# A NEW KEYNESIAN ANALYSIS OF INDUSTRIAL EMPLOYMENT FLUCTUATIONS

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#### Abstract

This paper describes a model with sticky prices, search frictions and hours-clearing wages that provides firm differentiation across several dimensions: price, output, wage, employment and hours per worker. The connection between pricing and hiring decisions results in firm-level employment fluctuations that depend upon sticky prices, search costs, demand elasticity and labor supply elasticity. The calibrated model is able to match average US industrial employment volatility when assuming a small industrial size, providing one possible answer to Shimer (2005a)'s puzzle.

Keywords: search frictions, sticky prices, industrial employment.

JEL codes: E3, J2, J3, and J4.

#### 1 Introduction

The aggregation procedure used to obtain macroeconomic series implies the compensation between positive and negative entries. Such "smoothing action" leaves out information on fluctuations that cancel out each other. Therefore, the aggregation procedure can be misleading for the business cycle analysis because it provides a downward-biased measure of short-run fluctuations. Let us see this point with a simple example.

One economy with one million workers is formed by only two industries, A and B. Each industry initially employs half a million workers. During the same period of time, A industry suffers a destruction of 5,000 jobs while B industry is able to create the same amount of 5,000 jobs. Thus, we would

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say that the level of total (aggregate) employment remains constant at 1 million workers, with a 0% variability. However, since employment fell by 1% in A industry and rose by 1% in B industry, the average variability of employment is 1%. That is the result obtained using the industrial perspective. If we do it in terms of aggregate employment, we would say that variability of employment was 0%. Apparently, the former conclusion is more correct than the latter because there were actually 10,000 people changing their job status, which by the way is 1% of total laborforce in the example.

Obviously, business cycle statistics obtained from model-based simulations can suffer from the same volatility reduction if the analysis lacks from any industrial disaggregation. This paper examines the implications of calculating statistics of volatility using series of industrial employment instead of aggregate employment. We do it both in the US data and in a calibrated New Keynesian model. Our results show that the standard deviation of employment is significantly undervalued both in US data and in the model when measured from aggregate employment compared with the values obtained as the average standard deviation of industrial employment.

Shimer (2005a) argues that employment volatility is unrealistically low in a model with search and matching frictions of the kind obtained by Mortensen and Pissarides (1994). This criticism is known as Simer's puzzle. The New Keynesian model with search frictions may be able to boost employment fluctuations through the price dispersion that results from sticky prices. However, seminal papers such as Walsh (2005) and Trigari (2009) assume a representative firm that makes no distinction of employment levels across firms.<sup>1</sup> Unlike most related literature, this paper describes extensively the model outlined in Casares (2008) where pricing and hiring decisions are connected at the firm level.<sup>2</sup> As a result, firm-specific employment dynamics depend upon firm's relative price.

Such firm-specific employment equation was used here to study the business cycle properties of industrial employment. This paper explores the determinants of volatility on firm-specific employment dynamics from a variety of factors that include search frictions, price rigidities and household's preferences. More concretely, firm-level employment variability depends positively on the rate of job destruction, the degree of price stickiness (bounded to some upper threshold) and the elasticity of demand. On the contrary, employment volatility falls with a higher elasticity of search costs and also

<sup>&</sup>lt;sup>1</sup>The absence of employment differentiation across firms leads to a theoretical inconsistency. How can it be assumed that there are costs of search and matching when all firms are identically either hiring or firing workers? Can search frictions cause unemployment fluctuations if all firms have the same employment dynamics? If all of them did wish to hire more workers search costs might be negligible whereas in times of job destruction searching for a job must be really useless. The representative firm model does not seem to be compatible with a search-and-matching theory of unemployment. Firm differentiation is required to explain why workers change from one industry to another. In other words, hiring and employment dynamics must be firm specific. This is one basic point of this paper.

<sup>&</sup>lt;sup>2</sup>A remarkable contribution by Thomas (2008) discusses how the lack of connections between pricing and employment dynamics in New Keynesian models is due to assuming a retailer-producer dual structure.

with an increase in the elasticity of labor supply.

Hence, this paper proposes a New Keynesian model in which firms differentiate in many dimensions: they have a specific selling price, they have a different number of employees, they offer a particular number of vacancies, they produce a different quantity of output, they organize different shifts of hours at work, and they pay a different nominal wage. Such a model is built assuming nominal rigidities (sticky prices) and real rigidities (search frictions) in a way that provides this multi-dimension dispersion at firm level. The model delivers both macroeconomic and microeconomic relationships. The macro model may serve for the conventional business cycle analysis or monetary/fiscal implications that is not analyzed here. The focus of the paper is located on the firm-level relationships that serve for a sectorial analysis, in particular to find out the determinants of industrial employment fluctuations. The results of this quantitative analysis can help to understand the volatility observed in US industrial employment.

The rest of the paper contains five more sections. Section 2 is empirical; it describes short-run fluctuations of industrial employment taken from recent US data. As documented there, employment variability across US industries is high and mildly procyclical. Section 3 describes the details and derivation of the New Keynesian model discussed in Casares (2008) that combines sticky prices with search frictions and hours-clearing wage setting. Section 4 introduces a baseline calibration that is used in the study of firm-specific and industrial employment fluctuations. Section 5 examines the volatility and cyclical correlation of employment in a double perspective: either taking a single measure of aggregate employment or by looking at industrial employment upon alternative industrial sizes. The analysis includes a comparison with the characteristics of employment fluctuations observed in US data. Section 6 concludes the paper with the review of major results.

## 2 US employment fluctuations

Despite of the generally accepted view (the so-called "Great Moderation"), US industrial data show that the last decade and a half has been a period of high employment variability. The Bureau of Labor Statistics (BLS) provides a wide range of sectorial employment data, with several levels of disaggregation.<sup>3</sup> According to the published data, there have been two short business cycles in the past fifteen years: the Information Technologies (IT) expansion and decline from 1994 through 2002 and the real-estate boom during the low interest-rate period (2002-2006) which led to the financial crisis in 2007. Thus, the number of employed people in the industry of *Computer systems design and related services* rose in 400,000 persons in 1998-2000; however, half of those jobs were destroyed during the IT crisis suffered in 2001 and 2002. The second wave of employment expansion came

<sup>&</sup>lt;sup>3</sup>Data are available from the BLS website at http://www.bls.gov/data/#employment.

with the housing boom that created almost 1,2 million jobs in the construction sector from 2003 to 2005. However, the burst of the housing bubble swept away almost 1 million jobs during the years 2007 and 2008. Besides, 47,200 jobs were destroyed in real estate agencies and 174,000 jobs in the Credit intermediation and related activities industry. Meanwhile, steady employment creation has been observed in industries related to education, health, and also in services provided at bars and restaurants. Many of these changes are lost out of track when building the aggregate series of Total Private Employment (TPE).

I did download BLS monthly employment data from industries that occupied more than 400,000 workers in 1994. The sample covers 67 industries that account for approximately 89% of the series of TPE also reported in the BLS website. Most of them correspond to the 3-digit categories defined by the North American Industry Classification System (NAICS). For a business cycle analysis, quarterly series were obtained by making three-month average values. Then, the series were logged and detrended using a Hoddrick-Prescott (HP) filter. Analogously, aggregate series of output and employment were obtained by taking logs and then making the HP filter to original series of Real Gross Domestic Product and TPE.<sup>4</sup> The business cycle volatility can be measured by the standard deviation of the resulting series relative to the standard deviation of output, whereas the coefficient of linear correlation with output provides the extent of procyclicality. These two dimensions were examined in US data taking a direct measure of aggregate employment (TPE) or, alternatively the average across industrial employment.<sup>5</sup> Table 1 reports the results:

Table 1. US employment fluctuations, 1994-2008

Total Private Employment (TPE)	
Standard deviation, relative to output:	1.07
Correlation with output:	0.78

#### $Industrial\ Employment$

Average standard deviation, relative to output: 1.67 Average correlation with output: 0.35

The numbers are different depending upon the treatment given to employment data. When numbers of employed workers are just added up to obtain TPE, the standard deviation of employment is similar to that of output (just 7% higher as shown in Table 1) while both series present a strong positive correlation. These numerical findings can be confirmed by looking at Figure 1 that plots the

<sup>&</sup>lt;sup>4</sup>The series of Real GDP was retrieved from the FRED database of the Federal Reserve Bank of St. Louis, http://research.stlouisfed.org/fred2/ .

<sup>&</sup>lt;sup>5</sup>A weighted average of the numbers obtained in the 67 US industries was computed by defining individual weights as the fraction of Total Private Employment that takes each particular industry. This applies to both averages of the standard deviation and the correlation with output.

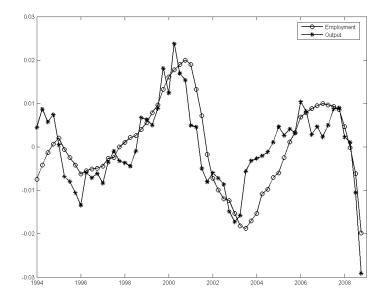


Figure 1: Fluctuations of Total Private Employment (TPE) and Real GDP (Output) in the US during the period 1994:1-2008:4. Both series have been logged and HP-filtered to extract the cyclical component.

business cycle series of US employment and output.

The results are quite different in the case of taking industrial employment data. On the one hand, the standard deviation rises substantially (by 56% from 1.07 to 1.67 as indicated in Table 1). On the other hand, the coefficient of correlation with output falls. Therefore, the average values of the standard deviation and the output correlation do not coincide with the numbers obtained using aggregate employment. As expected, the lack of a smoothing action coming from the overall aggregation makes employment be both more volatile and less procyclical. Figure 2 displays a selection of US industrial employment fluctuations in several plots that collect industries of the same sector. The numbers of the complete set of industries are provided in Appendix I.

There are industries where employment presents a much higher variability than output and others where it is the opposite. This information is blurred away when doing the aggregation. Hence, it can be observed in Figure 2 that the metal, electronics or motor industries have much greater employment fluctuations than output. Other industries that show high employment variability belong to the Construction sector or to Professional Services. By contrast, there are industries with low employment variability, such as the ones related to Education and Health as displayed in Figure 2. The complete industrial employment data (see Appendix I) show ten industries with a standard deviation more than

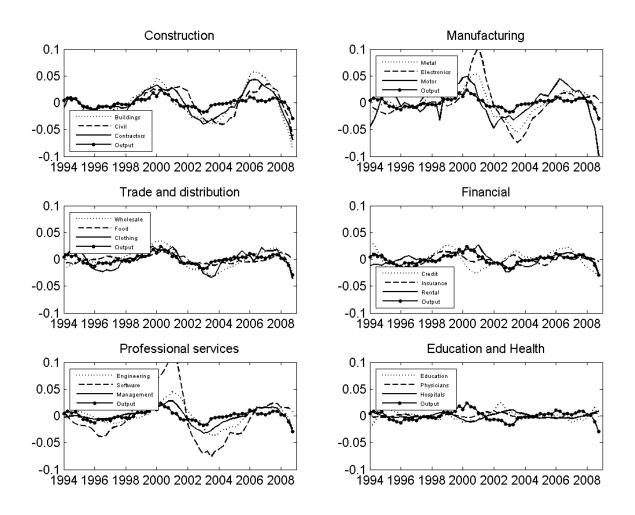


Figure 2: Employment business cycle fluctuations across selected sectors and industries, 1994-2008.

three times higher than the one of output, while quantitatively important industries, such as Hospitals and Food Manufacturing, report a much lower standard deviation that is approximately one half of that of output. The weighted average comes up with a number equal to 1.67, significantly higher than 1.07 obtained with the series of aggregate employment (TPE).

Regarding the correlation of industrial employment with output, we find again many differences across US industries. There are strongly procyclical industries (such as Furniture, Truck Transportation and Clothing); a large group of industries are mildly procyclical and there is also a few industries where employment is countercyclical (notably, Hospitals and Educational Services). Actually, there are 9 industries out of the total of 67 that get a negative coefficient of correlation between fluctuations of employment and aggregate output. Once the weighted average is computed, the mean correlation is at 0.35, substantially lower than 0.78 obtained when using the measure of aggregate employment (TPE).

Summarizing, the traditional way of looking at business cycle fluctuations with aggregate macromagnitudes looses information that is quantitatively relevant for the business cycle analysis. The industrial data analyzed here shows that the aggregation procedure downsizes employment volatility and rises employment procyclicality.

## 3 A New Keynesian model with firm-specific employment

I will now introduce a model that allows employment differentiation across firms with the objective of use it later for the analysis of industrial employment fluctuations. The model was already sketched in Casares (2008) and this section is devoted to carefully describe the elements of the model and the way structural equations are obtained. The key ingredients that determine employment dynamics in the model are sticky prices a la Calvo (1983) and search-and-matching frictions as in Mortensen and Pissarides (1994). The rest of the model belongs to the standard New Keynesian framework. The supply-side of the economy is formed by many heterogeneous firms that operate in monopolistic competition as described in Dixit and Stiglitz (1977). Firms may set an specific price while the amount of output produced is constrained by the Dixit-Stiglitz demand curve

$$y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} y_t,\tag{1}$$

where  $y_t(i)$  is output produced at firm i,  $P_t(i)/P_t$  is the ratio of price set by firm i over the average price level,  $y_t$  is the level of aggregate output, and  $\theta$  is a constant elasticity parameter. Firms have two forms of varying labor input: at the extensive margin with a one-period lag of adjustment (number of workers employed,  $n_{t+1}(i)$ ) and in the intensive margin during the current period (number of hours

per worker demanded,  $h_t^d(i)$ ). Assuming constant capital, and the same marginal productivity on both margins, the production function of the i firm is

$$y_t(i) = \left(\exp(z_t)h_t^d(i)n_t(i)\right)^{1-\alpha},\tag{2}$$

with  $0 < \alpha < 1$  and  $z_t$  denoting an AR(1) technology shock. After substituting (2) into (1), the amount of output produced by the *i*-th firm is demand determined as follows

$$\left(\exp(z_t)h_t^d(i)n_t(i)\right)^{1-\alpha} = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} y_t.$$
(3)

Unlike seminal New Keynesian models with unemployment such as Walsh (2005) and Trigari (2009), pricing and hiring decisions are not separated in this model.<sup>6</sup> The same monopolistically competitive firm that sets prices is making decisions on vacancies and new jobs. Hiring new workers is costly for the firm as in the Mortensen-Pissarides literature. In particular, a firm must post a vacancy in the market and wait for a matching of that vacancy with some available worker. Thus, job creation has the following search cost function that depends on the number of vacancy postings,  $v_t(i)$ ,

$$c(v_t(i)) = c_0 (v_t(i))^{1+c_1}$$
.

Many papers assume  $c_1 = 0.0$  to imply linear search costs (Walsh, 2005; Christoffel and Kuester, 2008). In addition, the hiring process requires time for the worker to find the job and for the firm to fill the vacancy. This is typically modeled by saying that new hires are incorporated at the firm one period after the matching takes place. The matching function delivers the number of new jobs that result from the search process of unemployed workers for vacancy postings

$$m_t = (u_t)^{\xi} (v_t)^{1-\xi},$$

where  $m_t$  is the number of economy-wide new matchings,  $u_t = 1 - n_t = 1 - \int n_t(i)di$  is the number of unemployed people seeking for a job,  $v_t = \int v_t(i)di$  is the total number of vacancies posted, and  $0 < \xi < 1$  is a technology parameter that provides the relative contribution of the pool of unemployed workers in making a match. Subsequently, the probability for a firm of making a new hiring out of a vacancy posting is

$$q_t = \frac{m_t}{v_t} = \frac{(u_t)^{\xi} (v_t)^{1-\xi}}{v_t} = \left(\frac{u_t}{v_t}\right)^{\xi}.$$

Meanwhile, job destruction is determined by a constant separation rate.<sup>7</sup> It means that all firms must face some exogenous job destruction at the separation rate, s. In turn, employment evolves for the

<sup>&</sup>lt;sup>6</sup>As shown by Casares (2008) and Thomas (2008), the lack of separation between pricing and hiring is not an innocuous assumption.

<sup>&</sup>lt;sup>7</sup>Hall (2005b) and Shimer (2005b) claim that the separation rate is quite stable in the US and has little effect on employment fluctuations.

i-th firm as indicated by the following dynamic equation

$$(1-s)n_t(i) + q_t v_t(i) = n_{t+1}(i), (4)$$

which says that next period's employment is the predetermined sum of the jobs that remain after current period,  $(1-s)n_t(i)$ , plus the number of new hirings,  $q_tv_t(i)$ , obtained as the product of the number of vacancies posted by its probability of filling them with a match.

The impossibility of instantaneous hiring obliges the firm to call for changes in the amount of hours,  $h_t^d(i)$ , when output must be adjusted to meet current demand conditions. The other inputs of the production function (2) cannot be used to adjust the level of production because they are either exogenous (the technology shock,  $z_t$ ) or predetermined (employment,  $n_t(i)$ ). If market conditions are favorable and the firm seeks to increase production right away, current employees will work more hours until the arrival of new employes in the next period. For the specific i firm, the demand for hours is obtained by turning (3) around to yield

$$h_t^d(i) = \frac{1}{n_t(i)} \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\theta}{1-\alpha}} \frac{y_t^{\frac{1}{1-\alpha}}}{\exp(z_t)}.$$
 (5)

Households are identical and large as in Merz (1995). The representative household supplies a continuum of differentiated labor services. Those household members working pool their labor income to be split up evenly in a way that conveys the same consumption for the employed members as for the unemployed members. With a separable utility function specification

$$U(c_t, n_t(i), h_t^s(i)) = \frac{c_t^{1-\sigma}}{1-\sigma} - \Psi \int_0^1 n_t(i) \frac{[h_t^s(i)]^{1+\gamma}}{1+\gamma} di$$

the supply of hours is

$$h_t^s(i) = \left(\frac{W_t(i)}{P_t}\lambda_t\right)^{\frac{1}{\gamma}},\tag{6}$$

where  $\lambda_t$  is the marginal utility of consumption and  $\frac{1}{\gamma}$  is the Frisch labor supply elasticity.

Wage determination takes place at the intensive margin of current employees. As in Casares (2008), nominal wages are firm-specific and linked to the supply and demand of hours. The "hours-clearing" wage is the hourly rate that equates the willingness of the worker to spend time at the firm with the need of workhours for the firm.<sup>8</sup> This is a different treatment from the standard Nash-bargained wage setting in the extensive margin that is used in the Mortensen-Pissarides literature.<sup>9</sup> Making

<sup>&</sup>lt;sup>8</sup>Krause and Lubik (2007) use the expression "notional wage" for the Nash-bargained nominal wage.

<sup>&</sup>lt;sup>9</sup>The Nash-bargained wage setting is also present in recent New Keynesian models that incorporate search frictions such as Walsh (2005), Krause and Lubik (2007), Christoffel and Kuester (2008), and Trigari (2009).

 $h_t^d(i) = h_t^s(i)$  with (5) and (6) and solving for the hours-clearing nominal wage,  $W_t(i)$ , it is obtained

$$W_t(i) = \frac{P_t}{\lambda_t} \left( \frac{1}{n_t(i)} \left( \frac{P_t(i)}{P_t} \right)^{-\frac{\theta}{1-\alpha}} \frac{y_t^{\frac{1}{1-\alpha}}}{\exp(z_t)} \right)^{\gamma}, \tag{7}$$

which reveals that the hours-clearing nominal wage depends on two firm-specific variables, the price and the level of employment, in both cases affecting negatively due to the reduction in the demand for hours.

Next, it is time to examine the optimizing behavior of monopolistically competitive firms. All firms seek to maximize intertemporal profits. The i-th firm maximizes

$$E_{t} \sum_{j=0}^{\infty} \beta_{t,t+j} \left[ \left( \frac{P_{t+j}(i)}{P_{t+j}} \right)^{1-\theta} y_{t+j} - \frac{W_{t+j}(i)}{P_{t+j}} h_{t+j}^{d}(i) n_{t+j}(i) - c_{0} \left( v_{t+j}(i) \right)^{1+c_{1}} \right]$$

subject to constraints (3) and (4) in period t and future periods. Future profits are discounted at the stochastic discount factor  $\beta_{t,t+j}$  for j > 0. The nominal wage is firm-specific and determined by (7). Supposing that the i firm receives the Calvo signal to price optimally, the first order condition that must satisfy is

$$E_{t}^{\eta} \sum_{j=0}^{\infty} \beta_{t+j} \eta^{j} \left[ (1-\theta) \left( \frac{(1+\pi)^{j} P_{t}^{*}(i)}{P_{t+j}} \right)^{-\theta} \frac{y_{t+j}(i)}{P_{t+j}} + \theta \psi_{t+j}(i) \left( \frac{(1+\pi)^{j} P_{t}^{*}(i)}{P_{t+j}} \right)^{-\theta-1} \frac{y_{t+j}(i)}{P_{t+j}} \right] = 0,$$
(8)

where  $\eta$  is the Calvo probability of not being able to price optimally,  $E_t^{\eta}$  is the rational expectation conditional to the lack of optimal pricing in future periods,  $P_t^*(i)$  is the optimal price set in period t, and  $\psi_{t+j}(i)$  is the firm-specific real marginal cost in period t+j. The first order condition regarding the choice of  $n_{t+1}(i)$  is

$$-E_{t}\beta_{t,t+1} \left[ \left( \frac{\frac{\partial W_{t+1}(i)}{\partial n_{t+1}(i)}}{P_{t+1}} h_{t+1}^{d}(i) + \frac{\partial h_{t+1}^{d}(i)}{\partial n_{t+1}(i)} \frac{W_{t+1}(i)}{P_{t+1}} \right) n_{t+1}(i) + \frac{W_{t+1}(i)}{P_{t+1}} h_{t+1}^{d}(i) \right] + E_{t}\beta_{t,t+1} \frac{\partial y_{t+1}(i)}{\partial n_{t+1}(i)} \psi_{t+1}(i) - \varphi_{t}(i) + E_{t}\beta_{t,t+1} \left[ (1-s)\varphi_{t+1}(i) \right] = 0, \quad (9)$$

where  $\varphi_t(i)$  and  $\psi_{t+1}(i)$  are the Lagrange multiplier respectively attached to constraints (4) in period t and (3) in period t+1. The optimality condition on the demand for hours,  $h_t^d(i)$ ,

$$\frac{W_t(i)}{P_t}n_t(i) - \frac{\partial y_t(i)}{\partial h_t^d(i)}\psi_t(i) = 0,$$

serves to identify  $\psi_t(i)$  as the real marginal cost

$$\psi_t(i) = \frac{\frac{W_t(i)}{P_t} n_t(i)}{\frac{\partial y_t(i)}{\partial h_t^d(i)}}.$$
 (10)

Meanwhile, the partial derivatives  $\frac{\partial W_{t+1}(i)}{\partial n_{t+1}(i)}$ ,  $\frac{\partial h_{t+1}^d(i)}{\partial n_{t+1}(i)}$ , and  $\frac{\partial y_{t+1}(i)}{\partial n_{t+1}(i)}$  can be computed using equations (7), (5), and (3) referring to period t+1. All yield

$$\frac{\partial W_{t+1}(i)}{\partial n_{t+1}(i)} = -\gamma \frac{W_{t+1}(i)}{n_{t+1}(i)}, \quad \frac{\partial h_{t+1}^d(i)}{\partial n_{t+1}(i)} = -1 \frac{h_{t+1}^d(i)}{n_{t+1}(i)}, \quad \text{and} \quad \frac{\partial y_{t+1}(i)}{\partial n_{t+1}(i)} = (1-\alpha) \frac{y_{t+1}(i)}{n_{t+1}(i)}.$$

The substitution in equation (9) of these three partial derivatives and also the expression for  $\psi_{t+1}(i)$ , obtained when moving (10) one period ahead, leads, after a massive simplification, to the following optimality condition for employment

$$E_{t}\beta_{t,t+1}\left[(1+\gamma)\left(h_{t+1}^{d}(i)\frac{W_{t+1}(i)}{P_{t+1}}\right)\right] = \varphi_{t}(i) - E_{t}\beta_{t,t+1}\left[(1-s)\varphi_{t+1}(i)\right]. \tag{11}$$

The interpretation of  $\varphi_t(i)$  can be extracted from the first order condition on the number of vacancies posted in period t,  $v_t(i)$ , which says

$$-c_0(1+c_1)(v_t(i))^{c_1} + \varphi_t(i)q_t = 0,$$

defining the shadow value of a job,  $\varphi_t(i)$ , as the marginal costs of a vacancy divided by the probability of making a match:

$$\varphi_t(i) = \frac{c_0(1+c_1)\left(v_t(i)\right)^{c_1}}{q_t}.$$
(12)

Thus, the interpretation of the optimal hiring equation (11) can be done in standard microeconomic terms: the marginal benefit expected for a new job on the left-hand side (measured as the expected saving of hours to accommodate the new employee) must be equal to the marginal cost saved when dropping the last vacancy posted on the right-hand side.

Equations (8), (11) and (12) jointly determine the dynamic behavior of prices and employment. Loglinearizing techniques can be used to find linear approximations that collect the period-to-period evolution of these variables. Using standard notation, the hat symbol on top of a variable refers to the log deviation of that variable from the steady-state level. For example,  $\hat{n}_{t+1} = \log\left(\frac{n_{t+1}}{n}\right)$  represents the log deviation of aggregate employment from steady state in period t+1. In addition, tilde-topped variables denote relative variables measured as log deviations with respect to the current aggregate variable. Hence, the relative price of firm i in period t is written as  $\tilde{P}_t(i) = \log P_t(i) - \log P_t$ . After doing the algebra, log-linear structural equations that govern aggregate and firm-specific employment dynamics are (proof available in Appendix II)

$$-\left(\frac{(1+\rho)c_{1}}{(\rho+s)s} + \frac{(1-s)^{2}c_{1}}{(\rho+s)s}\right)\widehat{n}_{t+1} = E_{t}\widehat{h}_{t+1} + E_{t}\widehat{w}_{t+1} + \frac{(1+\rho)c_{1}(1-s)}{(\rho+s)s}\widehat{n}_{t} + \frac{(1-s)c_{1}}{(\rho+s)s}E_{t}\widehat{n}_{t+2} + \frac{(1+\rho)(1+c_{1})}{\rho+s}\widehat{q}_{t} + \frac{(1+\rho)}{\rho+s}E_{t}\widehat{\beta}_{t+1} - \frac{(1-s)(1+c_{1})}{\rho+s}E_{t}\widehat{q}_{t+1}.$$

$$(13)$$

and

$$\widetilde{n}_{t+1}(i) = \tau_2 \widetilde{n}_t(i) - \tau_3 \widetilde{P}_t(i), \tag{14}$$

with the following analytical expressions for the undetermined coefficients  $\tau_2$  and  $\tau_3$ 

$$\tau_2 = \frac{\frac{(1+\rho)c_1(1-s)}{(\rho+s)s}}{1+\gamma + \frac{(1+\rho)c_1}{(\rho+s)s} + \frac{(1-s)^2c_1}{(\rho+s)s} - \tau_2 \frac{(1-s)c_1}{(\rho+s)s} - \tau_1(1-\eta)\left(\frac{(1+\gamma)\theta}{1-\alpha} + \tau_3\frac{(1-s)c_1}{(\rho+s)s}\right)}, \text{ and}$$
(15a)

$$\tau_{3} = \frac{\eta\left(\frac{(1+\gamma)\theta}{1-\alpha} + \tau_{3}\frac{(1-s)c_{1}}{(\rho+s)s}\right)}{1+\gamma + \frac{(1+\rho)c_{1}}{(\rho+s)s} + \frac{(1-s)^{2}c_{1}}{(\rho+s)s} - \tau_{2}\frac{(1-s)c_{1}}{(\rho+s)s} - \tau_{1}(1-\eta)\left(\frac{(1+\gamma)\theta}{1-\alpha} + \tau_{3}\frac{(1-s)c_{1}}{(\rho+s)s}\right)}.$$
(15b)

As for pricing dynamics, the relative price for a firm that was able to set the price optimally in period t is

$$\widetilde{P}_t^*(i) = \widetilde{P}_t^* - \tau_1 \widetilde{n}_t(i), \tag{16}$$

with the following analytical solution for the  $\tau_1$  coefficient (see proof available in Appendix II)

$$\tau_1 = \frac{\gamma(1-\beta\eta)}{(1-\beta\eta\tau_2)\left(1+\frac{\theta(\gamma+\alpha)}{1-\alpha} - \frac{\gamma\beta\eta\tau_3}{1-\beta\eta\tau_2}\right)}.$$
(17)

Fluctuations of the aggregate price level are determined by the following equation (see proof in Appendix II)

$$\widetilde{P}_{t}^{*} = \frac{1 - \beta \eta}{1 + \frac{\theta(\gamma + \alpha)}{1 - \alpha} - \frac{\gamma \beta \eta \tau_{3}}{1 - \beta \eta \tau_{2}}} E_{t} \sum_{j=0}^{\infty} \beta^{j} \eta^{j} \widehat{\psi}_{t+j} + E_{t} \sum_{j=1}^{\infty} \beta^{j} \eta^{j} \pi_{t+j}.$$

$$(18)$$

Combining (18) with  $\widetilde{P}_t^* = \frac{\eta}{1-\eta}\pi_t$  from the Calvo pricing scheme leads to

$$\pi_t = \frac{(1-\eta)(1-\beta\eta)}{\eta \left(1 + \frac{\theta(\gamma+\alpha)}{1-\alpha} - \frac{\gamma\beta\eta\tau_3}{1-\beta\eta\tau_2}\right)} E_t \sum_{j=0}^{\infty} \beta^j \eta^j \widehat{\psi}_{t+j} + \frac{1-\eta}{\eta} E_t \sum_{j=1}^{\infty} \beta^j \eta^j \pi_{t+j},$$

where one can do  $\pi_t - \beta E_t \pi_{t+1}$  to reach the following New Keynesian Phillips curve

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\beta\eta)(1-\eta)}{\eta \left(1 + \frac{\theta(\gamma+\alpha)}{1-\alpha} - \frac{\gamma\beta\eta\tau_3}{1-\beta\eta\tau_2}\right)} \widehat{\psi}_t.$$
 (19)

As pointed out in Casares (2008), hours-clearing wages and search frictions influence inflation dynamics through the additional term in brackets entering the denominator of (19), i.e.  $-\frac{\gamma\beta\eta\tau_3}{1-\beta\eta\tau_2}$ , which might be either positive or negative depending upon convexity ( $c_1 > 0.0$ ) or concavity ( $c_1 < 0.0$ ) on the search costs function introduced above.

The joint dynamics of pricing and employment decisions bring the employment equation (13) and the inflation equation (19). Other equations of the model are the IS curve obtained from the utility function with constant consumption elasticity ( $\sigma$ ) introduced above

$$\widehat{c}_t = E_t \widehat{c}_{t+1} - \frac{1}{\sigma} (R_t - E_t \pi_{t+1}), \tag{20}$$

a monetary policy rule that provides reactions of the nominal interest rate to both the rate of inflation and the real marginal cost with a component of interest-rate smoothing <sup>10</sup>

$$R_{t} = (1 - \mu_{R}) \left[ \mu_{\pi} \pi_{t} + \pi_{y} \widehat{\psi}_{t} \right] + \mu_{R} R_{t-1}, \tag{21}$$

the definition of log deviations of the aggregate real marginal cost,

$$\widehat{\psi}_t = \widehat{w}_t + \widehat{n}_t - \widehat{y}_t + \widehat{h}_t, \tag{22}$$

log fluctuations of output around the steady-state level implied by the production technology (2)

$$\widehat{y}_t = (1 - \alpha) \left( \widehat{n}_t + \widehat{h}_t + z_t \right), \tag{23}$$

log fluctuations of the aggregate real wage consistent with hours-clearing wages and constant elasticities of consumption  $(\sigma)$  and hours  $(\gamma)$  in the utility function

$$\widehat{w}_t = \gamma \widehat{h}_t + \sigma \widehat{c}_t, \tag{24}$$

log fluctuations of unemployment from the loglinearization of  $u_t = 1 - n_t$ 

$$\widehat{u}_t = -\frac{n}{n}\widehat{n}_t,\tag{25}$$

where  $\frac{n}{u}$  is the employment-to-unemployment ratio in steady state; log fluctuations on aggregate vacancies obtained from the aggregation of the loglinear version of (4)

$$\widehat{v}_t = \frac{1}{s}\widehat{n}_{t+1} - \frac{1-s}{s}\widehat{n}_t - \widehat{q}_t, \tag{26}$$

the loglinear probability of posting a successful vacancy obtained from taking logs in the definition of  $q_t = \left(\frac{u_t}{v_t}\right)^{\xi}$ 

$$\widehat{q}_t = \xi \widehat{u}_t - \xi \widehat{v}_t, \tag{27}$$

the log-linearized overall resources constraint

$$\widehat{y}_t = \frac{c}{y}\widehat{c}_t + \frac{c_0(v)^{1+c_1}}{y}(1+c_1)\widehat{v}_t, \tag{28}$$

and the definition of log deviations of the intertemporal discount factor

$$E_t \widehat{\beta}_{t,t+1} = -(R_t - E_t \pi_{t+1}). \tag{29}$$

Thus, the macro model consists of twelve equations, (13) and the set (19)-(29); that provide solution paths for the twelve endogenous variables:  $\hat{y}_t$ ,  $\hat{c}_t$ ,  $R_t$ ,  $\pi_t$ ,  $\hat{\psi}_t$ ,  $\hat{w}_t$ ,  $\hat{n}_{t+1}$ ,  $\hat{h}_t$ ,  $\hat{u}_t$ ,  $\hat{v}_t$ ,  $\hat{q}_t$ , and  $\hat{\beta}_{t,t+1}$ . Firmspecific equations (14) and (16) together with  $\tilde{P}_t^* = \frac{\eta}{1-\eta}\pi_t$  determine paths for firm-level employment and optimal pricing,  $\tilde{n}_{t+1}(i)$  and  $\tilde{P}_t^*(i)$ .

<sup>&</sup>lt;sup>10</sup>Fluctuations of the real marginal cost are proportional to the output gap in canonical New Keynesian models.

## 4 Determinants of firm-specific employment fluctuations

For a baseline calibration of the structural parameters, numerical values are borrowed from the recent literature. Regarding search frictions technology, I follow Walsh (2005) to set a 10% separation rate, s = 0.10, the matching technology share is at  $\xi = 0.5$ , and the function that measures the costs of posting new vacancies is slightly convex and close to the linear case with  $c_1 = 0.25$ . As for the scale parameter  $c_0$ , I set the value  $c_0 = 0.085$  in order to imply that search costs take 1% of total GDP as in Gertler and Trigari (2009).

In the utility function, consumption elasticity is quite standard at  $\sigma = 2$ . Meanwhile, the elasticity on labor disutility is also held at  $\gamma = 2.0$  which leads to a low Frisch labor supply elasticity ( $\gamma^{-1} = 0.5$ ) as suggested by numerous empirical studies (see Altonji ,1986; Card, 1994; and Blundell and Macurdy, 1999). The steady-state rate of intertemporal preference is assumed at  $\rho = 0.005$ , which implies a 2% annualized real interest rate in steady state.

The production function (2) incorporates the parameters of the real business cycle literature. Therefore, we take the usual labor-share coefficient,  $\alpha = 0.36$ , with the AR(1) technology shock,  $z_t$ , characterized by a 95% serial correlation and a 0.7% standard deviation of the innovations. Such calibration results in output volatility similar to that observed in US business cycle.<sup>11</sup>

Pricing conditions are governed by Calvo probability of non-optimal pricing set at  $\eta=2/3$ , which leads to having an average frequency of posting optimal prices equal to nine months, as suggested by the empirical evidence reported in a recent paper by Nakamura and Steinsson (2009).<sup>12</sup> The Dixit-Stiglitz demand elasticity is  $\theta=11.0$  to imply a 10% mark-up in steady state. Finally, the Taylor-type monetary policy rule is implemented with the original coefficients suggested by Taylor (1993),  $\mu_{\pi}=1.5$  and  $\mu_{y}=0.5/4$ , together with a high interest-rate smoothing coefficient,  $\mu_{R}=0.8$ .

Recalling (17), (15a) and (15b), the baseline calibration gives rise to the following numbers for  $\tau_1$ ,  $\tau_2$  and  $\tau_3$ 

$$\tau_1 = 0.04$$
,  $\tau_2 = 0.74$ , and  $\tau_3 = 2.32$ ,

which bring a weak and negative dependence of employment on optimal pricing (setting  $\tau_1 = 0.04$  in equation 16), a moderate employment inertia (setting  $\tau_2 = 0.74$  in equation 14), and a strong negative influence of the relative price on next period's employment (setting  $\tau_3 = 2.32$  in equation 14).

The model can illustrate the determinants of firm-specific employment fluctuations. In particular, it could be examined how the values of  $\tau_2$  and  $\tau_3$  entering (14) are influenced by changes in the numerical values assigned to structural parameters. Due to the rich interactions embedded in the model, employment fluctuations depend upon a variety of elements such as labor market rigidities,

<sup>&</sup>lt;sup>11</sup>The standard deviation of output in the model is 0.98% while in US data plotted in Figure 1 is 0.93%.

<sup>&</sup>lt;sup>12</sup>Bils and Klenow (2004) find a significantly shorter price duration of around 5 months.

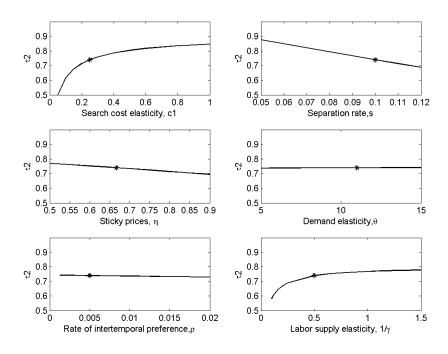


Figure 3: Determinants of inertia ( $\tau_2$ ) in firm-specific employment fluctuations. The position of '\*' corresponds to the baseline calibration.

pricing conditions and household's preferences. Looking at the analytical solutions (15a) and (15b),  $\tau_2$  and  $\tau_3$  receive some influence from variations in search costs elasticity  $(c_1)$ , the separation rate (s), price rigidities  $(\eta)$ , Dixit-Stiglitz demand elasticity  $(\theta)$ , the rate of intertemporal preference  $(\rho)$ , labor supply elasticity  $(\gamma^{-1})$ , and the production technology coefficient  $(\alpha)$ . Except for  $\alpha$ , the sensitivity of these structural parameters on the persistence and variability of firm-specific employment is examined next. Figure 3 shows the effects of moving the structural parameters on the inertia coefficient  $\tau_2$  (which collects the response of next period employment to current employment in relation 14). The elements of the model that bring frictions to the labor market, i.e. the search costs elasticity and the job destruction rate, are the most influential on the determination of persistence of firm-specific employment fluctuations. If the labor market presents higher search costs (increase in  $c_1$ ) there is more inertia on employment dynamics. Intuitively, since hiring is more costly new jobs last longer. On the other hand, a faster job destruction (an increase in s) reduces firm's employment inertia. A high separation rate makes hiring needs and vacancy postings more frequent, which drives firm-specific employment less dependent from the past. Interestingly, neither sticky prices nor demand elasticity shape significantly the inertial component of firm's employment (see central cells of Figure

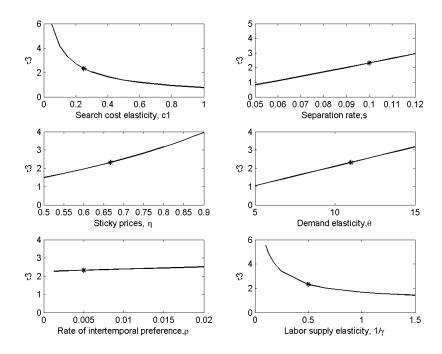


Figure 4: Determinants of sensitivity to relative prices  $(\tau_3)$  in firm-specific employment fluctuations. The position of '\*' corresponds to the baseline calibration.

3). Finally, labor supply elasticity plays some role on firm-specific employment. A higher labor supply elasticity increases employment inertia because wages loose variability and the incentives for employment changes are smaller.

The influence of relative prices on firm-specific employment is measured by the  $\tau_3$  coefficient in (14). Figure 4 shows numerical values of this coefficient at some intervals of the structural parameters. As observed there, a rise in price stickiness  $(\eta)$ , the job destruction rate (s), or the Dixit-Stiglitz demand elasticity  $(\theta)$  increase  $\tau_3$  making relative employment more sensitive on relative prices. These three elements augment the (negative) responsiveness of job creation to relative prices. Thus, stickier prices lead to higher dispersion on prices, output and also a greater differentiation in hiring decisions across firms. Similarly, a high Dixit-Stiglitz elasticity requires larger supply-side changes in response to relative prices. By contrast, any increase in labor supply elasticity  $(\gamma^{-1})$  or search cost elasticity  $(c_1)$  reduces the impact of relative prices on relative employment because the former cuts the marginal benefit of hiring (due to less real wage variability) and the latter raises the marginal cost of hiring (due to higher costs on vacancy posting).

Variability of firm-specific employment is the result of combining its inertial component with the

sensitivity to relative prices. A direct measure of firm-specific volatility can be obtained by computing the unconditional standard deviation of relative employment. Recalling Calvo pricing and the firm-level dynamic equations (14) and (16), one can obtain the following expression for the standard deviation of relative employment<sup>13</sup>

$$std(\widetilde{n}) = \sqrt{\frac{\tau_3^2 + 2\tau_2\tau_3^2\eta(1-\eta\tau_2)^{-1}}{1-\tau_2^2 - 2\tau_1\tau_2\tau_3(1-\eta)(1-\eta\tau_2)^{-1}}} std\left(\widetilde{P}\right).$$

Using Woodford's (2003, pages 694-696) result on Calvo-type staggered pricing, the unconditional standard deviation of relative prices is approximated by the following expression

$$std\left(\widetilde{P}\right) = \sqrt{\frac{\eta}{(1-\eta)^2}} std\left(\pi\right),$$

where  $std(\pi)$  is the unconditional standard deviation of economy-wide inflation. The last two equations can be used to obtain numerically the standard deviation of firm-specific employment depending on  $\tau_2$ ,  $\tau_3$ ,  $\eta$  and the standard deviation of inflation. Figure 5 provides the results.

As dhown in Figure 5, increasing search costs implies a significant reduction in the standard deviation of relative employment as hiring is more costly. By contrast, the separation rate affects in the opposite direction. A higher separation rate induces greater employment variability because job duration is shorter and workers change jobs more often. Price stickiness and demand elasticity also shape upwards employment volatility (see central plots of Figure 5). When price rigidities are increased, firm-specific employment gains volatility due to the greater variability of firm-specific output, hours and wages. This finding is reversed when nominal rigidities are so severe that overall prices and inflation have little volatility (as the value of  $\eta$  approaches its right-side end in Figure 5). At a minor extent, relative employment volatility also rises with a higher demand elasticity that would increase dispersion on the quantities of output, hours and hiring policies across firms. Finally, the structural parameters that bring household's preferences have some impact on relative employment volatility. Thus, a higher labor supply elasticity would cut employment volatility (due to less wage variability) whereas a higher rate of intertemporal preference would slightly increase the size of employment fluctuations.

## 5 A New Keynesian analysis of industrial employment fluctuations

Let us define one industry as a group of firms. Thus, industrial employment can be obtained as the average employment across all firms that belong to that particular industry. Using the employment dynamic equation (14),  $\tilde{n}_{t+1}(i) = \tau_2 \tilde{n}_t(i) - \tau_3 \tilde{P}_t(i)$ , the fluctuations of industrial employment can be

<sup>&</sup>lt;sup>13</sup>See Appendix III for the proof.

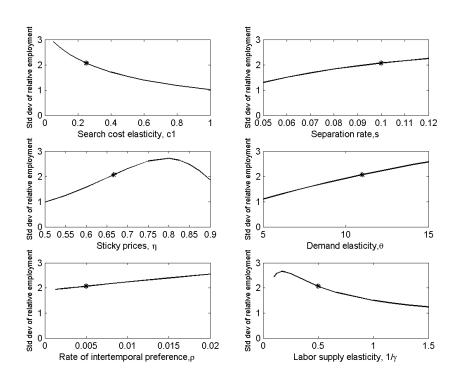


Figure 5: Determinants of volatility in relative employment. The position of '\*' corresponds to the baseline calibration.

obtained as the average fluctuation of all fims that belong to that industry. The number of firms per industry determines the size of each industry.

Using a computer, I can create artificial series taking random draws from a normal distribution to generate the technology shock. For each observation, I assume that there are 10,000 firms in the economy, all of them pending from the Calvo market signal that determines whether the firm is able to set the optimal price or not. Thus, 10,000 Calvo signals can be randomly generated from a (0,1) uniform distribution. Every firm receives a single draw. Recalling the baseline Calvo probability  $\eta = 2/3$ , if the number received is higher than 2/3, the firm could price optimally according to (16)

$$\widetilde{P}_t^*(i) = \widetilde{P}_t^* - \tau_1 \widetilde{n}_t(i) = \frac{\eta}{1-\eta} \pi_t - \tau_1 \widetilde{n}_t(i).$$

Otherwise, the firm would raise its price by the steady-state rate of inflation which would leave its relative price as follows

$$\widetilde{P}_t(i) = \widetilde{P}_{t-1}(i) - \pi_t.$$

The outcome on relative prices will be inserted in (14) to calculate next period's relative employment of the firm. The aggregation of all relative employments of the firms that belong to the same industry would yield the measure of relative industrial employment. As a introductory exercise, it may be interesting to take a look at Figure 6. It is a graphical display of a simulated 140-quarter economy where there are industrial employment fluctuations in four possible types of industries: very small single-firm industries (10,000 industries), small 10-firm industries (1,000 industries), mediumsize 100-firm industries (100 industries), and large 1000-firm industries (10 industries). Obviously, the large number of industries makes their employment fluctuations overlap in the plot. The "smoothing action" becomes clear; employment volatility falls as more firms are collected in one industry (from top to bottom in Figure 6). More firms per industry, deeper smoothing when making the industrial average and less employment variability as a result. Hence, the scale of the vertical axis shrinks as the industrial size is raised, i.e. industrial variability falls. By contrast, employment in smaller industries brings more markedly fluctuations. In the example displayed in Figure 6, technology improved in the second half of the sample. The most severe changes in employment occur in 1-firm industries that cut employment by almost 50% during that technology boom. This situation represents the evolution of employment in those firms that were not able to set the optimal price during the periods of the technological innovation. Subsequently, they were posting high relative prices which made them be countercyclical as they cut production and employment. The bottom plot of Figure 6 shows fluctuations of employment when there are only 10 industries in the economy (holding 1,000 firms each industry). In this case, we can see how most industries increase employment during the technology-driven expansion (periods 70 through 100). The aggregation procedure makes employment fluctuations be more procyclical.

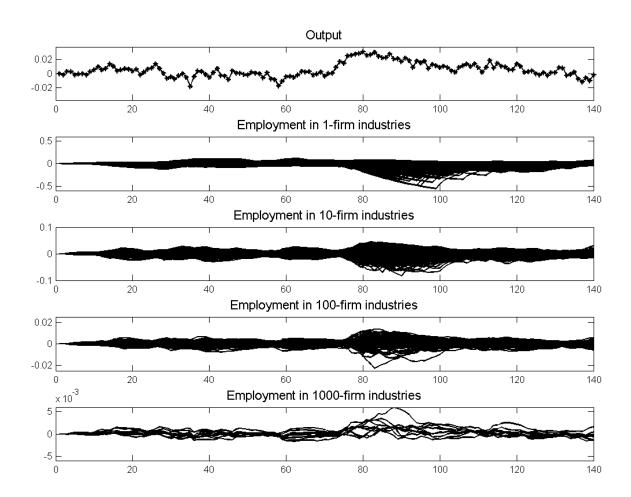


Figure 6: Output and industrial employment fluctuations in one simulated economy with different industrial sizes.

Next, we will examine employment fluctuations both at aggregate and industrial level (using the alternative industrial sizes introduced above). The results will be compared with the piece of empirical evidence of the US economy discussed in Section 2. The economy contains 10,000 firms that are assigned to some industry under four industrial sizes 1/10000, 10/10000, 100/10000 and 1000/10000. Computer-generated technology shocks shape business cycles for 140 quarters. The simulation exercise was done 500 times and the means of both the average standard deviations (relative to output) and the average coefficients of correlation with output were calculated across industries for the four alternative industrial sizes. I also computed the standard deviations of aggregate employment (relative to output) and its coefficient of correlation with output. Table 2 displays the results:

Table 2. Model-based employment fluctuations.

Aggregate Employment	
Standard deviation, relative to output:	0.04
Correlation with output:	0.78

#### Industrial Employment

	Industrial size			
	1/10000	10/10000	100/10000	1000/10000
Average standard deviation, relative to output:	2.35	0.76	0.25	0.09
Average correlation with output:	0.06	0.05	0.12	0.33

After reviewing Table 2, it is clear that the decision on whether to study employment fluctuations from aggregate employment or from industrial employment is relevant for the business cycle analysis. Thus, the industrial disaggregation of employment fluctuations increases employment volatility and reduces procyclicality compared to the standard treatment of aggregate employment fluctuations. Using aggregate employment, the results indicate that industrial employment has little variability (0.04 relative to output) and a high cyclical correlation (0.77). These numbers are in deep contrast with US data (see Table 1 for the comparison). That is the case of the New Keynesian model with search frictions and no firm differentiation. Then, employment volatility is low—as documented by Shimer (2005a)'s puzzle—15 and there is an excessive correlation with output due to the lack of industrial segmentation.

If the approach switches to using industrial employment, the New Keynesian model provides a better fit to the data. As a general finding, industrial employment is more volatile and less procyclical

<sup>&</sup>lt;sup>14</sup>The initial 40 observations are discarded in order to make a random start for the calculation of second-moment statistics.

<sup>&</sup>lt;sup>15</sup>Shimer (2005a) claims that the standard search and matching model is not able to generate the observed volatility of vacancies relative to unemployment. The standard deviation of the vacancy-unemployment ratio is around 20 times larger than the standard deviation of productivity in the US economy and the model brings a much lower number.

than aggregate employment. Moreover, small-sized industries increase employment standard deviation and reduce the correlation with output. For example, Table 2 reports that in an economy that had industries with just one firm the average standard deviation of industrial employment is 2.39 times that of output, whereas the average cyclical correlation is positive and close to zero (0.06). As the industrial size is raised the standard deviation of industrial employment falls and the correlation with output tends to increase. The limit case would be a single industry formed by all (10,000) firms in which numbers would coincide with those obtained for aggregate employment, showing very little employment volatility and a strong correlation with output.

Table 3. Model-based employment fluctuations with less price stickiness ( $\eta = 0.50$ ).

	With loss piles stidinges (i) (i)
Aggregate Employment	
Standard deviation, relative to output:	0.05
Correlation with output:	0.76

Industrial Employment

	Industrial size			
	1/10000	10/10000	100/10000	1000/10000
Average standard deviation, relative to output:	1.09	0.36	0.12	0.06
Average correlation with output:	0.08	0.12	0.30	0.62

As a final exercise, employment fluctuations in the model have been examined under weaker price rigidities. To do that, I did repeat the simulation procedure setting the Calvo probability at a lower value,  $\eta = 0.50$ , which shortens the average time without optimal pricing from three to two quarters. Table 3 provides statistics on employment fluctuations under less price rigidities. The results indicate that reducing price stickiness has two effects on industrial employment fluctuations. First, volatility falls due to much less variability in relative prices (firms tend to set more similar prices with less setting rigidities). Secondly, the correlation between industrial employment and output rises, especially in cases with large industries.

### 6 Conclusions

US employment data show that the "smoothing action" applied by the aggregation procedure reduces employment volatility and increases the correlation with output. Therefore, the average volatility of industrial employment is higher than that of aggregate employment, while the correlation with output turns lower taking data of industrial employment than using the series of aggregate employment. This paper has shown that the "smoothing action" found in the data is also present in employment fluctuations obtained from model-based simulations. Actually, the business cycle analysis of employment

from an industrial perspective in a New Keynesian model is more successful than the one made with the standard approach that ignores firm-specific employment dynamics. Such conventional New Keynesian model with search frictions is not capable to replicate that degree of employment dispersion which goes in line with Shimer (2005a)'s puzzle on search models. Besides, it predicts an excessive cyclical correlation of employment. Nevertheless, the modified model with firm-level employment differentiation provides a better fit to US data. Then, the variability of average industrial employment obtained depends on the industrial size assumed in a range that goes from lower volatility than in US data (with small industrial size) to higher volatility than in US data (assuming large industrial size). Reducing price stickiness lowers industrial employment volatility and increases the correlation with output.

Appendix I. US industrial employment data.

Volatility and cyclicality of US industrial employment ranked from high to low volatility, 1994-2008

Employment services (0.0339)  Computer systems design and related services (0.0111)	St.Deviation relative to output 4.98 4.97 4.05	Correlation with output 0.88 0.60
	4.98 4.97	0.88
	4.97	
Computer systems design and related services (0.0111)		0.60
	4.05	
Telecommunications (0.0119)		0.39
Home health care services $(0.0076)$	3.80	-0.40
Computer and electronic products (0.0163)	3.62	0.45
Securities, commodity contracts, investments (0.0077)	3.20	0.57
Construction of buildings (0.0163)	3.11	0.78
Motor vehicles and parts $(0.0121)$	3.07	0.73
Air transportation (0.0056)	3.06	0.37
Wood products (0.0059)	3.05	0.78
Furniture and related products (0.0063)	2.84	0.88
Machinery (0.0138)	2.81	0.49
Couriers and messengers (0.0059)	2.75	0.25
Primary metals (0.0058)	2.74	0.63
Mining (0.0059)	2.69	0.03
Electrical equipment and appliances (0.0054)	2.64	0.57
Accounting and bookkeeping services (0.0086)	2.57	0.47
Specialty trade contractors (0.0427)	2.57	0.80
Fabricated metal products (0.0168)	2.55	0.67
Heavy and civil engineering construction (0.0094)	2.51	0.67
Electronics and appliance stores (0.0054)	2.46	0.64
Apparel (0.0048)	2.39	0.35
Management and technical consulting services (0.0072)	2.23	0.25
Nonmetallic mineral products (0.0054)	2.09	0.77
Architectural and engineering services (0.0126)	2.02	0.52

Note: Numbers in parenthesis indicate the weights used to calculate the average. These weights are defined as the ratio between the sample mean of industrial employment and Total Private Employment.

(Cont.). Volatility and cyclicality of US industrial employment ranked from high to low volatility, 1994-2008

	St.Deviation	Correlation	
	relative to output	with output	
Warehousing and storage (0.0055)	2.00	0.74	
Business support services (0.0078)	1.95	0.35	
Truck transportation (0.0141)	1.93	0.75	
Plastic and rubber products (0.0090)	1.89	0.74	
Motion picture and sound recording industries (0.0038)	1.84	-0.04	
Building material and garden supply stores $(0.0120)$	1.82	0.57	
Publishing industries except Internet (0.0098)	1.80	0.62	
Wholesale trade of durable goods $(0.0318)$	1.79	0.71	
Support activities for transportation (0.0054)	1.74	0.70	
Sporting goods, hobby, book, and music stores (0.0067)	1.74	0.47	
Credit intermediation and related activities $(0.0275)$	1.74	0.27	
Clothing and clothing accessories stores $(0.0139)$	1.73	0.79	
Accommodation (0.0186)	1.72	0.76	
Department stores $(0.0171)$	1.71	0.36	
Printing and related support activities (0.0077)	1.67	0.67	
Furniture and home furnishings stores (0.0055)	1.62	0.77	
Management of companies and enterprises (0.0183)	1.51	0.76	
Electronic markets and agents and brokers (0.0069)	1.47	0.52	
Arts, entertainment, and recreation (0.0182)	1.29	0.44	
Rental and leasing services (0.0065)	1.20	0.69	
Paper and paper products (0.0058)	1.14	0.44	
Real estate (0.0140)	1.12	0.57	
Utilities (0.0063)	1.08	-0.26	
Services to buildings and dwellings (0.0165)	1.08	0.74	
Motor vehicle and parts dealers (0.0189)	1.03	0.49	
Health and personal care stores (0.0201)	1.02	0.41	

Note: Numbers in parenthesis indicate the weights used to calculate the average. These weights are defined as the ratio between the sample mean of industrial employment and Total Private Employment.

(Cont.). Volatility and cyclicality of US industrial employment ranked from high to low volatility, 1994-2008

, , , , , , , , , , , , , , , , , , , ,	1 0	0 0/
	St.Deviation	Correlation
	relative to output	with output
Social assistance (0.0095)	1.02	0.20
Membership associations and organizations $(0.0282)$	0.97	-0.38
Repair and maintenance (0.0125)	0.95	0.25
Educational services (0.0261)	0.89	-0.56
Plastic and rubber products (0.0095)	0.88	0.24
Insurance carriers and related activities $(0.0232)$	0.84	0.35
Chemicals (0.0098)	0.84	0.48
Legal services (0.0113)	0.78	0.35
Wholesale trade of nondurable goods $(0.0211)$	0.73	0.60
Food and beverage stores $(0.0302)$	0.73	0.37
Food services and drinking places (0.0876)	0.62	0.38
Hospitals (0.0428)	0.55	-0.58
Nursing and residential care facilities $(0.0276)$	0.55	-0.71
Food manufacturing (0.0159)	0.52	-0.17
Personal and laundry services (0.0129)	0.39	0.21
Offices of physicians (0.0197)	0.36	-0.46
<b>117</b> . 1 . 1	1.67	0.95
Weighted averages	1.67	0.35

Note: Numbers in parenthesis indicate the weights used to calculate the average. These weights are defined as the ratio between the sample mean of industrial employment and Total Private Employment.

Appendix II. Derivation of employment and price equations at both firm-specific and aggregate levels.

Optimality in firm-level hiring decisions is determined by equations (11) and (12), subject to the employment accumulation constraint (4). The substitution of both (12) and the equation correspondent to (12) for period t + 1 in equation (11) yields

$$E_{t}\beta_{t,t+1}\left[\left(1+\gamma\right)\left(h_{t+1}^{d}(i)\frac{W_{t+1}(i)}{P_{t+1}}\right)\right] = \frac{c_{0}(1+c_{1})(v_{t}(i))^{c_{1}}}{q_{t}} - E_{t}\beta_{t,t+1}\left[\frac{(1-s)c_{0}(1+c_{1})(v_{t+1}(i))^{c_{1}}}{q_{t+1}}\right],\tag{A1}$$

which can be loglinearized to reach

$$E_{t}\widehat{h}_{t+1}^{d}(i) + E_{t}\left(\widehat{W}_{t+1}(i) - \widehat{W}_{t+1}\right) + E_{t}\widehat{w}_{t+1} = \frac{1+\rho}{\rho+s}\left[c_{1}\widehat{v}_{t}(i) - \widehat{q}_{t} - E_{t}\widehat{\beta}_{t+1}\right] - \frac{1-s}{\rho+s}\left[c_{1}E_{t}\widehat{v}_{t+1}(i) - E_{t}\widehat{q}_{t+1}\right],$$
(A2)

where  $\widehat{w}_{t+1} = \widehat{W}_{t+1} - \widehat{P}_{t+1}$  is the log deviation of the aggregate real wage in period t+1 from its steady-state level, and both  $\rho$  and s are the steady-state rates of discount and exogenous separation.<sup>16</sup> Meanwhile, taking logs in (7) and subtracting the aggregated log of the nominal wage vield<sup>17</sup>

$$\widetilde{W}_t(i) = -\gamma \widetilde{n}_t(i) - \frac{\theta \gamma}{1 - \alpha} \widetilde{P}_t(i), \tag{A3}$$

where tilde-topped variables denote relative variables measured as log deviations with respect to aggregate levels, for example,  $\widetilde{W}_t(i) = \widehat{W}_t(i) - \widehat{W}_t = \log W_t(i) - \log W_t$ . Similarly, taking logs in (5) and subtracting the aggregated log of hours leads to the following relative demand for hours

$$E_t \widehat{h}_{t+1}^d(i) = -\widetilde{n}_t(i) - \frac{\theta}{1 - \alpha} \widetilde{P}_t(i) + E_t \widehat{h}_{t+1}. \tag{A4}$$

Moving (A3) and (A4) one period forward leads to expressions for  $E_t \widehat{h}_{t+1}^d(i)$  and  $E_t \widetilde{W}_{t+1}(i) = E_t \left(\widehat{W}_{t+1}(i) - \widehat{W}_{t+1}\right)$  such as

$$E_t \widehat{h}_{t+1}^d(i) = -\widetilde{n}_{t+1}(i) - \frac{\theta}{1-\alpha} E_t \widetilde{P}_{t+1}(i) + E_t \widehat{h}_{t+1}, \text{ and}$$

$$E_t \left(\widehat{W}_{t+1}(i) - \widehat{W}_{t+1}\right) = -\gamma \widetilde{n}_{t+1}(i) - \frac{\theta \gamma}{1-\alpha} E_t \widetilde{P}_{t+1}(i),$$

which can be inserted in the loglinear optimality condition (A2) to obtain

$$-(1+\gamma)E_{t}\widetilde{n}_{t+1}(i) - \frac{(1+\gamma)\theta}{1-\alpha}E_{t}\widetilde{P}_{t+1}(i) + E_{t}\widehat{h}_{t+1} + E_{t}\widehat{w}_{t+1} = \frac{1+\rho}{\rho+s} \left[c_{1}\widehat{v}_{t}(i) - \widehat{q}_{t} - E_{t}\widehat{\beta}_{t+1}\right] - \frac{1-s}{\rho+s} \left[c_{1}E_{t}\widehat{v}_{t+1}(i) - E_{t}\widehat{q}_{t+1}\right].$$
 (A5)

Log deviations of current vacancies,  $\hat{v}_t(i)$ , determine those on the number of new employees through the employment accumulation equation (4). Loglinearizing (4) and rearranging terms, it is reached

$$\widehat{v}_{t}(i) = \frac{1}{s}\widehat{n}_{t+1}(i) - \frac{1-s}{s}\widehat{n}_{t}(i) - \widehat{q}_{t} = \frac{1}{s}(\widetilde{n}_{t+1}(i) + \widehat{n}_{t+1}) - \frac{1-s}{s}(\widetilde{n}_{t}(i) + \widehat{n}_{t}) - \widehat{q}_{t}.$$
 (A6)

Both (A6) and its corresponding expression one period ahead for  $E_t \hat{v}_{t+1}(i)$  are substituted in (A5) to yield

$$-\left[\left(1+\gamma\right)+\frac{(1+\rho)c_{1}}{(\rho+s)s}+\frac{(1-s)^{2}c_{1}}{(\rho+s)s}\right]\widetilde{n}_{t+1}(i)+\frac{(1+\rho)c_{1}(1-s)}{(\rho+s)s}\widetilde{n}_{t}(i)+\frac{(1-s)c_{1}}{(\rho+s)s}E_{t}\widetilde{n}_{t+2}(i)$$

$$-\frac{(1+\gamma)\theta}{1-\alpha}E_{t}\widetilde{P}_{t+1}(i)+E_{t}\widehat{h}_{t+1}+E_{t}\widehat{w}_{t+1}=$$

$$\left(\frac{(1+\rho)c_{1}}{(\rho+s)s}+\frac{(1-s)^{2}c_{1}}{(\rho+s)s}\right)\widehat{n}_{t+1}-\frac{(1+\rho)c_{1}(1-s)}{(\rho+s)s}\widehat{n}_{t}-\frac{(1-s)c_{1}}{(\rho+s)s}E_{t}\widehat{n}_{t+2}-\frac{(1+\rho)(1+c_{1})}{\rho+s}\widehat{q}_{t}-\frac{(1+\rho)}{\rho+s}E_{t}\widehat{\beta}_{t+1}+\frac{(1-s)(1+c_{1})}{\rho+s}E_{t}\widehat{q}_{t+1}.$$
(A7)

<sup>16</sup> It should be noticed that in steady state  $\beta_{t,t+1} = (1+\rho)^{-1}$ .

<sup>&</sup>lt;sup>17</sup>This results was already reported in Casares (2008) without any published proof.

This last expression implies a certain relationship between firm-specific employment and pricing. In particular, next period's relative employment is negatively related to the firm's expected relative price.<sup>18</sup> In addition, there is some inertial pattern for employment accumulation that makes next period's hiring be dependant upon the current level of employment, with a positive sign of influence. Therefore, it is fair to guess that relative employment dynamics are governed by one expression of the following kind

$$\widetilde{n}_{t+1}(i) = \tau_2 \widetilde{n}_t(i) - \tau_3 \widetilde{P}_t(i), \tag{A8}$$

where  $\tau_2$  and  $\tau_3$  are undetermined coefficients to be found below. Using (A8) to infer  $E_t \tilde{n}_{t+2}(i)$ , it is obtained

$$E_t \widetilde{n}_{t+2}(i) = \tau_2 \widetilde{n}_{t+1}(i) - \tau_3 E_t \widetilde{P}_{t+1}(i),$$

which after being plugged in (A7) results in

$$-\left[\left(1+\gamma\right)+\frac{(1+\rho)c_{1}}{(\rho+s)s}+\frac{(1-s)^{2}c_{1}}{(\rho+s)s}-\tau_{2}\frac{(1-s)c_{1}}{(\rho+s)s}\right]\widetilde{n}_{t+1}(i)+\frac{(1+\rho)c_{1}(1-s)}{(\rho+s)s}\widetilde{n}_{t}(i)$$

$$-\left(\frac{(1+\gamma)\theta}{1-\alpha}+\tau_{3}\frac{(1-s)c_{1}}{(\rho+s)s}\right)E_{t}\widetilde{P}_{t+1}(i)+E_{t}\widehat{h}_{t+1}^{d}+E_{t}\widehat{w}_{t+1}=$$

$$\left(\frac{(1+\rho)c_{1}}{(\rho+s)s}+\frac{(1-s)^{2}c_{1}}{(\rho+s)s}\right)\widehat{n}_{t+1}-\frac{(1+\rho)c_{1}(1-s)}{(\rho+s)s}\widehat{n}_{t}-\frac{(1-s)c_{1}}{(\rho+s)s}E_{t}\widehat{n}_{t+2}-\frac{(1+\rho)(1+c_{1})}{\rho+s}\widehat{q}_{t}-\frac{(1+\rho)}{\rho+s}E_{t}\widehat{\beta}_{t+1}+\frac{(1-s)(1+c_{1})}{\rho+s}E_{t}\widehat{q}_{t+1}.$$

$$(A9)$$

Moreover, it is assumed that firm-specific relative optimal pricing is connected to economy-wide relative optimal prices and relative employment through the following equation<sup>19</sup>

$$\widetilde{P}_t^*(i) = \widetilde{P}_t^* - \tau_1 \widetilde{n}_t(i), \tag{A10}$$

which implies that the relative optimal price expected for next period is  $E_t \widetilde{P}_{t+1}^*(i) = E_t \widetilde{P}_{t+1}^* - \tau_1 \widetilde{n}_{t+1}(i)$  and that the expected relative price  $E_t \widetilde{P}_{t+1}(i)$  that appears in (A9) is<sup>20</sup>

$$E_{t}\widetilde{P}_{t+1}(i) = \eta \left(\log P_{t}(i) - E_{t} \log P_{t+1}\right) + (1-\eta)E_{t}\widetilde{P}_{t+1}^{*}(i) = \eta \left(\widetilde{P}_{t}(i) - E_{t}\pi_{t+1}\right) + (1-\eta)\left(E_{t}\widetilde{P}_{t+1}^{*} - \tau_{1}\widetilde{n}_{t+1}(i)\right),$$
(A11)

where  $E_t \pi_{t+1} = E_t \log P_{t+1} - \log P_t$  is expected next period's inflation. Calvo pricing also implies  $\widetilde{P}_t^* = \frac{\eta}{1-\eta} \pi_t$  and, subsequently,  $E_t \widetilde{P}_{t+1}^* = \frac{\eta}{1-\eta} E_t \pi_{t+1}$  that can be used in (A11) to yield

$$E_t \widetilde{P}_{t+1}(i) = \eta \widetilde{P}_t(i) - \tau_1 (1 - \eta) \widetilde{n}_{t+1}(i),$$

<sup>&</sup>lt;sup>18</sup>The reader can see that both  $E_t \widetilde{n}_{t+1}(i)$  and  $E_t \widetilde{P}_{t+1}(i)$  are premultiplied by a negative sign on the left hand side of (A7).

<sup>&</sup>lt;sup>19</sup>A high level of employment would reduce the hours-clearing nominal wage as indicated by (7), which would bring lower real marginal costs and a lower optimal price set by the firm.

<sup>&</sup>lt;sup>20</sup>Recalling the aggregation of Calvo-type sticky prices  $P_t = \left[ (1-\eta) \left[ P_t^* \right]^{1/(1-\theta)} + \eta \left[ (1+\pi) P_{t-1} \right]^{1/(1-\theta)} \right]^{1-\theta}$ , where  $P_t^* = \int P_t^*(i) di$  is the average optimal price.

which it is inserted in (A9) to obtain

$$-\left[\left(1+\gamma\right)+\frac{(1+\rho)c_{1}}{(\rho+s)s}+\frac{(1-s)^{2}c_{1}}{(\rho+s)s}-\tau_{2}\frac{(1-s)c_{1}}{(\rho+s)s}-\tau_{1}(1-\eta)\left(\frac{(1+\gamma)\theta}{1-\alpha}+\tau_{3}\frac{(1-s)c_{1}}{(\rho+s)s}\right)\right]\widetilde{n}_{t+1}(i)+\frac{(1+\rho)c_{1}(1-s)}{(\rho+s)s}\widetilde{n}_{t}(i)$$

$$-\eta\left(\frac{(1+\gamma)\theta}{1-\alpha}+\tau_{3}\frac{(1-s)c_{1}}{(\rho+s)s}\right)\widetilde{P}_{t}(i)+E_{t}\widehat{h}_{t+1}^{d}+E_{t}\widehat{w}_{t+1}=\left(\frac{(1+\rho)c_{1}}{(\rho+s)s}\right)\widehat{n}_{t+1}-\frac{(1+\rho)c_{1}(1-s)}{(\rho+s)s}\widehat{n}_{t}-\frac{(1-s)c_{1}}{(\rho+s)s}E_{t}\widehat{n}_{t+2}-\frac{(1+\rho)(1+c_{1})}{\rho+s}\widehat{q}_{t}-\frac{(1+\rho)}{\rho+s}E_{t}\widehat{\beta}_{t+1}+\frac{(1-s)(1+c_{1})}{\rho+s}E_{t}\widehat{q}_{t+1}.$$
(A12)

The aggregation of (A12) over the continuum of firms leads to the macro relationship that determines employment fluctuations

$$\left(\frac{(1+\rho)c_1}{(\rho+s)s} + \frac{(1-s)^2c_1}{(\rho+s)s}\right)\widehat{n}_{t+1} = E_t\widehat{h}_{t+1}^d + E_t\widehat{w}_{t+1} + \frac{(1+\rho)c_1(1-s)}{(\rho+s)s}\widehat{n}_t + \frac{(1-s)c_1}{(\rho+s)s}E_t\widehat{n}_{t+2} + \frac{(1+\rho)(1+c_1)}{\rho+s}\widehat{q}_t + \frac{(1+\rho)}{\rho+s}E_t\widehat{\beta}_{t+1} - \frac{(1-s)(1+c_1)}{\rho+s}E_t\widehat{q}_{t+1}, \tag{A13}$$

which is equation (13) in the main text. Another consequence of (A12) is that the analytical expressions for the undetermined coefficients  $\tau_2$  and  $\tau_3$ , consistent with the assumed relationship (A8), are

$$\tau_{2} = \frac{\frac{(1+\rho)c_{1}(1-s)}{(\rho+s)s}}{1+\gamma+\frac{(1+\rho)c_{1}}{(\rho+s)s}+\frac{(1-s)^{2}c_{1}}{(\rho+s)s}-\tau_{2}\frac{(1-s)c_{1}}{(\rho+s)s}-\tau_{1}(1-\eta)\left(\frac{(1+\gamma)\theta}{1-\alpha}+\tau_{3}\frac{(1-s)c_{1}}{(\rho+s)s}\right)}, \text{ and}$$

$$\tau_{3} = \frac{\eta\left(\frac{(1+\gamma)\theta}{1-\alpha}+\tau_{3}\frac{(1-s)c_{1}}{(\rho+s)s}\right)}{1+\gamma+\frac{(1+\rho)c_{1}}{(\rho+s)s}+\frac{(1-s)^{2}c_{1}}{(\rho+s)s}-\tau_{2}\frac{(1-s)c_{1}}{(\rho+s)s}-\tau_{1}(1-\eta)\left(\frac{(1+\gamma)\theta}{1-\alpha}+\tau_{3}\frac{(1-s)c_{1}}{(\rho+s)s}\right)},$$

that respectively become expressions (15a) and (15b) in the main text.

For the price dynamics equation, we start by making a log-linear approximation to (8) that renders

$$\widehat{P}_t^*(i) = (1 - \beta \eta) E_t^{\eta} \sum_{j=0}^{\infty} \beta^j \eta^j \left( \widehat{P}_{t+j} + \widehat{\psi}_{t+j}(i) \right), \tag{A14}$$

where log deviations from steady state of firm-specific real marginal costs can be obtained from (10) as follows

$$\widehat{\psi}_{t+j}(i) = \widehat{W}_{t+j}(i) - \widehat{P}_{t+j} + \widehat{n}_{t+j}(i) - \widehat{y}_{t+j}(i) + \widehat{h}_{t+j}^{d}(i).$$
(A15)

Subtracting log deviations of the aggregate real marginal cost,  $\hat{\psi}_{t+j} = \int \hat{\psi}_{t+j}(i)di$ , from (A15), it yields

$$\widehat{\psi}_{t+j}(i) = \widehat{\psi}_{t+j} + \widetilde{W}_{t+j}(i) + \widetilde{n}_{t+j}(i) - \widetilde{y}_{t+j}(i) + \widetilde{h}_{t+j}^d(i),$$

where using (A3) for  $\widetilde{W}_{t+j}(i)$ , the log-linear version of (1) for  $\widetilde{y}_{t+j}(i)$ , and (A4) for  $\widetilde{h}_{t+j}^d(i)$ , we get

$$\widehat{\psi}_{t+j}(i) = \widehat{\psi}_{t+j} - \gamma \widetilde{n}_{t+j}(i) - \frac{\theta(\gamma + \alpha)}{1 - \alpha} \widetilde{P}_{t+j}(i). \tag{A16}$$

Using (A16) in (A14), it is obtained

$$\widehat{P}_{t}^{*}(i) = (1 - \beta \eta) E_{t}^{\eta} \sum_{j=0}^{\infty} \beta^{j} \eta^{j} \left( \widehat{P}_{t+j} + \widehat{\psi}_{t+j} - \gamma \widetilde{n}_{t+j}(i) - \frac{\theta(\gamma + \alpha)}{1 - \alpha} \widetilde{P}_{t+j}(i) \right),$$

where subtracting the log of the aggregate price level,  $\hat{P}_t$ , on both sides of the equation, we reach

$$\widetilde{P}_{t}^{*}(i) = (1 - \beta \eta) E_{t}^{\eta} \sum_{j=0}^{\infty} \beta^{j} \eta^{j} \left( \widehat{\psi}_{t+j} - \gamma \widetilde{n}_{t+j}(i) - \frac{\theta(\gamma + \alpha)}{1 - \alpha} \widetilde{P}_{t+j}(i) + \sum_{k=1}^{j} \pi_{t+k} \right).$$
 (A17)

The rational expectation of future relative prices, conditional to optimal pricing in t and the lack of optimal price adjustments in the future, is  $E_t^{\eta} \widetilde{P}_{t+j}(i) = \widehat{P}_t^*(i) - E_t \widehat{P}_{t+j} = \widehat{P}_t^*(i) - \widehat{P}_t + \widehat{P}_t - E_t \widehat{P}_{t+j} = \widetilde{P}_t^*(i) + E_t \sum_{k=1}^j \pi_{t+k} = \widetilde{P}_t^*(i) + E_t \sum_{k=1}^j \pi_{t+k}$ . Using this result, (A17) becomes

$$\left(1 + \frac{\theta(\gamma + \alpha)}{1 - \alpha}\right) \widetilde{P}_t^*(i) = (1 - \beta \eta) E_t^{\eta} \sum_{j=0}^{\infty} \beta^j \eta^j \left(\widehat{\psi}_{t+j} - \gamma \widetilde{n}_{t+j}(i) + \left(1 + \frac{\theta(\gamma + \alpha)}{1 - \alpha}\right) E_t \sum_{k=1}^j \pi_{t+k}\right),$$

which is equivalent to

$$\left(1 + \frac{\theta(\gamma + \alpha)}{1 - \alpha}\right) \widetilde{P}_t^*(i) = (1 - \beta \eta) E_t^{\eta} \sum_{j=0}^{\infty} \beta^j \eta^j \left(\widehat{\psi}_{t+j} - \gamma \widetilde{n}_{t+j}(i)\right) + \left(1 + \frac{\theta(\gamma + \alpha)}{1 - \alpha}\right) E_t \sum_{j=1}^{\infty} \beta^j \eta^j \pi_{t+j}.$$
(A18)

Next, the expected future stream of relative employment, conditional to the lack of optimal pricing can be related to both the relative values of the optimal price and employment. Thus, it can be observed that applying (14)

$$E_t^{\eta} \sum_{j=0}^{\infty} \beta^j \eta^j \widetilde{n}_{t+j}(i) = \widetilde{n}_t(i) + E_t^{\eta} \sum_{j=1}^{\infty} \beta^j \eta^j \widetilde{n}_{t+j}(i) = \widetilde{n}_t(i) + \beta \eta E_t^{\eta} \sum_{j=0}^{\infty} \beta^j \eta^j \left( \tau_2 \widetilde{n}_{t+j}(i) - \tau_3 \widetilde{P}_{t+j}(i) \right),$$

which gets reduced to

$$(1 - \beta \eta \tau_2) E_t^{\eta} \sum_{i=0}^{\infty} \beta^j \eta^j \widetilde{n}_{t+j}(i) = \widetilde{n}_t(i) - \beta \eta \tau_3 E_t^{\eta} \sum_{i=0}^{\infty} \beta^j \eta^j \widetilde{P}_{t+j}(i). \tag{A19}$$

Applying again the conditional expectation on future relative prices,  $E_t^{\eta} \widetilde{P}_{t+j}(i) = \widetilde{P}_t^*(i) + E_t \sum_{k=1}^j \pi_{t+k}$ , we have

$$(1 - \beta \eta \tau_2) E_t^{\eta} \sum_{j=0}^{\infty} \beta^j \eta^j \widetilde{n}_{t+j}(i) = \widetilde{n}_t(i) - \beta \eta \tau_3 (1 - \beta \eta)^{-1} \left( \widetilde{P}_t^*(i) + E_t \sum_{j=1}^{\infty} \beta^j \eta^j \pi_{t+j} \right),$$

so that it is used in (A19) to yield

$$\left(1 + \frac{\theta(\gamma + \alpha)}{1 - \alpha} - \frac{\gamma \beta \eta \tau_3}{1 - \beta \eta \tau_2}\right) \widetilde{P}_t^*(i) = -\frac{\gamma(1 - \beta \eta)}{1 - \beta \eta \tau_2} \widetilde{n}_t(i) + (1 - \beta \eta) E_t \sum_{j=0}^{\infty} \beta^j \eta^j \widehat{\psi}_{t+j} + \left(1 + \frac{\theta(\gamma + \alpha)}{1 - \alpha} - \frac{\gamma \beta \eta \tau_3}{1 - \beta \eta \tau_2}\right) E_t \sum_{j=1}^{\infty} \beta^j \eta^j \pi_{t+j}.$$
(A20)

Recalling the proposed relation  $\widetilde{P}_t^*(i) = \widetilde{P}_t^* - \tau_1 \widetilde{n}_t(i)$ , the analytical solution for the undetermined coefficient  $\tau_1$  consistent with equation (A20) is

$$\tau_1 = \frac{\gamma(1-\beta\eta)}{(1-\beta\eta\tau_2)\left(1+\frac{\theta(\gamma+\alpha)}{1-\alpha}-\frac{\gamma\beta\eta\tau_3}{1-\beta\eta\tau_2}\right)},$$

that is expression (17) in the main text. Meanwhile, fluctuations of the aggregate relative optimal prices are determined by

$$\widetilde{P}_{t}^{*} = \frac{1 - \beta \eta}{1 + \frac{\theta(\gamma + \alpha)}{1 - \alpha} - \frac{\gamma \beta \eta \tau_{3}}{1 - \beta \eta \tau_{2}}} E_{t} \sum_{j=0}^{\infty} \beta^{j} \eta^{j} \widehat{\psi}_{t+j} + E_{t} \sum_{j=1}^{\infty} \beta^{j} \eta^{j} \pi_{t+j}, \tag{A21}$$

which corresponds to equation (18) in the main text

Appendix III. Derivation of the standard deviation of firm-specific relative employment.

From equation (14), the computation of the variance of relative employment gives

$$var(\widetilde{n}) = (1 - \tau_2^2)^{-1} \left[ \tau_3^2 var(\widetilde{P}) - 2\tau_2 \tau_3 cov(\widetilde{n}, \widetilde{P}) \right].$$

The covariance between relative employment and the relative price can be obtained from (16), the Calvo-style pricing and  $\widetilde{P}_t^* = \frac{\eta}{1-\eta}\pi_t$ 

$$cov(\widetilde{n}, \widetilde{P}) = E\left[\widetilde{n} * \widetilde{P}\right] = E\left[\widetilde{n}\left(\eta(\widetilde{P}_{-1} - \pi) + (1 - \eta)\left(\widetilde{P}^* - \tau_1\widetilde{n}\right)\right)\right] = E\left[\widetilde{n}\left(\eta\widetilde{P}_{-1} - (1 - \eta)\tau_1\widetilde{n}\right)\right].$$

Recalling the dynamics of relative optimal prices  $\widetilde{P}^*$  given by (16), the previous expression yields

$$cov(\widetilde{n}, \widetilde{P}) = E\left[\left(\tau_2 \widetilde{n}_{-1} - \tau_3 \widetilde{P}_{-1}\right) \eta \widetilde{P}_{-1} - (1 - \eta)\tau_1 \widetilde{n}^2\right],$$

which implies

$$cov(\widetilde{n}, \widetilde{P}) = -(1 - \eta \tau_2)^{-1} \left[ \eta \tau_3 var(\widetilde{P}) + (1 - \eta) \tau_1 var(\widetilde{n}) \right].$$

Substituting  $cov(\widetilde{n}, \widetilde{P})$  in the expression for  $var(\widetilde{n})$  results in

$$var(\tilde{n}) = (1 - \tau_2^2)^{-1} \left[ \tau_3^2 var(\tilde{P}) + 2\tau_2 \tau_3 (1 - \eta \tau_2)^{-1} \left( \eta \tau_3 var(\tilde{P}) + (1 - \eta) \tau_1 var(\tilde{n}) \right) \right]$$

or, alternatively,

$$\left[1 - \tau_2^2 - 2\tau_1\tau_2\tau_3(1 - \eta)(1 - \eta\tau_2)^{-1}\right]var(\widetilde{n}) = \left[\tau_3^2 + 2\tau_2\tau_3^2\eta(1 - \eta\tau_2)^{-1}\right]var(\widetilde{P}).$$

Taking the square root of the last expression leads to the expression for the standard deviation of relative employment

$$std(\widetilde{n}) = \sqrt{\frac{\tau_3^2 + 2\tau_2\tau_3^2\eta(1 - \eta\tau_2)^{-1}}{1 - \tau_2^2 - 2\tau_1\tau_2\tau_3(1 - \eta)(1 - \eta\tau_2)^{-1}}} std\left(\widetilde{P}\right),$$

that is displayed in Section 4 of the text.

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