WAGE SETTING ACTORS, STICKY WAGES, AND OPTIMAL MONETARY POLICY

Miguel Casares

D.T.2007/01

Wage Setting Actors, Sticky Wages, and Optimal Monetary Policy*

Miguel Casares†
Universidad Pública de Navarra

February 2007

Abstract

Following Erceg *et al.* (2000), sticky wages are generally modelled assuming that households set wage contracts *à la* Calvo (1983). This paper compares that sticky-wage model with one where wage contracts are set by firms, assuming flexible prices in any case. The key variable for wage dynamics moves from the marginal rate of substitution (households set wages) to the marginal product of labor (firms set wages). Optimal monetary policy in both cases fully stabilizes wage inflation and the output gap after technology or preference innovations. However, nominal shocks make the assumption on who set wages relevant for optimal monetary policy.

JEL codes: E24, E32, E52.

Keywords: Wage-setting households, Wage-setting firms, Optimal monetary policy.

1 Introduction

In a highly influential article, Erceg *et al.* (2000) show how sticky wages can be modeled in an optimizing framework by giving households a fixed probability *à la* Calvo (1983) to optimally reset their wage contract.[1] Thus, each household (or union as a group of households) owns some differentiated labor service and may decide the nominal wage associated with its labor supply.

*This line of research started in 2006 while I was a visiting fellow at the Federal Reserve Bank of St. Louis. I feel completely grateful to the Federal Reserve Bank of St. Louis for its kind invitation and hospitality, and I also thank Bill Gavin, James Bullard, and Ben McCallum for insightful comments and suggestions for this paper. Financial support was provided by the Spanish Ministry of Education and Science (Postdoc Fellowships Program and Research Project SEJ2005-03470/ECON).

†Departamento de Economía, Universidad Pública de Navarra, 31006, Pamplona, Spain. E-mail: mcasares@unavarra.es (Miguel Casares).

[1]The assumption of providing households with market power to set wages had already been taken in Blanchard (1986) and Rankin (1998).

The optimal wage can be reset only when receiving a market signal that arrives with a constant probability. As a result, the dynamics of either wage inflation or the real wage can be formulated in a single equation. Fluctuations of wage inflation (and the real wage) are governed by the gap between the aggregate marginal rate of substitution of households and the real wage. This sticky-wage structure is becoming very popular among New Keynesian researchers in recent times (Amato and Laubach, 2003; Smets and Wouters, 2003; Woodford, 2003; Giannoni and Woodford, 2004; Christiano *et al.*, 2005; Levin *et al.*, 2005; Casares, 2007).[2]

As one alternative sticky-wage variant, this paper examines the implications of moving the decision-making on the nominal wage from households to firms. Thus, the nominal wage contract may also be the one that maximizes profit of a monopsonistically competitive firm subject to a labor supply constraint.[3] Nominal rigidities can be readily introduced as Calvo-style contracts where firms are the wage setting actors. In turn, the paper shows how fluctuations of wage inflation are explained by a forward-looking equation that depends on the gap between the marginal product of labor and the real wage.

The consequences of nominal rigidities on the optimal design of monetary policy were first examined assuming that prices were sticky (Rotemberg and Woodford, 1997; Clarida *et al.,* 1999). Such analysis was extended to the case of economies where both prices and wages were sticky by Erceg *et al.* (2000).[4] Our contribution on this regard is to derive the optimal monetary policy in an optimizing macro model where the only source for nominal frictions is wage stickiness.[5] In that respect, the analysis will distinguish the implications of our two variants on wage setting actors for optimal monetary policy within a general equilibrium economy with flexible prices, and sticky wages. Using the targeting rules approach introduced by Svensson (1999a, 1999b) and Woodford (1999), the optimal monetary policy can be obtained by solving a central bank optimizing program subject to a set of model equations. Furthermore, the central-bank objective function can be written as an approximation to social welfare based on the average utility value. This paper also

[2] Analogously, a number of papers use Taylor (1980) staggered wage contracts set by households in a very similar optimizing framework. As representative examples, see Ascari (2000), Huang and Liu (2004), and Huang *et al.* (2004).

[3] Early contributions by Azariadis (1975), Hoehn (1988), and Flodén (2000) also put the wage setting decision on profit-maximizing firms.

[4] See Woodford (2003, ch. 6), and Amato and Laubach (2004) for discussions on optimal monetary policy under other model settings.

[5] A number of empirical papers recently argue that prices are not as sticky as generally assumed (Golosov and Lucas, 2003; Bils and Klenow, 2004) and others put emphasis on sticky wages as the source of nominal rigidities in the economy (Christiano *et al.*, 2005; Levin *et al.*, 2005).

compares the welfare-theoretic loss function for the central bank in each variant of our sticky-wage model.[6] We find that both cases agree on having variability of wage inflation and the output gap as the only monetary policy targets in a sticky-wage, flexible-price economy. However, their optimal policy reaction is distinct. If households set wages the rate of wage inflation must fall when there is a positive change in the output gap whereas the reaction must be of opposite sign in the case of firms acting as wage setters.

The rest of the paper is organized as follows. The sticky-wage model where households set wages is described in Section 2. The case where firms are wage setting actors is introduced in Section 3 as another sticky-wage variant. Section 4 is devoted to the theoretical monetary policy analysis as the optimal monetary policy rules are computed and discussed. The analysis is completed in Section 5 with simulation exercises such as impulse-response functions, calculation of second-moment statistics, and variance decomposition. Finally, Section 6 reviews the main conclusions of the paper.

2 Sticky wages set by households

Since the well-known paper by Erceg *et al.* (2000), sticky wages have been typically incorporated in the New Keynesian model by allowing households to set the nominal wage in the labor market. Hence, each household owns a differentiated labor service that supplies at her specific nominal wage. In this setup, firms demand bundles of labor services to be employed in their production processes. A labor bundle is obtained using the aggregation scheme first described by Dixit and Stiglitz (1977)

$$n_t = \left[\int_0^1 (n_t(h))^{(\theta_h - 1)/\theta_h} \, dh \right]^{\theta_h/(\theta_h - 1)}, \tag{1}$$

where the time period is indicated in the subscript of the variables, $\theta_h > 1.0$ is a constant parameter, and $n_t(h)$ is the labor service provided by the h-th household. A competitive labor agency assembles differentiated labor services from households to get labor bundles that will sell to firms. The maximum-profit condition for such labor agency leads to the following demand function (regarding the h-th labor service)[7]

$$n_t(h) = \left[\frac{W_t(h)}{W_t} \right]^{-\theta_h} n_t, \tag{2}$$

[6] To derive the welfare-theoretic loss functions, we borrow the computational methodology used by Erceg *et al.* (2000) and Woodford (2003, ch. 6).

[7] See Erceg *et al.* (2000) for details.

in which $W_t(h)$ is the nominal wage set by the h-th household, W_t is the Dixit-Stiglitz aggregate nominal wage, and θ_h gives the elasticity of substitution across labor services. Also as in Erceg *et al.* (2000), let us assume that a separable CRRA utility function ranks the preferences of the h-th representative household over consumption, c_t, and the supply of labor, $n_t(h)$,

$$U_t(h) = \frac{\exp(\chi_t)\,(c_t)^{1-\sigma}}{1-\sigma} - \Psi\frac{(n_t(h))^{1+\gamma}}{1+\gamma}, \tag{3}$$

where χ_t is an AR(1) shock on consumption preference, $\chi_t = \rho_\chi \chi_{t-1} + \varepsilon_t^\chi$ with $\varepsilon_t^\chi \sim N(0, \sigma_{\varepsilon^\chi})$. Units of consumption do not refer to the type of household because there are complete financial markets that ensure the same consumption across differentiated households. Intertemporal utility is maximized subject to a budget constraint and a demand for labor constraint as displayed in the following optimizing program

$$Max\ E_t \sum_{j=0}^{\infty} \beta^j U_{t+j}(h)$$

$$s.to\ : \quad \frac{W_{t+j}(h)}{P_{t+j}} n_{t+j}(h) = c_{t+j} + (1+R_{t+j})^{-1}\frac{B_{t+1+j}(h)}{P_{t+j}} - \frac{B_{t+j}(h)}{P_{t+j}} \quad for\ j = 0,1,2,...$$

$$and\ to\ : \quad n_{t+j}(h) = \left[\frac{W_{t+j}(h)}{W_{t+j}}\right]^{-\theta_h} n_{t+j} \quad for\ j = 0,1,2,...$$

The budget constraint is expressed in real magnitudes; labor real income may be spent on units of consumption and on net purchases of bonds.[8] If nominal wages can be optimally reset every period, we would obtain, in period t, this first order condition with respect to $W_t(h)$

$$\varsigma_t \frac{n_t(h)}{P_t} + \theta_h \xi_t(h)\frac{n_t(h)}{W_t(h)} = 0,$$

that includes the Lagrange multipliers on the budget constraint, ς_t, and on the labor-demand constraint $\xi_t(h)$. The first order conditions for consumption and labor imply the following values of the multipliers[9]

$$\varsigma_t = U_{c_t} \text{ and } \xi_t(h) = -\left(U_{n_t(h)} + \varsigma_t \frac{W_t(h)}{P_t}\right), \tag{4}$$

which can be substituted into the nominal wage optimality condition to yield

$$\frac{W_t(h)}{P_t} = \frac{\theta_h}{\theta_h - 1}\frac{-U_{n_t(h)}}{U_{c_t}}. \tag{5}$$

The optimal nominal wage sets the real wage as a mark-up over the marginal rate of substitution (mrs) between disutility of labor and utility of consumption.

[8] Regarding notation, P_t is the aggregate price level in period t, $B_{t+1}(h)$ is the nominal amount of bonds purchased in period t to be reimbursed in period $t+1$, and R_t is the nominal interest rate attached to such purchase.

[9] For simplicity in notation, U_{c_t} represents the consumption marginal utility and $U_{n_t(h)}$ the marginal disutility of labor.

Following the assumption by Calvo (1983), nominal rigidities on wage setting can be easily introduced assuming that households only reset optimally the wage contract in states of nature that arrive with a constant probability $1-\eta$. Therefore, households are not able to lay out the optimal wage contract with η probability. As in Casares (2007), the fraction of households that cannot set the optimal wage will apply the following stochastic indexation rule

$$W_t(s) = (1+\pi^{ss}+\upsilon_t)\,W_{t-1}(s) \tag{6}$$

referred to some s-th suboptimal household. The indexation factor in (6) depends on the steady-state rate of inflation, π^{ss}, and also on the stochastic element, υ_t, which follows the AR(1) process $\upsilon_t = \rho_\upsilon \upsilon_{t-1} + \varepsilon_t^\upsilon$ with $\varepsilon_t^\upsilon \sim N\left(0, \sigma_{\varepsilon^\upsilon}^2\right)$. The υ_t term can be interpreted as a cost-push shock that will ultimately affect the rate of economy-wide wage inflation (as it will be shown below).

Incorporating sticky wages *à la* Calvo, the nominal wage optimality condition becomes

$$E_t \sum_{j=0}^{\infty} \beta^j \eta^j \left[\varsigma_{t+j} \frac{\left(\Pi_{k=1}^{j}\left(1+\pi^{ss}+\upsilon_{t+k}\right)\right) n_{t+j}(h)}{P_{t+j}} + \theta_h \xi_{t+j}(h) \frac{n_{t+j}(h)}{W_t(h)} \right] = 0, \tag{7}$$

whereas the first order condition on the desired supply of labor for period $t+j$ turns out to be

$$\xi_{t+j}(h) = -\left(U_{n_{t+j}(h)} + \varsigma_{t+j} \frac{\left(\Pi_{k=1}^{j}\left(1+\pi^{ss}+\upsilon_{t+k}\right)\right) W_t(h)}{P_{t+j}} \right). \tag{8}$$

Substituting (7) into (8), and then loglinearizing around steady state yield

$$\widehat{W}_t(h) = (1-\beta\eta) E_t \sum_{j=0}^{\infty} \beta^j \eta^j (\widehat{P}_{t+j} + \widehat{mrs}_{t+j}(h)) - E_t \sum_{j=1}^{\infty} \beta^j \eta^j \upsilon_{t+j}, \tag{9}$$

where $\widehat{W}_t(h)$ and $\widehat{P}_{t+j}$ represent the log of the optimal nominal wage and the log of the aggregate price level, and $\widehat{mrs}_{t+j}(h)$ represents the log of the mrs, i.e. $\widehat{mrs}_{t+j}(h) = -\widehat{U}_{n_{t+j}(h)} - \widehat{U}_{c_{t+j}}$. Equation (9) can be transformed to present the optimal wage contract depending exclusively on aggregate magnitudes (see Appendix A for the proof)

$$\widehat{W}_t(h) = \widehat{W}_t + \frac{(1-\beta\eta)}{1+\gamma\theta_h} E_t \sum_{j=0}^{\infty} \beta^j \eta^j \left(\widehat{mrs}_{t+j} - \widehat{w}_{t+j}\right) + E_t \sum_{j=1}^{\infty} \beta^j \eta^j \left(\pi_{t+j}^W - \upsilon_{t+j}\right), \tag{10}$$

with $\widehat{W}_t$ denoting the log of the aggregate nominal wage, $\widehat{mrs}_t$ the log of the aggregate mrs, and $\widehat{w}_t$ the log of the aggregate real wage, $\widehat{w}_t = \widehat{W}_t - \widehat{P}_t$. In addition, π_t^W is the notation for the rate of wage inflation, $\pi_t^W = \widehat{W}_t - \widehat{W}_{t-1}$. The optimal wage contract has three determinants in (10): the aggregate nominal wage, the current and future expected gaps between the aggregate mrs and the real wage, and the expected future rates of wage inflation once the indexation shock is deducted. The relative wage taken from (9) can be replaced by $\widehat{W}_t(h) - \widehat{W}_t = \frac{\eta}{1-\eta}\left(\pi_t^W - \upsilon_t\right)$ as obtained by loglinearizing the aggregate nominal wage ($\widehat{W}_t = (1-\eta)\widehat{W}_t(h) + \eta\widehat{W}_{t-1} + \eta\upsilon_t$). Then, by computing $\beta\eta E_t\pi_{t+1}^W$ and calculating $\pi_t^W - \beta\eta E_t\pi_{t+1}^W$, we get to the following wage inflation equation

$$\pi_t^W = \beta E_t\pi_{t+1}^W + \frac{(1-\beta\eta)(1-\eta)}{\eta(1+\gamma\theta_h)}(\widehat{mrs}_t - \widehat{w}_t) + (1-\beta\rho_\upsilon)\upsilon_t. \tag{11}$$

Wage inflation dynamics evolve depending on three arguments: expected wage inflation, $E_t\pi_{t+1}^W$, the gap between the mrs and the real wage, $\widehat{mrs}_t - \widehat{w}_t$, and the exogenous indexation shock, υ_t. This is the wage inflation equation commonly used in the New Keynesian model with sticky wages (Erceg *et al.*, 2000; Woodford, 2003, ch. 3; Smets and Wouters, 2003; Christiano *et al.*, 2005) with the novelty of the indexation shock, υ_t.

With respect to prices, we suppose that they fully adjust every period to clear the labor market. The flexible-price assumption guarantees that the labor market is always in equilibrium. Thus, firms can choose their demand for labor bundles that implies maximum profit. In turn, the flexible-price condition yields

$$\widehat{w}_t = \widehat{mpl}_t, \tag{12}$$

where $\widehat{mpl}_t$ is the log of the marginal product of labor. Combining (11) and (12), it is obtained

$$\pi_t^W = \beta E_t\pi_{t+1}^W + \frac{(1-\beta\eta)(1-\eta)}{\eta(1+\gamma\theta_h)}(\widehat{mrs}_t - \widehat{mpl}_t) + (1-\beta\rho_\upsilon)\upsilon_t. \tag{13}$$

For illustrative purposes, we would rather present the wage inflation equation as a function of the output gap, $\widetilde{y}_t$, which is defined as the difference between the log of current output and the log of potential output

$$\widetilde{y}_t = \widehat{y}_t - \widehat{\overline{y}}_t. \tag{14}$$

As suggested by Woodford (2003), potential (natural-rate) output is the amount of output obtained in a perfect-competition economy with both flexible prices and wages. If that is the case, it is well-known that loglinear fluctuations on the mrs must be equal to those on the marginal product of labor, $\widehat{mrs}_t = \widehat{mpl}_t$, since both variables coincide with the real wage (Erceg *et al.*, 2000).[10] As standard, let us assume that output is produced using a Cobb-Douglas production technology

$$F(z_t, n_t) = [\exp(z_t)n_t]^{1-\alpha}, \tag{15}$$

where $0 < \alpha < 1$, and z_t is an AR(1) technology shock $z_t = \rho_z z_{t-1} + \varepsilon_t^z$ with $\varepsilon_t^z \sim N(0, \sigma_{\varepsilon^z})$. The condition $\widehat{mrs}_t = \widehat{mpl}_t$ for one economy with the utility function (2) and the production function

[10] This can be easily verified by setting a null Calvo probability for non-optimal wage contracts ($\eta = 0.0$).

(15) leads to the following equation for potential output fluctuations

$$\widehat{\widehat{y}}_t = \tfrac{1-\alpha}{\sigma(1-\alpha)+\alpha+\gamma}\left[(1+\gamma)\, z_t + \chi_t\right], \tag{16}$$

which implies

$$\widehat{mrs}_t - \widehat{mpl}_t = \left(\tfrac{\sigma(1-\alpha)+\alpha+\gamma}{1-\alpha}\right)\widetilde{y}_t.$$

The last result can be inserted in (13) to yield

$$\pi_t^W = \beta E_t \pi_{t+1}^W + \kappa_h \widetilde{y}_t + (1-\beta\rho_\upsilon)\upsilon_t, \tag{17}$$

with $\kappa_h = \frac{(1-\beta\eta)(1-\eta)}{\eta(1+\gamma\theta_h)}\frac{\sigma(1-\alpha)+\alpha+\gamma}{1-\alpha}$. The model with sticky wages set by households implies a positive relationship between wage inflation and the output gap.[11] As we just showed above, a positive output gap means that the mrs is higher than the marginal product of labor, and also higher than the real wage (recall the flexible-price condition $\widehat{w}_t = \widehat{mpl}_t$). Under that circumstance, households wish to work less hours and those that are able to reset their labor contract will choose a higher nominal wage that deliver less hours when entering (2). As a result, the aggregate nominal wage and the rate of wage inflation will rise.

3 Sticky wages set by firms

This section introduces another way of modeling sticky wages assuming that firms are the wage setting actors instead of households. An economy with wage setting firms requires some degree of labor demand differentiation to claim heterogeneity on wage contracts. Thus, in somehow a symmetric manner to the model with household setting wages, each firm acts as a monopsonistically competitor in the labor market because it is the only employer for a differentiated type of labor service. Meanwhile, households' disutility of labor depends on how many bundles of labor services they supply, obtained from the Dixit-Stiglitz aggregator

$$n_t = \left[\int_0^1 n_t(f)^{\frac{1+\theta_f}{\theta_f}} df\right]^{\frac{\theta_f}{1+\theta_f}} \tag{18}$$

where $\theta_f > 0.0$, and $n_t(f)$ represents the type of labor service supplied to the f-th firm. The instantaneous utility function (3) can be rewritten for this model variant as follows

$$U_t = \frac{\exp(\chi_t) c_t^{1-\sigma}}{1-\sigma} - \Psi \frac{n_t^{1+\gamma}}{1+\gamma}, \tag{19}$$

[11] The presence of $E_t \pi_{t+1}^W$ in (17) makes all expected future output gaps also affect positively for the determination of current wage inflation.

where n_t is given by (18). Unlike the model with household setting wages, households are identical and they decide the same amounts for consumption and labor services since they act both as price takers and wage takers. The aggregate nominal wage is defined by $W_t = \left[\int_0^1 W_t(f)^{1+\theta_f} df\right]^{\frac{1}{1+\theta_f}}$ so that the amount of nominal labor income obtained from all differentiated labor services supplied is the same as the income obtained by supplying bundles of labor, $\int_0^1 W_t(f) n_t(f) df = W_t n_t$. Taking this into account for the kind of budget constraint presented in the previous section, we can compute the following household's first order conditions with respect to the number of labor bundles, n_t, and the supply of the f-th type of labor service, $n_t(f)$

$$\begin{aligned} -\Psi n_t^{\gamma} - \varsigma_t \frac{W_t}{P_t} &= 0, \\ -\Psi n_t^{\gamma} n_t^{-\frac{1}{\theta_f}} (n_t(f))^{\frac{1}{\theta_f}} - \varsigma_t \frac{W_t(f)}{P_t} &= 0. \end{aligned}$$

The value of the Lagrange multiplier, ς_t, obtained from the first equation can be substituted out onto the second equation to yield

$$n_t(f) = \left[\frac{W_t(f)}{W_t}\right]^{\theta_f} n_t, \tag{20}$$

which represents the supply of labor constraint faced by the f-th firm when setting its nominal wage (analogous to (2) for the case of households setting wages). Note that the elasticity of substitution of the household across differentiated labor services is now positive at the constant parameter θ_f.

Turning to the firms' optimizing behavior on setting wages, let us suppose that they produce output using the Cobb-Douglas technology (15). Assuming that firms seek to maximize intertemporal profits, the optimizing program for the f-th representative firm can be written as

$$\begin{aligned} &Max \quad E_t \sum_{j=0}^{\infty} \beta^j \left(F(z_{t+j}, n_{t+j}(f)) - \frac{W_{t+j}(f)}{P_{t+j}} n_{t+j}(f) \right) \\ &s.to \qquad n_{t+j}(f) = \left[\frac{W_{t+j}(f)}{W_{t+j}}\right]^{\theta_f} n_{t+j} \quad for\ j = 0, 1, 2, ... \end{aligned}$$

With no wage rigidity, the optimality condition on $W_t(f)$ is

$$-\frac{n_t(f)}{P_t} - \varphi_t(f)\theta_f \frac{n_t(f)}{W_t(f)} = 0, \tag{21}$$

where $\varphi_t(f)$ is the Lagrange multiplier of the supply of labor constraint in period t. The value of $\varphi_t(f)$ is defined by the first order condition on the demand for labor $n_t(f)$

$$\varphi_t(f) = \frac{W_t(f)}{P_t} - mpl_t(f), \tag{22}$$

where $mpl_t(f)$ denotes the marginal product of the f-th type of labor. Combining the last two expressions and rearranging terms, we obtain

$$W_t(f) = \frac{\theta_f}{1+\theta_f} P_t mpl_t(f).$$

The optimal wage contract is a fraction of the market-valued marginal product of labor. Therefore, the value of the wage contract is obtained by applying the mark-down, $\frac{\theta_f}{1+\theta_f}$, over the market value of labor productivity.

Introducing both wage rigidities *à la* Calvo and the wage indexation rule (6) adapted to the firm, the optimality condition (21) for the wage contract set in period t changes to

$$E_t \sum_{j=0}^{\infty} \beta^j \eta^j \left[\frac{\left(\Pi_{k=1}^{j} \left(1 + \pi^{ss} + \upsilon_{t+k}\right) \right) n_{t+j}(f)}{P_{t+j}} + \varphi_{t+j}(f) \theta_f \frac{n_{t+j}(f)}{W_t(f)} \right] = 0,$$

where inserting (22) in the place of $\varphi_t(f)$, and $\varphi_{t+j}(f) = \frac{\left(\Pi_{k=1}^{j}(1+\pi^{ss}+\upsilon_{t+k})\right) W_t(f)}{P_{t+j}} - mpl_{t+j}(f)$ for future $\varphi_{t+j}(f)$ terms conditional to no optimal wage resetting, we find (after loglinearizing)

$$\widehat{W}_t(f) = (1-\beta\eta) E_t \sum_{j=0}^{\infty} \beta^j \eta^j (\widehat{P}_{t+j} + \widehat{mpl}_{t+j}(f)) - E_t \sum_{j=1}^{\infty} \beta^j \eta^j \upsilon_{t+j}. \tag{23}$$

The log of the optimal wage contract depends on current and expected future values of the log of the price level and the log of the specific marginal product of labor. Using some algebra, the optimal wage can also be written in terms of aggregate magnitudes as follows (see Appendix A for the proof)

$$\widehat{W}_t(f) = \widehat{W}_t + \frac{(1-\beta\eta)}{(1+\alpha\theta_f)} E_t \sum_{j=0}^{\infty} \beta^j \eta^j \left(\widehat{mpl}_{t+j} - \widehat{w}_{t+j} \right) + E_t \sum_{j=1}^{\infty} \beta^j \eta^j \left(\pi_{t+j}^{W} - \upsilon_{t+j} \right). \tag{24}$$

The interpretation of (24) is straightforward. The optimal nominal wage contract departs from the aggregate nominal wage whenever current or expected future labor productivity values are above the real wage, $\widehat{mpl}_{t+j} - \widehat{w}_{t+j} > 0.0$, and also if the rate of wage inflation is expected to be higher than the innovation on the indexation rate, $\pi_{t+j}^{W} - \upsilon_{t+j} > 0.0$. In order to find the rate of wage inflation equation for this economy, we can combine (24) with the implied relationship from the Calvo-type aggregation scheme, $\pi_t^W = \frac{1-\eta}{\eta}(\widehat{W}_t(f) - \widehat{W}_t) + \upsilon_t$, to reach

$$\pi_t^W = \beta E_t \pi_{t+1}^W + \frac{(1-\eta)(1-\beta\eta)}{\eta(1+\alpha\theta_f)} \left(\widehat{mpl}_t - \widehat{w}_t \right) + (1-\beta\rho_\upsilon)\upsilon_t. \tag{25}$$

The rate of wage inflation is a purely forward-looking variable whose quarter-to-quarter fluctuations are governed by the gap between the aggregate marginal product of labor and the real wage, as well as by the innovation on the wage indexation rule. When households set wages (equation 11), wage inflation was reacting to the gap between the household's marginal rate of substitution and the real wage. Hence, the labor productivity becomes the driving variable if firms become the wage setting actors while that was the marginal rate of substitution when households set wages.

Recalling the flexible-price scenario introduced above, prices will entirely adjust as needed to clear the labor market. It implies

$$\widehat{w}_t = \widehat{mrs}_t, \tag{26}$$

since the supply of labor bundles decided by households is optimal when their mrs equates the real wage. This flexible-price condition (26) can be plugged into (25) to obtain

$$\pi_t^W = \beta E_t \pi_{t+1}^W + \frac{(1-\eta)(1-\beta\eta)}{\eta(1+\alpha\theta_f)} \left(\widehat{mpl}_t - \widehat{mrs}_t\right) + (1-\beta\rho_\upsilon)\upsilon_t, \tag{27}$$

and then substituting $\widehat{mrs}_t - \widehat{mpl}_t = \frac{\sigma(1-\alpha)+\alpha+\gamma}{1-\alpha}\widetilde{y}_t$, it yields

$$\pi_t^W = \beta E_t \pi_{t+1}^W - \kappa_f \widetilde{y}_t + (1-\beta\rho_\upsilon)\upsilon_t, \tag{28}$$

with $\kappa_f = \frac{(1-\eta)(1-\beta\eta)}{\eta(1+\alpha\theta_f)} \frac{\sigma(1-\alpha)+\alpha+\gamma}{1-\alpha}$. Remarkably, the sticky-wage model where firms are wage setting actors delivers a negative relationship between the output gap and wage inflation, i.e., the opposite sign to that obtained in the sticky-wage model where households set wages. When the output gap is positive, the marginal product of labor falls below the real wage and firms wish to hire less labor. Thus, firms that can decide on a new wage contract will drop the value of the nominal wage in order to reduce the level of labor implied by (20). Besides their opposite sign, the numerical values of the slope coefficients are also different. Thus, the ratio of the output gap slopes in the wage inflation equations (17) and (28) is $\frac{\kappa_h}{-\kappa_f} = -\frac{1+\alpha\theta_f}{1+\gamma\theta_h}$.

4 Optimal monetary policy

The optimal monetary policy can be derived for the two variants of the sticky-wage model with flexible prices discussed above. The two cases only differ on who are the wage setting actors, either households (as commonly assumed in the New Keynesian literature) or firms.

Following Woodford (2003, ch. 6), optimal monetary policy is obtained for some given model from the first order conditions that maximize a measure of social welfare approximated from the utility function of that model. Such approximation consists of writing the average value of the utility function across households as a second-order expression that depends on a few aggregate variables of the model and also on the underlying structure of that model (price/wage rigidities, technology, distorting taxes,...). Since the second-order terms typically have a negative impact on the measure obtained for social welfare, the central-bank objective function is presented as a loss function to be minimized. The Appendix B of this paper shows that the period loss function of the model where households set wages is[12]

$$L_t^h = \left(\pi_t^W\right)^2 + \lambda_h \left(\widetilde{y}_t - \widetilde{y}^*\right)^2, \tag{29}$$

where $\lambda_h = \frac{(1-\beta\eta)(1-\eta)}{\eta(1+\gamma\theta_h)} \frac{\sigma(1-\alpha)+\alpha+\gamma}{\theta_h(1-\alpha)^2}$ and $\widetilde{y}^*$ denotes the efficient level of the output gap in steady state.[13] Therefore, welfare losses depend on the weighted sum of variabilities of wage inflation and the output gap relative to its efficiency level.

For the case with firms setting wages, the welfare-theoretic loss function is (see also Appendix B for the proof)

$$L_t^f = \left(\pi_t^W\right)^2 + \lambda_f \left(\widetilde{y}_t - \widetilde{y}^*\right)^2, \tag{30}$$

with $\lambda_f = \frac{(1-\eta)(1-\beta\eta)}{\eta(1+\alpha\theta_f)} \frac{\sigma(1-\alpha)+\alpha+\gamma}{\theta_f(1-\alpha)^2}$. Coincidently, social utility of households is also damaged by variability of wage inflation and the output gap when firms are wage setting actors. Therefore, the case of having either firms or households setting the wage contracts does not affect the definition of the targeting variables on optimal monetary policy because they are wage inflation and the output gap in either way. However, the relative weight of the output gap is not exactly defined in (29) as in (30) which may lead to distinctive policy reactions that will be examined in Section 5.[14]

For a long-run commitment to an optimal plan, we borrow the "timeless perspective" criterion, proposed in Woodford (1999, page 18) and Woodford (2003, pages 538-539). Thus, the optimal plan for monetary policy in the case of having households setting wages is obtained by solving the following optimizing program:

$$Min \;\; E_t \sum_{j=0}^{\infty} \beta^j \left(\left(\pi_{t+j}^W\right)^2 + \lambda_h \left(\widetilde{y}_{t+j} - \widetilde{y}^*\right)^2 \right)$$

subject to the all-time sequence of wage inflation equations

$$\pi_{t+j}^W = \beta E_{t+j} \pi_{t+j+1}^W + \kappa_h E_{t+j} \widetilde{y}_{t+j} + (1-\beta\rho_\upsilon) E_{t+j} \upsilon_{t+j} \qquad j = ..., -2, -1, 0, 1, 2, ...$$

The first order conditions with respect to wage inflation and the output gap in period t are

$$\begin{aligned} 2\pi_t^W - \varphi_{t-1}^h + \varphi_t^h &= 0, \text{ and} \\ 2\lambda_h \left(\widetilde{y}_t - \widetilde{y}^*\right) - \kappa_h \varphi_t^h &= 0, \end{aligned}$$

[12] The reader can verify that this welfare-theoretic loss function is the particular flexible-price case of that derived by Woodford (2003, pages 443-445).

[13] The efficient output gap is the amount produced if markets were perfectly competitive with no distortion (see Woodford, 2003, pages 393-394).

[14] Obviously, the different model structure may also have effects on the design of optimal monetary policy.

in which φ_t^h is the Lagrange multiplier in period t. Taking $\varphi_t^h = \frac{2\lambda_h}{\kappa_h}(\widetilde{y}_t - \widetilde{y}^*)$ obtained from the second equation, together with its lagged expression for φ_{t-1}^h, and inserting both into the first equation result in

$$\pi_t^W = -\frac{\lambda_h}{\kappa_h}(\widetilde{y}_t - \widetilde{y}_{t-1}), \tag{31}$$

where $\frac{\lambda_h}{\kappa_h} = \frac{1}{\theta_h(1-\alpha)}$. According to the time-consistent plan (31), optimal monetary policy requires that the rate of wage inflation responds in the opposite direction to the first-difference of the output gap. This policy resembles the "leaning against the wind" recommendation obtained by Clarida *et al.* (1999) and Woodford (2003, ch. 7) for a New Keynesian framework with sticky prices *à la* Calvo. If the output gap is increasing from the previous period the central bank must adjust the nominal interest rate to turn the wage inflation downwards by a factor of $\frac{1}{\theta_h(1-\alpha)}$. Hence, the optimal monetary policy requires anticyclical reactions of wage inflation relative to the output gap.

Let us check if the alternative sticky-wage specification leads to the same condition for the optimal monetary policy behavior. With firms setting wages, the central-bank optimizing program is

$$Min \quad E_t \sum_{j=0}^{\infty} \beta^j \left(\left(\pi_{t+j}^W\right)^2 + \lambda_f (\widetilde{y}_{t+j} - \widetilde{y}^*)^2 \right)$$

subject to the all-time sequence of wage inflation equations

$$\pi_{t+j}^W = \beta E_{t+j}\pi_{t+j+1}^W - \kappa_f E_{t+j}\widetilde{y}_{t+j} + (1-\beta\rho_\upsilon)E_{t+j}\upsilon_{t+j} \qquad j = ..., -2, -1, 0, 1, 2, ...$$

The first order conditions on wage inflation and the output gap for period t are

$$\begin{aligned} 2\pi_t^W - \varphi_{t-1}^f + \varphi_t^f &= 0, \text{ and} \\ 2\lambda_f (\widetilde{y}_t - \widetilde{y}^*) + \kappa_f \varphi_t^f &= 0, \end{aligned}$$

where φ_t^f is now the Lagrange multiplier. Inserting $\varphi_t^f = -\frac{2\lambda_f}{\kappa_f}(\widetilde{y}_t - \widetilde{y}^*)$ and its lagged expression in the wage inflation optimality condition, it yields

$$\pi_t^W = \frac{\lambda_f}{\kappa_f}(\widetilde{y}_t - \widetilde{y}_{t-1}). \tag{32}$$

with $\frac{\lambda_f}{\kappa_f} = \frac{1}{\theta_f(1-\alpha)}$. Comparing (32) with (31), it seems that the optimal monetary policy is opposite depending on the wage setting assumption. Thus, if firms are wage setting actors, the central bank will pursue a monetary policy that adjusts wage inflation on the same direction to the change in the output gap. It could be said that wage inflation must be procyclical since it should react with the same sign to changes in the output gap. The reaction factor is $\frac{1}{\theta_f(1-\alpha)}$ which mimics that obtained for the variant with households setting wages.

What are the consequences of applying the optimal monetary policy for our two variants of a sticky-wage model? In the case where households set wages, the central-bank optimality condition (31) and its expected next period's expression can be substituted in the wage inflation equation (17) to yield (after little algebra)

$$\widetilde{y}_t = \frac{\lambda_h}{\lambda_h(1+\beta)+\kappa_h^2}\left[\beta E_t \widetilde{y}_{t+1} + \widetilde{y}_{t-1} - \frac{(1-\beta\rho_\upsilon)\kappa_h}{\lambda_h}\upsilon_t\right]. \tag{33}$$

The only source of variability for the output gap in (33) is the wage indexation shock υ_t. In particular, a positive indexation shock brings in a negative output gap. Real-side shocks such as technology or preference innovations have no impact on the output gap.[15] The nominal interest rate would adjust as necessary to bring current output back at its potential level. Therefore, the output gap will have no variability under the optimal policy in the absence of indexation shocks. Moreover, wage inflation would also stay constant as implied by equation (17). The optimal monetary policy would be fully efficient because the loss function would value zero at all times. The presence of (nominal-side) wage indexation shocks is necessary to evaluate optimal policy tradeoffs in our flexible-price model with sticky wages set by households. This result was already pointed out by Taylor (1979) and Clarida *et al.* (1999) as the lack of an output-inflation variability tradeoff in models without nominal shocks (also called cost-push shocks).

Turning to the case of firms acting as wage setters, the optimality condition (32) can be combined with the wage inflation equation (28) to obtain

$$\widetilde{y}_t = \frac{\lambda_f}{\lambda_f(1+\beta)+\kappa_f^2}\left[\beta E_t \widetilde{y}_{t+1} + \widetilde{y}_{t-1} + \frac{(1-\beta\rho_\upsilon)\kappa_f}{\lambda_f}\upsilon_t\right]. \tag{34}$$

As in the case where households set wages, the evolution of the output gap also depends here exclusively on wage indexation shocks, υ_t. Nevertheless, the output gap becomes positive in (34) after a positive realization of υ_t. Another implication of (34) is that both technology and preference shocks are also neutralized by the optimal monetary policy and leave no effect on the output gap. With no change in the output gap, those real-side shocks have no impact on wage inflation as implied by (28). Therefore, the model with firms setting wages also needs (nominal) indexation shocks for evaluating tradeoffs between output gap and wage inflation variabilities. Without such nominal perturbations, optimal monetary policy achieves full stabilization in both model variants.

[15] A preference shock can be considered a real shock because it shifts the aggregate labor supply curve.

5 Model simulations with alternative wage setting actors

Table 1 summarizes the key equations that show why the assumption on who are the wage setting actors is relevant for a sticky-wage model. There are three aspects of the model affected. First, wage inflation dynamics are different because its relationship to the output gap may be positive (households set wages) or negative (firms set wages). Secondly, the optimal monetary policy implies a central-bank reaction of opposite sign: the rate of wage inflation must decrease when there is a positive change in the output gap (households set wages) whereas wage inflation must be raised in reaction to that output gap change (firms set wages). At the third level of distinction shown in Table 1, the flexible-price assumption gives rise to different market-clearing conditions for the labor market. Either the (log of the) real wage must be equal to the (log of the) marginal product of labor (households set wages) or the (log of the) real wage equates the (log of the) marginal rate of substitution (firms set wages).[16]

So far, we have shown that having either households or firms as wage setters plays a role on the dynamic behavior of wage inflation and also on the design of the optimal monetary policy. Is this also influential on the short-run fluctuations of other key macro variables such as output or (price) inflation? To answer this question we need to build the rest of the model. For the demand sector, let us introduce the following IS equation

$$\widehat{y}_t = E_t \widehat{y}_{t+1} - \tfrac{1}{\sigma}(R_t - E_t \pi_{t+1}) + \tfrac{1-\rho_\chi}{\sigma}\chi_t, \tag{35}$$

which can be obtained by combining the first order conditions on consumption and bonds on the household optimizing program presented in Section 2. As typical in a New Keynesian framework, output fluctuations in (35) are demand-determined in response to changes in expected output, the real interest rate (with a negative impact), and the consumption preference shock. Finally, the definition of the economy-wide real wage, $w_t = \frac{W_t}{P_t}$, implies that the rates of price and wage inflation are related as follows

$$\pi_t = \pi_t^W - \widehat{w}_t + \widehat{w}_{t-1}. \tag{36}$$

Summarizing, we have two variants for a flexible-price model with sticky wages *à la* Calvo, and optimal monetary policy:

- A model where households are wage setting actors: The three equations on the left column of Table 1, plus common equations (14), (16), (35), and (36) comprise a set of seven equations that provide solution paths for seven variables $\widehat{y}_t$, π_t, π_t^W, $\widehat{w}_t$, R_t, $\widetilde{y}_t$, and $\widehat{\overline{y}}_t$.

[16] The loglinearized marginal rate of substitution displayed in Table 1 can be obtained by using the specification for the utility function (19), the Cobb-Douglas technology (15), and the market-clearing condition $\widehat{y}_t = \widehat{c}_t$.

- A model where firms are wage setting actors: The three equations on the right column of Table 1, plus common equations (14), (16), (35), and (36), comprise a set of seven equations that provide solution paths for seven variables $\widehat{y}_t$, π_t, π_t^W, $\widehat{w}_t$, R_t, $\widetilde{y}_t$, and $\widehat{\widetilde{y}}_t$.

For illustrative purposes, we will simulate these two sticky-wage variants by analyzing impulse-response functions and some business cycle statistics obtained from them. On that regard, some numerical values need to be assumed on the parameters of the models (see Table 2). Repeating the baseline quarterly calibration chosen by Erceg *et al.* (2000), we have $\beta = 0.99$, $\sigma = 1.5$, $\gamma = 1.5$, $\theta_h = 4.0$, $\alpha = 0.3$, and wage contracts last on average for one year, $\eta = 0.75$. The household's elasticity of substitution across labor services in the model where wages are decided by firms is set at $\theta_f = 4.0$ to have it equal to that elasticity for the other model variant.

As for the stochastic elements of the model, some reasonable numbers are arbitrarily assigned.[17] Thus, the standard deviations on the innovations of the real-side shocks are those that yield a standard deviation of potential output around 1.8%, and 3/4 of fluctuations of potential output are driven by technology shocks.[18] The innovations on the wage indexation shock have a standard deviation that provides significant reactions of wage inflation in the impulse-response analysis (around 0.5% in annualized terms). As common in the literature, the coefficient of autocorrelation of the technology shock is very high ($\rho_z = 0.95$) whereas preference shocks have less inertia ($\rho_\chi = 0.80$). The wage indexation shock also has a moderate coefficient of autocorrelation ($\rho_\upsilon = 0.80$) as assumed by Woodford (2003, page 496) for his analogous cost-push shock.

As a result of our numerical selection, the slope coefficients in the wage inflation equations of Table 1 are rather distinctive. If firms are wage setting actors, wage inflation is much more sensitive to the output gap because the slope coefficient is $\kappa_f = 0.159$, more than three times that slope when households set wages ($\kappa_h = 0.050$). A similar result is obtained when comparing the relative weight of the output gap variability in the central-bank loss function. Stabilizing the output gap appears to be more important for social welfare if firms set wages ($\lambda_f = 0.057$ *versus* $\lambda_h = 0.018$).

The business cycle properties of the two variants presented above can be examined by describing the reactions to a technology shock, z_t, a consumption preference shock, χ_t, and an indexation shock, υ_t. The technology shock represents a supply-side shock because it gives rise to output fluctuations due to exogenous changes in productivity. Meanwhile, the preference shock represents a demand-side shock since the desired level of consumption is exogenously altered as a result of a change in its marginal utility. The indexation shock can be viewed as one example of a cost-push shock used in the literature on optimal monetary policy to analyze preferences on macroeconomic

[17] The aim of this paper is to show theoretical aspects without any particular empirical fit to some actual economy.

[18] The latter was verified in the long-run variance decomposition.

stabilization. What are the influence of these shocks in the model variants? Is the assumption on who set wages critical for the business cycle patterns in sticky-wage models with flexible prices and optimal monetary policy? We will try to give answers to these questions by computing impulse-response functions and selected statistics from each model variant.

Throughout Figures 1-3, impulse response functions are plotted as fractional deviations from steady state for output, and the real wage, while annualized departures from their steady-state levels for price inflation, wage inflation, and the nominal interest rate. The shocks are normalized by their standard deviations provided in Table 2.

Technology shocks

Figure 1 shows the responses to a technology shock in the sticky-wage model where either households set wages or firms do it. Interestingly, all variables react in the same way with either households or firms setting wages as identically displayed in Figure 1. At first, the optimal monetary policy is able to fully stabilize both wage inflation and the output gap under either case.[19] A zero output gap implies that current output is at its potential level ($\widehat{y}_t = \widehat{\overline{y}}_t$), which yields a 0.5% sudden increase that incorporates long inertia. Putting the zero output gap differently, fluctuations of the aggregate marginal product of labor (of firms) are the same as those of the aggregate marginal rate of substitution (of households). Accordingly, the real wage reports in both cases an upward response consistent with the productivity hike (see Figure 1). Meanwhile, there is a sharp one-time inflation drop, deeper than 2% in annualized terms, required to clear the labor market. Finally, it is worth commenting on the optimal reaction of the nominal interest rate to a technology shock. As displayed in Figure 1, the nominal interest rate must be lowered at the time of the shock in order to help current (demand-determined) output to catch up with potential output and close down the output gap. This monetary policy ease has two characteristics: first, it represents a weak reaction because the interest-rate cut is only by 6 (annualized) basis points at most. The second characterizing aspect is that the nominal interest rate returns very slowly to its steady-state level. As shown in Figure 1, half of the initial interest-rate cut still remains in place fifteen quarters after the shock.

Summarizing, a supply-side technology shock that raises productivity brings about long-lasting increases in current output and the real wage, a one-time drop in inflation, and keep wage inflation and the output gap unchanged. This result is observed when applying the optimal monetary policy based on a gentle and persistent interest-rate cut. These responses are obtained independently

[19] This result was already discussed at the end of Section 4.

from the assumption on who set wages in the labor market.

Preference shocks

Figure 2 displays how a demand-side shock on consumption preference has the same impact on the two sticky-wage variants. Since this is another example of a real-side shock, the optimal monetary policy also achieves full stabilization for wage inflation and the output gap in both cases. Thus, output and potential output equally rise; a 0.4% increase at the time of the shock that fades out after 8-10 quarters. Meanwhile, the real wage drops due to the reduction in the mrs (or in the marginal product of labor). Inflation also reacts equally in both sticky-wage variants as it rises by more than 0.5% at the quarter of the shock although it returns to its steady state rate in the next quarter.

The optimal policy requires a strong monetary tightening: the nominal interest rate must be raised by 57 (annualized) basis points at the quarter of the shock (see Figure 2). The return of the nominal interest rate to its steady-state level should take 8-10 quarters. Therefore, the response of the nominal interest rate to a preference shock is larger, less persistent, and with opposite sign to that observed after a technology shock. This contractionary monetary policy is the necessary central-bank action to pull down current output onto its potential (natural-rate) level and therefore eliminate the output gap.

Indexation shocks

Unlike the two real-side shocks examined above, a (nominal) indexation shock allows different responses across our two variants for a sticky-wage model. As a matter of fact, the reactions differ significantly in terms of both output and price inflation. Hence, output slightly goes up if firms set wages whereas it drops by a much greater extent if households set wages (see Figure 3). Why do we obtain such differentiated output reactions? Since the wage indexation shock does not affect potential output, the response of the output gap is the same as the response of output in both models. If firms are wage setting actors, the negative relationship between wage inflation and the output gap in (28) makes the central bank to increase the output gap in order to reduce wage inflation as indicated by the optimal policy condition (32). When households are wage setters, the relationship between wage inflation and the output gap is positive in (17) and the optimal monetary policy (31) will create a negative output gap to stabilize wage inflation.

The optimal response of the nominal interest rate is also sensitive to the assumption on who set wages. As shown in Figure 3, the nominal interest rate drops around 50 basis points if firms set wages whereas there is much larger interest-rate cut (150 basis points) when households set wages.

Summarizing, the implementation of optimal monetary policy in a sticky-wage model with flexible prices and alternative wage setting actors leads to the same responses to real-side shocks but to significantly different reactions to nominal shocks.

Table 3 provides a selection of second-moment statistics computed from the two sticky-wage models. Output, the nominal interest rate, and, especially, the output gap are more volatile in the model with households setting wages as much higher standard deviations were found. This is a consequence of their larger reactions observed after a wage indexation shock. Wage inflation is much more stable than price inflation in both model cases.[20] Regarding inertia, output, the real wage, and the output gap show long time persistence, characterized by high coefficients of autocorrelation in Table 3. The nominal interest rate moves along with moderate inertia when applying the optimal monetary policy in both models (coefficients of autocorrelation around 0.6-0.7). The flexible-price assumption leads to coefficients of autocorrelation of inflation close to zero. The only discrepancy between the two model variants in terms of autocorrelations is that the coefficient on wage inflation is somewhat higher if households set wages.

As for the coefficients of correlation with output, Table 3 reports that price inflation, wage inflation, and the nominal interest rate are basically acyclical in the two variants, with coefficients close to zero in all the cases. The real wage is clearly procyclical only in the model with firms setting wages.

Finally, the variance decomposition in the long-run (200 periods ahead) is displayed in Table 4. Technology shocks are the source of most fluctuations in output, the real wage, and price inflation in both sticky-wage cases. Meanwhile, preference shocks are the driving force for changes in the nominal interest rate also in both variants, though especially in the model where firms set wages. Wage indexation shocks explain all the variations in the two targeting variables of monetary policy: wage inflation and the output gap. This result was anticipated above in the impulse-response functions analysis where the impact of technology and preference shocks on these variables were fully neutralized by optimal monetary policy. Indexation shocks also determine a significant fraction of long-run fluctuations on output and the nominal interest rate in the model where households are wage setting actors, and on price inflation and the real wage in the variant where firms set wages (around 0.2 in all cases).

[20] This result should be expected for flexible-price models where price inflation variability is not a targeting variable for the central bank.

6 Conclusions

The assumption on who set wages is not trivial for an optimizing macroeconomic model with sticky wages. If households may decide on the value of nominal wage contracts, the optimal contract will depend on the difference between the marginal rate of substitution and the real wage. By contrast, if firms can set the wage contract, they will determine its value looking at the difference between the marginal product of labor and the real wage.

In addition, we have shown that, in a sticky-wage, flexible-price economy with Calvo-type wage contracts, the rate of wage inflation is forward-looking on the output gap with a sign defined by who are the wage setting actors. Hence, when households set wages the relationship is of positive sign whereas when firms set wages that relationship turns negative. In addition, the assumption on wage setting actors distinguishes the analytical value of the output gap slope in the wage inflation equation $\left(\frac{\kappa_h}{-\kappa_f} = -\frac{1+\alpha\theta_f}{1+\gamma\theta_h}\right)$.

Finally, optimal (welfare-theoretic) monetary policy must minimize, in both variants for a sticky-wage model, a weighted sum of variabilities of the output gap and the rate of wage inflation. The central bank optimal plan leads to a different targeting rule on each case: wage inflation must react either positively (firms set wages) or negatively (households set wages) to the output gap. Despite their different targeting rule, optimal monetary policy in both cases achieves full stabilization of the two targeting variables in the presence of either technology or preference shocks. Nevertheless, the assumption on who act as wage setters is relevant for the optimal interest-rate responses to (nominal) wage indexation shocks. Such nominal shocks also give rise to distinctive second-moment statistics.

REFERENCES

Amato, J. D., and T. Laubach (2004). Implications of Habit Formation for Optimal Monetary Policy, *Journal of Monetary Economics* 51, 305-325.

Azariadis, C. (1975). Implicit contracts and underemployment equilibria, *Journal of Political Economy* 83, 1183-1202.

Bils, M., and P. J. Klenow (2004). Some Evidence on the Importance of Sticky Prices, *Journal of Political Economy* 112, 947-985.

Blanchard, O. J. (1986). The wage price spiral, *The Quarterly Journal of Economics* 101, 543-565.

Calvo, G. A. (1983). Staggered pricing in a utility-maximizing framework, *Journal of Monetary Economics* 12, 383-396.

Casares, M. (2007). Monetary policy rules in a New Keynesian Euro area model, *Journal of Money, Credit, and Banking*, forthcoming.

Christiano, L.J., M. Eichenbaum, and C.L. Evans (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy, *Journal of Political Economy* 113, 1-45.

Clarida, R., J. Galí, and M. Gertler (1999). The Science of Monetary Policy: a New Keynesian Perspective, *Journal of Economic Literature* 37, 1661-1707.

Dixit, A. K., and J. E. Stiglitz (1977). Monopolistic competition and optimum product diversity, *American Economic Review* 67, 297-308.

Erceg, C.J., D.W. Henderson, and A.T. Levin (2000). Optimal monetary policy with staggered wage and price contracts, *Journal of Monetary Economics* 46, 281-313.

Flodén, M. (2000). Endogenous monetary policy and the business cycle, *European Economic Review* 44, 1409-1429.

Giannoni, M. P., and M. Woodford (2004). Optimal Inflation-Targeting Rules, in Ben S. Bernanke and Michael Woodford, eds., The Inflation Targeting Debate, Chicago: University of Chicago Press, 93-162.

Golosov, M., and R. E. Lucas (2003). Menu Costs and Phillips Curves, NBER Working Papers, No.10187.

Hoehn, J G. (1988). Procyclical real wages under nominal-wage contracts with productivity variations, *Federal Reserve Bank of Cleveland Review* 24, No. 4, 11-23.

Huang, K. X. D., and Z. Liu (2002). Staggered price setting, staggered wage setting, and business cycle persistence, *Journal of Monetary Economics* 49, 405-433.

Huang, K. X. D., Z. Liu, and L. Phaneuf (2004). Why Does the Cyclical Behavior of Real Wages Change Over time?, *American Economic Review* 94, 836-856.

Levin, A. T., A. Onatski, J. C. Williams, and N. Williams (2005). Monetary Policy Under Uncertainty in Micro-Founded Macroeconometric Models, NBER Working Papers, No. 11523.

Rankin, N. (1998). Nominal rigidity and monetary uncertainty, *European Economic Review* 42, 185-199.

Rotemberg, J. J., and M. Woodford (1997). An optimizing-based econometric framework for the evaluation of monetary policy, NBER Macroeconomics Annual 1997, B.M. Bernanke and J.J. Rotemberg, eds., MIT Press, Cambridge, MA, 297-346.

Smets, F., and R. Wouters (2003). An estimated Dynamic Stochastic General Equilibrium model of the Euro area, *Journal of the European Economic Association* 1, 1123-1175.

Svensson, L. E. O. (1999a). Price Level Targeting vs. Inflation Targeting: A Free Lunch?, *Journal of Money, Credit, and Banking* 31, 277-295.

Svensson, L. E. O. (1999b). Inflation Targeting as a Monetary Policy Rule, *Journal of Monetary Economics* 43, 607-654.

Taylor, J. B. (1979). Estimation and Control of a Macroeconomic Model with Rational Expectations, *Econometrica* 47, 1267-1286.

Taylor, J. B. (1980). Aggregate dynamics and staggered contracts, *Journal of Political Economy* 88, 1-24.

Woodford, M. (1999). Commentary: How Should Monetary Policy Be Conducted in an Era of Price Stability?, in *New Challenges for Monetary Policy*: A Symposium Sponsored by the Federal Reserve Bank of Kansas City, Federal Reserve Bank of Kansas City.

Woodford, M. (2003). Interest and prices: Foundations of a theory of monetary policy, Princeton University Press, Princeton, NJ.

APPENDIX A. Derivation of the equation for fluctuations on the optimal nominal wage depending on aggregate variables.

i) Model with wages set by households.

Let us start from repeating equation (9) from the text

$$\widehat{W}_t(h) = (1-\beta\eta)E_t \sum_{j=0}^{\infty} \beta^j \eta^j (\widehat{P}_{t+j} + \widehat{mrs}_{t+j}(h)) - E_t \sum_{j=1}^{\infty} \beta^j \eta^j \upsilon_{t+j}, \tag{A1}$$

where $\widehat{mrs}_{t+j}(h)$ represents fluctuations on the mrs for the households who decide the optimal contract in period t and will not adjust it in future periods. From the definition of the (aggregate) real wage, we can substitute $\widehat{P}_{t+j} = \widehat{W}_{t+j} - \widehat{w}_{t+j} = \widehat{W}_t + \sum_{k=1}^{j} \pi^W_{t+k} - \widehat{w}_{t+j}$ into (A1) to obtain

$$\widehat{W}_t(h) = (1-\beta\eta)E_t \sum_{j=0}^{\infty} \beta^j \eta^j (\widehat{W}_t - \widehat{w}_{t+j} + \widehat{mrs}_{t+j}(h)) + E_t \sum_{j=1}^{\infty} \beta^j \eta^j \left(\pi^W_{t+j} - \upsilon_{t+j}\right). \tag{A2}$$

Recalling our utility function specification (3), the (log-linearized) relationship between the household-specific mrs and the aggregate mrs is

$$\widehat{mrs}_{t+j}(h) = \widehat{mrs}_{t+j} + \gamma(\widehat{n}_{t+j}(h) - \widehat{n}_{t+j}). \tag{A3}$$

Now we wish to write the relative mrs as a function of the relative wage. Using (2) in loglinear terms for the $t+j$ period conditional to the indexation rule (6), it yields

$$\widehat{n}_{t+j}(h) - \widehat{n}_{t+j} = -\theta_h(\widehat{W}_t(h) + \sum_{k=1}^{j} \upsilon_{t+k} - \widehat{W}_{t+j}), \tag{A4}$$

where it should be noticed that the conditional wage contract evolves as $\widehat{W}_t(h) + \sum_{k=1}^{j} \upsilon_{t+k}$ in any $t+j$ future period. Combining (A3) and (A4), it is obtained

$$\widehat{mrs}_{t+j}(h) = \widehat{mrs}_{t+j} - \gamma\theta_h(\widehat{W}_t(h) + \sum_{k=1}^{j} \upsilon_{t+k} - \widehat{W}_{t+j}),$$

where inserting $\widehat{W}_{t+j} = \widehat{W}_t + \sum_{k=1}^{j} \pi^W_{t+k}$ leads to

$$\widehat{mrs}_{t+j}(h) = \widehat{mrs}_{t+j} - \gamma\theta_h(\widehat{W}_t(h) - \widehat{W}_t - \sum_{k=1}^{j} \left(\pi^W_{t+k} - \upsilon_{t+k}\right)). \tag{A5}$$

Finally, the substitution of (A5) into (A2) yields

$$\widehat{W}_t(h) = (1-\beta\eta)E_t \sum_{j=0}^{\infty} \beta^j \eta^j ((1+\gamma\theta_h)\widehat{W}_t + \widehat{mrs}_{t+j} - \widehat{w}_{t+j} - \gamma\theta_h \widehat{W}_t(h) + (1+\gamma\theta_h)E_t \sum_{j=1}^{\infty} \beta^j \eta^j \left(\pi^W_{t+j} - \upsilon_{t+j}\right).$$

Putting terms together, we can write a dynamic equation for the wage contract that depends entirely on aggregate magnitudes

$$\widehat{W}_t(h) = \widehat{W}_t + \frac{(1-\beta\eta)}{1+\gamma\theta_h} E_t \sum_{j=0}^{\infty} \beta^j \eta^j \left(\widehat{mrs}_{t+j} - \widehat{w}_{t+j}\right) + E_t \sum_{j=1}^{\infty} \beta^j \eta^j \left(\pi_{t+j}^W - \upsilon_{t+j}\right),$$

which is equation (10) in the main text of the paper.

ii) Model with wages set by firms.

In section 3, we obtained equation (23) for fluctuations on the optimal nominal wage contract set in period t

$$\widehat{W}_t(f) = (1-\beta\eta) E_t \sum_{j=0}^{\infty} \beta^j \eta^j (\widehat{P}_{t+j} + \widehat{mpl}_{t+j}(f)) - E_t \sum_{j=1}^{\infty} \beta^j \eta^j \upsilon_{t+j}, \tag{A6}$$

where $\widehat{mpl}_{t+j}(f)$ denotes the marginal product of labor in case of no optimal wage adjustment over future periods. The (log-linear) Cobb-Douglas production technology (15) can be used to find the relative relationship between the marginal product of labor and demand for labor

$$\widehat{mpl}_{t+j}(f) - \widehat{mpl}_{t+j} = -\alpha(\widehat{n}_{t+j}(f) - \widehat{n}_{t+j}). \tag{A7}$$

Loglinearizing the labor supply constraint (20) for period $t+j$ and using the wage indexation rule (6), we get

$$\widehat{n}_{t+j}(f) = \widehat{n}_{t+j} + \theta_f \left(\widehat{W}_t(f) + \sum_{k=1}^{j} \upsilon_{t+k} - \widehat{W}_{t+j} \right). \tag{A8}$$

The combination of (A7) and (A8) leads to

$$\widehat{mpl}_{t+j}(f) = \widehat{mpl}_{t+j} - \alpha\theta_f \left(\widehat{W}_t(f) + \sum_{k=1}^{j} \upsilon_{t+k} - \widehat{W}_{t+j} \right),$$

which can be inserted in (A6) to reach

$$\widehat{W}_t(f) = (1-\beta\eta) E_t \sum_{j=0}^{\infty} \beta^j \eta^j \left(\widehat{P}_{t+j} + \widehat{mpl}_{t+j} - \alpha\theta_f \left(\widehat{W}_t(f) - E_t \widehat{W}_{t+j} \right) \right) - (1+\alpha\theta_f) E_t \sum_{j=1}^{\infty} \beta^j \eta^j \upsilon_{t+j}.$$

Terms on the aggregate price level are dropped by inserting $\widehat{P}_{t+j} = \widehat{W}_{t+j} - \widehat{w}_{t+j}$. Next, we can use $\widehat{W}_{t+j} = \widehat{W}_t + \sum_{k=1}^{j} \pi_{t+k}^W$ to find

$$(1+\alpha\theta_f) \widehat{W}_t(f) = (1-\beta\eta) E_t \sum_{j=0}^{\infty} \beta^j \eta^j (\widehat{mpl}_{t+j} - \widehat{w}_{t+j} + (1+\alpha\theta_f) \widehat{W}_t) + (1+\alpha\theta_f) E_t \sum_{j=1}^{\infty} \beta^j \eta^j \left(\pi_{t+j}^W - \upsilon_{t+j}\right),$$

which can be also written as our equation (24) of the text

$$\widehat{W}_t(f) = \widehat{W}_t + \frac{(1-\beta\eta)}{(1+\alpha\theta_f)} E_t \sum_{j=0}^{\infty} \beta^j \eta^j \left(\widehat{mpl}_{t+j} - \widehat{w}_{t+j} \right) + E_t \sum_{j=1}^{\infty} \beta^j \eta^j \left(\pi_{t+j}^W - \upsilon_{t+j}\right).$$

APPENDIX B. Derivation of the welfare-theoretic loss function for the variants of the sticky-wage model of the paper.

All the approximations taken here are based on second-order Taylor expansions used in Erceg *et al.* (2000, pages 307-312), and Woodford (2003, pages 692-696).

i) Model where households set wages.

For convenience, let us define $V_t = \frac{\exp(\chi_t)(c_t)^{1-\sigma}}{1-\sigma}$ and $S_t(h) = \Psi \frac{(n_t(h))^{1+\gamma}}{1+\gamma}$ from the utility function (3) such that

$$U_t(h) = V_t - S_t(h). \tag{B1}$$

After dropping out constant and exogenous terms, the second-order Taylor approximation of V_t yields

$$V_t \simeq U_c \left[c_t - c\right] + \frac{1}{2} U_{cc} \left[c_t - c\right]^2 + U_c \left[c_t - c\right] \left[\exp(\chi_t) - 1\right].$$

By inserting the equilibrium condition $c_t = y_t$, using $U_{cc} = -\frac{\sigma U_c}{c}$, and the approximation $[\exp(\chi_t) - 1] \simeq \chi_t$, it is obtained

$$V_t \simeq U_c \left[y_t - y\right] - \frac{1}{2} \frac{\sigma U_c}{y} \left[y_t - y\right]^2 + U_c \left[y_t - y\right] \chi_t. \tag{B2}$$

A second-order Taylor expansion for $\frac{y_t}{y}$ is

$$\frac{y_t}{y} \simeq 1 + \widehat{y}_t + \frac{1}{2} \widehat{y}_t^2, \tag{B3}$$

which implies the second-order approximations

$$\left[y_t - y\right] \simeq y \left(\widehat{y}_t + \frac{1}{2} \widehat{y}_t^2\right) \text{ and } \left[y_t - y\right]^2 \simeq y^2 \widehat{y}_t^2, \tag{B4}$$

that can be substituted in (B2) to reach

$$V_t \simeq y U_c \left(\widehat{y}_t + \frac{1}{2} \widehat{y}_t^2\right) - \frac{1}{2} y \sigma U_c \widehat{y}_t^2 + y U_c \left(\widehat{y}_t + \frac{1}{2} \widehat{y}_t^2\right) \chi_t.$$

Next, the term $y U_c \frac{1}{2} \widehat{y}_t^2 \chi_t$ is dropped for being of third order and terms are grouped to obtain the final second-order approximation of V_t

$$V_t \simeq y U_c \left(\widehat{y}_t + \frac{1}{2}(1 - \sigma) \widehat{y}_t^2 + \widehat{y}_t \chi_t\right). \tag{B5}$$

Let us turn to the second term in (B1), $S_t(h) = \Psi \frac{(n_t(h))^{1+\gamma}}{1+\gamma}$. Its second-order Taylor approximation becomes

$$S_t(h) \simeq U_n \left[n_t(h) - n\right] + \frac{1}{2} U_{nn} \left[n_t(h) - n\right]^2, \tag{B6}$$

where constant and exogenous terms were taken out. Using $U_{nn} = \frac{\gamma U_n}{n}$ and the approximations analogous to (B4) for $[n_t(h) - n]$ and $[n_t(h) - n]^2$ in (B6), it yields

$$S_t(h) \simeq nU_n \left(\widehat{n}_t(h) + \frac{1}{2}(1+\gamma)\widehat{n}_t^2(h) \right). \tag{B7}$$

Integrating the household-specific utility function (B1) over all differentiated households, we obtain the aggregate utility function

$$U_t = \int_0^1 U_t(h)dh = V_t - \int_0^1 S_t(h)dh, \tag{B8}$$

that represents the social utility function to be maximized by the central bank. Here, we need to aggregate the labor disutility across households. Using (B7), it yields

$$\int_0^1 S_t(h)dh \simeq nU_n \left(E_h\widehat{n}_t(h) + \frac{1}{2}(1+\gamma)\left[(E_h\widehat{n}_t(h))^2 + var_h\widehat{n}_t(h) \right] \right), \tag{B9}$$

where, following Woodford (2003, page 694), we used the definitions $E_h\widehat{n}_t(h) = \int_0^1 \widehat{n}_t(h)dh$ and $var_h\widehat{n}_t(h) = \int_0^1 \widehat{n}_t^2(h)dh - (E_h\widehat{n}_t(h))^2$. A Taylor series approximation to (1) implies the following equation for aggregate labor fluctuations

$$\widehat{n}_t \simeq E_h\widehat{n}_t(h) + \frac{1}{2}\frac{\theta_h - 1}{\theta_h} var_h\widehat{n}_t(h),$$

which can be used to eliminate both $E_h\widehat{n}_t(h)$ and $(E_h\widehat{n}_t(h))^2$ from (B9) and thus to obtain (after dropping terms of order higher than two)

$$\int_0^1 S_t(h)dh \simeq nU_n \left(\widehat{n}_t + \frac{1}{2}(1+\gamma)\widehat{n}_t^2 + \frac{1}{2}(\gamma + \theta_h^{-1})var_h\widehat{n}_t(h) \right). \tag{B10}$$

Now, both (B5) and (B10) can be substituted in the social utility function (B8) to get

$$U_t \simeq yU_c \left(\widehat{y}_t + \frac{1}{2}(1-\sigma)\widehat{y}_t^2 + \widehat{y}_t\chi_t \right) - nU_n \left(\widehat{n}_t + \frac{1}{2}(1+\gamma)\widehat{n}_t^2 + \frac{1}{2}(\gamma + \theta_h^{-1})var_h\widehat{n}_t(h) \right). \tag{B11}$$

The (log-linear) Cobb-Douglas production function (15) relates labor and output in the following way $\widehat{n}_t = (1-\alpha)^{-1}\widehat{y}_t - z_t$, which implies $\widehat{n}_t^2 = (1-\alpha)^{-2}\widehat{y}_t^2 + z_t^2 - 2(1-\alpha)^{-1}\widehat{y}_t z_t$. Taking into account these two relationships, (B11) can be rewritten in terms of output fluctuations and the dispersion of differentiated labor services

$$\begin{aligned} U_t \simeq\; & yU_c \left(\widehat{y}_t + \frac{1}{2}(1-\sigma)\widehat{y}_t^2 + \widehat{y}_t\chi_t \right) \qquad \text{(B12)} \\ & - nU_n \left((1-\alpha)^{-1}\widehat{y}_t + \frac{1}{2}(1+\gamma)\left[(1-\alpha)^{-2}\widehat{y}_t^2 - 2(1-\alpha)^{-1}\widehat{y}_t z_t \right] + \frac{1}{2}(\gamma + \theta_h^{-1})var_h\widehat{n}_t(h) \right), \end{aligned}$$

where exogenous terms were dropped. Following Woodford (2003, chapter 6), the steady-state solution of the model with zero inflation implies the relationship $nU_n = yU_c(1-\alpha)(1-\Phi_y)$ where

$1-\Phi_y$ represents the inverse of the steady-state markup of the real wage over the mrs.[21] This result can be used in (B12) to obtain

$$U_t \simeq yU_c\left(\Phi_y\widehat{y}_t-\frac{1}{2}\left(\tfrac{\sigma(1-\alpha)+\alpha+\gamma}{1-\alpha}\right)\widehat{y}_t^2+\widehat{y}_t\left(\chi_t+(1+\gamma)z_t\right)-\frac{1}{2}(1-\alpha)(\gamma+\theta_h^{-1})var_h\widehat{n}_t(h)\right), \tag{B13}$$

where the terms $\Phi_y\widehat{y}_t^2$, $\Phi_y\widehat{y}_tz_t$, and $\Phi_y var_h\widehat{n}_t(h)$ were neglected as justified by Woodford (2003, pages 393-394). Again,. following Woodford (2003, page 395), the difference between the output gap and its efficiency level is

$$\widetilde{y}_t-\widetilde{y}^*=\left(\widehat{y}_t-\widehat{\overline{y}}_t\right)-\widetilde{y}^*, \tag{B14}$$

where $\widetilde{y}^*$ is obtained as the fractional difference in steady-state between the level of output produced in a perfectly competitive economy and the level of potential output obtained in a monopolistically competitive economy, i.e. $\widetilde{y}^*=\log\left(\frac{y^*}{\overline{y}}\right)$. For the model described in the text, we have $\widetilde{y}^*=-\varpi\log(1-\Phi_y)\simeq\varpi\Phi_y$ where ϖ completely depends on the values of parameters regarding preferences and technology $\varpi=\frac{1-\alpha}{\sigma(1-\alpha)+\alpha+\gamma}$. Computing the square of the efficient output gap defined in (B14) and using the approximation $\widetilde{y}^*=\varpi\Phi_y$, the square output fluctuations, $\widehat{y}_t^2$, can be written as follows

$$\widehat{y}_t^2=\left(\widetilde{y}_t-\widetilde{y}^*\right)^2+2\widehat{y}_t\widehat{\overline{y}}_t+2\varpi\Phi_y\widehat{y}_t-2\varpi\Phi_y\widehat{\overline{y}}_t-\widehat{\overline{y}}_t^2-\left(\varpi\Phi_y\right)^2, \tag{B15}$$

where potential output fluctuations are $\widehat{\overline{y}}_t=\varpi\left(\chi_t+(1+\gamma)z_t\right)$ as in equation (16) of the text. Inserting (B15) into (B14), we obtain (after dropping exogenous terms)

$$U_t\simeq -yU_c\left(\frac{1}{2}\varpi^{-1}\left(\widetilde{y}_t-\widetilde{y}^*\right)^2+(1-\alpha)(\gamma+\theta_h^{-1})var_h\widehat{n}_t(h)\right). \tag{B16}$$

By loglinearizing (2), the variance on differentiated labor services can be expressed in terms of wage dispersion

$$var_h\widehat{n}_t(h)=\theta_h^2 var_h\widehat{W}_t(h). \tag{B17}$$

Applying the results of Woodford (2003, page 694-696) to our Calvo-style sticky-wage structure, we have

$$var_h\widehat{W}_t(h)\simeq\eta var_h\widehat{W}_{t-1}(h)+\frac{\eta}{1-\eta}\left(\pi_t^W\right)^2,$$

which implies

$$\sum_{j=0}^{\infty}\beta^jE_t var_h\widehat{W}_{t+j}(h)\simeq\frac{\eta}{(1-\eta)(1-\beta\eta)}\sum_{j=0}^{\infty}\beta^jE_t\left(\pi_{t+j}^W\right)^2 \tag{B18}$$

[21] In this particular sticky-wage model, $\Phi_y=\theta_h^{-1}$.

Inserting (B17) in (B16), it is obtained

$$U_t \simeq -yU_c\left(\frac{1}{2}\varpi^{-1}\left(\widetilde{y}_t-\widetilde{y}^*\right)^2+(1-\alpha)\theta_h(1+\gamma\theta_h)var_h\widehat{W}_t(h)\right). \tag{B19}$$

Finally, let us assume that the central bank maximizes the intertemporal (social) utility function $\sum_{j=0}^{\infty}\beta^j E_t U_{t+j}$. Combining (B19), and (B18), it yields

$$\sum_{j=0}^{\infty}\beta^j E_t U_{t+j} \simeq -\frac{1}{2}yU_c\sum_{j=0}^{\infty}\beta^j E_t\left(\varpi^{-1}\left(\widetilde{y}_{t+j}-\widetilde{y}^*\right)^2+\frac{\eta(1-\alpha)\theta_h(1+\gamma\theta_h)}{(1-\eta)(1-\beta\eta)}\left(\pi_{t+j}^W\right)^2\right). \tag{B20}$$

Rearranging terms in (B20), we have

$$\sum_{j=0}^{\infty}\beta^j E_t U_{t+j} \simeq -\Omega_h\sum_{j=0}^{\infty}\beta^j E_t\left(\left(\pi_{t+j}^W\right)^2+\varpi^{-1}\frac{(1-\eta)(1-\beta\eta)}{\eta(1-\alpha)\theta_h(1+\gamma\theta_h)}\left(\widetilde{y}_{t+j}-\widetilde{y}^*\right)^2\right),$$

with $\Omega_h=\frac{yU_c}{2}\frac{\eta(1-\alpha)\theta_h(1+\gamma\theta_h)}{(1-\eta)(1-\beta\eta)}$, which implies that the central-bank loss function for period t is

$$L_t^h=\left(\pi_t^W\right)^2+\lambda_h\left(\widetilde{y}_t-\widetilde{y}^*\right)^2,$$

with $\lambda_h=\frac{(1-\beta\eta)(1-\eta)}{\eta(1+\gamma\theta_h)}\frac{\sigma(1-\alpha)+\alpha+\gamma}{\theta_h(1-\alpha)^2}$ as defined in equation (29) of the text.

ii) Model where firms set wages.

In this case, the household utility function and the social utility function are the same because both consumption and the supply of bundles of labor services are identical across households. Recalling (19), the social utility function in period t is in this model

$$U_t=\frac{\exp(\chi_t)\left(c_t\right)^{1-\sigma}}{1-\sigma}-\Psi\frac{\left(n_t\right)^{1+\gamma}}{1+\gamma}=V_t-S_t. \tag{B21}$$

As shown above, the two terms of (B21) can be approximated by the Taylor series expansions

$$V_t \simeq yU_c\left(\widehat{y}_t+\frac{1}{2}(1-\sigma)\widehat{y}_t^2+\widehat{y}_t\chi_t\right), \text{ and} \tag{B22a}$$

$$S_t \simeq nU_n\left(\widehat{n}_t+\frac{1}{2}(1+\gamma)\widehat{n}_t^2\right). \tag{B22b}$$

The term on disutility from the supply of labor bundles, S_t, will be affected by the degree of wage dispersion through its impact on fluctuations of market-clearing labor, $\widehat{n}_t$. Thus, a Taylor series expansion on (20) yields

$$\widehat{n}_t \simeq E_f\widehat{n}_t(f)+\frac{1}{2}\left(\frac{1+\theta_f}{\theta_f}\right)var_f\widehat{n}_t(f), \tag{B23}$$

where $E_f\widehat{n}_t(f)$ and $var_f\widehat{n}_t(f)$ respectively denote the expected value of the demand for labor and its variance computed across the differentiated firms. Recalling the log-linear Cobb-Douglas technology (15) for the specific f-th firm, we obtain

$$\widehat{n}_t(f) = (1-\alpha)^{-1}\widehat{y}_t(f) - z_t,$$

which, aggregating over the f space, implies

$$\begin{aligned} E_f\widehat{n}_t(f) &= (1-\alpha)^{-1}E_f\widehat{y}_t(f) - z_t, \text{ and} \\ var_f\widehat{n}_t(f) &= (1-\alpha)^{-2}var_f\widehat{y}_t(f). \end{aligned} \tag{B24}$$

Aggregate output in this economy with differentiated firms is $y_t = \int_0^1 y_t(f)df$, which leads to the Taylor series approximation

$$\widehat{y}_t \simeq E_f\widehat{y}_t(f) + \frac{1}{2}var_f\widehat{y}_t(f). \tag{B25}$$

Substituting (B24) into (B23) and then the value of $E_f\widehat{y}_t(f)$ implied by (B25), it is obtained

$$\widehat{n}_t \simeq (1-\alpha)^{-1}\left[\widehat{y}_t - \frac{1}{2}var_f\widehat{y}_t(f)\right] - z_t + \frac{1}{2}\left(\frac{1+\theta_f}{\theta_f}\right)var_f\widehat{n}_t(f),$$

where inserting $var_f\widehat{y}_t(f) = (1-\alpha)^2 var_f\widehat{n}_t(f)$ from (B24) results in

$$\widehat{n}_t \simeq (1-\alpha)^{-1}\left[\widehat{y}_t - \frac{1}{2}(1-\alpha)^2 var_f\widehat{n}_t(f)\right] - z_t + \frac{1}{2}\left(\frac{1+\theta_f}{\theta_f}\right)var_f\widehat{n}_t(f).$$

Putting terms together and dropping the exogenous variable, we get

$$\widehat{n}_t \simeq (1-\alpha)^{-1}\widehat{y}_t + \frac{1}{2}\left(\alpha + \theta_f^{-1}\right)var_f\widehat{n}_t(f). \tag{B26}$$

The substitution of (B26) into (B22b) yields

$$S_t \simeq nU_n\left((1-\alpha)^{-1}\widehat{y}_t + \frac{1}{2}(\alpha+\theta_f^{-1})var_f\widehat{n}_t(f) + \frac{1}{2}(1+\gamma)\widehat{n}_t^2\right),$$

where using $\widehat{n}_t^2 = (1-\alpha)^{-2}\widehat{y}_t^2 - 2(1-\alpha)^{-1}\widehat{y}_t z_t$ from the log-linear Cobb-Douglas technology (15) results in

$$S_t \simeq nU_n\left((1-\alpha)^{-1}\widehat{y}_t + \frac{1}{2}(\alpha+\theta_f^{-1})var_f\widehat{n}_t(f) + \frac{1}{2}(1+\gamma)(1-\alpha)^{-2}\widehat{y}_t^2 - (1+\gamma)(1-\alpha)^{-1}\widehat{y}_t z_t\right). \tag{B27}$$

The steady-state relationship $nU_n = yU_c(1-\alpha)(1-\Phi_y)$, which was already used above, still holds in this model because it keeps the same preferences and technology.[22] Plugging that into (B27), it yields

$$S_t \simeq yU_c(1-\Phi_y)\left(\widehat{y}_t + \frac{1}{2}(1-\alpha)(\alpha+\theta_f^{-1})var_f\widehat{n}_t(f) + \frac{1}{2}(1+\gamma)(1-\alpha)^{-1}\widehat{y}_t^2 - (1+\gamma)\widehat{y}_t z_t\right). \tag{B28}$$

[22] Nevertheless, the value of Φ_y is now $\Phi_y = (1+\theta_f)^{-1}$.

Using (B22a) for V_t and (B28) for S_t, the social utility function can be approximated by

$$U_t \simeq yU_c \left(\Phi_y \widehat{y}_t - \frac{1}{2}\varpi^{-1}\widehat{y}_t^2 + \widehat{y}_t \left(\chi_t + (1+\gamma)z_t\right) - \frac{1}{2}(1-\alpha)(\alpha+\theta_f^{-1})var_f \widehat{n}_t(f) \right),$$

recalling $\varpi = \frac{1-\alpha}{\sigma(1-\alpha)+\alpha+\gamma}$ and where the terms $\Phi_y \widehat{y}_t^2$, $\Phi_y \widehat{y}_t z_t$, and $\Phi_y var_f \widehat{n}_t(f)$ were dropped as we did in the model where households set wages. The definition of the efficient output gap was used above to obtain (B15) which can be inserted in the last expression to find (after dropping exogenous terms and using $\widehat{\overline{y}}_t = \varpi\left(\chi_t + (1+\gamma)z_t\right)$)

$$U_t \simeq -\frac{yU_c}{2} \left(\varpi^{-1}\left(\widetilde{y}_t - \widetilde{y}^*\right)^2 + (1-\alpha)(\alpha+\theta_f^{-1})var_f \widehat{n}_t(f) \right). \tag{B29}$$

By loglinearizing (20), the variance on differentiated labor services can be expressed in terms of wage dispersion

$$var_f \widehat{n}_t(f) = \theta_f^2 var_f \widehat{W}_t(f),$$

which leaves (B29) as follows

$$U_t \simeq -\frac{yU_c}{2} \left(\varpi^{-1}\left(\widetilde{y}_t - \widetilde{y}^*\right)^2 + \theta_f(1-\alpha)(1+\alpha\theta_f)var_f \widehat{W}_t(f) \right).$$

Accordingly, the central-bank intertemporal (social) utility function $\sum_{j=0}^{\infty}\beta^j E_t U_{t+j}$ becomes

$$\sum_{j=0}^{\infty}\beta^j E_t U_{t+j} \simeq -\frac{yU_c}{2}\sum_{j=0}^{\infty}\beta^j E_t \left(\varpi^{-1}\left(\widetilde{y}_{t+j} - \widetilde{y}^*\right)^2 + \theta_f(1-\alpha)(1+\alpha\theta_f)var_f \widehat{W}_{t+j}(f) \right). \tag{B30}$$

Again, we can use Woodford (2003, pages 694-696) to have

$$var_f \widehat{W}_f(h) \simeq \eta var_f \widehat{W}_{t-1}(f) + \frac{\eta}{1-\eta}\left(\pi_t^W\right)^2,$$

which implies

$$\sum_{j=0}^{\infty}\beta^j E_t var_f \widehat{W}_{t+j}(f) \simeq \frac{\eta}{(1-\eta)(1-\beta\eta)}\sum_{j=0}^{\infty}\beta^j E_t\left(\pi_{t+j}^W\right)^2. \tag{B31}$$

Combining (B30) and (B31), it is obtained

$$\sum_{j=0}^{\infty}\beta^j E_t U_{t+j} \simeq -\frac{yU_c}{2}\sum_{j=0}^{\infty}\beta^j E_t \left(\varpi^{-1}\left(\widetilde{y}_{t+j} - \widetilde{y}^*\right)^2 + \frac{\theta_f(1-\alpha)(1+\alpha\theta_f)\eta}{(1-\eta)(1-\beta\eta)}\left(\pi_{t+j}^W\right)^2 \right),$$

which can be reorganized as follows

$$\sum_{j=0}^{\infty}\beta^j E_t U_{t+j} \simeq -\Omega_f \sum_{j=0}^{\infty}\beta^j E_t \left(\left(\pi_{t+j}^W\right)^2 + \varpi^{-1}\frac{(1-\eta)(1-\beta\eta)}{\theta_f(1-\alpha)(1+\alpha\theta_f)\eta}\left(\widetilde{y}_{t+j} - \widetilde{y}^*\right)^2 \right),$$

with $\Omega_f = \frac{yU_c}{2}\frac{\theta_f(1-\alpha)(1+\alpha\theta_f)\eta}{(1-\eta)(1-\beta\eta)}$. Our last result implies that the central-bank loss function for period t is

$$L_t^f = \left(\pi_t^W\right)^2 + \lambda_f\left(\widetilde{y}_t - \widetilde{y}^*\right)^2,$$

with $\lambda_f = \frac{\sigma(1-\alpha)+\alpha+\gamma}{1-\alpha}\frac{(1-\eta)(1-\beta\eta)}{\theta_f(1-\alpha)(1+\alpha\theta_f)\eta}$ as defined in equation (30) of the text.

Table 1. Key equations for a flexible-price, sticky-wage model with optimal monetary policy

	Households set wages	Firms set wages
Wage inflation:	$\pi_t^W = \beta E_t \pi_{t+1}^W + \kappa_h \widetilde{y}_t + (1-\beta\rho_\upsilon)\upsilon_t$	$\pi_t^W = \beta E_t \pi_{t+1}^W - \kappa_f \widetilde{y}_t + (1-\beta\rho_\upsilon)\upsilon_t$
Opt. monetary policy:	$\pi_t^W = -\frac{1}{\theta_h(1-\alpha)}(\widetilde{y}_t - \widetilde{y}_{t-1})$	$\pi_t^W = \frac{1}{\theta_f(1-\alpha)}(\widetilde{y}_t - \widetilde{y}_{t-1})$
Flexible prices:	$\widehat{w}_t = -\frac{\alpha}{1-\alpha}\widehat{y}_t + z_t$	$\widehat{w}_t = \left(\frac{\sigma(1-\alpha)+\gamma}{1-\alpha}\right)\widehat{y}_t - \gamma z_t - \chi_t$

Table 2. Numerical values of parameters

Utility function		
$\beta = 0.99$	$\sigma = 1.5$	$\gamma = 1.5$
$\rho_\chi = 0.80$	$\sigma_{\varepsilon^\chi} = 0.0140$	
Production function		
$\alpha = 0.3$	$\rho_z = 0.95$	$\sigma_{\varepsilon^z} = 0.0085$
Wage stickiness and Dixit-Stiglitz elasticities		
$\eta = 0.75$	$\theta_h = 4.0$	$\theta_f = 4.0$
Wage indexation rule		
$\pi^{ss} = 0.0$	$\rho_\upsilon = 0.80$	$\sigma_{\varepsilon^\upsilon} = 0.004$
Slope coefficients of the wage inflation eq.		
$\kappa_h = 0.050$	$\kappa_f = 0.159$	
Output gap weights for monetary policy design		
$\lambda_h = 0.018$	$\lambda_f = 0.057$	

Table 3. Business cycle statistics.

	$\widehat{y}$	π	π^W	$\widehat{w}$	R	$\widetilde{y}$
Annualized standard deviations (%):						
Households set wages:	2.58	2.62	0.74	2.18	2.22	1.88
Firms set wages:	1.88	4.01	0.33	3.06	1.22	0.63
Coefficients of autocorrelation:						
Households set wages:	.95	-.03	.57	.95	.63	.96
Firms set wages:	.93	.20	.37	.94	.71	.93
Correlation with output:						
Households set wages:	1.0	-.08	-.10	.30	-.14	.73
Firms set wages:	1.0	-.12	.06	.81	.17	.34

Table 4. Long-run variance decomposition.

	$\widehat{y}$	π	π^W	$\widehat{w}$	R	$\widetilde{y}$
Households set wages:						
Technology shocks, z_t	.62	.86	.00	.93	.01	.00
Preference shocks, χ_t	.20	.14	.00	.04	.73	.00
Indexation shocks, υ_t	.18	$<.01$	1.0	.03	.26	1.0
Firms set wages:						
Technology shocks, z_t	.74	.68	.00	.75	.01	.00
Preference shocks, χ_t	.24	.11	.00	.03	.94	.00
Indexation shocks, υ_t	.02	.21	1.0	.22	.05	1.0

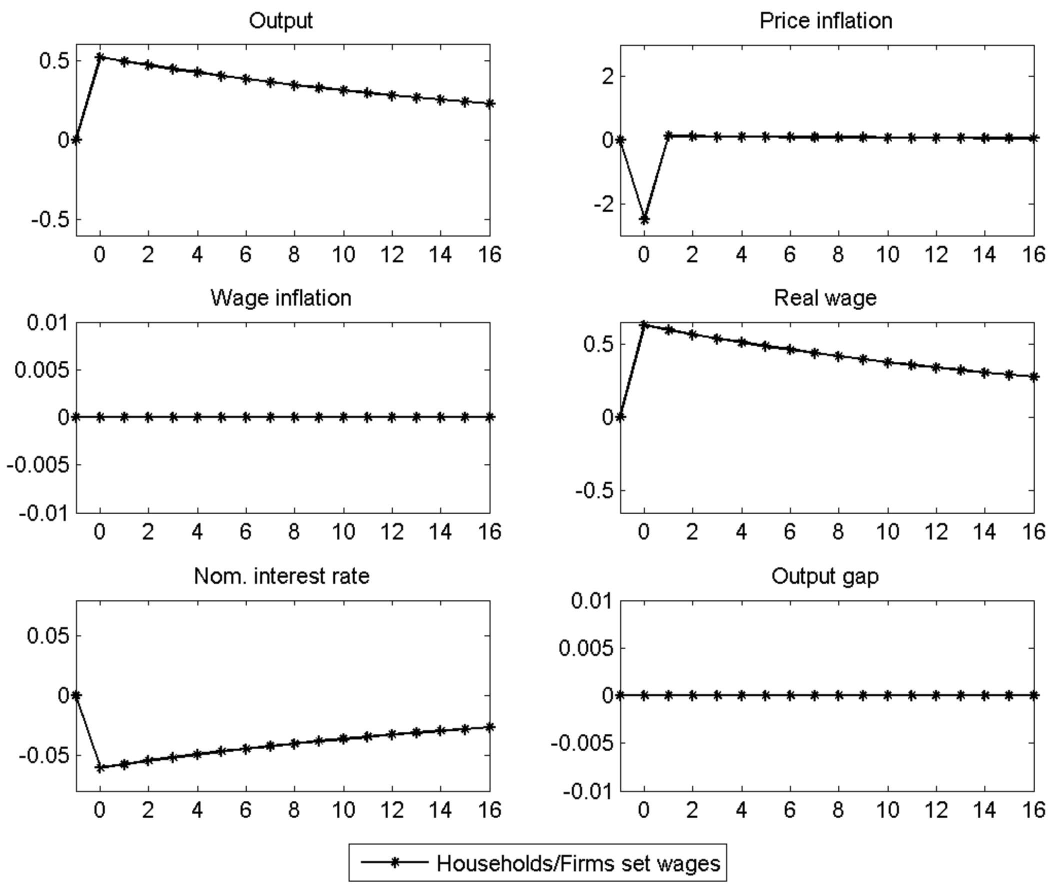

Figure 1: Impulse-response functions under optimal monetary policy. One standard deviation technology shock.

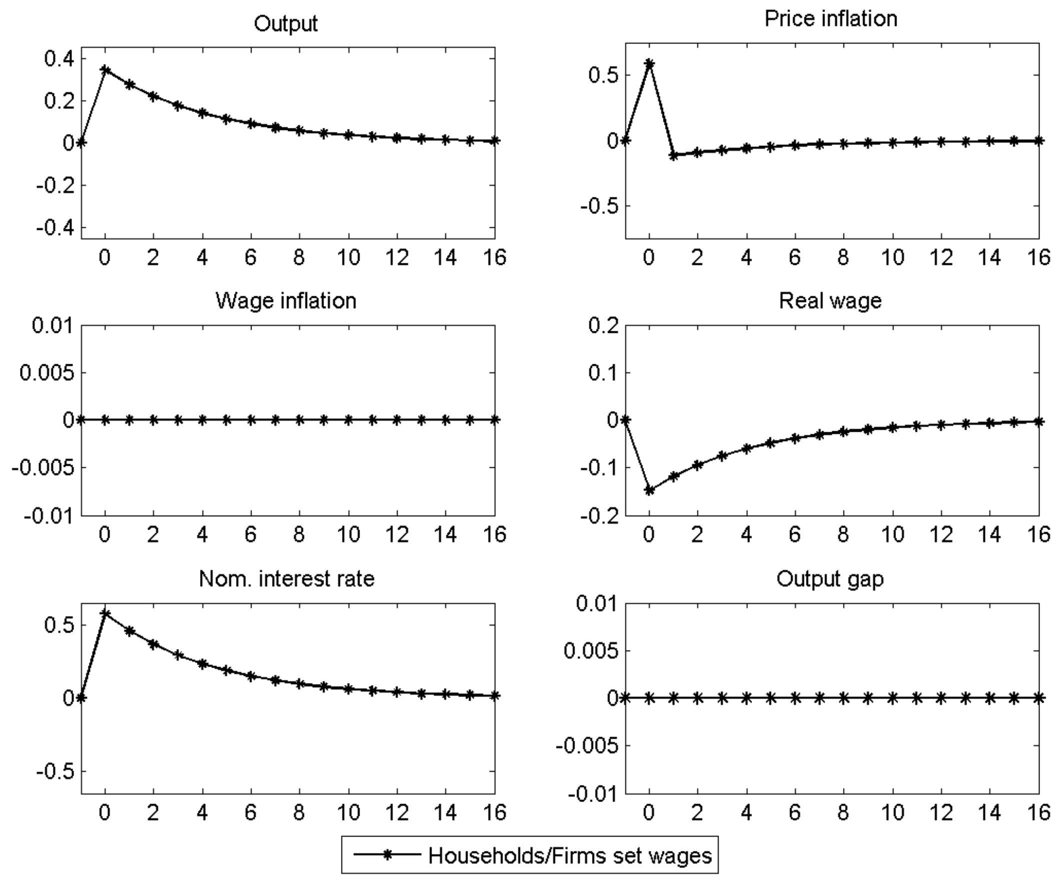

Figure 2: Impulse-response functions under optimal monetary policy. One standard deviation consumption preference shock.

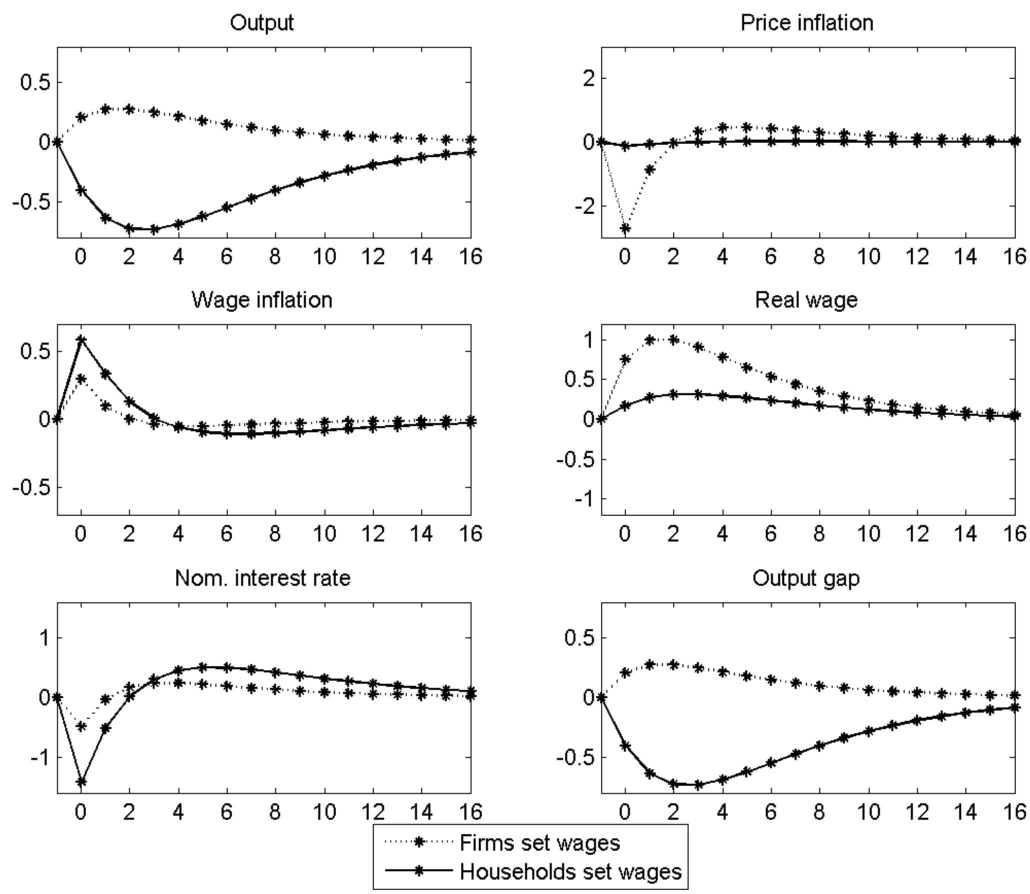

Figure 3: Impulse-response functions under optimal monetary policy. One standard deviation (nominal) wage indexation shock.