NIFS 1 (1995) 42-47

Handling multicriteria fuzzy decision-making problems based on intuitionistic fuzzy sets

H. Bustince
Departamento de Matemática e Informática
Universidad Pública de Navarra,31006
Campus Arrosadía, Pamplona, Spain
Phone: 3448169254, *Fax*: 3448169565,
E-mail: bustince@si.upna.es

Abstract: *In this paper we present a new tecnique for handling multicriteria decision making problems based on intuitionistic fuzzy sets. In this new tecnique we use the degree of reliability and of non-reliability of each criterion in relation to a set of alternatives. We also present a new score function in order to evaluate the degree of suitability of the choice of a certain alternative.*

Keywords:*Fuzzy set; intuitionistic fuzzy set; multicriteria fuzzy decision-making.*

1. Introduction

Interval valued fuzzy sets $IVFSs(X)$, ([9], [10], [11]) were introduced in the seventies , as a generalization of the concept of fuzzy set. K. Atanassov gave the definition of intuitionistic fuzzy set $IFSs(X)$, in ([1]), and afterwards established that, $IVFSs(X)$ and $IFSs(X)$ ([2]), are equipollent generalizations of the concept of fuzzy set.

An *intuitionistic fuzzy set* ([1]) in X, $X \neq \emptyset$ and Card(X)=K, is an expression A given by

$$A = \{< x, \mu_A(x), \nu_A(x) > | x \in X\}, \quad \text{where}$$
$$\mu_A \ : X \longrightarrow [0,1]$$
$$\nu_A \ : X \longrightarrow [0,1]$$

with the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for all $x \in X$

Numbers $\mu_A(x)$ and $\nu_A(x)$ denote respectively the degree of membership and the degree of non-membership of element x to set A. We will denote with $IFSs(X)(X)$ the set of all the intuitionistic fuzzy sets in X. A complete study of these sets can be found in ([1], [2], [4], [6]).

The following expressions are defined in ([1]), ([5]) for all $A, B \in X$

1. $A \leq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ & $\nu_A(x) \geq \nu_B(x) \quad \forall x \in X$

2. $A \preceq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ & $\nu_A(x) \leq \nu_B(x)$ $\forall x \in X$
3. $A \vee B = \{< x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) > |x \in X\}$
4. $A \wedge B = \{< x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) > |x \in X\}$

Operations 3 and 4 are easily generalizable to the case of n intuitionistic fuzzy sets.

5. $A = B$ if and only if $A \leq B$ & $B \leq A$
6. $A_c = \{< x, \nu_A(x), \mu_A(x) > |x \in X\}$

It is clear that $\mu_A(x)$ is the degree of membership of element x to the intuitionistic fuzzy set A and $\nu_A(x)$ is the degree of non-membership of x to set A, note that for all $x \in X$ it is verified that $\mu_A(x) + \nu_A(x) = 1$, then set A is fuzzy. It has been proved in ([6]) that fuzzy sets, which we will represent in any of the following ways:

$$A = \{< x, \mu_A(x) > |x \in X\} = \{< x, \mu_A(x), 1 - \mu_A(x) > |x \in X\}$$

are a particular case of the intuitionistic fuzzy sets.

In the past years fuzzy sets have been widely used for handling decision-making problems ([3], [7], [8], [12]). In this paper we present a new tecnique to for solving these problems using $IFSs(X)$. We will consider that the set of alternatives and the sets of criteria are intuitionistic fuzzy sets. Also, we will use as score function the distance between the functions of membership and non-membership od an element in relation to the set of criteria and to the set of alternatives. For that reason we remind that the adaptation of the well known *Hamming fuzzy distance* to the $IFSs(X)$ ([6]) gives us the following expression:

$$2 \cdot d_{H_{IFSs(X)}}(A, B) = \sum_{k=1}^{K} |\mu_A(x_k) - \mu_B(x_k)| + |\nu_A(x_k) - \nu_B(x_k)|$$

Let us consider $A, B \in IFSs(X)(x)$ and let x_i be an element of X, we will denote with $d(x_i)$ the following expression:

$$2 \cdot d(x_i) = |\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|$$

it is evident that:

$$d_{H_{IFSs(X)}}(A, B) = \sum_{i=1}^{K} d(x_i)$$

The previous definition will be of great importance in this paper, as we will later see.

2. Handling multicriteria fuzzy decision-making problems

Let $A = \{a_1, a_2, \cdots, a_m\}$ be an ordinary set of alternatives, we can write this set as an intuitionistic fuzzy set in the following way:

$$A = \{< a_1, 1.0, 0.0 >, < a_2, 1.0, 0.0 >, \cdots, < a_m, 1.0, 0.0 >\}.$$

Let $C_1, C_2, \cdots, C_n$ be sets of criteria, these sets are intuitionistic fuzzy sets on A, that is

$$C_i = \{< a_j, \mu_{C_i}(a_j), \nu_{C_i}(a_j) > | a_j \in A\}$$

where $i = 1, \cdots, n$, $j = 1, \cdots, m$, and $0 \leq \mu_{C_i}(a_j) + \nu_{C_i}(a_j) \leq 1$ for all $a_j \in A$.

It is important to point out that $\mu_{C_i}(a_j)$ indicates the degree in which the criterion C_i satisfies the alternative a_j and $\nu_{C_i}(a_j)$ indicates the degree in which the criterion C_i does not satisfy the alternative a_j.

We assume that there is a decision maker who wants to choose an alternative which satisfies criterion C_l and $C_r$and $\cdots$ and C_p or which satisfies the criterion C_q. In this case, we will call C the following intuitionisitc fuzzy set:

$$C = C_l \wedge C_r \wedge \cdots \wedge C_p \vee C_q \quad \text{where}$$

$$\mu_C(a_j) = \mu_{C_l}(a_j) \wedge \mu_{C_r}(a_j) \wedge \cdots \wedge \mu_{C_p}(a_j) \vee \mu_{C_q}(a_j)$$

$$\nu_C(a_j) = \nu_{C_l}(a_j) \vee \nu_{C_r}(a_j) \vee \cdots \vee \nu_{C_p}(a_j) \wedge \nu_{C_q}(a_i)$$

with $j = 1, 2, \cdots, m$.

We will use as score function to evaluate the degree of suitability that an alternative satisfies the decision-maker's requirement, the distance function $d(a_j)$ (for each $a_j \in A$) between the degree of memebership and the degree of non-membership of a_j to C and the degree of membership and the degree of non-membership of a_j to A, that is

$$2 \cdot d(a_j) = |1.0 - \mu_C(a_j)| + |0.0 - \nu_C(a_j)|$$

This way, we have a fuzzy set associated to the set of alternatives A given by the following expression:

$$D = \{< a_j, d(a_j) > | a_j \in A\}$$

with $j = 1, \cdots, m$.

The element of D that has the smallest membership, that is the least height, will mark the best choice of alternative, that is, the smallest of the values of $d(a_j)$ will indicate that the corresponding alternative a_j is the one that best satisfies the decision-maker's requirement, that is

$$d(a_1) = s_1 \in [0, 1]$$

$$d(a_2) = s_s \in [0, 1]$$

$$\cdots$$

$$d(a_m) = s_m \in [0, 1]$$

if $d(a_j) = s_j \in [0, 1]$ and s_j is the smallest value among the values $s_1, s_2, \cdots, s_m$, then the alternative a_j is our best choice.

Example

Let $A = \{a_1, a_2, a_3, a_4\}$ be a ordinary set of alternatives, this set can be represented as an intuitionistic fuzzy set in the following way:

$$A = \{< a_1, 1.0, 0.0 >, < a_2, 1.0, 0.0 >, < a_3, 1.0, 0.0 >, < a_4, 1.0, 0.0 >\}$$

and let C_1, C_2 and C_3 be three intuitionistic fuzzy sets of criteria on A.

We assume that these C_1, C_2 and C_3 are represented by the intuitionistic fuzzy sets shown as follows:

$$C_1 = \{< a_1, 0.0, 1.0 >, < a_2, 0.7, 0.1 >, < a_3, 0.2, 0.4 >, < a_4, 0.1, 0.9 >\}$$

$$C_2 = \{< a_1, 0.6, 0.4 >, < a_2, 1.0, 0.0 >, < a_3, 0.4, 0.5 >, < a_4, 0.3, 0.2 >\}$$

$$C_3 = \{< a_1, 0.5, 0.5 >, < a_2, 0.2, 0.6 >, < a_3, 0.9, 0.0 >, < a_4, 0.3, 0.3 >\}$$

and we assume that the decision-maker wants to choose an alternative which satisfies the criteria C_1 and C_2 or which satisfies the criterion C_3, then the decision-maker's requirement can be expressed by $C = C_1 \wedge C_2 \vee C_3$, that is

$$\mu_C(a_1) = \mu_{C_1}(a_1) \wedge \mu_{C_2}(a_1) \vee \mu_{C_3}(a_1) = 0.0 \wedge 0.6 \vee 0.5 = 0.5$$

$$\nu_C(a_1) = \mu_{C_1}(a_1) \vee \nu_{C_2}(a_1) \wedge \nu_{C_3}(a_1) = 1.0 \vee 0.4 \wedge 0.5 = 0.5$$

$$\mu_C(a_2) = \mu_{C_1}(a_2) \wedge \mu_{C_2}(a_2) \vee \mu_{C_3}(a_2) = 0.7 \wedge 1.0 \vee 0.2 = 0.7$$

$$\nu_C(a_2) = \mu_{C_1}(a_2) \vee \nu_{C_2}(a_2) \wedge \nu_{C_3}(a_2) = 0.1 \vee 0.0 \wedge 0.6 = 0.1$$

$$\mu_C(a_3) = \mu_{C_1}(a_3) \wedge \mu_{C_2}(a_3) \vee \mu_{C_3}(a_3) = 0.2 \wedge 0.4 \vee 0.9 = 0.9$$

$$\nu_C(a_3) = \mu_{C_1}(a_3) \vee \nu_{C_2}(a_3) \wedge \nu_{C_3}(a_3) = 0.4 \vee 0.5 \wedge 0.0 = 0.0$$

$$\mu_C(a_4) = \mu_{C_1}(a_4) \wedge \mu_{C_2}(a_4) \vee \mu_{C_3}(a_4) = 0.1 \wedge 0.3 \vee 0.3 = 0.3$$

$$\nu_C(a_4) = \mu_{C_1}(a_4) \vee \nu_{C_2}(a_4) \wedge \nu_{C_3}(a_4) = 0.9 \vee 0.2 \wedge 0.3 = 0.3$$

therefore

$$C = \{< a_1, 0.5, 0.5 >, < a_2, 0.7, 0.1 >, < a_3, 0.9, 0.0 >, < a_4, 0.3, 0.3 >\}$$

Now we calculate $d(a_1)$, $d(a_2)$, $d(a_3)$ and $d(a_4)$.

$$d(a_1) = \frac{|1.0 - 0.5| + |0.0 - 0.5|}{2} = 0.5$$

$$d(a_2) = \frac{|1.0 - 0.7| + |0.0 - 0.1|}{2} = 0.2$$

$$d(a_3) = \frac{|1.0 - 0.9| + |0.0 - 0.0|}{2} = 0.05$$

$$d(a_4) = \frac{|1.0 - 0.3| + |0.0 - 0.3|}{2} = 0.5$$

With these values we construct the following fuzzy set:

$$D = \{< a_1, 0.5 >, < a_2, 0.2 >, < a_3, 0.05 >, < a_4, 0.5 >\}$$

we can observe that element a_3 has the smallest membership to set D, that is, the least height, therefore, in accordance with the previous, alternative a_3 is our best choice.

3. Conclusions

In this paper , we have presented a new technique for handling multicriteria fuzzy decision-making problems, where the sets of criteria are intuitionistic fuzzy sets. This new tecnique allows us to handle the degree of reliability and the degree of non-reliability of each criterion in relation to each alternative. On the other hand, we take as score function the distance, and among all the alternatives we choose the one that has associated to it the smallest distance.

4. Future research

In the method presented in this paper we have assumed that the set of alternatives A was an ordinary set, however, it seems logical to study the case in which said set is also intuitionistic fuzzy. On the other hand, with the score function we take, that is $d(a_j)$, we construct a fuzzy set D, it also seems logical to analise the different numerical measures of information of this set (energy, entropy, etc...) as well as the correlation between A and D.

References

[1] K. Atanassov. Intuitionistic fuzzy sets, VII ITKR's Session, Deposed in Central Sci.- Techn. Library of Bulg. Acd. of Sci. Sofia June (1983) 1697-84.

[2] K. Atanassov and G. Gargov. Interval Valued Intuitionistic Fuzzy Sets. *Fuzzy Sets and Systems*, **31** (1989) 343-349.

[3] R. Bellman and L.A. Zadeh. Decision-making in a fuzzy environment. *Management Sci.*, **17** (1990) 141-164.

[4] P. Burillo and H. Bustince.Isoentropic methods for construction of IVFS. *CIFT'94* : 57-61, Trento, Italy, (1994) June 1-3.

[5] P. Burillo and H. Bustince. Estructuras Algebraicas en Conjuntos Intuicionistas Fuzzy, *II Congreso Español Sobre Tecnologías y Lógica Fuzzy,* Boadilla del Monte, Madrid, (1992) 135-146.

[6] H. Bustince,Conjuntos Intuicionistas e Intervalo valorados Difusos: Propiedades y Construcción. Relaciones Intuicionistas Fuzzy. Thesis, Universidad Pública de Navarra, (1994).

[7] S.M. Chen. A new approach to handling fuzzy decision-making problems, *IEEE Trans. Systems, Man, Cybern.*, **18** (1988) 1012-1016.

[8] W.J.M.Kickert. Fuzzy Theories on Decision- Making: A Critical review *Kluwet Boston INc.* (1978)

[9] C. Ponsard. Hirarchie des Places Centrales et Graphes Φ - Flous. *Enviro nment and Planning A*, Vol 9, (1977) 1233-1252.

[10] R. Sambuc. Functions Φ-Floues. Application à l'aide au Diagnostic en Pathologie Thyroidienne. *Thèse de Doctorat en Médecine*, Marseille, 1975.

[11] L. A. Zadeh. Fuzzy Sets. *Inform. and Control*, **8** (1965) 338-353.

[12] H. J. Zimmermann, Fuzzy sets, Decision Making, and Expert Systems, *Kluwer Academic Publihers*, Boston (1987).