



# Influence of the Driver and the Parameters of Damping in the torque at the Propeller Shaft after a Gear Shift

## **Master Thesis**

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Braunschweig (Germany), in June 2013

Technische Universität Braunschweig

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#### Declaration in Lieu of Oath

I hereby declare in lieu of oath that I composed the following thesis independently and with no additional help other than the literature and means referred to. Hiermit erklaere ich an Eides statt, dass ich die vorliegende Arbeit selbstaendig und ohne Benutzung anderer als der angegebenen Literatur und Hilfsmittel angefertigt habe. Braunschweig, June 2013.

Jose Angel Lacalzada Galilea





#### **ABSTRACT**

In this document, a description of the behaviour of the torque which affects the drive shafts of the vehicle, and consequently the wheels, is explained. For this purpose, it is used the 3D method developed in the Institute of Automotive Engineering of TU Braunschweig, which takes the name from the three dimensional parameter on which is based: driver, vehicle and road. First of all, the creation of an Oriented Speed Profile for the road dimension will be developed with the help of Aitor Diaz de Cerio Crespo and Miguel Pradini Aranda. After that, the work will be based on the description of the behavior of the torque by using some parameters of the clutch and the throttle which the driver controls, and some parameters of the second order system that produces the damping. This task is made with the help of a power train simulation and some programs created in Matlab/Simulink environment. Finally, the way in which the parameters affect the torque is presented, so that the previous work for further investigations is completed.





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## 1 Introduction

The customer demands for comfort represents a main task of automotive engineering nowadays. There are different criteria which are related and finally oriented to the improvement of the feeling that the customer could have while driving a vehicle. The development and optimization of alternative drive concepts, the active and passive safety of the vehicle, market specific emission regulations, the variety of environmental impacts and the study of the customer behaviour are some of the instruments which are used in order to develop a vehicle that responses to a defined customer profile.

Some methods have been developed so as to meet the customer expectations taking the criteria above into account. For this task, extensive measurements of different vehicles parameters and drivers have been made to identify the driver behaviour. Using the experiences for these measurements, customer oriented requirements can be predicted and designed for the implementation in new vehicle concepts and new components [1].

For the customer-oriented development of the vehicles and their components, a wide and precise knowledge of the customer behaviour as well as the knowledge of the car components operation and the driving environs is required. In this regard, researchers at the Institute of Automotive Engineering (IAE) have developed the 3D method, which is based on the study of three parameters which allows to difference between a great amount of possibilities. The three axes of the 3D parameter space are the driver behaviour, the driving environs, and the driven vehicle.

The driver behaviour describes the driving style, which is based in the interaction of the driver and vehicle components. For example depending on the type of driver, it can be distinguished between mild, average and sporty driver, where mild would be a predictive and intelligent driver who forecasts the situations; sporty would be an aggressive driver and average would be in the middle of this classification.

The driving environs mainly influence is the road, which can be affected by some parameters like climatic conditions, current weather situation and traffic. The 3D method classifies the road into various types, taking into account the slope, curvatures, coefficient of adhesion and traffic management. One example of such a classification would be dividing the driving environs between urban, extra urban, mountain and autobahn. However, such a classification always depends on the proposal of the study.

Finally, the driven vehicle includes the structural parts of the regarded vehicle and its components. It is not the same when a compact car or a very powerful car is driven, even if



the driver is the same for the two cars. For the component analysis, different parameters have been proved to have a mayor importance, and they mainly depend on the orientation of the study. Another example of this classification could be the weight that the vehicle bears in its chassis, heavy weight, average weight and only the driver.

The combination of all of these parameters would result in a big number of customer types, who contains minor portions of other customer types. The share of these proportions also depends on several variables that have to be taken into account. For example, if the driven vehicle is divided depending on the chassis load, maybe the power of the vehicle is different for the measurements.

In order to determine some requirements of the vehicles, a complex simulation model is required, and this one will require some vehicle parameters. It will be used a statistical driver model as a central element, which is parameterised by means of the statistically processed measuring data of the database.

The statistical road model continuously generates an orientation speed profile and the road gradient profile using the simulation. The vehicle simulation model implements the driver actions and allows the determination of interesting measurement variables. The vehicle model is constructed with a depth enough to get the results with sufficient accuracy. [2]

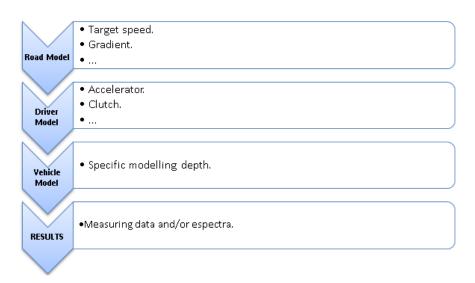


Figure 1.1: Process of a statistical 3D method customer simulation



#### CHAPTER 1. INTRODUCTION

In order to face the challenges in the vehicle development process, the 3D method represents a powerful tool to provide information on the expectable customer use already in early development phases. In addition, it concerns topology and system optimizations as well as representative component design and vehicle testing.



## 2 State of the Art of the Clutch Models

The friction modelling constitutes the base of all clutch models and must be chosen taking into account the purposes of use. In some cases it is desirable to have a clutch model which can provide an insight into the physical mechanisms of the friction interface, whereas in other cases a model which can predict the global, qualitative behaviour of a system with friction. [3]

The friction clutch models were developed based on existing models of friction, but some simplifications are made because there are some aspects which are not relevant for the purpose of the thesis:

- The thermal effects are not taken into account because they have a low dynamic and are not relevant enough.
- The friction torques are computed using a normalized command signal as a fraction of the maximum torque, which represents the position of the clutch pedal.

$$T_{cfd} = com \cdot T_{fmaxd} \tag{2.1}$$

$$T_{cfs} = com \cdot T_{fmaxs} \tag{2.2}$$

where  $T_{cfd}$  is the dynamic friction torque and  $T_{cfs}$  is the static friction torque transmitted through the clutch, com is the command signal,  $T_{fmaxd}$  and  $T_{fmaxs}$  are the maximum dynamic and static torques, respectively.

What is more, the maximum friction torques can be computed with expressions in which the friction radius of the clutch disks, the friction coefficient and the number of friction surfaces are included.

As a stage of documentation in this thesis, a study of the different clutch models was made. Now some of the current available theoretical models of friction clutches are going to be described, and after this, the one proposed and used in the simulation.



#### Coulomb friction model

The most commonly friction model used is the Coulomb friction model, which can be formulated for the clutch with the following:

$$T_c = \begin{cases} T_{cfd} \cdot sign(\omega_r) & \text{if } \omega_r \neq 0 \\ T_{app} \cdot n & \text{if } \omega_r = 0 \end{cases}$$
 (2.3)

where  $T_c$  is the torque transmitted through the clutch,  $w_r$  is the relative speed and  $T_{app}$  is the torque applied on the clutch plates.

The effects of this model are shown in the figure 2.1.

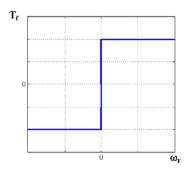


Figure 2.1: Friction force variation as function of relative velocity for Coulomb friction model

It can be easily implemented using Matlab/Simulink, as it is shown in the figure 2.2:

#### Combined Coulomb and viscous friction model

A viscous friction model can be used instead a Coulomb friction model for some applications because with the last one the equation of motion for dynamic systems is strongly non-linear. The viscous friction model is considerably easy to simulate, but the representation of the friction is not desirably for the clutch modelling. For this reason, a combination of the viscous friction model and the Coulomb friction model would be advantageous. Such a model would have the following form:

$$T_c = \begin{cases} T_{cfd} \cdot min(2 \cdot \omega_r/\omega_o, 1) & \text{if } \omega_r \ge 0 \\ T_{cfd} \cdot max(2 \cdot \omega_r/\omega_o, -1) & \text{if } \omega_r < 0 \end{cases}$$
(2.4)



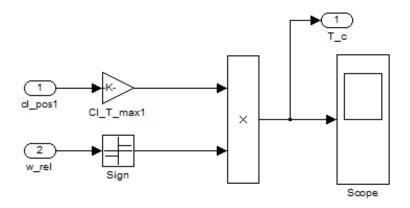


Figure 2.2: Simulink top level diagram of a Coulomb friction model

where  $\omega_o$  is a parameter that determines the speed of the transition from -1 to +1.

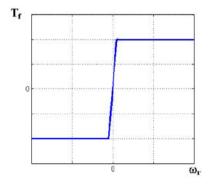


Figure 2.3: Friction force variation as function of relative velocity for combined Coulomb and viscous friction model

As the Coulomb friction model, this one can also be easily implemented using Matlab/Simulink, as it is shown in the figure 2.4 using the saturation block.

#### Hyperbolic tangent model

This model uses a tanh function to ensure the transition through zero and limit the torque. This model behaves like the previous one, the combined Coulomb and viscous friction model, but with the difference that is more numerically stable due to the use of a



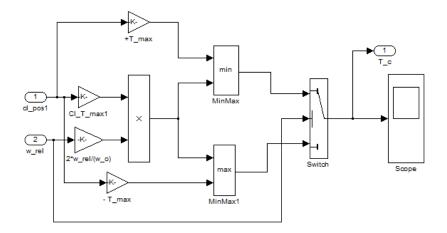


Figure 2.4: Simulink top level diagram of a combined Coulomb and viscous friction model perfectly continuous function. The equation for this model is the following:

$$T_c = T_{cfd} \cdot \tanh(2 \cdot \omega_r / \omega_o) \tag{2.5}$$

In the figure 2.5 the variation of the friction force as fuction of the relative velocity for the tanh friction model is represented.

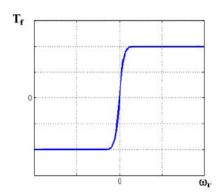


Figure 2.5: Friction force variation as function of relative velocity for *tanh* friction model It can also be easily modeled with Matlab/Simulink, as in the figure 2.6.



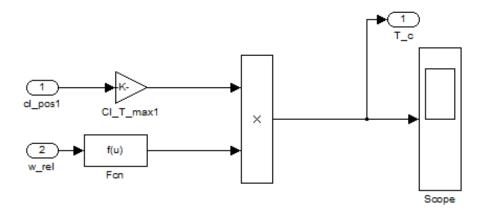


Figure 2.6: Simulink top level diagram of a tanh friction model

#### Stick-Slip phenomenon

These examples which have been presented above do not represent the stick-slip phenomenon, which can occur while two objects are sliding over each other. This effect happens due to the difference between the static and kinetic friction coefficients. When a state of static friction is overcome, then a sudden jump in the velocity of the movement can be experimented. This is because the kinetic friction coefficient is smaller than the static friction coefficients. In the following figure the stick-slip effect is represented.

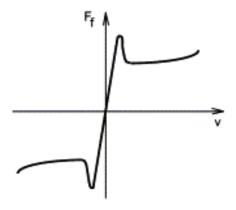


Figure 2.7: Representation of the stick-slip phenomenon



## 3 Oriented Speed Profile

The main task of this section is to be able to detect and predict the customer manoeuvres, with the final objective of being able to anticipate to the driver behaviour as well as improving the feeling which the customer has while driving a vehicle. First of all, it needs to be stated that this task was developed with the help of another two students, Miguel Pradini Aranda and Aitor Díaz de Cerio Crespo, and also with the guidance of our supervisor Benedikt Weiler.

#### 3.1 Detection of the customer's manoeuvers

A driving manoeuver is a move or series of moves which are used in order to get to a different situation of speed while the customer is driving. For example, the fact that a vehicle is moving at 55 km/h and accelerates so as to rise 100 km/h would be considered a driving manoeuver. Although it is very difficult to drive at constant velocity, so the manoeuvers would be useless as a vehicle is never driven constant, the detection of these is used in future calculations and can be very helpful for giving solution to some customer demands. A compromise between the real data and what is consider a manoeuver in this master thesis has to be made in order to get the results.

Even though a driving manoeuver mainly depends on the speed (and acceleration) of the vehicle, some other parameters and variables have to be taken into account and each specific case must be studied separately in order to have a more detailed definition of what each manoeuver represents and what the customer or driver would like to do in each moment.

Then, with a defined profile of each driver, vehicles, and roads (environs), a deeply study of the customer behaviour can be made, and so as a better solution for the problems related to the customer demands be found.

This previous work can be divided in two main tasks which will be the pillars for the identification of the driving manoeuvers, always taking into account some external actions which will be explained in the following pages. These two tasks are:

■ The identification of the global changes of speed, where the driver accelerates or decelerates so as to reach a constant speed.



■ The identification of the changes in the intensity, with which the customer accelerates or decelerates. As an easy example, it is not the same manoeuver when a vehicle is accelerated first at 2  $m/s^2$  and then starts to accelerate at 4  $m/s^2$ . This transition point from 2 to 4  $m/s^2$  needs to be identified.

#### 3.2 Explanation of the Oriented Speed Profile Program

At this point, the developing of the program that is built is going to be explained. For the construction of the program, Matlab software is used. Consequently, all the figures shown in the thesis and the programming language will be from Matlab. The formulation of the program will be added to the annexe 1.

Now the different steps which have been followed in order to build the oriented speed profile are explained in detail.

#### 3.2.1 Maximums, Minimums and Zeros

First of all, the local extreme points of the real data are detected, as well as the points where the velocity changes from zero to another value or from another value to zero. These points are also very important because many of them will be the limits of the customer's manoeuvers. For example, when a vehicle which is stopped because of a stop signal, and then starts again to roll, this point will be a zero and will mean the beginning of a manoeuver. Then the vehicle accelerates constant until a speed, and then it decrease its speed. Here another limit of the manoeuver is reached, a local maximum.

As it is written above, a compromise has to be made between the real data and the approximation of the profile. All the maximums, minimums and zeros cannot be taken into account because in the real life the speed oscillation is normal and some of these local extremes can be found in an interval of 1 km/h, in gear shifts,...For this reason some filters that will be explained at the end of the chapter have been created.

An example of the maximums, minimums and zeros which are founded in the speed function of the vehicle, is shown in the figure reflocalextreme.

#### 3.2.2 Changes of tendency

At this point, most of the points which can be relevant in order to detect the driver manoeuvers are found. However, not all of them are valid. Not always when the customer starts to accelerate means a change of manoeuver. As an example, if the vehicle starts to decelerate more or less constantly, then accelerates for one second, and then starts to decelerate with the same intensity as before until it reaches a determined speed. That would be considered only one manoeuver because the final intention of the driver is always to decelerate. This short period of positive acceleration could have been due to the traffic or



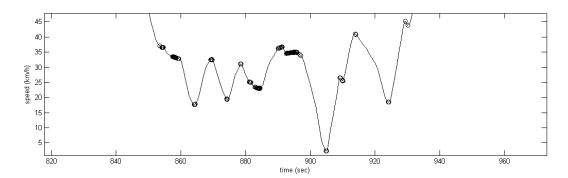


Figure 3.1: Identification of the local extreme points and zeros in the real speed data

something else that can also affect.

In this section is where the compromise between the real speed and the profile which has been built is made. A limit about what is considered a manoeuver must be stated and they have been considered as accurate as possible. For this reason a speed value as a limit is selected. If two relevant points do not differ more than this speed value more than one manoeuvre cannot be considered.

In order to achieve this objective, in the program which is built, a candidate point is selected. Once that a change of tendency is confirmed, the candidate point is allowed to be selected.

An example of all of this is shown in the figure refchangetendency.

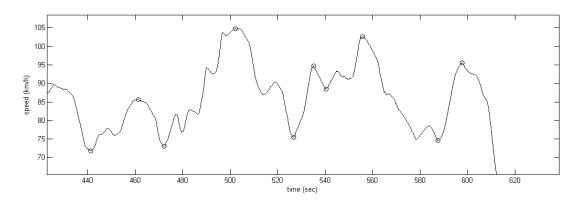


Figure 3.2: Selection of the maximums, minimums and zeros which may be relevant for the changes of tendency

It can be observed in the figure refchangetendency that, even though there are some oscillations in the speed, not all the extreme points are selected. Only the important ones,



that are considered to be relevant in order to limit the manoeuvers.

#### 3.2.3 Changes of Intensity in the Acceleration

There are some actions of the driver related to the acceleration which can also be considered as different manoeuvers. For example, the vehicle can accelerate from 40 km/h to 100 km/h, but from 40 km/h to 70 km/h with a strong acceleration, and from 70km/h to 100 km/h with a small one. These two different movements of the vehicle have to be differentiated. This comes to mean that the derivative function of the speed, in spite of having the same sign, has quite different values for the two stretches. This has to be taken into account for the purpose of the program.

Probably, these situations do not correspond to a maximum or minimum value. For this reason, the speed vector must be gone over again once that the candidate values that rule the major variations of speed are already identified. Then, an auxiliary line that connects all these points is constructed, which represents the average change of speed between two consecutive points. Consequently, the distance between the points to the line is calculated and compared. The maximums and minimums of that subtraction are the ones which are interesting to be studied.

In the figure refmiguel an illustrative example of which has been told above is shown.

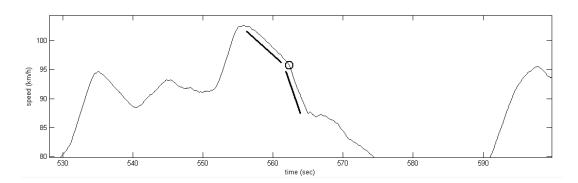


Figure 3.3: Point where a change of intensity in the deceleration occurs

Once this is done, the maximal and minimal differences to the line that has been built are identified. However, this will result in more points than the ones which are really necessary. Therefore, some filters that will be explained in the next section are used in order to eliminate the unnecessary points.

In the figure refmiguel2, another example of this method is presented.



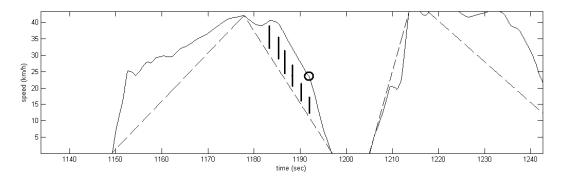


Figure 3.4: The dashed line represents the auxiliary vector that has been constructed in order to calculate the average change of speed between two consecutive points.

#### 3.2.4 Filters

At this point, a vector of maximums, minimums, zeros, changes of tendency and changes of intensity of the acceleration has been constructed. Nevertheless, there are some of these points that must be eliminated. Consequently, no more points will be added to the profile, but will be subtracted.

The process of filtering can be divided in three sections:

- Filtering points in the gear shifts.
- Filtering points which mean a change of intensity in the acceleration.
- Filtering points which are too close to each other.

Then, these filters will be explained briefly.

#### **Identification of the Gear Shifts**

First of all, it is important to identify where a gear shift is made. This is relevant because, in spite of the variation of the speed in the gear shift, this cannot be considered a point which separates two manoeuvers. For example, if the customer wants to drive from 30 km/h to 100 km/h with a more or less constant acceleration, probably he would have to make at least two gear shifts. In these gear shifts it can be observed that there is a little decrease of the speed for one second approximately, which is the time when the clutch separates the two parts of the power train, so the wheels do not receive torque from the engine. This reduction of velocity cannot be considered an independent manoeuver, as the intention of the customer is always to reach the final speed with a constant acceleration, and the decreasing is due to the mechanical part of the vehicle.



From now to the end of the section the real data for the speed of the vehicle will be represented in blue. In the figure 3.5, an example of the gear shift situation is shown.

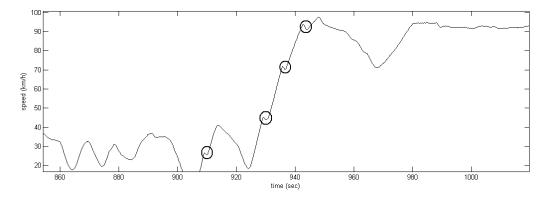


Figure 3.5: Identification of the gear shifts in an acceleration manoeuvre

Once all the gear shifts are taking into account in order to build the final profile, the next filter is explained.

#### Filter for the Acceleration Change of Intensity Points

In this filter the points which no represent a major change of acceleration are eliminated. For this purpose, a comparison between the variation of average derivative function and the nearest selected point is made.

#### Filter for Points which are Close to each other

Finally, points which are near each other between some established limits are deleted. The strategy to achieve this task is to assign a likelihood value to each point taking into account the values of the surrounding points.

#### 3.3 Results and Future Work

Finally, following all these steps, the oriented speed profile is constructed. An example of this profile is presented in the figure 3.6.

In the development of the final profile, only the speed and acceleration have been taking into account. However, there are some other variables which definitely influence in what is considered as a manoeuvre. For example, the position of the brake pedal and the



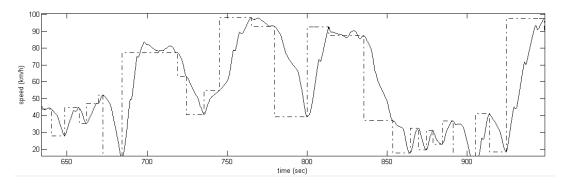


Figure 3.6: Oriented speed profile represented in dash dotted line

position of the acceleration pedal may also be taken into account.

Some strategies in order to include these two variables have been attempted. Nevertheless, at the end, they have been discarded because the final result was not as good as the one obtained with the method which has been used. Some of the reasons for the inclusion of these two variables will be explained in the following lines.

#### 3.3.1 Brake pedal position

The brake pedal, unless exceptions, is used always for deceleration or braking manoeuvers. If this variable is taken into account, it is easier to identify when the driver is decelerating the vehicle. On the other hand, not always in the deceleration manoeuvers has the break to be pushed. Sometimes the customer uses the engine brake without using the break pedal. For this reason, not always is easy to distinguish the cases.

Consequently, a lot of work can be developed in order to include this variable, and would be interesting to build the oriented speed profile regarding to this.

#### 3.3.2 Throttle position

The accelerator pedal position is as important as complicated in order to describe the manoeuvres of the vehicle. Another external factors influence in this variable such as slopes, curves, aerodynamics · · · Besides, not each vehicle has the same power neither the same engine map. In a powerful car, with a 40% of the throttle can develop more torque in the wheels that other car with 80% of the throttle pushed. An intuitive way to think is to believe that always when the throttle is pushed seems to be an accelerating manoeuver. However, it is not always like this.

For all these reasons, is complicated to generate the profile taking the throttle position



into account. The main reason is to create a system which is valid for different car models, since the corresponding of the throttle position and the power is not the same, so different accelerations are achieved.

Including the acceleration pedal position as a variable in the program, is the process that most need to be improved. As soon as a new method adapting this variable to the program is developed, the final result would be more accurate. For this task, may be some new measurements need to be included as well as the technical characteristics of the vehicle which is measured.



## 4 Physical Modelling

In this section, a short review of how to make mathematical models for dynamic systems will be given. To simulate a model we want a system written on the form:

$$\frac{dx(t)}{dt} = f(x(t), u(t)) \tag{4.1}$$

$$y(t) = h(x(t), u(t))$$
 (4.2)

When a system is modelled, three phases must be followed:

#### 1. Structuring the problem

In this phase, the system must be divided into subsystems and the main relations between them must be decided. The variables which are important and the influence that have in each other must be stated. It is in this phase where one decides the level of complexity and the degree of approximation for the model, always depending on the final purpose of it. What is more, the person who builds the model must know the physics. Therefore the model builder must be documented and must have the necessary knowledge to start working. This part of the modeling ends in some sort of block diagram describing the main function of each subsystem and the interaction between them.

#### 2. Setting Up the mathematical equations

The objective of this phase is to describe the subsystems and blocks from phase 1, but in an extensive manner. The relations between the variables and constants in the subsystems are stated. This task, for physical systems, must be done using the laws of nature and physical equations that are valid for the model simulated, such as Newton laws for mechanics and Kirchoff's and Ohm's laws for electricity.

#### 3. Compiling the State Space Model



After the phases above are completed, the model is actually finished. Nevertheless, the equations need to be organized in the form of equation 4.2, so they can be simulated in a real time simulation. This work can be done manually, or in a computer program for modeling. In Matlab/Simulink this phase is automatized by the connections between the subsystems and blocks, so that the model builder can avoid the error prone work and concentrate only in the first two phases.

For more information about the physical modelling, there are books as [4].

#### 4.1 Basic Equations

The driveline of a vehicle is represented in figure 4.1. It consists of an engine, clutch, transmission, propeller shaft, final drive or differential, drive shafts and wheels. In this section the fundamental equations for the powertrain will be explained in order to have a better knowledge of what is going to be simulated. What is more, some basic equations regarding the forces which actuate on the wheels are obtained. These equations are influenced by the complete dynamics of the vehicle, which means that the effects, for example, from the vehicle mass and from the aerodynamics will be described by the equation describing the wheels.[5]

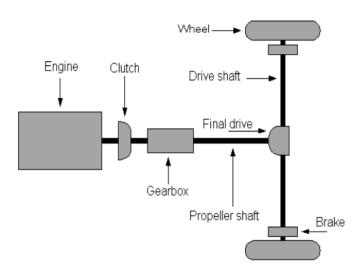


Figure 4.1: Driveline for a rear-driven vehicle

**Engine:** The output torque of the engine is characterized by the driving torque resulting from the combustion ( $T_e$ ), which in this case is controlled by the accelerator pedal



position and the speed of the engine, the internal friction from the engine ( $T_{fe}$ ), and the external load from the clutch ( $T_c$ ). The generalized Newton's second law of motion gives the following model:

$$J_e \cdot \dot{\omega}_e = T_e - T_{fe} - T_c \tag{4.3}$$

where  $J_e$  is the moment of inertia of the engine and  $\dot{\theta}_e$  is the angular acceleration of the flywheel.

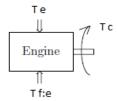


Figure 4.2: Subsystem of a vehicular engine with its input and output torque

**Clutch:** A friction clutch in vehicles equipped with a manual transmission consists of the clutch disks which connect the flywheel of the engine and the transmission input shaft. The equation that gives the relation between the torque that enters to the clutch and the one which goes to the transmission is the following:

$$J_c \cdot \dot{\omega}_c = T_c - T_{ti} \tag{4.4}$$

where  $J_c$  is the moment of inertia of the clutch and  $T_{ti}$  is the torque of the transmission input.

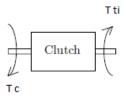


Figure 4.3: Subsystem of a vehicular friction clutch with its input and output torque



**Transmission:** A transmission has a set of gears (normally five or six for common cars), and each one has its conversion ratio  $i_t$ . This ratio gives the following relation between the input and output torque of the transmission:

$$T_{ti} \cdot i_t = T_{to} \tag{4.5}$$

$$\omega_{ti} = \omega_{to} \cdot i_t \tag{4.6}$$

where  $T_{to}$  is the torque of the transmission output and  $\omega_{ti}$  and  $\omega_{to}$  is the rotational speed of the transmission input and output, respectively.

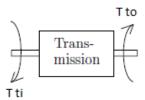


Figure 4.4: Subsystem of a vehicular transmission with its input and output torque

**Propeller shaft:** The propeller shaft connects the transmission output with the differential. In this thesis is assumed to be rigid and no friction between the parts is assumed. Therefore it has no effect in the torques equation.

**Differential:** The differential is characterized in the same way as the transmission, but has only one conversion ratio  $i_{fd}$  for all the gears. No friction is assumed into the final drive, which gives the following model for the torques input and output:

$$T_{fdi} \cdot i_{fd} = T_{fdo} \tag{4.7}$$

$$\omega_{fdi} = \omega_{fdo} \cdot i_{fd} \tag{4.8}$$

where  $T_{fdi}$  is the torque of the differential input and  $T_{fdo}$  is the torque of the differential output.  $w_{fdi}$  and  $w_{fdo}$  are the rotational velocity of the differential input and output, respectively.

**Drive shafts:** The drive shafts connect the wheels to the final drive. In this model it is assumed that the speed is the same for the two drive shafts and the two wheels. Consequently, they are modeled as if it was only one drive shaft. In a real case, when a vehicle



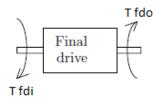


Figure 4.5: Subsystem of a vehicular final drive with its input and output torque

is making a curve, the speed differs between the wheels because the wheels which are in the interior have less distance to make due to the smaller radio. However, for this thesis, this effect is not taking into account. No friction between the wheels and the drive shafts gives the model equation:

$$T_d = T_w (4.9)$$

where  $T_d$  is the torque in the drive shafts and  $T_w$  is the torque in the wheels which are connected to them.

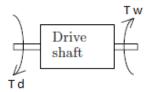


Figure 4.6: Subsystem of the drive shafts with its input and output torque

In this thesis, from now to the end, the drive shafts will be named as propeller shaft. Consequently, when propeller shaft is written for referring to the torque, it will be the torque at the drive shafts, which it will has been seen to be approximately the same as the torque in the wheels.

Wheels and car body: The second part of the car is considered to have one inertia for the wheels and the entire car body (including the transmission and the final drive). It is for that reason why the following equation is formulated.

$$J_{car} \cdot \dot{\omega}_w = T_{fdo} - T_w \tag{4.10}$$



where  $J_{car}$  is the moment of inertia of the car body, and  $\dot{\theta}_w$  is the rotational acceleration of the wheels. Therefore, with this last variable, the acceleration and the velocity of the vehicle can be calculated.

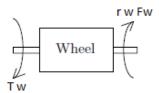


Figure 4.7: Subsystem of the wheels with its input and output torque

Once the torque of the wheels is calculated, an analysis of the external forces which actuate in the car and wheel must be done in order to calculate parameters such as the speed or the acceleration of the vehicle.

Regarding to the figure, the forces equilibrium is formulated:

$$F_w = m_{car} \cdot a_{car} + F_a + F_r + m_{car} \cdot g \cdot \sin \alpha \tag{4.11}$$

where  $F_w$  is the force which actuate in the wheels due to the torque that is transferred.  $F_a$  is the force due to the aerodynamics of the vehicle,  $F_r$  is the rolling resistance and  $m_{car} \cdot g \cdot \sin \alpha$  the gravitational force. Each of these forces will be explained in a more extensive way in the section in which the car body is simulated. [6]



## 5 Vehicle Simulation

In the last few years virtual dynamic system simulation has become very important in the design and development stage, as the vehicle procedures can be examined without expensive measurements and with reduced time. The driveline is a fundamental part of the vehicle and its dynamics has been modeled in different ways depending on the purpose. It is the driveline which transforms the energy from the combustion in the engine to kinetic energy of the vehicle. It is of major importance that the driveline is built as efficient as possible, in order to have a better performance and lower fuel consumption. What is more, the driveability and comfort must be high for the driver. To reach that it is important to to simulate its behaviour. A complete simulation of the powertrain of a vehicle is going to be described. The main elements of the powertrain include the engine, clutch, transmission and the car body.

In this part of the thesis a study of a driveline modeling is made. A modular programming approach in Matlab/Simulink environment is used for the simulation. Components from the Matlab/Simulink library for this purpose have not been used (like, for instance, built engine or transmission models). Instead of this, the whole group of components have been defined by mathematical equations together with standard components from the library.

Nowadays the demand for development of efficient vehicles is increasing. Accurate offline models can be used in vehicle design and development, which can provide advantages such as reducing cost and time. The power train provides the driving torque necessary for vehicle acceleration and handling. Although the model is developed and validated for a specific engine (taking into account the data which have been provided), is generic enough to be used for a wide range of spark ignition engines.

Most of the transmission models are based on dynamic clutch torques. But a lot of data required for such models is not available and is not always possible to find for the power train used in the vehicle under investigation. Consequently, a compromise has to be made between comprehensive and simple models. The available power train models in are mostly used for control and diagnostics, but in this work is used for predicting longitudinal dynamics after the gear shifting. Therefore, the integration of several subsystem models to build the power train model is a new attempt in this work.

The main objetive in the simulation is being able to describe the behaviour of the torque which is transmitted through the clutch, as well as the drive shafts torque, just after a new gear is engaged. For that, five input parameters are going to be used:  $t_c$ ,  $t_a$ ,  $s_c$ ,  $s_a$ ,  $f_a$ . Now



these parameters are going to be explained.

- 1.  $t_c$ : this parameter represents the time instant when the clutch pedal starts to be released again after the gear shift, so the driver starts to lift off the clutch pedal.
- 2.  $t_a$ : this parameter represents the instant of time when the driver starts to press the accelerator pedal, which can be before, after or at the same time when the clutch disks start to engage.
- 3.  $s_c$ : this parameter represents the correlation between the position of the clutch pedal and the time which is necessary to make the complete movement from one position of the pedal to the final position of the pedal, where the clutch disks are completely engaged.
- 4.  $s_a$ : this parameter represents the correlation between the position of the throttle pedal and the time which is necessary to make the movement from one position of the pedal to another position of the pedal, which is not necessarily to be when the throttle is 100% pressed.
- 5.  $f_a$ : this parameter represents final pedal position which is reached by the throttle. It is assumed that the customer tries at the end to drive with this throttle position.

For these two last parameters the most important is the difference between them. It does not really matter if a gear shift is made in the second 40 or the second 250, but the start times must be for the same gear shift. Consequently, it is going to be the same if the  $t_c$  takes a value of 40 and the  $t_a$  of 41 and if the  $t_c$  takes a value of 250 and the  $t_a$  of 251, but always if all the other parameters are the same, as the gear shift is going to be identical.

Now that the input parameters have been explained, the next step is to make a description in the next few pages from the different parts and subsystems of the powertrain model which are going to be used in the simulation.



#### 5.1 Vehicle Simulation

In this section, the simulation of the vehicle is explained. Real data have been taking into account. Therefore, the simulation is adjusted to the data as accurate as possible. The steps which are followed to calculate the parameters from the real vehicle are also shown in this section.

For the purpose of this master thesis, the simulation of the car is divided in four main parts: engine, clutch, transmission and car body, which will be explained and connected. In addition, a dynamic system to introduce a PT2 system and improving the dynamics results describing the oscillations which are produced at the torque, is connected to our clutch

#### 5.1.1 Engine

The engine model is developed by applying the NewtonÂ's second law to the rotational dynamics of the engine, as seen in the equation 4.3.

In the present work, the friction torque from the engine is assumed to be proportional to the engine torque. However, in the reality, it depends on several parameters such as the temperature at the engine and the engine speed.

A physically justified model for spark ignited engines is used for the engine torque, which is parameterized to capture the followings features:

- The maximum torque.
- The decrease of the torque when high speed is reached in the engine.
- Idle speed control in order that the speed of the engine does not decrease more than 800 rpm.

Instead of using equations, a simple look-up table is used to represent the data of the relationship between engine torque, throttle opening and the engine speed in revolutions per minute. Torque values for different engine speeds and throttle opening positions are entered into a look up table in Matlab/Simulink based on an engine map for the data which are going to be analyzed. These values are obtained from previous experiments and the engine speeds are between 800 and 7000 rpm, and for the throttle opening between 0 and 100%. The torque values are obtained by interpolation in Matlab/Simulink to obtain the torque at any other speed and throttle position. As the engine is not available in the data provided, an aproximation has been made in order to build the table with the torques, speeds and throttle positions.



What is more, the torque developing engine dynamics is modelled as a first order system with a time constant of 0.1 s for the torque in the engine map.

$$T_e = \frac{1}{0.1s + 1} \cdot T_{demand} \tag{5.1}$$

 $T_{demand}$  is the nonlinear function of the engine speed and the accelerator pedal position between 0 and 1, from the driver.

The idle speed is set to 800 rpm where  $T_{demand}$  would be 0 Nm.

The rotational acceleration of the engine can be also calculated by using a derivative function in Matlab/Simulink and applying it to the rotational velocity that is used as an input data.

Once all these parameters are known equation 4.3 can be filled and therefore the torque which is transmitted to the clutch (not the one which the clutch transmits) can be calculated, and the next step is reached.

The block diagram of the engine model developed in Simulink is shown in the figure 5.1:

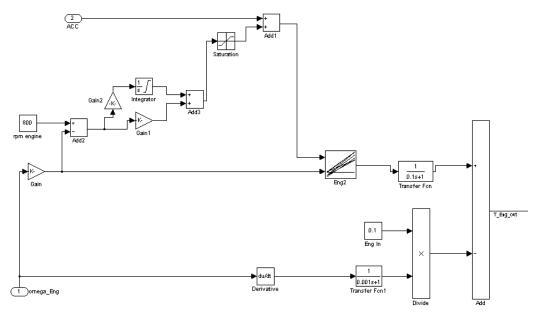


Figure 5.1: Block diagram of the engine subsystem with the idle control in Matlab/Simulink environment



#### 5.1.2 Clutch

#### **Built model**

Taking chapter 2 into account, the creation of a new clutch model with Matlab/Simulink is decided, and now it is going to be described. As it is shown previously, the torque which is transmitted through the clutch depends on the relative angular speed. The idea is to use a simple look-up table with one dimension to represent the data of the relationship between the torque transmitted and the relative angular speed. The vector of the input values would be the rotational relative speed and would take the values -10, -5, -1, 0, 1, 0, and 0 rad/s and then the corresponding table data. These data would be the factor that multiplies the maximum torque of the engine and its value would be between 0 and 0. This 0 represents the stick-slip phenomenon, since if it was not considered, the factor would take a value of 0. The factor values for different relative speeds which are entered in the look up table will be calculated by interpolation in Matlab/Simulink. The result of the relation between the factor that multiplies the friction torque and the difference of speeds between the engine and the transmission input is presented in the figure 0.

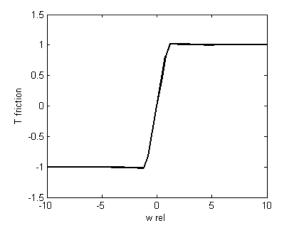


Figure 5.2: Relation between the fraction of friction torque transmitted and the relative angular speed

Finally, once the clutch model is built, all the torques and the velocities must be recalculated. Consequently, the block diagram of the clutch model built in Matlab/Simulink is shown in the figure 5.3

#### 5.1.3 Transmission

In this section a closer look on the design and function of the manual gear box and the transmission is going to be taken. This is not only the gear box, but also the differential or final drive is going to be taken into account. Consequently, this sub model will represent the part of the power train which goes from the transmission input, just after the clutch,



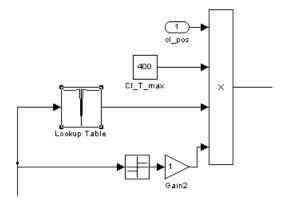


Figure 5.3: Block diagram of the clutch model created in Matlab/Simulink environment

to the final drive.

First of all, it is known from the data that our gear box is a manual transmission of six gears, which is nowadays typical from the cars. In the next step the transmission and differential ratios must be calculated for the data given. This task is going to be performed with the help of the engine velocity and the mean speed of the front wheels. If the correlation between these two parameters is represented graphically, the ratios for each gear are easily found when a gear shift is not made. While a gear shift is made, there are two parts of the drive line which move independently. One is composed by the engine and the first part of the clutch, and the other one is the second part of the clutch until the car body, going through the transmission, differential, propeller shaft and wheels. In this case, there is not a relation between the two movements, therefore the ratios cannot be observed. The equation which is used to calculate the ratios of the vehicle given is the following:

$$ratio = \frac{n_e}{(n_{lfw} + n_{rfw})/2} \tag{5.2}$$

where  $n_e$  is the rotational speed of the engine,  $n_{lfw}$  is the rotational speed of the left front wheel and  $n_{rfw}$  is the rotational speed of the right front wheel. The ratios will be the product of  $i_t \cdot i_{fd}$ . The mean speed of the wheel is calculated by the mean of the front wheels, but it can also be calculated by the mean of the roar wheels or even by the mean of the four wheels of the vehicle.

In figure 5.4 a graphic of the ratios which have been obtained is represented. The number of the gear, the relation between the engine velocity and the mean speed of the front



wheels, and the ratios calculated, are represented in the diagram in Matlab.

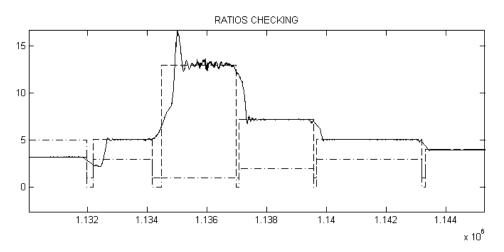


Figure 5.4: Diagram for a visual testing of the ratios which have been obtained. The relation between the engine velocity and the mean speed of the front wheels is represented in solid line. The ratios calculated are represented in dashed line and the gear is in dash-dotted line.

The modelling of the transmission must be only for the changing gear ratio, as we have a clutch sub model which models the effect of the synchronization. To keep the complexity down for faster simulation, some simplifications have been done. There is one gear efficiency and bearing friction all together for the whole transmission and that will be the mechanical efficiency for the transmission. It is going to be constant for all the different cases and it will be proportional to the torque that is transmitted through the clutch.

Once the ratios are calculated another look up table is built for the transmission model so as to construct the relation between each gear and its corresponding ratio. Then, the torque which exits the transmission can be also calculated, as well as the speed which goes into it. For this task, equations 4.5, 4.6, 4.7 and 4.8 are used.

Finally the final torque which is the same as the input for the drive shaft is calculated applying the NewtonÂ's second law to the rotational dynamics.

The block diagram of the entire transmission and differential model built in Matlab/Simulink is shown in the figure 5.5.

#### 5.1.4 Car Body

In this section, the last part of our vehicle simulation will be explained. Consequently, the forces acting on a vehicle with mass m and speed v will be shown. Newton's second law in the longitudinal direction is going to be used in order to make a description of



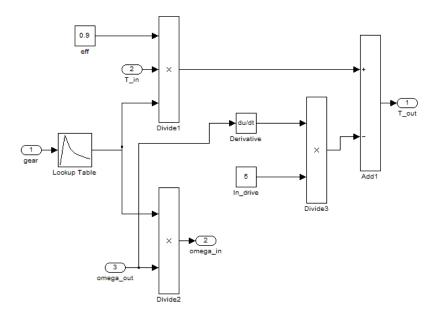


Figure 5.5: Simulink top level diagram of the transmission model

the dynamics with the help of the equation 4.11. If an analysis is made to the wheel and vehicle, these are the forces which actuate in them.

The force  $F_w$  that comes from the torque which acts on the wheels, can be calculated by multiplying the torque input of the car body, which is the one that the wheels have and goes out the transmission sub block, per the effective radius of the wheel. In this case, with the data provided, it is possible to calculate the radius. As the rotational velocity of the wheel, as well as the longitudinal speed of the wheels are provided, the external radius of the tires can be calculated with the following relation:

$$radius_w = \frac{(v_{lfw} + v_{rfw})/2}{(\omega_{lfw} + \omega_{rfw})/2}$$
(5.3)

where  $v_{lfw}$  and  $v_{rfw}$  are the longitudinal speeds of the front left and right wheels, respectively;  $\omega_{lfw}$  and  $\omega_{rfw}$  are the rotational speeds of the front left and right wheels, respectively; and  $radius_w$  is the radius of the wheel.

Furthermore, Fw can be described by the sum of the following forces:

•  $F_a$  , the air drag: It represents the braking force from the air drag and it is proportional to the squared velocity. Under normal weather conditions the influence form



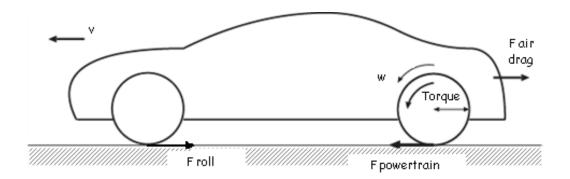


Figure 5.6: Longitudinal forces acting on a vehicle

the wind speed is small compared to the vehicle speed, but it depends on the aero-dynamics of the vehicle.  $F_a$  is approximated by:

$$F_a = \frac{1}{2} \cdot c_w \cdot A_a \cdot \rho_a \cdot v^2 \tag{5.4}$$

where  $c_w$  is the drag coefficient,  $A_a$  the maximum vehicle cross section area, and these two parameters depend on the vehicle.  $\rho_a$  is the air density and  $v^2$  is the squared velocity. However, effects from, for instance, open or closed windows will make the theoretical force more difficult to model and less accurately.

•  $F_r$ , the rolling resistance: This force originates from tires deformation. The centre of normal pressure is shifted in the direction of rolling, which produce a torque about the axis of rotation of the tire, the rolling resistance moment or torque. In a free rolling torque the applied wheel torque is zero. Consequently, a horizontal force at the tire ground contact patch must exit to maintain equilibrium. This horizontal force is generally known as the rolling resistance.

Here the rolling resistance will be treated as force acting through the axis of the wheel. It depends on the speed of the vehicle, and it will be modelled by the following equation:

$$F_r = 150 \cdot \tanh 1000v$$
 (5.5)



where the number 150 is a constant value for the rolling resistance, which will be modified by the function tanh of the vehicle speed.

•  $m \cdot g \cdot \sin \alpha$ , the gravitational force:  $\alpha$  is the slope of the road and m the mass of the vehicle. In the data provided it is not shown the slope of the road. However, it is known that this force is considerably smaller than the other two above, therefore it is considered zero for all the cases in this thesis.

In the figure 5.7, these forces are represented in the environment of Matlab/Simulink, and are connected to the torque which comes from the transmission sub block.

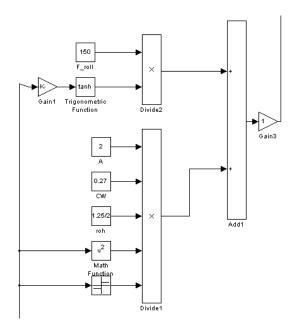


Figure 5.7: Simulink top level diagram of the longitudinal forces acting on a vehicle

Then, once these parameters of the equation are known, the acceleration of the vehicle can be calculated by introducing all the values, and the speed and position of the car are easier to calculate by adding one integrator for the velocity, and another one for the position in the sub model.

With the velocity of the car, the speed of the different parts of the model can be recalculated and a complete recirculation until the engine, which is the first part of our model, can be made.

## 5.1.5 Dynamics of the gear shifting

An auxiliary block is added in the model in order to describe the damping oscillations produced in the torque function during the gear shift. With the help of the PT2 data



which have been provided, these oscillations are calculated and represented.

The inputs of the subsystem will be the number of the gear, which will be used with the help of the data and look up tables to calculate the damping parameters for describing the oscillations; the torque that has been calculated before and comes out the clutch subsystem, which is the transmission input torque and will be the function which oscillates; and lastly a signal which will show if the clutch disks are moving with the same velocity or not.

The output of the system is the torque transmitted through the clutch with all the oscillations included.

The dynamic block with the inputs and the output signals is represented in the figure 5.8.

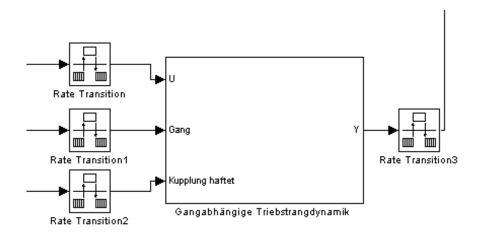


Figure 5.8: Simulink top level diagram of the auxiliary block for describing the oscillations produced in the torque transmitted through the clutch during a gear shift. The torque clutch calculated before, the number of the gear and the size of the relative speed of the clutch disks are the input signals.

In the figure 5.9 it can be observed how the oscillations are produced due to the dynamic system introduced in the model. The most important for this thesis is the one which occurs just after the clutch disks are engaged, which will be a reference point for the torque description.

At this point, an explanation of the PT2 system is going to be made in order to describe the damping of the system modelled. Generally, damped harmonic oscillators satisfy the second-order differential equation:

$$T^2\ddot{y}(t) + 2DT\dot{y}(t) + y(t) = u(t)$$
 (5.6)



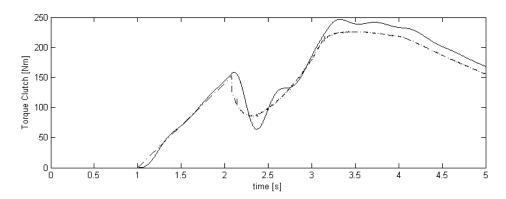


Figure 5.9: Representation of the torque transmitted through the clutch. In solid line it is represented this torque with the oscillations pruduced by the PT2 system. In dash-dotted line it is represented the torque without using the system for the PT2.

This equation for the homogeneous case comes from a mass, which is attached to a spring and a damper, with coefficients k and c respectively. Consequently, applying the Newton's second law, the total force on the body is:

$$F_{tot} = F_s + F_d \tag{5.7}$$

$$m\ddot{x} = -kx - c\dot{x} \tag{5.8}$$

Therefore, the differential equation for the homogeneous case must be rearranged in the following form:

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0 \tag{5.9}$$

At this point, the following parameters are defined, natural frequency, with units radians/second,  $\omega_0$  and damping ratio (dimensionless), D. These parameters will play important roles in defining second-order system responses:



$$\omega_0 = \sqrt{\frac{k}{m}}$$

$$D = \frac{c}{2\sqrt{mk}}$$
(5.10)

$$D = \frac{c}{2\sqrt{mk}} \tag{5.11}$$

Finally, the equation 5.12 is obtained.

$$\ddot{x} + 2D\omega_0 \dot{x} + \omega_0^2 x = 0 \tag{5.12}$$

Now the theoretical part has been explained, our PT2 system will be described in detail. For this purpose parts from the model in our system will be described separately and after that joined. In the figure 5.10 the Simulink model of the PT2 system is presented.

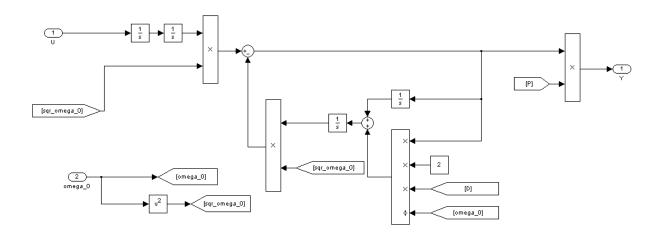


Figure 5.10: Representation of the PT2 system built in the Simulink environment

First of all, it can be observed that the system is of the same form as the one shown in the figure 5.11

This system will be solved in the following way.

$$u - G(s)y = y (5.13)$$

$$u = y(1 + G(s)) (5.14)$$

$$y = \frac{1}{1 + G(s)}u ag{5.15}$$



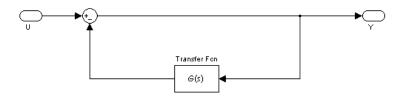


Figure 5.11: Representation of a simple diagram of a transfer function

Once this has been solved, G(s) is calculated in the system provided, and the result which is obtained has the following form:

$$G(s) = \frac{2D\omega_0}{s} + \frac{\omega_0^2}{s^2}$$
 (5.16)

Consequently the following transfer function for the complete system is obtained:

$$y = \frac{\omega_0^2}{s^2 + 2D\omega_0 s + \omega_0^2} u \tag{5.17}$$

The behaviour of this system will depend on the values of the two fundamental parameters, the natural frequency,  $\omega_0$ , and the damping ratio, D. Particularly, not the quantitative but the qualitative behaviour will depend on the value of the D, which can be D=1,  $0 \le D < 1$  and D>1. The three cases will be explained in the following lines.

- lacktriangle Critical damping, D=1: The system is said to be critically damped. A critically damped system converges to the value as fast as possible without oscillating.
- Over damping, D > 1: The system is said to be over damped. The process to converge is slower than in a system with critical damping.
- Under damping,  $0 \le D < 1$ : The system is said to be under damped. The system will oscillate at the damping frequency, which is a function of the natural frequency and the damping ratio.



An example of each different situation of critical, over and under damping is represented in the figure 5.12.

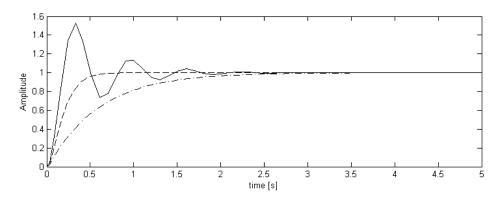


Figure 5.12: Representation of the damping function for a value of  $\omega_0=10$  and for different values of D. In solid line it is represented the under damping effect with D=0.2. In dashed line it is represented the critical damping effect, with D=1. In dash dotted it is represented the over damping, with D=3.

For a better understanding of the damping in the model, some easy simulations will be made changing the parameters of damping ratio and the natural frequency in order to obtain a visual example.

First of all, the source of the system created will be a step signal, so as to simplify the calculations. Then, the transfer function of the PT2 system is included. The Simulink model created is presented in the figure 5.13.

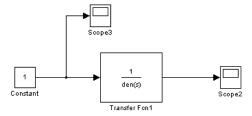


Figure 5.13: Simulink diagram of the system created in order to explain the simulations

The first parameter whose influence will be studied is  $\omega_0$  or the natural frequency. For this purpose, a constant value for the damping ratio is stated of 0.2. In the figure 5.14 is represented the damping signal for different values of  $\omega_0$ .



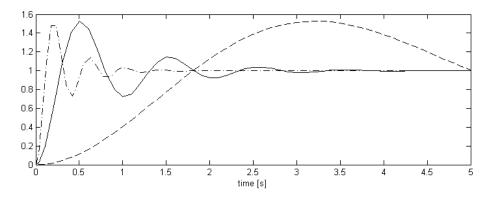


Figure 5.14: Behaviour of the damping function for a constant damping ratio of 0.2. In solid line it is represented for a value of  $\omega_0 = 2\pi$ , in dashed line for a value of  $\omega_0 = 1$ , and in dash-dotted for  $\omega_0 = 5\pi$ .

As it can be observed in the figure 5.14, the natural frequency of the system has its greatest influence in the period of the oscillation, as the system is oscillating with the damped natural frequency for the under damped systems.

The damped natural frequency is defined with in the equation 5.18:

$$\omega_d = \omega_0 \sqrt{1 - D^2} \tag{5.18}$$

Once this description has been done, it is time to describe what happens in the signal when  $\omega_0$  is maintained constant and the changes are produced in the damping ratio D. Since in the model which has been built, the case is under damping, different values for D < 1 will be taken and represented. In this way, the influence of the damping ratio in the damping signal can be detected. For this purpose, the figure 5.15 is represented.

In the figure 5.15, it can be observed that the higher is the value of the damping ratio, the smaller are the peaks in the oscillations of the damping function. Consequently, it can be stated that the damping ratio has influence in the amplitude of the oscillations. What is more, for the under damped systems, D has also a grat influence in the period of oscillation, as it has been observed in the equation 5.18. If the equation of under damping for a step response is analysed, the maximum points for the damping function are obtained when the value of the  $\cos$  is -1 and the minimum peaks are obtained for a value of 1. With this relation the tendency of the maximums and minimums of the damping function is observed and represented in the figure 5.16.

What is more, the equation from which the values of the maximums and minimums peaks for a step signal response are obtained is refpeak.

$$y(t) = K \mp \frac{K}{\sqrt{1 - D^2}} \cdot e^{-D\omega_0 \cdot t}$$

$$(5.19)$$



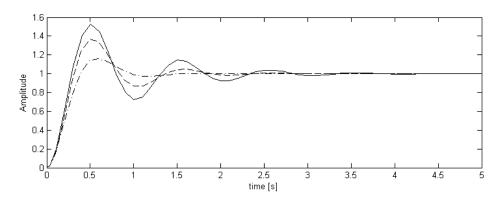


Figure 5.15: Behaviour of the damping function for a constant natural frequency of  $2\pi$ . In solid line it is represented for a value of D=0.2, in dashed line for a value of D=0.3, and in dash-dotted for D=0.5.

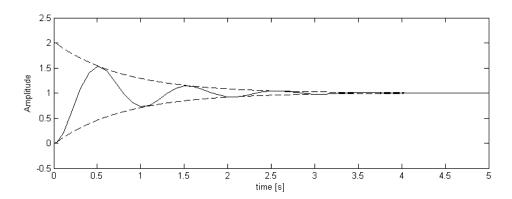


Figure 5.16: Behaviour of the damping function for a constant natural frequency of  $2\pi$  and a damping ratio of 0.2 represented in solid line. In dashed line they are represented the functions of the tendency of the maximums and minimums of the oscillations.

This equation is obtained from the equation of a step signal response for the under damped systems.

Although these examples have been stated for a step response of a second order system, this is not valid for the signal which is going to be introduced in the PT2 system in the model, as the torque which comes from the clutch does not have a step shape. Due to this fact, a study of how the second order systems behave for different signals must be done in order to simplify the calculations.

The characteristic responses of a second order system to the ramp, step and impulse fuctions are presented in this thesis for the case of under damping ( $0 \le D < 1$ ):



• Unit ramp response, u(t) = t:

$$y_r(t) = t - \frac{2D}{\omega_n} \left[ \frac{2D}{\omega_n} \cos \omega_d t + \frac{2D^2 - 1}{\omega_d} \sin \omega_d t \right]$$
 (5.20)

• Unit step response, u(t) = 1:

$$y(t) = \left[1 - \frac{1}{\sqrt{1 - D^2}} \cdot e^{-D\omega_0 \cdot t} \cdot \sin\left[\sqrt{1 - D^2}\omega_0 \cdot t + \arctan\frac{D}{\sqrt{1 - D^2}}\right] \quad (5.21)$$

■ Impulse response,  $u(t) = \delta(t)$ :

$$y(t) = \frac{e^{-D\omega_0 t}}{\sqrt{1 - D^2}} \sin \omega_d t \tag{5.22}$$

Where the damped natural frequency  $\omega_d = \omega_0 \sqrt{1 - D^2}$ .

As it can be observed in the equations, the parameters D and  $\omega_0$  have different effect in proportion depending on the signal which is introduced in the PT2 systems. That is why each case must be studied separately. A more extensive study of the PT2 systems will be made in chapter 7.

With this block all the parts of the model have been explained, and the next step is to connect all of them in order to calculate all the signals in real time. This will be explained in the next section.

## 5.1.6 The complete powertrain model

In this section the parts described above will be connected to each other, in order to build a complete powertrain model together with driver and vehicle model. All the sub blocks will be connected in Matlab/Simulink in the way which is going to be explained in the following text.

First of all, the variables that the driver controls in the model must be connected. These are the accelerator pedal position and the clutch pedal position.

As is it explained before, the clutch pedal has two parameters, which are the slope and the start time. A variable for the final value of the clutch would have no sense as it is always considered that the clutch is finally 100% engaged for all the cases.

In the other hand, the throttle has another three parameters which are the slope, the time when the driver starts pushing it, and the final value of the throttle position, which



will be constant until the end of the simulation.

The accelerator pedal position is connected to the look up table of the engine. Therefore, with this parameter and the rotational speed of the engine the torque can be calculated.

The clutch pedal position is connected to the clutch model in order to calculate the torque which is transmitted through the clutch.

Then the engine, clutch, transmission and car body are connected:

• To connect the engine to the clutch:

$$\omega_e = \omega_{ci} \tag{5.23}$$

$$T_e = T_{ci} (5.24)$$

• To connect the clutch to the transmission:

$$\omega_{co} = \omega_{ti} \tag{5.25}$$

$$T_{co} = T_{ti} (5.26)$$

■ To connect the transmission to the wheels or car body:

$$\omega_{to} = \omega_w \tag{5.27}$$

$$T_{to} = T_w (5.28)$$

As in this thesis there is not a difference between the wheels and the car body, this two parts do not need to be connected.

The complete diagram of the simulik model is presented in chapter 9 in figure 9.6.



#### 5.2 Validation of the vehicle model

Once the simulation model is built, the next step is to compare the simulation results with the real data, in order to know that the vehicle model works. What is more, some corrections of the parameters which cannot be acquired from the data will be corrected so that the results will be as accurate as possible. In spite of not having important things such as the engine map from the car, it will be observed that the simulation results will not differ so much from the real data.

For this purpose, some real gear shifts are selected from the real data and are studied separately. One of them is going to be presented in this section, and some of the others will be included in the final program and in chapter 9. In these gear shifts, some variables are going to be represented so as to compare the simulation with the data provided.

First of all, the parameters which the customer controls must be adjusted. These are the ones which have been presented in the first section of this chapter:  $s_c$ ,  $t_c$ ,  $s_a$ ,  $t_a$  and  $f_a$ .

In order to adjust  $t_c$ , it is important to see at what time the clutch starts to transmit torque. As the clutch simulated has not been made with play, at the moment the clutch pedal starts being released, the torque starts being transmitted. Consequently, it is a mistake to fix the time when the clutch start to be released regarding to this same time in the real data.

Once the time has been adjusted, it is the turn of  $s_c$ . In this case, some differences exist between the different gear shifts. It is not the same the way of dealing with the clutch pedal for a change to the first gear than for a change to the fifth gear for example. In addition, the behaviour of each clutch must be studied in detail because it is going to be different for one vehicle and for another. In this case, the gradient is not going to be constant. Instead of this, the evolution of the clutch position will be divided more or less in three stages: The first one with a constant slope, the second one approximately a constant value and the last one another slope until its final position.

In the figure 5.17 the real data and the simulation of the clutch are presented.

At this point, it is time to adjust the parameters of the throttle, which will be easier than the parameters of the clutch, since the gradient is approximately constant and the final position too.

As in the other case, the first parameters to be adjusted will be the  $t_a$  and  $s_a$ , but in this case it will not suppose so hard as in the clutch. Finally, the last parameter to be adjusted will be the throttle final position. It is important to notice that the throttle can also have play, and this is an important point to be taken into account.

The real data and the simulation are also shown in the figure 5.18.



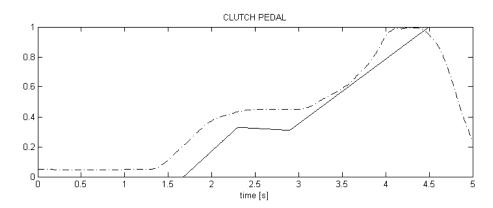


Figure 5.17: Representation of the evolution of the clutch pedal along the time. In solid line it is represented the simulation and in dash-dotted it is represented the real data of the gear shift chosen. Number 1 of y label represents the 100% of the clutch pedal released while 0 represents the pedal completely pressed.

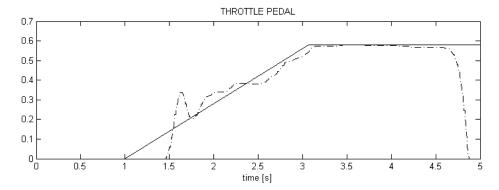


Figure 5.18: Representation of the evolution of the throttle along the time. In solid line it is represented the simulation and in dash-dotted it is represented the real data of the gear shift chosen. In this case, number 1 of y label represents the 100% of the throttle pressed and 0 the customer does not press the pedal.

Once all the parameters have been adjusted, the values of different variables will be presented in order to make a comparison and to verify that the vehicle parameters have been adjusted properly.

In order to represent the torque transmitted in the clutch and the torque at the propeller shaft, the dynamic sub model where the PT2 data are introduced is used. With it, all the oscillations which occurred at the torque are represented. Once the torque at the clutch is calculated, it is only necessary to apply the equations explained above in order to calculate the wheels torque. In the figures 5.19 and 5.20 the torque which is obtained in the simulation and in the real data are represented.



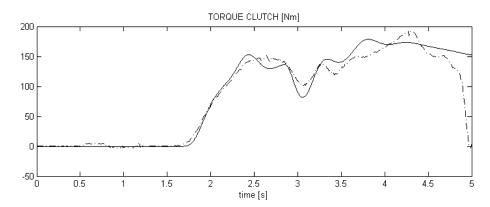


Figure 5.19: Evolution of the torque transmitted through the clutch. The torque simulated is represented in solid line and the real data are represented in dash-dotted line.

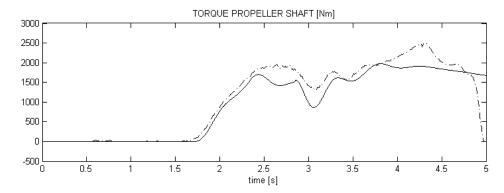


Figure 5.20: Evolution of the torque at the wheels. The torque simulated is represented in solid line and the real data are represented in dash-dotted line.

As it can be observed, although the values are not exactly the same, the behaviour of the torque function is approximately equal. However, it is difficult to adjust all the parameters of the vehicle when the data available are not so big. Consequently the results are considered as valid.

At this point, it is turn to represent both the transmission input speed and the engine speed, which will have a very important role at the time of adjusting the parameters. As the pedals of the vehicle possibly have play, and in our simulation model it does not exist, it is important to match these variables in order to adjust properly the throttle and clutch pedal parameters. The result of the simulation and the data provided are represented in the figure 5.21. It can also be seen that although they do not match perfectly, the result is very good in spite of not having the engine map of the vehicle that is being analysed.



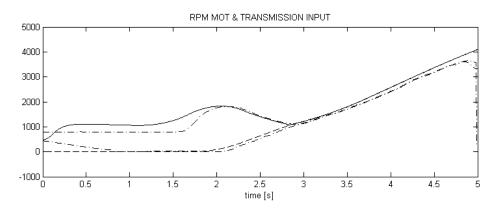


Figure 5.21: Evolution speeds of the transmission input and the speed of the engine. The simulated velocities are represented in solid (transmission input) and dashed (engine) lines and the real data are represented in dash-dotted line.

Finally, the evolution of the speed of the vehicle will be represented in order to check that the velocity as well as the acceleration have the same behaviour in both the simulation and in the real data. This is represented in the figure 5.22. As it can be observed, the results are very good. The approximation of the speeds and also the gradient of increasing is very similar.

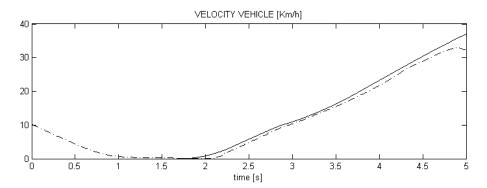


Figure 5.22: Evolution of the speed of the vehicle. The simulated speed is represented in solid line and the real data are represented in dash-dotted line.

The torque at the engine is also important to compare, but in this case, as the engine map is not available and has been estimated, it is not going to match. However, in case that an engine map of the vehicle is provided, it can be compared and all the other variables will be better adjusted.

Nevertheless, the results match good enough to consider the simulation model as well constructed. The important point is that now, with the Matlab/Simulink model built, new data can be introduced in the vehicle model by changing its parameters in the Mat-



lab/Simulink environment. Consequently, new simulations for different vehicles can be made easily. Besides, the more the data available, the easier will be the parameters adjusted.

For the validation of the model, the parameter of slope clutch constant cannot be used. As it has been seen, for the first gear, the intensity of releasing the clutch pedal is far from being constant. Therefore, the behaviour of the torque will vary if it is used a no constant slope or a constant one. A compromise in order to adjust more some parameters will be made, so some variables will match better than others.

However, for the later use of the simulation some compromises need to be made. This is the reason why the clutch pedal slope and the throttle slope are approximated to constant values, because this will simplify the calculations. In the next section the behaviour of the propeller shaft torque will be explained for different values of the parameters.

The graphics obtained for another gear shift which has been simulated are included in chapter 9.



# 6 Influence of the Driver Parameters in the Torque

In this section the propeller shaft torque after a gear shift will be described with the help of the clutch pedal and the accelerator pedal. In the thesis, the gear shift from the neutral gear to the first gear will be studied, but always with the vehicle with velocity 0 at the beginning of the measurements. Consequently, when the real data are observed and simulated, this initial condition must be taken into account and it is important to be sure that the vehicle is stopped before the first gear has been engaged.

Each gear shift behaves in a different manner, as the customer does not control the parameters in the same way for different situations. Therefore, each case needs to be considered and studied separately. However, the behaviour of the propeller shaft torque should be similar, although with different proportion.

What is more, this effect is accentuated when the driver uses different gears, different roads, or different vehicles. For different gears, as it has been explained along the thesis, there are different damping ratio and natural frequency in the damping. Therefore, the dynamic behaviour of the torque will be also different. According to this, the purpose of this description will be to have a general knowledge of the influence of the parameters used in the study.

The parameters which will be taken into account in order to explain the wheels torque have been explained in chapter 5:

- $s_c$ : The intensity with which the clutch pedal is released (slope of the clutch pedal).
- $t_c$ : The time when the clutch pedal starts to be released (start time of the clutch pedal).
- $s_a$ : The intensity with which the throttle is pushed (slope of the accelerator pedal).
- $t_a$ : The time when the throttle starts to be pushed (start time of the accelerator pedal).
- $f_s$ : The final position of the throttle, which will be considered to be constant (limit of the throttle position).



Note that the two parameters of time can be reduced in only one parameter. It is not important the exact moment of time when the clutch and the throttle start to actuate, but the difference between these two instants. For example, if the throttle starts to be pushed 1 s after the measurement begins, and the clutch 1.5 s after, a propeller shaft torque is going to be generated. However, this function of the torque will be the same as if the throttle starts to actuate 2 s and the clutch 2.5 s after. The main difference between these two cases is that the signal of the second one will be delayed 1 s, but the shape as well as the values will be the same. In the figures 6.1 and 6.2 an example of this is observed:

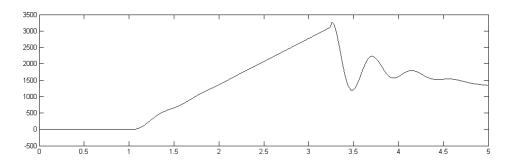


Figure 6.1: Propeller shaft torque with parameters  $t_c$  = 1 s and  $t_a$  = 0.5.

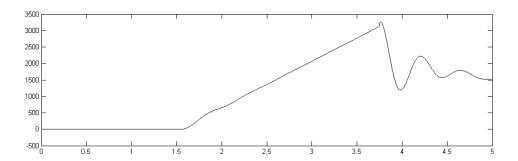


Figure 6.2: Propeller shaft torque with parameters  $t_c$  = 1.5 s and  $t_a$  = 1.

Consequently, the importance of the time parameters will lie in the difference between them, and not the value of each one separately.

For a better understanding of the behaviour of the clutch model and then of the propeller shaft torque after a gear shift, there are some basics which need to be explained before the description. Two main cases can be differentiated when the torque is transmitted from the engine to the transmission through the clutch.

The first situation is when the clutch is being closed but the speed of the transmission input is not the same as the speed of the first disk of the clutch, which has approximately the speed of the engine. Then the clutch is transmitting a proportional part of the max-



imum torque that it is capable to transmit, which can be more than the torque of the engine as this is a function of its size, friction characteristics, and the normal force that is applied. The equation that governs the physics of the torque that the clutch is capable to transmit is the following:

$$(T_f)_{max} = \int \int_A \frac{r \times F_f}{A} da = \frac{2}{3} \cdot R \cdot F_n \cdot \mu$$
 (6.1)

where  $(T_f)_{max}$  is the maximum friction torque which the clutch disks are capable to transmit,  $F_n$  is the normal force which actuates at the clutch disks, R is the equivalent net radious and  $\mu$  is the coefficient of friction.

However, the data available to calculate the maximum friction torque are not enough. Consequently, the adjustment of this value is done with the simulation help and the gear shifts which have been observed.

The second situation is when the speed of the transmission input and the speed of the engine are the same (with a minimum tolerance). Then the clutch is closed and the torque which is transmitted through it is the one which comes from the engine and does not depend on the clutch pedal position, which is assumed to be enough so as to maintain the two parts of the powertrain engaged.

Once these two situations have been stated, the method used in order to make the description will be in the following lines explained. The description will be developed with the help of some variables, which are:

- $T_{max}$ : The maximum torque which is reached after the two parts of the driveline match their speeds. It depends also on the parameters of damping. Although without the damping this point is reached at the time when the two speeds match, with the damping this maximum point will be reached a little bit later.
- $tim(T_{max})$ : The time in which this maximum torque is reached, also just after the clutch is completely engaged.
- T<sub>p</sub>: An approximation of the mean torque of the wheels, once the speed of the transmission input and the speed of the engine are the same, which is the one that the customer uses in order to accelerate the vehicle. This one is truly which represents the customer intentions of driving after the gear shift. However, it is difficult to set the exact point in which this variable starts being calculated.

With the help of Matlab, a program in order to calculate these three variables has been created. The steps which have been followed and the conditions used in its development



will be explained.

The first variables which are calculated are the  $T_{max}$  and  $tim(T_{max})$ . For this purpose, the first step is to identify all the maximums in the propeller shaft torque. After this, all the points in which the speeds of transmission input and engine match are recognised. As the speeds never have exactly the same value, a tolerance which has been chosen regarding to the data is used. Consequently, the difference between these two speeds must be less than this tolerance so that a point is recognised. Once this is done, only a few of the maximums calculated before are selected. For this selection, some limits of time are used, so only the maximums which are near the first point where the clutch is clothed are taken. At this point, the maximums which are near the point where the speeds start matching are saved in a vector. Then the maximum of all of these points,  $T_{max}$ , and the time when it occurs,  $tim(T_{max})$ , are easily calculated.

In order to calculate the  $T_p$  variable, the conditions which are selected are related to the position of the clutch and the slope of the torque in the propeller shaft, which must be less than a limit stipulated by regarding to real cases of the gear shift situation. Once all the points that verify these conditions are taken, the mean of them is calculated. This method could be changed and so its results by taking another conditions, but it depends on the experience of the researcher.

Once this is explained, a separately study of how each parameter influences the final wheel torque will be done in the following pages.



# 6.1 Influence of the Clutch Pedal Slope

In this section a study of the influence that the clutch slope,  $s_c$ , has in the torque at the propeller shaft will be done. For the realisation of the study, some measurements are made. These measurements are taken for different clutch slopes, remaining the other parameters constant and trying that the effects of these in the final results are as minor as possible or at least easily recognisable.

A good strategy to follow is to press the gas pedal until the customer's desire and reach this limit before the clutch pedal starts to be remained. Then, to the effect of  $s_c$  it is only added the effect of a constant value of the throttle position, which will be explained later.

As the different simulations have obtained similar behaviour, the results are considered good enough. One of the examples of the measurements which have been made is presented in this section. In this example the value of the acceleration pedal slope  $(s_a)$  is 2, the value of the clutch start time  $(t_c)$  is 1, the throttle start time  $(t_a)$  is 0.25, and a value of 0.4 for the throttle final position  $(f_a)$ .

In the table 9.1,which is in the chapter 9, the values of the variables  $T_{max}$ ,  $tim(T_{max})$  and  $T_p$  which have been obtained are presented.

First of all, the effect of the clutch slope in the maximum torque of the first oscillation,  $T_{max}$ , will be explained. The way in which the torque varies with the  $s_c$  is represented in the figure 6.3.

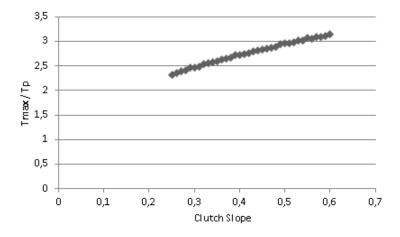


Figure 6.3: Maximum torque propeller shaft in the first oscillation  $(T_{max})$  divided per the final torque at the propeller shaft, for the different values of  $s_c$ , maintaining the other parameters constant.

In addition, the time when  $T_{max}$  is reached is also represented in the figure 6.4.



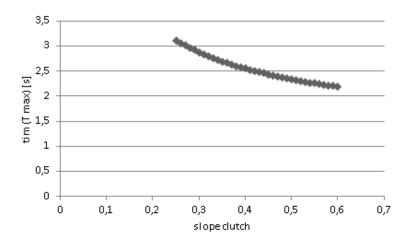


Figure 6.4: Time when the maximum torque propeller shaft in the first oscillation occurs  $tim(T_{max})$  for the different values of  $s_c$ , maintaining the other parameters constant.

As it can be observed, while the slope in the clutch pedal increases,  $T_{max}$  increases too. What is more,  $tim(T_{max})$  decreases. This effect is consistent with the physical explanation. While the transmission input and engine speeds are different, the torque transmitted through the clutch is proportional to the normal force that actuates in the clutch disks. This normal force is directly related to the position of the pedal. The more the pedal is released, the greater will be the normal force acting. Besides, it can be stated that, although the time is decreasing, the positive slope is enough to make  $T_{max}$  higher.

This effect is presented in the figure 6.5.

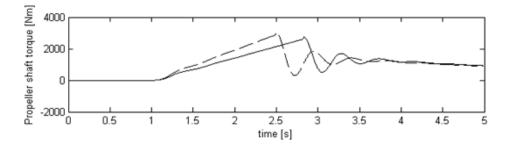


Figure 6.5: Evolution of the propeller shaft torque for two different cases. In solid is for  $s_c = 0.31$ , and in dashed for  $s_c = 0.42$ .

In the figure it is shown that for  $s_c$  = 0.42, the  $T_{max}$  is higher and is reached in less time than for  $s_c$  = 0.31.



Once this has been explained, the behaviour of the torque once the clutch is engaged  $(T_p)$  will be presented. First of all, the results which have been obtained are represented graphically in the figure 6.6.

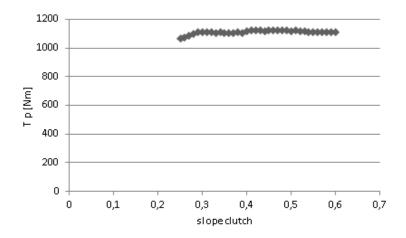


Figure 6.6:  $T_p$  obtained for different slopes of the clutch.

Since the values of  $T_p$  do not vary very much with the clutch slope, it can be assumed that  $s_c$  does not have influence in the value of the torque once the clutch is engaged. As it will be seen later, other parameters would have a greater influence in this variable. Even though in the table 9.1 some small changes can be seen, when the plots of the torque function are observed, it can be considered that the final torque is equal for the different values of  $s_p$ .

To resume, the intensity with which the clutch pedal is released affects the first part of the simulation, where the speeds of engine and transmission input do not match. The higher  $s_c$  is, the greater the slope of the torque transmitted through the clutch will be just after the gear shift. In consequence, it is demonstrated above how  $T_{max}$  and  $tim(T_{max})$  are affected.



# **6.2** Influence of the Throttle Slope

An analysis of the influence that the acceleration pedal slope  $(s_a)$  has in the variables explained above will be made in this part of the study. In this case it is a little more problematic to realize, because it is difficult to observe only the effect of this parameter trying that the others do not affect the final result.

Some measurements have been also taken in the experiment with the simulation model. Regarding to the data, it has been observed that  $s_a$  is strongly related to the parameters of start time. For example, the throttle slope does not have effect in  $T_{max}$  when the throttle starts to be pushed at the same time or a little bit later than the clutch. This is because while the vehicle is stopped and the engine rotates at 800 rpm, at the moment that the clutch starts to have effect the clutch disks start being engaged, so although the customer accelerates the clutch will finish to close at the same time, and so with the same maximum torque.

In the figures 6.7 and 6.8 an example of this situation is presented.

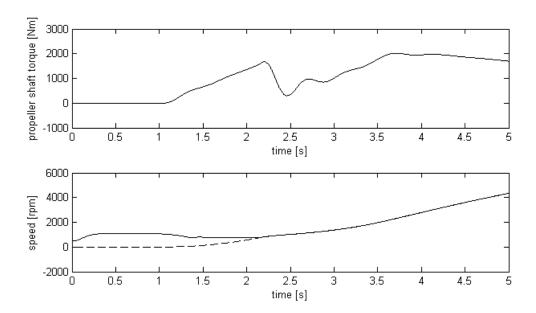


Figure 6.7: In the plot above it can be observed the torque at the propeller shaft for  $s_a$  = 0.25 and  $t_a$  = 1. In the plot below, the engine speed is in solid and the transmission input speed is in dashed line. The two disks of the clutch starts engaging at 1 s and are finally engaged at 2.2 s.

Even though the two throttle slopes differ notably, as the start times are too late, the parameter  $s_a$  has no effect in the  $T_{max}$  and its time. However, a difference after the clutch disks are engaged is observed. Since the engine speed for  $s_a = 0.51$  is higher, the torque



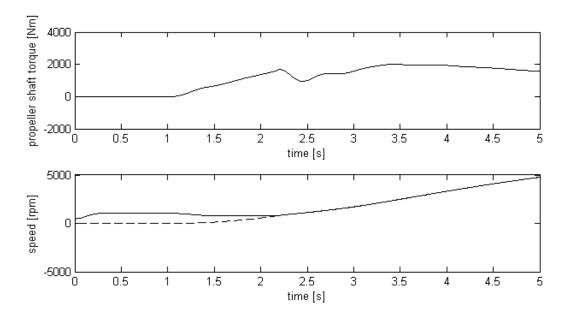


Figure 6.8: In the plot above it can be observed the torque at the propeller shaft for  $s_a$  = 0.51 and  $t_a$  = 1. In the plot below, the engine speed is in solid and the transmission input speed is in dashed line. The two disks of the clutch starts engaging at 1 s and are finally engaged at 2.2 s.

fall after  $tim(T_{max})$  is less, and consequently the low peak of the oscillation is attenuated.

Once this situation has been described, a strategy in order to take the measurements must be thought. The best way is to set a  $t_a$  small enough so that the effect of the gas pedal slope could be appreciated. In this way, simulations for different values of  $s_a$  have been made, and the results have been quite satisfactory. An example of the results obtained is presented in the table 9.2 in the chapter 9.

Two different situations can be distinguished in the figure 6.9. As it can be noted, in the first part of the figure, while the slope is increasing, a significant change in  $T_{max}$  occurred. What is more, the time where the engine speed and transmission input speed match is delayed with each change. In the second part, both  $tim_(T_{max})$  and  $T_{max}$  do not have significant changes while the slope is increasing. That is because the point where the clutch disks start to engage does not change in this area much with small changes of acceleration.

On the other hand, the influence of the acceleration pedal slope to the mean torque at the wheels  $(T_p)$ , will be explained in the following lines. This part is also a little difficult to justify, since the limits of when the customer is considered to drive with constant torque are subjective. This parameter should not have major effect in the final torque. While the gas pedal is increasing, a constant final torque should not be reached. The final torque should remain constant when the position of the throttle is approximately constant.



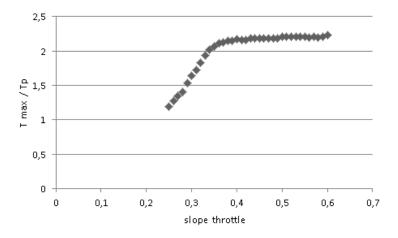


Figure 6.9: Maximum torque propeller shaft in the first oscillation  $(T_{max})$  divided per the final torque which is reached at the propeller shaft for the different values of  $s_a$ , maintaining the other parameters constant.

However, the bigger is  $s_a$  after the clutch disks have engaged, the faster the final position of the throttle and consequently the final torque will be reached. Therefore, the major influence of the throttle slope would be in the space of time which is between  $T_{max}$  occurs and the first point of  $T_p$ , as the oscillation will be attenuated with the increase of the acceleration pedal position. For this reason, the mean value of  $T_p$  differs from one experiment to another. While the acceleration pedal is constant, the torque is not, and it will decrease a little because of the external forces acting. As the simulation is only for 5 s, for small  $s_c$ , the final torque which the customer intends to drive will be measured for less time, and that is the reason for its higher values.

To resume, the accelerator pedal slope has influence in  $T_{max}$  and  $tim(T_{max})$  only when the clutch disks have not started engaging. Besides, it has also influence in the space of time between  $T_{max}$  and  $T_p$ , but only if the final position of the throttle has not been already reached by the time  $T_{max}$  occurs. In addition, the higher the value of  $s_a$  is, the sooner the constant value of  $T_p$  is reached. Once this explanation has been developed, the next parameter will be analysed.



# 6.3 Influence of the throttle final position

As in the other cases, a lot of simulations experiments have been developed in order to describe the effects of this parameter,  $f_a$ , in the propeller shaft torque.

Some measurements have been made so as to describe this effect. One example of these measurements is presented and discussed in the following lines and the reluts are included in the chapter 9.

The most intuitive effect that produces the throttle final position,  $f_a$ , in the wheels torque can be observed after the clutch is engaged. The final torque which the customer wants to drive with  $(T_p)$ , is observed to increase always with the final throttle position, for the same value of the other parameters.

This effect is consistent with the physical explanation, as the more the driver presses the throttle, the greater will be the torque at the engine (although it also depends on the speed of the engine). Since the area where  $T_p$  is developed is the one in which the clutch transmits the torque that comes directly from the engine, the higher the torque at the engine is, the bigger will be also in the propeller shaft. Consequently the acceleration of the car will increase.

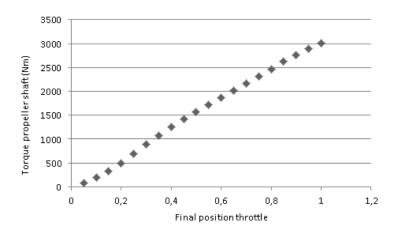


Figure 6.10: Final mean torque at the propeller shaft  $(T_p)$  for the different values of the final pedal position, maintaining the other parameters constant.

On the other hand, it can also be observed that the final pedal position has also influence in the maximum torque of the oscillation and the time when this one is reached. The more the gas is pressed, the higher will be the maximum torque and the time for it. This has also a physical explanation, as when the driver is accelerating, it is more difficult for the clutch to engage. Consequently the torque will be increasing for longer time and the maximum torque in the propeller shaft will be greater. In spite of this fact, this effect



can be only observed when the final position of the throttle has been reached while the clutch is not being engaged. If the clutch has started to engage but the final position of the throttle is not already reached, it has no effect here but only in the final torque the customer drives.

As before the action of this parameter it is the action of the acceleration pedal slope, if the clutch is closed during the action of  $s_a$  and not due to the final position of the throttle, then this last parameter will have effect only in the final torque developed.

What has been explained above will be presented in the following figures.

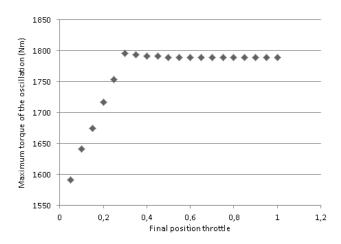


Figure 6.11: Maximum torque propeller shaft in the first oscillation for the different values of the final pedal position, maintaining the other parameters constant.

In the measurements, it is observed that from a final position of the throttle of 30% to 100%, more or less the same maximum torque is obtained in the oscillation. The explanation will be developed with the help of the figures 6.12 and 6.13.

In the figures 6.13 and 6.12 it is observed that the time when the speeds of the transmission input and the engine matches does not change, even though the final position of the throttle varies from 40% of the pedal to 90% of the pedal. Nevertheless, significant changes can be appreciated later, when the clutch has already engaged.

This fact is what has been explained above. In these two cases, it can be observed that the final throttle position is reached once the clutch disks have started engaging. Consequently the only difference that can be appreciated is in the torque after the engine speed and transmission input velocity match.

However, if the values 0.1 and 0.2 are taken for the analysis, the difference in  $T_{max}$  as well as  $tim(T_{max})$  will be appreciate, because the values 0.1 and 0.2 are reached before the



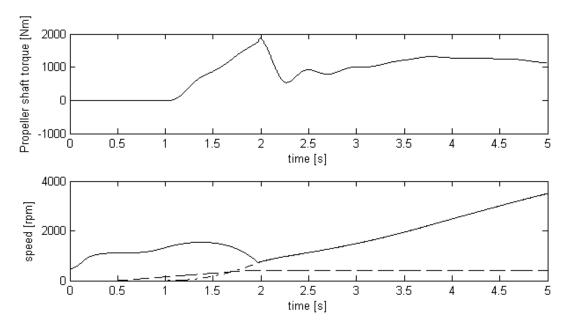


Figure 6.12: The plot above is the propeller shaft torque that the vehicle experiments for a final throttle position of 40%. In the graphic below, the engine speed is shown in solid, the transmission input speed in dot-dashed and the throttle position in dashed lines from 0 (0%) to 1000 (100%).

### clutch starts engaging.

To conclude, the main effect of the final throttle position is appreciated in  $T_p$ , which increases with the parameter  $f_a$ . On the other hand, its influence in  $T_{max}$  and  $tim(T_{max})$  depends on the instant when the value for  $f_a$  is reached. Normally it will have no influence.



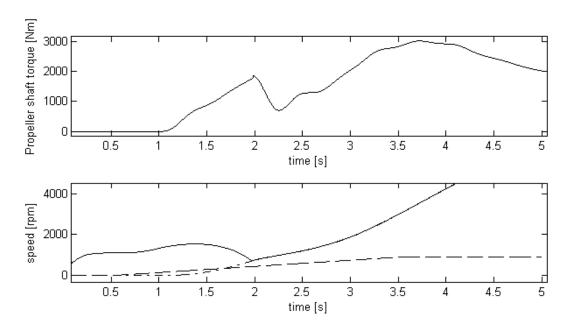


Figure 6.13: The plot above is the propeller shaft torque that the vehicle experiments for a final throttle position of 90%. In the graphic below, the engine speed is shown in solid, the transmission input speed in dot-dashed and the throttle position in dashed lines.



#### 6.4 Influence of the start times

In this section the influence of the times when the gas starts being pressed and the throttle starts being released in the propeller shaft torque is explained. As in the other cases, the effect of these parameters in the variables of maximum torque in the oscillation,  $T_{max}$ , time when this maximum torque takes place,  $tim(T_{max})$ , and torque in the propeller shaft,  $T_p$ ; will be described in detail.

Since it has been demonstrated in the first section of the chapter, the most important fact to describe the torque with this parameter is to observe the difference between the two start times, as it does not depend on each one separately, but in the interaction of them

Some measurements have been made so as to have an idea about what happens with the torque when this parameter is changed. For this reason, the other parameters have remained constant and the only parameter which has been changed in the simulation is the difference of times. As all the experiments behave in the same way, they are considered as valid. Consequently, an example of these measurements will be used in order to describe the variables.

The data are recollected in the table 9.4, which is presented in the chapter 9. With these measurements the following figures are obtained.

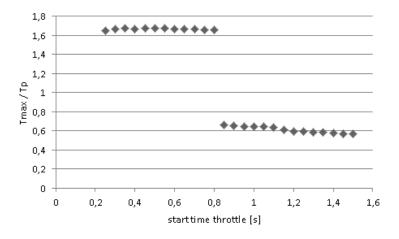


Figure 6.14: Maximum torque propeller shaft in the first oscillation divided per the final mean torque at the propeller shaf for the different values of the start time of the throttle, maintaining the other parameters constant.

As it can be observed in the measured data, the time difference between the clutch and the throttle has also a notable influence in the maximum torque of the first oscillation.



If the clutch starts to release at 1 s, the smaller the start time of the throttle is, the more difficult will be for the clutch to engage. The reason of this is that the limit of the acceleration pedal position will be reached earlier for small start times of the throttle. Therefore, when the clutch starts to have effect, the acceleration of the engine will be higher. Then, if the measurements are observed, it can be stated that until a throttle start time of 0.8 s the clutch disks are slipping for a longer time. When the throttle start time exceeds 0.8 s, the clutch disks are able to engage in a much less time, because now the acceleration is not so big.

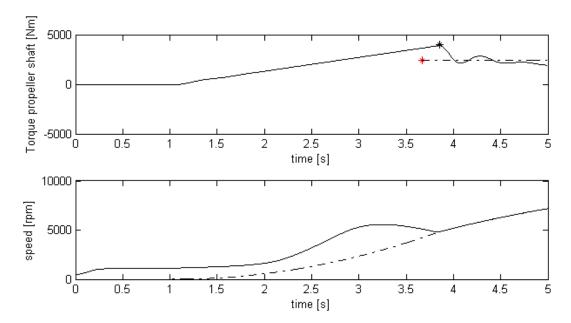


Figure 6.15: The plot above is the propeller shaft torque that the vehicle experiments for a throttle start time of 0.8 in solid. In dot dashed it is represented the mean torque the vehicle reach at the end,  $T_p$ . In the graphic below, the engine speed is shown in solid, the transmission input speed in dash-dotted and the throttle position in dashed lines.

The effect explained above is shown in the figures 6.15 and 6.16. In the first case the speeds of the transmission input and the engine match in the instant 3.87 s, while in the second case they match in 2.2 s.

In addition, if the measurement data are observed, minor changes are shown in the values  $T_{max}$ , but these ones are because of the acceleration intensity during the clutch engagement. If the throttle time is delayed enough with regard to the clutch start time, then the parameter of time will have effect only in the torque at the end of the simulation.

On the other hand,  $T_p$  variable must be also described. The effect of the start times in  $T_p$  is not as big as, for example, in the case of the throttle limit. But, if the start time of the



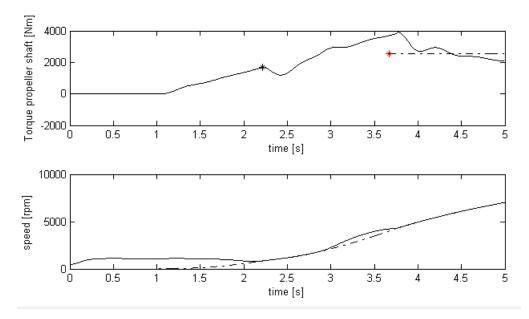


Figure 6.16: The plot above is the propeller shaft torque that the vehicle experiments for a throttle start time of 0.85 in solid. In dot dashed it is represented the mean torque the vehicle reach at the end,  $T_p$ . In the graphic below, the engine speed is shown in solid, the transmission input speed in dash-dotted and the throttle position in dashed lines.

throttle is much later than the start time of the clutch, then the torque at the end could be reached later because the limit of the throttle may be not reached yet. Nevertheless, if the start time of the throttle is soon enough, its final position is reached earlier. Consequently, the oscillation after  $T_{max}$  will be attenuated, since its low peak will not be so sharp.

As a result, it can be stated that the variables  $t_c$  and  $t_a$  influence together in the variables explained. While  $t_a$  is sooner enough than  $t_c$ , it will be more difficult to the transmission speed and the engine speed to match, so the clutch will be transmitting friction torque for a longer time. Consequently,  $T_{max}$  will rise and the  $tim(T_{max})$  will be higher. The same situation is presented for the influence of these parameters in  $T_p$ , the more the time of starting to press the throttle is, the later the  $T_p$  will be reached.



#### 6.5 Example of a Real Gear Shift

In this section, the behaviour of the torque in a gear shift and the influence of the parameters which have been explained along the thesis will be presented. First of all, and once all the characteristics of the vehicle simulated are adjusted, the parameters are settled for the real case. In this case, the gear shift chosen is produced at 1670 s in the measurements and has the following values for the parameters:

- $s_c = 0.35$
- $t_c = 1.42 \text{ s.}$
- $s_a = 0.28$
- $\bullet$   $t_a = 1 s$
- $f_s = 0.72$

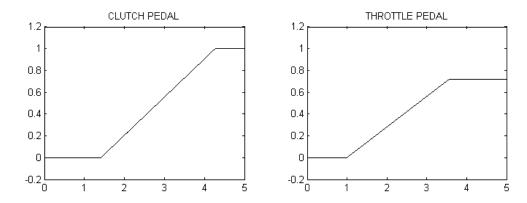


Figure 6.17: Representation of the clutch pedal and throttle parameters.

The torque at the wheels as well as the speeds for the engine and transmission input are represented in the figure 6.18.

As it can be observed, there are two areas with different behaviour. The first one is when the speeds of the transmission input and the engine do not match and it is from 0 s to 2.5 s approx. The other one is when the clutch disks are engaged and they move with the same velocity from 2.5 s to 5 s, the end of the simulation.

Firstly, the acceleration pedal starts to be pressed at 1s. It can be observed that the speed of the engine starts increasing a little by 1.2 seconds approximately. This is the effect due to the throttle, and as  $s_a$  is quite small, the increase of the velocity is slight too. As it has been demonstrated, the larger is the difference between the transmission input and the



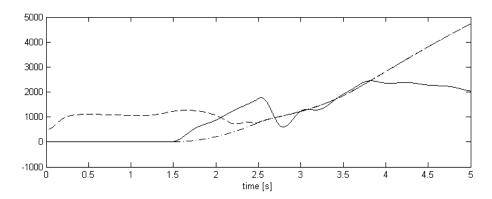


Figure 6.18: Propeller shaft torque [Nm] in solid line. Speed of the engine in dashed and speed of the transmission input in dash-dotted lines[rpm].

engine speeds, the bigger will be the time necessary for them to match.

After this, the clutch pedal starts to be released at 1.42 s. Because of this, the clutch disks start to transmit a friction torque which is proportional to the position of the pedal. This represents a fraction of the maximum normal force that can be actuating at the surfaces of the two clutch disks. This torque which is transmitted should be only proportional to the position of the pedal while the difference between the speeds of engine and transmission is still big enough. Therefore, the torque at the propeller shaft behaves approximately as a constant slope. Once the speeds match, the clutch does not transmit friction torque to the rest of the vehicle, but it will transmit the torque that has been generated by the engine. This is the point when the second area of behaviour which has been talked about starts.

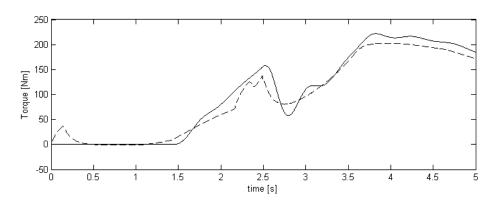


Figure 6.19: Torque transmitted through the clutch in solid and torque of the engine in dashed lines.

After 2.5 seconds, it can be seen that the torque transmitted through the clutch has approximately the same shape as the torque in the engine. The first thing which can be



observed are the oscillations that happen while the clutch torque is trying to equalize the torque in the engine. These oscillations are produced because of as abrupt effect of the torque decreasing and are modelled with the  $PT_2$  data for the dynamic system in Matlab/Simulink, which will be explained in the next chapter.

As the velocities of transmission input and engine have already matched, the clutch pedal slope has no more influence in the wheels torque. Consequently, all the effects produced will be because of the fact that the throttle is being used in a way or in another.

Since the gas pedal is still increasing its position, the effect of  $s_a$  is shown until 3.75 s approximately, which is the point where the position of the pedal starts remaining constant. What is more, an increase in the propeller shaft torque is observed until this point. This matches with the physical explanation, as the more the gas is being pressed, the more will be the torque at the engine while the vehicle is acelerating, and in this case also more torque at the propeller shaft.

Once the instant 3.75 s is reached, the throttle stops increasing and it remains constant until the end of the simulation. In this area  $f_a$  is actuating. As this parameter symbolizes the final pedal position, it is normal that the torque at the propeller shaft is more or less constant in the last area of our simulation.

To resume, the influence of the parameters in each area of the simulation is:

- 1. First area (0 to 1 s): the vehicle is stopped and no parameter is acting.
- 2. Second area (1 to 1.35 s):  $s_a$  is the parameter which makes the speed of the engine to increase.
- 3. Third area (1.2 to 2.5 s):  $s_c$  is the reason that the friction torque is transmitted through the clutch and  $s_a$  delays the moment when the speeds of the engine and transmission input match.
- 4. Forth area (2.5 to 3.75 s):  $s_a$  increases the torque at the propeller shaft proportionally to the slope value .
- 5. Fifth area (3.75 to 5 s):  $f_a$  is the parameter which maintains approximately constant the torque at the end of the simulation.



#### 6.6 Conclusions

For future development of systems in order to control the behaviour of the propeller shaft torque after a gear shift, it is important to recognise which parameters influence each part of the torque function. For this reason, a brief summary will be made using the variables explained above.

First of all, and except maybe particular cases, it can be assumed that  $T_p$  is only influenced by the final throttle position  $F_a$ , so its value will definitely depend only on this parameter for normal gear shifts.

More complicated will be the case of  $T_{max}$  and  $tim(T_{max})$ . They will depend on the combination of all the parameters without exception. The values of  $s_c$ ,  $s_a$ ,  $f_a$ , and the delay parameter of time will define the value of the maximum torque in the oscillation. Each case has been studied separately in the previous section.

Taking this into account, new studies about this particular behaviour of the torque after the gear shifting can be made, and consequently new systems in order to meet the customer expectations can be created.



# 7 Description of the Dynamic Behaviour of the Torque

Second order systems are often used in order to represent the exchange between mass and stiffness elements in mechanical systems. What is more, they are frequently used in the previous stages of design so as to establish the parameters of the energy storage and dissipation elements in order to achieve a satisfactory result.

In this power train model, a second order system is necessary to describe the oscillations which are produced in the torque due to the damping. As it has been presented in the previous description of the PT2 system, the oscillation system is characterized by decaying, growing or continuous oscillations. In this case, the oscillations will be decaying with time.

The main task of this section will be trying to describe the oscillations produced in the torque at the wheels when the clutch is being used, so that a relation between the torque and the parameters which the driver uses can be obtained.

First of all, an analysis of some responses of second order system will be made, so as to understand better the dynamics.

The first case which is going to be analysed is the step response for under damped systems. In the figure 7.1 an example of how will the system behave is presented.

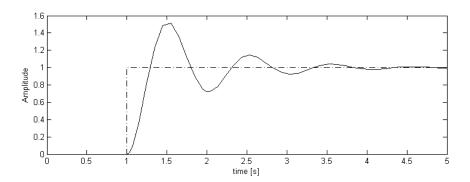


Figure 7.1: Step response of a second order system, for the underdamped case. In solid line it is represented the damping fuction and in dash-dotted it is represented the step fuction.



In this case the equation that must be solved is 7.1.

$$\frac{d^2y}{dt^2} + 2D\omega_0 \frac{dy}{dt} + \omega_0^2 y = u_s(t)$$
(7.1)

The solution to this equation is the sum of the homogeneous response and a particular solution.

$$y(t) = y_h(t) + y_p(t)$$
 (7.2)

$$= C_1 e^{-D\omega_0 + j\omega_0\sqrt{1 - D^2}} + C_2 e^{-D\omega_0 - j\omega_0\sqrt{1 - D^2}} + y_p(t)$$
(7.3)

At this point the constants must be chosen in order to satisfy the initial conditions and the particular solution also. For the example presented:

$$y(0) = C_1 + C_2 + \frac{1}{\omega_0^2} \tag{7.4}$$

$$\frac{dy}{dt}(t=0) = C_1(-D\omega_0 + j\omega_0\sqrt{1-D^2}) + C_2(-D\omega_0 - j\omega_0\sqrt{1-D^2})$$
 (7.5)

$$\omega_0^2 K = 1 \tag{7.6}$$

For the under damped case, the equation that governs the step response for the initial conditions of 0 is 7.7, which had already been presented in the chapter 5.

$$y(t) = \left[1 - \frac{1}{\sqrt{1 - D^2}} \cdot e^{-D\omega_0 \cdot t} \cdot \sin\left[\sqrt{1 - D^2}\omega_0 \cdot t + \arctan\frac{D}{\sqrt{1 - D^2}}\right]$$
 (7.7)

In addition, when the initial conditions are zero, some points from the damping function can be calculated easily, such as the maximum overshoot and the time when it occurs. The equations as well as the figure 7.2 are presented in the following lines.

$$maximum over shoot = e^{\frac{-D\pi}{\sqrt{1-D^2}}}$$
 (7.8)

$$peaktime = \frac{\pi}{\omega_0 \sqrt{1 - D^2}} \tag{7.9}$$

When the equation of the peak time is observed, it can be realised that the peak is reached in half a period when the system is oscillating with the damped natural frequency  $\omega_d = \omega_0 \sqrt{1 - D^2}$ . However, for this equation to have sense, the initial conditions must be



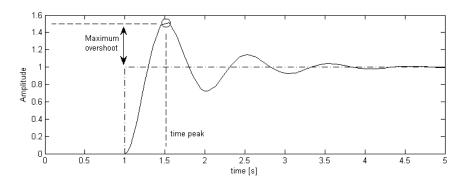


Figure 7.2: Step response of a second order system, for the underdamped case. In solid line it is represented the damping fuction and in dash-dotted it is represented the step fuction. The maximum overshoot and its time are also represented.

zero, as if not, the peak would be not reached in half a period ( $\pi$  at the numerator) [7].

Once the step response has been explained, it is interesting too the way in which the function behaves for a ramp response. This ramp response must be found by the integration of the step response. For initial conditions equal to zero, the equation which describes its behaviour is 7.10, which has been presented the chapter 5.

$$y_r(t) = t - \frac{2D}{\omega_n} \left[ \frac{2D}{\omega_n} \cos \omega_d t + \frac{2D^2 - 1}{\omega_d} \sin \omega_d t \right]$$
 (7.10)

In the figure 7.3 it can also be seen the way the oscillations behave.

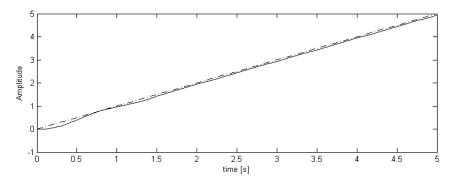


Figure 7.3: Ramp response of a second order system, for the underdamped case. In solid line it is represented the damping fuction and in dash-dotted it is represented the ramp fuction.

In the figure 7.3 it can be observed that the damping function oscillates around a line which is not the ramp function. The steady-state error which is committed for a ramp



response is described in the equation 7.11 and is a measurement of the system accuracy.

$$e_{ss} = \lim_{s \to 0} \frac{1}{sG(s)} = \frac{1}{K_v};$$
 (7.11)

where  $K_v = \lim_{s\to 0} sG(s)$  is defined as the velocity error constant. However, with the suppositions which will be made, this will not take so much importance [7].

Now the introduction of these ramp and step response has been made, in the next section a description of the wheels torque will be made using this information.



#### 7.1 Torque response of the PT2 system

In this section, the main task will be trying to describe the oscillations which are produced in the propeller shaft torque, and after that an approximation of the torque will be made in order to make easier the identification of the maximum torque in the first oscillation. At this point, a relation between the ramp and the maximum point at the first oscillation will be developed.

First of all an example of the torque at the clutch after the PT2 system is applied is presented in the figure 7.4.

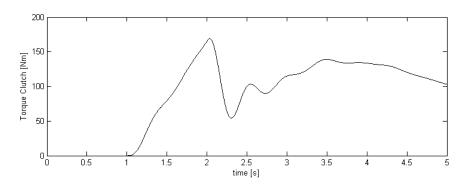


Figure 7.4: Torque at the transmission imput for the following values of driver parameters.  $s_c = 0.4, t_c = 1s, s_a = 0.35, t_a = 0.8$  and  $f_a = 0.4$ .

An approximation of the torque is made and it will be explained in the following lines. For this purpose, the figure 7.5 is represented.

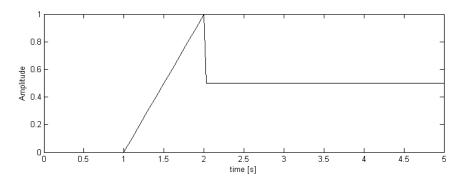


Figure 7.5: Approximation of the torque at the propeller shaft.

In the figure it can be observed how the torque function can be divided in two ways of behaviour. The first one is when the clutch is actuating and is the responsible for the ramp shape, until t=2s. The second part is when the clutch disks move at the same



velocity, and consequently the driver is trying to drive the vehicle with a constant torque, from t=2s to the end.

Regarding to this approximation, the behaviour of the damping oscillations can be explained with the equations that have been presented above. In the first part a ramp response can be observed, so it is necessary to apply the ramp response equation with initial conditions equal to zero.

In the second part of the damping function will be a step response for non-zero initial conditions, since at this point there is a slope and a position for the first point of the function. The equation which describes the behaviour of the step response for non-zero initial conditions is 7.12.

$$y(t) = e^{-D\omega_0 t} (A\cos(\omega_d t) + B\sin(\omega_d t)) + K$$
(7.12)

where A and B depend on the initial conditions and take the following form:

$$A = x(0) - K (7.13)$$

$$B = \frac{1}{\omega_d} (D\omega_0(x(0) - K) + \dot{x}(0)) \tag{7.14}$$

and K is the final constant value around which the damping function is going to oscillate. In the case presented above K=0.5

In the figure 7.6 it is represented the function with the corresponding damping oscillations.

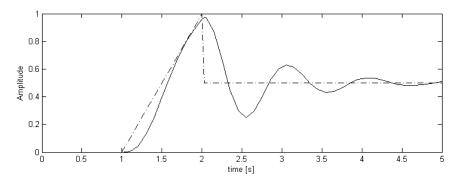


Figure 7.6: Representation of the torque approximation function in dash dotted line and the damping fuction in solid line. D=0.2 and  $\omega_0=2\pi$ .

If the values of time are introduced in the equations, it can be observed how the results match with the simulation.



However, this method is difficult to calculate and a relation between the maximum point at the oscillation and the slope cannot be easily made. For this reason, some simplifications are made in the model.

First of all, it is going to be considered that the system does not oscillate during the ramp, which is the period when the clutch is being released and the speeds of the transmission input and the speed of the engine do not match.

The second supposition is that for the initials conditions of the step response, the derivative initial condition of equation 7.14, is going to take the value of the slope of the torque, which will be proportional to the slope with which the clutch is released, regarding to the clutch parameters. Therefore, a relation between the maximum point at the first step oscillation and the slope of the clutch can be made.

In the figure 7.7 the same example but applying this suposition is presented.

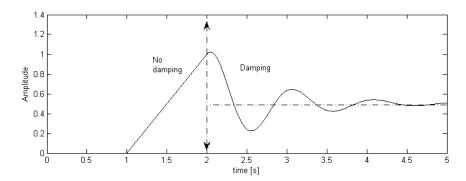


Figure 7.7: Representation of the torque approximation function in solid line, both the part which is damping and the one which is not. D = 0.2 and  $\omega_0 = 2\pi$ .

A comparison between the approximation with both the ramp and the step response, and the approximation with only the step without oscillations in the ramp is presented in figure 7.8 so as to check the error which is committed while doing this.

It can be observed that avoiding the oscillation at the ramp does not change very much the real result and it can simplify very much the calculations, as a relation between the clutch slope and the damping function can be stated. In the following lines it is explained the way of calculating the interesting points of this last function.

The problem now is that finding the maximum point of torque at the first oscillation is not going to be as easy as, for example in the step response case. In the step response the point when this maximum torque is reached is known, as it happens at half a period. In this case it is more difficult to detect the time, as the initial condition of position is not



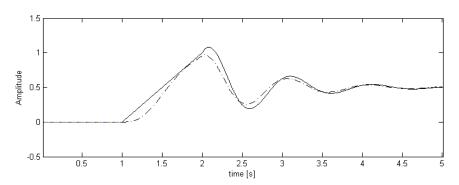


Figure 7.8: Representation of the torque approximation function with damping only in the second part of the fuction in solid line, and the torque approximation function with damping both in the ramp and in the step in dash dotted line. D=0.2 and  $\omega_0=2\pi$  for both cases.

zero and changes for each case. For this reason the way to find the maximum point is to derivate the function and make it equal to zero. This task will not be easy as the equation will have trigonometric expressions. However, it is a good way to relate the slope of the clutch to the maximum torque point.

As the function is oscillating and periodic, the first maximum must be selected because it will have an infinite number of them. What is more, it can be observed that the maximum torque will depend on the parameters of the second order equation, the damping ratio D and the natural frequency  $\omega_0$ .

When the initial position of the damping system changes with respect to the final torque constant value, the time in which the maximum torque is reached changes, so it cannot be stated that this is produced in a determined value of the period. On the other hand, it is indifferent for the time when the peak is reached to change the initial condition of slope. For example, if the maximum peak is reached at half a period for an initial slope of 1, it is going to be reached at half a period for an initial condition of slope of 0.5,2,3...



#### 7.2 Conclusions

The PT2 system will influence in the function of the torque by creating the oscillations of the system. The torque studied will respond to the under damped case, where the oscillations are attenuated with the time. In this system, there are two parameters which will control the second order system:  $\omega_0$ , which is natural frequency and is measured in rad/s, and D, the damping ratio.

The amplitude of the oscillations as well as the frequency with which they are produced will depend on these two parameters. The frequency of the second order under damped system will be  $\omega_d = \omega_0 \sqrt{1-D^2}$ . As it has been observed, the torque function is complicated, so it must be approximated in order to have an easier case and to try to make a relation between the parameters of the driver and the parameters of the damping. In this chapter some ideas for this purpose have been presented.



### 8 Conclusions

In this chapter the conclusions and the future work which must be done are presented. First of all, the conclusions of the master thesis are going to be stated.

The influence of the parameters of the driver in the torque at the propeller shaft are enumerated in the following lines:

- The torque which the driver is aimed to reach at the end, and consequently with which the car is going to be driven mostly depends on the final position of the throttle which will be remained constant.
- The maximum torque at the oscillation just after the clutch disks are engaged depends on all the parameters of the driver which have been studied in the thesis: the clutch slope, the time when it starts to be released, the throttle slope, also the time when it starts to be pressed, and the final position of the throttle.

The qualitative behaviour of the torque at the wheels will depend on the parameters of the driver and the PT system with which is excited. In this case, a second order system is used and for this reason the oscillations will behave in consequence.

The quantitative behaviour of the torque will depend on the parameters of the car and on the parameters of the second order system: the damping ratio and the natural frequency of the system. These parameters will be very important in order to describe the oscillations produced by the PT2 system.

The influence of these parameters in the behaviour of the torque has been presented some different sections of the thesis.

As the function of the torque is complicated, an approximation that describes the behaviour has been made and so that a relation between the clutch slope and the parameters of the second order system is developed. The relation will lie on the initial conditions of the equation that describes the oscillations, where the torque slope is introduced and it is proportional to the clutch ramp.

Once this relation is stated, no matters how the parameters of the damping are changed (while they are used under the limits of under damping), or the parameters of the car are



changed too, because the relation will work for all of them.

#### 8.1 Future Work

Once this point is reached, the task is to implement this information in order to build a model which completes the one which has been built in Simulink.

With this new model the objective is that, when a final torque at the propeller shaft is introduced, the final position of the throttle can be found. What is more, a factor that multiplies the final torque at the propeller shaft will symbolize the maximum torque which is reached in the oscillation. With this factor and the help of the parameters of damping the other parameters which the driver controls can be calculated.

To summarize, in the system which must be developed, the final torque that is reached at the wheels and the factor which has been explained, are introduced and will be the inputs, and the outputs will be the value of all the parameters which are needed in order to get the value of these points at the torque.

This will be very useful in order to be able to describe the behaviour of the torque during the gear shifting, and how the different profiles of drivers from the 3D parameter space behave in each situation.

It is important that the model works for different data. Consequently the relations which will be found must be related to the parameters of damping, so when they are changed, the model will work.

A good strategy to follow is to define the final torque at the propeller shaft with the final throttle position, and the clutch slope directly with the factor (maximum torque at the oscillation). Then with this data all the other parameters can be calculated. However, this is only an idea, but the way to find the solution will have probably multiple paths to follow.



## 9 Appendix

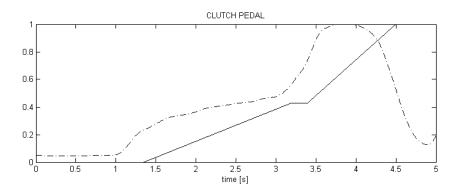


Figure 9.1: Representation of the evolution of the clutch pedal along the time. In solid line it is represented the simulation and in dash-dotted it is represented the real data of the gear shift chosen. Number 1 of y label represents the 100% of the clutch pedal released while 0 represents the pedal completely pressed.

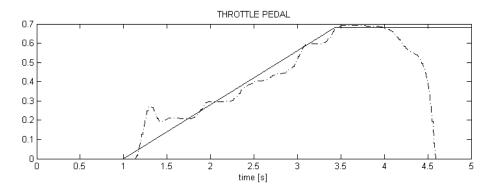


Figure 9.2: Representation of the evolution of the throttle along the time. In solid line it is represented the simulation and in dash-dotted it is represented the real data of the gear shift chosen. In this case, number 1 of y label represents the 100% of the throttle pressed and 0 the customer does not press the pedal.



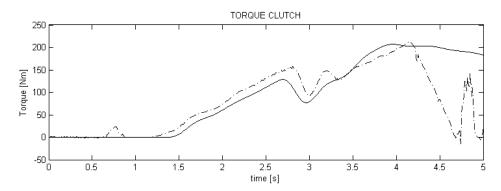


Figure 9.3: Evolution of the torque transmitted through the clutch. The torque simulated is represented in solid line and the real data are represented in dash-dotted line.

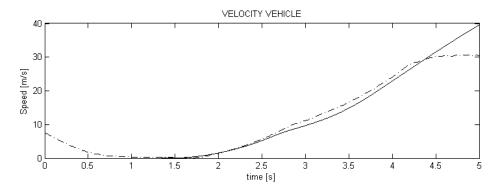


Figure 9.4: Evolution of the speed of the vehicle. The simulated speed is represented in solid line and the real data are represented in dash-dotted line.

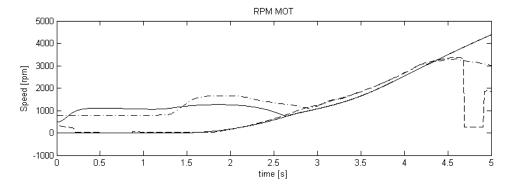


Figure 9.5: Evolution speeds of the transmission input and the speed of the engine. The simulated velocities are represented in solid (transmission input) and dashed (engine) lines and the real data are represented in dash-dotted line.



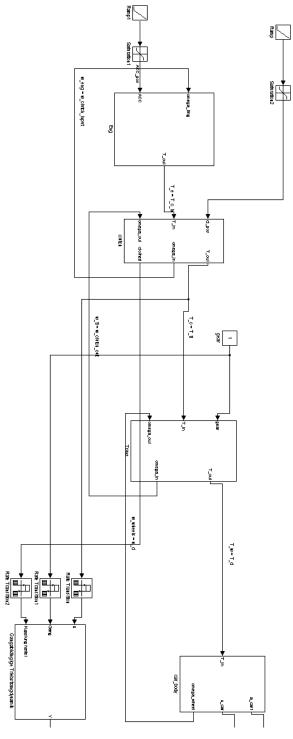


Figure 9.6: Representation of the Simulink top level diagram of the complete powertrain model with all the elements connected



Table 9.1: Measurements for different values of the clutch slope from 0.25 to 0.6.  $s_a$  = 2,  $t_a$  = 0.25,  $t_s$  = 1 and  $f_a$  = 0.4.

$s_c$	$T_{max}$ (Nm)	$tim(T_{max})$ (s)	$T_p$ (Nm)
0,25	2554,49506	3,116616893	1064,98513
0,26	2590,48759	3,061437256	1074,86815
0,27	2628,02401	3,011374398	1086,82879
0,28	2652,41636	2,962391934	1099,98133
0,29	2703,87493	2,920647175	1111,28213
0,3	2712,92342	2,875311579	1111,44498
0,31	2743,05935	2,835230792	1110,85984
0,32	2794,09794	2,8	1109,77808
0,33	2814,81449	2,761264094	1107,48676
0,34	2838,31247	2,72683954	1107,60369
0,35	2860,41867	2,694740385	1105,85884
0,36	2891,85808	2,662986852	1106,51526
0,37	2917,03126	2,631996967	1106,29124
0,38	2938,74599	2,604312499	1108,34478
0,39	2996,31066	2,580853396	1106,38491
0,4	3002,27953	2,55114764	1115,47975
0,41	3017,01263	2,526390324	1123,00566
0,42	3047,38786	2,502557783	1122,24736
0,43	3087,51848	2,48	1121,37267
0,44	3102,16729	2,46	1119,88828
0,45	3110,70592	2,434400247	1121,2461
0,46	3134,79903	2,413808088	1121,10888
0,47	3158,89478	2,394903244	1120,56749
0,48	3183,5906	2,375511207	1121,37166
0,49	3234,57983	2,36	1120,70927
0,5	3257,29239	2,34218695	1119,9015
0,51	3263,17011	2,322269142	1120,33002
0,52	3273,70856	2,305139769	1119,31096
0,53	3326,44362	2,292412126	1115,06245
0,54	3322,47457	2,273597201	1109,72506
0,55	3375,57728	2,261539448	1108,45117
0,56	3363,02906	2,243443456	1111,42024
0,57	3409,43065	2,231393582	1109,20683
0,58	3396,93073	2,215788508	1110,51476
0,59	3428,88142	2,201841026	1112,12404
0,6	3470,96618	2,193453986	1110,52108



Table 9.2: Measurements for different values of the throttle slope from 0.25 to 0.6.  $s_c$  = 0.4,  $t_a$  = 0.25,  $t_s$  = 1 and  $f_a$  = 0.8.

sc         Imax (Nm)         tim(Imax) (s)         Ip (Nm)           0,25         2685,0709         2,401752079         2254,64806           0,26         2824,94963         2,48172749         2223,27456           0,27         2969,6637         2,563789559         2208,44478           0,28         3097,60831         2,640732543         2197,09839           0,29         3265,67013         2,724457818         2121,60377           0,3         3418,62969         2,804199097         2078,2532           0,31         3541,66099         2,874752853         2056,01173           0,32         3679,80162         2,944431398         2009,87641           0,33         3806,13232         3,003438034         1970,14302           0,34         3913,45778         3,085897896         1915,11016           0,36         4015,87819         3,103165754         1900,65946           0,37         4035,96327         3,112360364         1894,63646           0,38         4063,50519         3,121038421         1889,45691           0,39         4049,7451         3,125107243         1888,9829           0,4         4083,26178         3,131054803         1879,82281           0,41         <		/NT \	1: (T)	/T /NT \
0,26         2824,94963         2,48172749         2223,27456           0,27         2969,6637         2,563789559         2208,44478           0,28         3097,60831         2,640732543         2197,09839           0,29         3265,67013         2,724457818         2121,60377           0,3         3418,62969         2,804199097         2078,2532           0,31         3541,66099         2,874752853         2056,01173           0,32         3679,80162         2,944431398         2009,87641           0,33         3806,13232         3,003438034         1970,14302           0,34         3913,45778         3,0532201         1938,49023           0,35         3965,47875         3,085897896         1915,11016           0,36         4015,87819         3,103165754         1900,65946           0,37         4035,96327         3,112360364         1894,63646           0,38         4063,50519         3,121038421         1889,45691           0,39         4049,7451         3,125107243         1888,9829           0,4         4083,26178         3,131054803         1879,82281           0,41         4073,38176         3,13336698         1881,009           0,43         <		$T_{max}$ (Nm)	$tim(T_{max})$ (s)	$T_p$ (Nm)
0,27         2969,6637         2,563789559         2208,44478           0,28         3097,60831         2,640732543         2197,09839           0,29         3265,67013         2,724457818         2121,60377           0,3         3418,62969         2,804199097         2078,2532           0,31         3541,66099         2,874752853         2056,01173           0,32         3679,80162         2,944431398         2009,87641           0,33         3806,13232         3,003438034         1970,14302           0,34         3913,45778         3,0532201         1938,49023           0,35         3965,47875         3,085897896         1915,11016           0,36         4015,87819         3,103165754         1900,65946           0,37         4035,96327         3,112360364         1894,63646           0,38         4063,50519         3,121038421         1889,45691           0,39         4049,7451         3,125107243         1888,9829           0,4         4083,26178         3,131054803         1879,82281           0,41         4073,38176         3,133366698         1880,76662           0,42         4072,57923         3,136023989         1881,009           0,43				
0,28         3097,60831         2,640732543         2197,09839           0,29         3265,67013         2,724457818         2121,60377           0,3         3418,62969         2,804199097         2078,2532           0,31         3541,66099         2,874752853         2056,01173           0,32         3679,80162         2,944431398         2009,87641           0,33         3806,13232         3,003438034         1970,14302           0,34         3913,45778         3,0532201         1938,49023           0,35         3965,47875         3,085897896         1915,11016           0,36         4015,87819         3,103165754         1900,65946           0,37         4035,96327         3,112360364         1894,63646           0,38         4063,50519         3,121038421         1889,45691           0,39         4049,7451         3,125107243         1888,9829           0,4         4083,26178         3,131054803         1879,82281           0,41         4073,38176         3,133366698         1881,009           0,43         4105,06903         3,14         1876,05455           0,44         401,39199         3,142397426         1873,51371           0,45         40	· ·	·	,	
0,29         3265,67013         2,724457818         2121,60377           0,3         3418,62969         2,804199097         2078,2532           0,31         3541,66099         2,874752853         2056,01173           0,32         3679,80162         2,944431398         2009,87641           0,33         3806,13232         3,003438034         1970,14302           0,34         3913,45778         3,0532201         1938,49023           0,35         3965,47875         3,085897896         1915,11016           0,36         4015,87819         3,103165754         1900,65946           0,37         4035,96327         3,112360364         1894,63646           0,38         4063,50519         3,121038421         1889,45691           0,39         4049,7451         3,125107243         1888,9829           0,4         4083,26178         3,133366698         1880,76662           0,42         4072,57923         3,136023989         1881,009           0,43         4105,06903         3,14         1876,05455           0,44         401,39199         3,142397426         1873,51371           0,45         4096,50052         3,143570162         1873,69653           0,46         40	<u> </u>	2969,6637	2,563789559	2208,44478
0,3         3418,62969         2,804199097         2078,2532           0,31         3541,66099         2,874752853         2056,01173           0,32         3679,80162         2,944431398         2009,87641           0,33         3806,13232         3,003438034         1970,14302           0,34         3913,45778         3,0532201         1938,49023           0,35         3965,47875         3,085897896         1915,11016           0,36         4015,87819         3,103165754         1900,65946           0,37         4035,96327         3,112360364         1894,63646           0,38         4063,50519         3,121038421         1889,45691           0,39         4049,7451         3,125107243         1888,9829           0,4         4083,26178         3,131054803         1879,82281           0,41         4073,38176         3,133366698         1880,76662           0,42         4072,57923         3,136023989         1881,009           0,43         4105,06903         3,14         1876,05455           0,44         4096,50052         3,143570162         1873,69653           0,45         4096,50052         3,143388783         1871,24373           0,47         4	0,28	3097,60831	2,640732543	2197,09839
0,31         3541,66099         2,874752853         2056,01173           0,32         3679,80162         2,944431398         2009,87641           0,33         3806,13232         3,003438034         1970,14302           0,34         3913,45778         3,0532201         1938,49023           0,35         3965,47875         3,085897896         1915,11016           0,36         4015,87819         3,103165754         1900,65946           0,37         4035,96327         3,112360364         1894,63646           0,38         4063,50519         3,121038421         1889,45691           0,39         4049,7451         3,125107243         1888,9829           0,4         4083,26178         3,131054803         1879,82281           0,41         4073,38176         3,133366698         1881,009           0,42         4072,57923         3,136023989         1881,009           0,43         4105,06903         3,14         1876,05455           0,44         4101,39199         3,142397426         1873,51371           0,45         4096,50052         3,143570162         1873,69653           0,46         4092,59554         3,143388783         1871,24373           0,47         4		3265,67013	2,724457818	2121,60377
0,32         3679,80162         2,944431398         2009,87641           0,33         3806,13232         3,003438034         1970,14302           0,34         3913,45778         3,0532201         1938,49023           0,35         3965,47875         3,085897896         1915,11016           0,36         4015,87819         3,103165754         1900,65946           0,37         4035,96327         3,112360364         1894,63646           0,38         4063,50519         3,121038421         1889,45691           0,39         4049,7451         3,125107243         1888,9829           0,4         4083,26178         3,131054803         1879,82281           0,41         4073,38176         3,133366698         1880,76662           0,42         4072,57923         3,136023989         1881,009           0,43         4105,06903         3,14         1876,05455           0,44         4101,39199         3,142397426         1873,51371           0,45         4096,50052         3,143570162         1873,69653           0,46         4092,59554         3,143388783         1871,24373           0,47         4091,76722         3,144382875         1876,45386           0,48 <td< td=""><td>0,3</td><td>3418,62969</td><td>2,804199097</td><td>2078,2532</td></td<>	0,3	3418,62969	2,804199097	2078,2532
0,33         3806,13232         3,003438034         1970,14302           0,34         3913,45778         3,0532201         1938,49023           0,35         3965,47875         3,085897896         1915,11016           0,36         4015,87819         3,103165754         1900,65946           0,37         4035,96327         3,112360364         1894,63646           0,38         4063,50519         3,121038421         1889,45691           0,39         4049,7451         3,125107243         1888,9829           0,4         4083,26178         3,131054803         1879,82281           0,41         4073,38176         3,133366698         1880,76662           0,42         4072,57923         3,136023989         1881,009           0,43         4105,06903         3,14         1876,05455           0,44         4101,39199         3,142397426         1873,51371           0,45         4096,50052         3,143570162         1873,69653           0,46         4092,59554         3,143388783         1871,24373           0,47         4091,76722         3,144382875         1876,45386           0,48         4095,94167         3,146130565         1877,94619           0,5	0,31	3541,66099	,	2056,01173
0,34         3913,45778         3,0532201         1938,49023           0,35         3965,47875         3,085897896         1915,11016           0,36         4015,87819         3,103165754         1900,65946           0,37         4035,96327         3,112360364         1894,63646           0,38         4063,50519         3,121038421         1889,45691           0,39         4049,7451         3,125107243         1888,9829           0,4         4083,26178         3,131054803         1879,82281           0,41         4073,38176         3,133366698         1880,76662           0,42         4072,57923         3,136023989         1881,009           0,43         4105,06903         3,14         1876,05455           0,44         4101,39199         3,142397426         1873,51371           0,45         4096,50052         3,143570162         1873,69653           0,46         4092,59554         3,143388783         1871,24373           0,47         4091,76722         3,144382875         1876,45386           0,48         4095,94167         3,146130565         1877,94619           0,5         4127,69402         3,15         1867,06164           0,51         4125,72	0,32	3679,80162	2,944431398	2009,87641
0,35         3965,47875         3,085897896         1915,11016           0,36         4015,87819         3,103165754         1900,65946           0,37         4035,96327         3,112360364         1894,63646           0,38         4063,50519         3,121038421         1889,45691           0,39         4049,7451         3,125107243         1888,9829           0,4         4083,26178         3,131054803         1879,82281           0,41         4073,38176         3,133366698         1880,76662           0,42         4072,57923         3,136023989         1881,009           0,43         4105,06903         3,14         1876,05455           0,44         4101,39199         3,142397426         1873,51371           0,45         4096,50052         3,143570162         1873,69653           0,46         4092,59554         3,143388783         1871,24373           0,47         4091,76722         3,144382875         1876,45386           0,48         4095,94167         3,146130565         1877,94619           0,49         4112,00742         3,15         1867,06164           0,51         4125,72789         3,150609171         1867,69837           0,52         4123	0,33	3806,13232	3,003438034	1970,14302
0,36         4015,87819         3,103165754         1900,65946           0,37         4035,96327         3,112360364         1894,63646           0,38         4063,50519         3,121038421         1889,45691           0,39         4049,7451         3,125107243         1888,9829           0,4         4083,26178         3,131054803         1879,82281           0,41         4073,38176         3,133366698         1880,76662           0,42         4072,57923         3,136023989         1881,009           0,43         4105,06903         3,14         1876,05455           0,44         4101,39199         3,142397426         1873,51371           0,45         4096,50052         3,143570162         1873,69653           0,46         4092,59554         3,143388783         1871,24373           0,47         4091,76722         3,144382875         1876,45386           0,48         4095,94167         3,146130565         1877,94619           0,49         4112,00742         3,15         1867,06164           0,51         4125,72789         3,150609171         1867,89837           0,52         4123,65171         3,152593261         1867,14607           0,53         4117	0,34	3913,45778	3,0532201	1938,49023
0,37         4035,96327         3,112360364         1894,63646           0,38         4063,50519         3,121038421         1889,45691           0,39         4049,7451         3,125107243         1888,9829           0,4         4083,26178         3,131054803         1879,82281           0,41         4073,38176         3,133366698         1880,76662           0,42         4072,57923         3,136023989         1881,009           0,43         4105,06903         3,14         1876,05455           0,44         4101,39199         3,142397426         1873,51371           0,45         4096,50052         3,143388783         1871,24373           0,46         4092,59554         3,143388783         1871,24373           0,47         4091,76722         3,144382875         1876,45386           0,48         4095,94167         3,146130565         1877,94619           0,49         4112,00742         3,15         1867,06164           0,51         4125,72789         3,150609171         1867,69837           0,52         4123,65171         3,152593261         1867,14607           0,53         4120,37002         3,153367378         1868,1289           0,55         4115,	0,35	3965,47875	3,085897896	1915,11016
0,38         4063,50519         3,121038421         1889,45691           0,39         4049,7451         3,125107243         1888,9829           0,4         4083,26178         3,131054803         1879,82281           0,41         4073,38176         3,133366698         1880,76662           0,42         4072,57923         3,136023989         1881,009           0,43         4105,06903         3,14         1876,05455           0,44         4101,39199         3,142397426         1873,51371           0,45         4096,50052         3,143570162         1873,69653           0,46         4092,59554         3,143388783         1871,24373           0,47         4091,76722         3,144382875         1876,45386           0,48         4095,94167         3,146130565         1877,94619           0,49         4112,00742         3,15         1867,06164           0,51         4127,69402         3,15         1867,06164           0,51         4125,72789         3,150609171         1867,69837           0,52         4123,65171         3,152593261         1867,14607           0,53         4120,37002         3,152505446         1866,43175           0,54         4117,64 <td>0,36</td> <td>4015,87819</td> <td>3,103165754</td> <td>1900,65946</td>	0,36	4015,87819	3,103165754	1900,65946
0,39         4049,7451         3,125107243         1888,9829           0,4         4083,26178         3,131054803         1879,82281           0,41         4073,38176         3,133366698         1880,76662           0,42         4072,57923         3,136023989         1881,009           0,43         4105,06903         3,14         1876,05455           0,44         4101,39199         3,142397426         1873,51371           0,45         4096,50052         3,143570162         1873,69653           0,46         4092,59554         3,143388783         1871,24373           0,47         4091,76722         3,144382875         1876,45386           0,48         4095,94167         3,146130565         1877,94619           0,49         4112,00742         3,15         1867,06164           0,51         4125,72789         3,150609171         1867,69837           0,52         4123,65171         3,152593261         1867,14607           0,53         4120,37002         3,152505446         1866,43175           0,54         4117,64         3,153367378         1868,1289           0,55         4115,25732         3,154154859         1866,48344           0,56         4112,220	0,37	4035,96327	3,112360364	1894,63646
0,4         4083,26178         3,131054803         1879,82281           0,41         4073,38176         3,133366698         1880,76662           0,42         4072,57923         3,136023989         1881,009           0,43         4105,06903         3,14         1876,05455           0,44         4101,39199         3,142397426         1873,51371           0,45         4096,50052         3,143570162         1873,69653           0,46         4092,59554         3,143388783         1871,24373           0,47         4091,76722         3,144382875         1876,45386           0,48         4095,94167         3,146130565         1877,94619           0,49         4112,00742         3,15         1867,06164           0,51         4127,69402         3,15         1867,06164           0,51         4125,72789         3,150609171         1867,69837           0,52         4123,65171         3,152593261         1867,14607           0,53         4120,37002         3,152505446         1866,43175           0,54         4117,64         3,153367378         1868,1289           0,55         4115,25732         3,154154859         1866,48344           0,56         4112,22071 <td>0,38</td> <td>4063,50519</td> <td>3,121038421</td> <td>1889,45691</td>	0,38	4063,50519	3,121038421	1889,45691
0,41         4073,38176         3,133366698         1880,76662           0,42         4072,57923         3,136023989         1881,009           0,43         4105,06903         3,14         1876,05455           0,44         4101,39199         3,142397426         1873,51371           0,45         4096,50052         3,143570162         1873,69653           0,46         4092,59554         3,143388783         1871,24373           0,47         4091,76722         3,144382875         1876,45386           0,48         4095,94167         3,146130565         1877,94619           0,49         4112,00742         3,15         1881,801           0,5         4127,69402         3,15         1867,06164           0,51         4125,72789         3,150609171         1867,69837           0,52         4123,65171         3,152593261         1867,14607           0,53         4120,37002         3,152505446         1866,43175           0,54         4117,64         3,153367378         1868,1289           0,55         4115,25732         3,154154859         1866,48344           0,56         4112,22071         3,153150192         1867,38929           0,57         4114,30763	0,39	4049,7451	3,125107243	1888,9829
0,42         4072,57923         3,136023989         1881,009           0,43         4105,06903         3,14         1876,05455           0,44         4101,39199         3,142397426         1873,51371           0,45         4096,50052         3,143570162         1873,69653           0,46         4092,59554         3,143388783         1871,24373           0,47         4091,76722         3,144382875         1876,45386           0,48         4095,94167         3,146130565         1877,94619           0,49         4112,00742         3,15         1881,801           0,5         4127,69402         3,15         1867,06164           0,51         4125,72789         3,150609171         1867,69837           0,52         4123,65171         3,152593261         1867,14607           0,53         4120,37002         3,152505446         1866,43175           0,54         4117,64         3,153367378         1868,1289           0,55         4115,25732         3,154154859         1866,48344           0,56         4112,22071         3,153150192         1867,38929           0,57         4114,30763         3,156378244         1874,08166           0,59         4136,99715	0,4	4083,26178	3,131054803	1879,82281
0,43         4105,06903         3,14         1876,05455           0,44         4101,39199         3,142397426         1873,51371           0,45         4096,50052         3,143570162         1873,69653           0,46         4092,59554         3,143388783         1871,24373           0,47         4091,76722         3,144382875         1876,45386           0,48         4095,94167         3,146130565         1877,94619           0,49         4112,00742         3,15         1867,06164           0,5         4127,69402         3,15         1867,06164           0,51         4125,72789         3,150609171         1867,69837           0,52         4123,65171         3,152593261         1867,14607           0,53         4120,37002         3,152505446         1866,43175           0,54         4117,64         3,153367378         1868,1289           0,55         4115,25732         3,154154859         1866,48344           0,56         4112,22071         3,153150192         1867,38929           0,57         4114,30763         3,155373001         1864,50736           0,58         4118,63574         3,158767904         1869,68939	0,41	4073,38176	3,133366698	1880,76662
0,44         4101,39199         3,142397426         1873,51371           0,45         4096,50052         3,143570162         1873,69653           0,46         4092,59554         3,143388783         1871,24373           0,47         4091,76722         3,144382875         1876,45386           0,48         4095,94167         3,146130565         1877,94619           0,49         4112,00742         3,15         1881,801           0,5         4127,69402         3,15         1867,06164           0,51         4125,72789         3,150609171         1867,69837           0,52         4123,65171         3,152593261         1867,14607           0,53         4120,37002         3,152505446         1866,43175           0,54         4117,64         3,153367378         1868,1289           0,55         4115,25732         3,154154859         1866,48344           0,56         4112,22071         3,153150192         1867,38929           0,57         4114,30763         3,155373001         1864,50736           0,58         4118,63574         3,158767904         1869,68939	0,42	4072,57923	3,136023989	1881,009
0,45         4096,50052         3,143570162         1873,69653           0,46         4092,59554         3,143388783         1871,24373           0,47         4091,76722         3,144382875         1876,45386           0,48         4095,94167         3,146130565         1877,94619           0,49         4112,00742         3,15         1881,801           0,5         4127,69402         3,15         1867,06164           0,51         4125,72789         3,150609171         1867,69837           0,52         4123,65171         3,152593261         1867,14607           0,53         4120,37002         3,152505446         1866,43175           0,54         4117,64         3,153367378         1868,1289           0,55         4115,25732         3,154154859         1866,48344           0,56         4112,22071         3,153150192         1867,38929           0,57         4114,30763         3,155373001         1864,50736           0,58         4118,63574         3,158767904         1869,68939	0,43	4105,06903	3,14	1876,05455
0,46         4092,59554         3,143388783         1871,24373           0,47         4091,76722         3,144382875         1876,45386           0,48         4095,94167         3,146130565         1877,94619           0,49         4112,00742         3,15         1881,801           0,5         4127,69402         3,15         1867,06164           0,51         4125,72789         3,150609171         1867,69837           0,52         4123,65171         3,152593261         1867,14607           0,53         4120,37002         3,152505446         1866,43175           0,54         4117,64         3,153367378         1868,1289           0,55         4115,25732         3,154154859         1866,48344           0,56         4112,22071         3,153150192         1867,38929           0,57         4114,30763         3,155373001         1864,50736           0,58         4118,63574         3,158767904         1869,68939	0,44	4101,39199	3,142397426	1873,51371
0,47         4091,76722         3,144382875         1876,45386           0,48         4095,94167         3,146130565         1877,94619           0,49         4112,00742         3,15         1881,801           0,5         4127,69402         3,15         1867,06164           0,51         4125,72789         3,150609171         1867,69837           0,52         4123,65171         3,152593261         1867,14607           0,53         4120,37002         3,152505446         1866,43175           0,54         4117,64         3,153367378         1868,1289           0,55         4115,25732         3,154154859         1866,48344           0,56         4112,22071         3,153150192         1867,38929           0,57         4114,30763         3,155373001         1864,50736           0,58         4118,63574         3,156378244         1874,08166           0,59         4136,99715         3,158767904         1869,68939	0,45	4096,50052	3,143570162	1873,69653
0,48         4095,94167         3,146130565         1877,94619           0,49         4112,00742         3,15         1881,801           0,5         4127,69402         3,15         1867,06164           0,51         4125,72789         3,150609171         1867,69837           0,52         4123,65171         3,152593261         1867,14607           0,53         4120,37002         3,152505446         1866,43175           0,54         4117,64         3,153367378         1868,1289           0,55         4115,25732         3,154154859         1866,48344           0,56         4112,22071         3,153150192         1867,38929           0,57         4114,30763         3,155373001         1864,50736           0,58         4118,63574         3,156378244         1874,08166           0,59         4136,99715         3,158767904         1869,68939	0,46	4092,59554	3,143388783	1871,24373
0,49         4112,00742         3,15         1881,801           0,5         4127,69402         3,15         1867,06164           0,51         4125,72789         3,150609171         1867,69837           0,52         4123,65171         3,152593261         1867,14607           0,53         4120,37002         3,152505446         1866,43175           0,54         4117,64         3,153367378         1868,1289           0,55         4115,25732         3,154154859         1866,48344           0,56         4112,22071         3,153150192         1867,38929           0,57         4114,30763         3,155373001         1864,50736           0,58         4118,63574         3,156378244         1874,08166           0,59         4136,99715         3,158767904         1869,68939	0,47	4091,76722	3,144382875	1876,45386
0,5         4127,69402         3,15         1867,06164           0,51         4125,72789         3,150609171         1867,69837           0,52         4123,65171         3,152593261         1867,14607           0,53         4120,37002         3,152505446         1866,43175           0,54         4117,64         3,153367378         1868,1289           0,55         4115,25732         3,154154859         1866,48344           0,56         4112,22071         3,153150192         1867,38929           0,57         4114,30763         3,155373001         1864,50736           0,58         4118,63574         3,156378244         1874,08166           0,59         4136,99715         3,158767904         1869,68939	0,48	4095,94167	3,146130565	1877,94619
0,51       4125,72789       3,150609171       1867,69837         0,52       4123,65171       3,152593261       1867,14607         0,53       4120,37002       3,152505446       1866,43175         0,54       4117,64       3,153367378       1868,1289         0,55       4115,25732       3,154154859       1866,48344         0,56       4112,22071       3,153150192       1867,38929         0,57       4114,30763       3,155373001       1864,50736         0,58       4118,63574       3,156378244       1874,08166         0,59       4136,99715       3,158767904       1869,68939	0,49	4112,00742	3,15	1881,801
0,52       4123,65171       3,152593261       1867,14607         0,53       4120,37002       3,152505446       1866,43175         0,54       4117,64       3,153367378       1868,1289         0,55       4115,25732       3,154154859       1866,48344         0,56       4112,22071       3,153150192       1867,38929         0,57       4114,30763       3,155373001       1864,50736         0,58       4118,63574       3,156378244       1874,08166         0,59       4136,99715       3,158767904       1869,68939	0,5	4127,69402	3,15	1867,06164
0,53       4120,37002       3,152505446       1866,43175         0,54       4117,64       3,153367378       1868,1289         0,55       4115,25732       3,154154859       1866,48344         0,56       4112,22071       3,153150192       1867,38929         0,57       4114,30763       3,155373001       1864,50736         0,58       4118,63574       3,156378244       1874,08166         0,59       4136,99715       3,158767904       1869,68939	0,51	4125,72789	3,150609171	1867,69837
0,54       4117,64       3,153367378       1868,1289         0,55       4115,25732       3,154154859       1866,48344         0,56       4112,22071       3,153150192       1867,38929         0,57       4114,30763       3,155373001       1864,50736         0,58       4118,63574       3,156378244       1874,08166         0,59       4136,99715       3,158767904       1869,68939	0,52	4123,65171	3,152593261	1867,14607
0,55       4115,25732       3,154154859       1866,48344         0,56       4112,22071       3,153150192       1867,38929         0,57       4114,30763       3,155373001       1864,50736         0,58       4118,63574       3,156378244       1874,08166         0,59       4136,99715       3,158767904       1869,68939	0,53	4120,37002	3,152505446	1866,43175
0,56       4112,22071       3,153150192       1867,38929         0,57       4114,30763       3,155373001       1864,50736         0,58       4118,63574       3,156378244       1874,08166         0,59       4136,99715       3,158767904       1869,68939	0,54	4117,64	3,153367378	1868,1289
0,57       4114,30763       3,155373001       1864,50736         0,58       4118,63574       3,156378244       1874,08166         0,59       4136,99715       3,158767904       1869,68939	0,55	4115,25732	3,154154859	1866,48344
0,58       4118,63574       3,156378244       1874,08166         0,59       4136,99715       3,158767904       1869,68939	0,56	4112,22071	3,153150192	1867,38929
0,59 4136,99715 3,158767904 1869,68939	0,57	4114,30763	3,155373001	1864,50736
0,59 4136,99715 3,158767904 1869,68939	0,58	4118,63574	3,156378244	1874,08166
0,6 4151,19979 3,161189609 1862,32927		4136,99715	3,158767904	1869,68939
	0,6	4151,19979	3,161189609	1862,32927



Table 9.3: Measurements maintaining  $s_c$  = 0.35,  $s_a$  = 0.35,  $t_a$  = 1 and  $t_c$  = 1.25 constant.  $f_a$  varies from 0 to 1 in 0.05.

111 0 00	0 10 1 111 0.031				
$f_a$	$T_{max}$ (Nm)	$tim(T_{max})$ (s)	$T_p$ (Nm)		
0,05	1590,62878	2,225436794	87,9382104		
0,1	1641,6486	2,258635234	193,678306		
0,15	1674,81997	2,282770823	326,417638		
0,2	1716,79709	2,314801051	490,983557		
0,25	1753,56587	2,343340361	684,090409		
0,3	1795,53519	2,362073829	883,645028		
0,35	1793,46067	2,362124571	1071,81993		
0,4	1791,73607	2,362201722	1252,11191		
0,45	1790,79458	2,361930494	1415,63466		
0,5	1789,14465	2,36110117	1567,78052		
0,55	1789,14465	2,36110117	1719,00498		
0,6	1789,14465	2,36110117	1869,59337		
0,65	1789,14465	2,36110117	2019,98839		
0,7	1789,14465	2,36110117	2171,51091		
0,75	1789,14465	2,36110117	2319,71896		
0,8	1789,14465	2,36110117	2470,0903		
0,85	1789,14465	2,36110117	2621,99971		
0,9	1789,14465	2,36110117	2759,70908		
0,95	1789,14465	2,36110117	2891,55959		
1	1789,14465	2,36110117	3005,38363		



Table 9.4: Values of the variables  $T_{max}$ ,  $tim(T_{max})$  and  $T_p$  for different times of the throttle start time from 0.25 to 1.5. The values of the other parameters are:  $t_s$  = 1,  $s_c$  = 0.3,  $s_a$  = 0.5 and  $f_a$  = 1.

$t_a$	$T_{max}$ (Nm)	$tim(T_{max})$ (s)	$T_p$ (Nm)
0,25	4157,29411	3,897740511	2517,60009
0,3	4140,95047	3,887820474	2487,36405
0,35	4135,28004	3,881139961	2473,19347
0,4	4115,49175	3,876668228	2466,10822
0,45	4109,88577	3,874415911	2454,35154
0,5	4111,9337	3,873613454	2458,39045
0,55	4116,13855	3,872535787	2458,94167
0,6	4120,40669	3,870672064	2474,71945
0,65	4097,44721	3,866655973	2458,57132
0,7	4092,8083	3,863402562	2455,1331
0,75	4098,83266	3,862536191	2468,30177
0,8	4075,0117	3,854421459	2461,09547
0,85	1692,27434	2,215166888	2540,72556
0,9	1677,55229	2,204985833	2573,06853
0,95	1679,04515	2,208997591	2590,93863
1	1680,17834	2,208373223	2600,93843
1,05	1681,28874	2,20890262	2605,44035
1,1	1682,43441	2,21	2657,46711
1,15	1677,09081	2,205422061	2760,98771
1,2	1667,07019	2,203492388	2808,93807
1,25	1671,69491	2,2026321	2818,59367
1,3	1670,67176	2,201810601	2841,046
1,35	1670,56904	2,203755359	2862,17625
1,4	1669,51346	2,2	2880,79055
1,45	1671,1205	2,202136799	2919,47766
1,5	1671,41224	2,201641276	2948,43962





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