

# Optimal strain gauge configurations for the estimation of mechanical loads in the main shaft of a HAWT

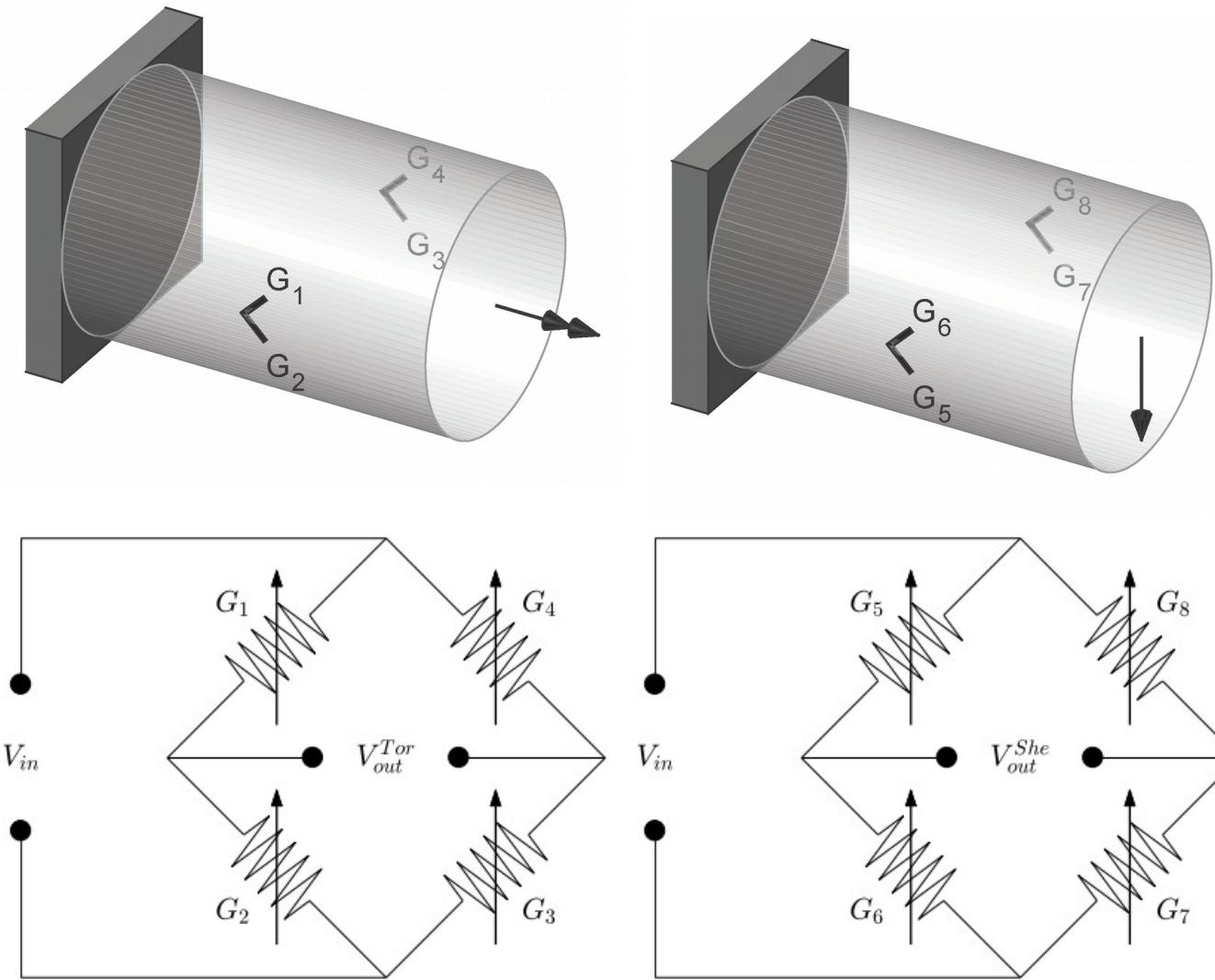
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## Motivation

The standard configurations for measuring individual loads in a shaft use four strain gauges connected in a full Wheatston-Bridge. If the whole wrench is desired, six full bridges have to be mounted.

As shown in the Figures below, gauge configurations for torsion and shear measurements are equivalent while they appear connected differently in the electrical circuit.



This motivated the idea of using the same set of strain gauges to measure different mechanical loads. This requires to measure the strain of individual gauges in quarter-bridges.

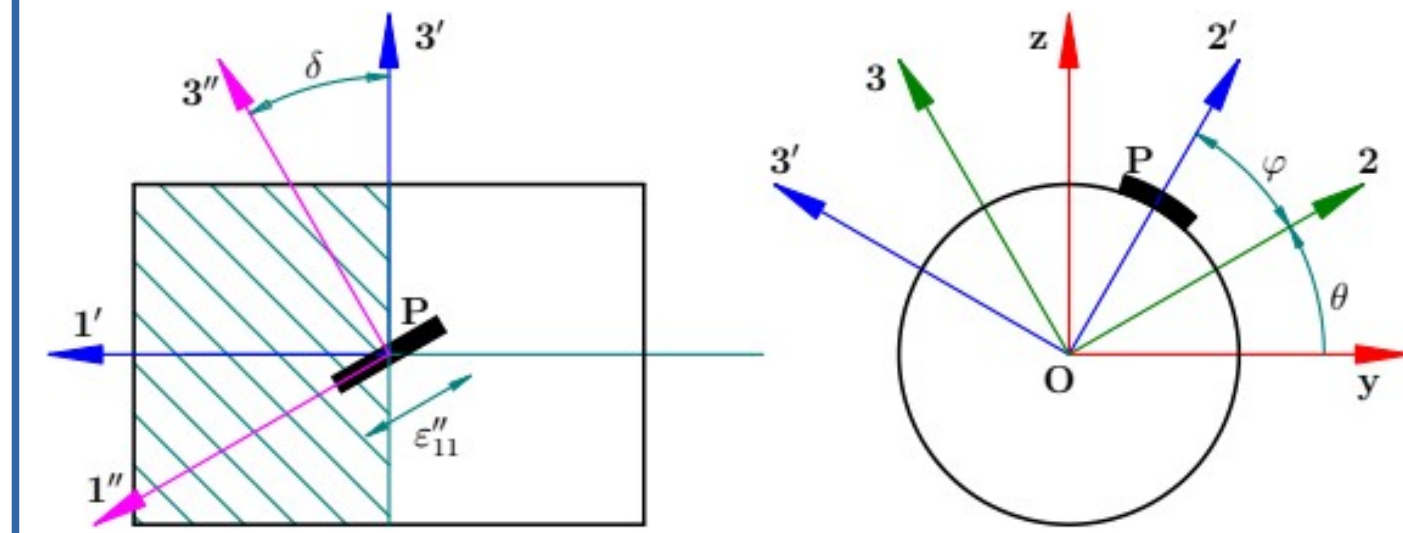
As a result, it would be possible to measure a group of mechanical load components (or the whole wrench) with a single set of gauges.

In this paper this possibility is explored.

## Development

In order to get gauge configurations to measure mechanical loads in terms of their strain, the first step is to express the strain of an arbitrary gauge in terms of the loads apply to the shaft.

Two coordinates determine the location of the position and orientation of the strain gauge:



For a cylindrical hollow shaft, the stress tensor at P can be written in terms of the mechanical loads applied to a section of the shaft at point O.

$$[\bar{\sigma}(\mathbf{t}')]_{1'2'3'} = \begin{bmatrix} -\frac{F'_1}{A} + \frac{M'_3}{w} & 0 & -\frac{M'_1 R}{I_P} - \frac{F'_3}{k A} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{1'2'3'}$$

Assuming a linear, elastic and isotropic material, the strain of an arbitrary gauge in 1'' direction is explicitly written in terms of the mechanical loads written in base 123, the location of the gauge and the material properties

$$\varepsilon = \mathbf{w}(\varphi, \delta) \mathbf{t} = \begin{bmatrix} \frac{(1+\nu) \sin^2 \delta - 1}{EA} \\ -\frac{2(\nu+1) \cos \delta \sin \delta \sin \varphi}{kEA} \\ \frac{2(\nu+1) \cos \delta \sin \delta \cos \varphi}{kEA} \\ \frac{R(\nu+1) \sin 2\delta}{EI_P} \\ \frac{((1+\nu) \sin^2 \delta - 1) \sin \varphi}{Ew} \\ -\frac{((1+\nu) \sin^2 \delta - 1) \cos \varphi}{Ew} \end{bmatrix}^T \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ M_1 \\ M_2 \\ M_3 \end{bmatrix}$$

## Determination of optimal configurations

Writing the strain of  $n$  gauges in terms of the mechanical load components, one gets the next system of  $n$  linear equations and 6 unknowns.

$$\varepsilon = \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{Bmatrix} = \begin{bmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \\ \vdots \\ \mathbf{w}_n \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ M_1 \\ M_2 \\ M_3 \end{Bmatrix} = \mathbf{W}(\varphi, \delta) \mathbf{t}$$

For fixed  $\mathbf{W}$ , solving this system of equations by least squares provides an estimation of the mechanical loads in terms of the strain measurements.

$$\hat{\mathbf{t}} = (\mathbf{W}^T \mathbf{W})^{-1} \mathbf{W}^T \varepsilon_m$$

Matrix  $\mathbf{W}$  depends on the locations of the  $n$  gauges. The optimal gauge locations (for some constraints) are those that minimize the so called *D-optimality criterion*.

$$\mathcal{F}(\mathbf{W}) = -\log(\det(\mathbf{W}^T \mathbf{W}))$$

$$(\varphi^{opt}, \delta^{opt}) = \arg_{(\varphi, \delta)} \min(\mathcal{F}(\mathbf{W}(\varphi, \delta)))$$

If the configurations are desired to be robust to temperature variations, assuming the thermal strain of all the gauges is the same, the next modified  $\mathbf{W}$  matrix can be used instead.

$$\varepsilon_m = \varepsilon + \varepsilon_T + \mathbf{e}$$

$$\varepsilon_m = \mathbf{W} \mathbf{t} + \mathbf{1} \varepsilon_T + \mathbf{e}$$

$$\varepsilon_m = [\mathbf{W} \quad \mathbf{1}] \begin{Bmatrix} \mathbf{t} \\ \varepsilon_T \end{Bmatrix} + \mathbf{e}$$

## Resulting optimal configurations

If  $n=6$  gauges are used, the optimal configuration to estimate all mechanical loads using 120° rosettes is:

$$\varphi^{opt} = (0^\circ, 0^\circ, 120^\circ, 120^\circ, 240^\circ, 240^\circ)$$

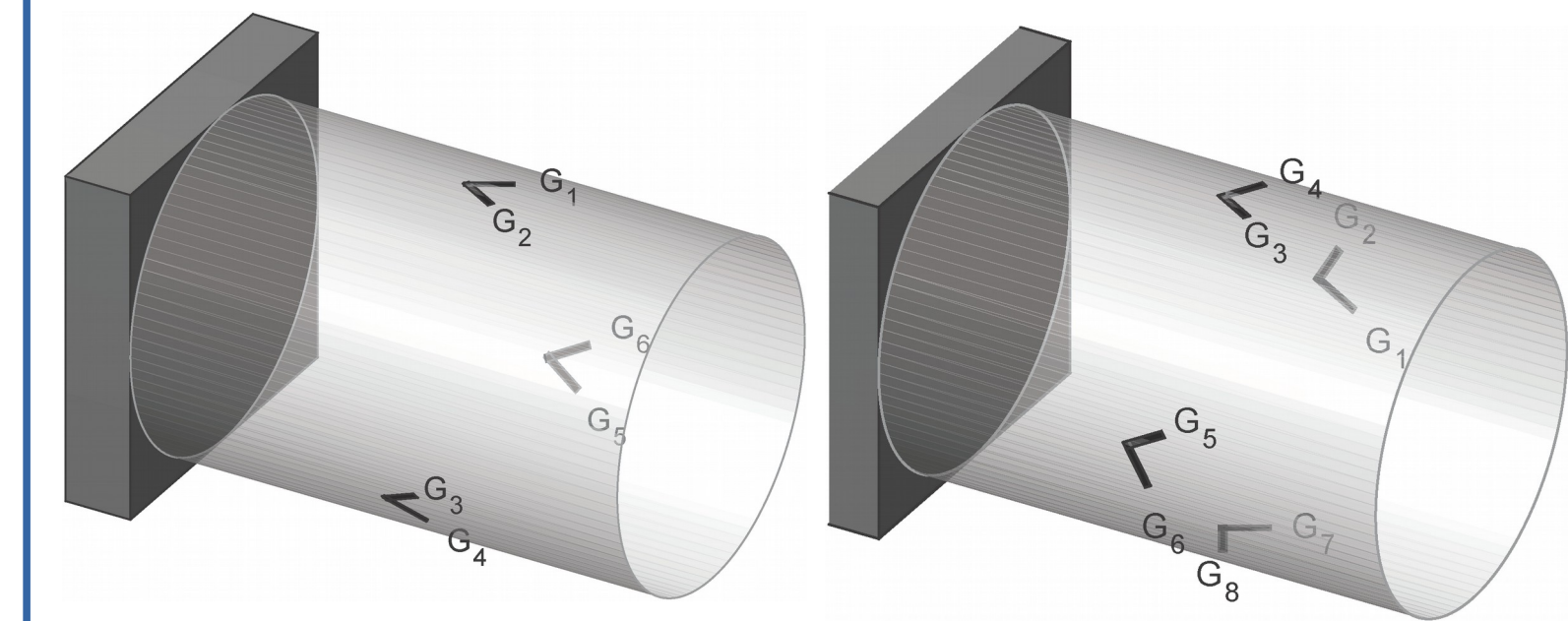
$$\delta^{opt} = (30^\circ, -30^\circ, 30^\circ, -30^\circ, 30^\circ, -30^\circ)$$

If  $n=8$  gauges are used, the optimal configuration to estimate all mechanical loads (with temperature compensation) using 90° rosettes is:

$$\varphi^{opt} = (0^\circ, 0^\circ, 90^\circ, 90^\circ, 180^\circ, 180^\circ, 270^\circ, 270^\circ)$$

$$\delta^{opt} = (60^\circ, -30^\circ, 30^\circ, -60^\circ, 60^\circ, -30^\circ, 30^\circ, -60^\circ)$$

These configurations are shown in the next figures:



## Conclusions

Measuring the strain of single gauges in Wheatstone quarter-bridges enables to estimate the 6 mechanical load components in a shaft with a single set of gauges. The configurations are optimal as they maximize the observability of the load components.

Different optimal configurations can be obtained for different constraints and number of strain gauges.

The configurations can compensate thermal strains.