Slow-light and enhanced sensitivity in a displacement sensor using a lossy fiber-based ring resonator

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Abstract— **Along the last years, much debate has been done on the efficiency of slow-light phenomena in order to enhance light-matter interactions, especially for sensing purposes. This improvement could be key to develop more compact and sensitive devices. In this work we develop an all-fiber sub-micrometric displacement sensor using slow-light sensitivity enhancement in a lossy ring resonator. In the proposed structure the losses produced by the displacement of a mechanic transducer can be translated into strong variations of group index and therefore strong transmittance variations. We show that this effect is strictly related to slow light, and not related to confinement effects or any other.**

Index Terms— **slow light, ring resonators, group delay, group index, Kerr effect.**

I. INTRODUCTION

Light-matter interactions are normally weak in most optical media in conventional conditions. Slow light structures allow a large reduction of the group velocity of the light signals travelling through them, and, as a consequence, a great enhancement of light-matter interactions [1]. These interactions include various linear and nonlinear effects.

From the point of view of fiber sensors, the possibility of enhancing light-matter interactions may favor the development of more compact and sensitive devices. Some research effort has been devoted to understanding the exact enhancement values given by slow light in different light-matter interactions. For instance, the role of slow light in enhancing nonlinear effects has been theoretically and experimentally investigated [2, 3]. The enhancement factor in Kerr effect has been shown to scale as n_g^2 (n_g being the group index of the structure). This enhancement effect has been explained as the combined effect of two contributions: the longer transit time of light pulses in the medium and the higher energy density due to spatial pulse compression. It has also been theoretically and experimentally proved that the extreme dispersion of slow light can lead to an enhancement in the spectral sensitivity of interferometers given by ng [4]. The role of slow light in enhancing gyroscope performance [5] and Beer-Lambert-Bouguer (BLB) absorption [6, 7] has also been theoretically and/or experimentally investigated. Generally speaking, light-matter interactions have been found to be only enhanced in structural slow light systems (e.g. coupled resonators, Bragg gratings, etc.), i.e. systems relying on multiple interferences. Material slow light systems (e.g. stimulated Brillouin/Raman scattering, parametric amplification, etc.) do not follow the same rules, the origin of this difference being that the electromagnetic energy velocity in material slow light does not depend on the group index [8].

From the fiber sensing point of view, it is therefore interesting to find structural slow light media, i.e. passive structures in which the group index could be tuned widely. Such a system could be considered as a platform for the development of slow-light assisted sensors. In this paper we demonstrate theoretically and experimentally a wide range tuning of the group delay in a lossy fiber-based ring resonator. This structure is shown to exhibit strong group delay changes close to the critical coupling regime. The strong group index variations observed are accompanied by strong absorption sensitivity enhancements, following similar scaling laws in both cases. We show that the sensitivity enhancement effect is not related to field enhancement in the cavity or to an increase in the photon lifetime (these enhancement effects only occur in high-Q resonators). For demonstrative purposes, we develop a simple displacement sensor with sub-micrometric resolution. We believe that this structure could become a basic

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building block in future slow-light-assisted fiber optic sensors.

II. THEORY

A. Basic Analysis of Lossy Ring Resonators and their attenuation sensitivity

In this section we perform a theoretical study of slow and fast light in lossy fiber ring resonators. We show that, depending on the coupling ratio and the loss in the resonator, the group delay of the ring resonator can be tuned from strong delay to strong advancement, in particular when the losses in the resonator is tuned close to the critical coupling regime.

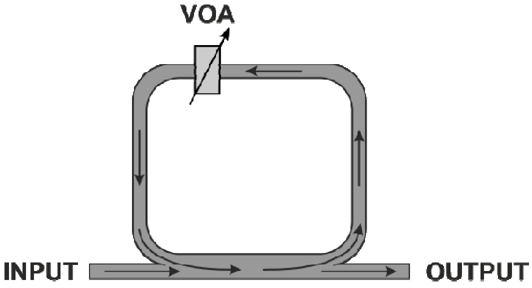

Fig. 1. Fiber-based lossy ring resonator considered.

We consider the structure depicted in Figure 1, i.e. a fiber ring resonator with a variable loss element inside. Although it is not shown, a polarization controller ensures that the polarization evolution over one round-trip remains constant. Our analysis of this structure is similar to the analysis found in [9] for resonator-coupled waveguides. A transfer function can be found:

$$\frac{E_o}{E_i}=\frac{\kappa\exp\left(\frac{i\omega nL}{c}\right)-a}{\exp\left(\frac{i\omega nL}{c}\right)-\kappa a} \tag{1}$$

where κ and a are, respectively, the coupling ratio (square root of the fraction of recirculated power) and the field transmission coefficient in the resonator (in unitless form). As expected, this transfer function leads to resonances in the spectrum with a periodicity given by c/nL. These resonances exhibit steep dispersion slopes close to the center, leading to large group delays, especially when tuned close to the critical coupling case. We can theoretically evaluate the group delay in the resonances by finding the phase of expression (1), obtaining the derivative with respect to ω, and evaluating the obtained expression at the resonant frequencies. The resulting expression for the group delay is:

$$\tau_g=\frac{nL}{c}\cdot\frac{a\left(1-\kappa^2\right)}{\kappa\left(a^2+1\right)-a\left(\kappa^2+1\right)} \tag{2}$$

We can therefore see that the delay in the resonator can be expressed as the regular single-pass delay of the resonator (nL/c) multiplied by a factor that may be varied continuously by changing only the attenuation and/or coupling ratio in the resonator. It can be easily shown that, when the losses exceed the coupling (undercoupling), negative group delays can be found at the resonances, while they remain positive in the opposite case (overcoupling). A discontinuity in function (2) is found for the critical coupling case (a = κ), where the group delay function displays a vertical asymptote, taking values of -∞ when $a \to \kappa-$ and +∞ when $a \to \kappa+$. The evolution of the group delay close to the critical coupling can be described by $\tau_g \propto (\kappa-a)^{-1}$. The qualitatively different behavior of both regimes is depicted in Fig. 2.

We can now also evaluate the transmission coefficient of the resonator in the center of the resonance as a function of the attenuation and coupling coefficients. The power transmission coefficient can be written as:

$$T=\left|\frac{a-\kappa}{\kappa a-1}\right|^2 \tag{3}$$

As it can be seen, in the transmission coefficient, the case a = κ (critical coupling) corresponds to the case in which the transmission at the resonance center goes to zero. It is interesting now to evaluate the sensitivity of the absorption/attenuation in the structure to small absorption/attenuation changes in the ring:

$$S=-\frac{d\ln T}{da}=\frac{2}{\kappa-a}+\frac{2}{\kappa a-1} \tag{4}$$

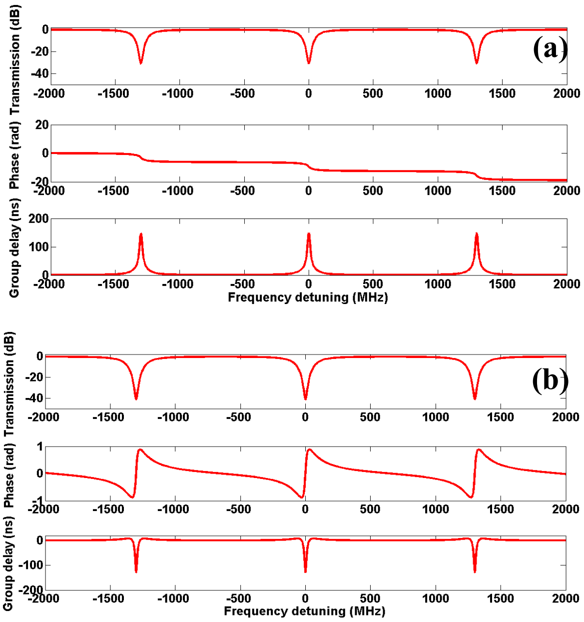

Fig. 2. Phase, modulus and group delay response vs. frequency in a 1.59 meter and κ=0.7 (≈1.55 dB) fiber loop with (a) 1 dB attenuation and (b) 2 dB attenuation.

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In this expression, the sensitivity S has been defined as the attenuation sensitivity of a traveling-wave system (derivative of the log transmission).

As it is visible, the sensitivity S to small variations in a can be extremely large under two conditions: (1) when the losses and the coupling ratio are equal ($a = \kappa$) or (2) when the $a\kappa \rightarrow 1$, which can only be achieved when the losses are low and the re-circulated intensity in the loop is maximized. This last case (high Q resonators) has been conventionally used in sensing, as strong field enhancements and long photon lifetimes can be obtained in the cavity. However, extremely high sensitivities can also be obtained in low-Q resonators by exploiting the region close to critical coupling ($a = \kappa$). This region has been far less exploited in the literature, and is the region in which the group delay can be made extremely large as shown before in this section.

B. *On the origin of sensitivity enhancement close to the critical coupling*

It has been shown above that strong sensitivity enhancements occur close to critical coupling, and also strong group index changes. One may then wonder if this enhancement effect is related to the well-known field enhancement within a resonator or a photon lifetime increase in the cavity. However, as mentioned, these situations are only observed in high-Q resonators. The intensity recirculating in the cavity in resonance can be written as:

$$\left|\frac{E_{circ}}{E_i}\right|^2 = \left|\frac{1-\kappa^2}{1-a\kappa}\right|^2 \qquad (5)$$

As it is visible, the circulating intensity decays monotonically as the losses are larger, and only diverging in the high-Q case ($a\kappa \rightarrow 1$, i.e. when the losses are zero and the recirculating power is maximized). In the critical coupling, the re-circulating power and the input power are equal. This is consistent with the fact that the transmission of the structure in this case is zero (all the energy is "kept" in the circulator). This storage picture is also consistent with the qualitative picture of slow light in a medium with infinite group delay.

In addition to the previous discussion, we may now turn our attention to the photon lifetime in the cavity, which can be written as:

$$\tau = \frac{nL/c}{\delta} = \frac{nL/c}{\left(1-a^2\kappa^2\right)} \qquad (6)$$

Where δ represents the fractional power loss in one round-trip. As it is visible, the photon lifetime also decays monotonically with the loss inside the cavity, and does not present any special behavior in critical coupling case.

We can find another interesting relationship with slow light by evaluating the expression of the sensitivity (4) and the group delay (2) close to the critical coupling. We can see that both expressions hold similar evolutions close to critical coupling:

$$S\Big|_{critical} \propto (\kappa - a)^{-1} \propto \tau_g\Big|_{critical} \qquad (7)$$

To sum up, we have performed a theoretical study that demonstrates the capability of lossy fiber ring resonator to tune between slow- and fast-light regimes by simply varying the coupling ratio or losses of the structure. Around the critical coupling, strong group delays can be observed which are expected to coincide with strong sensitivity enhancements to attenuation or absorption in the ring for a fixed coupling ratio. The next sections explore these concepts experimentally, proving the aforementioned analysis and proposing a simple low-cost implementation of a displacement sensor based upon these concepts.

III. EXPERIMENTAL DEMONSTRATIONS

The previous theoretical analysis has been experimentally validated through high-resolution measurements of amplitude transmission and phase response of the structure. For this purpose, an optical vector analyzer (OVA) has been used, obtaining both the spectral response and the group delay variation as a function of the loss~~es~~ inside the ring. Further to this, a functional setup is proposed which could measure sub-micrometric displacements in a simple setup with an assumable cost.

A. *Experimental validation of the theory*

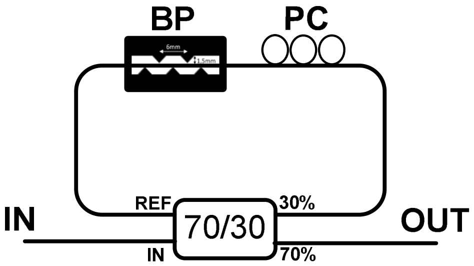

Fig. 3. Experimental setup for the fiber ring under study. The structure is illuminated by "IN" port and the response is detected by "OUT". BP: Bending Plates; PC: Polarization Controller.

Figure 3 illustrates the experimental setup for the lossy ring resonator structure used in this work. A fiber coupler, a polarization controller and a high precision mechanic attenuator based on stress-inducing plates, (labeled BP in the Figure 3), were used for creating the lossy fiber ring. The main component in the proposed structure is the 70:30 optical coupler. A polarization controller was placed inside the ring to adjust the polarization transfer function of the ring. The attenuation in the ring is varied by controlling the

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displacement of the bending plates through a high precision actuator. The attenuation response induced by the plates on the fiber is plotted in Figure 4. The total length of fiber in the loop is ~1.59 meters. The OVA was then used to measure the transmission and group delay response of the structure.

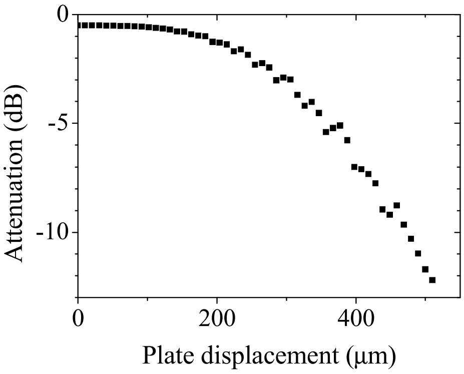

Fig. 4. Attenuation profile of the bending plates versus the displacement.

Figure 5 and 6 depicts two sample transmission and group delay spectra obtained for two different attenuation values. The central wavelength for the analysis was set at ~1550 nm with 10 pm (1.25 GHz) measuring span. The separation between resonances (Fig. 5 and 6) correspond to ~130 MHz as expected when ~1.59 m loop length is used. Figure 5 illustrates the group delay (above) and transmission (below) spectra when the attenuation was set at ~1.43 dB, corresponding to the case $a<\kappa$. As expected from the previous analysis, this configuration gives positive group delays (slow-light regime), peaking in the resonances. In Figure 6 the attenuation was increased until ~-1.7 dB which corresponds to the case $a>\kappa$ (fast light regime). In comparison of the response in Figure 5, the peaks show negative values referenced to the ~150 ns mean delay, corresponding to the expected fast-light response.

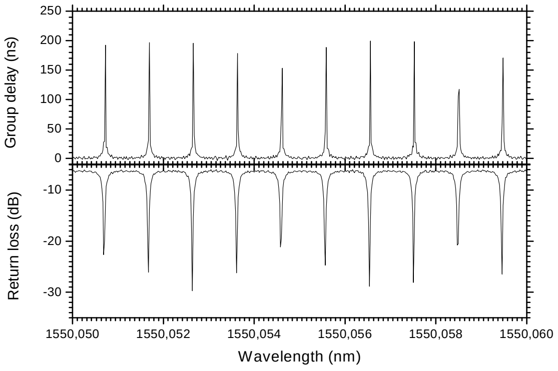

Fig. 5. Group delay and power transmission in the ring vs. frequency when ~1.43 dB attenuation is inside the ring.

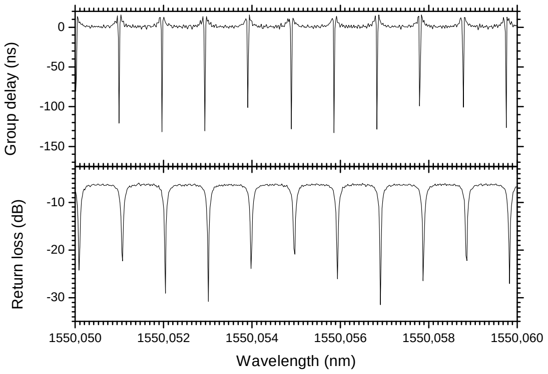

Fig. 6. Group delay and power transmission in the ring vs. when ~-1.7 dB attenuation is inside the ring.

In order to compare the theory with the experimental results, the group delay and transmission in the resonances have been tracked as the ring loss was varied by shifting the plates. The results are depicted in Figure 7 and 8. The dots are the experimental data and the lines correspond to the theoretical expectation obtained from the model developed in section 1. A good agreement can be seen between the results obtained experimentally and the theoretical model. At critical coupling (~1.55 dB attenuation), the transition between positive group delay and negative group delay is visible (slow-fast-light switch). Close to this point, the sensitivity to small absorption/attenuation is clearly visible, small absorption changes providing large transmission changes. It is important to highlight the enhanced loss sensitivity. In the following sub-section, we have taken advantage of this characteristic for its use in a fiber optic sensing application.

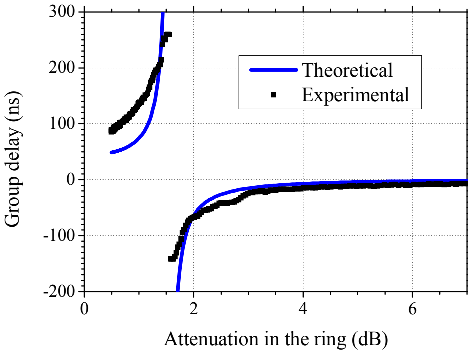

Fig. 7. Measured peak group delay as a function of the attenuation induced in the ring.

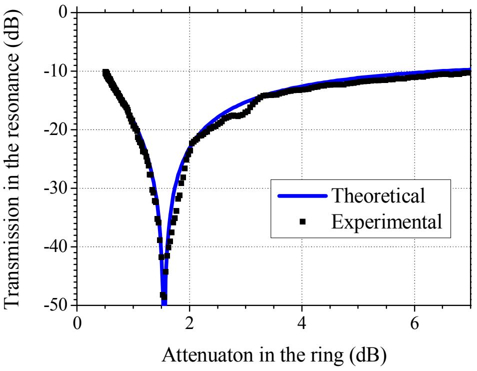

Fig. 8. Measured peak loss as a function of the attenuation induced in the ring.

B. *Enhanced sensitivity sub-micrometric fiber optic displacement sensor proposal*

One of the main objectives of this work is to provide a functional and practical application of the previous structure. The measurements of the previous section already show that strong absorption sensitivity can be achieved in this structure. Translating the previous results in terms of displacement resolution, with this structure it is possible to measure sub-micron displacements of the plates using a simple transmission measurement in the resonances. However, the use of an OVA for the interrogation of the structure is a very expensive solution. Thus, a functional and cost-effective solution for the interrogation is crucial. The main idea is to track the transmission spectral dips as a function of the displacement of the plates. Therefore, high resolution tuning of a laser is necessary to scan over several resonances. Linearity, stability, and narrow line-width of the laser is crucial to ensure that the transmission transfer function is correctly measured. In our case, a conventional external-cavity tunable laser with a line-with of 5 MHz is used as the sweeping source. The output power is detected by a simple narrowband photodetector. Figure 9 illustrates the results obtained with the proposed functional setup (black dots) in comparison with the OVA results (blue line). Full measurement range traces are depicted in order to compare the cost efficient setup and the OVA results. The results again clearly point out to a large sensitivity enhancement close to the critical coupling point. Small oscillations due to cladding mode coupling effects are visible in the tunable laser measurement. Other non-idealities of the results can also be due to the much larger linewidth of this source in comparison with the OVA source and the imperfect stability of the polarization. Still, the proposed setup shows reasonably high sensitivity around the critical coupling regime. In the particular case of the structure demonstrated in this work, the displacement sensitivity close to the critical coupling point exceeds 1 dB/µm (in the region between ~170 and ~200 µm). For comparison, this sensitivity value is more than two times the sensitivity achieved in [10] using the same bending plates in an interferometric structure. With low-cost acquisition methods, this sensitivity can easily allow sub-micron resolution in displacement values.

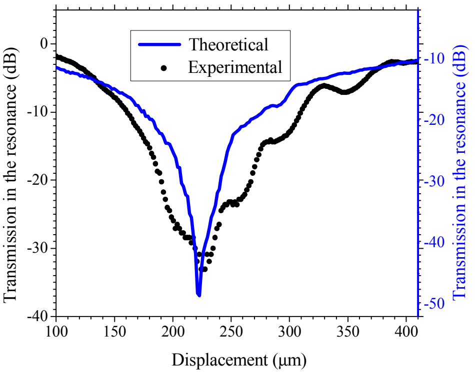

Fig. 9. Power transmission as a function of the displacement for theoretical and experimental measurements.

IV. CONCLUSIONS

A versatile all-fiber structure allowing high-resolution attenuation/absorption sensing based on a lossy fiber ring resonator has been theoretically and experimentally demonstrated. The theoretical and the experimental results show good agreement between them. Changing the attenuation or the coupling ratio of the ring, the group delay and the sensitivity of the structure is easily tuned. The proposed structure can be used for enhancing the sensitivity of different intensity-based sensors. We have also proposed a functional setup which interrogates a sub-micrometric displacement sensor based on bending losses in the fiber inside the ring. The proposed slow-light structure could be used for enhancing the sensitivity of different kinds of intensity sensors.

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