

Orderings in the referential set induced by an Intuitionistic Fuzzy Relation

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Abstract:In this paper we present different orderings in the referential set induced by intuitionistic fuzzy relations of ordering. We can see how these induced orders justify the definition given of intuitionistic fuzzy antisymmetry and the definition of partial enclosure.

Keywords: Intuitionistic fuzzy relation; composition of intuitionistic fuzzy relation; fuzzy relation; intuitionistic ordering relation; intuitionistic similarity relation; intuitionistic dissimilarity relation; antisymmetrical intuitionistic property; partially included relation.

1. Introduction

In this paper we present two different ways of inducing orders in the referential set X through intuitionistic fuzzy relations of ordering.

The paper consists of three parts. In the first part, we start by remembering the definitions given by K. Atanassov about intuitionistic fuzzy sets ([1]) and the one about the operator ([3]). Next, we give the definition of composition of intuitionistic fuzzy relations which was studied in ([6]). We finish this section remembering the most important properties of intuitionistic fuzzy relations in a set. We also mention the partially included relations, which, as we know, were introduced in ([6], [7]) to guarantee the conservation of the transitive property through Atanassov's operators. We will see that these relations are also used in the third part of the paper.

The second part of the paper is dedicated to present an ordering induced in the referential set X , through an intuitionistic fuzzy relation of ordering. The importance of this item is such that it justifies the definition of intuitionistic antisymmetrical relation, so that with this definition we will be able to guarantee an intuitionistic fuzzy relation of ordering.

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The third part of the paper is dedicated to the study of new induced orderings, but this time we will use the property of partial enclosure of relations and K. Atanassov's operators.

2. Preliminaries

Let X, Y and Z be ordinary finite non-empty sets.

Let $X \neq \phi$ be a given set. ([1]) An *intuitionistic fuzzy set* in X is an expression A given by

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \} \quad \text{where}$$

$$\mu_A : X \longrightarrow [0, 1]$$

$$\nu_A : X \longrightarrow [0, 1]$$

with the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1, \forall x \in X$

The numbers $\mu_A(x)$ and $\nu_A(x)$ denote respectively the degree of membership and the degree of non-membership of the element x in the set A . *IFSs* will denote the set of all the intuitionistic fuzzy sets in X . Obviously, when $\nu_A(x) = 1 - \mu_A(x)$ for every x in X , the set A is a fuzzy set. *FSs* will denote the set of all the fuzzy sets in X .

We will call $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ *intuitionistic index* of the element x in the set A .

The following expressions are defined in ([2], [4], [5], [8]) for every $A, B \in IFSs$

1. $A \leq B \iff \mu_A(x) \leq \mu_B(x) \text{ and } \nu_A(x) \geq \nu_B(x) \quad \forall x \in X$
2. $A \preceq B \iff \mu_A(x) \leq \mu_B(x) \text{ and } \nu_A(x) \leq \nu_B(x) \quad \forall x \in X$
3. $A = B \iff A \leq B \text{ and } B \leq A$
4. $A_c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle \mid x \in X \}$

In 1986, K. Atanassov established different ways of changing an intuitionistic fuzzy set into a fuzzy set and he defined the following operator:

If $E \in IFSs$ then

$$D_p(E) = \{ \langle x, \mu_E(x) + p \cdot \pi_E(x),$$

$$1 - \mu_E(x) - p \cdot \pi_E(x) \rangle \mid x \in X \}$$

with $p \in [0, 1]$. Obviously $D_p(E) \in FSs$.

A study of the properties of this operator, (we will call it *Atanassov's operator*), is made in ([3], [5], [8]).

Let E be an intuitionistic fuzzy set and D_p the operator given in the previous definition, then the family of all the fuzzy sets associated to E through the

operator D_p , will be denoted by $\{D_p(E)\}_{p \in [0,1]}$. It is clear that $\{D_p(E)\}_{p \in [0,1]}$ is a totally ordered family of fuzzy sets.

We know that an *intuitionistic fuzzy relation* is an intuitionistic fuzzy subset of $X \times Y$, that is, is an expression R given by

$$R = \{ \langle (x, y), \mu_R(x, y), \nu_R(x, y) \rangle \mid x \in X, y \in Y \}$$

where

$$\mu_R : X \times Y \longrightarrow [0, 1]$$

$$\nu_R : X \times Y \longrightarrow [0, 1]$$

satisfy the condition $0 \leq \mu_R(x, y) + \nu_R(x, y) \leq 1$ for every $(x, y) \in X \times Y$.

$IFR(X \times Y)$ will denote the set of all the intuitionistic fuzzy subsets in $X \times Y$.

The most important properties of intuitionistic fuzzy relations are studied in ([4], [6], [7], [8]).

Besides, we know that the composition of intuitionistic fuzzy relations is given by

Definition 1. Let $\alpha, \beta, \lambda, \rho$ be t -norms or t -conorms not necessarily dual two-two, $R \in IFR(X \times Y)$ and $P \in IFR(Y \times Z)$. We will call *composed relation* $P \overset{\alpha, \beta}{\underset{\lambda, \rho}{\circ}} R \in IFRS(X \times Z)$ to the one defined by

$$P \overset{\alpha, \beta}{\underset{\lambda, \rho}{\circ}} R = \{ \langle (x, z), \mu_{P \overset{\alpha, \beta}{\underset{\lambda, \rho}{\circ}} R}(x, z), \nu_{P \overset{\alpha, \beta}{\underset{\lambda, \rho}{\circ}} R}(x, z) \rangle \mid x \in X, z \in Z \}$$

where

$$\mu_{P \overset{\alpha, \beta}{\underset{\lambda, \rho}{\circ}} R}(x, z) = \alpha \{ \beta [\mu_R(x, y), \mu_P(y, z)] \}$$

$$\nu_{P \overset{\alpha, \beta}{\underset{\lambda, \rho}{\circ}} R}(x, z) = \lambda \{ \rho [\nu_R(x, y), \nu_P(y, z)] \}$$

whenever

$$0 \leq \mu_{P \overset{\alpha, \beta}{\underset{\lambda, \rho}{\circ}} R}(x, z) + \nu_{P \overset{\alpha, \beta}{\underset{\lambda, \rho}{\circ}} R}(x, z) \leq 1 \quad \forall (x, z) \in X \times Z.$$

Notice that the symbols α, β which are in the upper place of "o" are applied to the functions of membership and the symbols λ and ρ which are in the lower place are applied to the non-membership functions. We have proved in ([6]) that for $\alpha = \vee, \beta$ t -norm, $\lambda = \wedge$ and ρ t -conorm, the composition of intuitionistic fuzzy relations satisfies the largest number of properties.

In this paper we will take $\alpha = \vee, \beta = \wedge, \lambda = \wedge$ and $\rho = \vee$.

Now, we will remember the main properties of intuitionistic fuzzy relations in a set, that is, in $X \times X$. A complete study of these relations is made in ([6], [7], [8]).

Definition 2. We will say that $R \in IFR(X \times X)$ is:

1) *Reflexive*, if for every $x \in X$, $\mu_R(x, x) = 1$. Just notice that for every x in X , $\nu_R(x, x) = 0$.

2) *Antireflexive*, if for every $x \in X$, then $\begin{cases} \mu_R(x, x) = 0 \\ \nu_R(x, x) = 1, \end{cases}$ that is to say, if its

complementary R_c is reflexive.

3) *Symmetric*, if $R = R^{-1}$, that is, if for every (x, y) of $X \times X$

$$\begin{cases} \mu_R(x, y) = \mu_R(y, x) \\ \nu_R(x, y) = \nu_R(y, x) \end{cases}$$

in the opposite way we will say that it is *asymmetric*.

4) *Antisymmetrical intuitionistic*, if

$$\forall (x, y) \in X \times X, x \neq y \text{ then } \begin{cases} \mu_R(x, y) \neq \mu_R(y, x) \\ \nu_R(x, y) \neq \nu_R(y, x) \\ \pi_R(x, y) = \pi_R(y, x) \end{cases}$$

5) *Perfect antisymmetrical intuitionistic*, if for every $(x, y) \in X \times X$ with $x \neq y$ and

$$\begin{cases} \mu_R(x, y) > 0 \\ \text{or} \\ \mu_R(x, y) = 0 \text{ and } \nu_R(x, y) < 1, \end{cases}$$

then

$$\begin{cases} \mu_R(y, x) = 0 \\ \text{and} \\ \nu_R(y, x) = 1. \end{cases}$$

6) *Transitive*, if $R \geq R \overset{\vee, \beta}{\underset{\wedge, \rho}{\circ}} R$.

7) *C-transitive*, if $R \leq R \overset{\wedge, \rho}{\underset{\vee, \beta}{\circ}} R$.

8) *Partially included*, if for every $x, y, z \in X$ with $\mu_R(x, z) \neq \mu_R(z, y)$,

$$\text{Sign}(\mu_R(x, z) - \mu_R(z, y)) = \text{Sign}(\nu_R(z, y) - \nu_R(x, z))$$

The justification of the definition of partially included relation takes place from the study of the conservation of the transitive property of intuitionistic fuzzy relations through Atanassov's operators for any value of $p \in [0, 1]$. Next Theorem is proved in ([7]):

Theorem 1. Let's take $R \in IFR(X \times X)$ and let it be partially included.

R is transitive if and only if $D_p(R)$ is transitive fuzzy for every $p \in [0, 1]$.

Notice that not only the sign of inequality changes in items 6) and 7), but also the order of \vee , β , \wedge and ρ .

It is worth pointing out that the definition of intuitionistic perfect antisymmetric relation does recover the definition given by Zadeh ([10]) of fuzzy perfect antisymmetry if R is fuzzy.

The given definition of intuitionistic fuzzy antisymmetry will be justified in this paper, as we will see later on.

The following structures of intuitionistic fuzzy relations are defined in ([6], [7]).

Definition 3. An intuitionistic fuzzy relation R on the cartesian set $(X \times X)$, is called :

- 1) an intuitionistic preorder if it is reflexive and transitive
- 2) an intuitionistic order if it is reflexive, transitive and antisymmetrical intuitionistic
- 3) an intuitionistic perfect ordering if it is reflexive, transitive and perfect antisymmetrical intuitionistic
- 4) an intuitionistic strict order if R is antireflexive, transitive and antisymmetrical intuitionistic
- 5) an intuitionistic similarity relation on $X \times X$ if R is reflexive, symmetric and transitive
- 6) an intuitionistic dissimilarity relation on $X \times X$ if R is symmetric, antireflexive and transitive.

3. Ordering in X induced by an IFR

Intuitionistic fuzzy relations can induce different relations in the universal set X . Now we are going to study one of them.

Definition 4. Let R be an element of $IFR(X \times X)$, we define a relation \preceq_R in X through

$$x \preceq_R y \text{ if and only if } \begin{cases} \mu_R(y, x) \leq \mu_R(x, y) \\ \nu_R(y, x) \geq \nu_R(x, y) \end{cases}$$

with $x, y \in X$.

Theorem 2. If $R \in IFR(X \times X)$ is of intuitionistic ordering, then \preceq_R is of ordinary ordering in X .

Proof. i) \preceq_R is reflexive because

$$\begin{cases} \mu_R(x, x) \leq \mu_R(x, x) \\ \nu_R(x, x) \geq \nu_R(x, x) \end{cases}$$

ii) \preceq_R is antisymmetrical because if

$$\begin{cases} x \preceq_R y \\ y \preceq_R x \end{cases} \text{ then } \begin{cases} \mu_R(y, x) = \mu_R(x, y) \\ \nu_R(y, x) = \nu_R(x, y) \end{cases} \text{ therefore } x = y$$

iii) \preceq_R is transitive, so that if

$$\begin{cases} x \preceq_R y \text{ with } x \neq y \\ y \preceq_R z \text{ with } y \neq z, \end{cases}$$

we get

$$\begin{cases} \mu_R(y, x) \leq \mu_R(x, y) \\ \nu_R(y, x) \geq \nu_R(x, y) \end{cases} \quad \begin{cases} \mu_R(z, y) \leq \mu_R(y, z) \\ \nu_R(z, y) \geq \nu_R(y, z) \end{cases}$$

firstly, let's see that they cannot occur at the same time

$$\begin{cases} \mu_R(z, x) \geq \mu_R(x, y) \\ \text{and} \\ \mu_R(z, x) \geq \mu_R(y, z) \end{cases}$$

In order to see it we suppose that if they occur at the same time

$$\begin{aligned} \mu_R(x, y) &= \mu_R(z, x) \wedge \mu_R(x, y) \leq \\ &\leq \bigvee_t [\mu_R(z, t) \wedge \mu_R(t, y)] = \mu_R(z, y). \end{aligned}$$

$$\begin{aligned} \mu_R(y, z) &= \mu_R(y, z) \wedge \mu_R(z, x) \leq \\ &\leq \bigvee_t [\mu_R(y, t) \wedge \mu_R(t, x)] = \mu_R(y, x), \end{aligned}$$

so

$$\begin{aligned} \mu_R(y, x) &\leq \mu_R(x, y) \leq \mu_R(z, y) \leq \mu_R(y, z) \leq \mu_R(y, x) \\ \mu_R(z, y) &\leq \mu_R(y, z) \leq \mu_R(y, x) \leq \mu_R(x, y) \leq \mu_R(z, y), \end{aligned}$$

therefore

$$\begin{aligned} \mu_R(y, x) &= \mu_R(x, y) = \mu_R(z, y) = \mu_R(y, z) = \mu_R(y, x) \\ \mu_R(z, y) &= \mu_R(y, z) = \mu_R(y, x) = \mu_R(x, y) = \mu_R(z, y), \end{aligned}$$

that is to say

$$\mu_R(x, y) = \mu_R(y, x) = \mu_R(z, y) = \mu_R(y, z)$$

and as R is antisymmetric intuitionistic, we get $x = y$ and $y = z$ in opposition to the hypothesis, from where it is deduced that only one of the following possibilities can occur:

i) $\mu_R(z, x) < \mu_R(x, y)$

or

ii) $\mu_R(z, x) < \mu_R(y, z)$

from i) it is deduced that

$$\begin{aligned} \mu_R(z, x) &= \mu_R(z, x) \wedge \mu_R(x, y) \leq \\ &\leq \bigvee_t [\mu_R(z, t) \wedge \mu_R(t, y)] = \mu_R(z, y) \leq \mu_R(y, z), \end{aligned}$$

so

$$\begin{aligned} \mu_R(z, x) &\leq \mu_R(x, y) \wedge \mu_R(y, z) \leq \\ &\leq \bigvee_t [\mu_R(x, t) \wedge \mu_R(t, z)] \leq \mu_R(x, z) \end{aligned}$$

from ii) we get

$$\begin{aligned} \mu_R(z, x) &= \mu_R(z, x) \wedge \mu_R(y, z) \leq \\ &\leq \bigvee_t [\mu_R(y, t) \wedge \mu_R(t, x)] = \mu_R(y, x) \leq \mu_R(x, y), \end{aligned}$$

therefore

$$\mu_R(z, x) \leq \mu_R(x, y) \wedge \mu_R(y, z) \leq \mu_R(x, z).$$

If we take a reasoning analogous to the previous one, we get for the non-membership functions that:

$$\left\{ \begin{array}{l} \nu_R(z, x) \leq \nu_R(x, y) \\ \text{and} \\ \nu_R(z, x) \leq \nu_R(y, z) \end{array} \right.$$

they cannot occur at the same time, so it happens that

i) $\nu_R(z, x) > \nu_R(x, y)$

or

ii) $\nu_R(z, x) > \nu_R(y, z)$

from i) we deduce that

$$\begin{aligned} \nu_R(z, x) &= \nu_R(z, x) \vee \nu_R(x, y) \geq \\ &\geq \bigwedge_t [\nu_R(z, t) \vee \nu_R(t, y)] = \nu_R(z, y) \geq \nu_R(y, z), \end{aligned}$$

therefore

$$\nu_R(z, x) \geq \nu_R(x, y) \vee \nu_R(y, z) \geq \nu_R(x, z).$$

From ii), reasoning in an analogous way, it is deduced that $\nu_R(z, x) \geq \nu_R(x, z)$. \square

Notice that our definition of intuitionistic antisymmetry does not recover the one by A. Kaufmann ([9]) for the fuzzy case. However, the advantage of the definition established by us is that, with this definition, if the intuitionistic relation is an intuitionistic fuzzy ordering, we can induce the ordering \preceq_R in X , as we have proved in the previous Theorem. This fact does not happen with A. Kaufmann's definition, because the demonstration given by him to prove this fact in ([9]) is not correct.

4. Orderings in X induced by D_p operators

In the previous item we have studied a relation induced in the referential X through a relation of intuitionistic ordering. We present now a new relation induced in X using the sets of level $q \in [0, 1]$, associated to the fuzzy relation $D_p(R)$, obtained from the relation of perfect intuitionistic ordering R through D_p operators.

Starting from the sets of the level $q \in [0, 1]$

$$\left(\bar{D}_p(R) \right)_q = \{ (x, y) \mid \mu_{D_p(R)}(x, y) \geq q \},$$

we define the following relation in $X \times X$.

Definition 5.

$$x \preceq_{\left(\bar{D}_p(R) \right)_q} y \text{ if and only if } (x, y) \in \left(\bar{D}_p(R) \right)_q \text{ if and only if } \mu_{D_p(R)}(x, y) \geq q.$$

We will use, for the composition, the t-norms and t-conorms $\alpha = \vee, \beta = \wedge, \lambda = \wedge$ and $\rho = \vee$ in the following Theorems.

Theorem 3. Let $R \in IFR(X \times X)$ be partially included, for every $p, q \in [0, 1]$ it is verified that:

- i) If R is intuitionistic preorder, then $\preceq_{\left(\bar{D}_p(R)\right)_q}$ is preorder in X .
- ii) If R is of intuitionistic similarity, then $\preceq_{\left(\bar{D}_p(R)\right)_q}$ is of similarity in X .

Proof.

a) If R is reflexive, then $\preceq_{\left(\bar{D}_p(R)\right)_q}$ is also reflexive, because

$$\mu_{D_p(R)}(x, x) = \mu_R(x, x) + p \cdot (1 - \mu_R(x, x) - \nu_R(x, x)) = \mu_R(x, x) = 1 \text{ then}$$

$$\mu_{D_p(R)}(x, x) = 1 \geq q \text{ therefore } x \preceq_{\left(\bar{D}_p(R)\right)_q} x.$$

b)

$$x \preceq_{\left(\bar{D}_p(R)\right)_q} y \text{ then } \mu_{D_p(R)}(x, y) \geq q$$

$$y \preceq_{\left(\bar{D}_p(R)\right)_q} z \text{ then } \mu_{D_p(R)}(y, z) \geq q$$

as R is reflexive, transitive and partially included, we have

$$\begin{aligned} \mu_{D_p(R)}(x, z) &= \mu_{D_p(R) \overset{\vee, \wedge}{\underset{\wedge, \vee}{\circ}} D_p(R)}(x, z) = \\ &= \bigvee_t \{ \wedge [\mu_{D_p(R)}(x, t), \mu_{D_p(R)}(t, z)] \} \geq \\ &\geq \wedge [\mu_{D_p(R)}(x, y), \mu_{D_p(R)}(y, z)] \geq q \wedge q = q. \end{aligned}$$

c) If R is symmetric, $x \preceq_{\left(\bar{D}_p(R)\right)_q} y$ then $\mu_{D_p(R)}(x, y) =$

$$\mu_R(x, y) + p \cdot (1 - \mu_R(x, y) - \nu_R(x, y)) = \mu_R(y, x) + P \cdot (1 - \mu_R(y, x) - \nu_R(y, x)) =$$

$$= \mu_{D_p(R)}(y, x) \geq q \implies y \preceq_{\left(\bar{D}_p(R)\right)_q} x. \quad \square$$

Theorem 4. Let $R \in IFR(X \times X)$ be partially included. Then for every $p \in [0, 1]$ and for every $q \in (0, 1]$, it is verified:

If R is an ordering respecting the intuitionistic perfect antisymmetry, then $\preceq_{\left(\bar{D}_p(R)\right)_q}$ is an ordering in X .

Proof. Reflexivity and transitivity are proved in the same way as in the previous Theorem. Let's see antisymmetry.

$$x \preceq_{\left(\bar{D}_p(R)\right)_q} y \text{ then } \mu_{D_p(R)}(x, y) \geq q$$

$$y \preceq_{\left(\bar{D}_p(R)\right)_q} x \text{ then } \mu_{D_p(R)}(y, x) \geq q$$

as R is perfect intuitionistic antisymmetric, through paper ([7]), $D_p(R)$ is fuzzy perfect antisymmetric for every $p \in [0, 1]$, as $q \neq 0$, we get

$$\mu_{D_p(R)}(x, y) > 0 \text{ and } \mu_{D_p(R)}(y, x) > 0,$$

so $x = y$. \square

Theorem 5. Let $p, p_1, p_2, \in [0, 1]$, $q, q_1, q_2 \in (0, 1]$. The relation $R \in IFR(X \times X)$, partially included and of ordering respecting the perfect intuitionistic anti-symmetry. Then

i) If $p_1 \leq p_2$ then $\preceq_{\left(\bar{D}_{p_1}(R)\right)_q} \leq \preceq_{\left(\bar{D}_{p_2}(R)\right)_q} \quad \forall q \in (0, 1]$

ii) If $q_1 \geq q_2$ then $\preceq_{\left(\bar{D}_p(R)\right)_{q_1}} \leq \preceq_{\left(\bar{D}_p(R)\right)_{q_2}} \quad \forall p \in [0, 1]$

iii) If $p_1 \leq p_2, q_1 \geq q_2$ then $\preceq_{\left(\bar{D}_{p_1}(R)\right)_{q_1}} \leq \preceq_{\left(\bar{D}_{p_2}(R)\right)_{q_2}}$.

Proof.

i) $x \preceq_{\left(\bar{D}_{p_1}(R)\right)_q} y$ then $\mu_R(x, y) + p_1 \cdot (1 - \mu_R(x, y) - \nu_R(x, y)) \geq q$ as

$p_1 \leq p_2$ then $\mu_{D_{p_2}(R)}(x, y) \geq \mu_{D_{p_1}(R)}(x, y) \geq q \implies x \preceq_{\left(\bar{D}_{p_2}(R)\right)_q} y$.

ii) $x \preceq_{\left(\bar{D}_p(R)\right)_{q_1}} y$ then $\mu_{D_p(R)}(x, y) \geq q_1 \geq q_2$ therefore $x \preceq_{\left(\bar{D}_p(R)\right)_{q_2}} y$.

iii) Through the previous items we have got that:

$$\preceq \left(\bar{D}_{p_1}(R) \right)_{q_1} \leq \preceq \left(\bar{D}_{p_2}(R) \right)_{q_1} \leq \preceq \left(\bar{D}_{p_2}(R) \right)_{q_2} . \quad \square$$

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