A Dynamic Model of COVID-19: Contagion and Implications of Isolation Enforcement

Miguel Casares
Universidad Pública de Navarra

Hashmat Khan
Carleton University

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Universidad Pública de Navarra

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A Dynamic Model of COVID-19: Contagion and Implications of Isolation Enforcement

Miguel Casares[*]

Universidad Pública de Navarra

Hashmat Khan[†]

Carleton University

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Comments Welcome

Abstract

We present a dynamic model that produces day-to-day changes in key variables due to the COVID-19 contagion: the number of ever infected people, currently infected, deaths, healed, and infected people who require hospitalization. The model is carefully calibrated to Spanish data and we conduct simulation exercises to study the role of isolation measures to contain the virus spread. We find that virus containment from isolation exhibits increasing returns. Our model simulations show that the State of Alarm intervention of the Spanish government on March 14th, 2020 reduces deaths by almost 85%, and lowers the maximum number of infected people who need daily hospitalization by a factor of 1/12. The simulations also indicate that both the timing and the intensity of the isolation enforcement are key for the evolution of the virus spread and the smoothing of the hospitalization needs.

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[*]Departamento de Economía and INARBE, Universidad Pública de Navarra, 31006, Pamplona, Spain. E-mail: mcasares@unavarra.es (Miguel Casares).

[†]Department of Economics and CMFE, Carleton University, Ottawa, ON K1S 5B6, Canada. E-mail: hashmat.khan@carleton.ca (Hashmat Khan).

1 Introduction

On March 11th 2020, the World Health Organization declared the Coronavirus Disease 2019 (COVID-19) outbreak a pandemic—a worldwide spread of the disease. Figure 1 shows that as of March 25th, there are 18,565 confirmed deaths due to COVID-19 worldwide and the six countries with over 800 confirmed deaths, namely, Italy (6820), China (3287), Spain (2696), Iran (1934), France (1100) and the US (801). The total number of confirmed cases is over 416,916 (Roser et al. (2020)).

Figure 1: **Total Confirmed Deaths as of March 25th, 2020.**
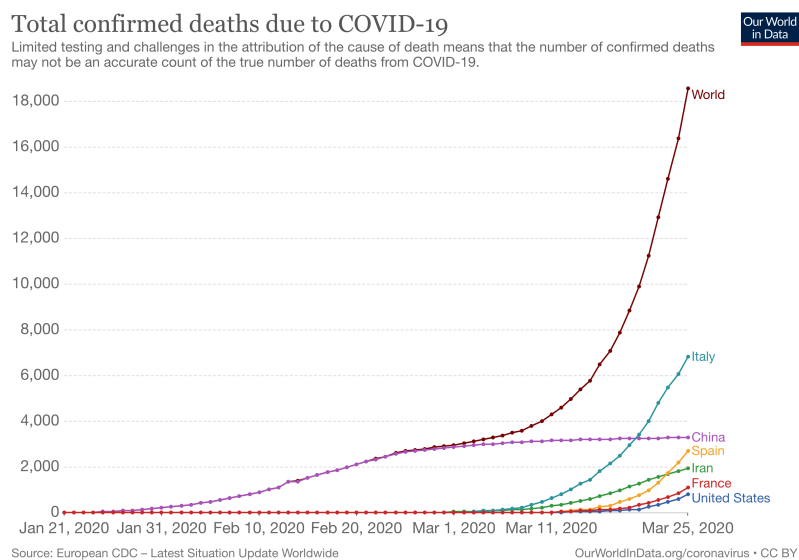
Unfortunately, the pandemic is still in progress unleashing a global health crisis and putting enormous pressure on health care systems. In addition to the travel related source of virus spread there is now a full-blown 'community spread' where the initial source of the infection remains unidentified. Governments and public authorities are implementing mandatory actions to contain the virus spread such as travel restrictions, lockdowns, closures of public spaces, institutions, and businesses, social (and physical) distancing, and self-isolation.

Drawing on the epidemiological SIR methodology, pioneered by Kermack and McKendrick (1927), we present a simple dynamic model to study the COVID-19 contagion. Even though the model is simple, it captures the main characteristics of the contagion process and provides insights valuable for policy orientation. The model can help understand the effects

of changes in medical and social policies. Due to its simplicity, the model may also be suitable for communicating the response of health and political authorities to the public.

As one clear-cut applied exercise, we calibrate the model parameters to Spanish data and present simulations to show the dramatic implications of enforcing mobility constraints over the COVID-19 spread in Spain.[1] Our paper is related to three recent contributions. Wang et al. (2019) estimates the evolution of the COVID-19 cases in Wuhan, China, while Atkeson (2020) investigates the impact of social distancing for the virus spread in the US, and Ferguson et al. (2020) analyze the impact of non-pharmaceutical interventions to contain the virus expansion in the UK. We use the global epidemiological data provided in Anderson et al. (2020) for the calibration of some of the model parameters.

2   Model description

For any given day t, we have the decomposition

$$N = x_t + z_t$$

where N is the total population on the arrival day of the first person infected by COVID-19, x_t is the accumulated number of people infected by COVID-19 on day t and z_t is the accumulated number of people never infected on day t.[2] On day 1, $x_1 = 1$ and $z_1 = N - 1$. For any future day t, the law of motion for x_t is

$$x_t = x_{t-1} + \alpha y \frac{\widetilde{x}_{t-1}}{N - k_{t-1}} z_{t-1} \tag{1}$$

that adds up to its value on the previous day, x_{t-1}, the number of newly infected people $\alpha y \frac{\widetilde{x}_{t-1}}{N-k_{t-1}} z_{t-1}$. In the latter term, $0 < \alpha < 1$ is the contagion probability on each encounter between one non-infected person and one infected person, $y > 0$ is the number of people each person meets per day, $\widetilde{x}_{t-1}$ is the number of people currently infected as of day $t - 1$,

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[1]While our focus is on studying the effects of immediate mobility controls in dealing with the ongoing health crisis, their unavoidable drastic effects on economic activity are underway. Eichenbaum et al. (2020) embed an epidemiological model in a macroeconomic general equilibrium model to study the tradeoff between the severity of decline in output and lives saved.

[2]We assume that the initial population N remains constant in this decomposition to consider that all deaths caused by the virus infection will determine the fatality rate of the virus. The same result would be obtained if there where no migration flows and the daily natality rate would be the same as the mortality rate not-related to COVID-19. Given the short time horizon of the analysis and the focus of the paper, we have decided to keep N fixed.

and k_{t-1} is the number of deaths caused by COVID-19 as of day $t-1$.[3] The ratio $\frac{\widetilde{x}_{t-1}}{N-k_{t-1}}$ provides the share of currently infected people with respect to the surviving population at the end of day $t-1$, which determines the probability of meeting someone infected. Thus, the product of the number of encounters by the rate of infected people, $y\frac{\widetilde{x}_{t-1}}{N-k_{t-1}}$, is the number of infected people every person meets on day t. Once we multiply it by the contagion rate on each encounter, we have $\alpha y\frac{\widetilde{x}_{t-1}}{N-k_{t-1}}$ as the effective daily contagion rate per person. The number of people who have *never* been infected at the end of day $t-1$ is z_{t-1}, and they are the potential newly infected people. Therefore, the second term on the right side of (1), $\alpha y\frac{\widetilde{x}_{t-1}}{N-k_{t-1}}z_{t-1}$ is the number of newly infected people on day t. It explains how the number of new cases depends on both the contagion rate α, and on the intensity at which the disease spreads in the matching between infected and non-infected individuals, $y\frac{\widetilde{x}_{t-1}}{N-k_{t-1}}z_{t-1}$.

The difference between x_t and the number of people (still) currently infected, $\widetilde{x}_t$ comes from the fact that the COVID-19 disease is neither chronic nor necessarily lethal. Let us assume, for simplicity and taking a realistic average, that the duration of the disease is T days and the incubation period is T_i days with $T_i < T$. Thus, T days after catching the virus the individual either recovers (with an associated survival probability $0 < 1 - \lambda < 1$) or dies (with an associated fatality probability $0 < \lambda < 1$). Both outcomes together reduce $\widetilde{x}_t$ by one. On day t, the number of infected people who will either get cured or die are $x_{t-T} - x_{t-T-1}$, i.e. those who were infected between day $t-T-1$ and day $t-T$. Therefore, the law of motion for the number of people currently infected by COVID-19 is

$$\widetilde{x}_t = \widetilde{x}_{t-1} + \alpha y\frac{\widetilde{x}_{t-1}}{N-k_{t-1}}z_{t-1} - (x_{t-T} - x_{t-T-1}) \tag{2}$$

where $\widetilde{x}_t$ is the number of people still infected on day t and $(x_{t-T} - x_{t-T-1})$ is the number of newly diagnosed cases between days $t-T$ and $t-T-1$ who drop out on day t as their disease outcome (either cured or death) is realized.[4]

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[3]For simplicity, the contagion probability α is both constant over time and identical for all meetings, which ignores heterogeneity in the meeting duration, the degree of physical contact, the viral load of the counterpart, etc. Hence, α is considered to represent contagion probability under average circumstances. We also assume the number of daily social contacts, y, is constant and exogenous, which must be interpreted as the behavior of the representative individual.

[4]Since the outcome of the disease is known T days after being infected, people who are excluded from the number of people holding the virus are the sum of those who survive and those who die:

$$(x_{t-T} - x_{t-T-1}) = \lambda\,(x_{t-T} - x_{t-T-1}) + (1 - \lambda)\,(x_{t-T} - x_{t-T-1})$$

With the model elements that have been introduced, we can also define the total number of deaths on day t as

$$k_t = k_{t-1} + \lambda \left(x_{t-T} - x_{t-T-1} \right)$$

where λ is the (constant and exogenous) fatality rate. Naturally, the total number of healed people, h_t, is

$$h_t = h_{t-1} + (1 - \lambda) \left(x_{t-T} - x_{t-T-1} \right)$$

Since $x_t = h_t + k_t + \sum_{j=0}^{T} \left(x_{t-j} - x_{t-j-1} \right)$, from $N = x_t + z_t$ we also get

$$N = h_t + k_t + \sum_{j=0}^{T} \left(x_{t-j} - x_{t-j-1} \right) + z_t \tag{3}$$

which means that total population, N, comprises four groups of people: those who have already healed, h_t, those who have already died, k_t, those who are infected with their outcome not yet known, $\sum_{j=0}^{T} \left(x_{t-j} - x_{t-j-1} \right)$, and those who have never been infected, z_t.

COVID-19 is an infectious virus that typically causes mild symptoms similar to the common flu, and only a minor fraction of infected people who test positive need hospitalization.[5] Nevertheless, the contagion rate of COVID-19 is very high and the capacity of hospitals to give treatment to sick people is severely constrained. In the model, we assume that a fraction θ of the infected people who have passed the incubation period, T_i, suffer from severe complications (typically, respiratory difficulties and pneumonia) and need hospitalization. Thus, the number of hospital beds, b_t, required to treat COVID-19 positive people on day t is

$$b_t = \theta \sum_{j=T_i}^{T} \left(x_{t-j} - x_{t-j-1} \right)$$

where $\sum_{j=T_i}^{T} \left(x_{t-j} - x_{t-j-1} \right)$ is the total number of infected people who have passed the incubation period on day t.

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[5]In fact, some of the people infected with COVID-19 are asymptomatic, which makes the spreading out of the epidemic more difficult to prevent and control by the health authorities.

To summarize, we have a dynamic system of 6 equations as follows:

$$x_t = x_{t-1} + \alpha y \frac{\widetilde{x}_{t-1}}{N - k_{t-1}} z_{t-1}$$

$$\widetilde{x}_t = \widetilde{x}_{t-1} + \alpha y \frac{\widetilde{x}_{t-1}}{N - k_{t-1}} z_{t-1} - (x_{t-T} - x_{t-T-1})$$

$$N = x_t + z_t$$

$$k_t = k_{t-1} + (1 - \lambda)(x_{t-T} - x_{t-T-1})$$

$$h_t = h_{t-1} + \lambda(x_{t-T} - x_{t-T-1})$$

$$b_t = \theta \sum_{j=T_i}^{T} (x_{t-j} - x_{t-j-1})$$

which, given initial values, determine the evolution of the 6 endogenous variables $\{x_t, \widetilde{x}_t, z_t, k_t, h_t, b_t\}$.

3 Model calibration for Spain

The baseline calibration is aimed at representing the outbreak of COVID-19 in a medium-size country. We take the case of Spain as one representative country and assume realistic values for the medical parameters that provide the epidemiological characteristics of COVID-19 based on Anderson et al. (2020). Table 1 provides the calibration values for the seven model parameters.

The total population is $N = 47$ million people to coincide approximately with the population of Spain in 2020. For the fatality rate, λ, we follow Anderson et al. (2020) who provide an estimated range of Case Fatality Rate (CFR) between 0.3% and 1% with reference to the

Table 1: Calibration of model parameters for Spain

1.	Population	$N = 47 \times 10^6$
2.	Fatality rate	$\lambda = 0.0075$
3.	Disease duration (days)	$T = 16$
4.	Incubation period (days)	$T_i = 5$
5.	Hospitalization rate	$\theta = 0.0528$
6.	Daily meetings per person	$y = 25$
7.	Contagion probability	$\alpha = 0.016$

data released by the World Health Organization.[6] Typically, the Infection Fatality Rate (IFR) (= confirmed COVID-19 deaths/(confirmed+unconfirmed cases)) is lower than the CFR as some of the cases are not reported because they are either asymptomatic or the tests have not been taken. Our model produces the IFR and this would recommend a lower value for λ than the range suggested by Anderson et al. (2020). However, Spain may experience a relatively higher IFR due to the population aging (in 2019 people over 75 years old represented 9.54% of the total population in Spain) and the much stronger severity of COVID-19 on the elderly.[7] Balancing out both arguments, we set $\lambda = 0.0075$ (0.75%), slightly above the median value of the range suggested by Anderson et al. (2020).

Anderson et al. (2020) report that the incubation period for COVID-19 is about 5 or 6 days and there is an average period of 10 days or more (longer than a common flu) of confrontation between the immune system and the virus.[8] Therefore, we set an average disease duration at $T = 16$ days and the incubation period to last for 5 days, $T_i = 5$.

Ferguson et al. (2020) estimate the COVID-19 hospitalization rate for the population of the Great Britain using a subset of cases obtained from China. Their estimate is set at 4.4%. For Spain, since the population has a higher fraction of elderly people than in either Great Britain or China, we set the hospitalization rate 20% higher at $\theta = 0.0528$ (5.28%).

The number of two-people encounters per day is subject to heterogeneity because it clearly depends on the specific social and economic characteristics of the individuals: type of job, social/leisure activities, age, etc, as well as on the social norms and habits of a country or territory. For the case of Spain, we set $y = 25$ meetings to represent an average behavior of a Spanish citizen, though recognizing the uncertainty and variance that affect this model parameter.

The contagion probability α measures the speed at which the virus spreads. Typically, this speed is calculated in the data with the time it takes for the infected people to pass on the infection to the same amount of people: the doubling time (also called serial interval). According to Anderson et al. (2020), COVID-19 is spreading more rapidly than the 2009 Influenza A H1N1 pandemic with a doubling time between 4.4 and 7.5 days (similar to SARS). In particular, COVID-19 is showing exponential growing patterns in Spain with doubling

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[6]Recently, Wu et al. (2020) have lowered the estimate of the case fatality risk (measured as the the probability of dying after developing symptoms) of COVID-19 in Wuhan to 1.4%.

[7]For comparative purposes, we found that in the UK the percentage of population over 75 years old was 8.29% in 2018.

[8]Anderson et al. (2020) cast some doubts about the length of the disease after the COVID-19 incubation as they say "*...perhaps lasting for 10 days or more after the incubation period*".

times of confirmed cases around 3.5 days and of deaths close to 2 days.[9] Hence, we search for a value of α required in model simulations to match the speed of 2 days of doubling time that has characterized the initial COVID-19 pattern of deaths observed in Spain.[10] This search determined setting $\alpha = 0.016$.

4   Simulations

We have programmed simulations in MatLab.[11] For initial values, we consider that on day 1, $t = 1$, there is one imported contagion and one person gets infected while the rest of the population had no virus, i.e. $x_1 = \widetilde{x}_1 = 1$. Then, we run the calibrated six-equation model forward over the next 200 days. Since COVID-19 is a seasonal virus, the effects observed after 200 days could tentatively be discarded due to a half-year seasonal change (from Winter to Summer) that would likely kill the virus due to warmer temperatures. Similarly, a future period of no further contagion could be considered if the COVID-19 vaccine were clinically tested successful and vaccinations could be administered to all the individuals.[12] In order to test the effects of policies aimed at people isolation and restrictions to mobility, we check the effects of having different values for y, i.e. the number of people each person meets per day.

Before commenting on the results, we briefly discuss the role of y in the dynamics of the model. If the number of individual contacts y is high there is also a high effective daily contagion rate per person, $\alpha y \frac{\widetilde{x}_{t-1}}{N-k_{t-1}}$, which accelerates the growth in both the accumulated number of infected people (Equation (1)) and the number of currently infected people (Equation (2)). The downward phase is also fast. As T days pass, there will be many infected people that will turn (most of them) healed and (some of them) will die. In both cases, these people

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[9]Ferguson et al. (2020) assume a doubling time of the confirmed cases of COVID-19 at 5 days.

[10]A direct observation of the contagion probability α is not possible because the incubation period of COVID-19 is typically long (5 or 6 days). Additionally, the contagion probability may depend on many circumstances such as the length of the meeting, the extent of physical contact and body proximity involved, the viral load of the transmitter, etc. These difficulties justify the criterion chosen to calibrate α based on the matching between *ex post* observations of model simulations and the data.

[11]The MatLab code written to carry out the model simulations is available upon request.

[12]An alternative model setup could have included $\overline{T}$ as the number of days after the first infected person from which the virus cannot spread out any further due to either climate conditions or a universal administration of vaccinations. This would imply $\alpha = 0$ if $t \geqslant \overline{T}$, and we could rewrite the low of motion for x_t and $\widetilde{x}_t$ as follows:

$$x_t = \left\{ \begin{array}{ll} x_{t-1} + \alpha y \frac{\widetilde{x}_{t-1}}{N-k_{t-1}} z_{t-1} & \text{if } t < \overline{T} \\ x_{t-1} & \text{if } t \geqslant \overline{T} \end{array} \right\}$$

$$\widetilde{x}_t = \left\{ \begin{array}{ll} \widetilde{x}_{t-1} + \alpha y \frac{\widetilde{x}_{t-1}}{N-k_{t-1}} z_{t-1} - (x_{t-T} - x_{t-T-1}) & \text{if } t < \overline{T} \\ \widetilde{x}_{t-1} - (x_{t-T} - x_{t-T-1}) & \text{if } t \geqslant \overline{T} \end{array} \right\}$$

are being excluded from the number of currently infected people (Equation 2). If there is a policy oriented to isolate people and y falls, these patterns of the virus spread become both slower and milder.

4.1 Reducing social interaction

Our first exercise illustrates the impact of reducing social interaction on the extent and length of the virus spread. We have simulated the model under three alternative values for the number of daily physical contacts among individuals: the value assigned in the calibration ($y = 25$) and two lower values that result from initially subtracting 8 daily meetings ($y = 17$) and additionally 8 more meetings subtracted ($y = 9$). Figures 2-4 and Table 2 show the results.

Figure 2: **Simulated series of COVID-19 infected people in Spain depending on social interaction**
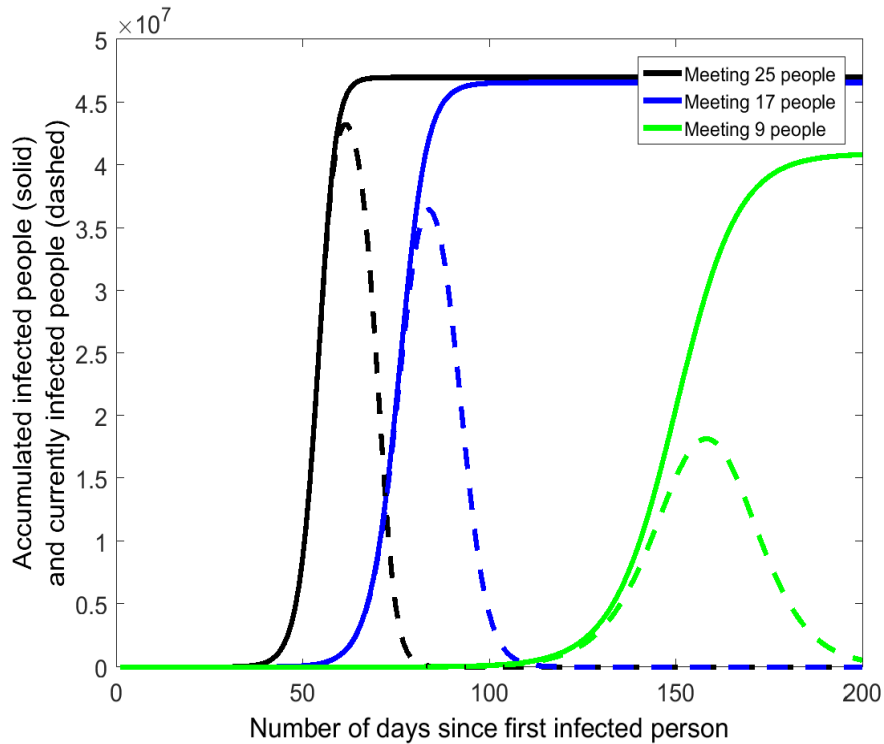
The case with the highest social interactions ($y = 25$ meetings) displays a fast and sharp contagion pattern. As Figure 2 displays, there is a rapid increase of infected cases around day 40 and the epidemic is over by day 80 (black line). Almost all the population get infected

(46.97 million). Table 1 reports 352 thousand deaths (0.75% of all the accumulated infected population) and the maximum number of people who need hospitalization on a single day is more than 2 million (observed on day 62). If we considered that the Spanish people have less social interaction and, therefore, a lower number of daily encounters at $y = 17$ meetings per individual, the contagion pattern is slower and less pronounced (dark blue line). Figure 2 shows the number of COVID-19 cases increasing around day 55, reaching a maximum daily value around day 85 and falling to close to zero levels by day 115. Still, the epidemic turns out to have very severe consequences, with most of the population getting infected (46.56 million) and 349 thousand deaths (see Table 2). The peak value of health coverage needs is reported on day 84 when nearly 1.6 million infected people must be hospitalized.

Figure 3: **Simulated series of COVID-19 hospitalization needs in Spain depending on social interaction**
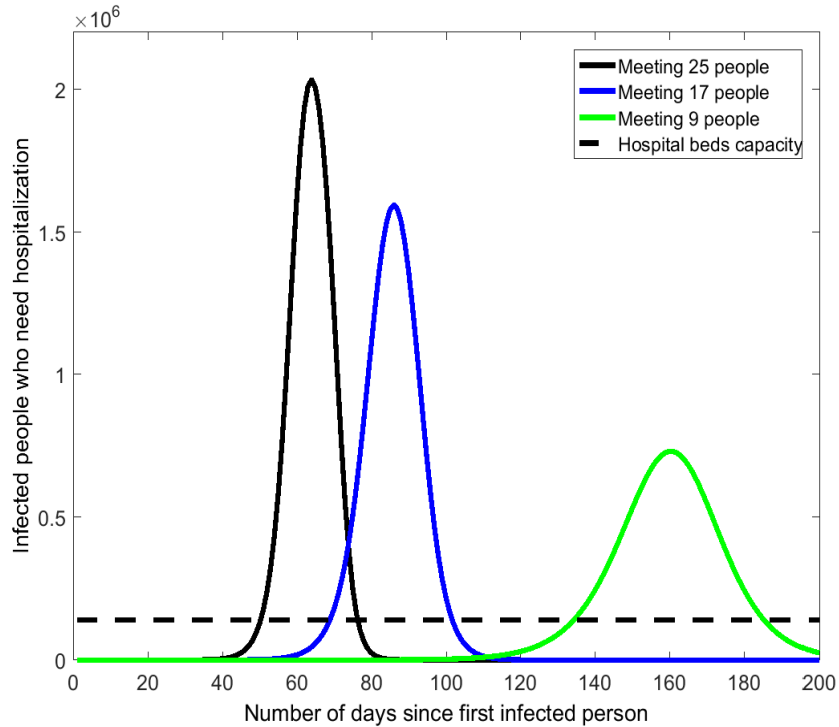
If we have a second same-sized reduction in the number of daily meetings to $y = 9$ in the model, the results show even a greater containment of the COVID-19 outbreak. The "*flattening of the curve*" is clearly observed in the green lines of Figure 2 both in terms of the slowing down in the pace of the total cases and also in the number of people who currently

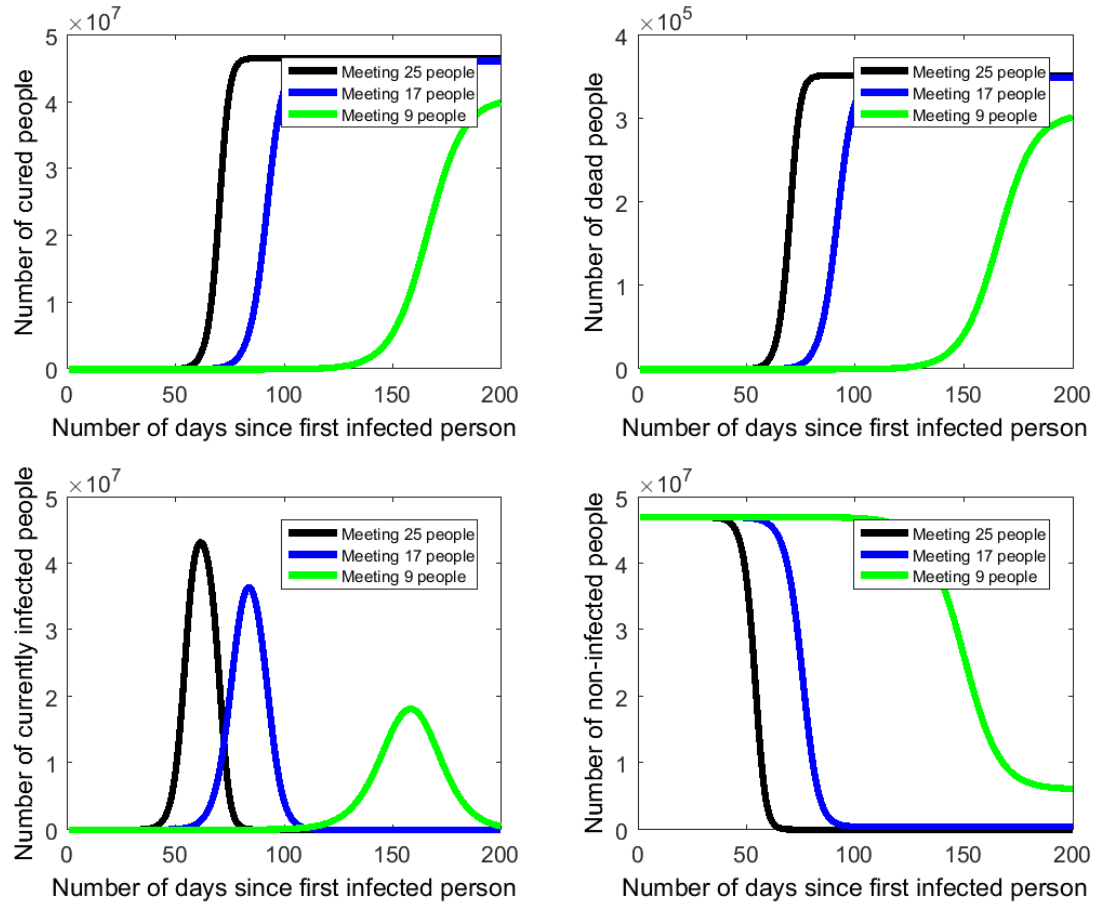Figure 4: **Distribution of the Spanish population following the COVID-19 outbreak depending on social interaction**

Table 2: Model simulation results with alternative social interaction regimes

	$y = 25$	$y = 17$	$y = 9$
• Accumulated infected people, millions	46.97	46.56	40.83
• Total deaths, thousands	352	349	302
• Daily peak of hospitalized people, thousands	2034	1596	731
Peak day	62	84	158

suffer the infection.[13] As reported in Table 2, we find still the number of deaths is very high at 302 thousand people and almost 41 million people (out of 47 million) get infected. The hospitalization needs turn clearly lower with a maximum of 731 thousand hospitalized people (around 1/3 the number reported when $y = 25$ and 1/2 of the number when $y = 17$). Notably, the slow down of the contagion process is really remarkable. The daily peak on the hospitalization of infected people takes place on day 158, which means 5 months after the start of the outbreak. It is quite likely that the climate conditions are favorable for the virus containment.

Figure 3 shows the needs for hospitalization and treatment depending on the three scenarios of social interactions. Spain has approximately 300 hospital beds per 100,000 people (below the EU average of about 372 beds), which brings an overall amount of around 141,000 units. We represent this as the horizontal line of hospitals bed capacity in Figure 3. The three scenarios of social interaction clearly give a number of people who need hospitalization that exceed the Spanish capacity (though the deficit is significantly smaller in the least social interaction case). This result indicates the urgent need of a policy action in Spain to reduce the number of daily meetings below $y = 9$ to prevent the health care system from collapsing.

Figure 4 illustrates the distribution of the initial population between the four possible daily states indicated in Equation 3): in the top left-hand side, the people who have already healed, h_t; in the top right-hand side, the people who have already died, k_t; in the bottom left-hand side, the people who are still infected, $\sum_{j=0}^{T} \left(x_{t-j} - x_{t-j-1} \right)$, and in the bottom right-hand side, the people who have never been infected, z_t. The impact of social distancing is very significant in the population allocation. As individuals meet less people, the speed of contagion is lower, people get infected later and, subsequently, people either recover or die later. Although there are fewer infected people as y is lower, the number of days with currently infected people is higher (see wider curves in the infected people cell of Figure 4).

Remarkably, all these effects are substantially more pronounced when social distancing switches from moderate to strict ($y = 17$ to $y = 9$) than from loose to moderate ($y = 25$ to $y = 17$). Therefore, we find increasing returns to isolation, which is confirmed with the numbers reported in Table 2. Hospitalization needs are reduced by 438 thousand (a 21.5% reduction) when the number of interactions is reduced to moderate. However, for the same number of reductions in interactions, a move from moderate to strict lowers hospitalization needs by 865 thousand (a 54.2% reduction).

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[13]See, for example, https://www.flattenthecurve.com/.

4.2 Policy intervention

On March 14th, 2020, the Spanish government declared the "State of Alarm" in response to the COVID-19 outbreak in Spain. The decree contemplated mobility restrictions, activity suspensions and home confinement for the population whose jobs were not related to either health care or basic needs. This is a natural response of most countries to the COVID-19 expansion worldwide. The calibrated model can capture the State of Alarm as one policy intervention that significantly reduces the number of physical contacts among citizens.[14]

This subsection analyzes the effects of the Spanish government intervention to alter social distancing following the COVID-19 outbreak. People gatherings for social and economic activities are quite common in Spain. The population density is moderately high at 93.53 inhabitants per square kilometer and slightly below the density in Europe.[15] Thus, we have initially characterized Spain in our calibration with a high degree of social interaction, $y = 25$, having each citizen 25 face-to-face contacts per day. In the model, once the COVID-19 infection arrives in Spain, we can simulate the effects of the State of Alarm intervention as a switch from $y = 25$ daily meetings per person to an isolation regime with $y = 3$ meetings.

The choice of the day in which the isolation is enforced can be crucial for the posterior extension of the disease (as we will document below). Thus, we paid special attention to selecting the day of our simulated series when the policy intervention took place. The State of Alarm decree was published on March 14th when the coronavirus death toll in Spain passed 100 people (121 confirmed deaths at the end of that day and 84 the day before). We have used this information in the calibrated model with $y = 25$ and searched for the day on which the number of deaths in the simulations surpasses the 100 threshold. This is day 46th (with 128 deaths in the simulation of the calibrated model), which we consider as the moment for the State of Alarm declaration in Spain.[16] We evaluate the impact of such intervention and two alternative timings as described in the following list:

- Four days earlier: day 42th with 33 deaths in the model (28 deaths in the Spanish data).
- State of Alarm: day 46th with 128 deaths in the model (121 deaths in the Spanish data).

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[14]It could be argued that a policy intervention of governments or health authorities following the COVID-19 outbreak would reduce the primary contagion probability α. Since the product $\alpha \cdot y$ determines the evolution of the newly infected people, a decrease of α would result equivalent to an increase of y. For example, setting $y = 3$ and keeping $\alpha = 0.016$ is equivalent to setting $y = 4$ and lowering α at $\alpha = 0.012$.

[15]Population density in the World is 14.7 persons per square kilometer. Europe (excluding Russia, Azerbaijan and Georgia) has a population density of 103 persons per square kilometer.

[16]On January 29, a German tourist tested positive of COVID-19 in La Gomera (Canary Island) who was the first confirmed infected person in Spain. Coincidently, if January 29th is considered day #1, the State of Alarm declaration day (March 14th) is day #46.

○ Four days later: day 50th with 489 deaths in the model (491 deaths in the Spanish data).

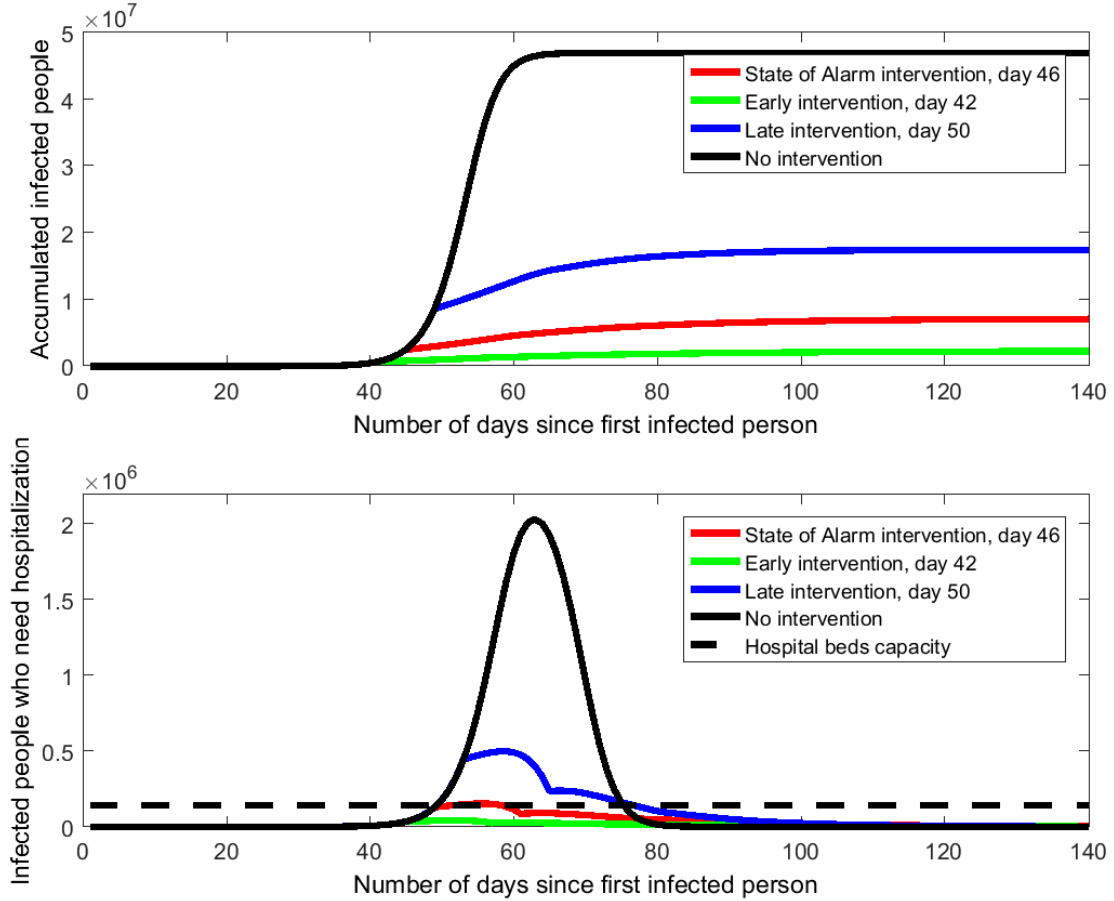Figure 5: **Alternative timings for the isolation policy in Spain following the COVID-19 outbreak**

Figure 5 and Table 3 show the results. The benchmark case for comparison is the "no intervention" scenario, keeping $y = 25$ which would lead to having almost all the Spanish people infected and over 350 thousand deaths.

The effects of the actual State of Alarm intervention is displayed as red lines in Figure 5. In comparison to the no intervention case (black lines), the curves of infected people and hospitalized people shift down and widen up as a clear example of the "flattening of the curve" pattern. Thus, the State of Alarm intervention of the Spanish government cuts by almost 85% the number of infected people (from 46.97 million to 7.16 million), while the maximum number of people who need daily hospitalization is reduced by 92% relative to

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the no intervention case (from slightly more than 2 million people to 153.6 thousand people). Although the numerical effects present a sharp contrast, the slowdown in the virus expansion is not observed. The peak day of hospitalization needs is slightly earlier (day 56 with respect to day 62). The reason is that there is a tradeoff between the increasing pattern on the number of people who got infected less than $T = 16$ days ago and the decreasing pattern of the number of new people who get infected. The former goes up because the daily change in new infected people was increasing over the days before the State of Alarm, while the latter goes down because not-infected people are less likely to meet the virus due to the isolation regime. The trade-off is favorable to the pressure of formerly infected people from day 46 to day 56 and from that time onwards the curve bends down.

Figure 6: **Estimated effects of the State of Alarm intervention on the number of COVID-19 infected people who require hospitalization**
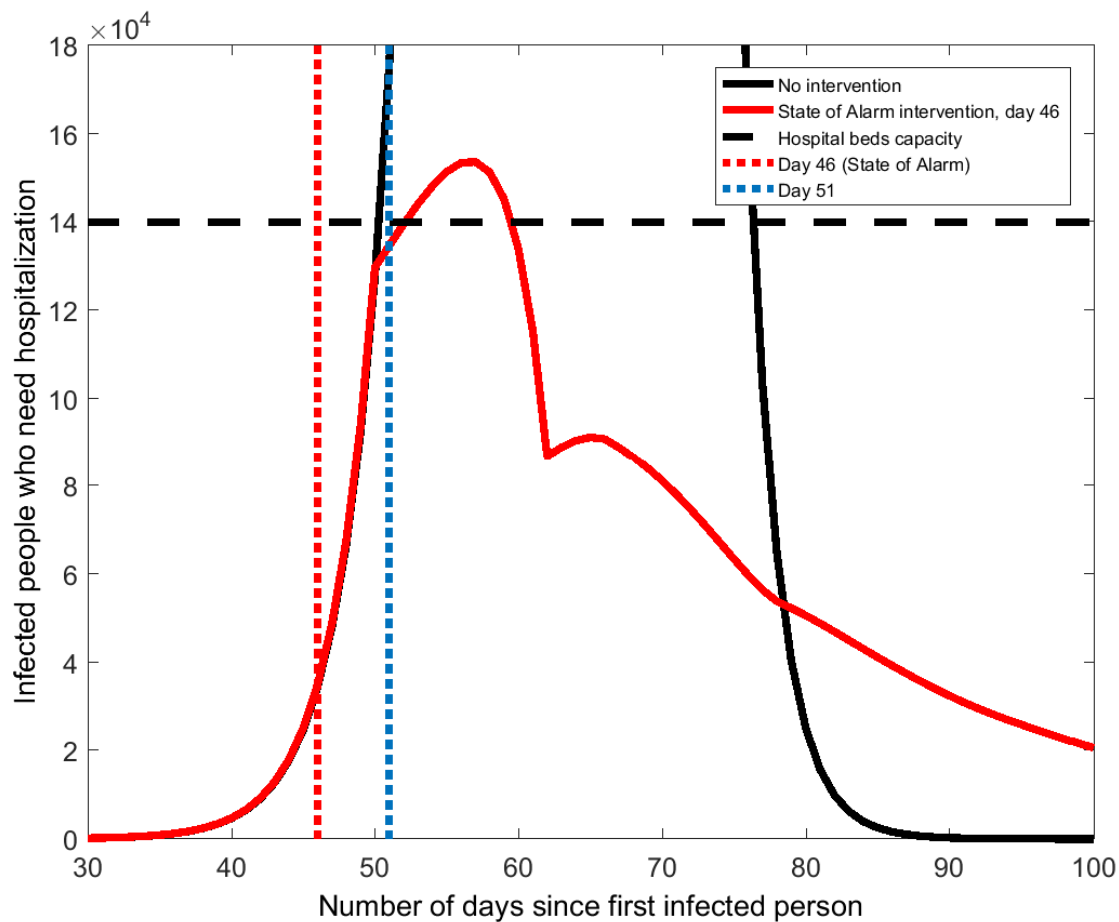
Table 3: Simulation results of the timing of social distancing in Spain

	No intervention	Day 42	Day 46 (S. of A.)	Day 50
• Accumulated infected people, millions	46.97	2.32	7.16	17.48
• Deaths, thousands	352	17.4	53.7	131.1
• Daily peak of hospitalized people, thousands	2034	42.2	153.6	499.0
Peak day	62	52	56	59

For a better look of the red line of Figure 5, the reader can look at the red line of Figure 6 which is a substantial zoom-in of the State of Alarm effects. The first five days of the State of Alarm show no apparent deviation in the number of people who need to be hospitalized from the no intervention scenario because the reduction in the newly infected people is not noticed before the end of the incubation period (5 days). Precisely, it is day 51 when the slope of the curve flattens as the change in there is a fall in the number of newly infected people who have developed symptoms and need hospitalization. The model also shows that there will be seven days (from day 52 to day 58, both included) of a deficit of hospitalization capacity because the number of people who need to be hospitalized is greater than the 141,000 hospital beds available in Spain. The downward phase is fast for some days after the peak day but it turns slower on day 62 onwards (coinciding with the 16 day duration of the infection we have assumed in the calibraton). To illustrate the slow pace on the recovery path predicted by the model, on day 100 the simulation indicates that there are around 20,000 infected people treated at the hospitals (13% of the number of people found on peak day).

A 4-day earlier intervention (day 42) would have been prevented many infections and reduced the number of deaths and the hospitalization needs (see the flattening and pushing down of the green lines in Figure 5 relative to the red line). Numbers reported in Table 3 support an important point on undertaking early action. If the social distancing enforcement would have been passed just 4 days earlier the model produces a reduction by over 65% in the number of infected people and deaths. Moreover, the number of required hospitalizations drops from 153,600 to 42,200 which can be totally covered by the Spanish health care system.

The 4-day postponement of the intervention to day 50 would increase infected people and deaths by a factor higher than 2. The situation would have been catastrophic for the health assistance of nearly half a million people people who need medical treatment when this number is more than 3 times the Spanish hospitalization capacity.

In short, the simulation results indicate that the choice of the day for setting the enforce-

ment of social distancing has critical consequences on the evolution of the virus spread.

Figure 7: **Alternative intensities for the isolation policy in Spain following the COVID-19 outbreak**
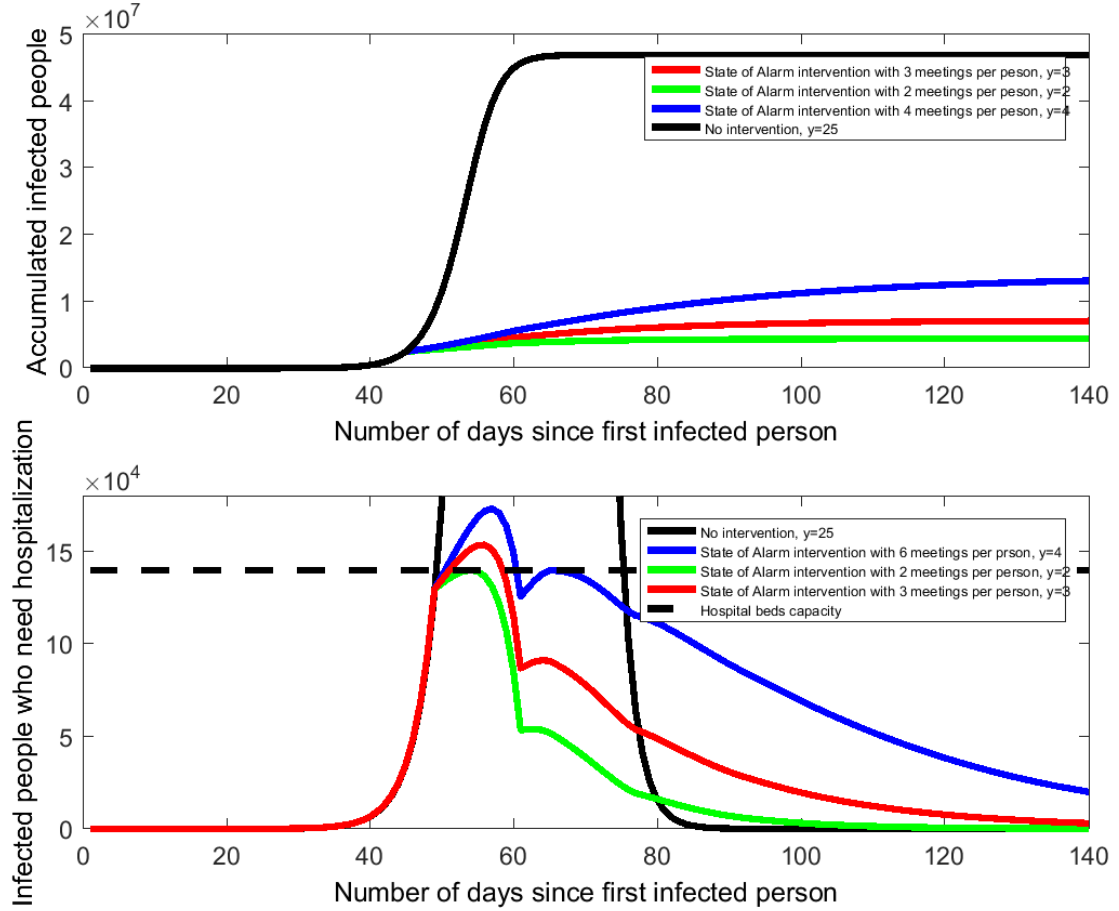
Finally, we examine the effects of different degrees of intensity of the social distancing action taken by the Spanish government. Figure 7 and Table 4 document the sensitivity of the results to assuming either a stronger or a weaker enforcement in the State of Alarm. Thus, we compare the cases of $y = 2$ (more intensity on isolation) and $y = 4$ (less intensity on isolation) to the calibrated setting of $y = 3$ for the State of Alarm procurement. Once again, the quantitative effects are very large. The numbers reported in Table 4 indicate that only reducing the State of Alarm enforcement in one more meeting would produce an estimated decrease in the number of deaths by 38% and in the peak number of people who need hospitalization by 9% (which would place the curve always below the capacity line as shown

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Table 4: Simulation results of the intensity of social distancing in Spain

	No intervention	$y = 2$	$y = 3$ (S. of A.)	$y = 4$
• Accumulated infected people, millions	46.97	4.40	7.16	13.75
• Deaths, thousands	352	33.0	53.7	103
• Daily peak of hospitalized people, thousands	1902	140.0	153.6	173.2
Peak day	60	54	56	77

in Figure 7). By contrast, a looser implementation of the State of Alarm with an increase of daily meetings to $y = 4$, would have a very important cost in human lives (number of deaths would almost double) and on the number of people who need hospitalization (on peak day around 20,000 more people). Actually, the health care system would be on the verge of collapsing because for 10 consecutive days (from day 51 to day 60, both included) that would require more hospital beds than the installed capacity.

A caveat to bear in mind is that the quantitative results obtained from the simulations of the model are sensitive to the choice of the calibrated parameters. Moreover, there might be changes in the early patterns of the COVID-19 spread which recommends some recalibration of the model parameters. For example, the spread speed is, fortunately, showing signs of decreasing in the latest available data from Spain: the 2-day doubling time for deaths that had been informed at the beginning of the spread (which we used as the criterion in the calibration of the contagion probability of the model) is rising to 3 days.[17] Thus, this change could be incorporated into the model prediction as a reduction of the calibrated value of the primary contagion probability, α, which could be capturing the response of Spanish citizens to the State of Alarm health recommendations to wear a protective gear (masks, gloves) when exposed to physical contacts. The slow down in the pace of the contagion would recommend a substantially downward revision on the estimates of the numbers of infected people, deaths and hospitalized people after the State of Alarm declaration in Spain (Tables 3 and 4). Some guidance on the numerical effects under this scenario of a lower contagion rate is reported in the column of Table 4 with lower meetings per person, $y = 2$, which would replicate the equivalent results of moving down α from $\alpha = 0.016$ to $\alpha' = 0.01067$ keeping the State of Alarm average meetings per person at $y = 3$.[18] Together with the quantitative effects under

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[17]The latest observation of accumulated deaths, released on the afternoon of March 25th is 3,434 deaths, which implies a 3-day doubling time because on March 22nd there were 1,720 deaths officially reported.

[18]The equivalency can be proved by having the same calibration for the $\alpha \cdot y$ product at $\alpha \cdot y = 0.016 \cdot 2 =$

alternative scenarios, the qualitative results of our empirical simulations clearly show the significant effects of different decisions of social interaction and policy actions to contain the COVID-19 spread.

5 Conclusions

We presented a dynamic model of the COVID-19 spread. The model provides information on six variables relevant to the ongoing containment efforts. We carefully calibrate the parameters of the model to Spanish data. A key advantage of the model is that it can provide quantitative answers to many ongoing containment efforts. Three general results emerge from the simulations. First, isolation efforts significantly slow down the speed of the contagion. Second, isolation reduces both the total number of people infected and deaths. Third, isolation exhibits increasing returns. The model provides a clear interpretation of the forces that produce the "flattening of the curve" of infected people towards the maximum capacity of the health care system.

In the simulation exercises conducted to examine the COVID-19 spread in Spain, the calibrated model shows that the actions of social distancing have tremendous effects on the evolution of the disease. According to the model results, no intervention to reduce people face-to-face interactions would condemn almost all the population to get infected and more than 350,000 Spanish people would die. The State of Alarm, characterized in the model by a reduction in the number of face-to-face contacts among individuals from 25 to 3 per day, is estimated to cut the number of deaths by 85% and the number of hospital beds needed by 92%. Significantly larger cuts would have been found with an earlier policy intervention or with a tighter social distancing action.

Finally, our model can be calibrated to other countries' data to quantify the impacts of isolation measures in the face of the COVID-19 pandemic.

$0.01067 \cdot 3 = 0.032$.

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