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A Strategic Multistage Tactical Two-Stage Stochastic Optimization Model for the Airline Fleet Management Problem.

Adrian Serrano-Hernandez[a,b,*], Luis Cadarso[a], and Javier Faulin[b]

[a] *European Institute for Aviation Training and Accreditation (EIATA), Rey Juan Carlos University, Fuenlabrada, Madrid, 28943, Spain*

[b] *Institute of Smart Cities, Public University of Navarre, Pamplona, 31006, Spain*

Abstract

This work proposes stochastic optimization for the airline fleet management problem, considering uncertainty in the demand, operational costs, and fares. In particular, a multistage tree is proposed, compounded of strategic and tactical nodes. At the former ones, fleet composition decisions are made, while at the latter ones, aircraft assignment decisions are formulated. Computational experiments are based on a small air network with seven strategic nodes and fourteen tactical nodes (i.e., seasons) where two fleet types are available to be included: Airbus 320, and Boeing 737. These results provide the optimal fleet planning and assignment at both strategic and tactical scopes. Finally, it is shown the superior performance of the stochastic version of this problem against the deterministic one.

Keywords: Airline fleet management problem; stochastic optimization; mixed integer linear programming

1. Introduction

The Airline Fleet Management Problem consists of different subproblems which feature different planning horizons and decisions. One of the most important decisions to be made by an airline is the fleet mix to be used in its flight schedules. This decision mostly fixes the supply side of the system, and therefore, it heavily constrains revenues. The Airline Fleet Planning problem, i.e., deciding about fleet composition, comprises a long planning horizon, even decades, and, roughly speaking, consists of the following description: decide when, which, and how many aircrafts to

* Corresponding author. Tel.: +34-948 16 9213. *E-mail address:* adrian.serrano@unavarra.es

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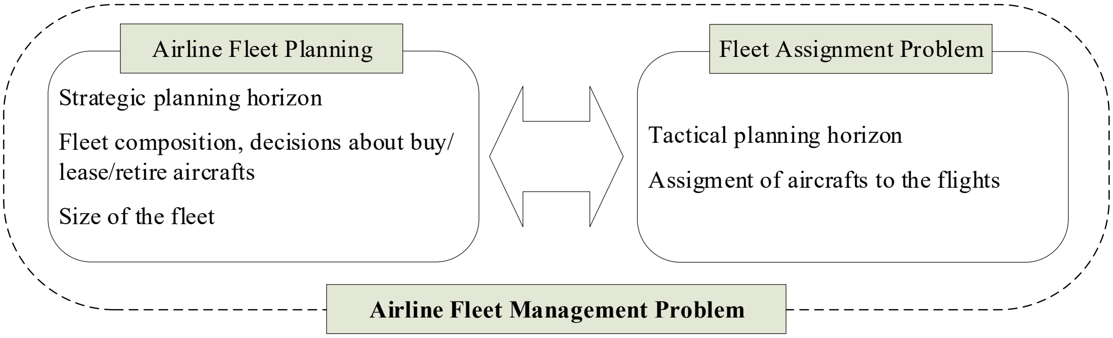
buy or lease, and also when, which, and how many of them have to be retired. Note that for such long-time periods, uncertainty is prominent (e.g., the socio-economic situation may fluctuate, making unknown demand figures, fares and flight schedules). The other important subproblem is the Fleet Assignment Problem, which consists of allocating aircrafts to flights in schedule while matching expected passenger demand and maximizing profits. This subproblem features a shorter planning horizon, e.g., several months. Figure 1 shows a visual description of these problems. Although both subproblems are characterized by different planning horizons (i.e., strategic and tactical) and decisions, they are heavily interrelated, and once they are solved, most of revenues and profits are also fixed. It is well-known that the airline strategic decisions related to fleet management have a huge impact on their profits: assigning a smaller aircraft implies some passengers are unable to travel and, on the other hand, a greater aircraft may suppose empty seats.

Figure 1. The Airline Fleet Management Problem



Scientific literature has addressed the topic from many points of view. This past research has long focused on the Fleet Assignment Problem, defining a convenient objective function (Dumas et al., 2009), developing efficient algorithms to solve such a complex problem (Yan et al. 2008), or investigating the dynamic nature of the problem (Jiang and Barnhart, 2009). The stochastic nature of the problem has also been investigated in Cadarso and de Celis (2017), and robustness issues in Jiang and Barnhart (2013). However, the Airline Fleet Planning problem has received less attention, and the integration of the two subproblems, namely Airline Fleet Management Problem, even less. Note that both depend on each other, and deciding in an isolated way, on any of them, may produce overall suboptimal plans. Closer to our research, Safak et al. (2018) propose a mixed integer three stage stochastic nonlinear programming model for the airline scheduling problem. There, the stochasticity is placed on the passanger demands whilst it is considered non-cuise times as well.

Therefore, we propose a strategic multistage tactical two-stage stochastic optimization model for the Airline Fleet Management Problem to fill the gap in the research literature. The model considers stochasticity in various parameters, i.e., demand, operational costs, and fares, and decides on fleet compositions and assignments (the Airline Fleet Planning and the Fleet Assignment Problem). This modelling approach allows the airline to respond and adapt to the changes on its environment, which features the aforementioned uncertainties, in such a way its profits are maximized.

2. Methodology

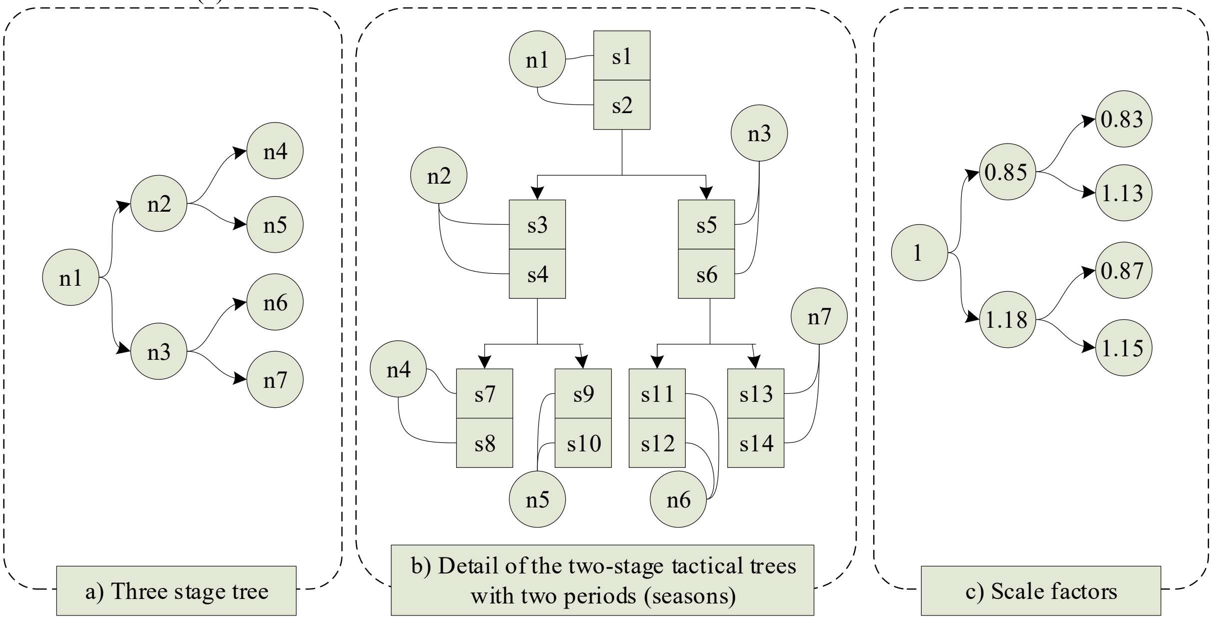
In this work, we consider a stochastic optimization model to cope with the uncertainties proposed (Cadarso et al. 2018), i.e. demand, operational costs, and fares evolution. In this sense, a multistage tree is proposed, compounded of strategic and tactical nodes. A full description of stochastic optimization is provided in Shapiro et al. (2009).

At strategic nodes, fleet composition decisions are made, while at tactical nodes aircraft assignment decisions are formulated. Figure 2a shows a three-stage tree featuring seven strategic nodes, composed of two tactical nodes or periods each of one, namely winter and summer seasons, as shown in Figure 2b. The selection of the number of stages and nodes depends on the problem characteristics. Firstly, the stage accounts for a sufficiently long time horizon in which the uncertain parameters will not change (in our case, the parameters related to the economic evolution).

Secondly, a (strategic) node is where the (strategic) decisions are made, based on the actual realization of the uncertainties (that is, the decisions related to the airline fleet planning depends on the economic evolution). Additionally, our problem considers also tactical nodes (i.e. the periods) in which the tactical decisions are made (in this case, the decisions related to the fleet assignment problem, but taking into account we are still in the same strategic node so the strategic decision is the same for both periods).

Demands, fares and costs evolutions are branched according to pessimistic (branches on the top) and optimistic (branches on the bottom) economist behavior. The optimist economic behavior correspond to economic growth prices go up (inflation) and the flights demand increases due to higher disposable income; and vice-versa, pessimistic economic behavior leads to recession, deflation and reduction of flights demand. This is performed using the scale factors proposed in Figure 2c. It starts from an initial node that represents the initial period (scale factor equals to 1). On the contrary, for the successive periods, the ancestor node is branched, and the parameters of the problem are rescaled. In addition, the scale factors are cumulative, for instance, for the fourth strategic node the parameters will be scaled by 0.85 and by 0.83 (70.55% of the initial values). Finally, the model is complemented by an additional scale factor according to the summer season, in which parameters are 20% higher than in the winter one. Additional details are provided in the '*parameter setting subsection'*.

Figure 2. Three-stage tree (a) with two tactical nodes on each strategic node (b) and the scale factors for the economic evolution (c)



The strategic multistage tactical two-stage stochastic optimization model can be formulated as a mixed integer linear programming model with the sets, parameters, and variables defined in the Tables 1, 2 and 3, respectively.

Table 1. Sets descriptions

Set	Description
F	Set of fleets $f \in F$
L	Set of flights $l \in L$
I	Set of consolidated nodes $i \in I$
It	Set of flight itinerary $it \in It$
N	Set of strategic nodes $n \in N$
S	Set of tactical seasons $s \in S$
G	Set of ground arcs $g \in G$
$ArcsAirOut_{lifs}$	Subset of flights $l \in L$ departing from the node $i \in I$ using aircraft $f \in F$ in the season $s \in S$

$ArcsAirIn_{lifs}$	Subset of flights $l \in L$ arriving to the node $i \in I$ using aircraft $f \in F$ in the season $s \in S$
$ArcsGroundOut_{gis}$	Subset of departure ground arcs. It is defined as the ground arc $g \in G$, outcoming from node $i \in I$ in the season $s \in S$
$ArcsGroundIn_{gis}$	Subset of arrival ground arcs. It is defined as the ground arc $g \in G$, incoming to node $i \in I$ in the season $s \in S$
$GActive_{gfs}$	Subset of ground arcs $g \in G$ that cross the counting line in any season $s \in S$ using any fleet $f \in F$
LS_{ls}	Subset of flights $f \in F$ to be made in each season $s \in S$
NS_n	Subset of seasons $s \in S$ belonging to strategic node $n \in N$
$PrecedeNode_{n-1,n}$	Subset of any node $n \in N$ and its ancestor
M_l	Subset of mandatory flights $f \in F$
NM_l	Subset of nonmandatory flights $f \in F$

Table 2. Parameters descriptions

Parameter	Description
$airplaneCost_f$	Daily cost of owning the aircraft $f \in F$
$rentCost_f$	Daily cost of renting (leasing) aircraft $f \in F$
$\delta_{it,l}$	1 if itinerary $it \in I$ belongs to flight $l \in L$
$fare_{it,s}$	Average fare for $it \in I$ in the season $s \in S$
$dit_{it,s}$	Demand for itinerary $it \in It$ in the season $s \in S$
c_{lfs}	Operation cost for flight $l \in L$ and aircraft $f \in F$ in the season $s \in S$
cap_f	Capacity (number of seats) of aircraft $f \in F$
dl_{ls}	Demand for flight $l \in L$ in season $s \in S$
pax_{lfs}	Passengers in flight $l \in L$ and aircraft $f \in F$ in the season $s \in S$
R_s	Revenue in the season $s \in S$ such that $R_s = fare_{lf,s} dit_{it,s}, \forall s \in S$
w_n	Weight (probability) of node $n \in N$

Table 3. Variables descriptions

Variable	Description
Z_n	Profits obtained in strategic node $n \in N$
X_{lfs}	1 if flight $f \in F$ is operated by aircraft $f \in F$ in the season $s \in S$
Y_{gs}	Number of aircrafts in ground arc $g \in G$ in the season $s \in S$
BA_{fs}	Number of aircrafts $f \in F$ owned by the airline in the season $s \in S$
RA_{fs}	Number of aircrafts $f \in F$ leased by the airline in the season $s \in S$
$P_{it,s}$	Unattended demand for itinerary $it \in It$ in the season $s \in S$

Note that we defined the set I as the set of consolidated nodes (please, do not confuse these nodes with the strategic/tactical nodes): each departure and arrival could be treated as a node, and the connections between nodes would be the air and ground arcs. However, if the model were treated according to this criterion, the number of nodes and arcs would be too high. But, applying the consolidation of nodes, it is possible to considerably reduce the number of nodes and arcs. It is also convenient to remark that any itinerary is made of one (direct) or more flights (with stops) in such a way a number of flights may produce a much higher number of itineraries.

All in all, the mixed integer linear problem consists of the following:

$$Max \sum_{n\in N} w_n \, Z_n \tag{1}$$

Such that,

$$Z_n = \sum_{s\in NS} \left[R_s - \sum_{\substack{l\in LS \\ f\in F}} c_{lfs} \, X_{lfs} - \sum_{f\in F} airplaneCost_f \, BA_{fs} - \sum_{f\in F} rentCost_f \, RA_{fs} - \sum_{it\in It} fare_{it,s} \, P_{it,s} \right], \forall n \in N \tag{2}$$

$$\sum_{f\in F} X_{lfs} = 1, \forall l \in M, s \in S \tag{3}$$

$$\sum_{f\in F} X_{lfs} \le 1, \forall\, l \in NM, LS, \; s \in S \tag{4}$$

$$\sum_{\substack{l \in arcsAirIn \\ l \in LS}} X_{lfs} + \sum_{g \in arcsGroundIn} Y_{gs} = \sum_{\substack{l \in arcsAirOut \\ l \in LS}} X_{lfs} + \sum_{g \in arcsGroundOut} Y_{gs}, \forall i \in I, s \in S \quad (5)$$

$$\sum_{g \in gActive} Y_{gs} \leq \sum_{\substack{n,n-1 \in PrecedeNode \\ n,n-1 \in NS}} BA_{f,s-1} + \sum_{n \in NS} RA_{fn}, \forall f \in F, s \in S \quad (6)$$

$$BA_{f,n-1} \leq BA_{fn}, \forall f \in F, n \in PrecedeNode \quad (7)$$

$$RA_{f,n-1} \leq RA_{fn}, \forall f \in F, n \in PrecedeNode \quad (8)$$

$$\sum_{it \in It} \delta_{it,l} P_{its} \geq dl_{ls} - \sum_{f \in F} cap_f \; X_{lfs}, \forall l \in L, s \in S \quad (9)$$

$$P_{it,s} \leq dit_{it,s}, \forall it \in It, s \in S \quad (10)$$

Where objective function (1) maximizes the auxiliary functions Z_n, weighted by the probability of each strategic node $n \in N$. These Equations (2) consist of the profit of the airline considering the revenues and the expenses for each strategic node. Constraints (3) impose compulsory flight operations, whereas Constraints (4) establish the noncompulsory flight. Constraints (5) state for the balanced behavior of the flights, that is departures and arrivals are coherent. Constraints (6) model the availabilities of aircrafts during the time horizon. That is, if the airline buys an aircraft, it will not be available up to the next season, whereas if the airline leases it, it will be immediately available. Additionally, Constraints (7) and Constraints (8) say that bought/leased aircrafts in any node $n \in N$ should be at least the same as the ones in the following node, as a consequence of the Equations (6). Equations (9) define the unattended passengers, and the Constraints (10) constraint the unattended demand to be lower than the actual demand.

3. Computational results

Parameter setting

Computational experiments are based on a small air network where two fleet types are available to be included: Airbus 320, which features 144 seats, and a Boeing 737, which features 108 seats. Seven strategic nodes and fourteen tactical nodes (i.e., seasons) are considered (see the Figure 2b). Note that strategic nodes may represent a year or a set of years if the socio-economic situation is not considered to vary significantly.

The selected air network is described in Table 4 which accounts for 11 flights and 19 itineraries covering the airports of Madrid (MAD), Seville (SVQ) and Barcelona (BCN), in Spain. In the Table 4, penultimate column shows the operation cost of operating the flight using an Airbus 320 (A) or a Boeing 737 (B). Additionally, flights 403 and 503 correspond to optional itineraries that are offered during the summer. Finally, we consider the following prices for the aircrafts: M€95 (€2054.8 on a daily basis for a 10-year useful life) the Airbus 320 and M€ 75 (€2602.7 daily for a 10-year useful life) the Boeing 737; with a daily leasing cost of €4000 for the Airbus 320 and €5200 the Boeing 737. Note that all costs elements as well as the demands and fares are referred to the initial node, so these will evolve according the factors proposed in the Figure 2c) depending on the economic evolution. Finally, we propose equiprobability for the strategic nodes that belong to the same stage in the tree, thus $w_{n1} = 1$; $w_{n2} = w_{n3} = 0.5$; $w_{n4} = w_{n5} = w_{n6} = w_{n7} = 0.25$. The last column gives M for a mandatory flight and O for an optional one. The sources for the data are diverse. Firstly, the air network (that includes flights, itineraries, fares, demand, and operational costs) was provided by a local airline as a case study; secondly, aircraft prices are available from Airbus and Boeing websites.

Results

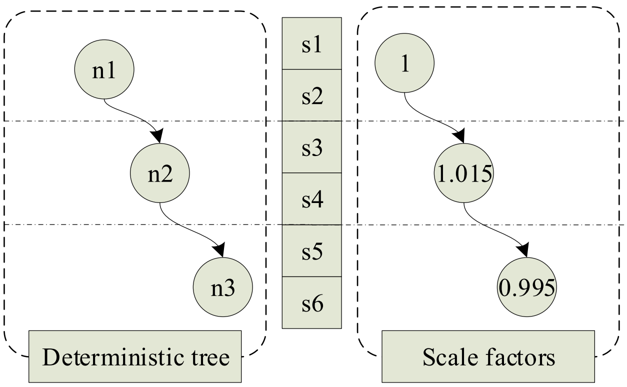
The mathematical model is coded in GAMS® and solved using CPLEX® on an Intel® Core™ i5-3570 CPU @ 3.40 GHz with 8 RAM GB.

As the Airline Fleet Management Problem is compounded of the Airline Fleet Planning and the Fleet Assignment Problem (see Figure 1), the results are also provided in the context of these two subproblems. Nevertheless, note that both subproblems are solved in an integrated approach. Additionally, since we are dealing with stochastic programming, there will be a deterministic equivalent model to be compared with, as shown in Figure 3. In that deterministic tree, we do not have uncertainty in the realization of the stochastic parameters. Therefore, it just shows one path, from n1 to n3 leading to a unique scenario. In this case, still the strategic nodes have two seasons each and the scale factors are the expect values from the stochastic model.

Table 4. Description of the air network used in this work

Itiner.	Flight(s)	Origin	Stop	Destination	Fare (€)	Demand (PAX)	Operation Costs (€)	M/O
1	101	MAD	-	SVQ	110	115	A=5000/ B=3000	M
2	102	SVQ	-	MAD	115	79	A=4500/ B=2500	M
3	201	SVQ	-	BCN	220	88	A=7000/ B=5000	M
4	202	BCN	-	SVQ	200	93	A=6900/ B=4900	M
5	301	BCN	-	MAD	100	85	A=4000/ B=2000	M
6	302	MAD	-	BCN	120	65	A=5000/ B=3000	M
7	401	BCN	-	SVQ	200	70	A=7200/ B=5200	O
8	402	SVQ	-	MAD	115	90	A=6500/ B=4500	O
9	403	MAD	-	BCN	120	85	A=4600/ B=2600	O
10	501	BCN	-	MAD	100	100	A=4500/ B=2500	O
11	502	MAD	-	SVQ	110	50	A=5000/ B=3000	O
12	503	SVQ	-	BCN	220	105	A=7800/ B=5800	O
13	102- 302	SVQ	MAD	BCN	200	52		
14	301- 101	BCN	MAD	SVQ	180	95		
15	501- 502	BCN	MAD	SVQ	150	40		
16	401- 503	BCN	SVQ	BCN	280	39		
17	301- 403	BCN	MAD	BCN	170	85		
18	102- 403	SVQ	MAD	BCN	210	90		
19	302- 202	MAD	BCN	SVQ	250	105		

Figure 3. The deterministic equivalent model



Firstly, the solution of the Airline Fleet Planning problem is displayed in Table 5 (the stochastic solution on the left and deterministic one on the right). As it can be observed, the solutions are similar in terms of fleet planning, but it is remarked that the tendency to acquire Airbus 320 aircrafts for positive economic situations is lost in the deterministic model. This finding is the first evidence of the loss of precision that presents the deterministic model against the stochastic.

The optimal fleet planning and assignment is provided in Table 6 in which A stands for the Airbus, B for the Boeing. The hyphen means an optional flight that is not operated. The last row in the Table 6 reports the expected value of the objective function, i.e., profits. Again, Table 6 shows the optimal fleet planning and assignment in the deterministic version of the problem. The solution differs in both, optimal solution and the value of the objective function. While differences are not significant at the earlier seasons, the further in time the greater differences can be observed. Additionally, the expected value of the objective function in the stochastic model is around 2% better than in the deterministic version.

Table 5. Optimal Airline Fleet Planning in the stochastic and deterministic model

	Stochastic Approach				Deterministic Approach			
	Bought Boeing	Bought Airbus	Leased Boeing	Leased Airbus	Bought Boeing	Bought Airbus	Leased Boeing	Leased Airbus
N1	0	1	1	1	0	1	1	1
N2	0	1	1	1	0	1	1	1
N3	0	1	1	1	0	1	2	1
N4	0	1	2	1				
N5	0	1	2	1				
N6	0	1	2	1				
N7	0	1	1	2				

Table 6. Optimal fleet assignment (A-Airbus, B-Boeing) in the stochastic version

	N1		N2		N3		N4		N5		N6		N7	
Flight\Season	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14
101	B	A	A	A	A	A	B	A	A	B	A	B	A	A
102	A	B	A	A	A	A	A	A	A	A	A	A	A	A
201	A	A	A	A	A	A	B	A	B	B	B	B	A	A
202	A	B	A	A	A	A	A	A	A	A	A	A	A	A
301	B	A	A	A	A	A	B	A	A	B	A	B	A	A
302	A	B	A	A	A	A	A	A	A	A	A	A	A	A
401	-	-	-	B	-	B	B	B	B	B	B	B	A	A
402	-	-	-	B	-	B	B	B	B	B	B	B	B	A
403	-	-	-	A	-	A	-	A	-	A	-	A	-	A
501	-	-	-	-	-	-	-	B	-	A	-	A	-	B
502	-	-	-	-	-	-	B	B	B	A	B	A	B	A
503	-	-	-	-	-	-	B	B	A	A	A	A	A	A
Profits (€)								590,110						

Table 7. Optimal fleet assignment (A-Airbus, B-Boeing) in the deterministic version

	N1		N2		N3	
Flight\Season	S1	S2	S3	S4	S5	S6
101	B	A	A	A	A	B
102	A	B	A	A	A	A
201	B	A	A	A	B	B
202	A	B	A	A	A	A
301	B	A	A	A	A	B
302	A	B	A	A	A	A
401	-	-	-	B	B	B
402	-	-	-	B	B	B
403	-	-	-	A	-	A
501	-	-	-	-	-	A
502	-	-	-	-	B	A
503	-	-	-	-	A	A
Profits (€)			579,930			

Nevertheless, looking at the four different strategic scenarios when solving the problem, i.e., s8-path, s10-path, s-12-path, and s14-path (see Figure 2b), we realize that the deterministic model differs more than a 10%, on average, in comparison with the stochastic version as it can be seen in Table 8. Notice that the results from that Table 8 has been obtained by simulating the solutions in the stochastic tree using the solution obtained in the deterministic version of the problem. By doing so, we can realize the superior performance of the stochastic approach when dealing with the uncertainties in the aeronautical sector. Actually, average profits are 10% higher using the stochastic approach than in the deterministic one.

Table 8. Comparison of solutions (profits -€) from the stochastic tree paths (scenarios)

	S8-path	S10-path	S12-path	S14-path
Stochastic	399,272	512,249	645,908	802,995
Deterministic	361,287	434,712	559,106	675,135
Percentual Change	9.51%	15.14%	13.44%	15.92%

4. Conclusions and future work

This work considered the Airline Fleet Planning and the Fleet Assignment Problem as an in integrated approach for solving the Airline Fleet Management Problem. Additionally, we included uncertainty in the demand and economic-related parameters. Following this approach, it is possible to valuate different scenarios, leading to a richer information for better decision making.

Thus, we have proposed a stochastic model for coping with the uncertainties that airlines are constantly dealing with, showing a superior performance of the stochastic solution, and encouraging the use of stochastic optimization for long run planning and strategic decision-making processes

This model is the starting point for future research. To this respect, the consideration of a greater air network is the logical next step. Additionally, greater trees with a wider range of scenarios is also of utmost interest for the airline fleet management. Nevertheless, considering more flights, fleets, and nodes will increase exponentially the complexity of the problem leading to unhandled models for exact methods. At this point, techniques such as Bender decompositions as well as the development of math -heuristics will reduce the computational time for such a huge model (Mansi et al., 2012).

Acknowledgements

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