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Essays in Macroeconomics

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Ph.D. Thesis

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To Amaia,

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Summary

This thesis consists of three chapters, each of them analyzing relevant issues in the field of Macroeconomics. The research conducted in all three chapters relies on the methodology of dynamic general equilibrium models. The microeconomic foundations are at the core of these type of models, in which rational, forward-looking agents face an uncertain environment. This methodology has proved to be a powerful tool to address a broad array of economic problems. The first chapter, *"Risk Aversion, Entrepreneurship and Wealth Distribution"*, explores the role of risk aversion heterogeneity on entrepreneurship, saving behavior and the wealth distribution. The distribution of wealth in the economy is one of the most debated issues in contemporary politics, popular media, as well as among academic scholars. In particular, the large concentration of wealth in the upper tail of the distribution observed in many advanced economies has come under the spotlight. The relevance of this issue has not been undermined despite the difficulty in estimating the actual levels of wealth held in the hands of the richest households. Supported by empirical grounds, the theoretical framework to analyze the wealth distribution should include both entrepreneurship and preference heterogeneity. To this aim, an occupational choice model with heterogeneous agents is developed in which households vary in their risk aversion, face uninsurable idiosyncratic shocks, are financially constrained, and decide to become workers or entrepreneurs. This decision is based on a combination of individual risk aversion, occupation-specific skills and wealth holdings. It is found that risk aversion has both a direct and indirect effect on the savings and occupation decision of agents. On one hand, higher risk aversion deters risky entrepreneurship, and hence the savings aimed at overcoming financial constraints may be lower. On the other hand, higher risk aversion also leads to larger precautionary savings which, indirectly, attenuate financial constraints and makes entrepreneurship more attractive. In this framework, the latter effect dominates, and more risk averse agents hold higher levels of wealth and also become entrepreneurs. The most important contribution of this model is that it allows to disentangle the effects that risk aversion and financial frictions have on entrepreneurship, which is a key driver of wealth inequality.

The second chapter is titled *"Long-term business relationships, bargaining and monetary policy"* and it is a joint work with Mirko Abbritti and Tommaso Trani. Its motivation stems from the growing empirical literature documenting the importance of long-term business relationships and bargaining for price rigidity and firms' dynamics. This paper introduces long-term business-to-business (B2B) relationships and price bargaining into a standard monetary DSGE model. The model is based on two assumptions. First, both wholesale and retail producers need to spend resources to form new business relationships. Second, once a B2B relationship is formed, the price is set in a bilateral bargaining between firms. The model provides a rigorous framework to study the effect of long-term business relationships and bargaining on monetary policy and business cycle dynamics. It shows that these relationships reduce both the allocative role of intermediate prices and the real effects of monetary policy shocks. We also find that the model does a good job in replicating the second moments and cross-correlations of the data, and that it improves over the benchmark New Keynesian model in explaining some of them.

The third and last chapter of this thesis, *"On Staggered Prices and Optimal Inflation"*, is coauthored with one my advisors, Miguel Casares. This paper computes the steady-state optimal rate of inflation assuming two different sticky-price specifications, Calvo (1983) and Taylor (1980), in a model with monopolistic competition. We find that the optimal rate of inflation in steady state is always positive. This result is robust to changes in the degree of price stickiness. In both cases of staggered prices, the optimal rate of inflation is approximately equal to the ratio between the rate of discount and the Dixit-Stiglitz elasticity. For standard calibrations, the welfare cost of inflation is quantitatively small but significantly higher under Calvo pricing than under Taylor pricing.

Resumen

Esta tesis consta de tres capítulos, cada uno de los cuales analiza temas relevantes en el campo de la Macroeconomía. La investigación realizada en los tres capítulos se basa en la metodología de los modelos dinámicos de equilibrio general. Los fundamentos microeconómicos se encuentran en el núcleo de este tipo de modelos, en los que los agentes forman sus expectativas sobre el futuro de manera racional. Esta metodología ha demostrado ser una herramienta poderosa para abordar un gran conjunto de problemas económicos. El primer capítulo, *"Risk Aversion, Entrepreneurship and Wealth Distribution"*, explora el efecto de la heterogeneidad en la aversión al riesgo sobre el emprendimiento, el comportamiento del ahorro y la distribución de la riqueza. La distribución de la riqueza es uno de los temas más debatidos no solo en la política contemporánea, si no también en los medios de comunicación y en ámbitos académicos. En particular, la gran concentración de riqueza en la parte superior de la distribución, que se observa en muchas economías avanzadas, se ha situado en el centro del debate. La relevancia de este problema no se ha visto socavada a pesar de la dificultad que supone estimar los niveles de riqueza acumulada en manos de los hogares más ricos. Justificado por razones empíricas, el marco teórico para analizar la distribución de la riqueza debe incluir tanto el emprendimiento como la heterogeneidad de las preferencias. Con este objetivo, en el primer capítulo se desarrolla un modelo de elección ocupacional con agentes heterogéneos en el que los hogares varían en su aversión al riesgo, se enfrentan a perturbaciones idiosincrásicas no asegurables, tienen restricciones financieras y deben decidir si convertirse en trabajadores o en emprendedores. Esta decisión se basa en una combinación de la aversión al riesgo individual, las habilidades específicas en cada ocupación y los niveles de riqueza acumulados. Como resultado se obtiene que la aversión al riesgo tiene un efecto directo e indirecto en la decisión de ahorro y en la ocupación de los agentes. Por un lado, una mayor aversión al riesgo disuade a los agentes de convertirse en emprendedores y, por lo tanto, los ahorros destinados a superar las restricciones financieras son menores. Por otro lado, una mayor aversión al riesgo también conduce a un mayor ahorro por motivo de precaución que, indirectamente, atenúa las restricciones financieras y hace que la

opción de convertirse en emprendedor sea más atractiva. En el modelo desarrollado, el último efecto domina, y los agentes con mayor aversión al riesgo acumulan mayores niveles de riqueza y también se convierten en emprendedores. La contribución más importante de este modelo es que permite desentrañar los efectos que la aversión al riesgo y las fricciones financieras tienen sobre el emprendimiento, que es un factor clave para explicar la desigualdad en la acumulación de la riqueza.

El segundo capítulo se titula *“Long-term business relationships, bargaining and monetary policy”* y es un trabajo conjunto con Mirko Abbritti y Tommaso Trani. Su motivación se deriva de la creciente literatura empírica que documenta la importancia de las relaciones comerciales a largo plazo y los procesos de negociación sobre la rigidez de los precios y las dinámicas empresariales. Este artículo introduce relaciones comerciales entre empresas (B2B) a largo plazo y procesos de negociación de precios en un modelo monetario de equilibrio general dinámico estocástico (DSGE) estándar. El modelo se basa en dos suposiciones. Primero, tanto los productores mayoristas como los minoristas necesitan gastar recursos para formar nuevas relaciones comerciales. Segundo, una vez que se forma una relación comercial, el precio se establece en una negociación bilateral entre las empresas. El modelo proporciona un marco riguroso para estudiar el efecto de las relaciones comerciales a largo plazo y los procesos de negociación sobre la política monetaria y la dinámica del ciclo económico. Además, se demuestra que estas relaciones comerciales reducen tanto el papel de asignación de los precios intermedios como los efectos reales de las perturbaciones de la política monetaria. También encontramos que el modelo hace un gran trabajo al replicar los segundos momentos y las correlaciones cruzadas de los datos, y que mejora con respecto al modelo de referencia Neo Keynesiano en explicar dichos estadísticos.

El tercer y último capítulo de esta tesis, *“On Staggered Prices and Optimal Inflation”*, es coautoreado con uno de mis directores, Miguel Casares. En este artículo se calcula la tasa de inflación óptima en estado estacionario bajo dos especificaciones de rigidez de precios diferentes, Calvo (1983) y Taylor (1980), en un modelo con competencia monopolística. Encontramos que la tasa óptima de inflación en estado estacionario es siempre positiva. Este resultado es robusto a los cambios en el grado de rigidez de los precios. En ambos casos con rigidez de precios, la tasa de inflación óptima es aproximadamente igual al cociente entre la tasa de descuento y la elasticidad de Dixit-Stiglitz. Para calibraciones estándar, el coste de bienestar de la inflación es cuantitativamente pequeño, pero significativamente más alto si los precios son rígidos *à la* Calvo que si son rígidos *à la* Taylor.

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1.1 Introduction

Inequality, its causes and consequences have always been of great interest for economists. Adam Smith (1776) affirms that "*No society can surely be flourishing and happy, of which the far greater part of the members are poor and miserable.*". Yet, despite its timeless appeal, it has been only in recent years that inequality in earnings, income and wealth has become among the most debated issues in contemporary politics and economics.[1] Empirically, a larger concentration of wealth in the upper tail of the distribution has been observed in many advanced economies. In 2014, the average shares of wealth in the OECD countries held by the richest 10%, 5% and 1% of the population were 50.91%, 37.57% and 17.78%, respectively.[2,3] Among these countries the one with the highest concentration in the top was the U.S. in 2016, where the richest 10%, 5% and 1% of the population held 79.47%, 68.05% and 42.48% of the wealth, respectively. Understanding why people save is not only necessary to explain why some people become rich while others remain poor, but also to provide an adequate guidance to policy makers on which instruments to use to tackle inequality.

The purpose of this paper is to study the role of risk aversion heterogeneity on entrepreneurship, saving behavior and the wealth distribution. The analysis is carried out in a model in which households vary in their risk aversion, face uninsurable idiosyncratic shocks, are financially constrained, and decide to become workers or entrepreneurs. Entrepreneurs are households who own and run a business, and receive its profits as compensation for their labor.

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[1] See for example Piketty (2014), Saez and Zucman (2016) and Kuhn and Rios-Rull (2016), among others. From the political side Senator Bernie Sanders, while campaigning for the Democratic presidential nomination in 2015, listed "Income and Wealth Inequality" as one of the most important issues for the American people.

[2] See Figure A1.1 in the Appendix.

[3] Average computed for year 2014 or latest available: Spain and Canada (2012), Estonia, Finland, Ireland and Portugal (2013), Denmark (2015), US (2016).

The most important contribution of this model is that it allows to disentangle the effects that risk aversion and financial frictions have on entrepreneurship, which is a key driver of wealth concentration. Households sort themselves between the different occupations based on their risk aversion, occupation-specific ability and wealth holdings. In accordance with empirical evidence, at the individual level, the entry decision into entrepreneurship is deterred by higher values of risk aversion, as well as by low wealth holdings.[4] This has important implications for the saving behavior. On one hand, more risk averse households are discouraged from entry into risky entrepreneurship, and hence the savings aimed at overcoming financial constraints may be lower. On the other hand, higher risk aversion also leads to larger precautionary savings. In this framework, the precautionary motives channel leads to higher savings which, indirectly, reduce the financial constraints and makes entrepreneurship a more profitable option. Consequently, at the aggregate level, more risk averse agents hold higher levels of wealth and also become entrepreneurs. This result is very relevant for two reasons. Firstly, it complements the existing literature on entrepreneurship and wealth distribution that argues that entrepreneurs save more to overcome borrowing constraints, explaining the higher concentration of wealth in the upper tail of the distribution. The introduction of risk aversion heterogeneity uncovers an additional channel that connects savings and entrepreneurship. The coexistence of individuals with different attitudes towards risk implies that, *ceteris paribus*, the more risk averse have higher precautionary savings, which indirectly reduce their borrowing constraints and makes them more prone to become entrepreneurs because they can run larger firms and therefore have higher profits. Moreover, this new channel also indicates that the distribution of risk aversion has an effect on the wealth distribution. In particular, distributions that display higher shares of more risk averse individuals are associated with lower levels of wealth inequality.

And secondly, the findings of this paper suggest that, in the presence of risk aversion heterogeneity and financial frictions, the decision to become an entrepreneur may not be fully determined by attitudes towards risk. One of the most prominent theories of entrepreneurship establishes that less risk averse individuals become entrepreneurs while the more risk averse become workers (Kihlstrom and Laffont, 1979; Feng and Rauch, 2015). This theory seems to be, to some extent, empirically supported by Barsky et al. (1997) who, using survey data from the Health and Retirement Study (HRS), estimate the predictive power of preferences over risky behaviors, such as the decisions to smoke and drink, to buy insurance, to be self-employed,

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[4] Barsky et al. (1997), Cramer et al. (2002), Ekelund et al. (2005), Dohmen et al. (2011) and Falk et al. (2018) among others, document a negative relation between risk aversion and entrepreneurship. A good survey of the empirical evidence on how borrowing constraints affect entrepreneurship can be found in Cagetti and De Nardi (2006).

and to hold stock. They find that individuals with higher risk tolerance are more likely to become entrepreneurs. However, none of these references take into account the existence of financial frictions, which has been proved to be an important determinant of entrepreneurship. Actually, the findings of this paper suggest that when risk aversion heterogeneity is considered in conjunction with financial frictions, the relevance of the latter on the decision to entry into entrepreneurship dominates and therefore the negative relation between risk aversion and entrepreneurship disappears at the aggregate level.[5] In other words, if one wants to understand who becomes an entrepreneur it is not enough to focus on the risk preferences of individuals.

Further motivation for this research is provided by the following arguments. One is to take into consideration the empirical evidence referenced above that documents how less risk averse individuals tend to become entrepreneurs more frequently than more risk averse individuals.[6] If individuals have to decide whether to become entrepreneurs or workers, it comes naturally that this decision is partly based on their risk preferences. Another argument is to provide a framework that can combine the two main purposes why people save, i.e., to smooth consumption through time and to overcome borrowing constraints. Models that only consider risk aversion heterogeneity but do not include entrepreneurship find that more risk averse individuals save more for precautionary motives and accumulate higher levels of wealth. Alternatively, models with entrepreneurship but without risk aversion heterogeneity find that households that want to become entrepreneurs accumulate more wealth to overcome borrowing constraints. The findings of this paper reconcile these previous results and show that, in the presence of risk aversion heterogeneity and financial frictions, precautionary savings are quantitatively substantial and, indirectly, attenuate borrowing constraints, which drives households into entrepreneurship, making them wealthier than workers. This implies that, at the aggregate level, there is a positive relation between wealth and risk aversion. This may seem counterintuitive but it is actually supported by the empirical finding of Paravisini et al. (2017) who estimate risk aversion from investors' financial decisions and find that wealthier investors are more risk averse in the cross section.

Bewley (1977) provides a powerful theoretical framework to quantitatively analyze wealth inequality, the literature to which this paper relates to. In a recent survey article, De Nardi and Fella (2017) argue that, within that class of models, two of the main forces that pro-

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[5] Falk et al. (2018) find that at the individual level, more risk tolerant individuals are more likely to become entrepreneurs. However, they also find that at the country level risk taking is not significantly positively related to entrepreneurial activities.

[6] Henceforth, I use the terms "entrepreneur" and "self-employed" interchangeably. In this I follow Quadrini (2000).

vide a better approximation of the shape of the wealth distribution are entrepreneurship and preference heterogeneity. The most influential papers connecting entrepreneurship and wealth inequality are Quadrini (2000) and Cagetti and De Nardi (2006). They show that introducing entrepreneurship is key to generate a higher concentration of wealth in the upper tail of the distribution, an empirical feature that the seminal paper of Aiyagari (1994) is not able to replicate. Actually, according to Kuhn and Rios-Rull (2016), business wealth and business income are the most important sources of wealth and income inequality in the U.S., accounting for about 20%. It is also worth noticing that, despite the fact that entrepreneurs only constitute about 8% of the population in the U.S., they hold approximately 40% of total net wealth, a rather large share (De Nardi and Fella, 2017).[7] Notwithstanding, none of these models include preference heterogeneity and therefore do not consider the effect that this may have on their results.

There is also a large empirical literature documenting preference heterogeneity among individuals and how these differences in preferences are associated with different behaviors. As mentioned before, Barsky et al. (1997) find substantial heterogeneity in risk preferences across individuals and that risk tolerance has predictive power for choice over risky behaviors.[8] These results are confirmed by Kimball et al. (2008, 2009), who additionally find that differences in risk preferences are persistent through time. This is important because if people are not choosing their occupation purely based on their productivity, in the presence of uninsurable risk, output may not be maximized (Schulhofer-Wohl, 2011).[9] Also, when studying the importance of precautionary savings for wealth accumulation, Cagetti (2003) finds that differences in wealth holdings can arise from differences in risk aversion and the degree of household patience.

Despite the fact that risk aversion and entrepreneurship seem to be closely related, to the best of my knowledge, I am the first to analyze how the combination of both can affect the wealth distribution.[10] The closest related papers to the study of this relation are Herranz et al. (2015) and Cozzi (2011). Nevertheless, both depart in important ways from the model proposed in this paper. On one hand, Herranz et al. (2015) analyze the interactions between risk aversion

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[7] Small businesses (fewer than 500 employees) play an important role in the U.S. economy. In 2008, small businesses produced 46% of the private nonfarm GDP (Kobe, 2012). According to the U.S. Small Business Administration, in 2014, small businesses employed 47.8% of the private workforce.

[8] Chiappori and Paiella (2011) and Paravisini et al. (2017) also find risk aversion heterogeneity.

[9] One could think of this as some type of labor missallocation. The most efficient outcome in terms of aggregate production would be obtained by matching households with the jobs they are most productive at, regardless of the risk these involve. If this is not the case, an increase in aggregate output could be obtained by reallocating workers to different jobs.

[10] Knight (1921) was one of the first to study this connection between risk aversion and entrepreneurship. For more recent work see Kihlstrom and Laffont (1979), Barsky et al. (1997), Kimball et al. (2008, 2009) and Hvide and Panos (2014).

heterogeneity and entrepreneurship to study firm size, capital structure and default, but their model does not provide any results on the wealth distribution, which is an essential issue in this paper. On the other hand, Cozzi (2011) develops a model with risk aversion heterogeneity, uninsurable idiosyncratic income risk, and (with or without) self-selection into risky jobs to quantify their effects on the distribution of wealth. However, the way the occupational choice is modeled in that paper is completely different from the one followed here. In his model there are no entrepreneurs, jobs only differ in their income risk and workers make a once-in-a-lifetime decision on their career. The absence of entrepreneurs is non-trivial because it eliminates the saving motive to overcome borrowing constraints. Without this mechanism, households only save for precautionary motives, which do not suffice to match the wealth holdings of the top 1% of the population, a fact that the model in this paper is perfectly able to replicate. Therefore, the model proposed below attempts to fill this gap and analyzes the effects that the interactions between risk aversion heterogeneity and entrepreneurship may have on saving behavior and the wealth distribution.

The structure of the rest of the paper is as follows. Sections 1.2 and 1.3 describe the quantitative model and the calibration strategy, respectively. The interactions between risk aversion heterogeneity and entrepreneurship, and their effect on the wealth distribution are discussed in Section 1.4. Section 1.5 addresses remaining work and future extensions. Finally, section 1.6 concludes.

1.2 The Model

The economy is populated by a continuum of households with unit mass, which are *ex ante* heterogeneous in terms of their risk aversion. Following the evidence documented by Kimball et al. (2008), it is assumed that the risk aversion of households is constant through time.[11] In each period, they have to decide whether they want to be an entrepreneur or a worker in the next period. The structure of the model is similar to Quadrini (2000) and Cagetti and De Nardi (2006), where there are two sectors of production, the entrepreneurial sector and the non-entrepreneurial or corporate sector. Also, households are heterogenous in their abilities as workers and entrepreneurs.

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[11] See Figure A1.2 in the Appendix.

1.2.1 Households

Households are heterogeneous in risk aversion (σ_i) and maximize the expected lifetime utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t^e, l_t^w; \sigma_i)$$

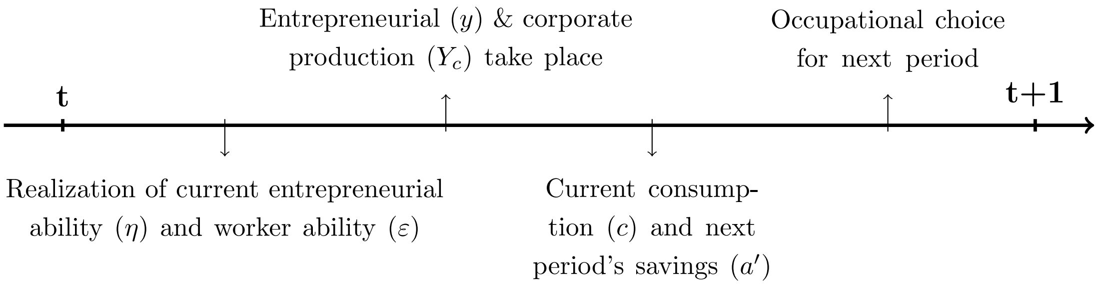
where $\beta \in (0,1)$ is the discount factor rate, c_t is consumption and l_t^e and l_t^w denote entrepreneurial and worker labor supplied, respectively. In particular, it is assumed that utility follows the CRRA specification $u(c_t, l_t^e, l_t^w; \sigma_i) = c_t^{1-\sigma_i}/(1-\sigma_i) - \varphi_e l_t^e - \varphi_w l_t^w$. Each household is endowed with one indivisible unit of time that can be either supplied to the market as a worker or employed in one's own business as an entrepreneur. This occupational choice decision was made in the previous period. The potentially different values of the disutility of each type of labor, φ_e and φ_w, can have different justifications. On one hand it could capture non-pecuniary benefits associated to entrepreneurship such as flexibility of hours or being one's own boss (see for example Hurst and Pugsley (2011) and Yurdagul (2017)). On the other hand there is empirical evidence suggesting that entrepreneurs have different Frisch elasticity from workers (Martinez et al., 2018).

Figure 1.1 describes the timing of events within a period. At the beginning of every period, each household is exogenously endowed with two types of abilities, one for each occupation. These abilities are stochastic, serially correlated, and uncorrelated with each other. The entrepreneurial ability (η) determines the level of productivity that the household can achieve from running his own business. The working ability (ε) captures the units of labor efficiencies. These units, which are equally productive in both sectors of production, are supplied to the labor market in exchange for the wage rate w. The assumption that households have to decide their next-period occupation without knowing their future abilities is made to introduce uncertainty and provides a more realistic scenario to study the combined effects of risk aversion and entrepreneurship on the accumulation of wealth. Each ability follows a first order Markov process where Γ_η and Γ_ε are the transition probability matrices of the entrepreneurial and worker ability, respectively.

1.2.2 Production

The production side of the economy is composed of two sectors. On one hand, the entrepreneurial sector represents the set of small firms in the economy, where each of them is owned by the household that runs it. On the other hand, the corporate sector is composed

Figure 1.1: Timing of events within a period



by large units of production. As in previous work, two important differences between sectors are the uninsurable risk and borrowing constraints, that only affect entrepreneurs. Firms from both sectors rent capital, which depreciates at a constant rate δ, and hire workers in competitive markets to produce an homogeneous good.

Entrepreneurial Sector

The entrepreneurial sector is composed by those households that in the previous period decided to run a business in the current period, and receive the profits as a compensation for their labor. The production function is given by

$$y = f(\eta, k, l) = \eta k^{\alpha} n^{\nu} \tag{1.1}$$

where $\alpha + \nu \in (0, 1)$, which implies diminishing returns to scale on capital and labor. This form of the production function serves a twofold motive. On one hand, diminishing returns to scale guarantee the existence of positive profits so entrepreneurs can be compensated for their labor. On the other hand, they imply a better description of the small-sized firms in the economy. The household-specific entrepreneurial ability is given by η, the amount of labor hired is n, and k is the amount of capital invested in the business.

The amount of capital that an entrepreneur can invest in his business is limited by his wealth holdings (a). Since the focus on this paper is on the effect of risk aversion on occupational choice, a simple specification of the borrowing constraint is assumed. Following Buera (2009), entrepreneurs cannot invest a capital amount higher than a multiple of their wealth, $k \leq \lambda a$, where $\lambda \geq 1$. If $\lambda = 1$ the entrepreneurs' wealth sets the upper limit on the capital they can invest in their own business. As λ increases, the borrowing constraint is relaxed and entrepreneurs can invest beyond their current wealth holdings.

Therefore, the static optimization problem of entrepreneurs is

$$\tilde{\Pi}(\eta, k, n, a; \lambda) = \max_{k,n} \{\eta k^{\alpha} n^{\nu} - (r+\delta)k - wn\}$$
$$s.t. \quad k \leq \lambda a$$

where r is the net risk-free rate between the previous and current period, and w is the wage rate. Given that the amount of labor hired is not constrained, the problem can be rewritten in terms of the entrepreneurial ability and capital

$$\Pi(\eta, k, a; \lambda) = \max_{k} \left\{\mathbb{A}_{\phi} \left(\eta k^{\alpha}\right)^{\phi} - (r+\delta)k\right\}$$
$$s.t. \quad k \leq \lambda a$$

where $\mathbb{A}_{\phi} = (1-\nu)(\nu/w)^{\nu/(1-\nu)}$ and $\phi = 1/(1-\nu)$. Notice that the borrowing constraint will not bind unless the unconstrained optimal level of capital, $k^{*}(\eta)$ given by

$$k^{*}(\eta) = arg \max_{k} \left\{\mathbb{A}_{\phi} \left(\eta k^{\alpha}\right)^{\phi} - (r+\delta)k\right\}$$

$$k^{*}(\eta) = \left(\frac{\alpha \phi A_{\phi} \eta^{\phi}}{r+\delta}\right)^{\frac{1}{1-\alpha\phi}}$$

is higher than λa. Hence, the capital and labor demand of each entrepreneur are given by:

$$k(\eta, a) = \begin{cases} k^{*} & \text{if } k^{*} \leq \lambda a \\ \lambda a & \text{if } k^{*} > \lambda a \end{cases} \qquad \text{and} \qquad n(\eta, k) = \left(\frac{\nu \eta k^{\alpha}}{w}\right)^{\frac{1}{1-\nu}}$$

This implies that wealth alone does not determine which entrepreneurs are going to be financially constrained, but a combination of wealth and entrepreneurial ability. Therefore, it is possible to have a situation where there are two entrepreneurs with different wealth levels and only the one with higher wealth is constrained. In other words, households with higher entrepreneurial entrepreneurial ability will also have a higher optimal level of capital, which might not be feasible despite having more wealth than a low-entrepreneurial-ability household, who is not financially constrained because it has a lower optimal level of capital. The necessity to overcome this financial friction is going to motivate entrepreneurs to save more and it is going to be an important element of the model to explain wealth accumulation. Hence, how much individuals decide to save is going to be affected by their occupation-specific ability, degree of risk aversion and wealth holdings.

Corporate Sector

The technology in the non-entrepreneurial sector is given by the standard constant returns to scale production function

$$Y_c = A_c K_c^{\Omega} L_c^{1-\Omega} \tag{1.2}$$

where K_c and L_c are the capital input and efficiency units of labor used, respectively. As opposed to the entrepreneurial sector, the corporate sector does not display financial frictions.

1.2.3 Entrepreneurs' Problem

At the beginning of the period, the set of state variables for an entrepreneur is (a, η, ε). Let $V(a, \eta, \varepsilon)$ denote the value function of an entrepreneur before the occupational choice is made, $V^w(a, \eta, \varepsilon)$ denote the value function of an entrepreneur today that has decided to become a worker in the next period and $V^e(a, \eta, \varepsilon)$ denote the value function of an entrepreneur today that has decided to continue being an entrepreneur in the next period. Hence, the beginning-of-period value function of an entrepreneur with risk aversion σ is[12]

$$V(a, \eta, \varepsilon; \sigma) = Max\left[V^e(a, \eta, \varepsilon; \sigma), V^w(a, \eta, \varepsilon; \sigma)\right] \tag{1.3}$$

The Bellman equation of an entrepreneur today that continues as an entrepreneur is the next period is[13]

$$\begin{gathered} V^e(a, \eta, \varepsilon; \sigma) = \max_{\{c, a', k\}} \{u(c; \sigma) + \beta \mathbb{E} V(a', \eta', \varepsilon'; \sigma)\} \\ s.t. \;\; c = a(1+r) + \Pi(\eta, k, a; \lambda) - a' \\ k \geq 0, \;\; a' \geq 0 \end{gathered}$$

where $\beta \in (0,1)$ is the discount factor. The expected value of the function is taken with respect to (η', ε'), conditional on (η, ε). The first constraint is the budget constraint, which includes the undepreciated capital and the profits from the entrepreneurs' own business, $\Pi(\eta, k, a; \lambda)$.

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[12] To be more rigorous, all the control variables should be indexed by the risk aversion of the individual. However, to keep notation simple I refrain from doing so.

[13] Technically, the instantaneous utility function depends not only on consumption but also on both types of labor. However, in order to provide a cleaner analysis of the effects of risk aversion on occupational choice, the labor disutility parameters φ_e and φ_w are set to zero and the utility function is rewritten as $u(c) = c^{1-\sigma}/(1-\sigma)$. This will be modified in future calibrations.

The problem of an entrepreneur that decides to become a worker in the next period is given by

$$\begin{aligned} V^w(a,\eta,\varepsilon;\sigma) = \max_{\{c,a',k\}} &\{u(c;\sigma) + \beta \mathbb{E} J(a',\eta',\varepsilon';\sigma)\} \\ s.t.\ \ c = a(1+r) &+ \Pi(\eta,k,a;\lambda) - a' \\ k \geq 0,&\ \ a' \geq 0 \end{aligned}$$

where $J(a',\eta',\varepsilon';\sigma)$ is the value function of a worker at the beginning of the period (before the occupational choice for the next period is made) and is the only difference between this and the previous problem. The details of this function $J(\cdot)$ are explained in the following paragraphs.

1.2.4 Workers' Problem

At the beginning of the period, the set of state variables for a worker is (a,η,ε), the same as for the entrepreneur. Let $J(a,\eta,\varepsilon;\sigma)$ denote the value function of a worker before the occupational choice is made, $J^w(a,\eta,\varepsilon;\sigma)$ denote the value function of a worker today that has decided to continue as a worker in the next period and $J^e(a,\eta,\varepsilon;\sigma)$ denote the value function of a worker today that has decided to become an entrepreneur in the next period. Hence,

$$J(a,\eta,\varepsilon;\sigma) = Max\left[J^e(a,\eta,\varepsilon;\sigma), J^w(a,\eta,\varepsilon;\sigma)\right] \tag{1.4}$$

The Bellman equation of a worker today that becomes an entrepreneur is the next period is

$$\begin{aligned} J^e(a,\eta,\varepsilon;\sigma) = \max_{\{c,a'\}} &\{u(c;\sigma) + \beta \mathbb{E} V(a',\eta',\varepsilon';\sigma)\} \\ s.t.\quad c = a(1+r) &+ w\varepsilon - a' \\ a' &\geq 0 \end{aligned}$$

Like in the entrepreneurs' problem the first constraint is the budget constraint and the only difference is that instead of profits and undepreciated capital the household receives the wage income.

If a worker continues being a worker in the next period his problem is given by

$$
\begin{aligned}
J^e(a,\eta,\varepsilon;\sigma) &= \max_{\{c,a'\}} \{u(c;\sigma) + \beta \mathbb{E} J(a',\eta',\varepsilon';\sigma)\} \\
s.t. \quad c &= a(1+r) + w\varepsilon - a' \\
a' &\geq 0
\end{aligned}
$$

1.2.5 Stationary Equilibrium

Suppose there are N_σ different types of risk aversion, each with mass ω_i. Therefore, we have $\sum_{i=1}^{N_\sigma} \omega_i = 1$. Let $x_i = (a_i, \eta_i, \varepsilon_i, \theta_i)$ be the state vector for a household with risk aversion σ_i, where $\theta_i \in \{\text{Worker, Entrepreneur}\}$ denotes current occupation. The state vector of the economy is $x = \sum_{i=1}^{N_\sigma} \omega_i x_i = (a, \eta, \varepsilon, \theta)$.

A steady state recursive competitive equilibrium for this economy consists of:

1. For each risk aversion type i, value functions $V(x_i)$, $V^e(x_i)$, $V^w(x_i)$, $J(x_i)$, $J^e(x_i)$ and $J^w(x_i)$.
2. For each risk aversion type i, decision functions $c(x_i)$, $a'(x_i)$, $k(x_i)$, $n(x_i)$ (entrepreneurs' labor demand) and occupational choice $\theta'(x_i)$.
3. Interest rate r and wage rate w.
4. Aggregate capital and labor demands for the corporate and entrepreneurial sector:
    i) K_c and $K_e = \sum_a \sum_\eta \sum_\varepsilon \sum_\theta k(a_i, \eta_i, \varepsilon_i, \theta_i)\mu_i$
    ii) L_c and $L_e = \sum_a \sum_\eta \sum_\theta n(a_i, \eta_i, \varepsilon_i, \theta_i)\mu_i$
5. For each risk aversion type, a function mapping the space of each households distribution μ_i into next period distribution: $Q_i(\mu)$ and invariant distribution μ_i^*.

Such that:

1. The decision rules $c(x_i)$, $a'(x_i)$, $k(x_i)$, $n(x_i)$ and $\theta'(x_i)$ solve the maximization problems described before.
2. Prices (w, r) are competitive, i.e. interest rate $r = MPK_c - \delta$ and wage rate $w = MPL_c$.
3. Capital and labor markets clear:
    i) $K_e + K_c = \sum_{i=1}^{N_\sigma} \sum_{a_i} \sum_{\eta_i} \sum_{\varepsilon_i} \sum_{\theta_i} \omega_i a'(x_i)\mu_i$

ii) $L_e + L_c = \sum_{\theta_i = W} \sum_{\varepsilon_i} \varepsilon_i \mu_i$

4. The distribution μ_i^* is a fixed point of the transition function Q_i where, given the subsets S_a, S_η, S_ε and S_θ, is defined as

$$Q_i = Q(S_a, S_\eta, S_\varepsilon, S_\theta; \sigma_i) = \mathbb{I}(x_i) \sum_{\eta' \in S_\eta} \sum_{\varepsilon' \in S_\varepsilon} \Gamma_\eta(\eta'/\eta)\Gamma_{\varepsilon'(\varepsilon'/\varepsilon)}$$

   where $\mathbb{I}(x_i) = 1$ if $a_i'(x)$ and $\theta_i'(x)$ are the optimal asset policy function and occupational choice, and 0 otherwise.

The computational algorithm is described in the appendix.

1.3 Calibration

The calibration of the model follows a parsimonious approach. As a general strategy, the objective is to match some features observed in U.S. data by choosing a reduced number of parameters. Parameters that are standard to many other models such as the elasticity of output with respect to capital, time preference, depreciation rate, etc., will be taken from the existing literature. No parameter will be chosen to directly match the distribution of wealth, which is something that should be obtained endogenously from the model. Given that one of the key issues in the model is entrepreneurship, it is important to find the empirical definition of entrepreneur that is as close as possible to the one portrayed in the theoretical model. Following Cagetti and De Nardi (2006), who use Survey of Consumer Finances (SCF) data, entrepreneurs are households declared as self-employed, own a business, and have an active management role in it.

Table 1.1: Benchmark Parameters

Parameter	**Value**	**Description**
β	0.895	Discount Factor
δ	0.06	Depreciation Rate
α	0.36	Entrep. Capital Income Share
ν	0.55	Entre. Labor Income Share
Ω	0.36	Corp. Capital Income Share
λ	2.00	Borrowing Constraint

The calibration strategy is as follows. First, a model in which there is only one type of risk aversion is calibrated to match some selected descriptive statistics of U.S. data. Then, these parameters are fixed in the model in which households have different levels of risk aversion.

The purpose of this strategy is to provide a clear picture of the contribution of introducing risk aversion heterogeneity. This process is done for different distributions of the risk aversion, and the differences between the results of each calibration are analyzed.

Preferences

Each period in the model corresponds to a year. The discount factor parameter is set at $\beta = 0.895$, as a compromise between the values used in Quadrini (2000) and Cagetti and De Nardi (2006), the two main references that analyze the effects of entrepreneurship on the wealth distribution, which are 0.934 and 0.865, respectively. As a compromise, in this model β is set between these two values and is equal to 0.895.[14] This delivers a wealth-to-output ratio between 2.5 and 3, as found in U.S. data.

In an expected utility framework, the parameter of the CRRA utility function σ captures the risk aversion. The use of these preferences is supported by the findings of Chiappori and Paiella (2011), Brunnermeier and Nagel (2008), Dohmen et al. (2011), and Sahm (2012), among others, who document that relative risk aversion is constant through time and independent of wealth.[15] Kimball et al. (2008), using Health and Retirement Study (HRS) data, estimate that the risk aversion parameter takes values between 4 and 10.4, which are much higher than the values used in the literature. Using Panel Study of Income Dynamics (PSID) data, the same authors estimate again the risk aversion parameter (Kimball et al., 2009) and find that it ranges from 1.4 to 6.7, which continue to be much higher than standard values.[16] If these values where to be used in this paper, the results found will significantly differ from those in the existing literature. However, these differences would mostly be explained by the higher values, not by their heterogeneity, which is the main object being studied here. Therefore, the calibration approach of the risk aversion is going to be very conservative, and the values used will be the most commonly assumed by researchers. This will obviously reduce the differences between the results of this paper and those from previous studies. However, it seems a reasonable starting point and, if anything, it will set a lower bound on the importance of the contribution of considering risk aversion heterogeneity.

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[14] In Bewley models in general equilibrium, $\beta(1 + r) < 1$ is satisfied. Otherwise individuals accumulate an infinitely large amount of assets.

[15] In the future, I plan to extend the model introducing Epstein and Zin (1989) preferences. The suitability of this preference specification stems from the fact that it breaks the relation between the risk aversion and the elasticity of intertemporal substitution imposed by other specifications such as the CRRA preferences. However, for the moment I work with CRRA preferences to make my results more relatable to those in the existing literature.

[16] Even though the lower bound (1.4) and the second lowest value (2.2) are close to the values commonly used, the individuals with these values represent a very small fraction, less that 20%.

Hence, for the version of the model without risk aversion heterogeneity (Model I), the unique value is $\sigma = 1.5$. This value is chosen not only because it is standard but also because it is the midpoint of the range of the most used values $[1, 2]$. Actually, the versions of the model with risk aversion heterogeneity only consider three possible values of risk aversion, $\sigma \in \{1, 1.5, 2\}$.[17] The model is solved for different distributions of the risk aversion, which are reported in Table 1.2. The distribution of Model II is chosen to analyzed the different results implied by having the same expected risk aversion as in Model I but including heterogeneity. This can be thought of as a mean preserving spread of the model without risk aversion heterogeneity. The distributions of models III, IV and V are motivated by the empirical studies mentioned before that estimate the risk tolerance of survey respondents and find that the majority of the population has a relatively high level of risk aversion.[18]

Table 1.2: Risk Aversion Distributions (household shares)

	$\omega_{\sigma=1}$	$\omega_{\sigma=1.5}$	$\omega_{\sigma=2}$
Model I	0	1	0
Model II	1/3	1/3	1/3
Model III	0.2	0.3	0.5
Model IV	0.1	0.2	0.7
Model V	0.05	0.1	0.85

Production Technologies

The capital depreciation rate δ is set to the standard value of 0.06. The elasticity of output with respect to capital in the entrepreneurial sector α and in the corporate sector Ω is equal to 0.36. Given that the entrepreneurial technology exhibits diminishing returns to scale, it is the case that $\alpha + \nu < 1$. Therefore, ν takes the value 0.55, which implies a degree of diminishing returns to scale of 9% and it is close to that chosen by Luo et al. (2010). The estimations of the borrowing constraint parameter λ in Buera (2009) are 1.01 and 4.90. As a compromise, an intermediate value of 2 is chosen, which implies that the amount of capital that entrepreneurs can borrow to invest in their own firm represents at most 50% of the total capital they use to produce.

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[17] This also implies and important reduction in the dispersion of the risk aversion compared to the findings of (Kimball et al., 2008, 2009).

[18] See Figure A1.2 in the Appendix.

Labor Abilities

An important element in the analysis are the stochastic processes for the labor skills. There are many estimated processes of the labor income of workers in the literature. Given that this paper builds on Cagetti and De Nardi (2006), their calibration of the worker ability is followed. In their paper it is assumed that the logarithm of the household's labor ability as worker ε follows an autoregressive process of order one with persistence equal to 0.95. They calibrate the variance of the AR(1) process targeting the average Gini coefficient for labor earnings found in the PSID, which is equal to 0.38. This process is approximated with a five-state Markov chain Γ_ε that takes the values $\varepsilon = \{0.2468, 0.4473, 0.7654, 1.3097, 2.3742\}$ and has the following transition matrix:

$$\Gamma_\varepsilon = \begin{bmatrix} 0.7375 & 0.2473 & 0.0150 & 0.0002 & 0.0000 \\ 0.1947 & 0.5555 & 0.2328 & 0.0169 & 0.0001 \\ 0.0113 & 0.2221 & 0.5332 & 0.2221 & 0.0113 \\ 0.0001 & 0.0169 & 0.2328 & 0.5555 & 0.1947 \\ 0.0000 & 0.0002 & 0.0150 & 0.2473 & 0.7375 \end{bmatrix}$$

Entrepreneurial abilities η are calibrated in a slightly different way from Cagetti and De Nardi (2006). The values of η are proposed to match the Gini coefficient for wealth in the U.S., which is equal to 0.80. This process is approximated with a two-state Markov chain Γ_η that takes the values $\eta = \{0, 2\}$. Following Quadrini (2000), the first component of the entrepreneurial ability is highly persistent. The corresponding transition matrix is:

$$\Gamma_\eta = \begin{bmatrix} 0.9840 & 0.0160 \\ 0.1500 & 0.8500 \end{bmatrix}$$

1.4 Results

1.4.1 Risk Aversion Heterogeneity and Entrepreneurship

Risk aversion is at the heart of many economic decisions. It affects how much individuals save and invest, their occupational choice, how much labor they supply, their portfolio composition, etc. As already mentioned before, the seminal article of Kihlstrom and Laffont (1979) was one of the first to develop a theory of entrepreneurship based on the risk aversion of individuals. In their framework, less risk averse agents become entrepreneurs while more risk averse agents decide to be workers. This is not only intuitive but also supported by the data. The most recent empirical study analyzing the relationship between risk aversion and entrepreneurship is the article of Falk et al. (2018), who find that, at the individual level, this relationship is negative. The relationship between risk aversion and entrepreneurship can also be important for macroeconomic dynamics. For example, Candian and Dmitriev (2016) show that the quantitative importance of financial shocks crucially depends on the risk aversion of entrepreneurs. In the same vein, Rampini (2004) finds that the risk aversion of entrepreneurs increases the volatility of economic activity.[19]

However, risk aversion is not the only variable affecting entrepreneurship. Both the empirical and theoretical literature, have considered that skills and wealth holdings are also among the most important determinants of entrepreneurship.[20] Therefore, I analyze the effects of the risk aversion distribution on entrepreneurship, in a model where the occupational choice decision depends on wealth endowments and on the relative skills between working and entrepreneurial activity. To this aim Figure 1.2 displays the occupational choice for the next period made by entrepreneurs, for different values of risk aversion, occupation-specific ability and wealth holdings.

Each row from Figure 1.2 corresponds to a different value of entrepreneurial ability. Hence, the first row represents the lowest realization of such ability. Each column varies the worker ability. Wealth is represented in the horizontal axis, and the range of values depicted on top and bottom panel is adjusted to make visible the threshold at which households switch their occupational choice for the next period.[21] The solid blue line represents the occupational choice decision of households with low risk aversion ($\sigma = 1$), the dashed red line represents that of households with medium risk aversion ($\sigma = 1.5$), and the dash-dot yellow line captures the choice of households with high risk aversion ($\sigma = 2$).

First, it can be observed that the entrepreneurial ability is key in the decision on whether to become a worker or stay as an entrepreneur. If the household has low entrepreneurial ability today, it is very likely that he will maintain the same ability in the next period. Consequently, the level of wealth holdings required to continue as an entrepreneur in the next period is much higher than if the household had high entrepreneurial ability today. This effect is reinforced by

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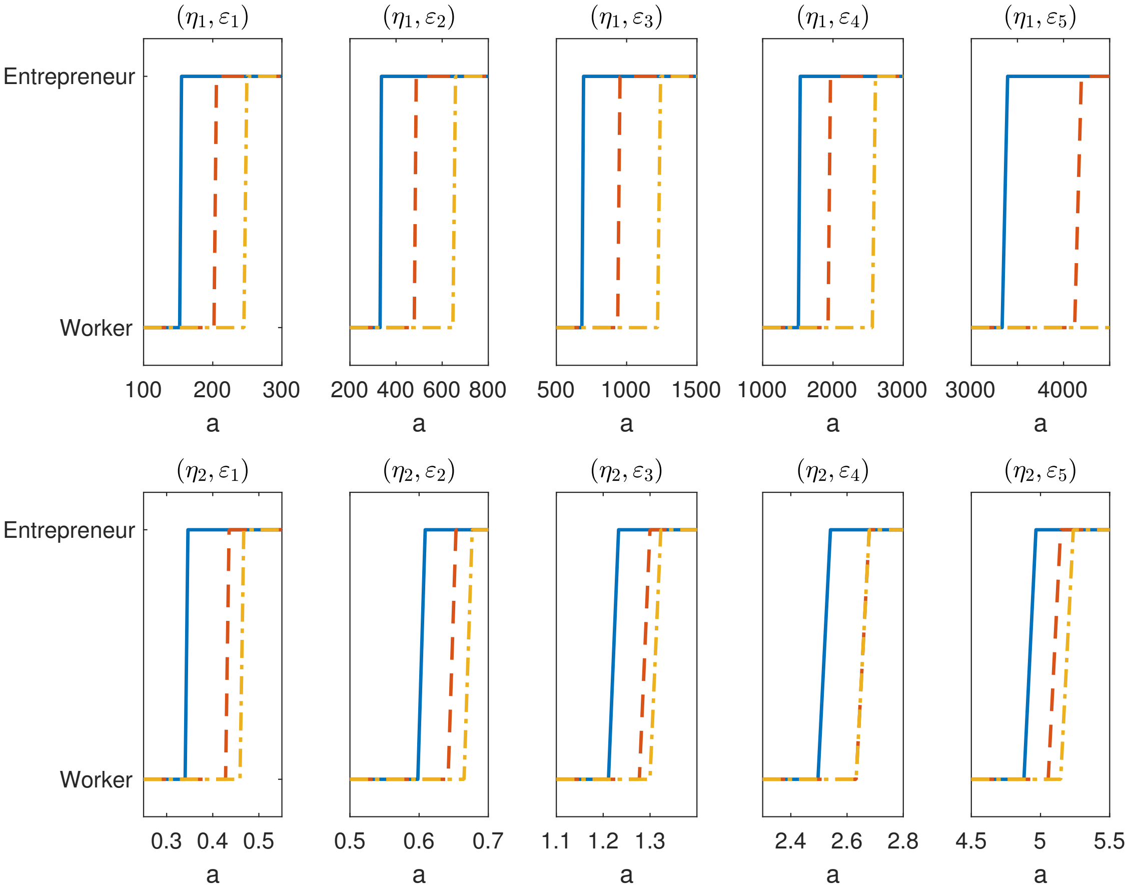
[19] Actually, the interest of risk aversion for macroeconomists extends well beyond its connection to entrepreneurship. Tallarini (2000) argues that risk aversion is important to understand asset market dynamics and the welfare costs of business cycles whereas Gourio (2012) claims that for high and time-varying levels of risk, risk aversion has strong effects in macroeconomic dynamics.

[20] Lucas (1978) develops a theory of entrepreneurship based on managerial talent where the high-skilled individuals become entrepreneurs. The relevance of the wealth holdings is justified by the presence of borrowing constraints. See the introduction for references on entrepreneurship and financial frictions.

[21] The occupational choice of workers for next period, which is very similar to that of entrepreneurs, can be found in the appendix in Figure A1.3.

Figure 1.2: Occupational Choice of Entrepreneurs



Notes: This figure corresponds to Model II, where risk aversion is uniformly distributed. Solid blue line: households with $\sigma = 1$; dashed red line: households with $\sigma = 1.5$; dash-dot yellow line: households with $\sigma = 2$.

the fact that low entrepreneurial ability is more persistent than high entrepreneurial ability.

Also, notice that the distance between wealth thresholds, above which the household remains as entrepreneur, for different levels of risk aversion is much coarser for the low entrepreneurial ability case. This implies, that the effect of risk aversion on the occupational choice decision is quite asymmetric for different realizations of entrepreneurial ability. In particular, this effect is much higher for the case where the household has low entrepreneurial ability. In other words, the occupational choice decision among households with high entrepreneurial ability depends less on their risk aversion than for households with low entrepreneurial ability.

Second, for a given level of risk aversion, the wealth threshold above which the household continues as an entrepreneur is increasing in the ability as a worker, which seems reasonable. As the alternative to entrepreneurship has a higher expected payoff, the decision to continue as an entrepreneur is deterred. Once again, the effect of risk aversion is significantly asymmetric and considerably larger for the households with low entrepreneurial ability. Actually, a high-risk-averse entrepreneur today with low entrepreneurial ability and high worker ability will never decide to continue as an entrepreneur in the next period, no matter what his wealth holdings are. From the findings of Figure 1.2, it can be concluded that, in line with the existing literature, at the individual level, there is a negative relationship between risk aversion and entrepreneurship.

Table 1.3: Share of Entrepreneurs by risk aversion type (%)

	Low RA ($\sigma = 1$)	**Medium RA** ($\sigma = 1.5$)	**High RA** ($\sigma = 2$)
Model II	5.17	6.41	7.31
Model III	4.31	5.96	7.32
Model IV	4.16	5.98	7.00
Model V	3.68	5.88	6.78

However, we find an alternative result in general equilibrium: the negative effect that risk aversion has on entry into entrepreneurship does not deter the most risk averse individuals from actually becoming entrepreneurs. By looking at Table 1.3, which shows the share of entrepreneurs by risk aversion type, we can see that the percentage of households who opt for entrepreneurship is greater for higher levels of risk aversion. In other words, at the aggregate level, the negative relationship between entrepreneurship and risk aversion found at the individual level does not hold. This happens because more risk averse households have higher precautionary savings, which indirectly relax the financial frictions and make entrepreneurship more attractive. This result holds for different model calibrations, where the household share that has the highest level of risk aversion is larger. In other words, the numbers in this table indicate that, even though higher risk aversion deters entry into entrepreneurship, this effect is not dominant.

A similar conclusion can be obtained by looking at Table 1.4. This table displays how risk aversion is distributed within entrepreneurs. Even though one might expect that, among entrepreneurs, those with lower risk aversion should represent a larger share, this is not the case. The fact that this happens for the case where risk aversion is uniformly distributed means that it is not originated by an *ad hoc* distribution in which the majority of the households has high risk aversion. Still, this result is reinforced for different risk aversion distributions with increasing number of high-risk-aversion households. What is even more stunning is that, as the share of high-risk-aversion households increases, so does the total share of entrepreneurs.

These results point towards an interesting finding: the negative effect that risk aversion has on entry into entrepreneurship is neutralized and surpassed by an opposite indirect effect,

Table 1.4: Distribution of Risk Aversion within Entrepreneurs (%)

	Low RA ($\sigma = 1$)	**Medium RA** ($\sigma = 1.5$)	**High RA** ($\sigma = 2$)	**Entrepreneurs Total (%)**
Model II	27.34	33.95	38.71	6.30
Model III	13.67	28.33	58.00	6.31
Model IV	6.39	18.36	75.25	6.51
Model V	2.81	8.99	88.19	6.54

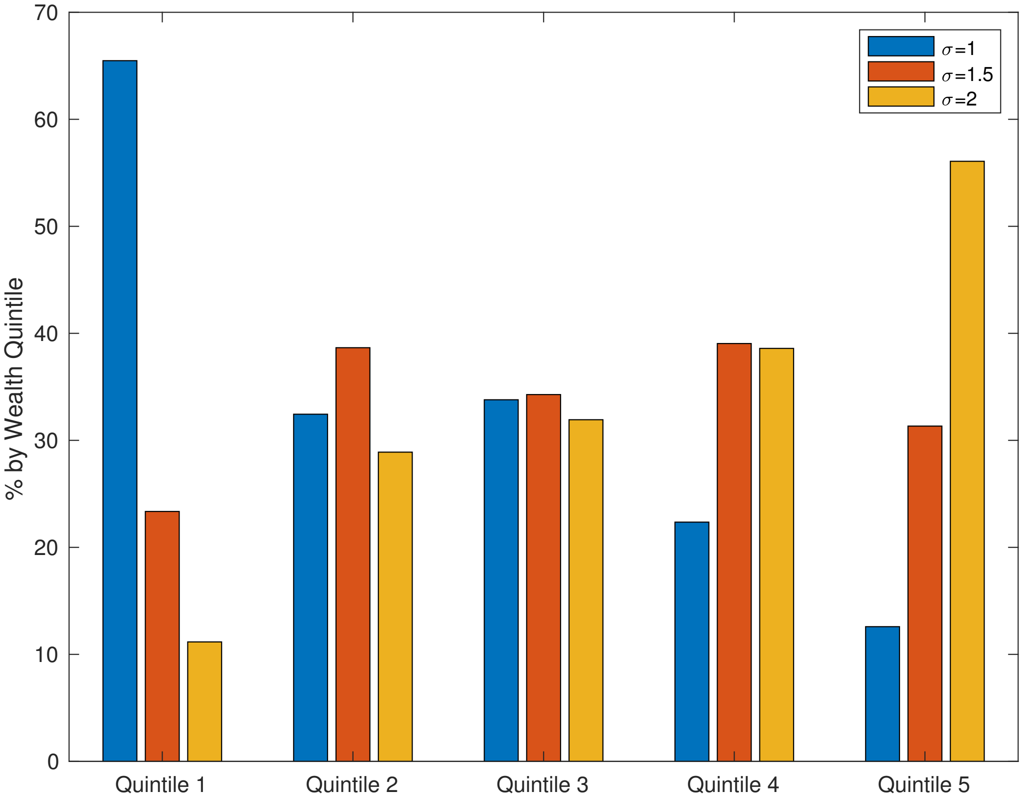
that eventually turns positive the relation between higher risk aversion and entrepreneurship at the aggregate level. In this model, this indirect effect comes from the higher precautionary savings of households with higher risk aversion. Since these save more than households with lower levels of risk aversion, the borrowing constraint is further relaxed for the former, which makes entry into entrepreneurship more profitable. The relevance of this result is twofold. Firstly, it complements the existing literature on entrepreneurship and wealth distribution that argues that entrepreneurs save more to overcome borrowing constraints, and explains the higher concentration of wealth in the upper tail of the distribution. The introduction of risk aversion heterogeneity in this model suggests that this effect is reinforced by the precautionary behavior of more risk averse households, which makes them more prone to become entrepreneurs because they can run larger firms and have higher profits. Secondly, the findings of this paper suggest that in the presence of risk aversion heterogeneity, entrepreneurship and financial frictions, the predictive power of attitudes toward risk that hold at the individual level do not necessarily hold at the aggregate level.

1.4.2 The Effects of Risk Aversion Heterogeneity and Entrepreneurship on the Wealth Distribution

This subsection examines the relation between risk aversion and wealth distribution. Understanding why people save is crucial to explain how wealth is distributed. If policy makers do not fully comprehend what motivates people to increase or reduce their savings, the instruments they may use to tackle inequality can have unintended and unwelcome consequences. The model in this paper combines the two main purposes why people save, which are consumption smoothing through time and the overcoming of borrowing constraints. The former is directly related to the risk aversion of individuals, hence the importance of introducing heterogeneity in this dimension. The latter is specially relevant for entrepreneurs, which are key to explain the higher levels of wealth concentrated in the upper tail of the distribution.

Figure 1.3 shows how risk aversion is distributed within wealth quintiles. Notice that most individuals in the first quintile have low risk aversion. In other words, the majority of the poorest 20% of the economy are low risk averse households. This share is pretty much decreasing as we move to higher wealth quintiles. Exactly the opposite happens with the most risk averse households, which represent the largest share in the fifth quantile. This result is in line with the empirical findings of Paravisini et al. (2017) who find that the wealthiest households are the most risk averse ones.

Figure 1.3: Risk Aversion with Wealth Quintiles



Notes: This figure corresponds to Model II, where risk aversion is uniformly distributed.

Table 1.5: Wealth Distribution

		Percentage Wealth in Top				
	Wealth Gini	**0.1%**	**1%**	**5%**	**20%**	**40%**
US Data	0.80	8	30	54	81	94
Model I	0.80	8	33	60	81	94
Model II	0.83	10	36	63	84	95
Model III	0.81	9	34	60	82	94
Model IV	0.78	8	31	57	79	92
Model V	0.77	8	30	55	77	92

Table 1.5 shows how wealth is distributed for different distributions of the risk aversion. Let

us recall that the only model that has actually been calibrated to match the Gini coefficient of wealth is Model I, where there is only one type of risk aversion with $\sigma = 1.5$ (i.e. medium risk aversion). Models II-V use the same parameter values as in Model I and the only difference is the introduction of risk aversion heterogeneity (see Table 1.2). Several interpretations can be made from this table. First, the model without risk aversion heterogeneity (Model I) does a good job replicating the wealth distribution, as previously found by Quadrini (2000) and Cagetti and De Nardi (2006).[22] Second, and more important for the purpose of this paper, introducing uniformly-distributed risk aversion heterogeneity (Model II) fails to match the wealth distribution. Not only its estimation of the Gini coefficient is higher, but also that of all the percentages of wealth held by the richest ones. Third, assuming that most households have medium-to-high risk aversion significantly improves the matching of the wealth distribution. In particular, the risk aversion distribution of Model IV provides a slightly better match of the wealth distribution than the calibration without risk aversion heterogeneity (Model I), though it does so at the cost of underestimating the Gini coefficient.[23] Fourth, the introduction of another source of heterogeneity does not necessarily increase wealth inequality. What seems to matter is how this new source of heterogeneity is distributed. More specifically, increasing the share of individuals that have higher risk aversion (as documented by empirical studies) reduces wealth inequality. Actually, comparing Models II and V, the Gini coefficient is six percentage point lower in the latter.

Table 1.6: Fraction of Entrepreneurs in a Given Wealth Percentile of the Overall U.S. Wealth Distribution

	Wealth Percentile, Top			
	1%	**5%**	**20%**	**40%**
US Data	54	39	32	22
Model I	55	42	38	24
Model II	50	42	35	23
Model III	48	40	34	23
Model IV	49	40	34	22
Model V	46	39	34	22

More evidence if favor of introducing risk aversion heterogeneity is presented in Table 1.6. This table shows the fraction of entrepreneurs in a given wealth percentile of the overall U.S.

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[22] One could interpret the fact that this calibration of the model replicates results from previous work as a validation and relevance of the results discussed in the previous section.

[23] If we take into account that the Models II to V have not been specifically calibrated, the fact that for some wealth percentiles they provide a better match than the model without risk aversion heterogeneity (which has actually been calibrated), suggests that introducing such heterogeneity can provide a better description of the real world.

wealth distribution, with data borrowed from Cagetti and De Nardi (2006).[24] The empirical definition of entrepreneur used is “Self-employed business owners”, since this is the one that more closely resembles the entrepreneurs in the model of this paper. With the exception of the fraction of entrepreneurs in the top 1% of the wealth distribution, the models risk aversion heterogeneity do a better job than the model without risk aversion heterogeneity. Actually, as Table 1.5 already documented, increasing the share of the population with high risk aversion improves the results.

Table 1.7: Percentage of Entrepreneurs in the Population and Corresponding Share of Total Wealth Held

	Percent in Population	**Share of Total Wealth**
Data	7.60	33.00
Model I	6.58	36.20
Model II	6.30	36.04
Model III	6.31	33.23
Model IV	6.51	31.24
Model V	6.54	30.43

Finally, Table 1.7 shows the percentage of entrepreneurs and their corresponding share of total wealth. All the different versions of the model, with and without risk aversion heterogeneity, underestimate the share of entrepreneurs in the economy, though all the values are similar to each other and pretty close to the data. Looking at these numbers, we see that introducing risk aversion heterogeneity has little effect on the share of the population that decides to entry into entrepreneurship. However, risk aversion heterogeneity, and more importantly its distribution, seems to have an effect on the share of total wealth held by the entrepreneurs.

1.5 Remaining Work and Possible Extensions

The results provided so far have relied on a parsimonious calibration, which has been made specifically for the model without risk aversion heterogeneity. Even though these results are already quite accurate, they may improve with a tailored calibration. Actually, it would be interesting to increase the range of values of risk aversion. Also, the role of financial frictions has yet to be explored given that it has been shown that introducing risk aversion heterogeneity has immediate effects on how the borrowing constraint affects households, which has a significant impact on both the occupational choice and saving behavior. Finally, an analysis of the effect of risk aversion heterogeneity on the firm size distribution may deliver interesting results.

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[24] See Table 3 in their paper.

One possible extension that can bring interesting results could be the following. In the baseline calibration of my model, it is assumed that the degree of risk aversion is not related with entrepreneurial ability. However, my framework is rich enough to allow me to parsimoniously add this relationship which naturally would increase the link between risk aversion and the decision of becoming an entrepreneur. This idea is supported by the empirical finding that cognitive skills are positively related with risk taking. Another fruitful avenue could be to assume Epstein and Zin (1989) preferences. The suitability of this preference specification stems from the fact that it breaks the relation between the risk aversion and the elasticity of intertemporal substitution imposed by other specifications such as the CRRA preferences. This may be important because, as documented by Barsky et al. (1997), risk tolerance and the elasticity of intertemporal substitution are essentially uncorrelated across individuals.[25]

1.6 Conclusions

There is a large empirical literature documenting substantial heterogeneity in risk preferences and its predictive power for many economic decisions, such as becoming an entrepreneur. Yet, these findings have not been properly considered by most of the previous work done on wealth inequality, of which entrepreneurship seems to be a key driving force. This paper explores the role of risk aversion heterogeneity on entrepreneurship, saving behavior and the wealth distribution. An occupational choice model is designed in which households vary in their risk aversion, face uninsurable idiosyncratic shocks, are financially constrained, and have to decide to become workers or entrepreneurs. This decision has different implications for savings, and agents base it on a combination of their individual risk aversion, occupation-specific skills and wealth holdings. The most important contribution of this is that it allows us to disentangle the effects that risk aversion and financial frictions have on entrepreneurship, which is a key driver of wealth inequality.

The results indicate that the negative effect that risk aversion has on entry into entrepreneurship is neutralized and surpassed by an opposite indirect effect, that eventually turns positive the relation between risk aversion and entrepreneurship. This net effect comes explained by the higher precautionary savings of households with higher risk aversion. The additional savings attenuate financial frictions, which makes entry into entrepreneurship more profitable. The consequences of these results are important for two reasons. On one hand, in-

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[25] Thanks to this property, the use of these preferences has gained popularity in macroeconomics in the last few years, see Tallarini (2000), Angeletos and Panousi (2011), Caldara et al. (2012), Van Binsbergen et al. (2012) and Rudebusch and Swanson (2012), among others.

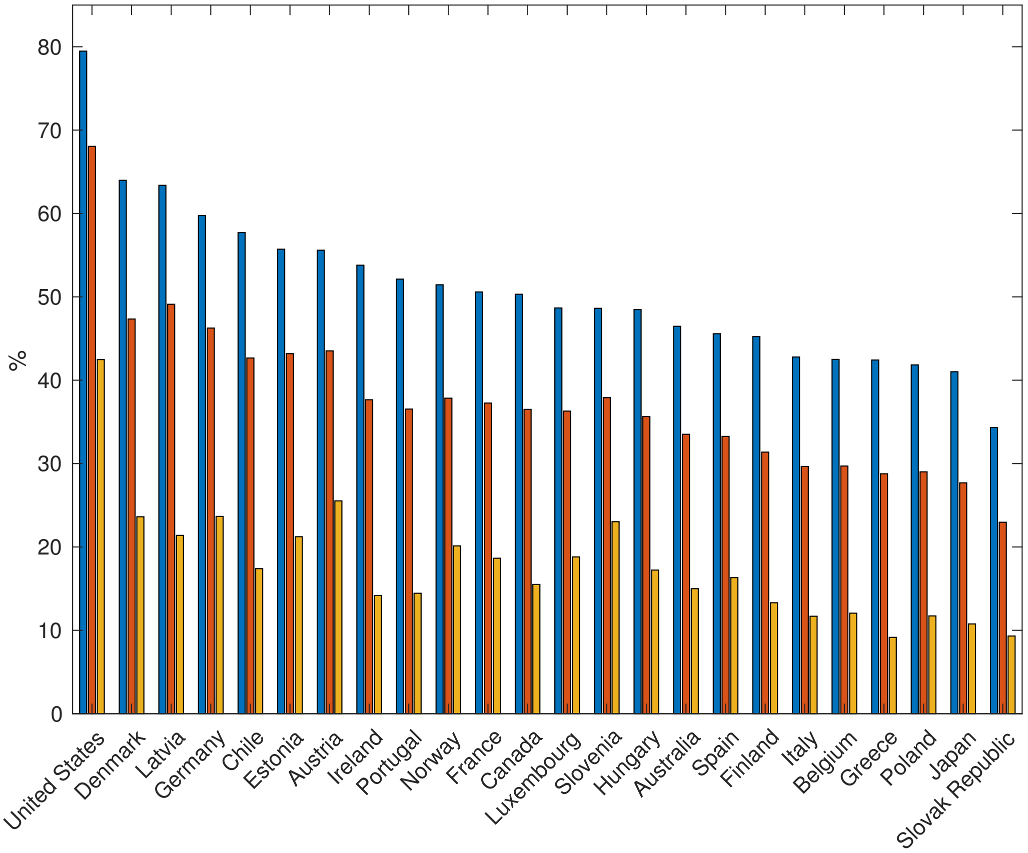
troducing risk aversion heterogeneity explains the higher concentration of wealth in the upper tail due to the precautionary behavior of more risk averse households, which makes them more prone to become entrepreneurs. On the other hand, risk aversion heterogeneity brings a discrepancy between the attitudes toward risk at the individual level and the general-equilibrium effects observed at the aggregate level. Finally, the distribution of risk aversion has an additional effect on the wealth distribution. In particular, there is a negative relationship between the share of individuals with high risk aversion and overall wealth inequality.

1.7 Appendix A

Distribution of Wealth in OECD Countries

This figure displays the shares of wealth held by the richest 10%, 5%, and 1% of the population, for each country. Despite differences across countries, wealth is highly concentrated in the upper tail of the distribution. Actually, on average more than half of the wealth in OECD countries is by only 10% of the population.

Figure A1.1: Shares of top 10%, 5%, and 1% of Wealth

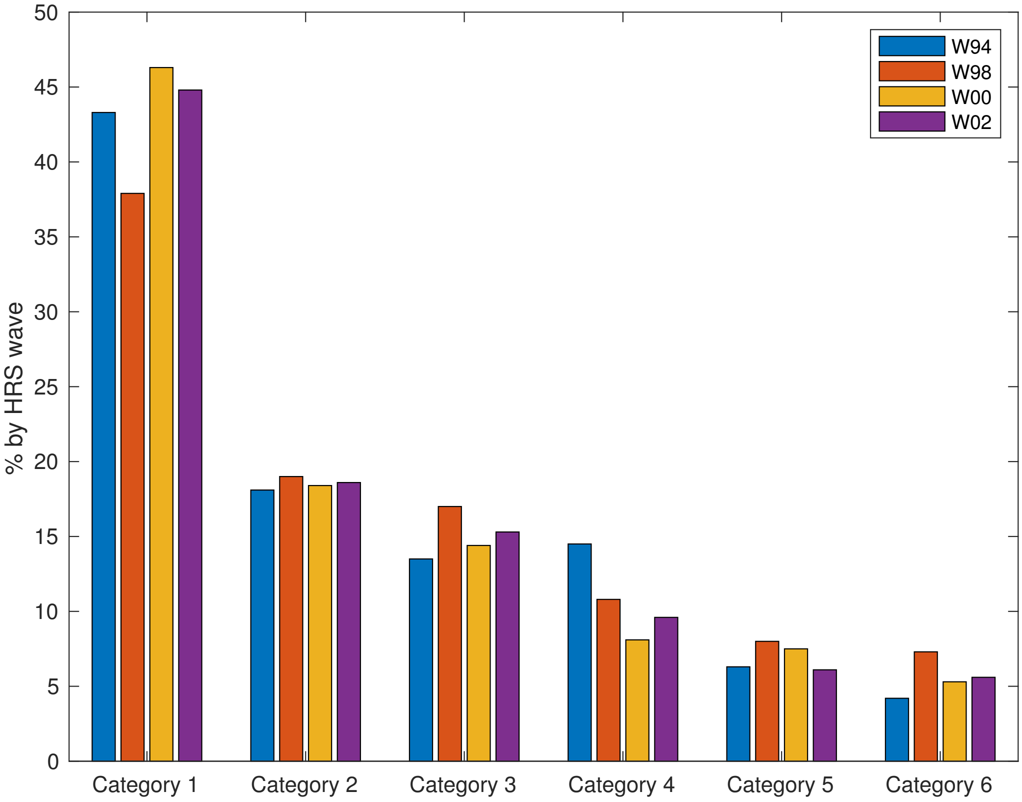


Source: OECD. Data corresponds to 2014 except Spain and Canada (2012), Estonia, Finland, Ireland and Portugal (2013), Denmark (2015), US (2016).

Distribution of Risk Tolerance

This figure is elaborated with the data obtained from Table 2 in Kimball et al. (2008). In their article they analyze the data from the Health and Retirement Study (HRS), a large-scale, biennial survey in which individuals where faced with hypothetical scenarios and had to choose between a job with certain payment and a risky job. Based on their responses they were classified in different categories. Individuals with the lowest risk tolerance (highest risk aversion) were place in Category 1 and individuals with the highest risk tolerance (lowest risk aversion) were place in Category 6. This survey was conducted in different waves, in particular in the years 1994, 1998, 2000 and 2002. The relative shares of each group change very little from wave to wave, exhibiting high persistence.

Figure A1.2: Distribution of Risk Tolerance

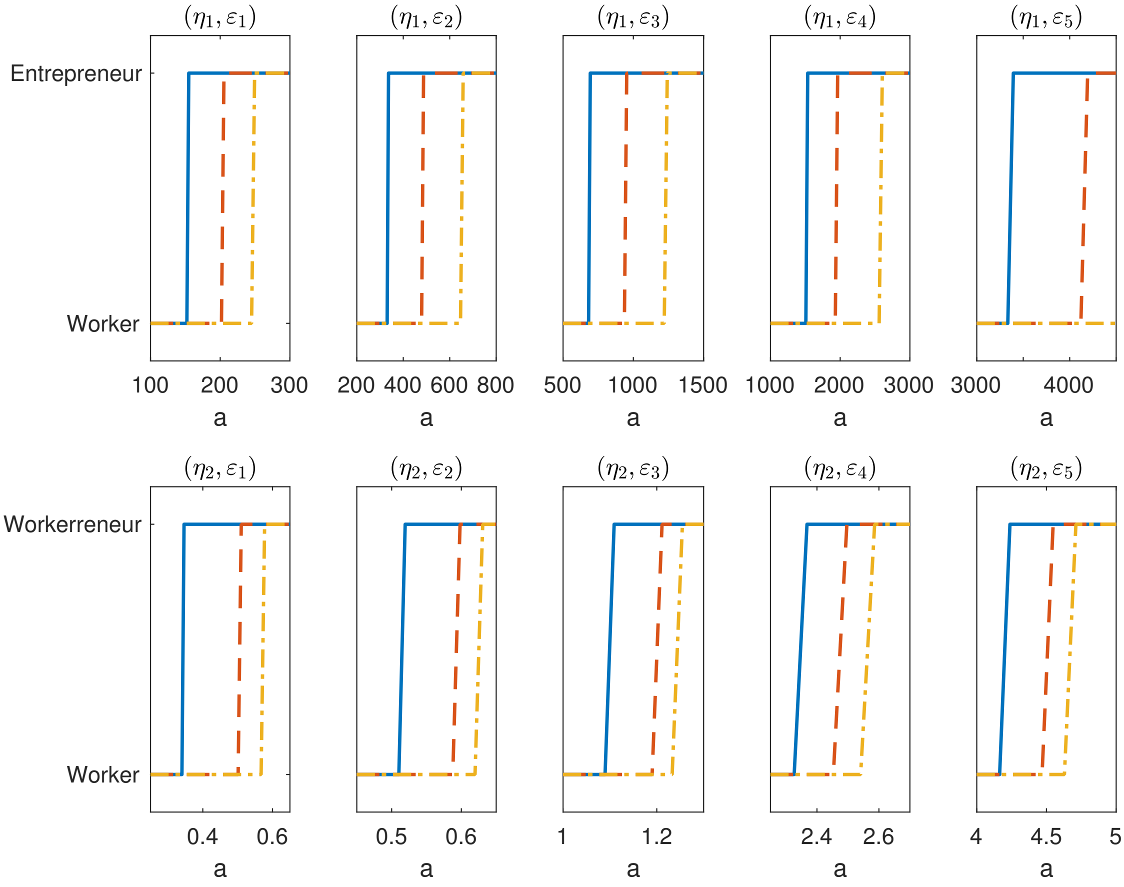


Source: Own elaboration based on data from Kimball et al. (2008).

Occupational Choice of Worker for Next Period

This figure displays the occupational choice for the next period made by workers, for different values of risk aversion, occupation-specific ability and wealth holdings.

Figure A1.3: Occupational Choice of Worker for Next Period



Notes: This figure corresponds to Model II, where risk aversion is uniformly distributed. Solid blue line: households with $\sigma = 1$; dashed red line: households with $\sigma = 1.5$; dash-dot yellow line: households with $\sigma = 2$.

1.8 Appendix B

Computational Algorithm

- **Step 1:** Construct a grid for asset holdings. The asset holdings interval is approximated by a log-spaced grid with 1000 points, so that the grid is finer for low levels of assets. The upper bound is set so that it is not binding for the household's saving policy function.

- **Step 2:**: For a given interest rate, solve the optimization problem using value function iteration.

- **Step 3:** To compute the invariant distribution use Monte Carlo simulation. Construct sample of 100,000 households. Give the same level of initial assets to all of them. Assign the combination of occupation-specific abilities (η, ε) according to the ergodic distribution of the kronecker product of the transition matrices of both abilities, $\Gamma = \Gamma_\eta \otimes \Gamma_\varepsilon$. For the initial occupation, assume that those households with high entrepreneurial ability are entrepreneurs and those with low entrepreneurial ability are workers. Assign randomly the (constant) level of risk aversion of each household, according to how it is distributed in the population. Update the state variable of each household in the sample by using the policy functions from step 2. For the next period occupation-specific abilities use a random number generator. Because of randomness, the share of households with each possible combination of abilities will not be equal to the ergodic share of Γ. Readjust these shares accordingly. Compute the mean and standard deviation of asset holdings, abilities and occupation. Iterate until the mean and standard deviation converge.

- **Step 4:** Compute the aggregate levels of capital (total asset holdings), entrepreneurial labor and worker labor implied by the invariant distribution. Compute the capital and (worker) labor demand of entrepreneurs. The capital and labor demand of the corporate sector is the difference between the total supply and those of the entrepreneurial sector.

- **Step 5:** Compute the capital-labor ratio in the corporate sector and compare it with the one implied by the interest rate guessed in step 2. If the difference is higher than 0.2% update the interest rate guess. Repeat until convergence of the capital-labor ratio in the corporate sector is reached.

2 — Long-term business relationships, bargaining and monetary policy

2.1 Introduction

The typical business environment often differs dramatically from the standard Dixit-Stiglitz monopolistic competition framework usually adopted in modern DSGE monetary models. As evidenced by empirical research, most firms engage mainly in long-term relationships with their customers, and most of their customers are other firms (see e.g. Blinder et al. (1998) for the US, Fabiani et al. (2006) for the Euro Area and Apel et al. (2005) for Sweden). Most of these long-term relationships are governed by implicit or explicit contracts, and these contracts last on average between one and two years. Therefore, negotiations of prices and quantities are the rule rather than the exception. In fact, in surveys firms report that the main reason they wish to keep prices stable is that they are concerned about losing customer relationships. For instance, Fabiani et al. (2006) find, on the basis of surveys conducted by nine Eurosystem national central banks, that the existence of implicit and explicit contracts with customers is the most important explanation for rigid prices. Zbaracki et al. (2004) find that customer communications and price negotiation costs account for almost 75% of the total price adjustment cost and are 20 times bigger than the size of the menu costs.

The repeated nature of the interactions between firms points toward an important issue: bargained intermediate prices may not be allocative, in the sense that they may not affect the final production of firms. For example, if the real intermediate price decreases, selling firms may decide not to adjust production if they expect buyers to compensate them in the future for the reduced profits incurred in the current period. In fact, as first shown by Barro (1977) for the labor market, the real effects of monetary policy when prices are sticky crucially depend on prices being allocative.[1]

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[1] See also Abbritti and Trani (2017) for a discussion.

Motivated by this literature, in this paper we introduce business-to-business (B2B) long-term relationships and price bargaining into a standard monetary DSGE model. In the model there are two types of firms, upstream producers (wholesalers) and downstream producers (retailers). Wholesalers produce intermediate goods, which are transformed by retailers into final goods and sold to households. The intermediate goods market is characterized by search and matching frictions *à la* Mortensen and Pissarides (1994). Both wholesalers and retailers need to spend time and resources to match and form long-term relationships with other firms. Once a business relationship is formed, the price is bargained between wholesalers and retailers according to a standard Nash bargaining protocol. In the future, this relationship will get destroyed with a certain probability, which is endogenous to the model. In other words, the main novelty is that the change in B2B relationships is determined by the model not only through a process of endogenous creation, but also through the endogenous destruction of inefficient matches. Lastly, the presence of quadratic intermediate-price adjustment costs introduce nominal stickiness and gives a role to monetary policy, which is magnified by the existence of costly search and matching with endogenous separation. The model provides a rigorous framework to study the effect of long-term relationships and bargaining on monetary policy and business cycle dynamics.

We highlight three main results. First, we show that the model, once calibrated to capture the main structural features of the US product market, does a remarkably good job in replicating the second moments and cross-correlations of the data, and that it improves over the benchmark New Keynesian (NK) model in explaining some of them.[2] In particular, introducing B2B long-term relationships helps to improve the volatility of employment, intermediate prices and core inflation as well as the cross-correlation of intermediate prices and core inflation with output. It also provides better estimates for the cross-correlation of intermediate prices and final-price inflation with core inflation.

Second, we find that the presence of long-term B2B relationships and bargaining strongly affect the transmission mechanism of monetary policy shocks and the allocative power of the bargained intermediate prices. In particular, we show that the real effects of monetary policy are strictly related to the presence of an endogenous match destruction margin. If match separations are exogenous, a monetary stimulus has a negligible effect on economic activity - even though intermediate prices are sticky. On the contrary, if we allow for endogenous separations of inefficient matches, intermediate prices recover some of their allocative power and

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[2] To allow comparability between the two models, the benchmark New Keynesian model also has two sectors, a wholesale and retail sector, and sticky intermediate prices

positive monetary policy shocks lead to economic expansions. This happens because following a monetary expansion, firms find it optimal to satisfy the increased demand by reducing the endogenous separation rate and allowing more matches to survive.

Finally, we show that for a standard calibration of the product market, the effectiveness of monetary policy in a model with B2B is significantly lower than in the benchmark NK model. In particular, the real effects of an unexpected monetary policy shock are almost 40 percent lower in the B2B model than in the NK model.

Recent research has started to investigate the importance of long-term relationships between firms and customers for price and business cycle dynamics. The vast majority of these papers, however, focus on retail firms to consumers relationships, and do not allow for bilateral negotiations between the parties.[3] These are important distinctions, because the business environment in B2B transactions is very different from the one in business-to-consumer (B2C) transactions.

To the best of our knowledge, only three papers analyze the implications of B2B relationships and bargaining for price and business cycle dynamics. Drozd and Nosal (2012) introduce dynamic frictions of building market shares into an international real business cycle model and show that the model can account for several pricing puzzles of international macroeconomics. Mathä and Pierrard (2011) introduce two-sided search and matching between wholesalers and retailers into the standard RBC model to study the effect of long-term relationships on business cycle dynamics. Abbritti and Trani (2017) study incomplete pass-through and the allocative power of intermediate goods prices in a model with product market frictions and bargaining over intermediate prices and quantities.

Our paper differs from these three references in two main aspects: First, we endogenize the match destruction margin. Following the model of Krause and Lubik (2007) for the labor market, we assume that the productivity of each match is match-specific, and that inefficient matches are destroyed. Second, we allow for price adjustment costs in the bargaining problem between wholesalers and retailers. These costs, which are meant to capture customer communications and price negotiation costs, introduce nominal price stickiness and give a role to monetary policy. We show in the following that endogenous match destructions and sticky prices potentially play an important role in B2B relationships, pricing dynamics and the effectiveness of monetary policy.

The structure of the paper is as follows. Section 2 derives the theoretical B2B model.

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[3] See, e.g, Hall (2008), Arseneau and Chugh (2007), Kleshchelski and Vincent (2009), Ravn et al. (2010), Gourio and Rudanko (2014), Paciello et al. (2014), Den Haan (2013).

Section 3 describes a two-sectors New Keynesian model that we use as a benchmark. In Section 4 the calibration strategy is explained. Section 5 shows the main results of the paper and Section 6 concludes.

2.2 The Model

2.2.1 Firms and Product Market

The product market is composed by two different types of firms, wholesalers and retailers, and follows the search and matching structure developed by Mortensen and Pissarides (1994). In order to sell their products, wholesale producers need to establish long-term customer relationships with retailers. Once both types of firms meet they bargain over the intermediate price at which retailers buy intermediate goods from the wholesalers. The productivity of firms is match-specific and has both an aggregate component and an idiosyncratic one, which we denote as $a_t(i)$ and is drawn from a time-invariant distribution with c.d.f. $F\left(a_t\left(i\right)\right)$ and p.d.f. $f\left(a_t\left(i\right)\right)$. We assume that the aggregate number of business to business (B2B) relationships T_t, follows the law of motion $T_{t+1} = \left(1-\delta_{t+1}\right)\left(T_t + m_t\right)$ where m_t, the number of new B2B relationships at time t, is a constant returns to scale function of the search effort of retailers V_t (purchase managers) and the search effort of wholesalers S_t (advertising and marketing):

$$m_t = \tilde{m} S_t^{\xi} V_t^{1-\xi}$$

The separation rate is defined as $\delta_t = \delta_x + \left(1-\delta_x\right) F\left(\tilde{a}_t\left(i\right)\right)$, where $\tilde{a}_t\left(i\right)$ is an endogenously determined productivity threshold below which matches are not profitable and hence terminated.

Wholesalers

There is a continuum of wholesale producers with unit mass. Each wholesaler j maximizes the expected present value of future profits

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_{0,t} \left\{ \left(\frac{P_{It}\left(j\right)}{P_t} - c_t^W\left(j\right) \right) T_t\left(j\right) - \left(r_t + \delta_k\right) K_t(j) - w_t N_t\left(j\right) - \gamma_W S_t\left(j\right) \right\},$$

subject to the production function

$$Y_t^W\left(j\right) = q T_t\left(j\right) = A_t K_t(j)^{\alpha} N_t\left(j\right)^{1-\alpha}$$

with q being the quantity produced per match, and the law of motion of the customer base

$$T_t\left(j\right)=\left(1-\delta_t\left(j\right)\right)\left(T_{t-1}\left(j\right)+S_{t-1}\left(j\right)\mu_W\left(\theta_{t-1}\right)\right).$$

The term $\beta_{t,t+1}=\beta\left(C_{t+1}/C_t\right)^{-\sigma}$ denotes the household's stochastic discount factor, while $\gamma_W S_t$ denotes the search costs. Intuitively, the wholesaler chooses how much search effort, S_t, he will execute to find new buyers for his product. Think of this as the firm choosing the number of sales managers it is going to hire.[4] Each unit of effort will provide him with an average of $\mu_W\left(\theta_t\right)=\tilde{m}\theta_t^{(1-\xi)}$ retailers at the end of the period, where $\theta_t=V_t/S_t$ is the product market tightness. $P_{It}\left(j\right)$ denotes the price of the intermediate good, that is decided after the successful match in a bilateral bargain with the retailers. The term $c_t^W=\frac{\phi_W}{2}\left(P_{It}(j)/P_{It-1}(j)-\pi_I\right)^2$ captures quadratic price adjustment costs. We assume that this cost, which is intended to capture price negotiation and communication costs, is proportional to the number of B2B relationships T_t.

The wholesaler also decides how much capital, $K_t\left(j\right)$, and labor, $N_t\left(j\right)$, he is going to rent. A_t is an AR(1) TFP shock and, for simplicity, we normalize $q=1$. The real rate of interest is r_t, the depreciation rate of capital is δ_k and the real wage is denoted by $w_t=W_t/P_t$.

From the first-order necessary conditions we get that both the capital-labor ratio

$$\frac{K_t(j)}{N_t(j)}=\frac{\alpha}{1-\alpha}\frac{w_t}{r_t+\delta_k}$$

and the marginal cost

$$mc_t=\left(A_t\right)^{-1}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{r_t+\delta_k}{\alpha}\right)^{\alpha} \tag{2.1}$$

are equal across wholesalers. This is because *ex-ante* all the wholesale producers are identical since the match-specific productivity draws are not realized until the matches occur and intermediate prices are bargained.

Further combinations of the FOCs give us the following expression:

$$J_t^W\left(j\right)=\frac{P_{It}\left(j\right)}{P_t}-c_t^W\left(j\right)-mc_t+\mathbb{E}_t\beta_{t,t+1}\left(1-\delta_{t+1}\left(j\right)\right)J_{t+1}^W\left(j\right) \tag{2.2}$$

This equation captures the expected value (across matches) of a B2B relationship for wholesaler j. This depends positively on the intermediate price that the retailer pays him and negatively on the marginal cost of production. The last term, $\mathbb{E}_t\beta_{t,t+1}\left(1-\delta_{t+1}\left(j\right)\right)J_{t+1}^W$, captures the

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[4] See Gourio and Rudanko (2014).

expected continuation value of a match. This brings dynamic effects into the model coming from the fact that in the next period only a fraction equal to $(1-\delta_{t+1}(j))$ of the matches survives and both wholesalers and retailers benefit from them.

The optimal amount of search is chosen to equate the expected marginal cost and the marginal benefit of a new business relationship:

$$\frac{\gamma_W}{\mu_W(\theta_t)} = \mathbb{E}_t \beta_{t,t+1} (1-\delta_{t+1}(j)) J^W_{t+1}(j) \tag{2.3}$$

This equation makes it clear that the search effort is executed in one period but it does not pay off until the next period and only if the match resulting from it is not destroyed.

Retailers

There is a continuum of retail producers with unit mass that buy the intermediate goods from wholesalers and sell it to consumers in perfectly competitive markets. Each retailer draws a match-specific productivity from a time-invariant distribution with c.d.f. $F(a)$ and p.d.f. $f(a)$. We assume that the draw of productivity takes place after intermediate price bargaining. This timing assumption simplifies considerably the bargaining problem and the solution of the model because it implies that the bargained price is identical for every match. The total production of retailer i is given by

$$T_t(i) \int_{\tilde{a}}^{\infty} a \frac{f(a)}{1-F(\tilde{a})} da = T_t(i) H(\tilde{a}_t(i))$$

where $T_t(i)$ is the number of productive or functional matches and $H(\tilde{a}_t(i))$ is the conditional expectation of the idiosyncratic shock $\mathbb{E}[a \mid a \geq \tilde{a}_t(i)]$. The productivity threshold $\tilde{a}_t(i)$ is endogenously determined such that below it matches are not profitable and hence destroyed. In a similar way to the case of the wholesale producers, the number of B2B relationships of retailer i follows a law of motion that depends on the current-period separation rate and the previous-period number of functional matches and search effort exercised, $V_{t-1}(i)$

$$T_t(i) = (1-\delta_t(i))(T_{t-1}(i) + V_{t-1}(i)\mu_R(\theta_{t-1}))$$

where $\mu_R(\theta_t) = \tilde{m}\theta_t^{-\xi}$ is the average number of wholesalers attracted in the current period per unit of effort.

Retailers maximize the expected present value of profits *before* the realization of the idiosyncratic shock a, i.e. based on the expected output $\mathbb{E}_a Y_t^R(i) = T_t(i) H(\tilde{a}_{it})$. Specifically, every retailer i maximizes:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_{t,t+1} \left\{ T_t(i) H(\tilde{a}_t(i)) - \left(\frac{P_{It}(i)}{P_t} + c_t^R(i) \right) T_t(i) - \gamma_R V_t(i) \right\}$$

subject to the law of motion of the customer base. Retailers also face a cost of changing the bargained price, which is defined as $c_t^R = \frac{\phi_R}{2} \left(P_{It}(i)/P_{It-1}(i) - \pi_I \right)^2$ and it is also proportional to the number of B2B relationships $T_t(i)$. The last term of the equation captures the cost of search effort.

At the beginning of each period the retailer chooses the level of production and the search effort. The intermediate price P_{It} is decided after the successful match in a bilateral bargaining between retailers and wholesalers.

From the first-order necessary conditions we get the expected value (across matches) of a customer relationship for the retailer

$$J_t^R(i) = H(\tilde{a}_t(i)) - \left(\frac{P_{It}(i)}{P_t} + c_t^R(i) \right) + \mathbb{E}_t \beta_{t,t+1} (1 - \delta_{t+1}(i)) J_{t+1}^R(i) \tag{2.4}$$

The value of a match depends positively on its production and negatively on the marginal cost, which is the relative price the retailer has to pay to the wholesaler. Similar to the case of the wholesaler, the last term in the equation connects the value of the matches in two subsequent periods bringing the dynamic effects into the model. Although (most) variables are connected in general equilibrium, we can notice a *ceteris paribus* effect of the threshold on the value of the matches. In particular, a higher threshold implies a higher average value of the matches because the previously least productive matches are destroyed, leaving operative those with higher productivity.

In equilibrium, the expected cost of a new match in a given period equals the expected marginal benefit that will be realized in the subsequent periods:

$$\frac{\gamma_R}{\mu_R(\theta_t)} = \mathbb{E}_t \beta_{t,t+1} (1 - \delta_{t+1}(i)) J_{t+1}^R(i) \tag{2.5}$$

Endogenous separation

We assume that a successful match is endogenously destroyed whenever the realization of the idiosyncratic shock does not make it profitable for at least one of the parties. Since prices are determined before the realization of a_t, the value of a B2B relationship for a wholesaler, $J_t^W(j)$, does not depend on the idiosynchratic productivity of a match a_t, which affects only retailers. Let us define by $J_t^R(a_t)$ the marginal value for the retailer of a match with idiosyncratic productivity a_t. The threshold $\tilde{a}_t$ is endogenously determined as solution of $J_t^R(\tilde{a}_t) = 0$.[5] Combining this equation with the first-order conditions of the retailer the critical threshold below which matches are terminated is implicitly defined as:

$$\tilde{a}_t(i) = \left(\frac{P_{It}(i)}{P_t} + c_t^R(i) - \frac{\gamma_R}{\mu_R(\theta_t)} \right) \tag{2.6}$$

The threshold $\tilde{a}_t$ is increasing on the relative intermediate price and on the cost of changing prices because the higher these are the more profitable the match has to be to allow the retailer to pay for them.

Bargaining

After wholesalers and retailers are matched, intermediate prices are determined through a Nash bargaining scheme between them. Precisely, for each match v, intermediate goods prices are determined as the outcome of the following bargaining scheme

$$\max_{P_{It}} SU_t = \left[\left(J_t^W(v)\right)^{\eta} \left(J_t^R(v)\right)^{1-\eta} \right]$$

where η is the bargaining power of wholesalers.

We assume that prices are determined *before* the productivity draw of the retailers. Hence, the bargaining problem is the same across matches and the intermediate price will be unique. Let us denote by $\varphi_t = \frac{P_{It}}{P_t}$ the relative intermediate price. Dropping the subscript v, maximization gives:

$$\varphi_t \left[\eta J_t^R - (1-\eta) \left(1 - \frac{\partial H(\tilde{a}_t)}{\partial \tilde{a}_t} \right) J_t^W \right] = (1-\eta)\, \tau_t^R J_t^W + \eta \tau_t^W J_t^R \tag{2.7}$$

where

$$\tau_t^W = \phi_W (\pi_{It} - 1)\, \pi_{It} - \mathbb{E}_t \beta_{t,t+1} \left[(1 - \delta_{t+1}(j))\, \phi_W + (1-\delta_x) f(\tilde{a}_{t+1}) J_{t+1}^W \phi_R \right] (\pi_{It+1} - 1)\, \pi_{It+1}$$

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[5] See appendix for the proof of existence and uniqueness of the threshold.

and

$$\tau_t^R = \left(1 - \frac{\partial H\left(\tilde{a}_t\right)}{\partial \tilde{a}_t}\right) \Big\{ \phi_R \left(\pi_{It} - 1\right) \pi_{It} - \mathbb{E}_t \beta_{t,t+1} \Big[\left(1 - \delta_{t+1}\right) \left(1 - \frac{\partial H\left(\tilde{a}_{t+1}\right)}{\partial \tilde{a}_{t+1}}\right) + \left(1 - \delta_x\right) f\left(\tilde{a}_{t+1}(i)\right) J_{t+1}^R(i) \Big] \phi_R \left(\pi_{It+1} - 1\right) \pi_{It+1} \Big\}$$

capture the marginal costs of changing the intermediate price for the wholesalers and the retailers respectively.

Notice that if prices were flexible we would have $\tau_t^W = \tau_t^R = 0$ and equation (2.7) would resemble the standard solution by which each party gets a share of the surplus equal to their bargaining power:

$$\eta J_t^R = (1 - \eta) \left(1 - \frac{\partial H(\tilde{a}_t)}{\partial \tilde{a}_t}\right) J_t^W \tag{2.8}$$

The main difference from a standard solution is the presence of the term $\frac{\partial H(\tilde{a}_t)}{\partial \tilde{a}_t}$, which enters the bargaining solution because firms internalize the fact that a higher bargained price leads retailers to increase the endogenous separation threshold and the average productivity of a match.

2.2.2 Households

There is a representative household in the economy and his total lifetime utility is given by:

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\sigma}}{1 - \sigma} - \varkappa \frac{N_t^{1+\nu}}{1 + \nu} \right\}$$

which depends positively on consumption, c_t, and negatively on labor, N_t. The household faces a sequence of flow budget constraints which denoted in real terms can be written as:

$$c_t + \frac{b_{t+1}}{R_t} \pi_{t+1} + I_t = w_t N_t + b_t + (r_t + \delta_k) K_t + d_t \tag{2.9}$$

$$K_{t+1} = (1 - \delta_k) K_t + \left\{ 1 - \frac{\phi_I}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right\} I_t \tag{2.10}$$

where b_t denote purchases of bonds, R_t is the nominal interest rate on bonds, w_t is the real wage and d_t are the dividends net of lump sum taxes.

From the first-order necessary conditions we obtain the standard Euler Equation, the labor supply and the no arbitrage condition on the assets:

$$\begin{aligned} c_t^{-\sigma} &= \beta c_{t+1}^{-\sigma} R_t \pi_{t+1}^{-1} && (2.11)\\ w_t &= \varkappa N_t^{\nu} c_t^{\sigma} && (2.12)\\ Q_t &= \beta \left[\frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} (r_{t+1} + \delta_k) + Q_{t+1}(1-\delta_k) \right] && (2.13)\\ 1 &= Q_t \left[1 - \frac{\phi_I}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \phi_I \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right] \\ &+ \beta \mathbb{E}_t \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} Q_{t+1} \phi_I \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 && (2.14) \end{aligned}$$

Where Q_t denotes Tobin's Q. These equations will determine the level of consumption, the demand for bonds and physical capital and the supply of labor.

2.2.3 Aggregate Constraints and Prices

To close the model we need to aggregate the quantities and the markets to clear. The total output in the economy is the result of adding up the production of every match whose productivity draw was above the threshold:

$$Y_t = T_t H\left(\tilde{a}_t\right)$$

and

$$T_t = A_t K_t^{\alpha} N_t^{1-\alpha}$$

And finally notice that output can be either consumed, invested in physical capital or used to pay the cost of changing bargained prices and/or search efforts.

$$Y_t = c_t + I_t + \phi\left(\pi_{It} - 1\right)^2 T_t + \gamma_R V_t + \gamma_W S_t$$

where $\phi = \phi_R + \phi_W$.

From the definition of the relative price, $\varphi_t = P_{It}/P_t$, we are able to establish the relationship between Consumer Price Index and Producer Price Index inflations:

$$\frac{\pi_{It}}{\pi_t} = \frac{\varphi_t}{\varphi_{t-1}}$$

2.2.4 Monetary Policy

The monetary policy is described by a simple Taylor-type rule where the nominal interest rate set by the monetary authority depends on core inflation, output and the previous-period nominal interest rate:

$$\frac{R_t}{R} = \exp(-z_t)\left[\left(\frac{P_{It}}{P_{It-1}}\right)^{\phi_{\pi_I}}\left(\frac{Y_t}{Y_{t-1}}\right)^{\phi_Y}\right]^{1-\phi_R}\left(\frac{R_{t-1}}{R}\right)^{\phi_R}$$

where ϕ_R, ϕ_{π_I} and ϕ_Y are the relative weights on the previous period interest rate, current core (intermediate price) inflation and output growth, respectively, and z_t denotes an i.i.d. monetary policy shock.

2.3 A Benchmark Two-Sectors New Keynesian Model

To validate the importance of our contribution, we compare the results of our product market frictions (B2B) model with the ones of a benchmark New Keynesian (NK) model with monopolistic competition. To make the models comparable, we assume that in the benchmark model there are also two sectors of production, wholesalers and retailers. Wholesalers are monopolistically competitive and face quadratic price adjustment costs. Retailers combine the varieties of the intermediate goods in a single bundle and sell it to households.

Specifically, in the benchmark NK model retailers operate under perfect competition and flexible prices. Their production function is $y_{rt} = y_{It}$, where

$$y_{It} = \left[\int_0^1 y_{It}(j)^{\frac{\varepsilon_{NK}-1}{\varepsilon_{NK}}}\,dj\right]^{\frac{\varepsilon_{NK}}{\varepsilon_{NK}-1}}$$

is a bundle of intermediate varieties bought from different wholesalers. The optimal demand of each variety j is

$$y_{It}(j) = \left(\frac{P_{It}(j)}{P_t}\right)^{-\varepsilon_{NK}} y_{It} \tag{2.15}$$

Each wholesaler j operates under monopolistic competition and faces quadratic adjustment costs

$$c_t^P(j) = \frac{\psi_p}{2}\left(\frac{P_{It}(j)}{P_{It-1}(j)} - \pi\right)^2$$

Notice that this cost function is identical to the one faced by wholesalers and retailers in the

B2B model. Wholesaler j maximizes the expected present value of future profits

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_{0,t} \left\{ \left(\frac{P_{It}(j)}{P_t} - c_t^P(j) \right) y_{It}(j) - \left(r_t^k + \delta_k \right) K_t(j) - w_t N_t(j) \right\}$$

subject to the production function $y_{It}(j) = A_t K_t(j)^{\alpha} N_t(j)^{1-\alpha}$ and the demand for each variety $y_{It}(j)$. From the wholesaler's maximization problem we obtain the following FOCs:

$$mc_t = \frac{1}{A_t} \left(\frac{w_t}{1-\alpha} \right)^{1-\alpha} \left(\frac{r_t^k + \delta_k}{\alpha} \right)^{\alpha} \tag{2.16}$$

$$\frac{w_t}{r_t^k + \delta_k} = \frac{1-\alpha}{\alpha} \frac{K_t(j)}{N_t(j)} \tag{2.17}$$

$$\frac{P_{It}(j)}{P_t} = \frac{\varepsilon_{NK}}{\varepsilon_{NK} - 1} \left(mc_t + c_t^P(j) - \frac{\tau_{Pt}(j)}{\varepsilon_{NK}} \right) \tag{2.18}$$

where

$$\tau_{Pt}(j) = \psi_p \left(\pi_{It}(j) - \pi \right) \pi_{It}(j) - \beta_{t,t+1} \frac{y_{It+1}(j)}{y_{It}(j)} \left\{ \psi_p \left(\pi_{It+1}(j) - \pi \right) \pi_{It+1}(j) \right\}$$

denotes the marginal costs of changing prices. The first two equations capture the marginal costs and the capital-labor ratio. Equation (2.18) is instead a version of the Phillips curve relating present and future inflation rates to marginal costs. In fact, aggregating across firms and log-linearizing around the steady state one can rewrite equation (2.18) as:

$$\hat{\pi}_{It} = \beta \mathbb{E}_t \hat{\pi}_{It+1} + \frac{(\varepsilon_{NK} - 1)}{\psi_p} \left(\widehat{mc}_t \right)$$

where variables with hats denote log deviations from the steady state.

Importantly, the presence of sticky prices is not sufficient to generate intermediate price variability in the NK model. Notice in fact that in a symmetric equilibrium, equation (2.15) implies that the relative intermediate price is constant and equal to 1, $\varphi_t = \frac{P_{It}}{P_t} = 1$, and that PPI and CPI inflation are identical:

$$\hat{\pi}_{It} = \hat{\pi}_t$$

2.4 Calibration

We calibrate the model at the quarterly frequency, so we set the discount factor $\beta = 0.99$ to match a standard annualized interest rate of 4%. We use standard values also for the share of capital in production and the rate of capital depreciation. These are, respectively, $\alpha = 0.33$

and $\delta_k = 0.025$.

Our calibration of the search and matching with bargaining follows largely the strategy developed by Abbritti and Trani (2017). This is based on survey interviews to business managers from various sectors of the U.S. economy and on survey data on employment in sales-related activities. Given the average opinion of business managers, the most reasonable average duration of firm-to-firm relationships is between 1 and 2 years. This sets a target for the quarterly separation rate, which we calibrate to $\delta = 0.20$. The labor search literature has assumed that the exogenous component explains the most of the separation rate. For example, in Krause and Lubik (2007), the exogenous component is 3/4 of the overall separation rate. Here we adopt a more conservative approach and assume that $\delta_x = 0.60\delta$, which in turn implies that the endogenous component is $\delta_n = F(\tilde{a}) = 0.40\delta / [1 - 0.60\delta]$. By assumption, $F(\tilde{a})$ is a log-normal distribution. We normalize its mean, so that $\mu_{LN} = 1$, and set its volatility σ_{LN} to 0.175. Consequently, $\tilde{a}$ is equal to 0.78.

According to the evidence on sales-related activities, wholesalers' search S is 9% of intermediate goods output. Since in this model the volume of trade between firms coincides with the number of matches (i.e., there is only an extensive margin of trade), this means that wholesalers' search is close to 9% of GDP. This target allows us to determine both the search cost parameter and the matching efficiency. Therefore, assuming $\eta = \xi = 0.5$, we obtain $\gamma = 0.5726$ and $\tilde{m} = 2.8497$. The main justification for a conservative parametrization of the bargaining power η and elasticity of matching ξ is that there is no useful evidence for choosing them, so, setting them to 0.5, we can better relate our results to the endogenous separation of the matches and the other new features of the model.[6]

We then set the time spent producing goods N to 0.2, which implies that working time represents 20% of the total available time (see Mathä and Pierrard (2011)). Together with the elasticity of labor supply, this pins down the labor disutility $\varkappa$. We choose a labor elasticity equal to 1.6 by setting $\nu = 0.625$, which is broadly consistent with macroeconomic estimates (restated recently by Peterman (2016)). Regarding the calibration of the quadratic price adjustment costs, we follow Krause and Lubik (2007), who introduce one-sided price rigidity and calibrate its parameter to a value of 40. Since in our model (B2B) there is two-sided price rigidity, we equally distribute the price rigidity between both sides and set the parameters governing the degree of price rigidities to $\phi_W = \phi_R = 20$.

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[6] In a model that abstracts from nominal price rigidity and endogenous destruction, Abbritti and Trani (2017) show that one can choose η to approximate the volatility of the PPI in the data, with little consequence for the other moments.

Lastly, we describe our strategy for calibrating the monetary policy and the TFP shocks. We assume that the strength of the reaction of the Central Bank to core inflation is $\phi_{\pi_I} = 1.5$ and to output growth is $\phi_Y = 0.5/4$. The persistence of the interest rates is $\phi_R = 0.85$. The standard deviation of monetary policy shocks is set to the standard value of 0.1%. As far as the TFP shocks are concerned, we assume that their persistence is 0.9 and choose their volatility to match the volatility of U.S. GDP. The implied value is $\sigma_A = 0.975\%$. Conditional on these choices, we control the relative volatility of investment using the parameter ϕ_I which is set equal to 0.215 in our model.

To understand the role of long term B2B relationships and bargaining for business cycle dynamics, it will be instructive to compare the dynamics of B2B model with the ones of the benchmark NK model. To facilitate comparison, the calibration of the benchmark NK model is identical to the one of the B2B model. Specifically, to calibrate the degree of price rigidity in the NK model we have followed Krause and Lubik (2007) and set $\psi_p = 40$, which is equal to the sum of the price adjustment costs for wholesalers and retailers. The only additional parameter that we need to specify is the elasticity of demand ε_{NK}, which we set to 6 as, e.g., in Blanchard and Galí (2010).

2.5 Results

2.5.1 Steady State Analysis

In order to understand the role and the contribution of the endogenous separation rate, in this section we analyze the steady states of our model with and without endogenous separation rate, for different values of the bargaining power. The model with exogenous separation rate is obtained by simply setting to zero the variance of the match-specific productivity and keeping all the other parameters fixed at their baseline values.[7] Specifically, we compare steady-state equilibria for the following three values of the bargaining power of wholesalers: 0.3, 0.5, and 0.7. The results can be seen in Table 2.1, where the last column displays the ratio of the final price to total marginal cost, which is introduced as an approximation to the mark-up of producers.[8]

Considering that the model with endogenous separation is our baseline assumption, as well

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[7] When σ_{LN} is exactly equal to zero, the distribution of idiosyncratic productivities is degenerate and its c.d.f. evaluated at the threshold is also zero. Then, the separation rate becomes completely exogenous.

[8] The total marginal cost is computed as the ratio of wholesalers marginal cost to the average productivity of matches, i.e. $\chi = mc/H(\tilde{a})$.

Table 2.1: Steady State Analysis

	T	θ	$H(\tilde{a})$	Y	δ	mc	${}^{P}/_{\chi}$
			B2B Baseline				
$\eta = 0.3$	0.5832	2.3137	1.0034	0.5852	0.1276	0.9343	1.0740
$\eta = 0.5$	0.5795	0.9552	1.0281	0.5958	0.2004	0.9222	1.1148
$\eta = 0.7$	0.5491	0.3930	1.0705	0.5878	0.3430	0.8305	1.2890
			B2B with Exogenous Separation				
$\eta = 0.3$	0.5835	2.3333	1.0000	0.5835	0.1200	0.9352	1.0693
$\eta = 0.5$	0.5854	1.0000	1.0000	0.5854	0.1200	0.9406	1.0632
$\eta = 0.7$	0.5835	0.4286	1.0000	0.5835	0.1200	0.9352	1.0693

as our main contribution, let us start by analyzing its steady state. Since η measures the bargaining power of wholesalers it affects the share of the total surplus of a match that these retain. In particular, the higher it is, the higher the value of a match for wholesalers, and the more they will search. The opposite is true for retailers and this is what explains the observed values of the product market tightness, θ, which is defined as the ratio of the search effort of retailers to that of wholesalers. However, as we can see from equation (2.8), η is not the only determinant of the solution to the bargaining problem.[9] Actually, the (endogenous) productivity threshold, $\tilde{a}$, below which a match is terminated also affects how its surplus is shared between wholesalers and retailers. Intuitively, the fact that retailers have a direct control of the separation rate provides them with additional leverage on the bargaining problem.[10] Through this additional wedge, retailers are partially compensated for bearing most of the risk of an adverse realization of the idiosyncratic shock. This allows us to differentiate between the "bargaining power", which is exogenous and is fully captured by the parameter η, and the"*effective* bargaining power", which is endogenous and is jointly determined by η and $\tilde{a}$. In other words, whereas retailers can affect the number of B2B relations both through their search effort and the decision to separate (or not), wholesalers can only adjust through their search effort. This is the reason behind the negative relationship between η and the number of B2B relationships, T, which is driven by the fact that the lower the bargaining power of retailers the more they choose to separate. However, the matches being destroyed are the ones with lower productivity, which increases the average productivity of matches in the economy, $H(\tilde{a})$. This increase in average productivity implies a reduction in the marginal cost of wholesalers, which further reduces the total marginal cost and significantly increases the approximation of

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[9] Notice that equation (2.8) is the solution to the bargaining problem with flexible prices, so by simply removing the time subindices we obtain the solution to the bargaining problem is steady state for the baseline model (i.e. with sticky prices).

[10] The retailer is the one drawing the match-specific productivity and deciding whether a match survives or is terminated. Remember that the threshold below which matches are terminated is such that the marginal value of a match for a retailer is zero.

the mark-up of wholesalers, P/χ.

Next, let us analyze what happens to the steady-state equilibria with exogenous separation rate. If retailers cannot decide to terminate a B2B relationship, the term with $\tilde{a}$ in equation (2.8) disappears and we obtain the standard solution to the Nash bargaining problem where only the bargaining power, η, determines how the surplus of a match is shared between both parties. In this case, for the baseline calibration (i.e. $\eta = 0.5$), the Hosios (1990) condition is satisfied and the solution to the bargaining problem is constrained efficient. However, notice that while the number of B2B relationships is lower in the model with endogenous separation than in the model with exogenous separation, total production is higher in the former (for the same value of η). This is explained by the fact that the matches being destroyed in the model with endogenous separation are those with lowest productivity whereas in the exogenous separation model all the matches are equally productive. Another important difference between both model specifications is that in the model with exogenous separation rate different calibrations of the bargaining power do not seem to significantly affect the steady state values. Furthermore, it can be seen that its effect is symmetric. For example, output follows a symmetric inverse-U shape for different values of η, and it is maximized when wholesalers and retailers held the same bargaining power. This happens because wholesalers and retailers can only affect the number of B2B relationships through their search effort and, for this calibration, the search externality is fully internalized. This is not the case in the model with endogenous separation and η equal to 0.5, where the search externality is not fully internalized by producers, which leads to a congestion problem and hence the value of output is not the maximum possible. The nonlinear and symmetric behavior is also observed for the number of B2B relations T, the marginal cost of wholesalers, mc, and therefore in the approximate mark-up, P/χ, as well. Therefore, the results from this analysis indicate that the endogenous separation rate plays an important role in the steady-state equilibrium of the model and is a potential source of asymmetries that might be important for the transmission of different shocks.[11]

2.5.2 Second Moments

To assess the quantitative validity of our model, Table 2.2 shows selected second moments of different versions of the model and compare them with the ones of the U.S. data and the benchmark NK model. The data are collected from FRED and cover the period from 1975Q1

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[11] See sections 2.5.2 and 2.5.3 for an analysis of the contribution of the endogenous separation rate on the transmission of technology and monetary policy shocks.

to 2015Q2.[12] The simulations of the various economies, except for the one in the last column of the table, are instead based on the preferred calibration of our model, which is the case of the B2B model displayed in column B2B(I).

The B2B model with endogenous separations, column B2B(I), does a fairly good job in replicating most second moment statistics of the data. Specifically, it captures the relative volatilities of employment and intermediate prices, and the cross-correlations of most variables with GDP and PPI inflation. The model instead fails to match the relative volatility and cross-correlation of CPI inflation. This can be explained by the fact that, to clarify the mechanism of the model, we have assumed that retail prices are perfectly flexible.

Table 2.2: Second Moments

	Data	**B2B (I)**	**B2B (II)**	**NK (I)**	**NK (II)**
Volatility GDP	1.39	1.39	1.39	1.36	1.39
Vol(x)/Vol(GDP)					
Investment	3.49	3.49	3.36	5.87	3.49
Employment	0.96	0.84	1.19	0.62	0.66
Wages	0.42	0.93	1.23	0.84	0.87
Interm. Price	0.40	0.37	0.63	0.00	0.00
CPI Inflation	0.20	0.53	0.89	0.26	0.33
PPI Inflation	0.33	0.16	0.18	0.26	0.33
Corr(x, GDP)					
Investment	0.89	0.96	0.97	0.94	0.87
Employment	0.85	0.56	0.57	0.44	-0.07
Wages	0.85	0.73	0.70	0.92	0.92
Interm. Price	0.22	0.10	0.31	0.00	0.00
CPI Inflation	0.43	-0.20	-0.35	-0.01	-0.23
PPI Inflation	0.25	0.21	0.26	-0.01	-0.23
Corr(x,PPI Infl.)					
Interm. Price	0.28	0.40	0.31	0.00	0.00
CPI Inflation	0.25	0.08	-0.04	1.00	1.00

Notes: B2B (I) denotes the baseline model specification with endogenous separation rate. B2B (II) is the model specification with exogenous separation rate (computed by setting to zero the variance of the match-specific productivity). NK (I) denotes the benchmark New Keynesian model with the same calibration as the B2B model. NK (II) denotes the benchmark New Keynesian model with an alternative calibration that sets the investment adjustment costs and the standard deviation of TFP shocks to match output volatility and the relative investment volatility.

Column B2B(II) shows the results of a nested B2B model with an exogenous separation rate. A comparison between the two models reveals that closing down the match destruction margin strongly increases the relative volatilities of employment, wages, real intermediate prices and

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[12] For the intermediate price we use “PPI Final Demand Finished Goods Less Energy” and for the final price we use “CPI All Goods Less Energy”.

CPI inflation, while the volatility of output is not affected. The B2B model with endogenous separations also provides a better fit of the cross-correlation of PPI inflation with intermediate prices and CPI inflation. Overall, the fact that the B2B model with endogenous separation rate provides a better match of the relative volatility of the intermediate price suggests that allowing firms to decide whether they want to continue with a business relationship has important effects on price dynamics.

To provide a deeper understanding of the role of the endogenous separation rate, we compare the dynamics of models B2B(I) and B2B (II) following a TFP shock (see Figure B2.1 in Appendix). A positive TFP shock makes wholesalers more productive, increasing total production. In the B2B(I) model, the increase in the number of business relationships comes from two different sources. On one hand, the reduction of wholesalers' marginal costs increases the total value of each match and induces both wholesalers and retailers to increase their search efforts, which results in the creation of a higher number of matches. On the other hand, the threshold of the idiosyncratic productivity below which matches are destroyed declines, bringing down both the endogenous separation rate and the average productivity of matches. The high persistence of output is mainly driven by the high persistence of the number of B2B relationships, while the endogenous separation margin mainly affects the short-run response of output and B2B relationships. Overall, the shock reduces wholesalers' marginal costs and leads to lower relative intermediate prices and PPI inflation. Closing down the endogenous separation rate mainly affects the short-run ability of firms to adjust to the technology shock. In the B2B(II) model, since firms can adjust production only through the match creation margin, the reaction of search efforts, employment and intermediate prices are amplified, while the short-run response of production and the number of matches is reduced. The next section elaborates more on the contribution and importance of the endogenous separation rate of matches and how it shapes the transmission of monetary policy.

The second moments of the B2B model differ significantly from the ones of the benchmark New Keynesian model (see column NK (I) in Table 2.2). Adopting exactly the same calibration, the NK model generates similar volatility of output but a much higher volatility of investment. More importantly, as we mentioned before, the presence of sticky intermediate prices is not sufficient to generate intermediate price variability in the NK model, and implies a one to one relationship between PPI and CPI inflation. As a consequence, the relative volatility and cross correlation of intermediate prices are 0, and the relative volatilities and cross-correlations of PPI and CPI inflation with output are identical. In other words, the B2B model fits the data at least as well as the NK model, and this is not an artifact of an advantageous calibration. Indeed, column NK (II) shows the results of re-calibrating the NK model using the same calibration strategy as the B2B model. In particular, we reset the standard deviation of technology shocks and the investment adjustment costs of the NK model to match the U.S. output volatility and relative volatility of investment.[13] The fit of the NK model improves for what concerns investment, employment, CPI and PPI inflation volatility, but the cross-correlations of employment, CPI and PPI investment worsen and are even more counterfactually negative than in column NK(I).

Once again, we believe that it is interesting to compare the dynamics of the economy under the B2B and NK model following a TFP shock. In particular, we perform this comparison using the calibrations B2B(I) and NK (I) and obtain the results in Figure B2.2 of the Appendix. Our findings show that the presence of product market frictions and bargaining reduces the responses of investment, wages and PPI inflation, but amplifies the ones of employment and production, which is in line with intuition obtained from the second moments.

2.5.3 The Transmission of Monetary Policy Shocks

In this section we show that the presence of long-term business relationships and bargaining crucially determines both the real effects of monetary policy as well as its transmission mechanism. In our baseline model, the number of B2B relationships, and therefore overall production, changes through both the endogenous destruction and endogenous creation of matches. This implies that the presence of sticky intermediate prices can in principle affect production through both channels. On one hand, changes in the relative intermediate price have a direct effect on the separation threshold. By looking at equation (2.6) it can be seen that a lower relative intermediate price decreases the separation threshold, $\tilde{a}$. Therefore, both the separation rate and the average productivity of surviving matches decrease. Through this mechanism, intermediate prices have a direct allocative role on the number of B2B relationships and hence on final output. This implies that, in the presence of sticky prices, monetary policy can directly affect the number of endogenous separations, the average productivity of surviving business relationships and final and intermediate output.

On the other hand, the intermediate price also affects the value of a B2B relationship to wholesalers and retailers, modifying their incentives to engage in costly search activities. Moreover, it is straightforward to see from equations (2.2) and (2.4), that the change in the

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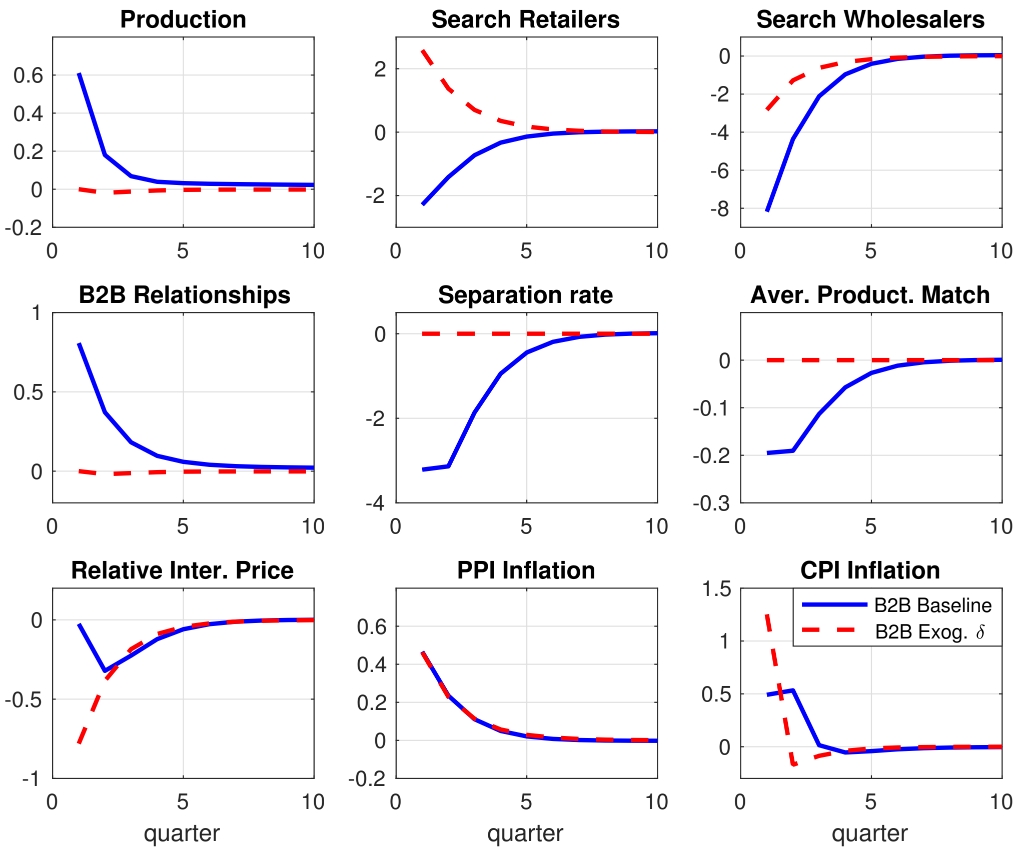
[13] The implied parameters for the NK model are $\sigma_A = 1.28\%$ and $\phi_I = 1.92$.

value of a match and hence in incentives is opposite across the two sides of the market: while a decrease in the relative intermediate price induces wholesalers to decrease their search effort, it also increases the search effort of retailers. In the end, the two effects tend to cancel out. The overall effect on the formation of new matches depends on the initial product market tightness, on the presence of search externalities and on the separation rate.

Figure 2.1: Comparison of the IRFs of the B2B Model With and Without Endogenous Separation Rate to a Monetary Policy Shock



To gauge the relative size of these two channels, Figure 2.1 compares the effects of an expansionary monetary policy shock in our model with endogenous separation rate and the same model but with exogenous separation rate. Let us consider first the model with endogenous separation. The monetary policy shock, which corresponds to a 0.25 percent reduction of the nominal interest rate, stimulates the economy increasing the levels of consumption and investment, and therefore aggregate demand. As a result, final and intermediate prices increase. However, since price rigidity occurs in the intermediate level, the final price increases more than the intermediate price and hence the relative intermediate price goes down. As previously explained, this change in the intermediate price leads to a decrease in the separation threshold which reduces the separation rate and increases the number of matches. In other words, to satisfy the increase in aggregate demand retail firms increase their production adjusting through the endogenous separation margin, that is by keeping alive matches with lower productivity. Nevertheless, wholesalers and retailers are aware of the transient nature of the shock and, anticipating the need to reduce their stock of B2B relationships in the future, both reduce their search effort. The overall effect is a short-lived increase in production and in the number of matches, which goes hand in hand with a reduction of the average productivity of matches.

A completely different pattern is observed when firms are not allowed to adjust production through the endogenous separation of inefficient matches. As before, the monetary policy shock leads to an increase in aggregate demand and a reduction of the relative intermediate price. Consequently, the value of a match for a wholesaler decreases, whereas that of the retailer increases. Accordingly, the change in search effort of retailers is positive whereas in the model with endogenous separation it was negative. This is key because it reveals the different transmission mechanisms of monetary policy between both models. The differences arising from the endogenous separation also imply that there is an endogenous response of the effective bargaining power. In the end, in the model with exogenous separation, the opposite effects in searching effort of wholesalers and retailers cancel out and there is no change at all neither in production nor on the number of B2B relationships. Actually, there is no change in any other real variable except for search effort. This implies that intermediate prices have (almost) no allocative role for output dynamics along the endogenous match creation margin, and monetary policy shocks have negligible real effects in a model with exogenous separations.

This analysis suggests that monetary policy shocks can still have real effects on output and consumption dynamics, but that these effects work almost entirely through the endogenous separation margin. But how big are these real effects? To answer this question, Figure 2.2 compares the effects of an unexpected monetary shock in the baseline B2B model with the ones in the B2B model with exogenous separation, the B2B model with endogenous separation and $\eta = 0.7$, and the benchmark NK model.

The comparison between the baseline B2B model ($\eta = 0.5$) and the same model with $\eta = 0.7$ is aimed at complementing the discussion provided in section 2.5.1 about the effect of the bargaining power on the equilibrium of the model. In particular we want to see how much the allocative role of prices changes with different values of η.[14] The first aspect we notice is

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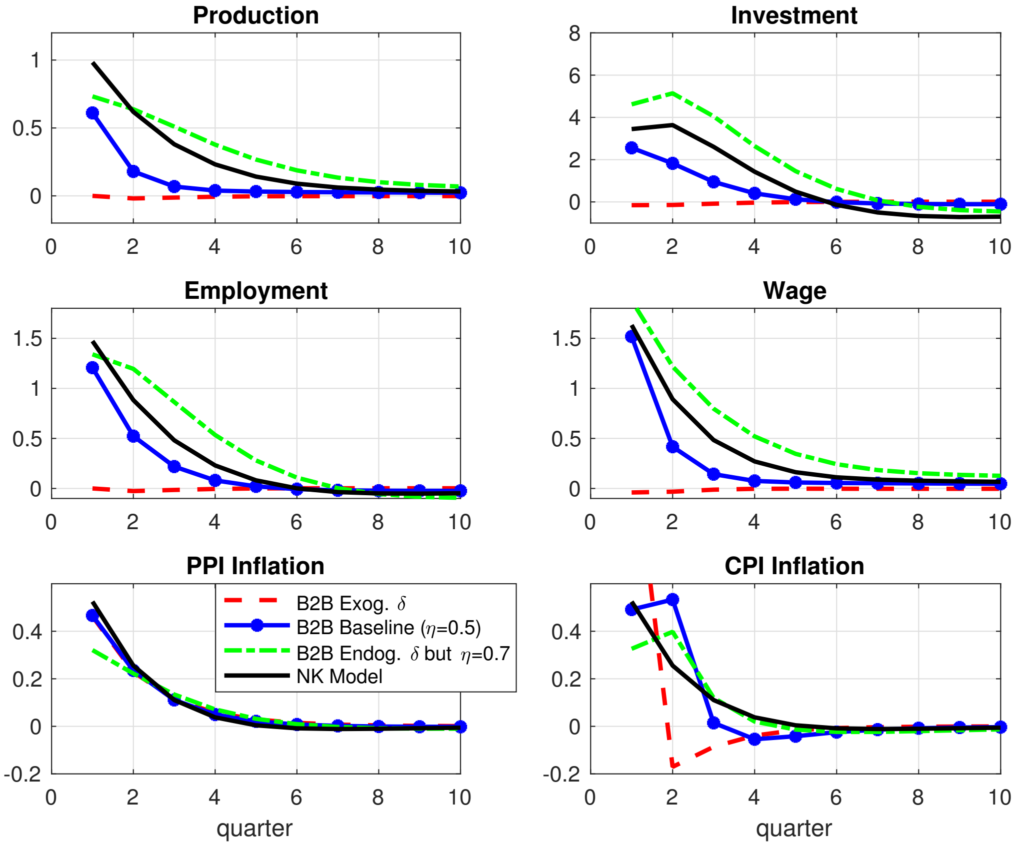
[14] We do not include a comparison with the baseline B2B with $\eta = 0.3$ because that calibration will provide too much bargaining power to retailers, considering that they are the ones deciding if a match should be destroyed or not.

Figure 2.2: Comparison of the IRFs of the B2B and NK Models to a Monetary Policy Shock



that, on impact, the effect of monetary policy on output is almost the same in both models. However, we observe that the shock is more persistent for the higher value of the bargaining power of wholesalers. This higher persistence is shared in other variables such as investment, employment and wages. Furthermore, the effect of a monetary policy shock on investment is twice as high on impact in the model with $\eta = 0.7$. This can be explained by the fact that now retailers have less bargaining power and therefore the endogenous separation margin (which is solely controlled by them) becomes more important and has a greater effect on the dynamics of the model. In other words, the real effects of monetary policy are increasing in the bargaining power of wholesalers.

Comparing both versions of the baseline B2B and the NK models we see that the effects of a monetary policy shock are qualitatively similar but quantitatively rather different. Most notably, while the responses of PPI and CPI inflation in the three models resemble each other, the responses of output, investment and employment are significantly larger - and more persistent - in the NK model than in the baseline B2B model. Overall, notwithstanding the relatively generous calibration of the endogenous separation margin, the real effects of monetary policy shocks on output dynamics are 40 percent smaller in the latter model than in the NK model.

This is consistent with the idea of a lower allocative role of intermediate prices in B2B relationships. However, notice that the real effects of monetary policy increase in the B2B model if the bargaining power of wholesalers is increased. Even though, on impact, the response of output and employment is lower in the B2B model with $\eta = 0.7$ than in the NK model, the effect of the monetary policy shock is more persistent in the former. The higher persistence B2B model with $\eta = 0.7$ is also observed in investment and wages which, on top of that, have a higher response on impact than the NK model. As explained above, the reason behind this is the increased use of the endogenous separation margin made by retailers as a response to their reduction in bargaining power.

2.6 Conclusions

A growing empirical literature shows that most transactions are firm-to-firm and that price rigidities mainly arise at the intermediate goods level, in relationships governed by implicit or explicit long-term contracts. This paper studies theoretically the implications of long-term business relationships and bargaining over sticky prices for monetary policy and business cycles dynamics. To this aim, it introduces search and matching frictions, endogenous separations and bargaining between firms into an otherwise standard monetary DSGE model. The different business environment has important effects on monetary policy and business cycle dynamics. The model outperforms the benchmark New Keynesian model in replicating some of the second moments and cross-correlations of US product market and business cycle data. We show that, in the presence of long-term business relationships and bargaining, monetary policy is less effective. This happens because, for standard calibrations, the long-term nature of the relationships between firms reduces the allocative role of intermediate good prices and the real effects of monetary policy shocks.

2.7 Appendix A

Bargaining Problem

Intermediate prices are determined through a Nash bargaining scheme between the retailer and the wholesaler. Precisely, for each match v, intermediate goods prices are determined as the outcome of the following bargaining scheme

$$\max_{P_{It}} SU_t = \left[\left(J_t^W(v)\right)^{\eta} \left(J_t^R(v)\right)^{1-\eta} \right]$$

where η is the bargaining power of retailers.

Recall the endogenous separation rate:

$$\tilde{a}_t(i) = \left[\frac{P_{It}(i)}{P_t} + \frac{\phi_R}{2}\left(\frac{P_{It}(i)}{P_{It-1}(i)} - 1\right)^2 - \mathbb{E}_t\beta_{t,t+1}\left(1-\delta_{t+1}(i)\right) J^R_{t+1}(i) \right] \tag{A2.1}$$

Necessary derivations for the bargaining problem:

$$\frac{\partial \tilde{a}_t(i)}{\partial P_{It}} = \left[\frac{1}{P_t} + \phi_R\left(\frac{P_{It}(i)}{P_{It-1}(i)}-1\right)\frac{1}{P_{It-1}(i)} - \mathbb{E}_t\beta_{t,t+1}\left[-\frac{\partial\delta_{t+1}(i)}{\partial P_{It}}J^R_{t+1}(i) + \left(1-\delta_{t+1}(i)\right)\frac{\partial J^R_{t+1}(i)}{\partial P_{It}}\right]\right]$$

Since

$$\begin{aligned}
\frac{\partial\delta_{t+1}(i)}{\partial P_{It}} &= \frac{\partial\delta_{t+1}(i)}{\partial\tilde{a}_{t+1}(i)}\frac{\partial\tilde{a}_{t+1}(i)}{\partial P_{It}} \\
&= (1-\delta_x)f\left(\tilde{a}_{t+1}(i)\right)\phi_R\left(\frac{P_{It+1}(i)}{P_{It}(i)}-1\right)\left(-\frac{P_{It+1}(i)}{P_{It}(i)^2}\right) \\
&= -(1-\delta_x)f\left(\tilde{a}_{t+1}(i)\right)\phi_R\left(\frac{P_{It+1}(i)}{P_{It}(i)}-1\right)\left(\frac{P_{It+1}(i)}{P_{It}(i)^2}\right)
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial J^R_{t+1}(i)}{\partial P_{It}} &= \frac{\partial H\left(\tilde{a}_{t+1}(i)\right)}{\partial\tilde{a}_{t+1}(i)}\frac{\partial\tilde{a}_{t+1}(i)}{\partial P_{It}} + \phi_R\left(\frac{P_{It+1}(i)}{P_{It}(i)}-1\right)\left(\frac{P_{It+1}(i)}{P_{It}(i)^2}\right) \\
&= \frac{\partial H\left(\tilde{a}_{t+1}(i)\right)}{\partial\tilde{a}_{t+1}(i)}\phi_R\left(\frac{P_{It+1}(i)}{P_{It}(i)}-1\right)\left(-\frac{P_{It+1}(i)}{P_{It}(i)^2}\right) + \phi_R\left(\frac{P_{It+1}(i)}{P_{It}(i)}-1\right)\left(\frac{P_{It+1}(i)}{P_{It}(i)^2}\right) \\
&= \left(1-\frac{\partial H\left(\tilde{a}_{t+1}(i)\right)}{\partial\tilde{a}_{t+1}(i)}\right)\phi_R\left(\frac{P_{It+1}(i)}{P_{It}(i)}-1\right)\left(\frac{P_{It+1}(i)}{P_{It}(i)^2}\right)
\end{aligned}$$

we obtain

$$\begin{aligned}
\frac{\partial \tilde{a}_t(i)}{\partial P_{It}} &= \Bigg\{ \frac{1}{P_t} + \phi_R \left(\frac{P_{It}(i)}{P_{It-1}(i)} - 1 \right) \frac{1}{P_{It-1}(i)} \\
&- \mathbb{E}_t \beta_{t,t+1} \Bigg[(1-\delta_x) f\left(\tilde{a}_{t+1}(i)\right) \phi_R \left(\frac{P_{It+1}(i)}{P_{It}(i)} - 1 \right) \left(\frac{P_{It+1}(i)}{P_{It}(i)^2} \right) J^R_{t+1}(i) \\
&+ \left(1-\delta_{t+1}(i)\right) \left(1 - \frac{\partial H\left(\tilde{a}_{t+1}(i)\right)}{\partial \tilde{a}_{t+1}(i)} \right) \phi_R \left(\frac{P_{It+1}(i)}{P_{It}(i)} - 1 \right) \left(\frac{P_{It+1}(i)}{P_{It}(i)^2} \right) \Bigg] \Bigg\}
\end{aligned}$$

Rearranging terms

$$\begin{aligned}
\frac{\partial \tilde{a}_t(i)}{\partial P_{It}} &= \Bigg\{ \frac{1}{P_t} + \phi_R \left(\frac{P_{It}(i)}{P_{It-1}(i)} - 1 \right) \frac{1}{P_{It-1}(i)} \\
&- \mathbb{E}_t \beta_{t,t+1} \Bigg[\left(1-\delta_{t+1}(i)\right) \left(1 - \frac{\partial H\left(\tilde{a}_{t+1}(i)\right)}{\partial \tilde{a}_{t+1}(i)} \right) \\
&+ (1-\delta_x) f\left(\tilde{a}_{t+1}(i)\right) J^R_{t+1}(i) \Bigg] \phi_R \left(\frac{P_{It+1}(i)}{P_{It}(i)} - 1 \right) \left(\frac{P_{It+1}(i)}{P_{It}(i)^2} \right) \Bigg\} \qquad \text{(A2.2)}
\end{aligned}$$

Recall

$$J^W_t(j) = \frac{P_{It}(j)}{P_t} - \frac{\phi_W}{2} \left(\frac{P_{It}(j)}{P_{It-1}(j)} - 1 \right)^2 - mc_t + \mathbb{E}_t \beta_{t,t+1} \left(1-\delta_{t+1}(j)\right) J^W_{t+1}(j)$$

Differentiating with respect to P_{It}

$$\frac{\partial J^W_t(j)}{\partial P_{It}} = \frac{1}{P_t} - \phi_W \left(\frac{P_{It}(j)}{P_{It-1}(j)} - 1 \right) \frac{1}{P_{It-1}(j)} + \mathbb{E}_t \beta_{t,t+1} \left[-\frac{\partial \delta_{t+1}(j)}{\partial P_{It}} J^W_{t+1}(j) + \left(1-\delta_{t+1}(j)\right) \frac{\partial J^W_{t+1}(j)}{\partial P_{It}} \right]$$

Since

$$\frac{\partial J^W_{t+1}(j)}{\partial P_{It}} = \phi_W \left(\frac{P_{It+1}(j)}{P_{It}(j)} - 1 \right) \left(\frac{P_{It+1}(j)}{P_{It}(j)^2} \right)$$

We obtain

$$\begin{aligned}
\frac{\partial J^W_t(j)}{\partial P_{It}} &= \frac{1}{P_t} - \phi_W \left(\frac{P_{It}(j)}{P_{It-1}(j)} - 1 \right) \frac{1}{P_{It-1}(j)} \\
&+ \mathbb{E}_t \beta_{t,t+1} \Bigg[(1-\delta_x) f\left(\tilde{a}_{t+1}(j)\right) \phi_R \left(\frac{P_{It+1}(j)}{P_{It}(j)} - 1 \right) \left(\frac{P_{It+1}(j)}{P_{It}(j)^2} \right) J^W_{t+1}(j) \\
&+ \left(1-\delta_{t+1}(j)\right) \phi_W \left(\frac{P_{It+1}(j)}{P_{It}(j)} - 1 \right) \left(\frac{P_{It+1}(j)}{P_{It}(j)^2} \right) \Bigg]
\end{aligned}$$

Rearranging terms

$$\begin{aligned}\frac{\partial J_t^W(j)}{\partial P_{It}} &= \frac{1}{P_t} - \phi_W\left(\frac{P_{It}(j)}{P_{It-1}(j)} - 1\right)\frac{1}{P_{It-1}(j)} \\ &+ \mathbb{E}_t\beta_{t,t+1}\left[(1-\delta_{t+1}(j))\,\phi_W\right. \\ &+ \left.(1-\delta_x)f\left(\tilde{a}_{t+1}(i)\right)J_{t+1}^W(j)\phi_R\right]\left(\frac{P_{It+1}(j)}{P_{It}(j)} - 1\right)\left(\frac{P_{It+1}(j)}{P_{It}(j)^2}\right) \qquad \text{(A2.3)}\end{aligned}$$

Also recall

$$J_t^R(i) = H\left(\tilde{a}_t(i)\right) - \left[\frac{P_{It}(i)}{P_t} + \frac{\phi_R}{2}\left(\frac{P_{It}(i)}{P_{It-1}(i)} - 1\right)^2\right] + \mathbb{E}_t\beta_{t,t+1}\left(1-\delta_{t+1}(i)\right)J_{t+1}^R(i)$$

Differentiating with respect to P_{It}

$$\begin{aligned}\frac{\partial J_t^R(i)}{\partial P_{It}} &= \frac{\partial H\left(\tilde{a}_t(i)\right)}{\partial \tilde{a}_t(i)}\frac{\tilde{a}_t(i)}{\partial P_{It}} - \left[\frac{1}{P_t} + \phi_R\left(\frac{P_{It}(i)}{P_{It-1}(i)} - 1\right)\frac{1}{P_{It-1}(i)}\right] \\ &+ \mathbb{E}_t\beta_{t,t+1}\left[-\frac{\partial\delta_{t+1}(i)}{\partial P_{It}}J_{t+1}^R(i) + (1-\delta_{t+1}(i))\frac{\partial J_{t+1}^R(i)}{\partial P_{It}}\right] \\ &= \frac{\partial H\left(\tilde{a}_t(i)\right)}{\partial \tilde{a}_t(i)}\left\{\frac{1}{P_t} + \frac{\phi_R}{P_{It-1}(i)}\left(\frac{P_{It}(i)}{P_{It-1}(i)} - 1\right) - \mathbb{E}_t\beta_{t,t+1}\left[(1-\delta_{t+1}(i))\left(1 - \frac{\partial H\left(\tilde{a}_{t+1}(i)\right)}{\partial \tilde{a}_{t+1}(i)}\right)\right.\right. \\ &+ \left.\left.(1-\delta_x)f\left(\tilde{a}_{t+1}(i)\right)J_{t+1}^R(i)\right]\phi_R\left(\frac{P_{It+1}(i)}{P_{It}(i)} - 1\right)\left(\frac{P_{It+1}(i)}{P_{It}(i)^2}\right)\right\} - \left[\frac{1}{P_t} + \frac{\phi_R}{P_{It-1}(i)}\left(\frac{P_{It}(i)}{P_{It-1}(i)} - 1\right)\right] \\ &+ \mathbb{E}_t\beta_{t,t+1}\left[(1-\delta_x)f\left(\tilde{a}_{t+1}(i)\right)\phi_R\left(\frac{P_{It+1}(i)}{P_{It}(i)} - 1\right)\left(\frac{P_{It+1}(i)}{P_{It}(i)^2}\right)J_{t+1}^R(i)\right. \\ &+ \left.(1-\delta_{t+1}(i))\left(1 - \frac{\partial H\left(\tilde{a}_{t+1}(i)\right)}{\partial \tilde{a}_{t+1}(i)}\right)\phi_R\left(\frac{P_{It+1}(j)}{P_{It}(i)} - 1\right)\left(\frac{P_{It+1}(i)}{P_{It}(i)^2}\right)\right]\end{aligned}$$

Rearranging terms

$$\begin{aligned}\frac{\partial J_t^R(i)}{\partial P_{It}} &= -\left(1 - \frac{\partial H\left(\tilde{a}_t(i)\right)}{\partial \tilde{a}_t(i)}\right)\left\{\frac{1}{P_t} + \frac{\phi_R}{P_{It-1}(i)}\left(\frac{P_{It}(i)}{P_{It-1}(i)} - 1\right)\right. \\ &- \mathbb{E}_t\beta_{t,t+1}\left[(1-\delta_{t+1}(i))\left(1 - \frac{\partial H\left(\tilde{a}_{t+1}(i)\right)}{\partial \tilde{a}_{t+1}(i)}\right)\right. \\ &+ \left.\left.(1-\delta_x)f\left(\tilde{a}_{t+1}(i)\right)J_{t+1}^R(i)\right]\phi_R\left(\frac{P_{It+1}(i)}{P_{It}(i)} - 1\right)\left(\frac{P_{It+1}(i)}{P_{It}(i)^2}\right)\right\} \qquad \text{(A2.4)}\end{aligned}$$

From the bargaining problem:

$$\eta \frac{\partial J_t^W}{\partial P_{It}} J_t^R = -\left(1-\eta\right) \frac{\partial J_t^R}{\partial P_{It}} J_t^W$$

$$\eta J_t^R \left(\begin{array}{c} \frac{1}{P_t} - \phi_W \left(\frac{P_{It}(j)}{P_{It-1}(j)} - 1 \right) \frac{1}{P_{It-1}(j)} \\ +\mathbb{E}_t \beta_{t,t+1} \left[\left(1-\delta_{t+1}(j)\right) \phi_W \right. \\ \left. + \left(1-\delta_x\right) f\left(\tilde{a}_{t+1}(i)\right) J_{t+1}^W(j) \phi_R \right] \\ \left(\frac{P_{It+1}(j)}{P_{It}(j)} - 1 \right) \left(\frac{P_{It+1}(j)}{P_{It}(j)^2} \right) \end{array} \right) = \left(1-\eta\right) J_t^W \left(\begin{array}{c} \left(1 - \frac{\partial H(\tilde{a}_t(i))}{\partial \tilde{a}_t(i)}\right) \left\{ \frac{1}{P_t} + \frac{\phi_R}{P_{It-1}(i)} \left(\frac{P_{It}(i)}{P_{It-1}(i)} - 1 \right) \right. \\ - \mathbb{E}_t \beta_{t,t+1} \left[\left(1-\delta_{t+1}(i)\right) \left(1 - \frac{\partial H(\tilde{a}_{t+1}(i))}{\partial \tilde{a}_{t+1}(i)}\right) \right. \\ \left. + \left(1-\delta_x\right) f\left(\tilde{a}_{t+1}(i)\right) J_{t+1}^R(i) \right] \\ \left. \phi_R \left(\frac{P_{It+1}(i)}{P_{It}(i)} - 1 \right) \left(\frac{P_{It+1}(i)}{P_{It}(i)^2} \right) \right\} \end{array} \right)$$

$$\eta J_t^R \left(\begin{array}{c} \varphi_t - \phi_W \left(\pi_{It} - 1\right) \pi_{It} \\ +\mathbb{E}_t \beta_{t,t+1} \left[\left(1-\delta_{t+1}\right) \phi_W \right. \\ \left. + \left(1-\delta_x\right) f\left(\tilde{a}_{t+1}\right) J_{t+1}^W \phi_R \right] \\ \left(\pi_{It+1} - 1\right) \pi_{It+1} \end{array} \right) = \left(1-\eta\right) J_t^W \left(\begin{array}{c} \left(1 - \frac{\partial H(\tilde{a}_t(i))}{\partial \tilde{a}_t(i)}\right) \left\{ \varphi_t + \phi_R \left(\pi_{It} - 1\right) \pi_{It} \right. \\ - \mathbb{E}_t \beta_{t,t+1} \left[\left(1-\delta_{t+1}\right) \left(1 - \frac{\partial H(\tilde{a}_{t+1}(i))}{\partial \tilde{a}_{t+1}(i)}\right) \right. \\ \left. + \left(1-\delta_x\right) f\left(\tilde{a}_{t+1}(i)\right) J_{t+1}^R(i) \right] \\ \phi_R \left(\pi_{It+1} - 1\right) \pi_{It+1} \\ \\ \end{array} \right)$$

$$\eta J_t^R \left(\varphi_t - \tau_t^W\right) = \left(1-\eta\right) J_t^W \left(\left(1 - \frac{\partial H(\tilde{a}_t)}{\partial \tilde{a}_t}\right) \varphi_t + \tau_t^R \right)$$

$$\left(\eta J_t^R - \left(1-\eta\right) \left(1 - \frac{\partial H(\tilde{a}_t)}{\partial \tilde{a}_t}\right) J_t^W \right) \varphi_t = \left(1-\eta\right) \tau_t^R J_t^W + \eta \tau_t^W J_t^R$$

where

$$\begin{aligned} \tau_t^W &= \phi_W \left(\pi_{It} - 1\right) \pi_{It} - \mathbb{E}_t \beta_{t,t+1} \left[\left(1-\delta_{t+1}\right) \phi_W + \left(1-\delta_x\right) f\left(\tilde{a}_{t+1}\right) J_{t+1}^W \phi_R \right] \left(\pi_{It+1} - 1\right) \pi_{It+1} \\ \tau_t^R &= \left(1 - \frac{\partial H\left(\tilde{a}_t\right)}{\partial \tilde{a}_t}\right) \left\{ \phi_R \left(\pi_{It} - 1\right) \pi_{It} - \mathbb{E}_t \beta_{t,t+1} \left[\left(1-\delta_{t+1}\right) \left(1 - \frac{\partial H\left(\tilde{a}_{t+1}\right)}{\partial \tilde{a}_{t+1}}\right) \right. \right. \\ &+ \left. \left. \left(1-\delta_x\right) f\left(\tilde{a}_{t+1}(i)\right) J_{t+1}^R(i) \right] \phi_R \left(\pi_{It+1} - 1\right) \pi_{It+1} \right\} \\ \varphi_t &= \frac{P_{It}}{P_t} \end{aligned}$$

2.8 Appendix B

The Role of Technology Shocks

Figure B2.1: Comparison of the IRFs of the B2B Model With and Without Endogenous Separation Rate to a Technology Shock

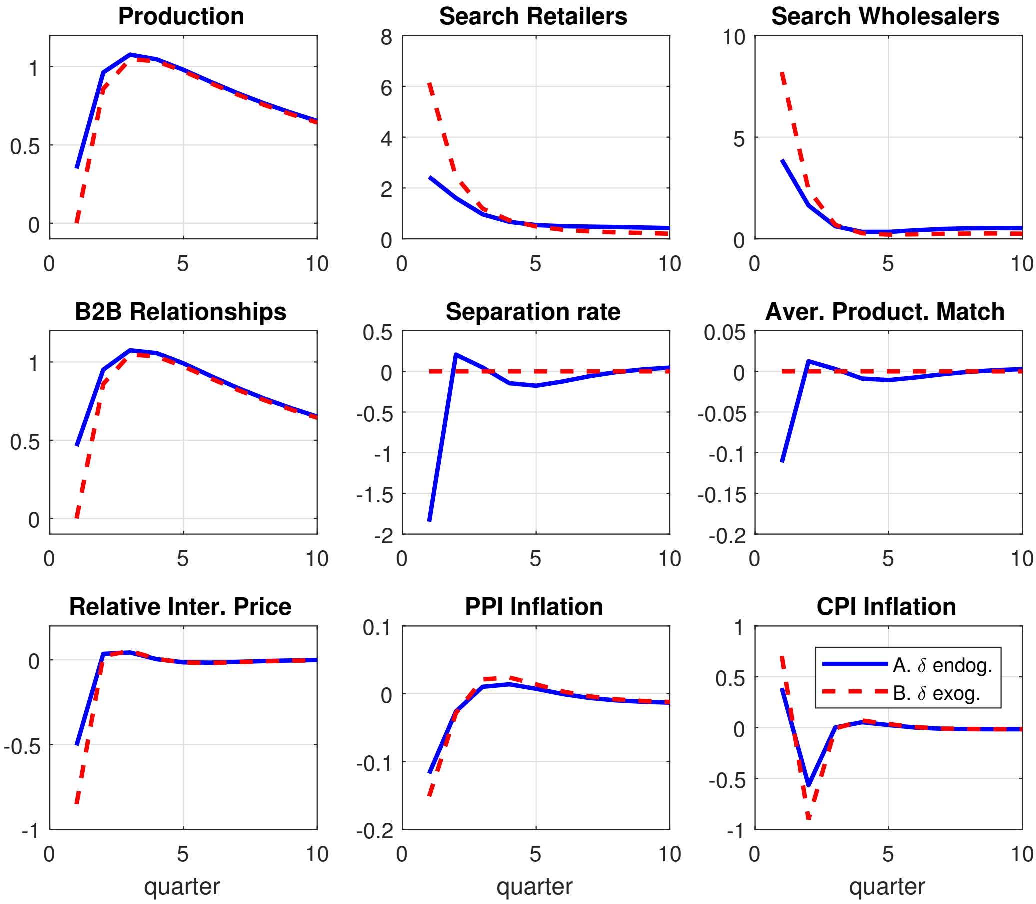
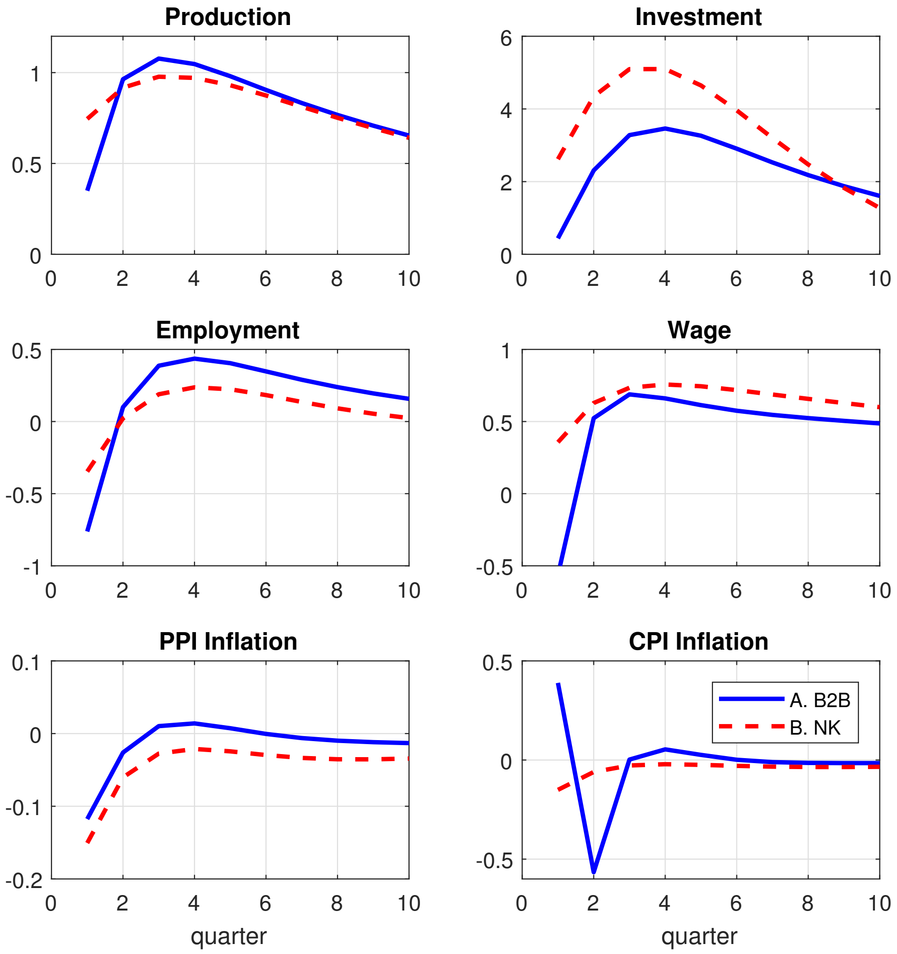


Figure B2.2: Comparison of the IRFs of the B2B and NK Models to a Technology Shock



3 — On Staggered Prices and Optimal Inflation

3.1 Introduction

The objective of this paper is to calculate the optimal rate of inflation in economies with monopolistic competition and sticky prices. On that purpose, two types of slow price-adjustment specifications will be introduced in a Dixit-Stiglitz monopolistically competitive framework: the Taylor (1980) staggered prices, and the Calvo (1983) partial adjustment based on fixed probabilities. Together they represent the bulk of recent literature on optimizing models with sticky prices.[1,2]

We compute the optimal monetary policy by finding the inflation rate that maximizes welfare in steady state. Intuitively, our optimality analysis is based on the steady-state link between inflation and the mark-up of prices over the marginal cost of production. This mark-up is recognized as a source of economic inefficiency that stems from monopolistic competition models.[3] Therefore, the rate of inflation that maximizes the utility of the representative individual in the economy is also the one that minimizes the mark-up in steady state. This becomes the optimal rate of inflation when abstracting from other channels for long-run effects. Neither price indexation nor transactions-facilitating money (shoe-leather costs) will feature in the analysis in order to isolate the steady-state effects of combining monopolistic competition with sticky prices.[4] These assumptions may be acceptable in economies with low and stable

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[1] Examples of papers using Calvo model are Yun (1996), King and Wolman (1996), Erceg et al. (2000), and Sbordone (2002). The Taylor model has been employed by in King and Wolman (1999), Chari et al. (2000), and Huang and Liu (2002).

[2] One alternative way of introducing price rigidity in models with monopolistic competition is by assuming an adjustment cost of price changes as in Rotemberg (1982). One example is Faia (2008), who also computes the optimal monetary policy and finds similar results to the ones obtained in this paper.

[3] It results in some welfare loss relative to the price-taking behavior of perfect competition as first pointed out by Blanchard and Kiyotaki (1987). Other papers that examine this issue are King and Wolman (1996, 1999) and Khan et al. (2003).

[4] See Dotsey and Ireland (1996), Lucas (2000), and Khan et al. (2003) for the welfare analysis of inflation

inflation.[5]

We find that the optimal rate of inflation in steady state is positive because it reduces the monopolistic competition distortions, minimizing the mark-up and hence maximizing welfare. This result is robust for different schemes and degrees of price stickiness. Moreover, the optimal rate of inflation is approximately equal to the ratio between the rate of discount and the Dixit-Stiglitz elasticity.

Finally, we document that even though the welfare cost of steady-state inflation is quantitatively small, it is significantly higher if prices are sticky *à la* Calvo than *à la* Taylor.

3.2 A Model With Monopolistic Competition and Sticky Prices

3.2.1 Households

There is a representative household in the economy who, in period t, seeks to maximize his expected intertemporal constant relative risk aversion (CRRA) utility given by:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left(\frac{c_{t+j}^{1-\sigma}}{1-\sigma} - \varphi \frac{n_{t+j}^{1+\gamma}}{1+\gamma} \right)$$

where $\mathbb{E}_t$ is the rational expectation operator in period t, $\beta = 1/(1+\rho)$ is the discount factor, with $\rho > 0$ as the household discount rate. The preference parameters are strictly positive, $\sigma, \gamma, \varphi > 0$. Household utility depends positively on consumption, c_{t+j}, and negatively on labor, n_{t+j}.[6] The household faces a sequence of flow budget constraints which, denoted in real terms for a given period t, can be written as:

$$w_t n_t + r_t^k k_t + d_t = c_t + k_{t+1} - (1-\delta) k_t + (1+r_t)^{-1} b_{t+1} - b_t$$

where k_{t+1} and b_{t+1} denote purchases of physical capital and bonds, respectively, and $0 < \delta < 1$ is the constant rate of capital depreciation. Accordingly, r_t^k and r_t are the real returns on capital and bonds. The real wage is w_t, and d_t are firm dividends.

The first-order necessary conditions with respect to consumption, bonds, physical capital

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in models with transactions-facilitating money, and Casares (2004) when there is price indexation.

[5] From 1995 to 2018, the average rate of inflation in the US has been 1.89% per year, whereas in the Euro Area it has been 1.53% per year.

[6] The household actually consumes a bundle of differentiated goods, i.e. $c_t = \left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$.

and labor supply are

$$\begin{aligned}
c_t^{-\sigma} - \lambda_t &= 0 \\
-\lambda_t \left(1 + r_t\right)^{-1} + \beta E_t \lambda_{t+1} &= 0 \\
-\lambda_t + \beta E_t \lambda_{t+1}(1 - \delta + r_{t+1}^k) &= 0 \\
-\varphi n_t^\gamma + \lambda_t w_t &= 0
\end{aligned}$$

where λ_t is the Lagrange multiplier of the budget constraint.

3.2.2 Producers

We begin with the monopolistic competition setup described in Dixit and Stiglitz (1977). There is a continuum of firms each producing a differentiated good and selling it in a monopolistically competitive market. Thus, the firm i sets the price $P_t(i)$ in quarter t, and the amount of output that will sell $y_t(i)$ is given by the Dixit-Stiglitz demand equation

$$y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} y_t,$$

where $\theta > 1$ is the elasticity of substitution between differentiated goods, P_t is the aggregate price level, and y_t is aggregate output. Regardless of the price-adjustment specification, in every period firms choose capital and labor to minimize the cost of production:

$$r_t^k k_t(i) + w_t n_t(i)$$

subject to the Cobb-Douglas production technology

$$k_t^\alpha(i) n_t^{1-\alpha}(i) = y_t(i)$$

where $0 \leq \alpha \leq 1$ is the capital elasticity with respect to output. The first-order necessary conditions with respect to capital and labor are

$$r_t^k - mc_t(i)\alpha \left(\frac{k_t(i)}{n_t(i)}\right)^{\alpha-1} = 0 \tag{3.1}$$

$$w_t - mc_t(i)(1-\alpha) \left(\frac{k_t(i)}{n_t(i)}\right) = 0 \tag{3.2}$$

where $mc_t(i)$ is the Lagrange multiplier of the technological constraint which coincides with the firm-level real marginal cost. Combining both (3.1) and (3.2), we obtain the capital-labor ratio for firm i

$$\frac{k_t(i)}{n_t(i)} = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t^k}$$

Notice that every firm has the same capital-labor ratio, which implies that the real marginal cost is also the same across firms and is given by

$$mc_t = \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{r_t^k}{\alpha}\right)^{\alpha}$$

Next, we will introduce two different price-adjustment schemes.

Optimal Price Under Calvo Scheme

Following Calvo (1983), let us assume that, in every period, there is a constant probability, $0 < \eta < 1$, that firms will not be able to change prices. If the reresentative firm i receives the Calvo signal to set the optimal price, the decision is made by maximizing the intertemporal profit function:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \eta^j \left[\left(\frac{P_t\left(i\right)}{P_{t+j}}\right)^{1-\theta} y_{t+j} - mc_{t+j}\left(\frac{P_t\left(i\right)}{P_{t+j}}\right)^{-\theta} y_{t+j}\right]$$

The first order condition for the price $P_t\left(i\right)$ is:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \eta^j \left[(1-\theta)\left(\frac{P_t\left(i\right)}{P_{t+j}}\right)^{1-\theta} \frac{1}{P_{t+j}} y_{t+j} + \theta mc_{t+j}\left(\frac{P_t\left(i\right)}{P_{t+j}}\right)^{-\theta-1} \frac{1}{P_{t+j}} y_{t+j}\right] = 0 \qquad (3.3)$$

After some algebra, equation (3.3) can be solved for the optimal price which, in a steady state, with a constant rate of inflation π, is given by[7]

$$P(i) = \frac{\theta}{\theta-1}\left[\frac{1-\beta\eta(1+\pi)^{\theta-1}}{1-\beta\eta(1+\pi)^{\theta}}\right] Pmc \qquad (3.4)$$

Optimal Price Under Taylor Scheme

Alternatively, it could be assumed that firms adjust the price with a constant frequency as proposed by Taylor (1980). In particular, firms decide new prices every J quarters, remaining unchanged meanwhile. Hence, the optimal-price decision for the representative firm is made

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[7] Variables with no time subscript indicate their value in steady state.

by maximizing the intertemporal profit function:

$$\mathbb{E}_t \sum_{j=0}^{J-1} \beta^j \left[\left(\frac{P_t(i)}{P_{t+j}} \right)^{1-\theta} y_{t+j} - mc_{t+j} \left(\frac{P_t(i)}{P_{t+j}} \right)^{-\theta} y_{t+j} \right]$$

The first order condition for $P_t(i)$ is:

$$\mathbb{E}_t \sum_{j=0}^{J-1} \beta^j \left[(1-\theta) \left(\frac{P_t(i)}{P_{t+j}} \right)^{1-\theta} \frac{1}{P_{t+j}} y_{t+j} + \theta mc_{t+j} \left(\frac{P_t(i)}{P_{t+j}} \right)^{-\theta-1} \frac{1}{P_{t+j}} y_{t+j} \right] = 0$$

As in the previous case, this equation can be solved for the optimal price which, in a steady state with constant inflation is given by

$$P(i) = \frac{\theta}{\theta-1} \left[\frac{1-\beta^J(1+\pi)^{J\theta}}{1-\beta^J(1+\pi)^{J(\theta-1)}} \frac{1-\beta(1+\pi)^{\theta-1}}{1-\beta(1+\pi)^{\theta}} \right] Pmc \qquad (3.5)$$

3.3 Results

3.3.1 Inflation, the Mark-up and the Welfare Cost

The market power that firms have under monopolistic competition produces a positive mark-up of prices over marginal cost. This mark-up creates an inefficient wedge between the marginal productivities of capital and labor and their corresponding marginal payments, i.e. the real wage, w, and the real rental rate on capital, r^k, respectively.[8] Consequently, the equilibrium levels of capital and labor, and hence output, under monopolistic competition are lower than in an economy with perfect competition. Therefore, a higher mark-up implies a lower output produced in the economy. This distortion, whose presence is solely stemming from monopolistic competition, is what motivates the search for policies that monetary and fiscal authorities could use to reduce the mark-up and, therefore, improve the aggregate economic activity. In particular, in this paper we look for the (constant) steady-state inflation rate that maximizes welfare.[9] Not surprisingly, this is equivalent to finding the inflation rate that minimizes the mark-up. Hence, in the following, an analysis of the effects that steady-state inflation has on the mark-up, and subsequently on welfare, will be carried out.

Even though the presence of a positive mark-up is independent from the existence of sticky prices, its size actually depends on both the scheme and degree of price stickiness. By looking

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[8] In other words, the real marginal cost in equations (3.1) and (3.2) is lower than one.

[9] This is what King and Wolman (1999) define as the monetary modified golden rule.

at equations (3.4) and (3.5), we notice that the optimal price is equal to the product of the firm-level mark-up and the nominal marginal cost. In turn, the firm-level mark-up can be decomposed into its constant component $\theta/(\theta-1)$, and a model-specific component that depends on the price stickiness parameter (η and J, respectively in the Calvo and Taylor specifications). In both variants, the firm-level mark-up is increasing in inflation. This seems intuitive, if there is positive inflation the firms that have the opportunity to adjust their price will increase it.

If we multiply both sides of equations (3.4) and (3.5) by $P/P(i)$, we find the aggregate price as a proportion, μ, of the nominal marginal cost: $P = \mu Pmc$. Therefore, μ is the average mark-up, which with Calvo sticky prices is

$$\mu = \frac{\theta}{\theta-1}\left[\frac{1-\beta\eta(1+\pi)^{\theta-1}}{1-\beta\eta(1+\pi)^{\theta}}\right]\frac{P}{P(i)} \tag{3.6}$$

while with Taylor staggered prices becomes

$$\mu = \frac{\theta}{\theta-1}\left[\frac{1-\beta^J(1+\pi)^{J\theta}}{1-\beta^J(1+\pi)^{J(\theta-1)}}\frac{1-\beta(1+\pi)^{\theta-1}}{1-\beta(1+\pi)^{\theta}}\right]\frac{P}{P(i)} \tag{3.7}$$

Notice that the average mark-up of the economy is the product of the firm-level mark-up and the inverse of the relative price, $P/P(i)$ which is a good indicator of price dispersion due to the price stickiness.[10] Computing the aggregate Dixit-Stiglitz price level, it can be seen that the relationship between steady-state inflation, π, and $P/P(i)$, depends on the price-adjustment specification. For Calvo price stickiness, we obtain

$$\frac{P}{P(i)} = \left[\frac{1-\eta}{1-\eta(1+\pi)^{\theta-1}}\right]^{\frac{1}{1-\theta}} \tag{3.8}$$

while for Taylor staggered contracts we have

$$\frac{P}{P(i)} = \left[\frac{1}{J}\frac{1-(1+\pi)^{J(\theta-1)}}{1-(1+\pi)^{\theta-1}}\right]^{\frac{1}{1-\theta}} \tag{3.9}$$

In both cases, (3.8) and (3.9) show that $P/P(i)$ falls with the inflation rate, π.

The steady-state relation between the average mark-up and inflation is finally obtained by inserting equations (3.8) and (3.9) into (3.6) and (3.7), respectively. This allows us to write down, for both the Calvo and Taylor cases, the steady-state average mark-up, $\mu = mc^{-1}$,

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[10] If $P/P(I)$ is different from one, there is price dispersion. With positive steady-state inflation, $\pi > 0$, the value of $P/P(I)$ is lower than one. With negative steady-state inflation, $\pi < 0$, the value of $P/P(I)$ is greater than one.

Table 3.1: Baseline Calibration

Parameter	Value	Description
ρ	0.005	Discount rate
θ	6	Dixit-Stiglitz Elasticity
σ	1.25	Risk Aversion
γ	2	Labor elasticity
φ	0.55	Labor Disutility
α	0.36	Capital Share
δ	0.025	Capital Depreciation Rate
η	0.75	Calvo Stickiness
J	4	Taylor Stickiness

as a function of the Dixit-Stiglitz elasticity parameter θ, the rate of discount ρ (through the discount factor $\beta = (1+\rho)^{-1}$), the level of price rigidity, either η or J, and the steady-state rate of inflation π

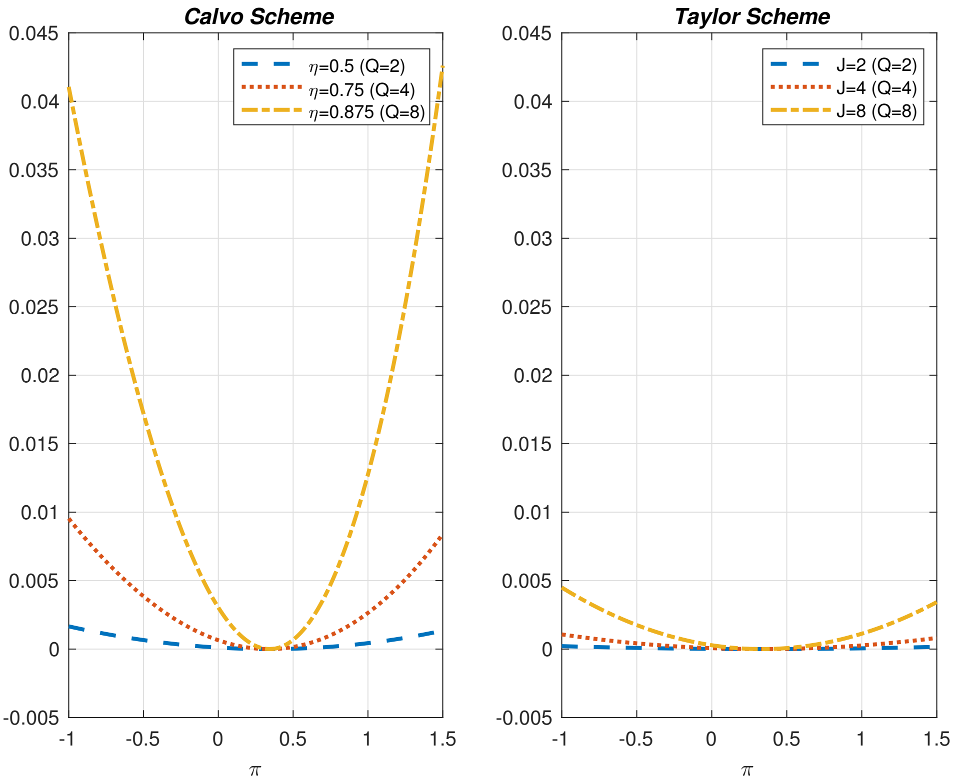
$$\mu = \frac{\theta}{\theta - 1} \frac{1 - \beta\eta(1+\pi)^{\theta-1}}{1 - \beta\eta(1+\pi)^{\theta}} \left[\frac{1-\eta}{1 - \eta(1+\pi)^{\theta-1}} \right]^{\frac{1}{1-\theta}} \tag{3.10}$$

$$\mu = \frac{\theta}{\theta - 1} \frac{1 - \beta^J(1+\pi)^{J\theta}}{1 - \beta^J(1+\pi)^{J(\theta-1)}} \frac{1 - \beta(1+\pi)^{\theta-1}}{1 - \beta(1+\pi)^{\theta}} \left[\frac{1}{J} \frac{1 - (1+\pi)^{J(\theta-1)}}{1 - (1+\pi)^{\theta-1}} \right]^{\frac{1}{1-\theta}} \tag{3.11}$$

As we just discussed, steady-state inflation has opposing effects on the average mark-up of the economy. On one hand, higher inflation leads to a higher firm-level mark-up, which increases the average mark-up. On the other hand, higher inflation leads to higher price dispersion (lower $P/P(i)$), which decreases the average mark-up. Our numerical simulations have found that the latter effect dominates over the former effect because the mark-up falls as the steady-state rate of inflation marginally rises from 0%. Therefore, the rate of inflation that minimizes the mark-up is slightly positive. Actually, we have checked that the rate of inflation that minimizes the mark-up is also the rate of inflation that maximizes both output and household utility (welfare). Hence, the optimal steady-state rate of inflation is not 0%, it is a small positive number.

Let us calibrate the parameters of the model and provide some numerical results. We assume that each period represents a quarter. In the baseline calibration, we set $\rho = 0.005$, so that the annual rate of discount is equal to 2%, and $\theta = 6$, which implies a mark-up of prices over the marginal cost approximately equal to 20%. Table 3.1 contains the calibration for the remaining parameters, borrowing usual values taken in the related literature. For comparative purposes, let the degree of price stickiness under Calvo and Taylor schemes (parameterized

Figure 3.1: Welfare Cost of Steady-State Inflation, % of Output



by η and J, respectively) be the same. Hence, Q denotes the average number of quarters without price adjustment. Thus the cases $\eta = [0.5, 0.75, 0.875]$ and $J = [2, 4, 8]$ represent three situations for both pricing specifications in which Q is two quarters (half a year), four quarters (one year) and eight quarters (two years).[11]

Figure 3.1 displays the optimal steady-state rate of inflation as the value that minimizes the welfare cost. The welfare cost is measured as the percent of output that represents the consumption equivalence, i.e. the required increase in the amount of consumption to reach the maximum household utility (welfare) obtained at the optimal inflation rate. These welfare costs are displayed in Figure 3.1 for different values of steady-state inflation (ranging from -1% to 1.5% in annualized terms), for both pricing schemes and for several degrees of price rigidity. Remarkably, all the sticky-price specifications give a minimum welfare cost of inflation at a steady-state rate of inflation between 0% and 0.5%, closer to 0.5% than to 0%. It means that neither the Chicago rule ($-400\frac{\rho}{1-\rho} \approx -2\%$) nor the 0% rate of inflation are optimal. The numerical solution of the model indicates that the optimal rate of inflation is very close to 0.33% for the three different levels of price stickiness (Q=2, Q=4, Q=8), with Calvo pricing as well as Taylor pricing. Therefore, the degree of price stickiness in either model has very

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[11] Note that under Calvo pricing, $Q = (1-\eta)^{-1}$, whereas under Taylor pricing $Q = J$.

little effect on the optimal rate of inflation in steady state.[12] However, both the type and the degree of price stickiness crucially determine the size of the welfare cost of inflation. With Calvo staggered prices the welfare losses are clearly larger than with Taylor prices because the mark-up increases much more rapidly when steady-state inflation moves from its optimal value.[13] In addition, the longer the average time without adjusting prices (Q), the larger the welfare cost is when inflation deviates from the optimal rate. Table 3.2 reports the welfare costs of 2%, 5% and 10% annual inflation in both specifications:

Table 3.2: Welfare Cost of Steady-State Inflation (Additional Results)

π(%)	**Calvo**	**Taylor**
-1	0.0095	0.0010
0	0.0006	0.0001
2	0.0177	0.0016
5	0.1737	0.0129
10	1.2001	0.0545

3.3.2 Staggered Prices and the Optimal Steady-State Inflation

After showing that the optimal rate of inflation in steady-state is robust to both different pricing schemes and different degrees of price stickiness, we conduct a sensitivity analysis for alternative calibrations of the Dixit-Stiglitz elasticity, θ, and the rate of discount, ρ. Our objective is to check whether the result found in the previous section holds for different values of these key parameters.

Figure 3.2 plots the optimal rate of inflation, in both the Calvo and Taylor specifications, for a reasonable range of values of θ and ρ. It is straightforward to notice that both graphs are almost identical to each other, which supports the fact that the optimal inflation rate is independent from the pricing scheme for any value on the Dixit-Stiglitz elasticity, θ, and the rate of discount, ρ. Also, it can be noticed that either higher discount rates or lower values of the Dixit-Stiglitz elasticity (i.e. higher monopolistic power) lead to a higher optimal rate of inflation. However, Figure 3.2 does not provide any information about how the degree of price stickiness influences the optimal rate of inflation. To overcome this, Table 3.3 reports the optimal inflation rates for the combinations of cases with $\theta = [4, 6, 10]$ and $\rho = [0.005, 0.01]$, for the Calvo and Taylor specifications and different degrees of price stickiness.

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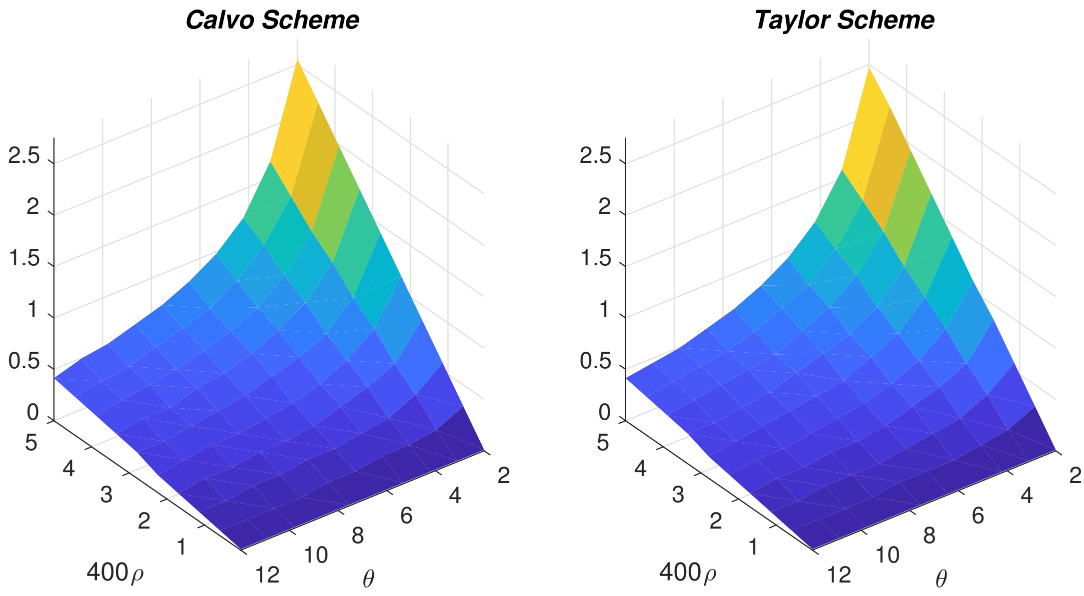
[12] This seems surprising because the steady-state relationships (3.10) and (3.11), include the price stickiness parameters η and J. Furthermore, these equations look significantly different yet yield nearly the same optimal rate of inflation.

[13] The same result has been found by Kiley (2002).

Figure 3.2: Optimal Steady-State Rate of Inflation (annualized, %) depending on ρ and θ



By looking at Table 3.3, we can conclude that once ρ and θ are set, not only the sticky-price specification (either *á la* Calvo or *á la* Taylor) does not matter for the optimal rate of inflation but also that both schemes provide very similar numbers for any degree of price stickiness, Q. There is just one minor difference. The Calvo pricing seems to give slightly higher rates of inflation than the Taylor one, especially when there is great price stickiness (see the cases with Q=8). However, the difference is quantitatively very small.

Table 3.3: Optimal Annual Rates of Inflation (%) for Different Sticky-Price Specifications

	$\rho = 0.005$ and $\theta = 4$			$\rho = 0.005$ and $\theta = 6$			$\rho = 0.005$ and $\theta = 10$		
	Calvo	Taylor	$400\frac{\rho}{\theta}$	Calvo	Taylor	$400\frac{\rho}{\theta}$	Calvo	Taylor	$400\frac{\rho}{\theta}$
Q=2	0.5015	0.4974	0.5000	0.3333	0.3333	0.3333	0.2012	0.2012	0.2000
Q=4	0.5055	0.4974	0.5000	0.3373	0.3333	0.3333	0.2012	0.2012	0.2000
Q=8	0.5135	0.4974	0.5000	0.3413	0.3333	0.3333	0.2052	0.2012	0.2000
	$\rho = 0.01$ and $\theta = 4$			$\rho = 0.01$ and $\theta = 6$			$\rho = 0.01$ and $\theta = 10$		
	Calvo	Taylor	$400\frac{\rho}{\theta}$	Calvo	Taylor	$400\frac{\rho}{\theta}$	Calvo	Taylor	$400\frac{\rho}{\theta}$
Q=2	1.0060	0.9979	1.0000	0.6736	0.6656	0.6666	0.4054	0.3973	0.4000
Q=4	1.0220	0.9979	1.0000	0.6816	0.6656	0.6666	0.4094	0.3973	0.4000
Q=8	1.0540	0.9979	1.0000	0.7057	0.6656	0.6666	0.4254	0.3973	0.4000

An additional remarkable result is that the optimal rate of inflation can be fairly well approximated by the ratio $400\frac{\rho}{\theta}$. Thus, the ratio of the annualized rate of discount (400ρ) over the Dixit-Stiglitz elasticity (θ) provides a very accurate approximation to the annual rate of inflation that would maximize welfare in steady state. Consequently, doubling the discount rate approximately doubles the optimal rate of inflation.

Summarizing, the optimal steady state rate of inflation is a slightly positive number, which is not determined by the price-adjustment scheme or the degree of price stickiness. Rather, it is well characterized by the ratio between two of the model parameters, the discount rate (ρ) in the numerator, and the Dixit-Stiglitz elasticity (θ) in the denominator.

3.4 Conclusions

In this paper we have computed the welfare-maximizing rate of inflation in the steady state of a monopolistic competition model under two different sticky-price specifications: the Calvo (1983) fixed probability and Taylor (1980) staggered contracts. The maximum welfare is obtained at a steady-state rate of inflation that leads to the minimum mark-up. This optimal rate of inflation is a positive number. Furthermore, its value is fairly well represented by the ratio between the annual rate of discount and the Dixit-Stiglitz elasticity. This result is remarkably robust to changes in either the pricing scheme or the level of price stickiness.

The welfare cost of steady-state inflation is rather small. In the baseline calibration with $\rho = 0.005$ and $\theta = 6$, the welfare cost of a 5% annual steady-state inflation is a permanent 0.17% of output with Calvo pricing and just 0.013% with Taylor contracts.

3.5 Appendix A

Steady State Equilibrium Conditions

We have a system of 10 equations for 10 endogenous variables (c, k, r, r^k, b, w, n, y, mc, μ).

- Overall Resources Constraint

$$y = c + \delta k - \frac{r}{1+r} b \tag{A3.1}$$

- Euler Equation

$$1 = \beta(1+r) \tag{A3.2}$$

- No arbitrage

$$r = r^k - \delta \tag{A3.3}$$

- Labor supply

$$\varphi n^{\gamma} = c^{-\sigma} w \tag{A3.4}$$

- Bond supply

$$b = 0 \tag{A3.5}$$

- Output

$$y = k^{\alpha} n^{1-\alpha} \tag{A3.6}$$

- Marginal cost

$$mc = \left(\frac{w}{1-\alpha}\right)^{1-\alpha} \left(\frac{r^k}{\alpha}\right)^{\alpha} \tag{A3.7}$$

- Capital-labor ratio

$$\frac{w}{r^k} = \frac{1-\alpha}{\alpha} \frac{k}{n} \tag{A3.8}$$

- Mark-up

$$\mu = mc^{-1} \tag{A3.9}$$

- Optimal pricing with either Calvo sticky prices

$$\mu = \frac{\theta}{\theta-1} \frac{1-\beta\eta(1+\pi)^{\theta-1}}{1-\beta\eta(1+\pi)^{\theta}} \left[\frac{1-\eta}{1-\eta(1+\pi)^{\theta-1}}\right]^{\frac{1}{1-\theta}} \tag{A3.10}$$

or Taylor staggered prices

$$\mu = \frac{\theta}{\theta - 1} \frac{1 - \beta^J (1+\pi)^{J\theta}}{1 - \beta^J (1+\pi)^{J(\theta-1)}} \frac{1 - \beta(1+\pi)^{\theta-1}}{1 - \beta(1+\pi)^{\theta}} \left[\frac{1}{J} \frac{1 - (1+\pi)^{J(\theta-1)}}{1 - (1+\pi)^{\theta-1}} \right]^{\frac{1}{1-\theta}} \tag{A3.11}$$

Overall Conclusions

This thesis has addressed three important issues in macroeconomics. First, it has analyzed the role of risk aversion heterogeneity on entrepreneurship, saving behavior and the wealth distribution. It has been shown that the introduction of risk aversion heterogeneity is key because it allows to disentangle the effects that risk aversion and financial frictions have on entrepreneurship, which is a key driver of wealth inequality. Second, it has been demonstrated that the presence of long-term business relationships and bargaining over sticky prices reduce both the allocative role of intermediate prices and the real effects of monetary policy shocks. And third, it has been found that the optimal rate of inflation in steady state, in a model with monopolistic competition under two prominent sticky-price specifications, is always positive, regardless of the degree of price stickiness.

The methodology used in the analysis of these three articles has been crucial for the validity and relevance of the results obtained. Throughout this thesis it has been demonstrated that dynamic general equilibrium models are a powerful analytical tool for economists because they allow addressing complex issues by offering a simplified and coherent version of reality.

Conclusiones Generales

Esta tesis ha abordado tres temas importantes en Macroeconomía. Primero, se ha analizado el efecto de la heterogeneidad en la aversión al riesgo sobre el emprendimiento, el comportamiento del ahorro y la distribución de la riqueza. Se ha demostrado que la introducción de la heterogeneidad en la aversión al riesgo es clave porque permite desentrañar los efectos que la aversión al riesgo y las fricciones financieras tienen sobre el emprendimiento, que es un factor clave para explicar la desigualdad en la acumulación de la riqueza. En segundo lugar, se ha demostrado que la presencia de relaciones comerciales a largo plazo y los procesos de negociación sobre precios con rigidez reducen el papel de asignación de los precios intermedios y los efectos reales de las perturbaciones de la política monetaria. Y tercero, se ha encontrado que la tasa óptima de inflación en estado estacionario, en un modelo con competencia monopolística y bajo dos especificaciones de rigidez de precios diferentes, siempre es positiva, independientemente del grado de rigidez de precios.

La metodología utilizada en el análisis de estos tres artículos ha sido crucial para la validez y relevancia de los resultados obtenidos. A lo largo de esta tesis, se ha demostrado que los modelos dinámicos de equilibrio general son una herramienta analítica poderosa para los economistas porque permiten abordar problemas complejos al ofrecer una versión simplificada y coherente de la realidad.

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