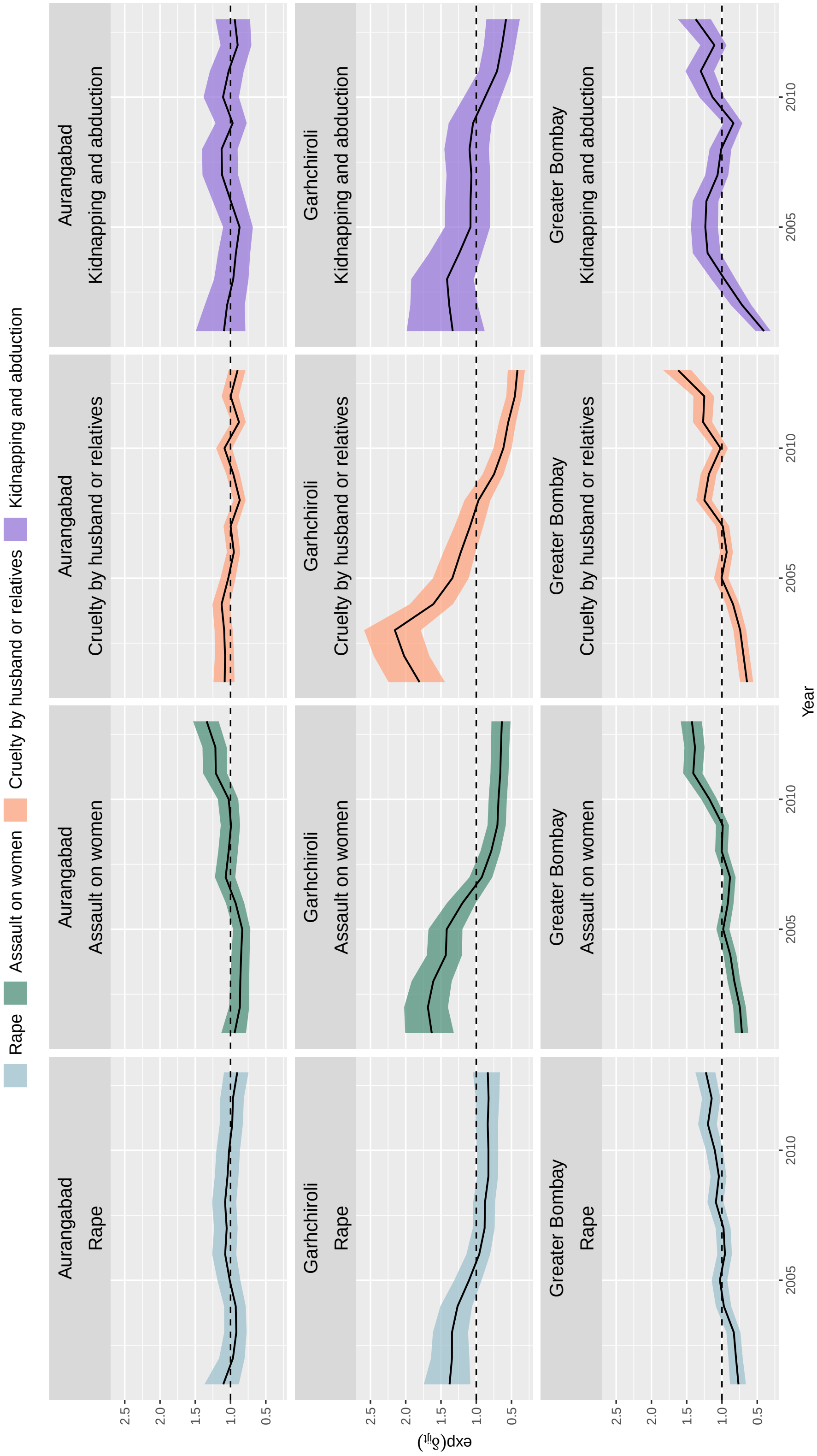

Figure B.2: Specific temporal trends (posterior median of $\exp(\delta_{ijt})$) for three selected districts: Aurangabad, Garhchiroli, and Greater Bombay.

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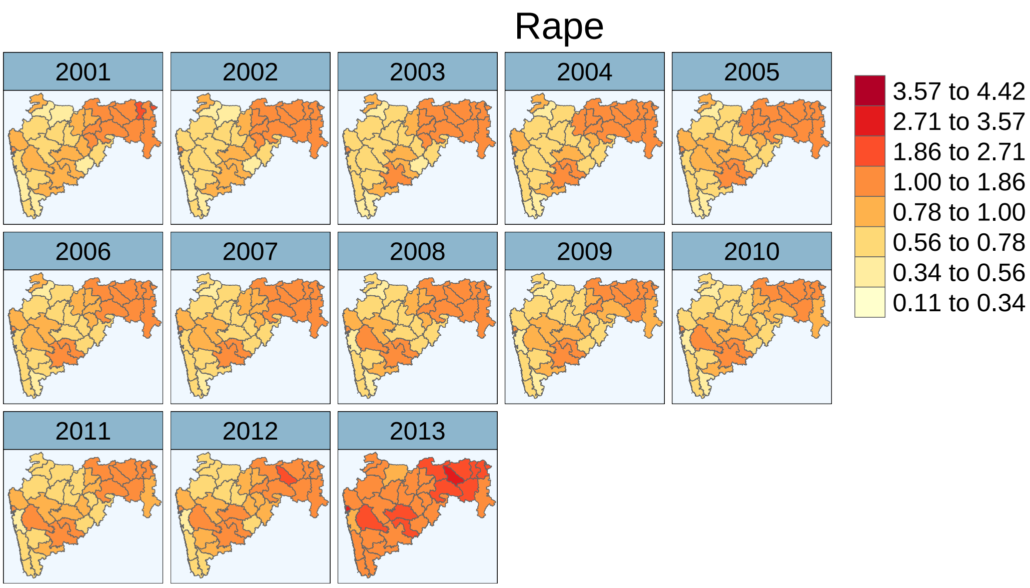

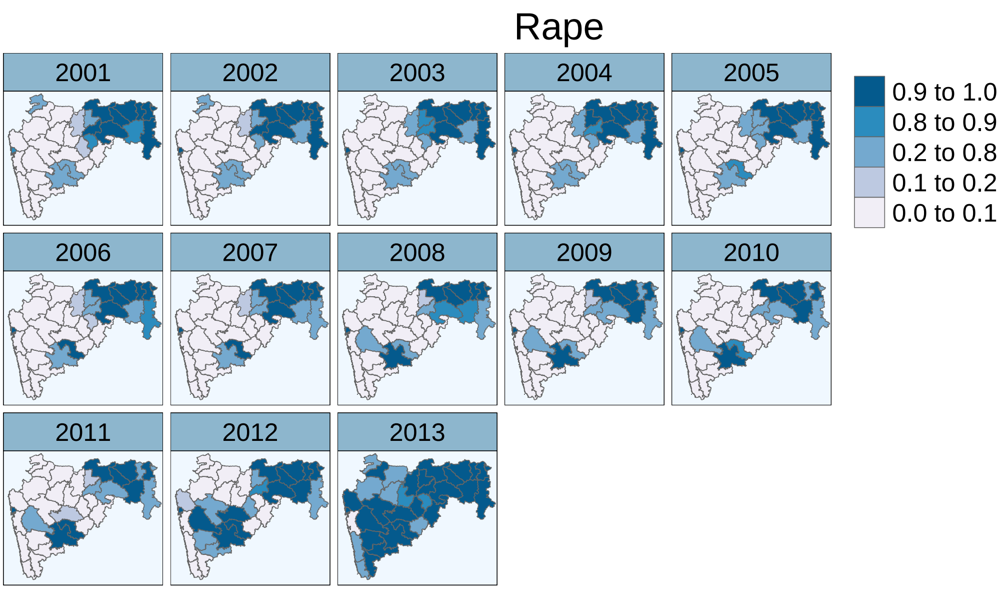

Figure B.3: Map of estimated incidence risks for rape (top) and posterior probabilities that the relative risk is greater than one in Maharashtra between 2001 and 2013.

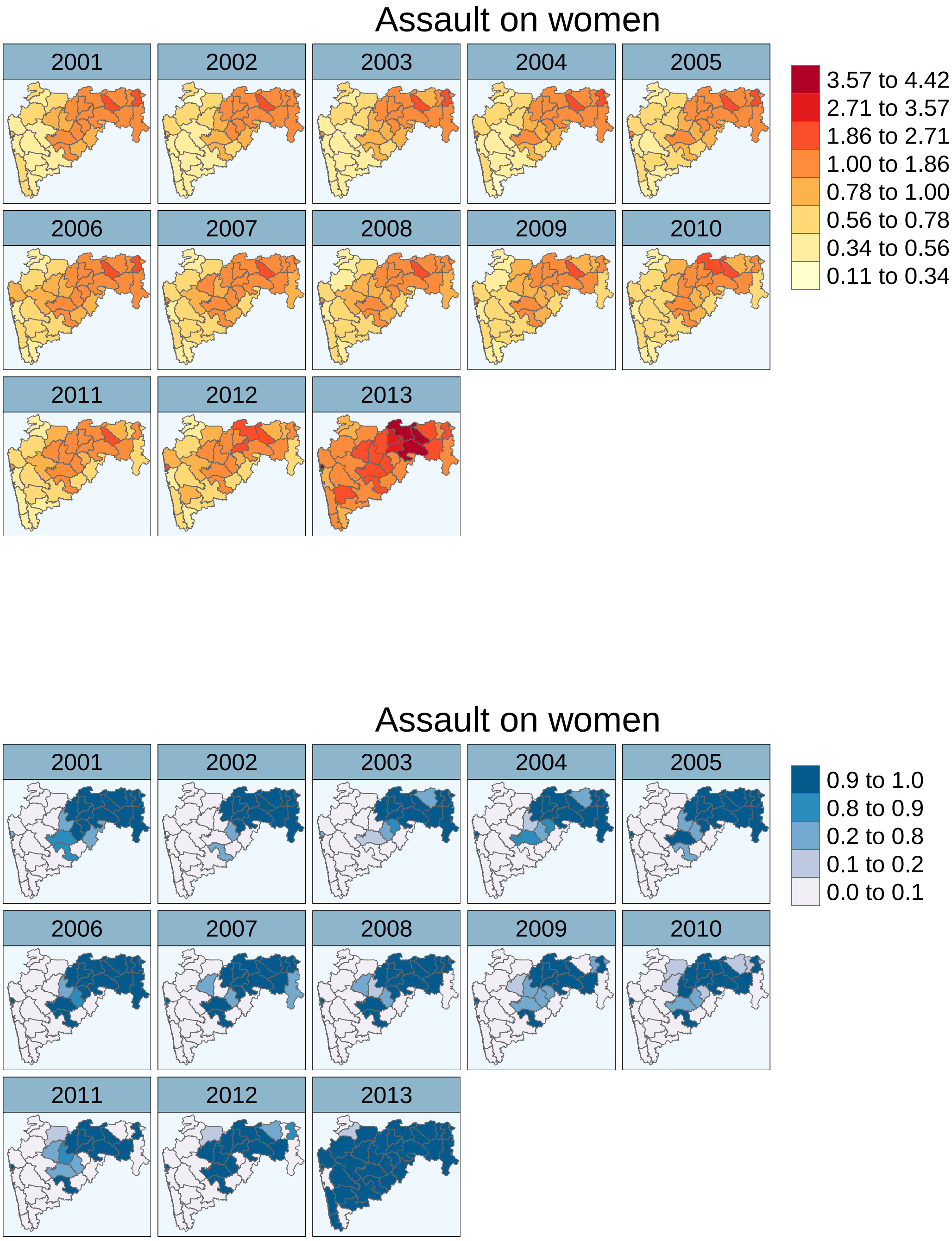

Figure B.4: Map of estimated incidence risks for assault on women (top) and posterior probabilities that the relative risk is greater than one in Maharashtra between 2001 and 2013.

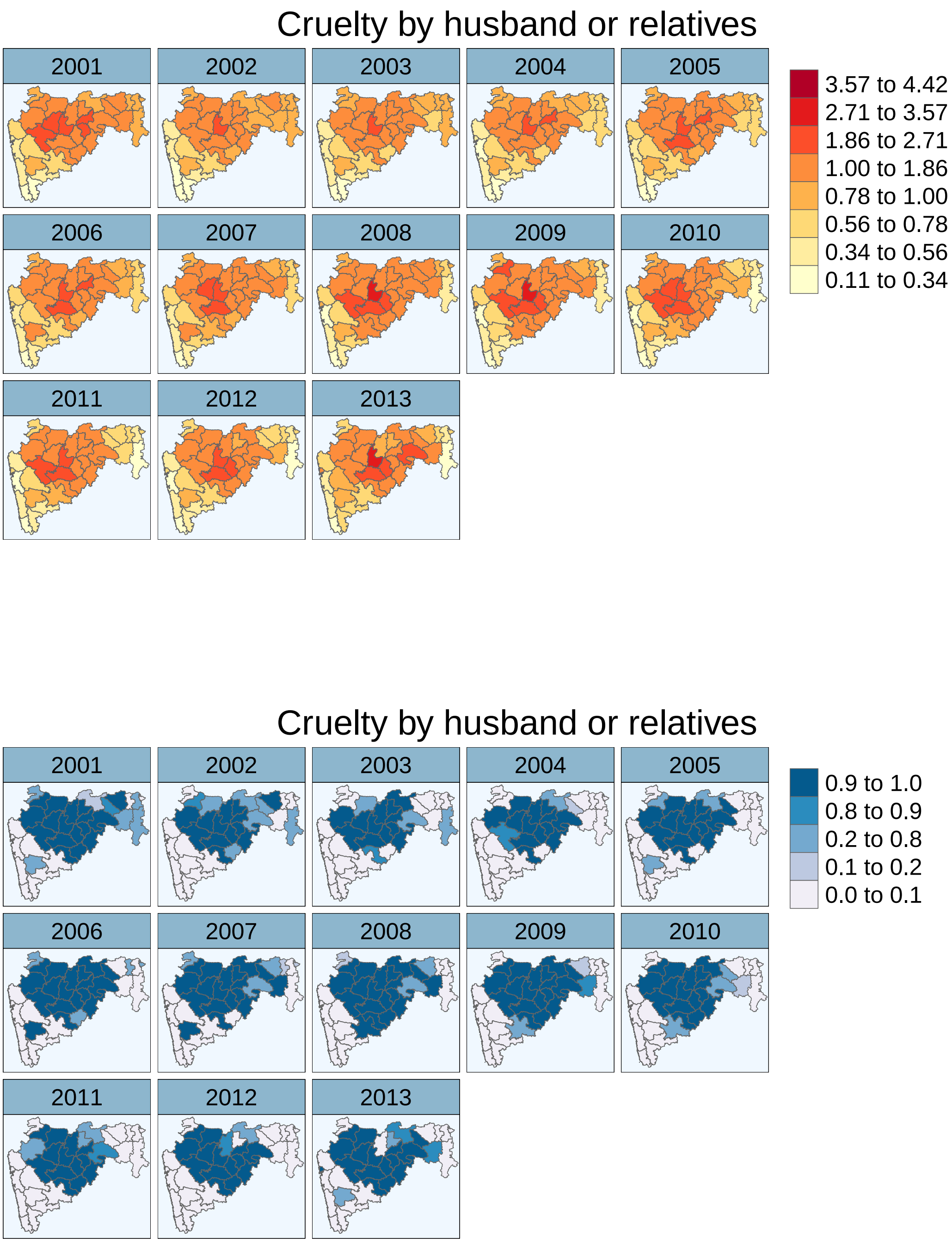

Figure B.5: Map of estimated incidence risks for cruelty by husband or relatives (top) and posterior probabilities that the relative risk is greater than one in Maharashtra between 2001 and 2013.

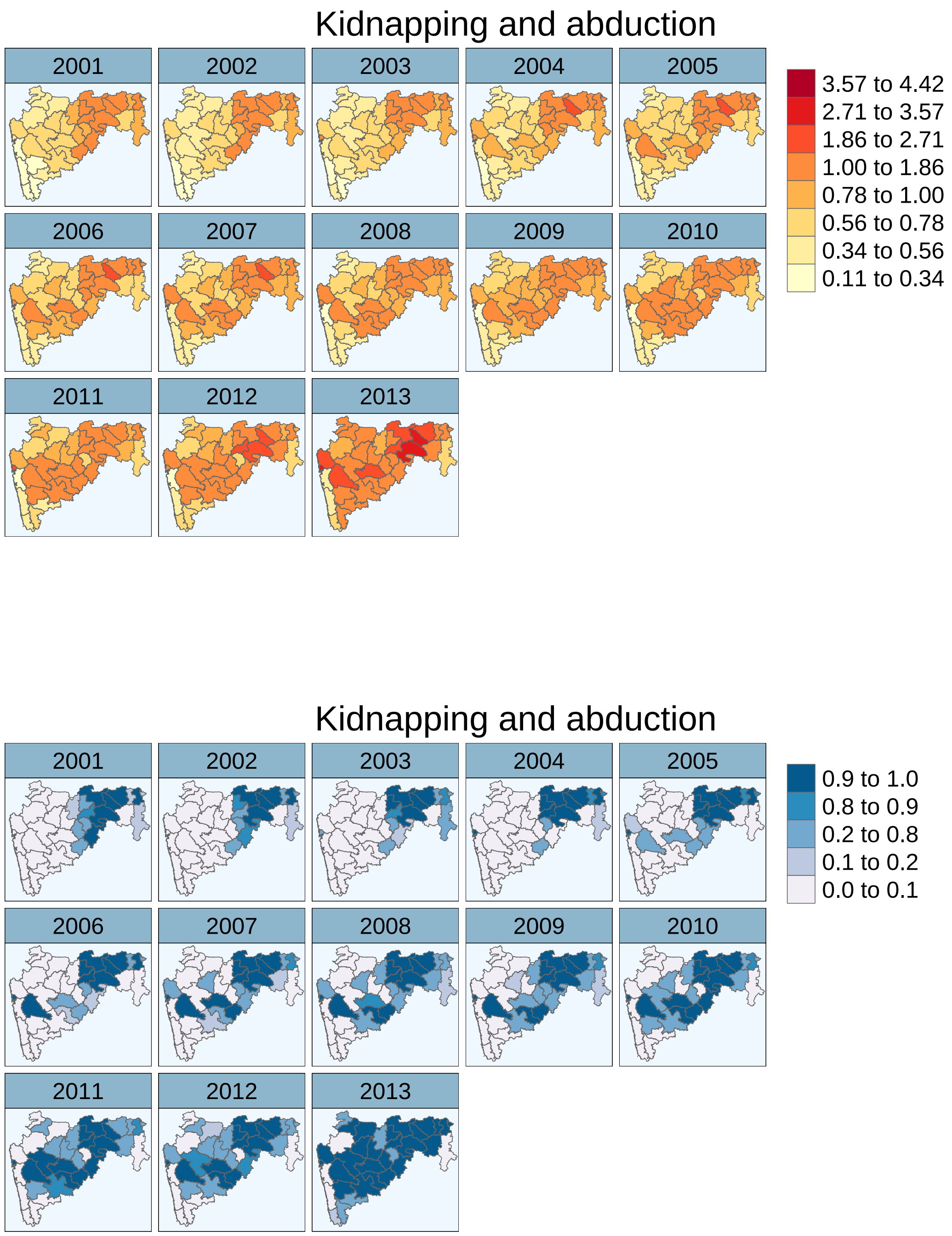

Figure B.6: Map of estimated incidence risks for kidnapping and abduction (top) and posterior probabilities that the relative risk is greater than one in Maharashtra between 2001 and 2013.

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Conclusions and further work

Spatio-temporal models for areal data have a long tradition to study mortality or incidence risk of certain diseases. However, its use to discover geographical patterns and temporal trends of crimes against women is scarce. In this dissertation, spatio-temporal areal models are proposed to shed light on this terrible and current problem in India, a country with hugely diverse traditions and complex social structures that hamper the comprehension of the phenomenon.

Chapter 2 compares univariate spatio-temporal models that include different spatial CAR priors to assess their effects on the final risk estimates. In particular, we focus on the LCAR, DCAR and BYM2 priors and their induced correlations between the regions of the graph. The DCAR and BYM2 priors induced an undesired negative correlation between regions located farther apart. On the other hand, the LCAR prior leads to a sensible pattern of correlation which tends to zero as distance between regions increases. In addition, the variability in the correlation of one region with its neighbours is reduced with this prior, something desirable as we expect similar correlation between neighbours. In the analysis of dowry deaths in Uttar Pradesh, neither the undesired effects of the induced correlation matrix of the DCAR and BYM2 nor the different set of hyperpriors change the final risk estimates. That is, the relative risk estimates are robust to the choice of spatial priors and hyperpriors. The analysis of dowry deaths in this chapter reveals interesting findings with important practical implications. First, our study reveals the association of sex-ratio and other general forms of crime with dowry deaths, and it also discovers, two clearly different zones in Uttar Pradesh, the eastern districts with low risk of dowry deaths, and the western districts with a high risk. Some hypothesis about the reasons underlying these differences in risk are highlighted.

The multifaceted nature of crimes against women and the difficulty in determining the relationships between crimes and socio-economic, demographic and religious

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factors, lead us to use multivariate spatio-temporal models to look for connections between different crimes that can shed light on this complex phenomenon. In Chapter 3, we introduce multivariate spatio-temporal models to study crimes against women. In particular, we propose a spatio-temporal extension of the spatial M-models presented by Botella-Rocamora et al. (2015). Our multivariate extension allows us to estimate correlations between spatial patterns and temporal trends of different crimes, something relevant from a practical point of view as these correlations may indicate that crimes share risk factors. This is what we observe in the joint analysis of dowry deaths and rapes in Uttar Pradesh, where a strong and positive correlation between the temporal trends indicates that certain policies, intervention programs or any governmental action are affecting both crimes. In this chapter, we also implement our multivariate proposal in INLA using generic functions. Though these functions are rather complex, they can be used for non-expert users as they are integrated in the usual syntax of INLA.

CAR priors have been widely used in univariate spatio-temporal models. As a natural extension, multivariate models are also based on CAR structures. However, other models, such as P-splines, have been proposed to model one response. In Chapter 4, we propose multivariate P-spline models to analyze different forms of crimes against women. The goal is to propose a new battery of multivariate models based on P-splines to see if they offer advantages over univariates counterparts. Our proposal accounts for the correlation of the spatial and temporal patterns of the different responses. The technique has been implemented in INLA and it is publicly available in the GitHub of our research group. Some complex generic functions have been created, and similar to the M-based models, can be included routinely in INLA. The methodology has been used to analyze four different crimes against women in the Indian state of Maharashtra. Our study shows that our multivariate P-spline models overcome the univariate counterparts in terms of precision and model selection criteria. The results show a strong relationship between sexual-type crimes, whereas cruelty by husband seems to behave differently. The computational cost of the proposed multivariate models, while not excessive, can become a problem when considering a relatively large number of crimes. In our case study, similar results have been obtained with a Gaussian and a simplified Laplace strategy. If the Gaussian strategy works well, it can be a solution to save computing time.

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Further work

In this dissertation we focus on multivariate models to discover spatial and temporal patterns of crimes against women and the relationship between them. The techniques provide valuable information to discover risk factors that may be related to the crimes. The next step is the inclusion of covariates in the multivariate models to stablish their relationships with the crimes in a formal way. This is not an easy task for several reasons. The first one is the difficulty to obtain relevant covariates of high quality. The second one has to do with theoretical aspects that require further research. Here, we highlight two relevant issues that should be addressed before including covariates in the multivariate models.

The first one is that one covariate may not be related with all the crimes under study or it relationship with each crime may be different. This requires further research about how to include different covariates for each crime in the multivariate models or how to estimate a different relationship of the covariate with the crimes. A research line could be to select a range of covariates with spatial and temporal patterns compatible with certain crimes. Then, fit univariate models as a first guess to estimate the fixed effects coefficients and finally, use these estimates to weight the covariates in the multivariate models.

The second, and possibly the most important issue is confounding fixed effects by random effects. It is well known that in spatial disease mapping, the effect of a covariate may be confounded with the spatial random effect leading to biased estimates of the fixed effects and to variance inflation (Reich et al., 2006; Hodges and Reich, 2010). Consequently, if a risk factor is included in the model, the estimation may not be valid (see for example Kelling et al., 2020). This is even worse in the spatio-temporal setting where confounding may be present due to the spatial, temporal, and the interaction random effects. Adin et al. (2020) are currently working on how to address this relevant issue in univariate spatio-temporal models, where a reparameterization is proposed. However, including this reparameterization to deal with confounding in the multivariate setting is not straightforward, as the spatial and temporal main effects become time and spatially varying random effects, and it is not clear how correlations between crimes should be incorporated and, more importantly, interpreted. Dealing with these two issues regarding covariates in multivariate models is indeed a challenge.

The high rate of underreporting in certain crimes against women is an issue that may hinder the true extent of the problem. Trying to estimate this underreporting or to explore ways to include it in the models is also relevant for future research.

Multivariate models are computing intensive. Though INLA is a technique for approximate Bayesian inference, computing burden may be a problem, particularly if the number of areas, time periods, and crimes increases. A further insight on how to implement the models to save computational burden is recommended.

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Finally, our research team has created a web application using Shiny that currently fits univariate spatial and spatio-temporal areal models. Implementing our multivariate proposals in this web application is also in our “to-do list”.

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