INEQUALITY FOUNDATIONS OF CONCENTRATION MEASURES: AN APPLICATION TO THE HANNAH-KAY INDICES*

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ABSTRACT

In this paper we provide a connection between concentration and inequality by showing that the inequality measures consistent with the whole class of Hannah-Kay concentration indices are the general entropy inequality indices. We isolate the inequality component underlying the concentration measures, obtaining an explicit additive decomposition of the change in concentration into the change in its two components: inequality and the number of firms. This relationship proves to be valid for the whole class of Hannah-Kay concentration indices, and embodies as particular cases other previously found in the literature. Finally, our proposed decomposition is shown by means of an empirical example, which illustrates the sources of a change in sectoral concentration between two points in time.

JEL Classification: L11, D63

Key Words: Concentration, inequality

1. Introduction

Concentration indices are traditional instruments in industrial economics, which provide a synthetic measure of market structure, and allow evaluating the degree of competition present in different industries. The indices are defined in such a way to incorporate the two relevant aspects of industry structure, namely the number of firms and size inequalities [see, e.g., Waterson (1984)].

The aim of this paper is to find which class of inequality measures is behind the concentration indices proposed by Hannah and Kay (1977). The relationship between both concepts has been previously noticed in the literature. As already pointed out by Hannah and Kay (1977), there is an ambiguous effect on concentration following a change in the number of firms into an industry, since the overall result would be also dependent on the change in inequality. In addition, the class of Hannah-Kay concentration indices would be founded on more solid grounds if the explicit trade-off between the inequality and the number of firms' components were formally derived.

An early attempt in the analysis of the implicit relationship between concentration and inequality was made by Marfels (1971). This author found a consistent relationship between the Herfindahl and entropy measures of concentration, and the corresponding inequality indices, the former being two particular cases of the more general Hannah-Kay class of concentration indices. Subsequently, Hannah and Kay (1977) showed the consistent relationship between the Atkinson index of inequality and a subset of the concentration measures proposed by them.

In this paper we go further from these partial relationships, trying to find out which kind of inequality indices are consistent with the whole class of Hannah-Kay concentration indices. We obtain that the general entropy inequality indices (up to any increasing transformation) are those which are consistent with the whole family of Hannah-Kay concentration indices, generalizing previous findings by other authors.

In addition, we will also provide an explicit additive decomposition of the change in concentration into the change in its two components: inequality and the number of firms. Finally, we will present an application to real data, which illustrates our approach.

The rest of the paper is organized as follows. The relationship between concentration and inequality indices is derived in section 2, and the decomposition of the change in concentration, together with the empirical example, is shown in section 3. The main conclusions are summarized in section 4.

2. Consistent relationships between concentration and inequality indices

Concentration indices are formally defined as a function C: $R^N \rightarrow R$ over a vector $s=(s_1,..,s_i,..,s_N)$, where s_i is the relative market share of the ith firm:

$$s_i = \frac{X_i}{\sum_{i=1}^{N} X_i} \qquad (1)$$

being X_i an indicator of the size of the ith firm (usually sales or employment).

Assuming an axiomatic derivation as in Hannah and Kay (1977) or Encaoua and Jacquemin (1980), industry concentration indices can be expressed as a function of two variables [see, e.g., Waterson (1984)]:

$$C = f(N, I) \qquad f_N < 0, f_I > 0 \qquad (2)$$

where N denotes the number of firms in the industry, and I is an inequality index of firm size I: $R^N \rightarrow R$, defined over the vector $X=(X_1,..,X_i,..,X_N)$. Under the classical "principle of transfers" (Dalton, 1920), I(·) must be strictly S-convex (Dasgupta, Sen and Starret, 1973).

More specifically, a new entrant into an industry might lead to an ambiguous effect on concentration. On the one hand, concentration directly falls due to the increased number of firms. But, on the other hand, the degree of inequality within the industry would be also affected, so that concentration could actually rise in the case that the entrant is big enough.

Our aim in this paper will be to try to disentangle both effects by building a bridge between concentration indices and the classical inequality indices. To this end, in this section we will focus our attention on the consistent derivation of the Hannah and Kay concentration indices from the general entropy inequality indices, as defined by Cowell (1977,1995):

$$I_{GE(c)} = \begin{cases} \dfrac{1}{N}\dfrac{1}{c(c-1)}\displaystyle\sum_{i=1}^{N}[(X_i/\overline{X})^c - 1], & \forall c \neq 0,1 \\ \dfrac{1}{N}\displaystyle\sum_{i=1}^{N}\ln(\overline{X}/X_i), & if\ c = 0 \\ \dfrac{1}{N}\displaystyle\sum_{i=1}^{N}[(X_i/\overline{X})\ln(X_i/\overline{X})], & if\ c = 1 \end{cases} \quad (3)$$

where, according to the income distribution literature, X_i denote the ith household income, $\overline{X}$ is the mean income across households, and N is the number of households. Notice that, for our purposes, the concept of income will be extended to define the analogous concept for the firm, so that X_i would apply to any indicator of the firm's size[1].

Formally, we propose the following definition. A concentration index C is consistent with (i.e., can be consistently derived from) an inequality index I if, given N, for any two vectors s^1 and s^2 the following equivalence is satisfied:

$$C(s^1) \geq C(s^2) \Leftrightarrow I(s^1) \geq I(s^2) \quad (4)$$

[1] Moreover, it can be shown that, since the inequality indices defined throughout the paper are relative (i. e., zero-degree homogeneous in the X variable) inequality indices, they can be interpreted alternatively in terms of relative shares, i. e., I(X)=I(s).

which is equivalent to the condition $f_I>0$ in equation (2). We will be concerned with the concentration indices that are homogeneous of degree minus one in N, i. e., the number of firms[2].

Next, we can write the Hannah and Kay class of concentration indices in the following way:

$$C_{HK(\alpha)} = \begin{cases} [\sum_{i=1}^{N} s_i^{\alpha}]^{\frac{1}{(\alpha-1)}} & \text{if } \alpha > 0, \alpha \neq 1 \\ \exp[\sum_{i=1}^{N} s_i \ln s_i] & \text{if } \alpha = 1 \end{cases} \tag{5}$$

Notice that $C_{HK(1)}$ is defined as the limit of $C_{HK(\alpha)}$ when $\alpha \to 1$, which coincides with the antilogarithm of (minus) the first-order entropy concentration index; see also Waterson (1984). Now, from the previous definition, we can derive in a consistent way the Hannah-Kay concentration indices from the general entropy inequality indices. In fact, equations (3) and (5) can be shown to be related through:

$$C_{HK(\alpha)} = \begin{cases} \dfrac{[1+\alpha(\alpha-1)I_{GE(\alpha)}]^{\frac{1}{\alpha-1}}}{N} & \text{if } \alpha = c > 0, \alpha = c \neq 1 \\ \dfrac{\exp[I_{GE(\alpha)}]}{N} & \text{if } \alpha = c = 1 \end{cases} \tag{6}$$

Equation (6) is the central result of the paper. From here, three particular cases previously noticed in the literature can be derived from our more general equation (6) [see, e.g.,

2 Notice that inequality indices are also influenced by population changes; in particular, all the indices used in this paper satisfy the population replication axiom. More specifically, the Atkinson inequality indices satisfy the marginal population replication axiom (Salas, 1998), so they are good candidates to perform well under changes in population size.

Marfels (1971) for the first two, and Hannah and Kay (1977) for the third]. First, $C_{HK(2)}$ (i.e., the Herfindahl concentration index C_H), is consistent with $I_{GE(2)}$:

$$C_H = \frac{1+2\, I_{GE(2)}}{N} \tag{7}$$

Second, $C_{HK(1)}$ is consistent with $I_{GE(1)}$ (i.e., the classical Theil 1 index):

$$C_{HK(1)} = \frac{\exp[\, I_{GE(1)}\,]}{N} \tag{8}$$

Third, for the case $0<\alpha<1$, the $C_{HK(\alpha)}$ indices are also consistent with the classical Atkinson indices $I_{A(\varepsilon)}$, defined for every $\varepsilon>0$ in the following way (Atkinson, 1970):

$$I_{A(\varepsilon)} = \begin{cases} 1 - [\frac{1}{N}\sum_{i=1}^{N}(\frac{X_i}{\overline{X}})^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}}, & \forall\, \varepsilon > 0, \varepsilon \neq 1 \\ 1 - \exp[\frac{1}{N}\sum_{i=1}^{N}\ln(\frac{X_i}{\overline{X}})], & if\ \varepsilon = 1 \end{cases} \tag{9}$$

so that, when $\varepsilon=1-\alpha$, the following equivalence holds[3]:

$$C_{HK(\alpha)} = \frac{[1 - I_{A(1-\alpha)}]^{\frac{\alpha}{\alpha-1}}}{N} \quad if\ 0 < \alpha < 1 \tag{10}$$

[3] Notice that a complete consistent equivalence between the Hannah-Kay and the general entropy (for all α, in equation (6)) and Atkinson (for all $\alpha<1$, in equation (10)) indices could be found by further generalizing the Hannah-Kay indices, if we extend the definition in equation (5) to

$$C_{HK(\alpha)} = -[\sum_{i=1}^{N} s_i^{\alpha}]^{\frac{1}{(\alpha-1)}} \quad \text{if } \alpha<0$$

3. Decomposing the change in concentration: an example

In this section we provide a decomposition of the change in concentration between the two sources identified in equation (2), i.e., the number of firms N and the degree of inequality I. Notice that equation (6) can be written in the general form:

$$C_{HK(\alpha)} = \frac{\varphi(I_{GE(\alpha)})}{N} \qquad \forall \alpha > 0 \tag{6'}$$

where $\varphi(I_{GE(\alpha)})$ is the component of inequality in $C_{HK(\alpha)}$, which is an increasing function of the general entropy inequality indices. From (6'), it is straightforward to see that the following additive expression can be derived:

$$\frac{\Delta C_{HK(\alpha)}}{C_{HK(\alpha)}} \approx \frac{\Delta\varphi(I_{GE(\alpha)})}{\varphi(I_{GE(\alpha)})} - \frac{\Delta N}{N} \qquad \forall \alpha > 0 \tag{11}$$

We illustrate this decomposition with an example taken from Bajo and Salas (1997). In that paper we computed a set of concentration indices for 68 sectors of the Spanish economy in 1993, using the Spanish Institute for Fiscal Studies' data set coming from the Profit Tax reports by more than 300,000 firms (i.e., providing an almost exhaustive coverage of both firms and sectors). Then, our decomposition was applied to the change in concentration between 1992 and 1993, for the Hannah-Kay indices with α=0.5, 1, 1.5, 2, and 2.5.

Notice that, according to equation (11), and for any particular α, concentration would unambiguously increase when:

$$\frac{\Delta\varphi(I)}{\varphi(I)} > \frac{\Delta N}{N}$$

which, in turn, would occur in any of the following cases:

$$(i)\Delta N < 0 \quad \text{and} \quad \Delta I > 0$$

$$(ii)\Delta N < 0, \Delta I < 0 \quad \text{and} \quad \frac{\Delta\varphi(I)}{\varphi(I)} > \frac{\Delta N}{N}$$

$$(iii)\Delta N > 0, \Delta I > 0 \quad \text{and} \quad \frac{\Delta\varphi(I)}{\varphi(I)} > \frac{\Delta N}{N}$$

On the other hand, for any particular α, concentration would unambiguously decrease when:

$$\frac{\Delta\varphi(I)}{\varphi(I)} < \frac{\Delta N}{N}$$

which would occur in any of the following cases:

$$(iv)\Delta N > 0 \quad \text{and} \quad \Delta I < 0$$

$$(v)\Delta N > 0, \Delta I > 0 \quad \text{and} \quad \frac{\Delta\varphi(I)}{\varphi(I)} < \frac{\Delta N}{N}$$

$$(vi)\Delta N < 0, \Delta I < 0 \quad \text{and} \quad \frac{\Delta\varphi(I)}{\varphi(I)} < \frac{\Delta N}{N}$$

In table 1 we present an example of the decomposition shown in equation (11). As the last column of the table shows, we are able to explain reasonably well the change in concentration during the period. From the 68 sectors in our previous study, we have selected nine industries, which cover the six cases stated above.

In six of the sectors, concentration increases. In Food industry, Textiles, and Banking, concentration rises due to both a lower number of firms and a higher inequality -i.e., case (i) above-. In Basic chemicals, concentration rises due to a lower number of firms and despite a lower inequality for α=0.5, 1 and 1.5 -i.e., case (ii) above-; however, for α=2 and 2.5, higher inequality would also lead to higher concentration -i.e., case (i) above-. Finally, in Chemicals and Precision instruments, concentration rises due to a higher inequality and despite a higher number of firms -i.e., case (iii) above-.

In the three remaining sectors, concentration decreases. In Air and sea transportation, concentration falls due to both a higher number of firms and a lower inequality -i.e., case (iv) above-. In Computing services, concentration falls due to a higher number of firms and despite a higher inequality for α=1, 1.5, 2 and 2.5 -i.e., case (v) above-; however, for α=0.5 lower inequality would also lead to lower concentration -i.e., case (iv) above-. Finally, in House renting, concentration falls due to a lower inequality and despite a lower number of firms -i.e., case (vi) above-.

Table 1 : Decomposition of the change in concentration, 1992-93

A) Index HK(0.5)

SECTOR	Rate of change in the concentration index (1)	Rate of change in the inequality component (2)	Rate of change in the number of firms (3)	Explained rate of change (4)=(2)-(3)	Percentage of explanation (5)=(4)/(1)*100
Basic Chemicals	2.00	-0.06	-2.02	1.96	97.98
Chemicals	2.81	2.94	0.12	2.82	100.12
Precision Instruments	0.41	3.57	3.14	0.43	103.14
Food Industry	6.15	5.19	-0.91	6.09	99.09
Textiles	9.83	1.33	-7.74	9.07	92.26
Air and Sea Transportation	-15.69	-12.63	3.63	-16.25	103.63
Banking	4.61	2.13	-2.36	4.50	97.64
Computing Services	-11.73	-2.20	10.80	-13.00	110.80
House Renting	-7.10	-9.20	-2.26	-6.94	97.75

B) Index HK(1)

SECTOR	Rate of change in the concentration index (1)	Rate of change in the inequality component (2)	Rate of change in the number of firms (3)	Explained rate of change (4)=(2)-(3)	Percentage of explanation (5)=(4)/(1)*100
Basic Chemicals	1.01	-1.02	-2.02	1.00	98.98
Chemicals	5.62	6.08	0.12	5.96	106.08
Precision Instruments	5.70	9.37	3.14	6.24	109.37
Food Industry	11.41	11.86	-0.91	12.76	111.86
Textiles	11.03	3.70	-7.74	11.44	103.70
Air and Sea Transportation	-52.50	-32.05	3.63	-35.67	67.95
Banking	5.14	2.93	-2.36	5.29	102.93
Computing Services	-10.63	0.15	10.80	-10.65	100.15
House Renting	-53.90	-36.49	-2.26	-34.23	63.51

C) Index HK(1.5)

SECTOR	Rate of change in the concentration index (1)	Rate of change in the inequality component (2)	Rate of change in the number of firms (3)	Explained rate of change (4)=(2)-(3)	Percentage of explanation (5)=(4)/(1)*100
Basic Chemicals	1.46	-0.50	-2.02	1.51	104.00
Chemicals	8.37	7.26	0.12	7.14	85.40
Precision Instruments	9.37	11.02	3.14	7.88	84.05
Food Industry	22.90	19.22	-0.91	20.13	87.89
Textiles	16.38	6.36	-7.74	14.10	86.06
Air and Sea Transportation	-31.73	-27.68	3.63	-31.31	98.66
Banking	4.52	1.72	-2.36	4.09	90.50
Computing Services	-6.83	2.88	10.80	-7.92	115.93
House Renting	-70.65	-69.46	-2.26	-67.21	95.12

Table 1 (continued)

D) Index H

SECTOR	Rate of change in the concentration index (1)	Rate of change in the inequality component (2)	Rate of change in the number of firms (3)	Explained rate of change (4)=(2)-(3)	Percentage of explanation (5)=(4)/(1)*100
Basic Chemicals	2.53	0.46	-2.02	2.48	97.98
Chemicals	10.62	10.75	0.12	10.63	100.12
Precision Instruments	9.15	12.58	3.14	9.44	103.14
Food Industry	34.34	33.12	-0.91	34.02	99.09
Textiles	19.83	10.55	-7.74	18.29	92.26
Air and Sea Transportation	-26.98	-24.33	3.63	-27.96	103.63
Banking	3.96	1.51	-2.36	3.87	97.64
Computing Services	-5.63	4.57	10.80	-6.23	110.80
House Renting	-78.34	-78.83	-2.26	-76.57	97.74

E) Index HK(2.5)

SECTOR	Rate of change in the concentration index (1)	Rate of change in the inequality component (2)	Rate of change in the number of firms (3)	Explained rate of change (4)=(2)-(3)	Percentage of explanation (5)=(4)/(1)*100
Basic Chemicals	3.57	1.48	-2.02	3.49	97.98
Chemicals	12.52	12.66	0.12	12.54	100.12
Precision Instruments	8.15	11.55	3.14	8.41	103.14
Food Industry	44.91	43.59	-0.91	44.50	99.09
Textiles	21.64	12.23	-7.74	19.97	92.26
Air and Sea Transportation	-23.84	-21.08	3.63	-24.70	103.63
Banking	3.75	1.29	-2.36	3.66	97.64
Computing Services	-4.50	5.81	10.80	-4.99	110.80
House Renting	-79.89	-80.35	-2.26	-78.09	97.74

Source: Bajo and Salas (1997)

4. Conclusions

In this paper we have derived a consistent relationship between the whole class of Hannah-Kay concentration indices and the classical general entropy inequality measures coming from the income distribution literature. We isolated the inequality component underlying these concentration measures, and then we provided an explicit additive decomposition of the change in concentration into the change in its two components: inequality and the number of firms. Our result proved to be valid for the whole class of Hannah-Kay concentration indices, and included as particular cases other previously found in the literature.

This decomposition might be useful in empirical work since it could help to identify the sources of a change in sectoral concentration between two points in time. We concluded by presenting an empirical application to the Spanish economy, which illustrated the procedure proposed in the paper.

As we have seen, the Hannah-Kay concentration indices are flexible enough to be consistent with a wide class of different inequality measures. However, this property does not hold regarding the number of firms' component of the indices, since they are always homogeneous of degree minus one in N. Further research on the subject might be addressed to develop more general concentration indices, which would allow for more flexibility in the N-component.

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