A GENERAL FRAMEWORK FOR THE MACROECONOMIC ANALYSIS OF MONETARY UNIONS*

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Abstract

The objective of this paper is to develop a general framework for the macroeconomic modelling of monetary unions, which could be useful for policy analysis, as well as for teaching purposes. Our starting point will be the standard two-country Mundell-Fleming model with perfect capital mobility, extended to incorporate the supply side, and modified so that the money market is common for two countries forming a monetary union. The model is presented in two versions: for a small and a big monetary union, respectively. After solving each model, we will derive multipliers for monetary, real (i. e., demand-side), supply, and external shocks, paying a special attention to the distinction between symmetric and asymmetric shocks. A graphical analysis is also provided.

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1. Introduction

The concept of "monetary union" has acquired a renewed interest in last years, as illustrated by the recent formation of the so-called Economic and Monetary Union (EMU) by 12 member countries of the European Union. The possibility of advancing towards a monetary union is also being discussed in other integrated economic areas, such as MERCOSUR or NAFTA.

From another point of view, establishing a monetary union has even been suggested as an alternative to a system of fixed exchange rates. As is well known, recent experiences (such as the crisis of the European Monetary System in 1992-1993, or the cases of Mexico and the South-Eastern Asian countries at the end of 1994 and 1997, respectively) have shown the increasing difficulty for a country to build the reputation needed to sustain a fixed exchange rate system. The ultimate reason is the spectacular growth of world capital markets, following the continuous liberalization and deregulation of capital movements that occurred in last years. So, if a government's compromise of maintaining a certain exchange rate is not believed as credible by financial markets, huge speculative attacks will take place, so that central banks will find extremely difficult to respond to a speculative attack at such a massive scale. All this has led to some authors (e. g., Obstfeld and Rogoff, 1995) to suggest that, in the near future, the choice faced by a country would be either maintaining a flexible exchange rate or adopting a common currency, rather than a fixed exchange rate, with other related countries.

However, macroeconomic models of monetary unions are not very frequent in the literature. Indeed, monetary unions are not properly described by either a fixed or a flexible exchange rate system, so that a specific framework would be required. The reason is that, on the one hand, the formation of a monetary union means the adoption of the same currency for all the countries concerned (which amounts to establishing a fixed exchange rate between the common currency and the old national currencies); but, on the other hand, the exchange rate between the common currency and the rest of currencies in the world will be (usually) flexible. This is particularly important in the face of country-specific or *asymmetric* shocks (i. e., those affecting to some of the monetary union's members, but not to others). However, as we will see, in the face of common or *symmetric* shocks (i. e., those equally affecting to all the monetary union's members), their macroeconomic effects will coincide with those derived from a conventional model in which the monetary union is taken as one country.

As mentioned above, attempts to provide macroeconomic models for monetary unions are not common in the literature. A pioneering contribution is Levin (1983), who develops a model for the analysis of stabilisation policy in a currency area, but only the demand-side of the model is considered. Also, Marston (1984) discusses the choice between a flexible exchange rate and an exchange rate union, following several alternative shocks, in a model where (unlike Levin's) the supply side is incorporated. A related analysis is that of Läufer and Sundararajan (1994), who develop a three-country model, with a fixed exchange rate among two of them, and a flexible exchange rate towards the third country, and study the international transmission of several economic disturbances (demand-side, monetary, and third-country shocks). A common feature to all these papers is that they consider the case of a small monetary union (or two small fixed exchange-rate countries, in the case of Läufer and Sundararajan), i. e., the case in which the rest of the world's variables are taken as exogenous.

A very interesting contribution to the modelling of monetary unions is De Bonis (1994), who discusses the effectiveness of monetary and fiscal policies in two alternative models designed, respectively, for a small and a big monetary union (i. e., when the rest of the world's variables are made endogenous). However, the supply-side of the model is not fully specified (in particular, price interactions between the union's member countries are omitted for simplicity). In addition, since the analysis of monetary and fiscal policies is mainly focused on their effects on the whole union's economy, the crucial distinction between symmetric and asymmetric shocks is neglected.

To summarise, and despite the growing (and potentially increasing) importance of monetary unions in the real world, the literature specifically addressed to the macroeconomic modelling of monetary unions is rather scarce and incomplete. The objective of this paper is to develop a general framework for the macroeconomic modelling of monetary unions, trying to make compatible both realism and tractability, which could be useful for policy analysis, as well as for teaching purposes.

Our starting point will be the standard two-country Mundell-Fleming model with perfect capital mobility, extended to incorporate the supply side in a context of rigid real wages [see Mundell (1964) and Sachs (1980)]. This basic model will be modified so that the money market becomes common for the two economies analysed, which form a monetary union, and presented in two versions: for a small and a big monetary union. After solving each model, we will derive multipliers for monetary, real (i. e., demand-side), supply, and external shocks, paying a special attention to the distinction between symmetric and asymmetric shocks.

The paper is organised as follows. The basic reference model is developed in section 2, and the analysis for a small and a big monetary union is presented in sections 3 and 4, respectively. The main conclusions of the paper are summarised in section 5.

2. The model

2. 1. Description of the model

The model in this section will be linear in logs, with Greek letters (all of them taken to be positive numbers) denoting multipliers, and asterisks denoting rest of the world's variables; time subscripts are omitted for simplicity. Perfect capital mobility is assumed, and the exchange rate is flexible. Regarding expectations, we will make the simplifying assumption that, for any variable *x*:

$$x^E = x_{-1}$$

that is, the expected value of a variable equals that variable in the previous period. Since we are in a static context, this assumption will allow us to identify real and nominal interest rates, and domestic and foreign interest rates (due to the assumption of perfect capital mobility; see below).

The demand side of the model is straightforward and given by:

$$y = -\alpha i + \beta(e + p^* - p) + \gamma y^* + f \quad (1)$$

$$m - p = \delta y - \varepsilon i \quad (2)$$

$$i = i^* = i_W \quad (3)$$

Equation (1) is the equilibrium condition in the goods market, where real output *y* depends negatively on the interest rate *i*, and positively on the real exchange rate $e + p^* - p$ (being *e*, p^* and *p*, respectively, the nominal exchange rate - defined as the domestic currency price of a unit of foreign currency-, and the foreign and domestic price levels), the rest of the world's output y^*, and a real, positive, aggregate demand shock *f* (which can include, for instance, the effect of a higher public sector deficit). Equation (2) is the equilibrium condition in the money market, where real money balances (being *m* the nominal money supply) equal the demand for money, which depend positively on output and negatively on the interest rate. Finally, equation (3) is the condition for perfect capital mobility, so that domestic and foreign interest rates would be equal to the world interest rate, i_W, common to all countries.

Regarding aggregate supply, we follow a "New Keynesian" approach [as in, e. g., Layard, Nickell and Jackman (1991) or Carlin and Soskice (1990)]. The supply side of the model includes a wage equation, a price equation, and a relationship between output and employment:

$$w = p_C^E - \varphi u + \phi prod + z^w \quad (4)$$

$$p = w - \phi prod + z^p \quad (5)$$

$$n = y - prod \quad (6)$$

Equation (4) shows that the nominal wage *w* is fully indexed to the expected value of the consumer price index p_C^E, and also depends negatively on the unemployment rate *u*, and positively on productivity *prod* and wage pressure factors summarised in z^w. According to equation (5), prices are set by adding a margin, which depends on several variables summarised in z^p, on average costs[1]. And equation (6) defines employment *n*, as the difference between real output and productivity.

[1] As usual, the coefficients on the variable *prod* are the same in the wage and price equations in order to avoid that productivity would affect unemployment in the long run; see Layard, Nickell and Jackman (1991).

On the other hand, as explained above, expectations on the consumer price index will equal the level of the consumption price index p_C in the previous period:

$$p_C^E = p_{C,-1} \qquad (7)$$

where the subscript –1 denotes the value of a variable the period before. The supply side of the model should be completed with the definitions of the consumer price index, as a weighted average of domestic and foreign prices, the latter valued in domestic currency:

$$p_C = \sigma p + (1-\sigma)(p^* + e) \qquad (8)$$

and the rate of unemployment, as the difference between active population l and employment:

$$u = l - n \qquad (9)$$

Now, from (4) to (9), we can get the expression of the aggregate supply equation:

$$y = \frac{1}{\varphi} p - \frac{\sigma}{\varphi} p_{-1} - \frac{(1-\sigma)}{\varphi}(p_{-1}^* + e_{-1}) - s \qquad (10)$$

where s is a contractionary supply shock summarising all the possible supply shocks considered above:

$$s = \frac{z^p + z^w}{\varphi} - l - prod$$

Finally, the long-run aggregate supply equation would be given by:

$$y = -\frac{(1-\sigma)}{\varphi}(p^* + e - p) - s \qquad (10')$$

In this way, our basic model is made up of equations (1) to (3), and (10) or (10'), for the short and the long run, respectively. The advantage of this specification is that it will allow us to study separately the immediate or impact effect of a shock, and its final effect, once prices have adjusted to the new equilibrium.

2. 2. A macroeconomic model for a monetary union

Now we will assume that the above model describes a monetary union, i. e., a group of countries that have decided abolishing their national currencies and adopting a new currency, common to all of them; the exchange rate against the rest of the world is assumed to be flexible. In order to make things as simple as possible, we will assume that the monetary union is made up of two symmetric countries, denoted by the subscripts 1 and 2; and that each variable of the union is a weighted average of the corresponding variables of countries 1 and 2, with the weights equal to ½. In other words, for any variable x:

$$x = \frac{1}{2}(x_1 + x_2)$$

Therefore, if we write the union's equations in terms of countries 1 and 2, the model for the monetary union will be given by[2]:

$$y_1 = -\alpha i_W + \beta(e + p^* - p_1) + \beta(p_2 - p_1) + \gamma y^* + \gamma(y_2 - y_1) + f_1 \qquad (11)$$

$$y_2 = -\alpha i_W + \beta(e + p^* - p_2) + \beta(p_1 - p_2) + \gamma y^* + \gamma(y_1 - y_2) + f_2 \qquad (12)$$

[2] As can be proved easily, from the weighted sum of equations (11) and (12), (14) and (15), and (14') and (15'), we can get, respectively, equations (1) (once replaced (3)), (10), and (10'); in turn, equation (13) would be a transformation of equation (2) (once replaced (3)).

$$m-\frac{1}{2}(p_1+p_2)=\frac{\delta}{2}(y_1+y_2)-\varepsilon i_W \qquad (13)$$

$$y_1=\frac{1}{\varphi}p_1-\frac{\sigma}{2\varphi}(p_{1,-1}+p_{2,-1})-\frac{(1-\sigma)}{\varphi}(p^*_{-1}+e_{-1})-s_1 \qquad (14)$$

$$y_2=\frac{1}{\varphi}p_2-\frac{\sigma}{2\varphi}(p_{1,-1}+p_{2,-1})-\frac{(1-\sigma)}{\varphi}(p^*_{-1}+e_{-1})-s_2 \qquad (15)$$

while, in the long run, equations (14) and (15) should be replaced by:

$$y_1=\frac{(2-\sigma)}{2\varphi}p_1-\frac{\sigma}{2\varphi}p_2-\frac{(1-\sigma)}{\varphi}(p^*+e)-s_1 \qquad (14')$$

$$y_2=\frac{(2-\sigma)}{2\varphi}p_2-\frac{\sigma}{2\varphi}p_1-\frac{(1-\sigma)}{\varphi}(p^*+e)-s_2 \qquad (15')$$

being $s_1=\frac{z_1^p+z_1^w}{\varphi}-l_1-prod_1$, and $s_2=\frac{z_2^p+z_2^w}{\varphi}-l_2-prod_2$.

If we take the rest of the world's variables (y^*, p^*, and i_W) as exogenous, the model given by equations (11) to (15) would represent the case of a *small monetary union*, i. e., when (in analogy to the small open economy case) the union is so small that is unable to affect the economic conditions of the rest of the world. In this case, the model of the monetary union has five endogenous variables: y_1, y_2, p_1, p_2, and e.

However, we could alternatively assume that the union is big enough to influence the economic conditions of the rest of the world. This would be the case of a *big monetary union*, where the rest of the world's variables become endogenous, so that the economy of the rest of the world should be explicitly modelled. Assuming an analogous framework to that of the union, and writing the union's variables in terms of countries 1 and 2, the rest of the world's equations would be:

$$y^*=-\alpha i_W-\beta(e+p^*)+\frac{\beta}{2}(p_1+p_2)+\frac{\gamma}{2}(y_1+y_2)+f^* \qquad (16)$$

$$m^*-p^*=\delta y^*-\varepsilon i_W \qquad (17)$$

$$y^*=\frac{1}{\varphi}p^*-\frac{\sigma}{\varphi}p^*_{-1}-\frac{(1-\sigma)}{2\varphi}(p_{1,-1}+p_{2,-1})+\frac{(1-\sigma)}{\varphi}e_{-1}-s^* \qquad (18)$$

with equation (18) replaced in the long run by:

$$y^*=\frac{(1-\sigma)}{\varphi}(p^*+e)-\frac{(1-\sigma)}{2\varphi}(p_1+p_2)-s^* \qquad (18')$$

being $s^*=\frac{z^{*p}+z^{*w}}{\varphi}-l^*-prod^*$.

In this way, the model of the big monetary union would be given by equations (11) to (18), and the eight endogenous variables would be: y_1, y_2, p_1, p_2, y^*, p^*, e, and i_W.

2. 3. Characterisation of the shocks

In this paper we are going to examine the effects of different shocks on the endogenous variables of the model presented above. To that end, we need to characterise the kind of shocks to be analysed in the rest of the paper.

From the perspective of the monetary union, shocks will be characterised following two criteria:

(i) In which sector of the economy the shock occurs, so that we can distinguish among *monetary*, *real* (i. e., aggregate demand), *supply*, and *external* shocks, when the shock occurs in the money market, the goods market, the supply side, and the foreign sector of the economy, respectively.

(ii) Whether the shock affects equally to all the countries belonging to the monetary union, i. e., the case of a common or *symmetric* shock; or, alternatively, whether the shock first occurs in a particular country, and so affects differently to every member country of the union, i. e., the case of a country-specific or *asymmetric* shock.

According to this classification, monetary and external shocks would be always symmetric; unlike real and supply shocks, which can be either asymmetric or symmetric[3].

Due to our assumption of perfectly symmetric countries, the effect of a symmetric shock would be the same both on every member country of the union, and on the union as a whole. As we will see in the next two sections, the effect of a symmetric shock in our models would be equivalent to the results derived from conventional models in which the monetary union is taken as one country [i. e., the Sachs (1980) model for the small monetary union case, and its two-country counterpart for the big monetary union case].

In turn, the effect of an asymmetric shock would be the same on any member country of the union where the shock first occurs, and also the same on any member country of the union where the shock is later transmitted; however, the effect would be different for the country of origin of the shock, when compared to the country where the shock is transmitted. The reason would be that an asymmetric shock first occurring in one of the countries of the union can be transmitted to the other either with the same sign (which is sometimes called the "locomotive effect"), or with the opposite sign (so that the shock would be "beggar-thy-neighbour"), depending on the channel of transmission (the aggregate demand, or the interest rate and the real exchange rate, respectively).

On the other hand, the effect of an asymmetric shock on the union as a whole would have the same sign than in the country where the shock first occurs, and would be equal to one half of the sum of the effects on every member country[4]. Notice that the effect on the union would be greater, in absolute value, than one half the effect on the country of origin of the shock, for the "locomotive effect" case; and lower, in absolute value, if the shock is "beggar-thy-neighbour".

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3 Strictly speaking, there can be asymmetric monetary shocks, affecting to the demand for money in only one of the member countries of the union (remember that money supply is commonly determined in a monetary union). However, it can be shown that the effect of this kind of shock would be the same on both countries and on the union as a whole, and equal to one half of the effect of a (symmetric) money supply shock. The ultimate reason would be that the money market is common to all the member countries of the union. In other words, an asymmetric money demand shock would work in practice as a symmetric shock.

4 Recall that this result derives from our assumption of perfectly symmetric countries. If the shock was "beggar-thy-neighbour", the higher the weight of the country to which the shock is transmitted, the lower would be the effect of the shock on the union as a whole; and, if the weight of that country was very high, the shock might be "beggar-thy-neighbour" even for the whole union.

Next, we will introduce some terminology. From now on, any shock d will be denoted

$$d \neq 0$$

when symmetric;

$$d_1 \neq 0, \quad d_2 = 0$$

when asymmetric, originated in country 1; and

$$d_1 = 0, \quad d_2 \neq 0$$

when asymmetric, originated in country 2.

On the other hand, we will always consider the case of positively signed shocks (the case of "negative" shocks would be analogous), which will be denoted as follows:

- Monetary shocks, as $m > 0$ (reflecting an increase in money supply or, alternatively, a decrease in money demand, occurring within the monetary union).
- Real shocks, as $f_1 > 0$ or $f_2 > 0$ if asymmetric, or $f_1 = f_2 = f > 0$ if symmetric (reflecting a higher public sector deficit or, alternatively, any other exogenous increase in aggregate demand, occurring within the monetary union).
- Supply shocks, as $s_1 > 0$ or $s_2 > 0$ if asymmetric, or $s_1 = s_2 = s > 0$ if symmetric (reflecting an exogenous increase in prices or wages, a fall in active population or productivity, and, in general, any contractionary shock affecting the supply side of the monetary union).
- External shocks, as $\Delta y^* > 0$, $\Delta p^* > 0$, or $\Delta i_W > 0$, for the small monetary union case (reflecting a positive shock to the trade balance through higher foreign output or prices, and a negative shock to capital movements through a higher world interest rate, respectively); or as $m^* > 0$, $f^* > 0$, or $s^* > 0$, for the big monetary union case (reflecting a positive foreign monetary shock, a positive foreign real shock, and a negative foreign supply shock, respectively, analogous to those previously defined for the monetary union).

In the next two sections we will analyse the effects of the different shocks considered above on the endogenous variables of the two models developed in this section, that is, those describing a small and a big monetary union, respectively. As in Sargent (1979), we will get reduced forms for the endogenous variables as a function of the shocks, and then derive multipliers, i. e., partial derivatives of the endogenous variables with respect to the shocks. Notice that, since our models are linear in logs, these multipliers will represent elasticities. For simplicity, we will only show the signs of the multipliers; their full expressions can be seen in the Appendix. A graphical analysis of the models will be also provided.

3. The model for a small monetary union

3. 1. Multipliers of the shocks

As shown in section 2, the model for a small monetary union would be given by equations (11) to (15), with five endogenous variables: y_1, y_2, p_1, p_2, and e. Recall that in this case the rest of the world's variables (y^*, p^*, and i_W) would be exogenous to the model. We will present first the short-run multipliers, obtained after solving the model given by equations (11) to (15), and then the long-run multipliers, obtained from the solution of the long-run version of the model, i. e., that given by equations (11) to (13), (14') and (15').

The multipliers would be, for the different shocks analysed:

A) Monetary shocks

$$\frac{\partial y_1}{\partial m} = \frac{\partial y_2}{\partial m} = \frac{\partial y}{\partial m} > 0, \frac{\partial p_1}{\partial m} = \frac{\partial p_2}{\partial m} = \frac{\partial p}{\partial m} > 0, \frac{\partial e}{\partial m} > 0$$

in the short run, and

$$\frac{\partial y_1}{\partial m} = \frac{\partial y_2}{\partial m} = \frac{\partial y}{\partial m} = 0, \frac{\partial p_1}{\partial m} = \frac{\partial p_2}{\partial m} = \frac{\partial p}{\partial m} = 1, \frac{\partial e}{\partial m} = 1$$

in the long run.

B) Real shocks

B. 1) Asymmetric real shocks

$$\frac{\partial y_1}{\partial f_1} = \frac{\partial y_2}{\partial f_2} > 0, \frac{\partial y_2}{\partial f_1} = \frac{\partial y_1}{\partial f_2} < 0, \frac{\partial y}{\partial f_1} = \frac{\partial y}{\partial f_2} = 0$$

$$\frac{\partial p_1}{\partial f_1} = \frac{\partial p_2}{\partial f_2} > 0, \frac{\partial p_2}{\partial f_1} = \frac{\partial p_1}{\partial f_2} < 0, \frac{\partial p}{\partial f_1} = \frac{\partial p}{\partial f_2} = 0$$

$$\frac{\partial e}{\partial f_1} = \frac{\partial e}{\partial f_2} < 0$$

in the short run, and

$$\frac{\partial y_1}{\partial f_1} = \frac{\partial y_2}{\partial f_2} > 0, \frac{\partial y_2}{\partial f_1} = \frac{\partial y_1}{\partial f_2} \underset{>}{\overset{<}{}} 0, \frac{\partial y}{\partial f_1} = \frac{\partial y}{\partial f_2} > 0$$

$$\frac{\partial p_1}{\partial f_1} = \frac{\partial p_2}{\partial f_2} \underset{>}{\overset{<}{}} 0, \frac{\partial p_2}{\partial f_1} = \frac{\partial p_1}{\partial f_2} < 0, \frac{\partial p}{\partial f_1} = \frac{\partial p}{\partial f_2} < 0$$

$$\frac{\partial e}{\partial f_1} = \frac{\partial e}{\partial f_2} < 0$$

in the long run.

B. 2) Symmetric real shocks

$$\frac{\partial y_1}{\partial f} = \frac{\partial y_2}{\partial f} = \frac{\partial y}{\partial f} = \frac{\partial p_1}{\partial f} = \frac{\partial p_2}{\partial f} = \frac{\partial p}{\partial f} = 0, \frac{\partial e}{\partial f} < 0$$

in the short run, and

$$\frac{\partial y_1}{\partial f} = \frac{\partial y_2}{\partial f} = \frac{\partial y}{\partial f} > 0, \frac{\partial p_1}{\partial f} = \frac{\partial p_2}{\partial f} = \frac{\partial p}{\partial f} < 0, \frac{\partial e}{\partial f} < 0$$

in the long run.

C) Supply shocks

C. 1) Asymmetric supply shocks

$$\frac{\partial y_1}{\partial s_1}=\frac{\partial y_2}{\partial s_2}<0,\ \frac{\partial y_2}{\partial s_1}=\frac{\partial y_1}{\partial s_2}\lessgtr 0,\ \frac{\partial y}{\partial s_1}=\frac{\partial y}{\partial s_2}<0$$

$$\frac{\partial p_1}{\partial s_1}=\frac{\partial p_2}{\partial s_2}>0,\ \frac{\partial p_2}{\partial s_1}=\frac{\partial p_1}{\partial s_2}\lessgtr 0,\ \frac{\partial p}{\partial s_1}=\frac{\partial p}{\partial s_2}>0$$

$$\frac{\partial e}{\partial s_1}=\frac{\partial e}{\partial s_2}\lessgtr 0$$

both in the short run and in the long run.

C. 2) Symmetric supply shocks

$$\frac{\partial y_1}{\partial s}=\frac{\partial y_2}{\partial s}=\frac{\partial y}{\partial s}<0,\ \frac{\partial p_1}{\partial s}=\frac{\partial p_2}{\partial s}=\frac{\partial p}{\partial s}>0,\ \frac{\partial e}{\partial s}\lessgtr 0$$

both in the short run and in the long run.

D) External shocks

D. 1) Foreign output shocks

$$\frac{\partial y_1}{\partial y^*}=\frac{\partial y_2}{\partial y^*}=\frac{\partial y}{\partial y^*}=\frac{\partial p_1}{\partial y^*}=\frac{\partial p_2}{\partial y^*}=\frac{\partial p}{\partial y^*}=0,\ \frac{\partial e}{\partial y^*}<0$$

in the short run, and

$$\frac{\partial y_1}{\partial y^*}=\frac{\partial y_2}{\partial y^*}=\frac{\partial y}{\partial y^*}>0,\ \frac{\partial p_1}{\partial y^*}=\frac{\partial p_2}{\partial y^*}=\frac{\partial p}{\partial y^*}<0,\ \frac{\partial e}{\partial y^*}<0$$

in the long run.

D. 2) Foreign price shocks

$$\frac{\partial y_1}{\partial p^*}=\frac{\partial y_2}{\partial p^*}=\frac{\partial y}{\partial p^*}=\frac{\partial p_1}{\partial p^*}=\frac{\partial p_2}{\partial p^*}=\frac{\partial p}{\partial p^*}=0,\ \frac{\partial e}{\partial p^*}=-1$$

both in the short run and in the long run.

D. 3) World interest rate shocks

$$\frac{\partial y_1}{\partial i_W}=\frac{\partial y_2}{\partial i_W}=\frac{\partial y}{\partial i_W}>0,\ \frac{\partial p_1}{\partial i_W}=\frac{\partial p_2}{\partial i_W}=\frac{\partial p}{\partial i_W}>0,\ \frac{\partial e}{\partial i_W}>0$$

in the short run, and

$$\frac{\partial y_1}{\partial i_W}=\frac{\partial y_2}{\partial i_W}=\frac{\partial y}{\partial i_W}<0,\ \frac{\partial p_1}{\partial i_W}=\frac{\partial p_2}{\partial i_W}=\frac{\partial p}{\partial i_W}>0,\ \frac{\partial e}{\partial i_W}>0$$

in the long run.

3. 2. Graphical analysis

Next, we will provide a graphical analysis of the effects derived from the different shocks considered. Our graphical apparatus will consist of:

(i) The *YY* and *LL* curves[5], linking the output levels of countries 1 and 2;

(ii) The aggregate demand functions of countries 1 and 2 AD_1 and AD_2, to be derived below; and

(iii) The short-run aggregate supply functions of countries 1 and 2 AS_1 and AS_2, that is, equations (14) and (15).

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5 These curves were first introduced by Levin (1983).

If we subtract (12) from (11), i. e., the equilibrium condition in the goods market of country 2 from the analogue for country 1, we get the *YY* curve:

$$y_1 = y_2 - \frac{3\beta}{1+2\gamma}(p_1 - p_2) + \frac{1}{1+2\gamma}(f_1 - f_2)$$

as a positively signed relationship between the output levels of countries 1 and 2. This would be so, since an increase (decrease) in country 2's output would lead to a worsening (improvement) in its trade balance, which would require, to keep the equilibrium, a depreciation (appreciation) of the exchange rate, leading in turn to an increase (decrease) in country 1's output. The slope of the curve would be equal to one given our assumption of symmetry regarding both countries' economic frameworks.

In turn, from (13), i. e., the equilibrium condition in the money market of the union, we get the *LL* curve:

$$y_1 = -y_2 - \frac{1}{\delta}(p_1 + p_2) + \frac{2}{\delta}m + \frac{2\varepsilon}{\delta}i_W$$

as a negatively signed relationship between the output levels of countries 1 and 2. This would be so, since an increase (decrease) in country 2's output would lead to an increase (decrease) in its demand for money, which would require, to keep the equilibrium, a decrease (increase) in the demand for money of country 1, and hence an decrease (increase) in that country's output. The slope of the curve would be again equal to one, now in absolute value.

If we replace y_2 from *LL* in *YY*, we get the aggregate demand function for country 1, AD_1:

$$y_1 = -\frac{1}{2}\frac{1+2\gamma+3\beta\delta}{\delta(1+2\gamma)}p_1 - \frac{1}{2}\frac{1+2\gamma-3\beta\delta}{\delta(1+2\gamma)}p_2 + \frac{1}{2}\frac{1}{1+2\gamma}(f_1 - f_2) + \frac{1}{\delta}m + \frac{\varepsilon}{\delta}i_W \quad (19)$$

and, in a similar way, replacing y_1 from *LL* in *YY*, we get the aggregate demand function for country 2, AD_2:

$$y_2 = -\frac{1}{2}\frac{1+2\gamma+3\beta\delta}{\delta(1+2\gamma)}p_2 - \frac{1}{2}\frac{1+2\gamma-3\beta\delta}{\delta(1+2\gamma)}p_1 + \frac{1}{2}\frac{1}{1+2\gamma}(f_2 - f_1) + \frac{1}{\delta}m + \frac{\varepsilon}{\delta}i_W \quad (20)$$

Finally, we will modify the *YY* and *LL* curves above, by eliminating p_1 and p_2. In this way, our modified *YY* curve follows after replacing $p_1 - p_2$ from (14) and (15):

$$y_1 = y_2 + \frac{1}{1+2\gamma+3\beta\varphi}(f_1 - f_2) - \frac{3\beta\varphi}{1+2\gamma+3\beta\varphi}(s_1 - s_2) \quad (21)$$

and, in a similar way, our modified *LL* curve follows after replacing $p_1 + p_2$ from (14) and (15):

$$y_1 = -y_2 - \frac{\sigma}{\delta+\varphi}(p_{1,-1} + p_{2,-1}) - \frac{2(1-\sigma)}{\delta+\varphi}(p^*_{-1} + e_{-1}) + \frac{2}{\delta+\varphi}m + \frac{2\varepsilon}{\delta+\varphi}i_W - \frac{\varphi}{\delta+\varphi}(s_1 + s_2)$$

(22)

Figure 1 shows the graphical apparatus used to discuss the effects of shocks in our model. The modified *YY* and *LL* curves [equations (21) and (22)] are represented in panel (a) of the figure, and the aggregate demand and aggregate supply functions AD_1 and AS_1 for country 1 [equations (19) and (14)], and AD_2 and AS_2 for country 2 [equations (20) and (15)], appear in panels (b) and (c), respectively; panel (d) is used to connect panels (a) and (c). In the rest of this section we will examine the effects of different shocks to the monetary union, in terms of the graphical apparatus shown in Figure 1. As a general rule, point 1 in the figures denotes the initial equilibrium before the shock; and points 2 and 3 denote, respectively, the short-run or transitory equilibrium after the shock occurs, and the long-run or final equilibrium following the completion of the full set of effects of the shock (due to the lags in price adjustment assumed in the supply side).

We begin with the case of an (always symmetric) expansionary monetary shock, $m > 0$, in Figure 2. Starting from point 1, this shock means an aggregate demand expansion in the short run, both in countries 1 and 2 and the union as a whole, due not only to the shock in itself, but also to the exchange rate depreciation; the LL, AD_1 and AD_2 curves shift to the right, and countries 1 and 2 stay at point 2 in the figure. However, the increase in prices in both countries, together with the exchange rate depreciation, lead to a later shift to the left of LL (fully offsetting the initial shift to the right), as well as of AS_1 and AS_2 (due to the higher wage aspirations in both countries). In the end, countries 1 and 2 move to point 3 in Figure 2, and in the long run the monetary shock would be neutral on output, with a price increase equal to the exchange rate depreciation (so that the real exchange rate would not change). The result would be the same in countries 1 and 2, as well as in the union as a whole.

The case of an asymmetric expansionary real shock in country 1, $f_1 > 0$, is depicted in Figure 3[6]. Aggregate demand expands in country 1 but contracts in country 2, since the shock has led to an exchange rate appreciation, which, despite the positive transmission of the shock through higher output in country 1, would reduce output in country 2; in turn, the exchange rate appreciation and the output contraction in country 2 would partially offset the initial expansion in country 1. In terms of the figure, the YY and AD_1 curves shift to the right, and AD_2 to the left, so that countries 1 and 2 move from point 1 to point 2. Notice, however, that the effect on the union would be nil since, due to our assumption of symmetry regarding countries 1 and 2, the demand expansion in country 1 would fully offset the demand contraction in country 2; in other words, for the union as a whole the exchange rate appreciation would fully counteract the expansionary demand shock that occurred in country 1. Next, the exchange rate appreciation, through its effect on wage setting, would lead to a rightward shift of LL, AS_1 and AS_2, and countries 1 and 2 would move to point 3 in Figure 3.

The final result would be an output increase with an ambiguous effect on prices in country 1, coupled with an ambiguous effect on output and a decrease in prices in country 2; we have assumed in the figure a final output contraction in country 2. The effect on country 2's output would depend on the relative weight of the real appreciation against the rest of the world, on the one hand, and the output expansion in country 1 together with the real depreciation against the latter, on the other hand. As for the union as a whole, output would increase (so that the expansion in country 1 would be greater than the eventual contraction in country 2, if the shock was beggar-thy-neighbour as in Figure 3) and prices would decrease (i. e., the price fall in country 2 would predominate even if prices go up in country 1).

We can derive the condition for an asymmetric real shock being beggar-thy-neighbour in the other member country of the union, from the multipliers in the Appendix.

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6 The case of an asymmetric expansionary real shock in country 2, $f_2 > 0$, would be fully analogous, with the results for countries 1 and 2 now happening in countries 2 and 1, respectively.

This condition can be expressed as a threshold value for σ, a parameter which would proxy the degree of openness of the member countries of the union (i. e., the greater σ, the lower the degree of openness), as:

$$\sigma > \frac{2\gamma + 2\beta\varphi}{2\gamma + 3\beta\varphi}$$

If the expansionary real shock would be symmetric rather than asymmetric, $f_1 = f_2 = f > 0$, we would have the situation given by Figure 4. Since the aggregate demand expansion would be exactly balanced by the exchange rate appreciation, the short-run equilibrium at point 2 in the figure will coincide with the initial equilibrium at point 1; the rightward shift of the *YY*, AD_1 and AD_2 curves due to the expansionary real shock would be fully offset by a leftward shift due to the exchange rate appreciation. As before, the appreciation of the exchange rate would shift rightwards *LL*, AS_1 and AS_2, so countries 1 and 2 would finish at point 3, with a long-run output expansion coupled with a fall in prices both for countries 1 and 2 and the union as a whole.

We turn now to the analysis of supply shocks, beginning with the case of an asymmetric contractionary supply shock in country 1, $s_1 > 0$, in Figure 5[7]. As output contracts and prices rise in country 1, the transmission of the shock to country 2 would be ambiguous, due to the uncertainty about the consequences of the shock on the exchange rate. The reason is that lower output and higher prices in country 1 would lead to opposite effects on the trade balance, and so on the exchange rate. Therefore, the effects of the shock on country 2 would be ambiguous, rendering also ambiguous the subsequent feedback effect on country 1. So, in Figure 5 the *LL*, *YY* and AS_1 curves shift to the left, accompanied with an ambiguous shift of AD_2; we have assume for simplicity no short-run effect on country 2. In any case, despite this ambiguity regarding country 2, the union as a whole would experience the same effects than country, i. e., lower output and higher prices. In the long run, higher prices would shift leftwards *LL*, AS_1 and AS_2, leading to an additional output fall and price increase in both country 1 and the union, as well as (in this case) in country 2; in general, the long-run effects on country 2 are also ambiguous.

As in the case of real shocks, we can derive the condition for an asymmetric supply shock being beggar-thy-neighbour in the other member country of the union. Expressed again as a threshold value for σ, this condition would be:

$$\sigma > \frac{2}{3}(1 - \gamma)$$

The case of a symmetric contractionary supply shock, $s_1 = s_2 = s > 0$, is shown in Figure 6. Now, output would fall and prices would rise unambiguously both in countries 1 and 2 and in the union, in the short run and in the long run; the effect on the exchange rate would be again ambiguous. In terms of the figure, the *LL*, AS_1 and AS_2 curves would shift leftwards in the short run (*YY* would simultaneously experience a leftward and a rightward shift, fully offsetting each other), and this shift would be reinforced in the long run following the rise in prices.

To conclude this section, we will refer briefly to external shocks. The effect of a positive shock to the trade balance following an increase in foreign output, $\Delta y^* > 0$, would

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7 Again, the case of an asymmetric contractionary supply shock in country 2, $s_2 > 0$, would be fully analogous, with the results for countries 1 and 2 now happening in countries 2 and 1, respectively.

be analogous to the case of a symmetric expansionary real shock shown in Figure 4. In turn, the effect of a negative shock to capital movements following an increase in the world interest rate, $\Delta i_W > 0$, would be analogous to the case of an expansionary monetary shock shown in Figure 2; the only difference would be that now the short run effects would be quantitatively smaller, so that an output fall would occur in the long run both in countries 1 and 2 and the union. Finally, a positive shock to the trade balance following an increase in foreign prices, $\Delta p^* > 0$, would have no effects both in the short run and in the long run, only leading to an exchange rate appreciation equal, in absolute value, to the initial increase in foreign prices.

4. The model for a big monetary union

4. 1. Multipliers of the shocks

Regarding the model for a big monetary union, this would be given by equations (11) to (18), with eight endogenous variables: y_1, y_2, p_1, p_2, y^*, p^*, e, and i_W. As in the case of the small monetary union, we will present first the short-run multipliers, obtained after solving the model given by equations (11) to (18), and then the long-run multipliers, obtained from the solution of the long-run version of the model, i. e., that given by equations (11) to (13), (14'), (15'), (16), (17), and (18'). The multipliers for the different shocks would be now:

A) Monetary shocks

$$\frac{\partial y_1}{\partial m} = \frac{\partial y_2}{\partial m} = \frac{\partial y}{\partial m} > 0, \ \frac{\partial p_1}{\partial m} = \frac{\partial p_2}{\partial m} = \frac{\partial p}{\partial m} > 0$$

$$\frac{\partial y^*}{\partial m} < 0, \ \frac{\partial p^*}{\partial m} < 0$$

$$\frac{\partial e}{\partial m} > 0, \quad \frac{\partial i_W}{\partial m} < 0$$

in the short run, and

$$\frac{\partial y_1}{\partial m} = \frac{\partial y_2}{\partial m} = \frac{\partial y}{\partial m} = 0, \ \frac{\partial p_1}{\partial m} = \frac{\partial p_2}{\partial m} = \frac{\partial p}{\partial m} = 1$$

$$\frac{\partial y^*}{\partial m} = 0, \ \frac{\partial p^*}{\partial m} = 0$$

$$\frac{\partial e}{\partial m} = 1, \quad \frac{\partial i_W}{\partial m} = 0$$

in the long run.

B) Real shocks

B. 1) Asymmetric real shocks

$$\frac{\partial y_1}{\partial f_1} = \frac{\partial y_2}{\partial f_2} > 0, \ \frac{\partial y_2}{\partial f_1} = \frac{\partial y_1}{\partial f_2} \lessgtr 0, \ \frac{\partial y}{\partial f_1} = \frac{\partial y}{\partial f_2} > 0$$

$$\frac{\partial p_1}{\partial f_1} = \frac{\partial p_2}{\partial f_2} > 0, \ \frac{\partial p_2}{\partial f_1} = \frac{\partial p_1}{\partial f_2} \lessgtr 0, \ \frac{\partial p}{\partial f_1} = \frac{\partial p}{\partial f_2} > 0$$

$$\frac{\partial y^*}{\partial f_1} = \frac{\partial y^*}{\partial f_2} > 0, \quad \frac{\partial p^*}{\partial f_1} = \frac{\partial p^*}{\partial f_2} > 0$$

$$\frac{\partial e}{\partial f_1} = \frac{\partial e}{\partial f_2} < 0, \quad \frac{\partial i_W}{\partial f_1} = \frac{\partial i_W}{\partial f_2} > 0$$

in the short run, and

$$\frac{\partial y_1}{\partial f_1} = \frac{\partial y_2}{\partial f_2} > 0, \ \frac{\partial y_2}{\partial f_1} = \frac{\partial y_1}{\partial f_2} \lessgtr 0, \ \frac{\partial y}{\partial f_1} = \frac{\partial y}{\partial f_2} > 0$$

$$\frac{\partial p_1}{\partial f_1} = \frac{\partial p_2}{\partial f_2} \lessgtr 0, \ \frac{\partial p_2}{\partial f_1} = \frac{\partial p_1}{\partial f_2} \lessgtr 0, \ \frac{\partial p}{\partial f_1} = \frac{\partial p}{\partial f_2} \lessgtr 0$$

$$\frac{\partial y^*}{\partial f_1} = \frac{\partial y^*}{\partial f_2} < 0, \quad \frac{\partial p^*}{\partial f_1} = \frac{\partial p^*}{\partial f_2} > 0$$

$$\frac{\partial e}{\partial f_1} = \frac{\partial e}{\partial f_2} < 0, \quad \frac{\partial i_W}{\partial f_1} = \frac{\partial i_W}{\partial f_2} > 0$$

in the long run.

B. 2) Symmetric real shocks

$$\frac{\partial y_1}{\partial f} = \frac{\partial y_2}{\partial f} = \frac{\partial y}{\partial f} > 0, \quad \frac{\partial p_1}{\partial f} = \frac{\partial p_2}{\partial f} = \frac{\partial p}{\partial f} > 0$$

$$\frac{\partial y^*}{\partial f} > 0, \quad \frac{\partial p^*}{\partial f} > 0$$

$$\frac{\partial e}{\partial f} < 0, \quad \frac{\partial i_W}{\partial f} > 0$$

in the short run, and

$$\frac{\partial y_1}{\partial f} = \frac{\partial y_2}{\partial f} = \frac{\partial y}{\partial f} > 0, \frac{\partial p_1}{\partial f} = \frac{\partial p_2}{\partial f} = \frac{\partial p}{\partial f} \lessgtr 0$$

$$\frac{\partial y^*}{\partial f} < 0, \quad \frac{\partial p^*}{\partial f} > 0$$

$$\frac{\partial e}{\partial f} < 0, \quad \frac{\partial i_W}{\partial f} > 0$$

in the long run.

C) Supply shocks

C. 1) Asymmetric supply shocks

$$\frac{\partial y_1}{\partial s_1} = \frac{\partial y_2}{\partial s_2} < 0, \frac{\partial y_2}{\partial s_1} = \frac{\partial y_1}{\partial s_2} \lessgtr 0, \frac{\partial y}{\partial s_1} = \frac{\partial y}{\partial s_2} < 0$$

$$\frac{\partial p_1}{\partial s_1} = \frac{\partial p_2}{\partial s_2} > 0, \frac{\partial p_2}{\partial s_1} = \frac{\partial p_1}{\partial s_2} \lessgtr 0, \frac{\partial p}{\partial s_1} = \frac{\partial p}{\partial s_2} > 0$$

$$\frac{\partial y^*}{\partial s_1} = \frac{\partial y^*}{\partial s_2} > 0 \, (\text{short run}), \frac{\partial y^*}{\partial s_1} = \frac{\partial y^*}{\partial s_2} < 0 \, (\text{long run}), \frac{\partial p^*}{\partial s_1} = \frac{\partial p^*}{\partial s_2} > 0$$

$$\frac{\partial e}{\partial s_1} = \frac{\partial e}{\partial s_2} \lessgtr 0, \quad \frac{\partial i_W}{\partial s_1} = \frac{\partial i_W}{\partial s_2} > 0$$

both in the short run and in the long run (except for the case of y^*).

C. 2) Symmetric supply shocks

$$\frac{\partial y_1}{\partial s} = \frac{\partial y_2}{\partial s} = \frac{\partial y}{\partial s} < 0, \frac{\partial p_1}{\partial s} = \frac{\partial p_2}{\partial s} = \frac{\partial p}{\partial s} > 0$$

$$\frac{\partial y^*}{\partial s} > 0 \, (\text{short run}), \frac{\partial y^*}{\partial s} < 0 \, (\text{long run}), \frac{\partial p^*}{\partial s} > 0$$

$$\frac{\partial e}{\partial s} \lessgtr 0, \quad \frac{\partial i_W}{\partial s} > 0$$

both in the short run and in the long run (except for the case of y^*).

D) External shocks

D. 1) Foreign monetary shocks

$$\frac{\partial y_1}{\partial m^*} = \frac{\partial y_2}{\partial m^*} = \frac{\partial y}{\partial m^*} < 0, \; \frac{\partial p_1}{\partial m^*} = \frac{\partial p_2}{\partial m^*} = \frac{\partial p}{\partial m^*} < 0$$
$$\frac{\partial y^*}{\partial m^*} > 0, \quad \frac{\partial p^*}{\partial m^*} > 0$$
$$\frac{\partial e}{\partial m^*} < 0, \quad \frac{\partial i_W}{\partial m^*} < 0$$

in the short run, and

$$\frac{\partial y_1}{\partial m^*} = \frac{\partial y_2}{\partial m^*} = \frac{\partial y}{\partial m^*} = \frac{\partial p_1}{\partial m^*} = \frac{\partial p_2}{\partial m^*} = \frac{\partial p}{\partial m^*} = 0$$
$$\frac{\partial y^*}{\partial m^*} = 0, \quad \frac{\partial p^*}{\partial m^*} = 1$$
$$\frac{\partial e}{\partial m^*} = -1, \quad \frac{\partial i_W}{\partial m^*} = 0$$

in the long run.

D. 2) Foreign real shocks

$$\frac{\partial y_1}{\partial f^*} = \frac{\partial y_2}{\partial f^*} = \frac{\partial y}{\partial f^*} > 0, \quad \frac{\partial p_1}{\partial f^*} = \frac{\partial p_2}{\partial f^*} = \frac{\partial p}{\partial f^*} > 0$$
$$\frac{\partial y^*}{\partial f^*} > 0, \quad \frac{\partial p^*}{\partial f^*} > 0$$
$$\frac{\partial e}{\partial f^*} > 0, \quad \frac{\partial i_W}{\partial f^*} > 0$$

in the short run, and

$$\frac{\partial y_1}{\partial f^*} = \frac{\partial y_2}{\partial f^*} = \frac{\partial y}{\partial f^*} < 0, \quad \frac{\partial p_1}{\partial f^*} = \frac{\partial p_2}{\partial f^*} = \frac{\partial p}{\partial f^*} > 0$$
$$\frac{\partial y^*}{\partial f^*} > 0, \quad \frac{\partial p^*}{\partial f^*} \lessgtr 0$$
$$\frac{\partial e}{\partial f^*} > 0, \quad \frac{\partial i_W}{\partial f^*} > 0$$

in the long run.

D. 3) Foreign supply shocks

$$\frac{\partial y_1}{\partial s^*} = \frac{\partial y_2}{\partial s^*} = \frac{\partial y}{\partial s^*} > 0, \quad \frac{\partial p_1}{\partial s^*} = \frac{\partial p_2}{\partial s^*} = \frac{\partial p}{\partial s^*} > 0$$
$$\frac{\partial y^*}{\partial s^*} < 0, \quad \frac{\partial p^*}{\partial s^*} > 0$$
$$\frac{\partial e}{\partial s^*} \lessgtr 0, \quad \frac{\partial i_W}{\partial s^*} > 0$$

in the short run, and

$$\frac{\partial y_1}{\partial s^*} = \frac{\partial y_2}{\partial s^*} = \frac{\partial y}{\partial s^*} < 0, \quad \frac{\partial p_1}{\partial s^*} = \frac{\partial p_2}{\partial s^*} = \frac{\partial p}{\partial s^*} > 0$$
$$\frac{\partial y^*}{\partial s^*} < 0, \quad \frac{\partial p^*}{\partial s^*} > 0$$

$$\frac{\partial e}{\partial s^*} \lessgtr 0, \quad \frac{\partial i_W}{\partial s^*} > 0$$

in the long run.

4. 2. Graphical analysis

The graphical representation of the model for the big monetary union will be analogous to the small monetary union case. First, the short-run aggregate supply functions AS_1 and AS_2, equations (14) and (15), will be used as before. Regarding the aggregate demand functions AD_1 and AD_2, we will make a modification in equations (19) and (20) above. If we get i_W from the money market equilibrium conditions for the union and the rest of the world, (13) and (17), and later replace $y + y^*$ from the goods market equilibrium conditions for the union and the rest of the world, (1) and (16), we get:

$$i_W = \frac{1}{4}\frac{(1-\gamma)}{\alpha\delta+\varepsilon(1-\gamma)}(p_1+p_2) + \frac{1}{2}\frac{(1-\gamma)}{\alpha\delta+\varepsilon(1-\gamma)}p^* + \\ + \frac{1}{4}\frac{\delta}{\alpha\delta+\varepsilon(1-\gamma)}(f_1+f_2) + \frac{1}{2}\frac{\delta}{\alpha\delta+\varepsilon(1-\gamma)}f^* - \frac{1}{2}\frac{(1-\gamma)}{\alpha\delta+\varepsilon(1-\gamma)}(m+m^*)$$

Replacing this expression for i_W in equations (19) and (20) we get the aggregate demand functions AD_1 and AD_2 to be used in this section:

$$y_1 = -\frac{1}{4}\frac{(1+2\gamma)(2\alpha\delta+\varepsilon(1-\gamma))+6\beta\delta(\alpha\delta+\varepsilon(1-\gamma))}{(1+2\gamma)\delta(\alpha\delta+\varepsilon(1-\gamma))}p_1 - \\ -\frac{1}{4}\frac{(1+2\gamma)(2\alpha\delta+\varepsilon(1-\gamma))-6\beta\delta(\alpha\delta+\varepsilon(1-\gamma))}{(1+2\gamma)\delta(\alpha\delta+\varepsilon(1-\gamma))}p_2 + \frac{1}{2}\frac{\varepsilon(1-\gamma)}{\delta(\alpha\delta+\varepsilon(1-\gamma))}p^* + \\ + \frac{1}{4}\frac{\varepsilon(1+2\gamma)+2(\alpha\delta+\varepsilon(1-\gamma))}{(1+2\gamma)(\alpha\delta+\varepsilon(1-\gamma))}f_1 + \frac{1}{4}\frac{\varepsilon(1+2\gamma)-2(\alpha\delta+\varepsilon(1-\gamma))}{(1+2\gamma)(\alpha\delta+\varepsilon(1-\gamma))}f_2 + \\ + \frac{1}{2}\frac{\varepsilon}{\alpha\delta+\varepsilon(1-\gamma)}f^* + \frac{1}{2}\frac{2\alpha\delta+\varepsilon(1-\gamma)}{\delta(\alpha\delta+\varepsilon(1-\gamma))}m - \frac{1}{2}\frac{\varepsilon(1-\gamma)}{\delta(\alpha\delta+\varepsilon(1-\gamma))}m^* \qquad (19')$$

and

$$y_2 = -\frac{1}{4}\frac{(1+2\gamma)(2\alpha\delta+\varepsilon(1-\gamma))+6\beta\delta(\alpha\delta+\varepsilon(1-\gamma))}{(1+2\gamma)\delta(\alpha\delta+\varepsilon(1-\gamma))}p_2 - \\ -\frac{1}{4}\frac{(1+2\gamma)(2\alpha\delta+\varepsilon(1-\gamma))-6\beta\delta(\alpha\delta+\varepsilon(1-\gamma))}{(1+2\gamma)\delta(\alpha\delta+\varepsilon(1-\gamma))}p_1 + \frac{1}{2}\frac{\varepsilon(1-\gamma)}{\delta(\alpha\delta+\varepsilon(1-\gamma))}p^* + \\ + \frac{1}{4}\frac{\varepsilon(1+2\gamma)+2(\alpha\delta+\varepsilon(1-\gamma))}{(1+2\gamma)(\alpha\delta+\varepsilon(1-\gamma))}f_2 + \frac{1}{4}\frac{\varepsilon(1+2\gamma)-2(\alpha\delta+\varepsilon(1-\gamma))}{(1+2\gamma)(\alpha\delta+\varepsilon(1-\gamma))}f_1 + \\ + \frac{1}{2}\frac{\varepsilon}{\alpha\delta+\varepsilon(1-\gamma)}f^* + \frac{1}{2}\frac{2\alpha\delta+\varepsilon(1-\gamma)}{\delta(\alpha\delta+\varepsilon(1-\gamma))}m - \frac{1}{2}\frac{\varepsilon(1-\gamma)}{\delta(\alpha\delta+\varepsilon(1-\gamma))}m^* \qquad (20')$$

The YY curve is given again by equation (21). Finally, replacing $p + p^*$ from the short-run aggregate supply functions for the union and the rest of the world, (10) and (18), in the above expression for i_W, we get:

$$i_W = \frac{1}{4}\frac{(1-\gamma)}{\alpha(\delta+\varphi)+\varepsilon(1-\gamma)}(p_{1,-1}+p_{2,-1}) + \frac{1}{2}\frac{(1-\gamma)}{\alpha(\delta+\varphi)+\varepsilon(1-\gamma)}p^*_{-1} +$$
$$+\frac{1}{4}\frac{\delta+\varphi}{\alpha(\delta+\varphi)+\varepsilon(1-\gamma)}(f_1+f_2) + \frac{1}{2}\frac{\delta+\varphi}{\alpha(\delta+\varphi)+\varepsilon(1-\gamma)}f^* -$$
$$-\frac{1}{2}\frac{(1-\gamma)}{\alpha(\delta+\varphi)+\varepsilon(1-\gamma)}(m+m^*) + \frac{1}{4}\frac{\varphi(1-\gamma)}{\alpha(\delta+\varphi)+\varepsilon(1-\gamma)}(s_1+s_2) + \frac{1}{2}\frac{\varphi(1-\gamma)}{\alpha(\delta+\varphi)+\varepsilon(1-\gamma)}s^*$$

which, once replaced in equation (22) gives us the expression for the LL curve to be used in this section:

$$y_1 = -y_2 - \frac{1}{2}\frac{1}{\delta+\varphi}\frac{2\sigma\alpha(\delta+\varphi)-(1-2\sigma)\varepsilon(1-\gamma)}{\alpha(\delta+\varphi)+\varepsilon(1-\gamma)}(p_{1,-1}+p_{2,-1}) -$$
$$-\frac{1}{\delta+\varphi}\frac{2(1-\sigma)\alpha(\delta+\varphi)+(1-2\sigma)\varepsilon(1-\gamma)}{\alpha(\delta+\varphi)+\varepsilon(1-\gamma)}p^*_{-1} - \frac{2(1-\sigma)}{\delta+\varphi}e_{-1} +$$
$$+\frac{1}{2}\frac{\varepsilon}{\alpha(\delta+\varphi)+\varepsilon(1-\gamma)}(f_1+f_2) + \frac{\varepsilon}{\alpha(\delta+\varphi)+\varepsilon(1-\gamma)}f^* + \qquad (22')$$
$$+\frac{1}{\delta+\varphi}\frac{2\alpha(\delta+\varphi)+\varepsilon(1-\gamma)}{\alpha(\delta+\varphi)+\varepsilon(1-\gamma)}m - \frac{1}{\delta+\varphi}\frac{\varepsilon(1-\gamma)}{\alpha(\delta+\varphi)+\varepsilon(1-\gamma)}m^* -$$
$$-\frac{1}{2}\frac{\varphi}{\delta+\varphi}\frac{2\alpha(\delta+\varphi)+\varepsilon(1-\gamma)}{\alpha(\delta+\varphi)+\varepsilon(1-\gamma)}(s_1+s_2) + \frac{\varphi}{\delta+\varphi}\frac{\varepsilon(1-\gamma)}{\alpha(\delta+\varphi)+\varepsilon(1-\gamma)}s^*$$

Next, we can examine the effects of the different shocks using our graphical representation. Beginning again with the case of an expansionary monetary shock within the union, $m > 0$, the effects of this shock on the countries of the union are fully analogous to those occurring in the small monetary union, shown in Figure 2 above. Now, the depreciation of the exchange rate of the union means an appreciation of the foreign exchange rate, which would lead to a demand contraction in the rest of the world, with falling output and prices. This in turn would mean a lower short-run expansion in the union, via lower external demand and a lower real exchange rate depreciation. In the long run, output would come back to its initial level in the union, with prices rising by the same amount than the exchange rate depreciation, whereas in the rest of the world the short-run demand expansion in the union would exactly offset the initial contraction. In other words, a monetary shock within the union would be beggar-thy-neighbour in the short run, and would have no effect on the rest of the world in the long run.

Now we turn to the case of an asymmetric expansionary real shock in country 1, $f_1 > 0$, as shown in Figure 7; again, the effects of an asymmetric expansionary real shock in country 2, $f_2 > 0$, would be their mirror image. Although the results for country 1 are similar to those for the small monetary union, the exchange rate appreciation means now a depreciation of the foreign exchange rate, leading to a demand expansion in the rest of the world. This would increase foreign output and prices so that, via higher external demand and a lower real exchange rate appreciation, the effect on country 2's output would be ambiguous; together with the increase in country 1's output, this would mean an increase in the union's output as well. In terms of Figure 7, the YY, LL and AD_1 curves shift to the right, being ambiguous the shift of AD_2; in the figure, however, we have assumed a leftward shift (which implies that LL shifts less than YY), so that country 2's output falls and countries 1 and 2 move from point 1 to point 2.

From the multipliers in the Appendix, we can derive the condition for an asymmetric real shock being beggar-thy-neighbour in the other member country of the union, as:

$$\sigma > \frac{\beta\varphi - 1}{3\beta\varphi - 1}$$

In the rest of the world, the exchange rate depreciation leads to a contraction in its aggregate supply, so that foreign output now falls, lowering the long-run output increase both in country 1 and in the union; the effect on country 2's output would be still ambiguous. In Figure 7, the LL, AS_1 and AS_2 curves shift rightwards and countries 1 and 2 would move to point 3[8].

The case of a symmetric expansionary real shock within the union, $f_1 = f_2 = f > 0$, is depicted in Figure 8. As in the case of the asymmetric shock, the exchange rate appreciation in the union means a depreciation of the foreign exchange rate, which leads to a demand expansion in the rest of the world, and hence to a short-run demand expansion in the union too. In the figure, the LL, AD_1 and AD_2 curves shift rightwards, countries 1 and 2 move from point 1 to point 2, and the appreciation of the exchange rate then shifts rightwards LL, AS_1 and AS_2, so countries 1 and 2 finish at point 3. As before, due to the final contraction in foreign output, the long-run output expansion in countries 1 and 2 and in the union would be lower than in the small monetary union; and the effect on prices would be ambiguous.

Therefore, as we have seen, and regarding their effects on the rest of the world, an (asymmetric or symmetric) real shock within the union would be locomotive in the short run, and beggar-thy-neighbour in the long run.

On the other hand, the effects of supply shocks within the union would be analogous to those analysed in the small monetary union case. As before, the condition for an asymmetric supply shock being beggar-thy-neighbour in the other member country of the union would be:

$$\sigma > \frac{\beta\varphi(1-\gamma) - (1+2\gamma)(1+\gamma)}{[3\beta\varphi - (1+2\gamma)](1+\gamma)}$$

And, regarding their effects on the rest of the world, they would coincide with those of real shocks, i. e., they would be locomotive in the short run, and beggar-thy-neighbour in the long run.

We conclude this section by analysing external shocks. An expansionary monetary shock in the rest of the world, $m^* > 0$, shown in Figure 9, leads to a foreign demand expansion together with an exchange rate depreciation in the rest of the world, i. e., an appreciation in the union, which contracts demand in the short run. So, the LL, AD_1 and

[8] Notice that, if y_2 would have increased in the short run, it would increase in the long run too, and the increase in y_1 would have been lower (AS_1 would have shifted by less). Otherwise, if y_2 decreases in the short run, the long run effect would be ambiguous, and the increase in y_1 greater (AS_1 would have shifted by more).

AD_2 curves shift to the left in the figure, and countries 1 and 2 move from point 1 to point 2, with lower output and prices. Later on, the short-run demand expansion in the rest of the world would lead to a demand expansion in the union fully offsetting the previous contraction, at the same time that foreign output comes back to its initial level and prices rise by the same amount than the foreign depreciation. In Figure 9 the LL, AD_1 and AD_2 curves come back to their initial position in the figure, so that output and prices in countries 1 and 2 and in the union would be unchanged in long run. Notice that the effects of this shock would be equivalent in the long run to those of an increase in foreign prices in the small monetary union case. Again, as regards the union, a monetary shock occurring in the rest of the world would have been beggar-thy-neighbour in the short run, with no effect on the union in the long run.

Figure 10 shows the case of an expansionary real shock in the rest of the world, $f^* > 0$. This shock would mean a foreign demand expansion coupled with an exchange rate appreciation in the rest of the world, i. e., a depreciation in the union. Demand expands in the union, leading to higher output and prices in the short run; the LL, AD_1 and AD_2 curves shift rightwards, and countries 1 and 2 move from point 1 to point 2. Next, the combination of higher prices in the union and in the rest of the world, and the exchange rate depreciation, contracts aggregate supply in the union and expands it in the rest of the world. Output falls and prices rise in the union, and output rises in the rest of the world, with an ambiguous effect on prices. In Figure 9 the LL, AS_1 and AS_2 curves shift to the left, and countries 1 and 2 finish in point 3. Notice that the effects of this shock would be equivalent to those of an increase in the world interest rates examined in the case of a small monetary union. And regarding their effects on the union, a real shock occurring in the rest of the world would have been locomotive in the short run, and beggar-thy-neighbour in the long run.

Finally, the effects of a contractionary supply shock in the rest of the world, $s^* > 0$, would be analogous to those of an expansionary real shock in the rest of the world, and can be followed from Figure 10.

5. Conclusions

In this paper we have developed a general framework for the macroeconomic modelling of monetary unions, which could be useful for policy analysis as well as for teaching purposes. The interest in modelling monetary unions can be justified since they have been suggested as an alternative to a system of fixed exchange rates (given the fragility of the latter in a world of very high capital mobility), as illustrated by the recent moving to EMU.

We use as reference framework the standard two-country Mundell-Fleming model with perfect capital mobility, extended to incorporate the supply side in a context of rigid real wages. This model is modified so that the money market becomes common for two symmetric countries that form a monetary union, and keep a flexible exchange rate against the rest of the world. The basic model is presented in two versions: for a small and a big monetary union (i. e., if the rest of the world's variables are taken as exogenous or not), and is solved in two stages: the short and the long run (i. e., if prices have adjusted to the final equilibrium or not). The solutions to the models are presented for the different shocks analysed (monetary, real, supply, and external), both algebraically and graphically.

The results of the paper are summarised in tables 1 and 2, for the small and the big monetary union, respectively. Recall that the crucial point of our results refers to the effect of asymmetric shocks. Unlike common or symmetric shocks, which lead, in the two countries of the union, to the same effects than in a conventional model where the monetary union is taken as one country, country-specific or asymmetric shocks lead to different effects in the two countries of the union. In particular, the effect of an asymmetric shock could be transmitted to the other country with either the same or the opposite sign, when compared to the country of origin of the shock, depending on the dominant channel of transmission of that shock (the aggregate demand, or the interest rate and the real exchange rate, respectively).

References

Carlin, W. and Soskice, D. (1990): *Macroeconomics and the wage bargain: A modern approach to employment, inflation and the exchange rate*, Oxford University Press, Oxford.

De Bonis, V. (1994): *Stabilisation policy in an exchange rate union. Transmission, coordination and influence on the union cohesion*, Physica-Verlag, Heidelberg.

Läufer, N. K. A. and Sundararajan, S. (1994): "The international transmission of economic shocks in a three-country world under mixed exchange rates", *Journal of International Money and Finance* 13, 419-446.

Layard, R., Nickell, S. and Jackman, R. (1991): *Unemployment: Macroeconomic performance and the labour market*, Oxford University Press, Oxford.

Levin, J. H. (1983): "A model of stabilisation policy in a jointly floating currency area", in Bhandari, J. S. and Putnam, B. H. (eds.): *Economic interdependence and flexible exchange rates*, The MIT Press, Cambridge, MA, 329-349.

Marston, R. C. (1984): "Exchange rate unions as an alternative to flexible rates: The effects of real and monetary disturbances", in Marston, R. C. and Bilson, J. F. O. (eds.): *Exchange rate theory and practice*, The University of Chicago Press, Chicago, 407-442.

Mundell, R. A. (1964): "A reply: Capital mobility and size", *Canadian Journal of Economics and Political Science* 30, 421-431.

Obstfeld, M. and Rogoff, K. (1995): "The mirage of flexible exchange rates", *Journal of Economic Perspectives* 9, 73-96.

Sachs, J. (1980): "Wages, flexible exchange rates, and macroeconomic policy", *Quarterly Journal of Economics* 94, 731-747.

Sargent, T. J. (1979): *Macroeconomic Theory*, Academic Press, New York.

Appendix

Small monetary union

Short run

$$y_1 = \frac{1}{\varphi+\delta} m + \frac{1}{2}\frac{1}{1+2\gamma+3\beta\varphi} f_1 - \frac{1}{2}\frac{1}{1+2\gamma+3\beta\varphi} f_2 -$$

$$-\frac{1}{2}\frac{\varphi(1+2\gamma+3\beta\delta+6\beta\varphi)}{(\varphi+\delta)(1+2\gamma+3\beta\varphi)} s_1 - \frac{1}{2}\frac{\varphi(1+2\gamma-3\beta\delta)}{(\varphi+\delta)(1+2\gamma+3\beta\varphi)} s_2 + \frac{\varepsilon}{\varphi+\delta} i_w -$$

$$-\frac{1}{2}\frac{\sigma}{\varphi+\delta} p_{1,-1} - \frac{1}{2}\frac{\sigma}{\varphi+\delta} p_{2,-1} - \frac{(1-\sigma)}{\varphi+\delta} p^*_{-1} - \frac{(1-\sigma)}{\varphi+\delta} e_{-1}$$

$$y_2 = \frac{1}{\varphi+\delta} m - \frac{1}{2}\frac{1}{1+2\gamma+3\beta\varphi} f_1 + \frac{1}{2}\frac{1}{1+2\gamma+3\beta\varphi} f_2 -$$

$$-\frac{1}{2}\frac{\varphi(1+2\gamma-3\beta\delta)}{(\varphi+\delta)(1+2\gamma+3\beta\varphi)} s_1 - \frac{1}{2}\frac{\varphi(1+2\gamma+3\beta\delta+6\beta\varphi)}{(\rho+\delta)(1+2\gamma+3\beta\varphi)} s_2 + \frac{\varepsilon}{\varphi+\delta} i_w -$$

$$-\frac{1}{2}\frac{\sigma}{\varphi+\delta} p_{1,-1} - \frac{1}{2}\frac{\sigma}{\varphi+\delta} p_{2,-1} - \frac{(1-\sigma)}{\varphi+\delta} p^*_{-1} - \frac{(1-\sigma)}{\varphi+\delta} e_{-1}$$

$$p_1 = \frac{\varphi}{\varphi+\delta} m + \frac{1}{2}\frac{\varphi(1+\delta)}{(\varphi+\delta)(1+2\gamma+3\beta\varphi)} f_1 - \frac{1}{2}\frac{\varphi(1+\delta)}{(\varphi+\delta)(1+2\gamma+3\beta\varphi)} f_2 +$$

$$+\frac{1}{2}\frac{\varphi^2(1+2\gamma+3\beta\varphi)+2\varphi\delta(1+2\gamma)}{(\varphi+\delta)(1+2\gamma+3\beta\varphi)} s_1 - \frac{1}{2}\frac{\varphi^2(1+2\gamma+3\beta\varphi)}{(\varphi+\delta)(1+2\gamma+3\beta\varphi)} s_2 + \frac{\varphi\varepsilon}{\varphi+\delta} i_w +$$

$$+\frac{1}{2}\frac{\delta\sigma}{\varphi+\delta} p_{1,-1} + \frac{1}{2}\frac{\delta\sigma}{\varphi+\delta} p_{2,-1} + \frac{\delta(1-\sigma)}{\varphi+\delta} p^*_{-1} + \frac{\delta(1-\sigma)}{\varphi+\delta} e_{-1}$$

$$p_2 = \frac{\varphi}{\varphi+\delta} m - \frac{1}{2}\frac{\varphi(1+\delta)}{(\varphi+\delta)(1+2\gamma+3\beta\varphi)} f_1 + \frac{1}{2}\frac{\varphi(1+\delta)}{(\varphi+\delta)(1+2\gamma+3\beta\varphi)} f_2 -$$

$$-\frac{1}{2}\frac{\varphi^2(1+2\gamma+3\beta\varphi)}{(\varphi+\delta)(1+2\gamma+3\beta\varphi)} s_1 + \frac{1}{2}\frac{\varphi^2(1+2\gamma+3\beta\varphi)+2\varphi\delta(1+2\gamma)}{(\varphi+\delta)(1+2\gamma+3\beta\varphi)} s_2 + \frac{\varphi\varepsilon}{\varphi+\delta} i_w +$$

$$+\frac{1}{2}\frac{\delta\sigma}{\varphi+\delta} p_{1,-1} + \frac{1}{2}\frac{\delta\sigma}{\varphi+\delta} p_{2,-1} + \frac{\delta(1-\sigma)}{\varphi+\delta} p^*_{-1} + \frac{\delta(1-\sigma)}{\varphi+\delta} e_{-1}$$

$$e=\frac{1+\beta\varphi}{\beta(\varphi+\delta)}m-\frac{1}{2\beta}f_1-\frac{1}{2\beta}f_2+$$

$$+\frac{1}{2}\frac{\varphi(\beta\delta-1)}{\beta(\varphi+\delta)}s_1+\frac{1}{2}\frac{\varphi(\beta\delta-1)}{\beta(\varphi+\delta)}s_2-\frac{\gamma}{\beta}y^*-p^*+\frac{\alpha(\varphi+\delta)+\varepsilon(1+\beta\varphi)}{\beta(\varphi+\delta)}i_w+$$

$$+\frac{1}{2}\frac{\sigma(\beta\delta-1)}{\beta(\varphi+\delta)}p_{1,-1}+\frac{1}{2}\frac{\sigma(\beta\delta-1)}{\beta(\varphi+\delta)}p_{2,-1}+\frac{(1-\sigma)(\beta\delta-1)}{\beta(\varphi+\delta)}p^*_{-1}+\frac{(1-\sigma)(\beta\delta-1)}{\beta(\varphi+\delta)}e_{-1}$$

Long run

$$y_1=\frac{1}{2}\frac{(1-\sigma)(2+2\gamma+3\beta\varphi)+\beta\varphi}{[(1-\sigma)+\beta\varphi](1+2\gamma+3\beta\varphi)}f_1+\frac{1}{2}\frac{(1-\sigma)(2\gamma+3\beta\varphi)-\beta\varphi}{[(1-\sigma)+\beta\varphi](1+2\gamma+3\beta\varphi)}f_2-$$

$$-\frac{1}{2}\frac{\beta\varphi[3(1-\sigma)+(1+2\gamma)+6\beta\varphi]}{[(1-\sigma)+\beta\varphi](1+2\gamma+3\beta\varphi)}s_1-\frac{1}{2}\frac{\beta\varphi[-3(1-\sigma)+(1+2\gamma)]}{[(1-\sigma)+\beta\varphi](1+2\gamma+3\beta\varphi)}s_2+$$

$$+\frac{\gamma(1-\sigma)}{(1-\sigma)+\beta\varphi}y^*-\frac{\alpha(1-\sigma)}{(1-\sigma)+\beta\varphi}i_w$$

$$y_2=\frac{1}{2}\frac{(1-\sigma)(2\gamma+3\beta\varphi)-\beta\varphi}{[(1-\sigma)+\beta\varphi](1+2\gamma+3\beta\varphi)}f_1+\frac{1}{2}\frac{(1-\sigma)(2+2\gamma+3\beta\varphi)+\beta\varphi}{[(1-\sigma)+\beta\varphi](1+2\gamma+3\beta\varphi)}f_2-$$

$$-\frac{1}{2}\frac{\beta\varphi[-3(1-\sigma)+(1+2\gamma)]}{[(1-\sigma)+\beta\varphi](1+2\gamma+3\beta\varphi)}s_1-\frac{1}{2}\frac{\beta\varphi[3(1-\sigma)+(1+2\gamma)+6\beta\varphi]}{[(1-\sigma)+\beta\varphi](1+2\gamma+3\beta\varphi)}s_2+$$

$$+\frac{\gamma(1-\sigma)}{(1-\sigma)+\beta\varphi}y^*-\frac{\alpha(1-\sigma)}{(1-\sigma)+\beta\varphi}i_w$$

$$p_1=m-\frac{1}{2}\frac{\delta(1-\sigma)(1+2\gamma+3\beta\varphi)-\varphi[(1-\sigma)+\beta\varphi]}{[(1-\sigma)+\beta\varphi](1+2\gamma+3\beta\varphi)}f_1-\frac{1}{2}\frac{\delta(1-\sigma)(1+2\gamma+3\beta\varphi)+\varphi[(1-\sigma)+\beta\varphi]}{[(1-\sigma)+\beta\varphi](1+2\gamma+3\beta\varphi)}f_2+$$

$$+\frac{1}{2}\varphi\frac{\delta\beta(1+2\gamma+3\beta\varphi)+(1+2\gamma)[(1-\sigma)+\beta\varphi]}{[(1-\sigma)+\beta\varphi](1+2\gamma+3\beta\varphi)}s_1+\frac{1}{2}\frac{\delta\beta(1+2\gamma+3\beta\varphi)-(1+2\gamma)[(1-\sigma)+\beta\varphi]}{[(1-\sigma)+\beta\varphi](1+2\gamma+3\beta\varphi)}s_2-$$

$$-\frac{\gamma\delta(1-\sigma)}{(1-\sigma)+\beta\varphi}y^*+\frac{\alpha\delta(1-\sigma)+\varepsilon[(1-\sigma)+\beta\varphi]}{(1-\sigma)+\beta\varphi}i_w$$

$$p_2 = m - \frac{1}{2}\frac{\delta(1-\sigma)(1+2\gamma+3\beta\varphi)+\varphi[(1-\sigma)+\beta\varphi]}{[(1-\sigma)+\beta\varphi](1+2\gamma+3\beta\varphi)} f_1 - \frac{1}{2}\frac{\delta(1-\sigma)(1+2\gamma+3\beta\varphi)-\varphi[(1-\sigma)+\beta\varphi]}{[(1-\sigma)+\beta\varphi](1+2\gamma+3\beta\varphi)} f_2 +$$

$$+\frac{1}{2}\varphi\frac{\delta\beta(1+2\gamma+3\beta\varphi)-(1+2\gamma)[(1-\sigma)+\beta\varphi]}{[(1-\sigma)+\beta\varphi](1+2\gamma+3\beta\varphi)} s_1 + \frac{1}{2}\frac{\delta\beta(1+2\gamma+3\beta\varphi)+(1+2\gamma)[(1-\sigma)+\beta\varphi]}{[(1-\sigma)+\beta\varphi](1+2\gamma+3\beta\varphi)} s_2 -$$

$$-\frac{\gamma\delta(1-\sigma)}{(1-\sigma)+\beta\varphi} y^* + \frac{\alpha\delta(1-\sigma)+\varepsilon[(1-\sigma)+\beta\varphi]}{(1-\sigma)+\beta\varphi} i_w$$

$$e = m - \frac{1}{2}\frac{\delta(1-\sigma)+\varphi}{(1-\sigma)+\beta\varphi} f_1 - \frac{1}{2}\frac{\delta(1-\sigma)+\varphi}{(1-\sigma)+\beta\varphi} f_2 + \frac{1}{2}\frac{\beta\delta\varphi-\varphi}{\beta\varphi+(1-\sigma)} s_1 + \frac{1}{2}\frac{\beta\delta\varphi-\varphi}{\beta\varphi+(1-\sigma)} s_2 -$$

$$-\frac{\gamma[(1-\sigma)+\beta\varphi]}{(1-\sigma)+\beta\varphi} y^* - p^* + \frac{\alpha[\delta(1-\sigma)+\varphi]+\varepsilon[(1-\sigma)+\beta\varphi]}{(1-\sigma)+\beta\varphi} i_w$$

Big monetary union

Short run

$$y_1 = \frac{1}{2}\frac{\varepsilon(1-\gamma)+2\alpha(\varphi+\delta)}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]} m + \frac{1}{4}\frac{2\alpha(\varphi+\delta)+\varepsilon(3+\beta\varphi)}{[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)](1+2\gamma+3\beta\varphi)} f_1 + \frac{1}{4}\frac{-2\alpha(\varphi+\delta)+\varepsilon(-1+4\gamma+3\beta\varphi)}{[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)](1+2\gamma+3\beta\varphi)} f_2 -$$

$$-\frac{1}{4}\varphi\frac{6\beta\delta[\alpha\delta+\varepsilon(1-\gamma)]+\varepsilon[(1-\gamma)(1+2\gamma)]+2\alpha[(\varphi+\delta)(1+2\gamma)]+9\beta\varphi[2\alpha\delta+\varepsilon(1-\gamma)]+12\alpha\beta\varphi^2}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)](1+2\gamma+3\beta\varphi)} s_1 +$$

$$+\frac{1}{4}\varphi\frac{6\beta\delta[\alpha\delta+\varepsilon(1-\gamma)]-\varepsilon[(1-\gamma)(1+2\gamma)]-2\alpha[(\varphi+\delta)(1+2\gamma)]+3\beta\varphi[2\alpha\delta+\varepsilon(1-\gamma)]}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)](1+2\gamma+3\beta\varphi)} s_2 -$$

$$-\frac{1}{2}\frac{\varepsilon(1-\gamma)}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]} m^* + \frac{1}{2}\frac{\varepsilon}{[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]} f^* + \frac{1}{2}\frac{\varepsilon\varphi(1-\gamma)}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]} s^* +$$

$$+\frac{1}{4}\frac{\varepsilon(1-\gamma)(1-2\sigma)-2\alpha\sigma(\varphi+\delta)}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]} p_{1,-1} + \frac{1}{4}\frac{\varepsilon(1-\gamma)(1-2\sigma)-2\alpha\sigma(\varphi+\delta)}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]} p_{2,-1} -$$

$$-\frac{1}{2}\frac{\varepsilon(1-\gamma)(1-2\sigma)-2\alpha(1-\sigma)(\varphi+\delta)}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]} p^*_{-1} - \frac{(1-\sigma)}{(\varphi+\delta)} e_{-1}$$

$$y_2=\frac{1}{2}\frac{\varepsilon(1-\gamma)+2\alpha(\varphi+\delta)}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}m+\frac{1}{4}\frac{-2\alpha(\varphi+\delta)+\varepsilon(-1+4\gamma+3\beta\varphi)}{[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)](1+2\gamma+3\beta\varphi)}f_1+\frac{1}{4}\frac{2\alpha(\varphi+\delta)+\varepsilon(3+\beta\varphi)}{[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)](1+2\gamma+3\beta\varphi)}f_2+$$

$$+\frac{1}{4}\varphi\frac{6\beta\delta[\alpha\delta+\varepsilon(1-\gamma)]-\varepsilon[(1-\gamma)(1+2\gamma)]-2\alpha[(\varphi+\delta)(1+2\gamma)]+3\beta\varphi[2\alpha\delta+\varepsilon(1-\gamma)]}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)](1+2\gamma+3\beta\varphi)}s_1-$$

$$-\frac{1}{4}\varphi\frac{6\beta\delta[\alpha\delta+\varepsilon(1-\gamma)]+\varepsilon[(1-\gamma)(1+2\gamma)]+2\alpha[(\varphi+\delta)(1+2\gamma)]+9\beta\varphi[2\alpha\delta+\varepsilon(1-\gamma)]+12\alpha\beta\varphi^2}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)](1+2\gamma+3\beta\varphi)}s_2-$$

$$-\frac{1}{2}\frac{\varepsilon(1-\gamma)}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}m^*+\frac{1}{2}\frac{\varepsilon}{[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}f^*+\frac{1}{2}\frac{\varepsilon\varphi(1-\gamma)}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}s^*+$$

$$+\frac{1}{4}\frac{\varepsilon(1-\gamma)(1-2\sigma)-2\alpha\sigma(\varphi+\delta)}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}p_{1,-1}+\frac{1}{4}\frac{\varepsilon(1-\gamma)(1-2\sigma)-2\alpha\sigma(\varphi+\delta)}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}p_{2,-1}-$$

$$-\frac{1}{2}\frac{\varepsilon(1-\gamma)(1-2\sigma)-2\alpha(1-\sigma)(\varphi+\delta)}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}p^*_{-1}-\frac{(1-\sigma)}{(\varphi+\delta)}e_{-1}$$

$$p_1=\frac{1}{2}\frac{\varphi[\varepsilon(1-\gamma)+2\alpha(\varphi+\delta)]}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}m+\frac{1}{4}\frac{\varphi[3\varepsilon+2\alpha(\delta+\varphi)+3\varphi+\varepsilon\beta]}{[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)](1+2\gamma+3\beta\varphi)}f_1+\frac{1}{4}\frac{\varphi[-\varepsilon-2\alpha(\delta+\varphi)+3\varphi\varepsilon\beta+4\varepsilon\gamma]}{[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)](1+2\gamma+3\beta\varphi)}f_2-$$

$$-\frac{1}{4}\varphi\frac{6\beta\delta\varphi[\varepsilon(1-\gamma)+\delta\alpha]+3\varepsilon\varphi(1-\gamma)(1+2\gamma)+3\beta\varphi^2[\varepsilon(1-\gamma)+2\delta\alpha]+2\alpha(1+2\gamma)[\delta(3\varphi+2\delta)+\varphi^2]+4\delta\varepsilon(1-\gamma)(1+2\gamma)}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)](1+2\gamma+3\beta\varphi)}s_1-$$

$$-\frac{1}{4}\varphi\frac{6\beta\delta\varphi[\varepsilon(1-\gamma)+\delta\alpha]-\varepsilon\varphi(1-\gamma)(1+2\gamma)-2\alpha\varphi[(\varphi+\delta)(1+2\gamma)]+3\beta\varphi^2[\varepsilon(1-\gamma)+2\delta\alpha]}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)](1+2\gamma+3\beta\varphi)}s_2-$$

$$-\frac{1}{2}\frac{\varphi\varepsilon(1-\gamma)}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}m^*+\frac{1}{2}\frac{\varphi\varepsilon}{[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}f^*+\frac{1}{2}\varphi\frac{\varepsilon\varphi(1-\gamma)}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\lambda)]}s^*+$$

$$+\frac{1}{4}\frac{2\delta\sigma[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]+\varepsilon\varphi(1-\gamma)}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}p_{1,-1}+\frac{1}{4}\frac{2\delta[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]+\varphi\varepsilon(1-\gamma)}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}p_{2,-1}+$$

$$+\frac{1}{2}\frac{2\delta(1-\sigma)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]+\varphi\varepsilon(1-\gamma)}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}p^*_{-1}-\frac{\delta(1-\sigma)}{(\varphi+\delta)}e_{-1}$$

$$p_2=\frac{1}{2}\frac{\varphi[\varepsilon(1-\gamma)+2\alpha(\varphi+\delta)]}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}m+\frac{1}{4}\frac{\varphi[-\varepsilon-2\alpha(\delta+\varphi)+3\varphi\varepsilon\beta+4\varepsilon\gamma]}{[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)](1+2\gamma+3\beta\varphi)}f_1+\frac{1}{4}\frac{\varphi[3\varepsilon+2\alpha(\delta+\varphi)+3\varphi+\varepsilon\beta]}{[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)](1+2\gamma+3\beta\varphi)}f_2-$$

$$-\frac{1}{4}\varphi\frac{6\beta\delta\varphi[\varepsilon(1-\gamma)+\delta\alpha]-\varepsilon\varphi(1-\gamma)(1+2\gamma)-2\alpha\varphi[(\varphi+\delta)(1+2\gamma)]+3\beta\varphi^2[\varepsilon(1-\gamma)+2\delta\alpha]}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)](1+2\gamma+3\beta\varphi)}s_1-$$

$$-\frac{1}{4}\varphi\frac{6\beta\delta\varphi[\varepsilon(1-\gamma)+\delta\alpha]+3\varepsilon\varphi(1-\gamma)(1+2\gamma)+3\beta\varphi^2[\varepsilon(1-\gamma)+2\delta\alpha]+2\alpha(1+2\gamma)[\delta(3\varphi+2\delta)+\varphi^2]+4\delta\varepsilon(1-\gamma)(1+2\gamma)}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)](1+2\gamma+3\beta\varphi)}s_2-$$

$$-\frac{1}{2}\frac{\varphi\varepsilon(1-\gamma)}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}m^{*}+\frac{1}{2}\frac{\varphi\varepsilon}{[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}f^{*}+\frac{1}{2}\varphi\frac{\varepsilon\varphi(1-\gamma)}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\lambda)]}s^{*}+$$

$$+\frac{1}{4}\frac{2\delta[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]+\varepsilon\varphi(1-\gamma)}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}p_{1,-1}+\frac{1}{4}\frac{2\delta\sigma[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]+\varphi\varepsilon(1-\gamma)}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}p_{2,-1}+$$

$$+\frac{1}{2}\frac{2\delta(1-\sigma)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]+\varphi\varepsilon(1-\gamma)}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}p_{-1}^{*}-\frac{\delta(1-\sigma)}{(\varphi+\delta)}e_{-1}$$

$$y^{*}=-\frac{1}{2}\frac{\varepsilon(1-\gamma)}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}m+\frac{1}{4}\frac{\varepsilon}{[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}f_{1}+\frac{1}{4}\frac{\varepsilon}{[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}f_{2}+$$

$$+\frac{1}{4}\frac{\varepsilon\varphi(1-\gamma)}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}s_{1}+\frac{1}{4}\frac{\varepsilon\varphi(1-\gamma)}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}s_{2}+$$

$$+\frac{1}{2}\frac{\varepsilon(1-\gamma)+2\alpha(\varphi+\delta)}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}m^{*}+\frac{1}{2}\frac{\varepsilon}{[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}f^{*}-\frac{1}{2}\frac{\varphi[2\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}s^{*}+$$

$$+\frac{1}{4}\frac{\varepsilon(1-\gamma)(1+2\sigma)-2\alpha(1-\sigma)(\varphi+\delta)}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}p_{1,-1}+\frac{1}{4}\frac{\varepsilon(1-\gamma)(1+2\sigma)-2\alpha(1-\sigma)(\varphi+\delta)}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}p_{2,-1}-$$

$$-\frac{1}{2}\frac{\varepsilon(1-\gamma)(1-2\sigma)-2\alpha\sigma(\varphi+\delta)}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}p_{-1}^{*}-\frac{(1-\sigma)}{(\varphi+\delta)}e_{-1}$$

$$p^{*}=-\frac{1}{2}\frac{\varepsilon\varphi(1-\gamma)}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}m+\frac{1}{4}\frac{\varphi\varepsilon}{[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}f_{1}+\frac{1}{4}\frac{\varphi\varepsilon}{[\alpha(\varphi+\delta)+(1-\gamma)]}f_{2}+$$

$$+\frac{1}{4}\frac{\varepsilon\varphi^{2}(1-\gamma)}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}s_{1}+\frac{1}{4}\frac{\varepsilon\varphi^{2}(1-\gamma)}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}s_{2}+$$

$$+\frac{1}{2}\varphi\frac{2\alpha(\varphi+\delta)+(1-\gamma)}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}m^{*}+\frac{1}{2}\frac{\varphi\varepsilon}{[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}f^{*}+\frac{1}{2}\frac{\varepsilon\varphi^{2}(1-\gamma)+2\delta\varphi[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}{[\varphi+\delta][\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}s^{*}+$$

$$+\frac{1}{4}\frac{\varepsilon\varphi(1-\gamma)+2\delta(1-\sigma)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}p_{1,-1}+\frac{1}{4}\frac{\varepsilon\varphi(1-\gamma)+2\delta(1-\sigma)[\alpha(\rho+\delta)+\varepsilon(1-\gamma)]}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}p_{2,-1}+$$

$$+\frac{1}{2}\frac{\varepsilon(1-\gamma)+2\delta\sigma[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}p_{-1}^{*}-\frac{\delta(1-\sigma)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}{(\varphi+\delta)[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}e_{-1}$$

$$e=\frac{2\varphi\beta+(1+\gamma)}{2\beta(\varphi+\delta)}m-\frac{1}{4\beta}f_1-\frac{1}{4\beta}f_2+\frac{1}{4}\frac{\varphi(1+\gamma-2\beta\delta)}{\beta(\varphi+\delta)}s_1+\frac{1}{4}\frac{\varphi(1+\gamma-2\beta\delta)}{[\beta(\varphi+\delta)]}s_2-$$

$$-\frac{2\varphi\beta+(1+\gamma)}{2\beta(\varphi+\delta)}m^*+\frac{1}{2\beta}f^*+\frac{1}{2}\frac{\varphi(1+\gamma-2\beta\delta)}{\beta(\varphi+\delta)}s^*+$$

$$+\frac{1}{4}\frac{(1-2\sigma)(1+\gamma-2\beta\delta)}{\beta(\varphi+\delta)}p_{1,-1}+\frac{1}{4}\frac{(1-2\sigma)(1+\gamma-2\beta\delta)}{\beta(\varphi+\delta)}p_{2,-1}-\frac{1}{2}\frac{(1-2\sigma)(1+\gamma-2\beta\delta)}{\beta(\varphi+\delta)}p^*_{-1}-\frac{(1-\sigma)(1+\gamma-2\beta\delta)}{\beta(\varphi+\delta)}e_{-1}$$

$$i_w=-\frac{1}{2}\frac{(1-\gamma)}{[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}m+\frac{1}{4}\frac{(\varphi+\delta)}{[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}f_1+\frac{1}{4}\frac{(\varphi+\delta)}{[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}f_2+$$

$$+\frac{1}{4}\frac{\varphi(1-\gamma)}{[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}s_1+\frac{1}{4}\frac{\varphi(1-\gamma)}{[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}s_2-$$

$$-\frac{1}{2}\frac{(1-\gamma)}{[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}m^*+\frac{1}{2}\frac{(\varphi+\delta)}{[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}f^*+\frac{1}{2}\frac{\varphi(1-\gamma)}{[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}s^*+$$

$$+\frac{1}{4}\frac{(1-\gamma)}{[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}p_{1,-1}+\frac{1}{4}\frac{(1-\gamma)}{[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}p_{2,-1}+\frac{1}{2}\frac{(1-\gamma)}{[\alpha(\varphi+\delta)+\varepsilon(1-\gamma)]}p^*_{-1}$$

Long run

$$y_1=\frac{1}{4}\frac{(1-\sigma)(1+2\gamma+3\beta\varphi)+2[\beta\varphi+(1-\sigma)(1+\gamma)]}{[\beta\varphi+(1-\sigma)(1+\gamma)](1+2\gamma+3\beta\varphi)}f_1+\frac{1}{4}\frac{(1-\sigma)(1+2\gamma+3\beta\varphi)-2[\beta\varphi+(1-\sigma)(1+\gamma)]}{[\beta\varphi+(1-\sigma)(1+\gamma)](1+2\gamma+3\beta\varphi)}f_2-$$

$$-\frac{1}{4}\frac{[(1-\sigma)(1+\gamma)+2\beta\varphi](1+2\gamma+6\beta\varphi)+[(1-\sigma)(1+\gamma)+3\beta\varphi]}{[\beta\varphi+(1-\sigma)(1+\gamma)](1+2\gamma+3\beta\varphi)}s_1-\frac{1}{4}\frac{[(1-\sigma)(1+\gamma)+2\beta\varphi](1+2\gamma)-[(1-\sigma)(1+\gamma)+3\beta\varphi]}{[\beta\varphi+(1-\sigma)(1+\gamma)](1+2\gamma+3\beta\varphi)}s_2+$$

$$+\frac{1}{2}\frac{(1-\sigma)}{[\beta\varphi+(1-\sigma)(1+\gamma)]}f^*-\frac{1}{2}\frac{(1-\sigma)(1+\gamma)}{[\beta\varphi+(1-\sigma)(1+\gamma)]}s^*$$

$$y_2=\frac{1}{4}\frac{(1-\sigma)(1+2\gamma+3\beta\varphi)-2[\beta\varphi+(1-\sigma)(1+\gamma)]}{[\beta\varphi+(1-\sigma)(1+\gamma)](1+2\gamma+3\beta\varphi)}f_1+\frac{1}{4}\frac{(1-\sigma)(1+2\gamma+3\beta\varphi)+2[\beta\varphi+(1-\sigma)(1+\gamma)]}{[\beta\varphi+(1-\sigma)(1+\gamma)](1+2\gamma+3\beta\varphi)}f_2-$$

$$-\frac{1}{4}\frac{[(1-\sigma)(1+\gamma)+2\beta\varphi](1+2\gamma)-[(1-\sigma)(1+\gamma)+3\beta\varphi]}{[\beta\varphi+(1-\sigma)(1+\gamma)](1+2\gamma+3\beta\varphi)}s_1-\frac{1}{4}\frac{[(1-\sigma)(1+\gamma)+2\beta\varphi](1+2\gamma+6\beta\varphi)+[(1-\sigma)(1+\gamma)+3\beta\varphi]}{[\beta\varphi+(1-\sigma)(1+\gamma)](1+2\gamma+3\beta\varphi)}s_2-$$

$$-\frac{1}{2}\frac{(1-\sigma)}{[\beta\varphi+(1-\sigma)(1+\gamma)]}f^*-\frac{1}{2}\frac{(1-\sigma)(1+\gamma)}{[\beta\varphi+(1-\sigma)(1+\gamma)]}s^*$$

$$p_1 = m + \frac{1}{4}\frac{\left[\varepsilon[(1-\sigma)(1+\gamma)+\beta\varphi]-\delta\alpha(1-\sigma)\right](1+2\gamma+3\beta\varphi)+2\rho\alpha[(1-\sigma)(1+\gamma)+\beta\varphi]}{\alpha[(1-\sigma)(1+\gamma)+\beta\varphi](1+2\gamma+3\beta\varphi)} f_1 +$$

$$+\frac{1}{4}\frac{\left[\varepsilon[(1-\sigma)(1+\gamma)+\beta\varphi]-\delta\alpha(1-\sigma)\right](1+2\gamma+3\beta\varphi)-2\rho\alpha[(1-\sigma)(1+\gamma)+\beta\varphi]}{\alpha[(1-\sigma)(1+\gamma)+\beta\varphi](1+2\gamma+3\beta\varphi)} f_2 +$$

$$+\frac{1}{4}\frac{\left[(1-\gamma)\varepsilon[(1-\sigma)(1+\gamma)+\beta\varphi]+\alpha\delta[(1+\gamma)(1-\sigma)+2\beta\varphi]\right](1+2\gamma+3\beta\varphi)+2(1+2\gamma)\varphi\alpha[(1-\sigma)(1+\gamma)+\beta\varphi]}{\alpha[(1-\sigma)(1+\gamma)+\beta\varphi](1+2\gamma+3\beta\varphi)} s_1 +$$

$$+\frac{1}{4}\frac{\left[(1-\gamma)\varepsilon[(1-\sigma)(1+\gamma)+\beta\varphi]+\alpha\delta[(1+\gamma)(1-\sigma)+2\beta\varphi]\right](1+2\gamma+3\beta\varphi)-2(1+2\gamma)\varphi\alpha[(1-\sigma)(1+\gamma)+\beta\varphi]}{\alpha[(1-\sigma)(1+\gamma)+\beta\varphi](1+2\gamma+3\beta\varphi)} s_2 +$$

$$+\frac{1}{2}\frac{\varepsilon[(1-\sigma)(1+\gamma)+\beta\varphi]+\delta\alpha(1-\sigma)}{\alpha[(1-\sigma)(1+\gamma)+\beta\varphi]} f^* + \frac{1}{2}\frac{\delta\alpha(1-\sigma)(1+\gamma)+\varepsilon(1-\gamma)[(1-\sigma)(1+\gamma)+\beta\varphi]}{\alpha[(1-\sigma)[1+\gamma]+\beta\varphi]} s^*$$

$$p_2 = m + \frac{1}{4}\frac{\left[\varepsilon[(1-\sigma)(1+\gamma)+\beta\varphi]-\delta\alpha(1-\sigma)\right](1+2\gamma+3\beta\varphi)-2\rho\alpha[(1-\sigma)(1+\gamma)+\beta\varphi]}{\alpha[(1-\sigma)(1+\gamma)+\beta\varphi](1+2\gamma+3\beta\varphi)} f_1 +$$

$$+\frac{1}{4}\frac{\left[\varepsilon[(1-\sigma)(1+\gamma)+\beta\varphi]-\delta\alpha(1-\sigma)\right](1+2\gamma+3\beta\varphi)+2\rho\alpha[(1-\sigma)(1+\gamma)+\beta\varphi]}{\alpha[(1-\sigma)(1+\gamma)+\beta\varphi](1+2\gamma+3\beta\varphi)} f_2 +$$

$$+\frac{1}{4}\frac{\left[(1-\gamma)\varepsilon[(1-\sigma)(1+\gamma)+\beta\varphi]+\alpha\delta[(1+\gamma)(1-\sigma)+2\beta\varphi]\right](1+2\gamma+3\beta\varphi)-2(1+2\gamma)\varphi\alpha[(1-\sigma)(1+\gamma)+\beta\varphi]}{\alpha[(1-\sigma)(1+\gamma)+\beta\varphi](1+2\gamma+3\beta\varphi)} s_1 +$$

$$+\frac{1}{4}\frac{\left[(1-\gamma)\varepsilon[(1-\sigma)(1+\gamma)+\beta\varphi]+\alpha\delta[(1+\gamma)(1-\sigma)+2\beta\varphi]\right](1+2\gamma+3\beta\varphi)+2(1+2\gamma)\varphi\alpha[(1-\sigma)(1+\gamma)+\beta\varphi]}{\alpha[(1-\sigma)(1+\gamma)+\beta\varphi](1+2\gamma+3\beta\varphi)} s_2 +$$

$$+\frac{1}{2}\frac{\varepsilon[(1-\sigma)(1+\gamma)+\beta\varphi]+\delta\alpha(1-\sigma)}{\alpha[(1-\sigma)(1+\gamma)+\beta\varphi]} f^* + \frac{1}{2}\frac{\delta\alpha(1-\sigma)(1+\gamma)+\varepsilon(1-\gamma)[(1-\sigma)(1+\gamma)+\beta\varphi]}{\alpha[(1-\sigma)[1+\gamma]+\beta\varphi]} s^*$$

$$y^* = -\frac{1}{4}\frac{(1-\sigma)}{[\beta\varphi+(1-\sigma)(1+\gamma)]} f_1 - \frac{1}{4}\frac{(1-\sigma)}{[\beta\varphi+(1-\sigma)(1+\gamma)]} f_2 - \frac{1}{4}\frac{(1-\sigma)(1+\gamma)}{[\beta\varphi+(1-\sigma)(1+\gamma)]} s_1 -$$

$$-\frac{1}{4}\frac{(1-\sigma)(1+\gamma)}{[\beta\varphi+(1-\sigma)(1+\gamma)]} s_2 + \frac{1}{2}\frac{(1-\sigma)}{[\beta\varphi+(1-\sigma)(1+\gamma)]} f^* - \frac{1}{2}\frac{2\beta\varphi+(1-\sigma)(1+\gamma)}{[\beta\varphi+(1-\sigma)(1+\gamma)]} s^*$$

$$p^* = \frac{1}{4}\frac{\varepsilon[(1-\sigma)(1+\gamma)+\beta\varphi]+\alpha\delta(1-\sigma)}{\alpha[(1-\sigma)(1+\gamma)+\beta\varphi]} f_1 + \frac{1}{4}\frac{\varepsilon[(1-\sigma)(1+\gamma)+\beta\varphi]+\alpha\delta(1-\sigma)}{\alpha[(1-\sigma)(1+\gamma)+\beta\varphi]} f_2 +$$

$$+\frac{1}{4}\frac{(1-\gamma)\varepsilon[(1-\sigma)(1+\gamma)+\beta\varphi]+\alpha\delta(1+\gamma)(1-\sigma)}{\alpha[(1-\sigma)(1+\gamma)+\beta\varphi]} s_1 + \frac{1}{4}\frac{(1-\gamma)\varepsilon[(1-\sigma)(1+\gamma)+\beta\varphi]+\alpha\delta(1+\gamma)(1-\sigma)}{\alpha[(1-\sigma)(1+\gamma)+\beta\varphi]} s_2 +$$

$$+m^* + \frac{1}{2}\frac{\varepsilon[(1-\sigma)(1+\gamma)+\beta\varphi]-\alpha\delta(1-\sigma)}{\alpha[(1-\sigma)(1+\gamma)+\beta\varphi]} f^* + \frac{1}{2}\frac{(1-\gamma)\varepsilon[(1-\sigma)(1+\gamma)+\beta\varphi]+\alpha\delta[(1+\gamma)(1-\sigma)+2\beta\varphi]}{\alpha[(1-\sigma)(1+\gamma)+\beta\varphi]} s^*$$

$$e = m - \frac{1}{4}\frac{\varphi+2\delta(1-\sigma)}{[(1-\sigma)(1+\gamma)+\beta\varphi]} f_1 - \frac{1}{4}\frac{\varphi+2\delta(1-\sigma)}{[(1-\sigma)(1+\gamma)+\beta\varphi]} f_2 - \frac{1}{4}\frac{\varphi[(1+\gamma)-2\delta\beta]}{[(1-\sigma)(1+\gamma)+\beta\varphi]} s_1 -$$

$$-\frac{1}{4}\frac{\varphi[(1+\gamma)-2\delta\beta]}{[(1-\sigma)(1+\gamma)+\beta\varphi]} s_2 - m^* + \frac{1}{2}\frac{\varphi+2\delta(1-\sigma)}{[(1-\sigma)(1+\gamma)+\beta\varphi]} f^* + \frac{1}{2}\frac{\varphi[(1+\gamma)-2\delta\beta]}{[(1-\sigma)(1+\gamma)+\beta\varphi]} s^*$$

$$i_w = \frac{1}{4\alpha} f_1 + \frac{1}{4\alpha} f_2 + \frac{(1-\gamma)}{4\alpha} s_1 + \frac{(1-\gamma)}{4\alpha} s_2 + \frac{1}{2\alpha} f^* + \frac{(1-\gamma)}{4\alpha} s^*$$

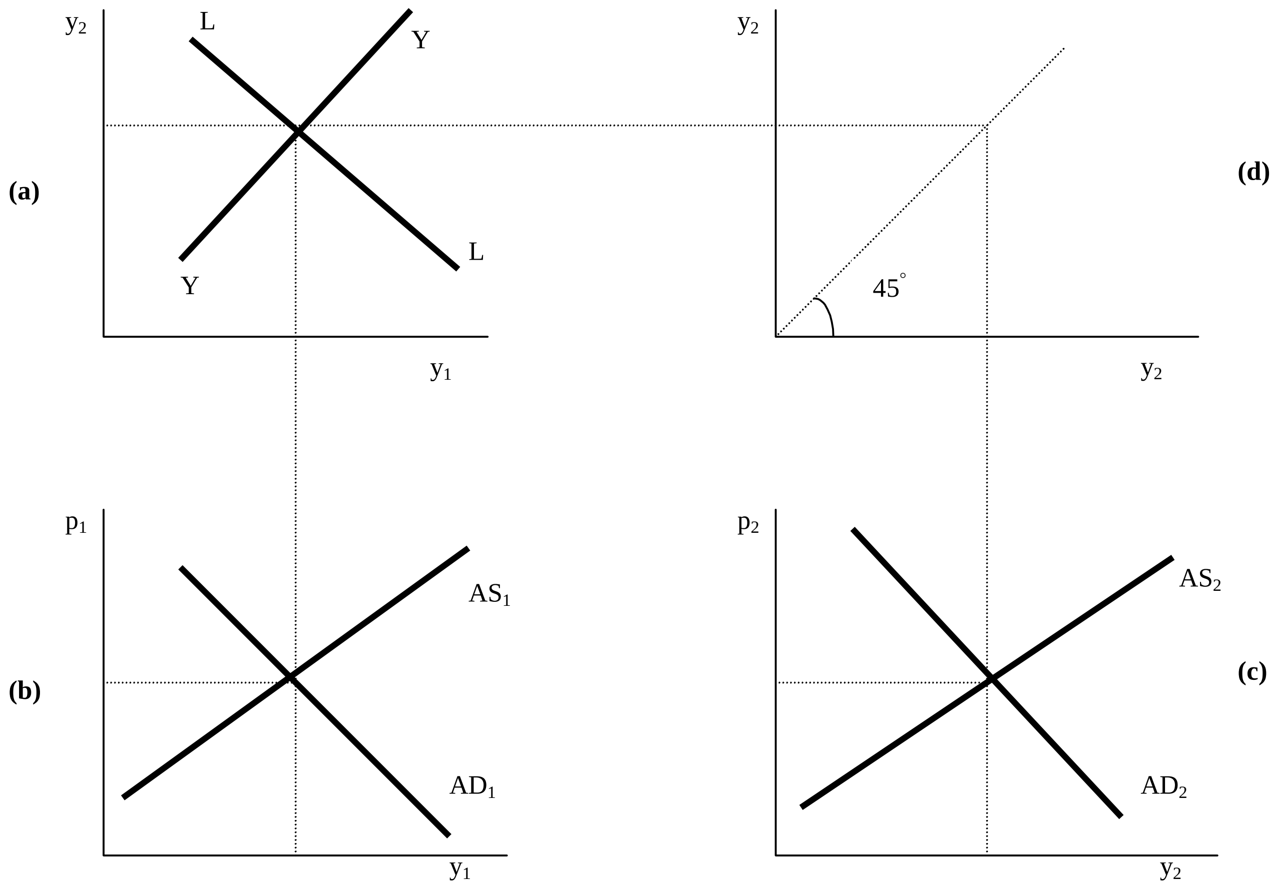
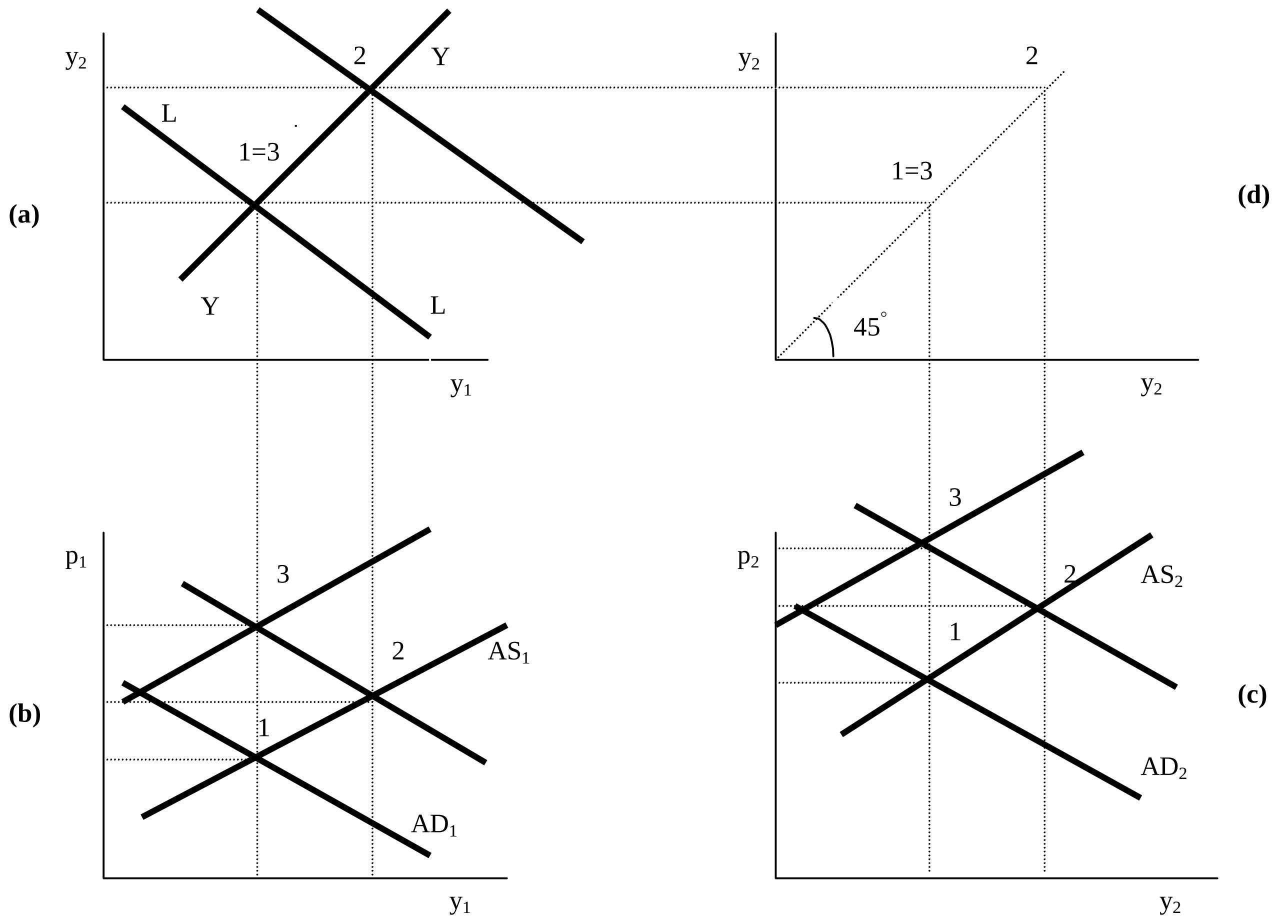


Figure 1



y2
2
Y
L
1=3
(a)
Y
L
y1
y2
2
1=3
(d)
45°
y2
p1
3
2
AS1
(b)
1
AD1
y1
3
p2
2
AS2
1
(c)
AD2
y2

Figure 2

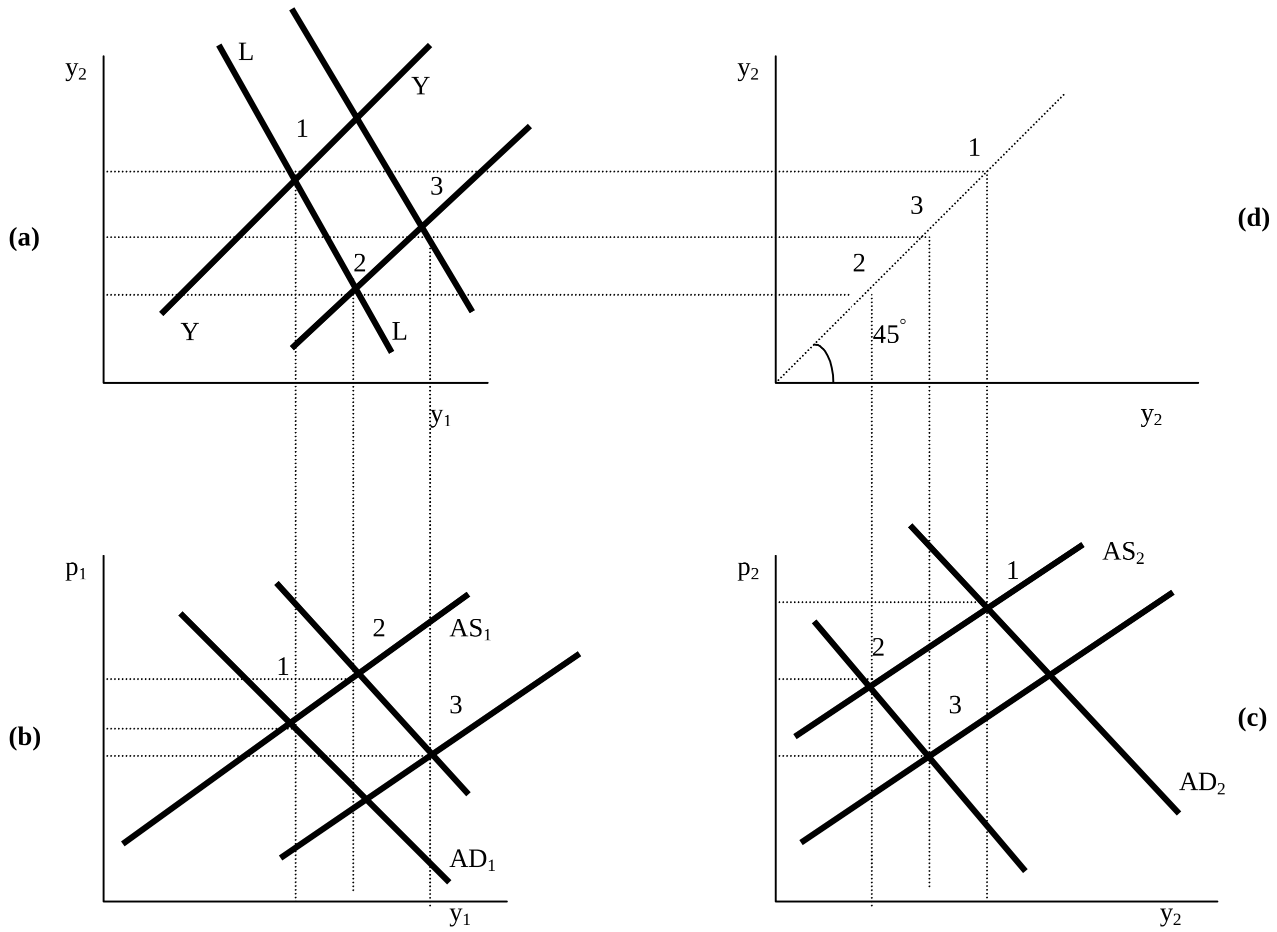
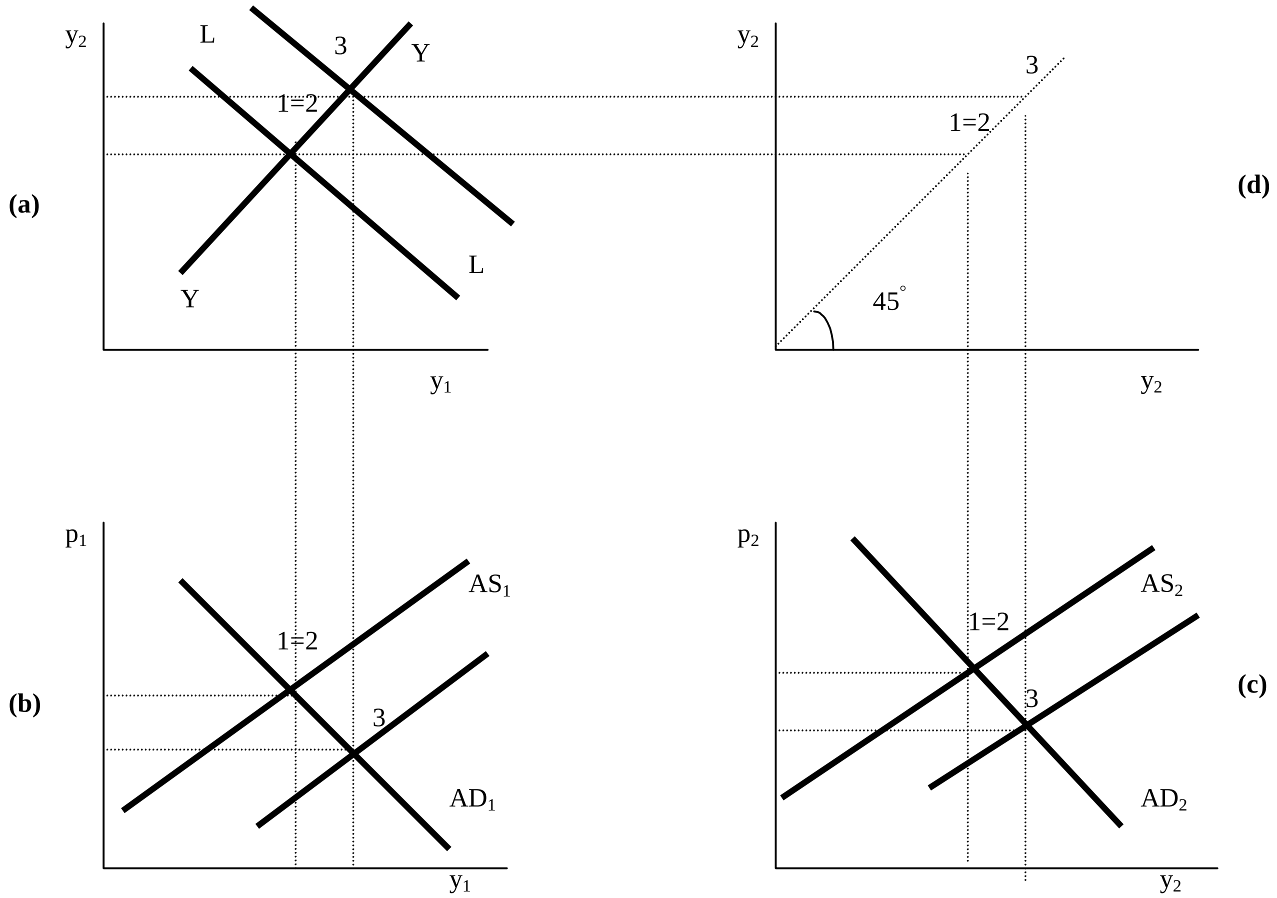
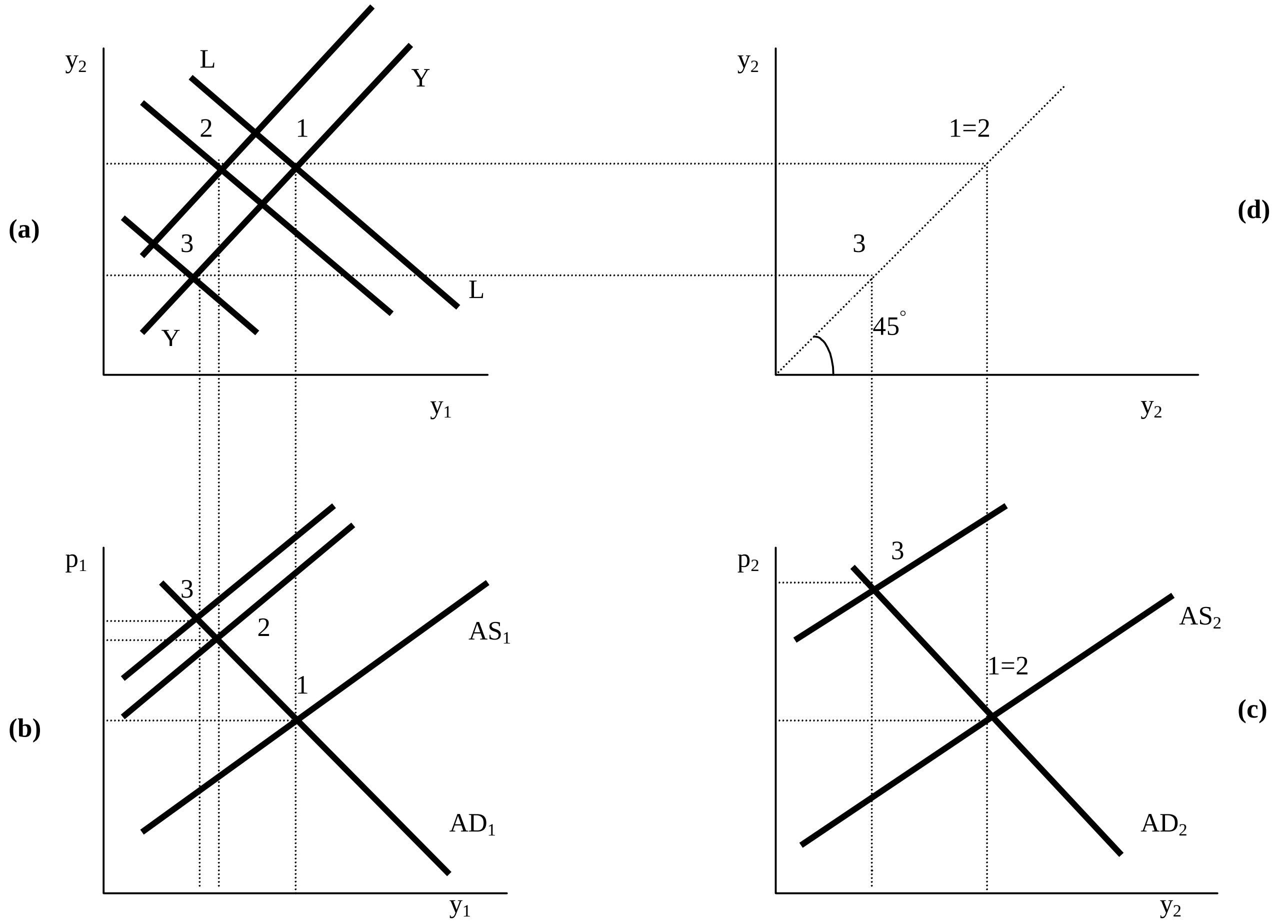


Figure 3



y2
L
3
Y
1=2
(a)
L
Y
y1
y2
3
1=2
(d)
45°
y2
p1
AS1
1=2
(b)
3
AD1
y1
p2
AS2
1=2
(c)
3
AD2
y2

Figure 4



(a)
y2
L
Y
2
1
3
L
Y
y1
(d)
y2
1=2
3
45°
y2
(b)
p1
3
2
1
AS1
AD1
y1
(c)
p2
3
AS2
1=2
AD2
y2

Figure 5

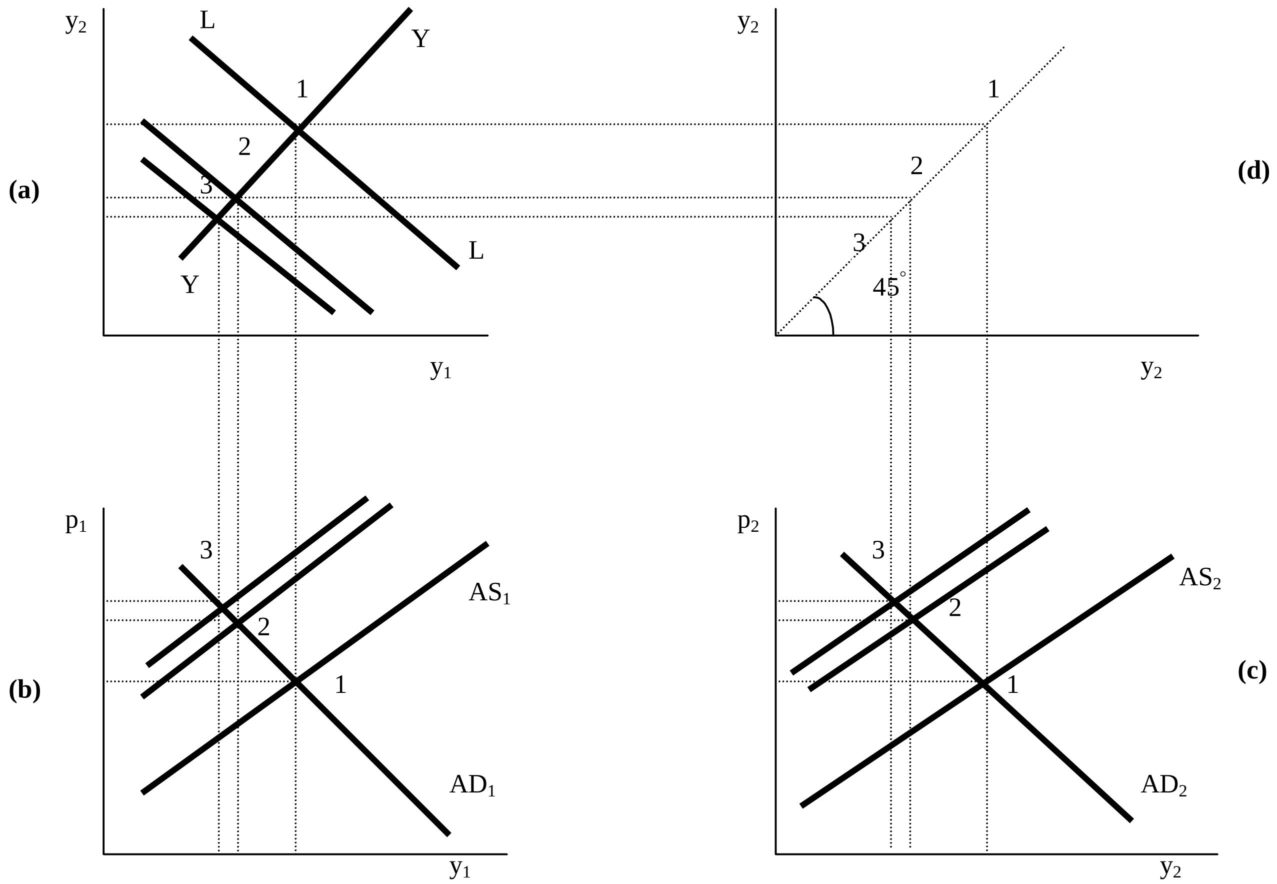
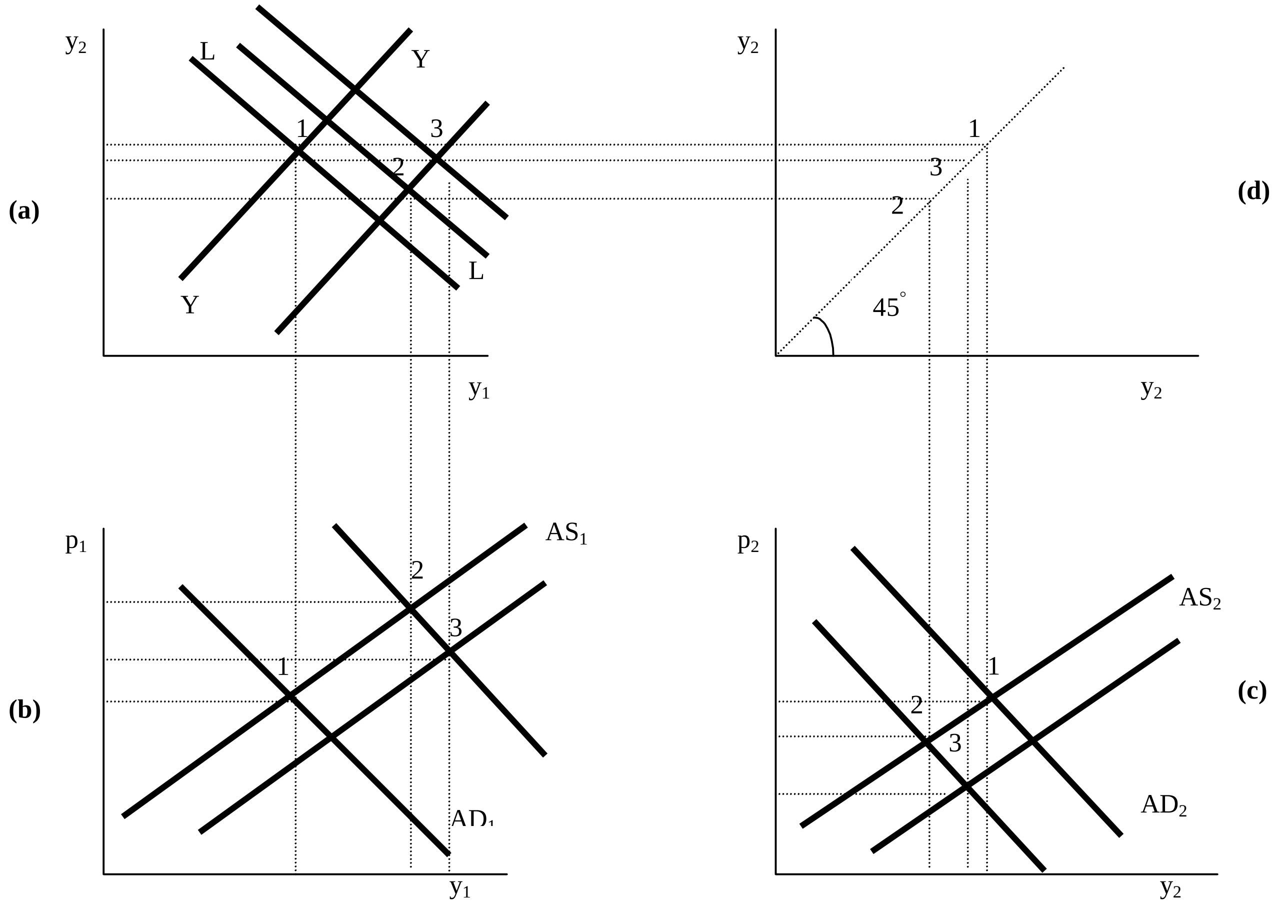
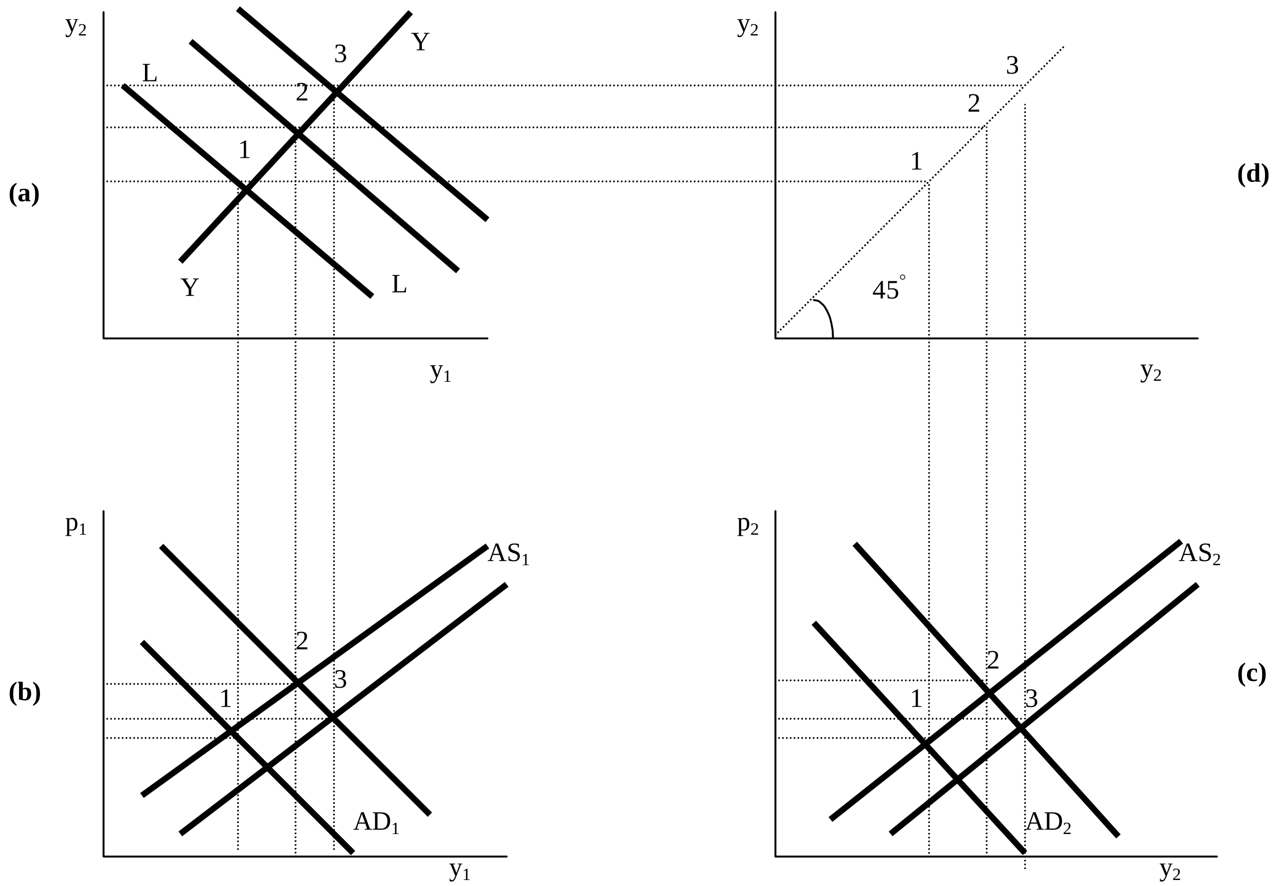

(a)
y2
L
Y
1
2
3
L
Y
y1
(d)
y2
1
2
3
45°
y2
(b)
p1
3
2
1
AS1
AD1
y1
(c)
p2
3
2
1
AS2
AD2
y2

Figure 6



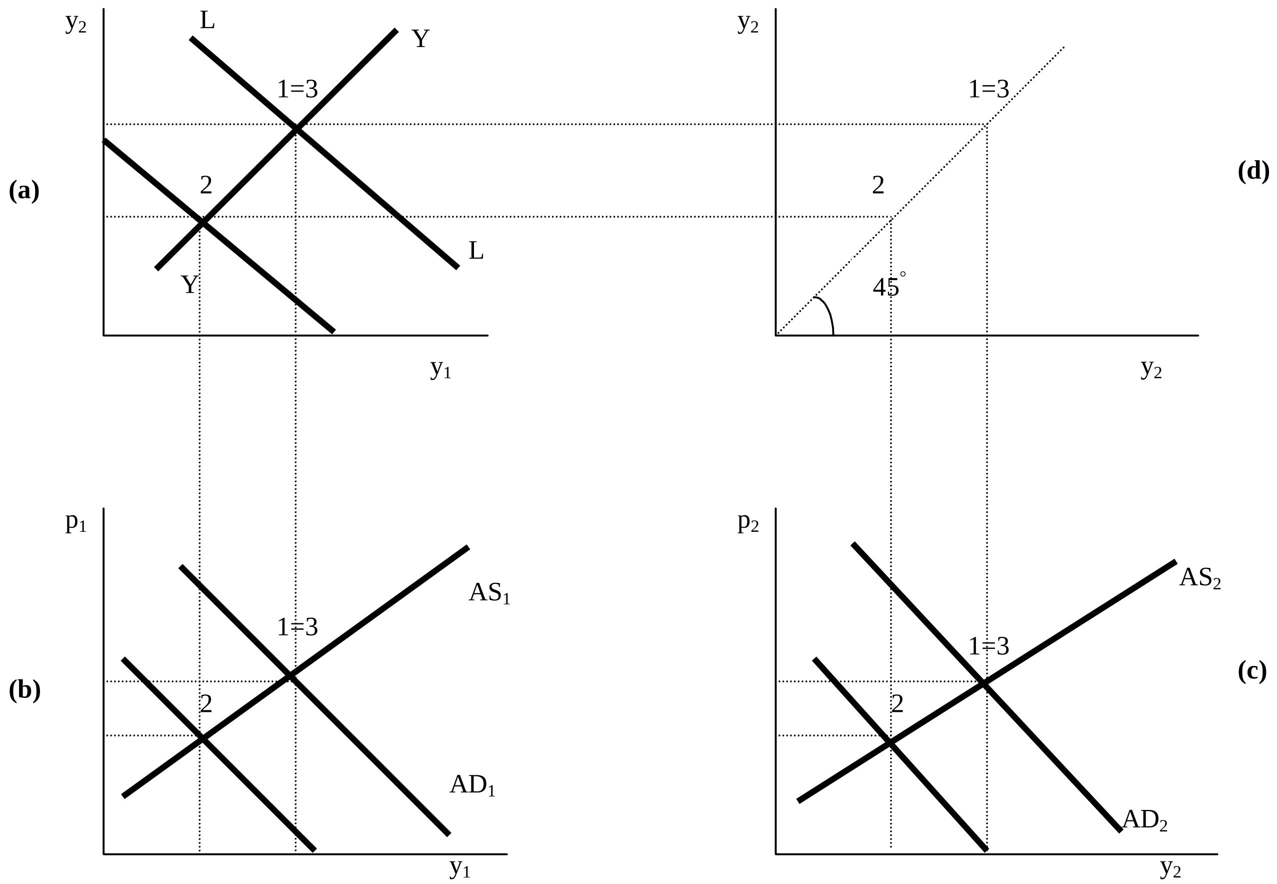
(a)
y2
L
Y
1
3
2
L
Y
y1
(d)
y2
1
3
2
45°
y2
(b)
p1
AS1
2
3
1
AD1
y1
(c)
p2
AS2
1
2
3
AD2
y2

Figure 7



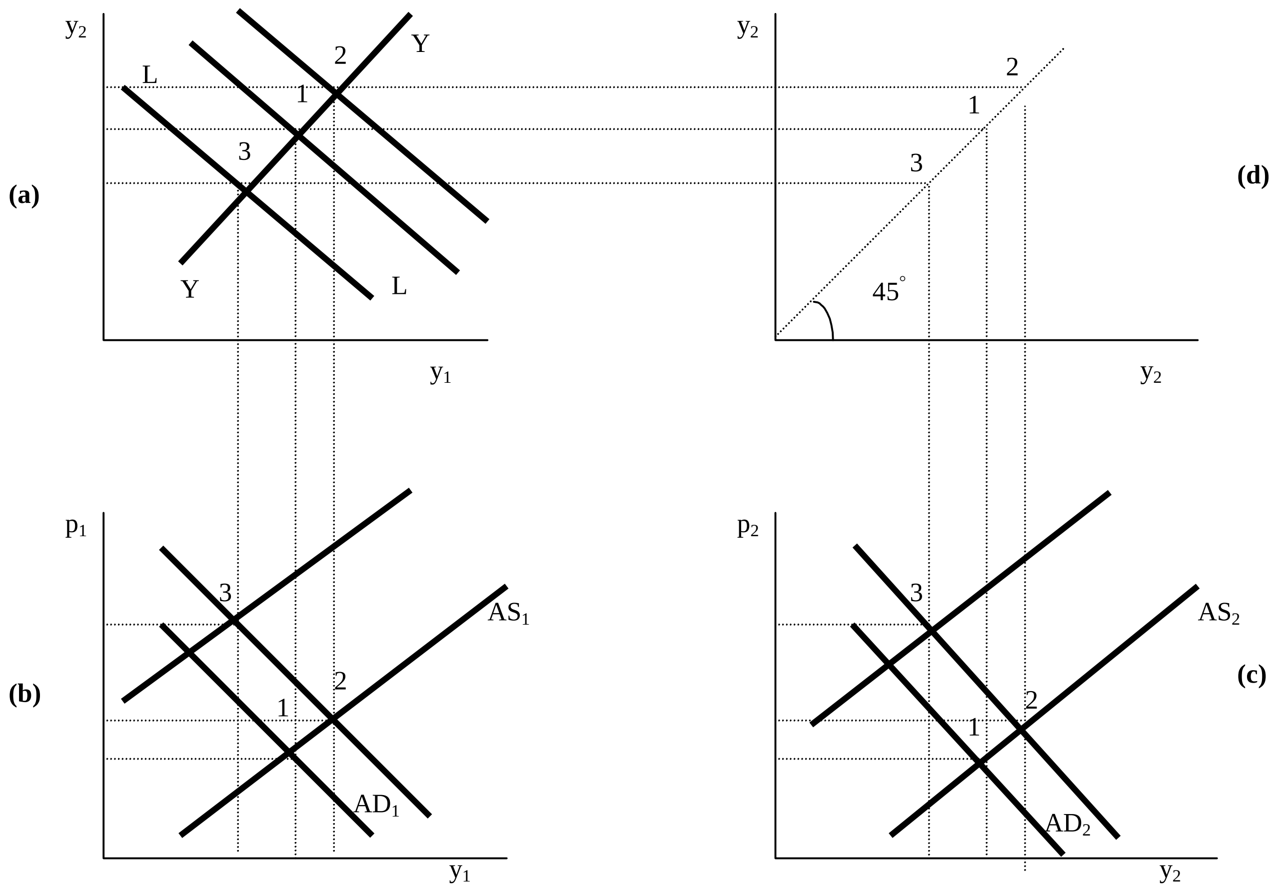
(a)
y2
L
Y
3
2
1
Y
L
y1
(d)
y2
3
2
1
45°
y2
(b)
p1
AS1
2
3
1
AD1
y1
(c)
p2
AS2
2
1
3
AD2
y2

Figure 8



y2
L
Y
1=3
2
Y
L
y1
(a)
y2
1=3
2
45°
y2
(d)
p1
AS1
1=3
2
AD1
y1
(b)
p2
AS2
1=3
2
AD2
y2
(c)

Figure 9



y2
L
2
Y
1
3
(a)
Y
L
y1
y2
2
1
3
(d)
45°
y2
p1
3
AS1
(b)
2
1
AD1
y1
p2
3
AS2
(c)
2
1
AD2
y2

Figure 10

Table 1.A. Small monetary union: symmetric shocks (d≠0)

		EFFECTS ON		
		OUTPUT	PRICES	EXCHANGE RATE
SHOCKS		$\frac{\partial y_1}{\partial d} = \frac{\partial y_2}{\partial d} = \frac{\partial y}{\partial d}$	$\frac{\partial p_1}{\partial d} = \frac{\partial p_2}{\partial d} = \frac{\partial p}{\partial d}$	$\frac{\partial e}{\partial d}$
MONETARY ***m***	sr	+	+	+
	lr	0	1	1
REAL ***f***	sr	0	0	–
	lr	+	–	–
SUPPLY ***s***	sr=lr	–	+	±
FOREIGN OUTPUT ***y****	sr	0	0	–
	lr	+	–	–
FOREIGN PRICES ***p****	sr=lr	0	0	–1
WORLD INTEREST RATE i_w	sr	+	+	+
	lr	–	+	+

Notes: (i) sr = short run, lr = long run
(ii) $d = m, f, s, y^*, p^*, i_w$

Table 1.B. Small monetary union: asymmetric shocks ($d_i \neq 0$, $d_j = 0$)

		EFFECTS ON						
		OUTPUT			PRICES			EXCH. RATE
SHOCKS		$\frac{\partial y_1}{\partial d_1} = \frac{\partial y_2}{\partial d_2}$	$\frac{\partial y_2}{\partial d_1} = \frac{\partial y_1}{\partial d_2}$	$\frac{\partial y}{\partial d_1} = \frac{\partial y}{\partial d_2}$	$\frac{\partial p_1}{\partial d_1} = \frac{\partial p_2}{\partial d_2}$	$\frac{\partial p_2}{\partial d_1} = \frac{\partial p_1}{\partial d_2}$	$\frac{\partial p}{\partial d_1} = \frac{\partial p}{\partial d_2}$	$\frac{\partial e}{\partial d_1} = \frac{\partial e}{\partial d_2}$
REAL f_1, f_2	sr	+	–	0	+	–	0	–
	lr	+	±	+	±	–	–	–
SUPPLY s_1, s_2	sr=lr	–	±	–	+	±	+	±

Notes: (i) sr = short run, lr = long run
(ii) $d_i, d_j = f_i, f_j, s_i, s_j$ ($i, j = 1, 2$)

Table 2.A. Big monetary union: symmetric shocks (d≠0)

		EFFECTS ON					
		OUTPUT		PRICES		EXCHANGE RATE	WORLD INT. RATE
SHOCKS		$\frac{\partial y_1}{\partial d}=\frac{\partial y_2}{\partial d}=\frac{\partial y}{\partial d}$	$\frac{\partial y^*}{\partial d}$	$\frac{\partial p_1}{\partial d}=\frac{\partial p_2}{\partial d}=\frac{\partial p}{\partial d}$	$\frac{\partial p^*}{\partial d}$	$\frac{\partial e}{\partial d}$	$\frac{\partial i_W}{\partial d}$
MONETARY ***m***	sr	+	–	+	–	+	–
	lr	0	0	1	0	1	0
REAL ***f***	sr	+	+	+	+	–	+
	lr	+	–	±	+	–	+
SUPPLY ***s***	sr=lr	–	+ (sr) – (lr)	+	+	±	+
FOREIGN MONETARY ***m****	sr	–	+	–	+	–	–
	lr	0	0	0	1	– 1	0
FOREIGN REAL ***f****	sr	+	+	+	+	+	+
	lr	–	+	+	±	+	+
FOREIGN SUPPLY ***s****	sr	+	–	+	+	±	+
	lr	–	–	+	+	±	+

Notes: (i) sr = short run, lr = long run
(ii) $d = m, f, s, m^*, f^*, s^*$

Table 2.B. Big monetary union: asymmetric shocks (d_i≠0, d_j=0)

		EFFECTS ON									
		OUTPUT				PRICES				EXCH. RATE	WORLD INT. RATE
SHOCKS		$\frac{\partial y_1}{\partial d_1}=\frac{\partial y_2}{\partial d_2}$	$\frac{\partial y_2}{\partial d_1}=\frac{\partial y_1}{\partial d_2}$	$\frac{\partial y}{\partial d_1}=\frac{\partial y}{\partial d_2}$	$\frac{\partial y^*}{\partial d_1}=\frac{\partial y^*}{\partial d_2}$	$\frac{\partial p_1}{\partial d_1}=\frac{\partial p_2}{\partial d_2}$	$\frac{\partial p_2}{\partial d_1}=\frac{\partial p_1}{\partial d_2}$	$\frac{\partial p}{\partial d_1}=\frac{\partial p}{\partial d_2}$	$\frac{\partial p^*}{\partial d_1}=\frac{\partial p^*}{\partial d_2}$	$\frac{\partial e}{\partial d_1}=\frac{\partial e}{\partial d_2}$	$\frac{\partial i_W}{\partial d_1}=\frac{\partial i_W}{\partial d_2}$
REAL f_1, f_2	sr	+	±	+	+	+	±	+	+	–	+
	lr	+	±	+	–	±	±	±	+	–	+
SUPPLY s_1, s_2	sr=lr	–	±	–	+ (sr) – (lr)	+	±	+	+	±	+

Notes: (i) sr = short run, lr = long run
(ii) d_i, d_j = f_i, f_j, s_i, s_j (i, j = 1, 2)