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Facultad de Ciencias Económicas y Empresariales

TRABAJO FIN DE GRADO EN
ECONOMÍA

MONETARY POLICY ANALYSIS IN A NEW KEYNESIAN MODEL

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Modulo: Análisis Económico

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14 de Mayo de 2020

ABSTRACT

The present paper is an initiation to research, in which the aim is to learn about the basic methodologies for the business cycle and monetary policy analysis in a dynamic macroeconomic model. First, the equations of a New Keynesian (sticky-price) model are presented, based on the rational behavior of households, firms and the Central Bank. All the equations undergo a log-linearization process, allowing to solve and simulate the initially non-linear model through Matlab and Dynare. Once the model is solved, the effects of three different shocks to technology, household preferences and inflation are examined by displaying the corresponding impulse-response functions. The work ends with a proposal for an optimal monetary policy for the Central Bank and a comparative analysis of its economic consequences. The criterion of optimality chosen is that of maximizing household welfare.

KEY WORDS

New Keynesian model, business cycle analysis, monetary policy

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1. INTRODUCTION

The present paper is an initiation to research, in which the aim is to learn about the common tools and methodologies for the monetary analysis of a macroeconomic model. This work specifically analyzes a New-Keynesian dynamic stochastic general equilibrium model (New-Keynesian DSGE model).

In order to work with this model, one must first proceed to its theoretical explanation. Therefore, in the first section I explain how the equations that form part of the model are constructed, together with the inclusion of the three external shocks that represent factors not explained by the model, but which influence the endogenous variables.

The first equations in the model are derived from the optimal choices of a representative household. The objective function (intertemporal household utility) is subject to budgetary and time constraints. Money is included as the medium-of-exchange asset that reduces transaction costs. Therefore, it plays a role in both budgetary and time constraints, with the latter representing the time available to households. In addition, an inflation equation is included that is derived from the optimal decision of firms, represented by the model's Phillips curve. This section clearly reflects the Keynesian nuance of the model, since firms are faced with two situations characteristic of these models: monopolistic competition and sticky prices.

As this is a model that analyzes the role of monetary policy, an equation is included that summarizes the behavior of the Central Bank in this theoretical economy. One of the instrumental variables of the Central Bank, the output gap, is obtained assuming a situation of fully flexible prices.

It is important to note that all equations have to go through a log-linearization process, where the logarithm of the variable in the stationary state is subtracted from the logarithm of the variable in the current period. As the variables have been log-linearized, a significant number of equations depend on the variables valued in the steady state. Before solving the dynamic model, it is necessary to solve the model in the (long-run equilibrium) steady state.

In the third section, the model is calibrated. That is, based on articles by macroeconomic researchers, empirical evidence and some arbitrary decisions on ratios that I consider to be coherent with the data, I establish the appropriate value of the parameters needed to define the structure of the model.

Once the model is calibrated, in Section 4 I proceed to perform an impulse-response function analysis following the 3 shocks of the model (technology shock, consumption preference shock and inflation shock), using the Dynare extension in Matlab.

Section 5 examines monetary policy. I proceed to analyze how the parameters that capture the behavior of the Central Bank would be modified if it had as an added objective to guarantee the maximum welfare of households.

Section 6 concludes the paper with a revision of its main findings.

2. THEORETICAL MODEL

2.1 Optimizing program for the representative household

2.1.1 Household's objective function and constraints

The equations of the model are micro-founded, i.e., depend on individual rational decisions. That is, each household is assumed to follow a rational approach to achieve its maximum benefit, and all households are identical. Let us assume that the household's welfare depends positively on both consumption (c) and leisure (l). Households maximize their utility according to time by the following function:

$$U(c_t, l_t) = e^{v_t} \frac{c_t^{1-\sigma}}{1-\sigma} + \Xi \frac{l_t^{1-\gamma}}{1-\gamma} \qquad [1]$$

where σ is the constant elasticity of the consumption's marginal utility $\left(\sigma = \frac{U_{C_c} c}{U_C}\right)$ and γ is the constant elasticity of the leisure's marginal utility $\left(\gamma = \frac{U_{l_l} l}{U_l}\right)$. The t subscript represents the current period and Ξ is the weighted factor applied to leisure in a representative household's total utility. The variable v_t represents a consumption shock and follows an autoregressive (AR [1]) dynamic evolution. This is represented as follows:

$$v_t = \rho_v v_{t-1} + \varepsilon_{v_t} \quad \text{where } \varepsilon_{v_t} \sim N(0, \sigma^2_{\varepsilon_{v_t}}) \text{ and } \rho_v < 1 \qquad [2]$$

This shock can be interpreted as an exogenous effect not considered by the model, in which consumption at time t is more valued by households. A consumption unit will generate a greater increase in the total utility than without the shock factor.

Once the welfare value of a household has been determined, I should analyze where the income comes from and what it is used on. In other words, I shall introduce the budget constraint the representative household is subject to when deciding what to employ its income in.

Households get their income both from hours of work (n_t^s), amount of invested capital (k_t) and transfers from the government (g_t). There are competitive markets for capital and labor that determine the equilibrium values for a real rental rate of capital per unit of capital (r_t^k) and a real wage per hour of work (w_t) . Households spend their income on the consumption of goods and services (c_t), the investment of new capital and, assuming a constant depreciation rate δ, the restitution of depreciated capital ($k_{t+1} - (1-\delta)k_t$), and the purchase of new government bonds for the next period $\left(\frac{b_{t+1}}{1+r_t} - b_t\right)$.

As mentioned at the beginning, the present work is focused on the analysis of monetary policy, and hence money must play a role in the model equations. An alternative for the household is, then, to keep the residual income (not used on consumption or investment) and increase its money holdings, instead of purchasing new government bonds. As the next period bond is divided by a real return (the interest rate, r_t), the amount of money must also be corrected for the level of inflation from de corresponding period. I use the following equations for money, inflation and interest rate.

$$m_t = \frac{M_t}{P_t} \quad [3]$$

$$\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}} \quad [4]$$

$$1 + r_t = \frac{1+R_t}{(1+E_t\pi_{t+1})} \quad [5]$$

Money (M_t) is calculated as the difference between the current stock and the stock from the previous period. Dividing this by the price level at the current time we obtain the real money (m_t, [3]). In addition to appearing in the budget constraint, real money will also play an important role in the time constraint that I will include below. Inflation is the rate of increase in prices (π_t, [4]). And a Fisher relationship establishes that real interest rate (r_t) is equal to the nominal interest rate (R_t) corrected by the expectation of inflation for the following period [5].

The budget constraint appears as follows

$$w_t n_t^s + r_t^k k_t + g_t = c_t + (k_{t+1} - (1-\delta)k_t) + \left(\frac{b_{t+1}}{1+r_t} - b_t\right) + \left(m_t - \frac{m_{t-1}}{1+\pi_t}\right) \quad [6]$$

2.1.2 The role of money: transaction costs and shopping time

There are different options for introducing money into this model. I include in this paper three different alternatives, finally choosing one of them. The first one introduces money as another variable to consider inside the utility function. As it happens with consumption and leisure, in this alternative money would not generate *per se* an increase of the household's welfare. This option does not sound accurate if we think about money's definition. Money is a medium of exchange, and as such does not have value in itself. Therefore, it does not seem rigorous to include it in this way in the model.

Another alternative is to include money as an extra constraint. Thus, households, in an extreme case, will not be able to consume any goods if they do not have cash. However, there are situations where money is not necessary to consume (canceling securities, shares, etc.). Consequently, this alternative is not a convincing one either.

The third and last alternative presented in this work is shown by McCallum (1989, chapter 3) and Walsh (2017, chapter 3). In this one cash plays a considerable role in speeding up the time dedicated to shopping, and therefore reducing transaction costs. It is evident that, with enough cash, the time dedicated to go to the bank or to make transferences will disappear. This is the option I choose, in which cash allows households to reduce transaction costs and frees part of their time, which can be destinated for other purposes.

In order to do this, I will present a shopping time function. This function will depend positively on the level of consumption, because a higher level of consumption means a longer time spent shopping. Conversely, it will depend negatively on the amount of real money. As I have already explained, cash speeds up the time dedicated to shopping and lowers transaction costs, reducing the time allocated to shopping by the household. Accordingly, the function for shopping time will be

$$s_t = s(c_{t,} m_t) \qquad [7]$$

where the partial derivative of shopping time in respect to consumption is positive $\left(\frac{ds_t}{dc_t} = s_{c_t} > 0\right)$ and the one in respect to real money is negative $\left(\frac{ds_t}{dm_t} = s_{m_t} < 0\right)$. Besides, the second crossed derivatives are also negative. Increasing real money reduces the effect of consumption on shopping time and vice versa $\left(\frac{ds_{c_t}}{dm_t} = s_{c_t m_t} < 0 \,\&\, \frac{ds_{m_t}}{dc_t} = s_{m_t c_t} < 0\right)$.

The following is the specific function for shopping time considering both consumption and real money:

$$s_t = \emptyset_0 + \emptyset_1 c_t \left(\frac{c_t}{m_t}\right)^{\emptyset_2} \qquad [8]$$

with $\emptyset_0, \emptyset_1, \emptyset_2 > 0$.

This function involves the following partial derivatives

$$s_{c_t} = \emptyset_1(1+\emptyset_2)\left(\frac{c_t}{m_t}\right)^{\emptyset_2} \quad [9]$$

$$s_{m_t} = -\emptyset_1\emptyset_2\left(\frac{c_t}{m_t}\right)^{1+\emptyset_2} \quad [10]$$

which will allow me to obtain the consumption, the labor offer and the money demand functions.

Once the shopping time function has been defined, I must determine the restriction associated to it. This function consists of a time distribution, so it will be subject to a time constraint showing how a representative household distributes its lifetime. In a schematic way we can consider that households spend their time working (n_t^s), in leisure activities (l_t) and shopping (s_t).The time constraint is as follows

$$T = l_t + n_t^s + s(c_{t,} m_t) \quad [11]$$

where T represents the time available for the households.

2.1.3 Household's equations

Once the two restrictions and the objective function have been defined, I can represent the optimizing program in the period t for the representative household below

$$Max_{l_t, c_t, n_t^s, k_{t+1}, b_{t+1}, m_t} \; E_t \sum_{j=0}^{\infty} \beta^j \left[e^{v_t} \frac{c_t^{1-\sigma}}{1-\sigma} + \Xi \frac{l_t^{1-\gamma}}{1-\gamma} \right]$$

subject to

$$E_t\, \beta^j \Big[w_{t+j} n_{t+j}^s + r_{t+j}^k k_{t+j} + g_{t+j} - c_{t+j} - \left(k_{t+j+1} - (1-\delta)k_{t+j}\right) - \left(\frac{b_{t+j+1}}{(1+r_{t+j})} - b_{t+j}\right) - \left(m_{t+j} - \frac{m_{t+j-1}}{(1+\pi_{t+j+1})}\right)\Big] = 0, \quad j = 0,1,2,\ldots,$$

and

$$E_t\, \beta^j \left[T - l_{t+j} - n_{t+j}^s - s(c_{t+j,} m_{t+j})\right] = 0, \quad j = 0,1,2,\ldots,$$

where E_t is the rational expectation operator: the expectation (which I assume rational) of the representative household in respect to following periods. The future, in addition to being conditioned by the expectation households have, is also influenced by the assessment that they make of its importance. Namely, they will consider the future in their decision, but give it less importance than the present. This evaluation is represented by β, a constant discount factor $\left(\beta = \frac{1}{1+\rho} < 1\right)$ with $\rho > 0$ as the intertemporal rate of discount.

By a process of substitution and reduction I arrive at the same result as in the option where money was explicitly included in the utility. We can think of this as a more realistic approach when analyzing the role of money, even if the same conclusions are reached as in the other alternatives.

Once the objective function and the two constraints (budget and time) are fixed, I can already calculate the first order conditions (FOCs) of the variables on the basis of which I optimize the model. The household optimizing program would therefore be left with the following first order conditions:

$$\Xi(l_t)^{-\gamma} - \varphi_t = 0 \qquad [l_t^{foc}]$$

$$e^{v_t}\, c_t^{-\sigma} - \lambda_t - \varphi_t s_{c_t} = 0 \qquad [c_t^{foc}]$$

$$w_t - \varphi_t = 0 \qquad [n_t^{s\,foc}]$$

$$-\lambda_t + \beta E_t\, \lambda_{t+1}\left(r_{t+1}^k + 1 - \delta\right) = 0 \qquad [k_{t+1}^{foc}]$$

$$-\frac{\lambda_t}{(1+r_t)} + \beta E_t\, \lambda_{t+1} = 0 \qquad [b_{t+1}^{foc}]$$

$$-\lambda_t - \frac{\beta E_t\, \lambda_{t+1}}{(1+E_t\, \pi_{t+1})} - \varphi_t s_{m_t} = 0 \qquad [m_t^{foc}]$$

$$\begin{aligned} w_t n_t^s + r_t^k k_t + g_t - c_t - (E_t k_{t+1} - (1-\delta)k_t) - \frac{E_t b_{t+1}}{(1+r_t)} &+ \\ b_t - m_t + \frac{m_{t-1}}{(1+\pi_t)} = 0 & \end{aligned} \qquad [\lambda_t^{foc}]$$

$$T - l_t - n_t^s - s(c_{t,} m_t) = 0 \qquad [\varphi_t^{foc}]$$

Where λ_t is the Lagrange multiplier associated with the budget restriction in time t, and φ_t is the Lagrange multiplier associated with the time restriction in period t.

From these first order conditions, I will be able to obtain the behavioral equations I am interested in. These will be the consumption equation [I], the leisure equation [III], the labor supply equation [IV], the money demand equation [VI] and two different equations from shopping time [II, V].

The consumption equation [I] will relate the level of present consumption with: the expected future consumption, the present and the future real wage, the marginal shopping cost (in terms of consumption) and the real interest rate.

In order to find this equation I am going to use the FOCs of consumption $[c_t^{foc}]$, labor $[n_t^{s\,foc}]$ and future government bonds $[b_{t+1}^{foc}]$.

First, I solve for φ_t in $[n_t^{s\,foc}]$, and substitute it in $[c_t^{foc}]$. After that I solve for λ_t:

$$\lambda_t = \frac{e^{v_t} c_t^{-\sigma}}{(1+w_t s_{c_t})} \qquad [12]$$

We see now the role played by transaction costs. By comparing it to the case without transaction costs we see the role they play in penalizing the marginal satisfaction of the consumption. This is in accordance to my appreciation that, as consumption involves a certain shopping time, the increase in marginal welfare by unit of consumption is smaller than without transaction costs. The λ_t (also called shadow value of consumption) represents the marginal satisfaction, and decreases when considering transaction costs.

Without transaction costs:

$$\lambda_t = \frac{dU(c_t, l_t)}{dc_t} \qquad [13]$$

With transaction costs:

$$\lambda_t = \frac{\frac{dU(c_t, l_t)}{dc_t}}{(1+w_t s_{c_t})} \qquad [14]$$

Once the shadow price is found, I reorder $[b_{t+1}^{foc}]$

$$\frac{\lambda_t}{(1+r_t)} = \beta E_t\, \lambda_{t+1} \qquad [15]$$

And, by substituting λ_t [12], I define the consumption equation:

$$\frac{e^{v_t}\, c_t^{-\sigma}}{(1+w_t s_{c_t})(1+r_t)} = \frac{\beta E_t\, e^{v_{t+1}} c_{t+1}^{-\sigma}}{(1+w_{t+1} s_{c_{t+1}})} \qquad [16]$$

We can understand this equation with the following scheme:

$$\frac{Present\ satisfaction}{(Transaction\ cost)(Opportunity\ cost)} = \frac{Expected\ future\ satisfaction}{Expected\ future\ transaction\ cost}$$

To facilitate the task of solving the dynamic system of equations, I proceed to log-linearize the original equations. The process of log-linearization consists in transforming the variables in a way that allows me to make the equation linear. Specifically, it consists in taking the fraction of the variable's current state over its long-term value (steady state), and applying a logarithm to it. In addition, I will use two approaches to facilitate the task of finding the log-linearized equation.

The process of log-transformation consists on applying logarithms to both sides of the equation, establish the same equation for the steady state, and subtract the latter from the former.

Definition of log-linearization and approximation:

1) $\widehat{x_t} = \log\left(\frac{x_t}{z}\right) = \log(x_t) - \log(x) \cong \frac{x_t - x}{x}$ (1)

2) $\log(1 + x_t) \cong x_t$ (2)

Both approximations are true for small (close to zero) values of z_t. I take the natural logarithms on both sides of the consumption equation

$$log\left[\frac{e^{v_t} c_t^{-\sigma}}{(1+w_t s_{c_t})(1+r_t)}\right] = log\left[\frac{\beta E_t\, e^{v_{t+1}} c_{t+1}^{-\sigma}}{(1+E_t\, w_{t+1} s_{c_{t+1}})}\right] \quad [17]$$

and because of the properties of logarithms

$$v_t - \sigma\log(c_t) - \log\left(1 + w_t s_{c_t}\right) - \log(1 + r_t) = E_t v_{t+1} + \log(\beta) - \sigma\log(E_t\, c_{t+1}) - \log\,(1 + E_t\, w_{t+1} s_{c_{t+1}}) \quad [18]$$

and in the steady state (with no growth)

$$-\sigma\log(c) - \log(1 + ws_c) - \log(1 + r) = \log(\beta) - \sigma\log(c) - \log\,(1 + ws_c) \quad [19]$$

It can be seen from the above equation that in the long-term external shocks disappear. s_{c_t} is a small number, as it represents the marginal cost of consumption in terms of time (shopping). Likewise, it can also be assumed that the real interest rate is a very small number. Therefore, I can apply to both the second approach mentioned above (2).

By taking the difference between the last two equations, and applying the definition of log-linearization and the properties of logarithms

$$v_t - \sigma\widehat{c_t} - \left(w_t s_{c_t} - ws_c\right) - (r_t - r) = E_t v_{t+1} - \sigma E_t\,\widehat{c_{t+1}} - (E_t\, w_{t+1} s_{c_{t+1}} - ws_c) \quad [19]$$

I can now use the first approximation (1), but in inverse order, to obtain the log-linearized variables

$$\frac{(w_t s_{c_t} - ws_c)}{ws_c} \cong \log\left(\frac{w_t s_{c_t}}{ws_c}\right) = \log\left(\frac{w_t}{w}\frac{s_{c_t}}{s_c}\right) = \log\left(\frac{w_t}{w}\right) + \log\left(\frac{s_{c_t}}{s_c}\right) = \widehat{w_t} + \widehat{s_{c_t}}$$

Reaching

$$\left[w_t s_{c_t} - ws_c = ws_c\left(\widehat{w_t} + \widehat{s_{c_t}}\right)\right] \quad [20]$$

$$\left[E_t\, w_{t+1} s_{c_{t+1}} - ws_c = ws_c(\,E_t\,\widehat{w_{t+1}} + E_t\,\widehat{s_{c_{t+1}}})\right] \quad [21]$$

which allows me to substitute the equation [19] and get the following

$$v_t - \sigma\widehat{c_t} - ws_c\left(\widehat{w_t} + \widehat{s_{c_t}}\right) - (r_t - r) = E_t v_{t+1} - \sigma E_t\, \widehat{c_{t+1}} - ws_c\left(E_t\, \widehat{w_{t+1}} + E_t\, \widehat{s_{c_{t+1}}}\right) \quad [22]$$

Clearing the present consumption, I arrive at the log-linearized consumption equation.

$$\widehat{c_t} = E_t\, \widehat{c_{t+1}} - \frac{ws_c}{\sigma}\left(\widehat{w_t} - E_t\, \widehat{w_{t+1}}\right) - \frac{ws_c}{\sigma}\left(\widehat{s_{c_t}} - E_t\, \widehat{s_{c_{t+1}}}\right) - \frac{(r_t - r)}{\sigma} - \frac{E_t v_{t+1} - v_t}{\sigma} \quad [23]$$

Knowing the definition of an autoregressive (AR [1]) process, I can rewrite the difference of the external shock

$$E_t\, v_{t+1} = \rho_v v_t + E_t\, \varepsilon_{v_{t+1}}\ ;\ E_t\, v_{t+1} - v_t = \rho_v v_t + E_t\, \varepsilon_{v_{t+1}} - v_t$$

Because of the nature of the noise ($\varepsilon_{v_{t+1}}$)

$$E_t\, v_{t+1} - v_t = (\rho_v - 1)v_t \quad [24]$$

The log-linearized consumption equation becomes:

$$\widehat{c_t} = E_t\, \widehat{c_{t+1}} - \frac{ws_c}{\sigma}\left(\widehat{w_t} - E_t\, \widehat{w_{t+1}}\right) - \frac{ws_c}{\sigma}\left(\widehat{s_{c_t}} - E_t\, \widehat{s_{c_{t+1}}}\right) - \frac{(r_t - r)}{\sigma} + \frac{(1-\rho_v)v_t}{\sigma} \quad [I]$$

In order to finish determining the consumption equation, I should also log-linearize s_{c_t}. I start from [9] and carry out a similar procedure as with consumption. Here is [9]:

$$s_{c_t} = \emptyset_1(1 + \emptyset_2)\left(\frac{c_t}{m_t}\right)^{\emptyset_2}$$

I apply the properties of logarithms and the log-linearization techniques and are left with the following equation

$$\widehat{s_{c_t}} = \emptyset_2(\widehat{c_t} - \widehat{m_t}) \quad [II]$$

that satisfies the positive relationship of consumption and negative relationship of cash with shopping time.

From this these results I can already extract the relationship of consumption with the level of wages and with the amount of cash. An increase in $\widehat{w_t}$ (keeping everything else constant) conveys a reduction in $\widehat{c_t}$ [I]: if the wage level increases, so does the value of time. This is because the transaction cost is measured in the time the individual spends shopping, time that is worth more to the household as wages increase. Therefore, the household will decide to reduce consumption so that the shopping time shortens. Furthermore, an increase in cash ($\widehat{m_t}$) will mean that one more unit of consumption means less time spent shopping, allowing for a higher level of consumption.

In contrast, the effect on consumption of expected (future) wages and cash is the opposite. If the household expects wages to increase in the next period, it will decide to consume more in time t, on the expectation of this greater future income and the increased value of time. If, on the contrary, it expects the money in the following period to increase, it will reduce its consumption in the current period, knowing that it will cost less if it postpones the purchase to t+1. Having assumed a ρ_v less than 1, the positive effect of the shock (v_t) on the level of consumption can also be seen [I].

Once this equation is found, I go on to analyze the process to get the leisure equation [III]. This equation will relate leisure (l_t) to consumption (c_t), wage (w_t) and the effect of consumption on shopping time $\left(s_{c_t}\right)$. From leisure I will be able to obtain the labor supply by resorting to the distribution of time by the households.

To find the equation of leisure, I use the FOCs of labor supply $\left[n_t^{s\,foc}\right]$, consumption $\left[c_t^{foc}\right]$ and leisure $\left[l_t^{foc}\right]$. From the FOCs of consumption and labor supply I have obtained the shadow value of consumption, λ_t[12]. By entering this value in $\left[n_t^{s\,foc}\right]$, I obtain φ_t in terms of consumption and salary. Using $\left[l_t^{foc}\right]$, clearing again φ_t and equating it to the previous equation I obtain the following equation for leisure

$$\frac{e^{v_t} c_t^{-\sigma}}{(1+w_t s_{c_t})} w_t = \Xi(l_t)^{-\gamma} \qquad [25]$$

which could be described as

$$Labor's\ marginal\ satisfaction = Leisure's\ marginal\ satisfaction$$

Let us proceed with the log-linearization, which produces the following result

$$v_t - \sigma\widehat{c_t} - \left(w_t s_{c_t} - w s_c\right) + \widehat{w_t} = -\gamma\widehat{l_t} \qquad [26]$$

After applying approximation (1) and [20] and clearing $\widehat{l_t}$, the leisure equation looks like this

$$\widehat{l_t} = \frac{1}{\gamma}\left[\sigma\widehat{c_t} - (1 - w s_c)\widehat{w_t} + w s_c \widehat{s_{c_t}} - v_t\right] \qquad [III]$$

From this equation we can understand the relationship of leisure with the rest of the variables. As consumption increases, its marginal utility decreases, $\left(U_{c_t} = \frac{dU_t}{dc_t} = e^{v_t}\, c_t^{-\sigma}\right)$, while that of leisure remains constant $\left(U_{l_t} = \frac{dU_t}{dl_t} = \Xi(l_t)^{-\gamma}\right)$. As a result, the value of leisure seems to increase and the household prefers to spend more on it. Increased consumption reduces the marginal utility of consumption. As the marginal utility of leisure remains constant to this change, the household comparatively values the effect of increased leisure more than consumption, leading it to spend more on leisure. This is called the welfare effect. In the same way, an increment in the temporal marginal cost of shopping would lead households to spend more time on leisure while consuming less.

On the other hand, from [III] we see that an increase in $\widehat{w_t}$ would cause a reduction in $\widehat{l_t}$: by raising wages households have an incentive to work more, increasing their labor supply and decreasing the level of leisure (this can clearly be seen in the time constraint T). The effect of shock is seen again in this equation. Having assumed a positive value for it, the effect on leisure will be the inverse of the effect on consumption. Including this shock means that households give a higher value to consumption than to leisure.

In order to find the equation for the labor supply I will use the equation for leisure together with the time constraint [11]:

$$T = l_t + n_t^s + s_t$$

I subtract its corresponding value in the steady state, divide by the total time available (T) and multiply and divide each fraction by its value in the long term (l, n^s, s).

Rearranging I have

$$0 = \frac{n_t^s - n^s}{n^s}\frac{n^s}{T} + \frac{l_t - l}{l}\frac{l}{T} + \frac{s_t - s}{s}\frac{s}{T} \qquad [27]$$

where it can be seen how the time of households is conditioned by the long term. Households distribute their available time bearing in mind the stationary state. Using the approximation (1) and clearing $\widehat{n_t^s}$, the labor supply takes the following form

$$\widehat{n_t^s} = -\frac{l}{n^s}\widehat{l_t} - \frac{s}{n^s}\widehat{s_t} \qquad [IV]$$

Before analyzing the relationships within the labor supply, it is convenient to log-linearize the shopping time function (s_t).

Based on the equation [46] of the Appendix (part 1), I transform the following function [8]:

$$s_t = \emptyset_0 + \emptyset_1 c_t \left(\frac{c_t}{m_t}\right)^{\emptyset_2}$$

I apply logarithms and calculate the log-linear transformation of s_t to obtain

$$\widehat{s_t} = \left(\frac{\emptyset_1 c \left(\frac{c}{m}\right)^{\emptyset_2}}{s}\right)\left((1+\emptyset_2)\widehat{c_t} - \emptyset_2 \widehat{m_t}\right) \qquad [V]$$

that clarifies the relationship of shopping time with consumption and amount of real money balances.

If I substitute the equations for leisure $\left(\widehat{l_t}\right)$ and shopping time $(\widehat{s_t})$ in the labor supply $\left(\widehat{n_t^s}\right)$ I will obtain the explicit relation I am after.

$$\widehat{n_t^s} = -\frac{l}{n^s\gamma}\left[\sigma\widehat{c_t} - (1 - ws_c)\widehat{w_t} + ws_c\widehat{s_{c_t}} - v_t\right] - \frac{s}{n^s}\left[(1 + \emptyset_2)\widehat{c_t} - \emptyset_2\widehat{m_t}\right] \quad [28]$$

In this way I am relating indirectly (through leisure) the labor supply to the level of consumption, the level of wages, the temporal marginal cost of consumption and the external shock. I have explained previously that an increase in consumption (and its welfare effect) or in its marginal time cost will increase the level of leisure, and from [IV] we see that this will reduce the supply of labor. Moreover, an increase on wages gives households an incentive to work, causing them to increase their labor supply. The external shock will also have an effect on the decision of how many hours households will decide to work. As, due to the shock, consumption is more highly valued, the household decides to work more in order to be able to consume more. In short, the external shock will have a positive effect on the labor supply.

Through the shopping time function I can relate the labor supply again to the level of consumption, as well as to the money level. Enhancing consumption will increase the time spent on shopping, therefore shortening the time spent on work. Conversely, increasing the level of cash will reduce the time spent shopping, allowing the household to work more hours.

I assume that for the labor's factor the condition of market clearing is met, where the supply of labor is equal to the demand for labor. That means

$$\widehat{n_t^s} = \widehat{n_t^d} = \widehat{n_t} \quad [29]$$

It follows from this equation that this model does not take into account the possibility of unemployment. It is a situation in which the amount of work demanded by the firms coincides with the amount of work offered by the workers. This assumption, far from reality, facilitates the analysis to a considerable extent. In any case, with the subsequent analysis of monopolistic competition and sticky-prices it will allow me to give the model a greater credibility.

In order to finish the section on households I end with the equation of money demand [VI]. For this purpose, I use the FOCs of labor supply $\left[n_t^{s\,foc}\right]$, money $\left[m_t^{foc}\right]$ and government bonds $[b_{t+1}^{foc}]$. Along with these, I use Fisher relationship, which establishes a function that determines the nominal interest rate (R_t) in terms of the real interest rate (r_t) and the inflation expectations $(E_t\pi_{t+1}\ )$ as follows

$$(1 + R_t\ ) = (1 + r_t)(1 + E_t\pi_{t+1}) \qquad [30]$$

By clearing φ_t from $[n_t^{sfoc}]$ and substituting it in $[m_t^{foc}]$ I can clear the shadow value (λ_t). I can clear it also in $[b_{t+1}^{foc}]$, and by comparing both equations get the following:

$$-1 + \frac{1}{(1+r_t)(1+E_t\pi_{t+1})} = w_t s_{m_t} \qquad [31]$$

that reordering and using Fisher's relationship leaves us

$$\frac{R_t}{(1+R_t\ )} = -w_t s_{m_t} \qquad [32]$$

which could be outlined as follows

$$Marginal\ opportunity\ cost\ of\ cash = Marginal\ profit\ of\ cash$$

Before performing the log-linearization I can substitute s_{m_t} by the result obtained in [10]. Furthermore, the correction $(1 + R_t\ )$ is presumably small, and therefore $\frac{R_t}{(1+R_t\ )} \cong R_t$. Once this is taken into account, the money equation is modified to

$$R_t\ = w_t \emptyset_1 \emptyset_2 \left(\frac{c_t}{m_t}\right)^{1+\emptyset_2} \qquad [33]$$

I proceed to log-linearize it, assuming the approximation (1) for the nominal interest rate, since, as a rate, I am interested in its value *per se*, not in its relation to its long-term value. I obtain

$$\frac{R_t - R}{R} = \widehat{w_t} + (1 + \emptyset_2)(\widehat{c_t} - \widehat{m_t}) \qquad [34]$$

I clear $\widehat{m_t}$ and get the log-linearized money equation

$$\widehat{m_t} = -\frac{1}{(1+\emptyset_2)R}(R_t - R) + \frac{1}{(1+\emptyset_2)}\widehat{w_t} + \widehat{c_t} \qquad [VI]$$

As can be seen, the money equation relates the amount of cash to the nominal interest rate, the wage level and the level of consumption.

Here we can appreciate how cash depends positively on consumption and on the level of wages. Logically, the higher the salary, the greater the amount of cash, and the higher the level of consumption, the greater the need for cash.

In contrast, the relationship with the interest rate is opposite. If we understand the nominal interest rate as the opportunity cost of cash, this relationship becomes logical. The interest rate represents the return the household could earn by putting the money into bonds, rather than owning it as cash. Increasing the interest rate gives households an incentive to invest, because they will get a higher return. By spending more of the money in bonds, the amount of cash decreases.

2.2 Firm optimizing program and the Phillips Curve

After obtaining the equations that represent the optimal decision of the households, I proceed to obtain the optimal decision of the firms. In this specific case, I will work with a New Keynesian model, in which there are price rigidities. I translate this into Calvo (1983) sticky-prices model, where firms set the optimal price with a certain level of probability and maintain the behavior of the previous period with the remaining level of probability. Unlike the Real Business Cycle (RBC) model, I work with a model that does not assume perfect competition, but rather there are rigidities that prevent the development of competition. These price rigidities will allow me to capture information about short-term fluctuations produced by the implemented policies, that models such as RBC do not allow me to observe.

In order to get the optimal price, let us start by looking at how firms set their prices. In this case I will assume that they set prices in a context of monopolistic competition as explained by Dixit and Stiglitz (1977). The quantity produced by firm i in time t is determined by the demand function:

$$y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\theta} y_t \qquad [35]$$

where $P_t = \left[\int_0^1 \lfloor P_t(i) \rfloor^{1-\theta} di\right]^{\frac{1}{1-\theta}}$ is the Dixit-Stiglitz aggregated price, $y_t = \left[\int_0^1 \lfloor y_t(i) \rfloor^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}$ is the Dixit-Stiglitz aggregated output and $\theta > 0$ is the elasticity of substitution between differentiated goods.

Being in monopolistic competition, firms can set the production objectives they want and demand the necessary work for it. They determine the level of production by setting the price, as they have the market power to do so. Therefore, the function to be optimized consists of the value of the revenue obtained from selling their production minus the cost of producing it. After some calculation the function to be optimized is

$$Max_{P_t(i)}\ E_t\ \sum_{j=0}^{\infty} \beta^j \left[\left[\frac{P_{t+j}(i)}{P_{t+j}}\right]^{1-\theta} y_{t+j} - w_{t+j} n_{t+j}^d(i)\right] \quad [36]$$

If there were no market rigidities, firms would obtain a constant mark-up with respect to the nominal value of the marginal labor cost:

$$P_t(i) = \frac{\theta}{\theta-1} P_t \psi_t(i) \quad [37]$$

where $\psi_t(i)$ is the real marginal cost of the firm i:

$$Marginal\ cost\ (\psi_t(i)) = \frac{Real\ wage\ (w_t)}{Labor\ marginal\ productivity\ \ (f_{n_t^d(i)})}$$

and $\frac{\theta}{\theta-1} > 1$ is the mark- up or the monopolistic profit of the firm i.

But, in addition to monopolistic competition the model adds price rigidities. I will introduce these rigidities by following Calvo (1983), where the firm has a constant probability (1-η) of setting the optimal price in t and η of continuing with the behavior of the previous period. Firms follow this inertial behavior by indexing the prices set in the previous period using long-term inflation. They therefore set the price in the current period based on what they did in the previous period and correct it using the inflation rate. In addition, this indexation component includes a shock (τ_τ), which represents an exogenous factor that makes companies adapt their indexing (since they no longer use only the long-term inflation rate, but are affected by the exogenous component). The price aggregation equation results in the following

$$P_t = \left[\eta[(1+\pi+\tau_\tau)P_{t-1}]^{1-\theta} + (1-\eta)[P_t(i)]^{1-\theta}\right]^{\frac{1}{1-\theta}} \quad [38]$$

where τ_τ is an inflation shock of autoregressive nature and with the following form:

$$\tau_t = \rho_\tau \tau_{\tau-1} + \varepsilon_{\tau_t} \text{ where } \varepsilon_{\tau_t} \sim N(0, \sigma_{\varepsilon_{\tau_t}}^2) \text{ and } \rho_\tau < 1 \quad [39]$$

This price aggregation equation causes firms to take into account the probability that in the future they will not set the optimal price but keep it due to lack of information. This conditional probability makes the FOC for $P_t(i)$, after some calculations:

$$\left[(1-\theta)\ E_t\ \sum_{j=0}^{\infty} \beta^j \eta^j \left[\frac{\prod_{k=0}^{j-1}(1+\pi+\tau_{\tau+k})P_{t+j}(i)}{P_{t+j}}\right]^{-\theta} \frac{y_{t+j}}{P_{t+j}} + \right.$$
$$\left. \theta\ E_t\ \sum_{j=0}^{\infty} \beta^j \eta^j\ \psi_{t+j}(i) \left[\frac{\prod_{k=0}^{j-1}(1+\pi+\tau_{\tau+k})P_{t+j}(i)}{P_{t+j}}\right]^{-\theta-1} \frac{y_{t+j}}{P_{t+j}}\right] = 0 \quad [40]$$

In order to facilitate calculations, I proceed to log-linearize the optimal price (whole procedure in Appendix, Part 1). The final equation [57] shows clearly how the price set by the firms depends positively on their future expectations on the evolution of prices and on the evolution of the aggregated real marginal labor cost.

As we know, the Phillips curve represents changes in the rate of inflation, not in prices. I will therefore have to find a relationship between the evolution of prices and the evolution of inflation. For a more enjoyable reading, this procedure is explained in the appendix without including the shock, as the demonstration including it would be too long and complex. Here I point out how the Phillips curve looks including the shock component:

$$\pi_t - \pi = (\beta E_t \pi_{t+1} - \pi) + \left[\frac{(1-\beta\eta)(1-\eta)}{\left(1+\frac{\theta\alpha}{1-\alpha}\right)\eta}\right]\widehat{\psi_t} + (1-\beta\rho_\tau)\tau_\tau \qquad [VII]$$

As can be seen from the Phillips curve, current inflation is explained by the expected inflation of the following period and the current fluctuation of the real marginal cost. It also reflects the effect of the inflation shock. Even if none of the endogenous variables change, firms may set a different price, affected by some factor not captured by the model. This effect is summarized within the exogenous component included in the Phillips curve.

2.3 Steady state and dynamic equations

As can be seen in several of the equations, some solutions depend on the long-term value of a variable, so I first solve the model in the stationary state. I indicate in the appendix the list of the 12 equations necessary to solve the model, together with the 12 variables to which they are associated. Additionally, I indicate the specific procedure for some of these equations.

Once the values in the stationary state of the variables have been calculated, the dynamic model is proposed. This model consists of 24 equations with their corresponding endogenous variables. Besides these variables, it depends on several parameters $(\sigma, \gamma, \emptyset_1, \emptyset_2, \beta, \theta, \alpha)$ and variables $(w, s_c, c, l, n, s, c, y, r, R, \pi)$ calibrated for the stationary state, some parameters that did not appear in the long term equation $(\eta, \mu_R, \mu_\pi, \mu_{\widetilde{y_t}}, \rho_v, \rho_\tau)$ and the external shocks (v_t, z_t, τ_t).

Here is the list of equations:

$$\widehat{c_t} = E_t \widehat{c_{t+1}} - \frac{ws_c}{\sigma}(\widehat{w_t} - E_t \widehat{w_{t+1}}) - \frac{ws_c}{\sigma}(\widehat{s_{c_t}} - E_t \widehat{s_{c_{t+1}}}) - \frac{(r_t - r)}{\sigma} + \frac{(1-\rho_v)v_t}{\sigma} \qquad [I]$$

$$\widehat{s_{c_t}} = \emptyset_2(\widehat{c_t} - \widehat{m_t}) \qquad [II]$$

$$\widehat{l_t} = \frac{1}{\gamma}\left[\sigma\widehat{c_t} - (1 - ws_c)\widehat{w_t} + ws_c\widehat{s_{c_t}} - v_t\right] \quad [III$$

$$\widehat{n_t} = -\frac{l}{n}\widehat{l_t} - \frac{s}{n}\widehat{s_t} \quad [IV]$$

$$\widehat{s_t} = \left(\frac{\emptyset_1 c\left(\frac{c}{m}\right)^{\emptyset_2}}{s}\right)(1 + \emptyset_2)\widehat{c_t} - \emptyset_2\widehat{m_t} \quad [V]$$

$$\widehat{m_t} = -\frac{R_t - R}{(1+\emptyset_2)R} + \frac{1}{(1+\emptyset_2)}\widehat{w_t} + \widehat{c_t} \quad [VI]$$

$$\pi_t - \pi = (\beta E_t\,\pi_{t+1} - \pi) + \left[\frac{(1-\beta\eta)(1-\eta)}{\left(1+\frac{\theta\alpha}{1-\alpha}\right)\eta}\right]\widehat{\psi_t} + (1 - \rho_\tau)\tau_\tau \quad [VII]$$

$$r_t - r = (R_t - R) - (E_t\pi_{t+1} - \pi) \quad [VIII]$$

$$\widehat{\psi_t} = \widehat{w_t} - z_t + \alpha\widehat{n_t} \quad [IX]$$

$$\widehat{y_t} = z_t + (1 - \alpha)\widehat{n_t} \quad [X]$$

$$\widehat{y_t} = \frac{c}{y}\widehat{c_t} \quad [XI]$$

$$R_t - R = \mu_R(R_{t-1} - R) + (1 - \mu_R)\left[\mu_\pi(\pi_t - \pi) + \mu_{\tilde{y}}\widetilde{y_t}\right] \quad [XII]$$

$$\widetilde{y_t} = \widehat{y_t} - \widehat{\widehat{y_t}} \quad [XIII]$$

which has 14 associated variables: $\widehat{c_t}$, $\widehat{l_t}, \widehat{n_t}, \widehat{s_t}, \widehat{s_{c_t}}, \widehat{w_t}, \widehat{\psi_t}, r_t - r, R_t - R, \widehat{m_t}, \pi_t - \pi, \widehat{y_t}$, $\widehat{\widehat{y_t}}$ and $\widetilde{y_t}$.

Equations [I] to [VI] correspond to the rational decision of households and [VII] to the rational decision of firms. Equations [VIII] and [IX] are the log-linearized Fisher relationship and marginal labor cost equations. Equation [X] is the log-linearized Cobb-Douglass function, explained in the Appendix (part 1).

It is necessary to explain in more detail the origin of the last three equations. The first one [XI], comes from log-linearizing the over-all resources constraint: $y_t = c_t + \delta k$, used in the steady state model solution (see Equation 80, Appendix 2). If a variable is the result of the sum of others, its corresponding log-linearized relationship can be obtained as the weighted sum of the variables (see Equation 46, Appendix 1). In order to simplify the calculations this model assumes that the capital is constant. Taking this into account, the value of the second summand (δk) will be zero and I arrive at the equation [XI].

The second one [XII], is a specific type of monetary policy rule that simulates the behavior of the Central Bank. This is Taylor-type monetary policy rule, which specifically includes an inertia component, named the interest-rate smoothing, similar to the case presented by Calvo (1983) for sticky-prices.

According to this equation, the Central Bank sets the interest rate with a probability μ_R of carrying the behavior it has carried out the previous year, setting the interest rate equal to that of t-1. In other words, as it happened with companies, other institutions are influenced by what happened the previous period.

The second part of the equation, with a probability of $(1-\mu_R)$, represents the situation in which the Central Bank is not affected by the inertia component and reacts to the given circumstances in the economy at time t. In this equation, we can see that the main concerns of this agent are the evolution of the inflation and the productive capacity of the country, and each of these summands is weighted according to the importance the Central Bank gives to them.

The evolution of the inflation is represented by the difference between the current value of inflation and its value in the long term. In turn, productive capacity is represented by the output gap, i.e., the difference between current output and potential output, which is presented in the last equation [XIII].

The potential output is the output that would be obtained if prices were not rigid. Within the price aggregation scheme, it corresponds to the situation in which firms always set the optimal price, without being affected by any kind of inertia. In the equation proposed by Calvo (1983), it corresponds to an η equal to 0. This causes a constant mark-up of prices in relation to marginal costs. Applying log-linearization techniques, this means that the real log-linearized marginal cost is equal to 0, as there is no difference between the short-term and long-term values.

In order to work with the potential output, it is necessary to analyze the situation with fully flexible prices. Therefore, I must take the previous set of dynamic equations and rename the variables, considering in this case the situation with no rigidities:

$$\widehat{\overline{c_t}} = E_t\,\widehat{\overline{c_{t+1}}} - \frac{ws_c}{\sigma}\left(\widehat{\overline{w_t}} - E_t\,\widehat{\overline{w_{t+1}}}\right) - \frac{ws_c}{\sigma}\left(\widehat{\overline{s_{c_t}}} - E_t\,\widehat{\overline{s_{c_{t+1}}}}\right) - \frac{(\overline{r_t}-r)}{\sigma} + \frac{(1-\rho_v)v_t}{\sigma} \quad \text{[XIV]}$$

$$\widehat{\overline{s_{c_t}}} = \emptyset_2\left(\widehat{\overline{c_t}} - \widehat{\overline{m_t}}\right) \quad \text{[XV]}$$

$$\widehat{\overline{l_t}} = \frac{1}{\gamma}\left[\sigma\widehat{\overline{c_t}} - (1-ws_c)\widehat{\overline{w_t}} + ws_c\widehat{\overline{s_{c_t}}} - v_t\right] \quad \text{[XVI]}$$

$$\widehat{\overline{n_t}} = -\frac{l}{n^s}\widehat{\overline{l_t}} - \frac{s}{n}\widehat{\overline{s_t}} \quad \text{[XVII]}$$

$$\widehat{s_t} = \left(\frac{\emptyset_1 c\left(\frac{c}{m}\right)^{\emptyset_2}}{s}\right)(1+\emptyset_2)\widehat{c_t} - \emptyset_2\widehat{m_t} \quad [XVIII]$$

$$\widehat{m_t} = -\frac{1}{(1+\emptyset_2)R}(\overline{R_t} - R) + \frac{1}{(1+\emptyset_2)}\widehat{w_t} + \widehat{c_t} \quad [XIX]$$

$$\overline{r_t} - r = (\overline{R_t} - R) - (E_t\overline{\pi_{t+1}} - \pi) \quad [XX]$$

$$0 = \widehat{w_t} - z_t + \alpha\widehat{n_t} \quad [XXI]$$

$$\widehat{y_t} = z_t + (1-\alpha)\widehat{n_t} \quad [XXII]$$

$$\widehat{y_t} = \frac{c}{y}\widehat{c_t} \quad [XXIII]$$

$$\overline{R_t} - R = \mu_R(\overline{R_{t-1}} - R) + (1-\mu_R)[\mu_\pi(\overline{\pi_t} - \pi)] \quad [XXIV]$$

which introduces 10 additional variables: $\widehat{c_t}, \widehat{w_t}, \widehat{s_{c_t}}, \overline{r_t} - r\ \widehat{l_t}, \widehat{n_t}, \widehat{s_t}, \widehat{m_t}, \overline{R_t} - R, \overline{\pi_t} - \pi.$

The system of equations of the dynamic model has a total of 24 equations with 24 associated variables. In addition to these variables, the system includes three external shocks, which represent changes coming from outside the model. There is a shock for consumption (v_t), for inflation (τ_τ) and for technology (z_t).

3. CALIBRATION

For the calibration process, I will set the values of the model's parameters. Most of them will be set based on what has been demonstrated in the economic literature by other authors as values that are more veridical.

I set the household rate of intertemporal preferences at $\rho = 0.005$, which implies a 2% annualized real interest rate in the steady state (consistent with an average real return of a risk-free bond in the long term). From this, we can deduce that the value of the constant discount factor $\beta = \frac{1}{1+\rho} = 0.995$, meaning that this model gives a 99.5% relevance to the next period with respect to the 100% with which the present moment is valued

In the utility function, the elasticity of the consumption marginal utility $\left(\varepsilon = \frac{dU_c}{dc} \times \frac{c}{U_c}\right)$ is set at $\sigma = 1.5$, similar to the estimated value found by Smets and Wouters (2003, 2007) using Bayesian econometrics in a DSGE model for both the Euro area and the US. Based on the previous, a 1% increase in consumption leads to a 1.5% reduction in the marginal utility of consumption. On the other hand, γ is the inverse of the Frisch labor supply elasticity, and empirical studies (Altonji, 1986; Card, 1994) have found a low labor supply elasticity in the data, therefore I will assume it to be 0.25

The value for T, the life-time, depends on the value we have placed on the labor supply. I parameterize the labor supply (n), normalizing it to 1. I assume that the time dedicated to work by households is a third of their available time, with the rest, except for a residual part for shopping, being spent on leisure. Therefore, the value of T in this case will be equal to 3. When setting the variable n, one of the model parameters was required to act provisionally as a variable. The parameter Ξ has been used, which indicates the weight of leisure in the utility of households. Once the model is solved in the steady state, the resulting value for Ξ is 22.887. After this the task can be reversed, converting again n into a variable and Ξ into a parameter.

The value of alpha can be obtained from the empirical evidence regarding the total distribution of income. The share of labor income is approximately 60% taking an average over industrialized countries (Guerriero, 2019). For a Cobb-Douglas production function, this share corresponds to the elasticity of labor, $1-\alpha$. Therefore, the value I set is $\alpha = 0.4$.

The calibrated $\theta = 10.0$ implies that the steady-state mark-up of prices over marginal costs is 10%, which has been assumed in New Keynesian papers like Chari, Kehoe, and McGrattan (2000) or Galí, Gertler, and López-Salido (2001). In addition, I have assumed for the model that long-term inflation (π) is 0.005, which implies a 2% annualized rate in the steady state solution of the model.

The calibration for η, the probability of firms' not being able to set the optimal price is 0.75 as frequently assumed in the New Keynesian literature (Erceg *et al.*, 2000). The parameters associated with Taylor's monetary policy rule have been taken from his papers. According to Taylor (1993) the value of the parameter associated with inflation (μ_π) is set at 1.5, while that of the annual output gap is set at 0.5. As I am working with quarterly periods, I divide the parameter associated with the annual output gap, setting $\mu_{\tilde{y}}$ at 0.125 (0.5/4). The parameter that includes such a smoothing component (μ_R), is set at a value between 0 and 1 (Clarida *et. al* 1998). I set it at 0.8. In other words, at time t, the Central Bank is conditioned at 80% by the behavior it carried out in time t-1.

The remaining three parameters are those included in the shopping time function. The initial values given to these parameters have been set arbitrarily. By calibrating these parameters, I have been able to adjust my model, making it more consistent with economic reality. The first parameter is $\emptyset_0 = 0.028$ for the constant coefficient in the shopping-time function. It represents the time that households would spend on shopping, even without the theoretical possibility of consuming. The second parameter is $\emptyset_1 = 0.049$, which is the linear coefficient, and the third one is $\emptyset_2 = 10$, which is the quadratic marginal effect of $\frac{c}{m}$ over shopping time.

As mentioned, by a series of calibrations I obtained these last parameters, which could be consider veridical, as the resulting ratios are alike those found in reality. Based on these parameters, the remaining two-thirds of the time available to households (when they are not working), is divided between leisure and shopping time in the following manner. The time dedicated to leisure is around 66% $\left(\frac{l}{T} = 0.657\right)$, while the time spent on shopping is approximately 1% of the available time $\left(\frac{s}{T} = 0.01\right)$. This last ratio corresponds to approximately 15 minutes a day dedicated to shopping by households of. Finally, the value of the parameters results in a cash to consumption ratio of 1.35 $\left(\frac{m}{c} = 1.3478\right)$. In other words, households spend 75% of the cash they have in consumption.

Table 1. Parameters for log-linearized dynamic model

$\rho = 0.005$	β=0.995	$\sigma = 1.5$	$\gamma = 4$
$\alpha = 0.4$	$\theta = 10$	$\emptyset_0 = 0{,}028$	$\emptyset_1 = 0.049$
$\emptyset_2 = 10$	$\Xi = 22.887$	$\eta = 0.75$	$\mathrm{T} = 3$
$\pi = 0.005$	$\mu_R = 0.8$	$\mu_\pi = 1.5$	$\mu_{\tilde{y}} = 0.125$

4. BUSSINES-CYCLE ANALYSIS

4.1 Previous setting

This section consists of observing what happens to the evolution of the most relevant variables of the model when faced with each type of shock included in it: technology (z_t), consumption (v_t) and inflation (τ_t). These external shocks explain aggregate (short-run) fluctuations of the endogenous variables around their steady-state (long-run) values. With the inclusion of these, in addition to analyzing which business cycle occurs in the short term due to a certain exogenous effect, it is possible to observe how the economy progressively returns to equilibrium, at values close to those of the long term.

The time series of the output $(\hat{y}_t)$ generated by the model is typically non-stationary. For the analysis to be more appropriate, the first difference of this variable is included, thus having a variable that collects the information on output and is also stationary (Integrated of order 0). As shown below, the first difference of the log-linearized output is roughly equivalent to its growth rate.

$$\hat{y}_t - \hat{y}_{t-1} = log\left(\frac{y_t}{y}\right) - log\left(\frac{y_{t-1}}{y}\right) = \log(y_t) - \log\,(y_{t-1}) \cong \frac{y_t - y_{t-1}}{y_{t-1}}$$

In addition to including this new variable, the calibration process will be streamlined once the results of the model made by Dynare in Matlab have been observed. As mentioned at the beginning of the section, the inclusion of shocks serves to generate variability in the model. The (linear) solution form obtained by Matlab provides information about the relationships between endogenous variables and between them and the exogenous components of the model. The shocks, although already explained in the inclusion of each one of them, follow the following generating process:

$$x_t = \rho_x x_{t-1} + \varepsilon_{x_t} \quad \text{where} \quad \varepsilon_{x_t} \sim N(0, \sigma^2_{\varepsilon_{x_t}})$$

As it is an autoregressive of order 1, it is necessary to fix the value that ρ will have and the standard deviation of the error in each shock. The value of ρ explains the level of inertia of the shock; that is, how strongly the behavior of the shock persists in the following periods. The standard deviation of the error gives information on the variability of the shock, which influences the magnitude of the economic cycle represented in the model.

To calibrate both ρ and the standard deviation values, I have to search for values that approximately replicate realistic values for the standard deviation of production and inflation. Collecting the data for the quarterly evolution of US production (in the World Bank) and calculating the standard deviation of the first differences in the logarithm of production gives a value of approximately 0.8%. The data for the variability of the inflation are slightly smaller than ¼ of the variability of the production. That is, between 0.2% and 0.25%.

In addition to including information for each variable, Matlab also includes the correlation that exists in the model of a variable with the rest. The focus is on the correlation between inflation and the production growth rate, and between the latter and the nominal interest rate, all of which are contemporary. The empirical evidence reflects how inflation shows a slight counter-cyclical pattern, meaning that inflation tends to rise when the rate of production growth falls. The other important correlation is between the nominal interest rate and the rate of production growth. Empirical evidence shows a value slightly above 0 for this correlation. That is, there seems to be some pro-cyclicality in the nominal interest rate.

Taking these four issues into account, the calibration of the components of the shocks is carried out. The values of the coefficients of autocorrelation and standard deviations that partially guarantee these four relationships are as follows:

Table 2. Parameters for the three external shocks:

$\rho_v = 0.8$	$\rho_\tau = 0.8$	$\rho_z = 0.95$
$\sigma_{\varepsilon_{v_t}} = 1.8$	$\sigma_{\varepsilon_{\tau_t}} = 0.25$	$\sigma_{\varepsilon_{z_t}} = 0.6$

Once the dynamic model has been correctly calibrated, it is possible to proceed with the analysis of the impulse-response functions of the shocks.

The variables considered to be the most important are presented. First, the economy's impulse response is presented in comparative terms in the same graph: when there are sticky-prices $(\hat{y}_t)$ and when there are flexible prices $(\hat{\bar{y}}_t)$. Along with this, I include the two variables targeted by the Central Bank: the output gap ($\tilde{y}_t$, which is deduced from the evolution of the previous comparative analysis) and the inflation rate $(\pi_t - \pi)$. Together with them, I include the tool used by the Central Bank, the nominal interest rate $(R_t - R)$ and its corresponding value taking into account inflation, the real interest rate $(r_t - r)$. As this work highlights the role of monetary policy in a New Keynesian model, the evolution of real money balances is also included $(\widehat{m_t})$. I also consider it important to include what happens with real wages $(\widehat{w_t})$ in order to understand the interrelationship of several of the variables represented. Along with these, I also add the three variables that give information on how the household distributes its time available: work hours $(\widehat{n_t})$, leisure $(\hat{l}_t)$ and shopping time $(\hat{s}_t)$.

The rest of the variables are not included because they can be easily deduced from the other variables represented in the graphs. Although consumption $(\hat{c}_t)$ is a primary variable for understanding the situation of households, in my model it is always a 75% of the income, of production [XI]. Similarly, the marginal cost $(\widehat{\psi_t})$ faced by firms is the (log) difference between the real wage and the marginal product of labor [IX]. For the same reason, the marginal cost of consumption when shopping is not included $(\widehat{s_{c_t}})$, as it is always given by the difference between consumption and real money [II]. Finally, I include potential production to see what would happen in the economy in the face of these shocks if there were no sticky prices. In summary, the evolution of the following 11 variables is represented in 10 different graphs: $\hat{l}_t, \widehat{n_t}, \hat{s}_t,, \widehat{w_t}, r_t - r, R_t - R, \widehat{m_t}, \pi_t - \pi, \hat{y}_t, \hat{\bar{y}}_t, \tilde{y}_t$.

The explanation is supported by the graphs included at the beginning of each section, which will show the disaggregated evolution of each variable in the face of a specific shock. It is important to note that in the first graph of all the figures, the evolution of the potential output is represented by the dashed line. Before the analysis, it is important to mention that I represent log-linearized variables, so the graphic evolution shows how each variable evolves with respect to its value in the long term.

4.2 Technology shock

Also called productivity shock, technology shock can be understood as a sudden improvement in the technology for the firms of the economy. This shock may be due to the role played by Research and Development spending, which is not represented in my model. In order to liven up the case, we can think that in the current period (quarter 1) a new technology is being designed that allows most workers to use much more powerful software than they have used up to now. With the design of more efficient technologies, the first impact on the economy will be a clear improvement in worker productivity.

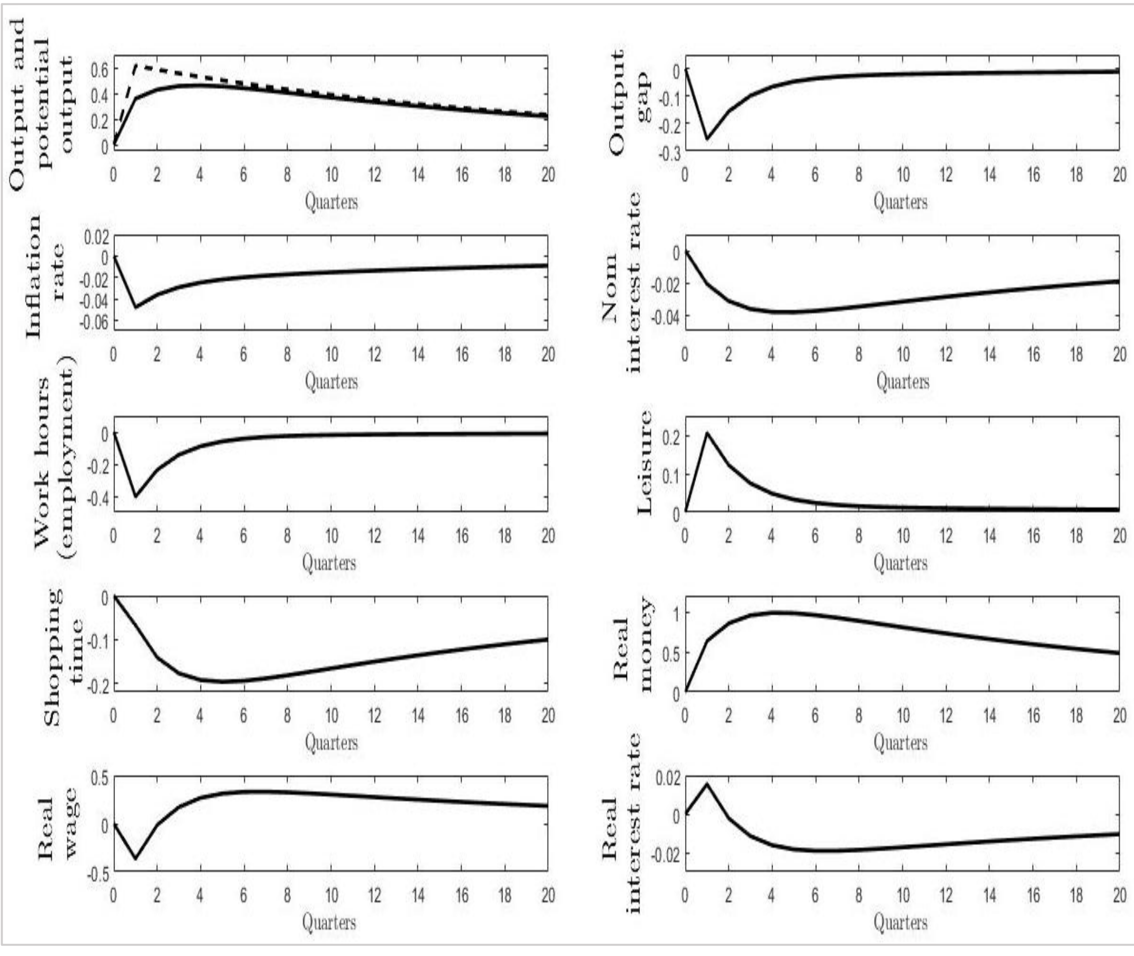

Figure 1. Impulse-response function – Technology shock

This improvement in labor productivity will be immediately observed by the firms, since they are the ones that have implemented this software in the workers' equipment. By being more productive, companies will not need the number of workers they needed in the previous period to set the same level of production. Therefore, the effect that the increment in productivity has corresponds to a reduction in the demand for labor (through layoffs, reductions in working hours, etc.). This reduction in hours worked does not translate into a reduction in production, as the initial increase in labor productivity allows firms to produce even more by hiring fewer workers. Moreover, by reducing the demand for labor, the real wages of workers initially decrease, although later they will be seen to approach productivity again.

On the other hand, the increment in productivity, together with the initial drop in real wages, means that the marginal labor cost observed by companies has been reduced. Firms, in order to maximize their profits, must change the price they set in response to this change in the cost of labor. Therefore, firms that have sufficient information reduce the price to bring it closer to the new optimal price. The lowering of prices carried out by the firms causes the increase in productivity to translate into an initial decrease in the level of inflation.

Now, what about the time distribution of households? The economy represented in the model assumes that the labor market is always in equilibrium, so the level of hours worked will be lower in this new period, thus increasing the time dedicated by households to leisure. However, the reduction in time spent working does not translate in the model into more time spent shopping. This is due to the different strength with which the evolution of consumption and real money balances influence the shopping time. Both consumption and real money balances increase at first.

In order to understand why consumption evolves in this way, I can analyze what happens in the first equation of the model [I]. During the quarter of the shock, the real interest rate has risen. This variable has an inverse relationship with consumption. Why then does consumption increase? Because this relationship between the variables is intertemporal, so in this case interest rate expectations (downwards) will influence households to a great extent, moving them to consume more in the first quarter.

The level of real money balance will also increase in this period. There are two reasons for this: the level of consumption has risen while the nominal interest rate has decreased. The decrease in the nominal interest rate will be explained later, analyzing the behavior of the Central Bank in this situation. The nominal interest rate represents the opportunity cost that households face when they have cash in hand. With the decrease in the nominal interest rate, households have less incentive to allocate part of their income to bonds, since the return they will get for them (the nominal rate) has decreased. This justifies the increase in the level of households' real money balances. Therefore, the decrease in the time spent on purchases is due to the fact that the effect of the increase in the real money balance dominates over the effect of an increase in consumption, causing the transaction cost to be reduced, and resulting in less time spent on shopping.

One has to ask what would have happened if the economy had fully flexible prices. Following the technology shock, there would even be a greater increase in the output produced. This difference causes the output gap to be reduced initially, as the economy has greater potential if it were to address the problem of price rigidities.

In short, the economy initially suffers from an expansive cycle due to the technological shock, where prices have fallen due to the reduction in labor costs, and which also has even greater possibilities due to the greater effect of this technological improvement on potential output. The Central Bank's target variables, inflation and the output gap, are initially reduced. How does the Central Bank respond immediately? It lowers the nominal rate, thus allowing the economy to grow even more, encouraging consumption and activity which in theory will lead to an increase in prices. This initial drop in the nominal rate corresponds, on the contrary, to an increase in the real interest rate. This is because initial expectations of a price decline dominate over this interest rate, due to its smoothing policy on interest rate adjustments. The Central Bank's strategy will be to maintain increasingly low interest rates until the problem of falling prices and the output gap has been solved. Therefore, during 3-4 quarters after the shock, we can see how the Central Bank cuts the nominal interest rate. What effect does this lowering of the nominal interest rate have on the economy?

Once it is observed that the situation has largely been resolved, both inflation and the output gap return to their pre-shock values. This behavior of the Central Bank causes most of the endogenous variables to slow down the behavior carried out in the first periods, getting closer and closer to long term levels. Consumption and real money balances will evolve downwards due to this increase in nominal rates, with the effect of the real money balance dominating over the time dedicated to shopping, which will increase towards values close to those of the long term.

In summary, the effects of the technological shock allow the Central Bank to carry out a stabilizing policy, taking advantage of the fact that the economy can grow even more (negative output gap and initial deflation). After this, the Central Bank returns to the initial nominal rates so that the economy can return to equilibrium several periods later.

4.3 Consumption preference shock

The consumption preference shock, also called consumption shock and demand shock, contrives the desire for a higher level of consumption, since it raises its marginal satisfaction (utility). One can imagine, in this very simplified economy, the following situation: the government carries out an advertising campaign during this period justifying the benefits and potential of an increase in aggregate demand, which includes household consumption. The government uses this discourse to encourage households to consume more. This campaign is a profound success and fits into the collective imagination. What are the effects that this campaign will have in the short term?

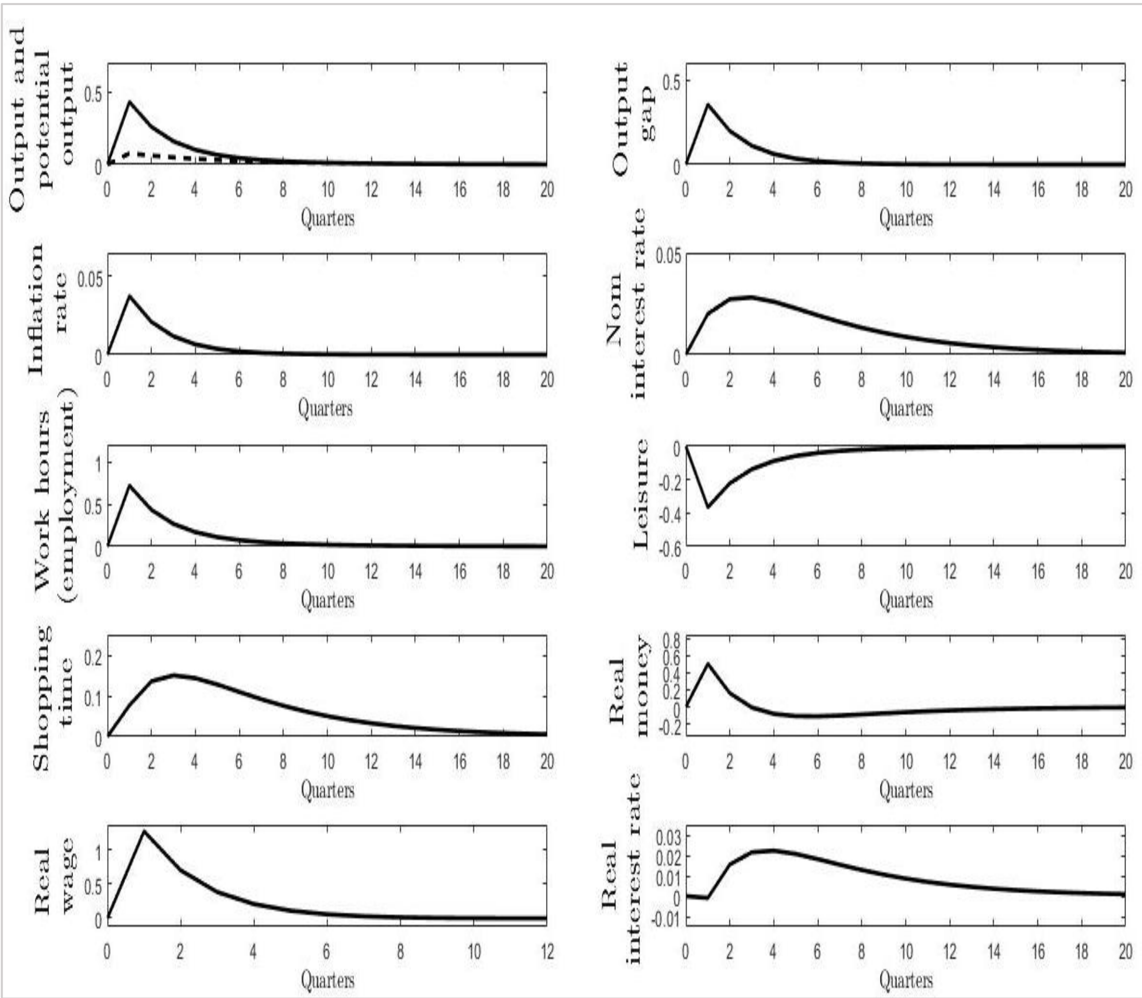

Figure 2. Impulse-response function – Consumption preference shock

This increase in households' propensity to consume has an immediate effect on the level of aggregate demand. Such increase in aggregate demand is captured by firms which, seeing that the population wants to consume more, set a higher level of production than before the demand shock. In order to increase the level of production, firms increase the demand for labor. This increment in demand has a direct, positive effect on real wages, as to ensure that equilibrium occurs in the labor market. Increased demand for labor means a higher number of hours worked for the same existing technology. As employment in this economy increases, worker productivity decreases due to the decreasing marginal return of labor. This decline in labor productivity, together with the increase in the level of wages of workers, has an immediate positive effect on the marginal cost of labor. After the shock, the cost of maintaining workers becomes more expensive for the firms. In order to maximize profits, firms faced with this increase in labor costs decide to set higher prices. As a considerable part of the firms have changed the price to a new higher optimum, the inflation rate increases in the face of the demand shock.

On the other hand, households redistribute the way they use their available time. It has already been explained that the demand shock has led to an increase in the level of employment. This increase in working hours is offset by a reduction in the time households spend on leisure. However, the effect on the time they spend shopping is not reduced by the additional time they are working in this period. This situation is again explained by the strength with which the level of consumption and the real money balances affect the time spent on shopping. The real money balances are increased in this case. Although the nominal interest rate turns higher (I will explain the Central Bank's behavior later), the increase in consumption will be such that it will dominate over the effect of the nominal interest rate, causing an increase in the demand for cash. For similar reasons, the increase in consumption will be so strong that it will initially dominate over the effect of the increase in real money balances with respect to the time spent on purchases. Even though households have more cash and this speeds up purchases, they increase consumption so much that the time they finally spend on shopping increases.

This demand shock will not have the same effect on potential output as on current output. The economy in which there are no price rigidities is very sensitive to technological changes, yet it is hardly affected by changes in household preferences. The output gap will therefore have increased in this period, and the economy will have reached levels that exceed its potential capabilities, because of the increase in the labor supply that allows to earn the income needed for the extra consumption. This widening of the output gap, together with rising inflation, are clear symptoms that the economy is overheating. The Central Bank, faced with this demand shock, will immediately raise the nominal interest rate so that the economy does not overheat too much. For its part, the real interest rate will hardly be modified initially, as the increase in the nominal rate will be offset by the effect of the increase in inflation expectations.

During the first quarters after the shock, the Central Bank will continue with its contractionary strategy, slightly increasing the nominal rate in each period. The effect of the nominal rate hike will soon have clear effects on the economy. Consumption, which had soared in the previous period, begins to decline rapidly, as households are unable to borrow as much as in the previous period, due to the rapid increase in the real interest rate in the following quarters. This fall in consumption therefore has the opposite effect to that explained during the shock period. The drop in aggregate demand will lower the production set by firms, and with it the level of contracting together with real wages. This will initially have a positive effect on the productivity of workers who remain employed. The increase in productivity, together with the fall in wages, will lead to a fall in the cost of labor. Firms will observe this and set lower and lower prices, causing the level of inflation to return to values closer to the time before the shock.

In addition, households will work fewer hours and spend more time on leisure. The effect of the rate increase will be strongly felt in the level of real money balances, which will fall to levels even below the pre-shock value. Such a sharp fall in real balances will dominate the fall in consumption, with repercussions for a longer shopping time in the early periods. Households consume less, but have much less cash, slowing down the time spent on each purchase.

On the other hand, the real interest rate will rise sharply, due to the increment in nominal rates and the reduction in inflation expectations. As mentioned in the previous section, the nominal interest rate does not influence the economy in such a way when there are no price rigidities. Therefore, output will fall more sharply than potential output, thus reducing the output gap.

As the economy returns to its steady-state values, the Central Bank may reset nominal interest rates to values close to those before the shock. As these nominal rates are lowered, most variables will continue to move in the direction of their long-term values, albeit more slowly. There are two sudden changes of direction: the evolution of real money balances, which again evolves positively, and the evolution of purchasing time, which decreases rapidly (as consumption continues to fall and real money balances are recovering), although both are also approaching their long-term values. Once again, the economy will be approaching initial equilibrium after the effects of the shock.

4.4 Inflation shock

The inflation shock can be interpreted as a sudden change in the way firms set their prices. According to the equation I use for price aggregation, with sticky-prices a la Calvo(1983), firms set the optimal price with a probability of (1-η), while with a probability of η they drag out the behavior carried out in the previous period, since they do not have enough information to set the optimal price. This inertial behavior is captured by the indexation of the current prices to the lagged (previous) prices. One way of understanding the inflation shock is the following example: let us suppose that the Organization of Petroleum Exporting Countries (OPEC) announces restrictions on oil supply that may increase the price of oil and energy costs. The companies that do not have all the information, upon learning of this release, change their way of indexing for the current period. Taking into account the price level of the previous period, companies are now indexing the price level upwards, above the long-term inflation level. This is because they suspect that the OPEC announcement will take effect in the current period. As mentioned, this behavior will be carried out by only a part of the companies, which cannot set the optimal price for the current period, and have to observe the past behavior to decide which price to set. Another part of the firms will know which price is optimal for the current period, having enough information and being indifferent to the news that has leaked in the newspapers. What effects will this news have in short-run fluctuations of the macroeconomic variables?

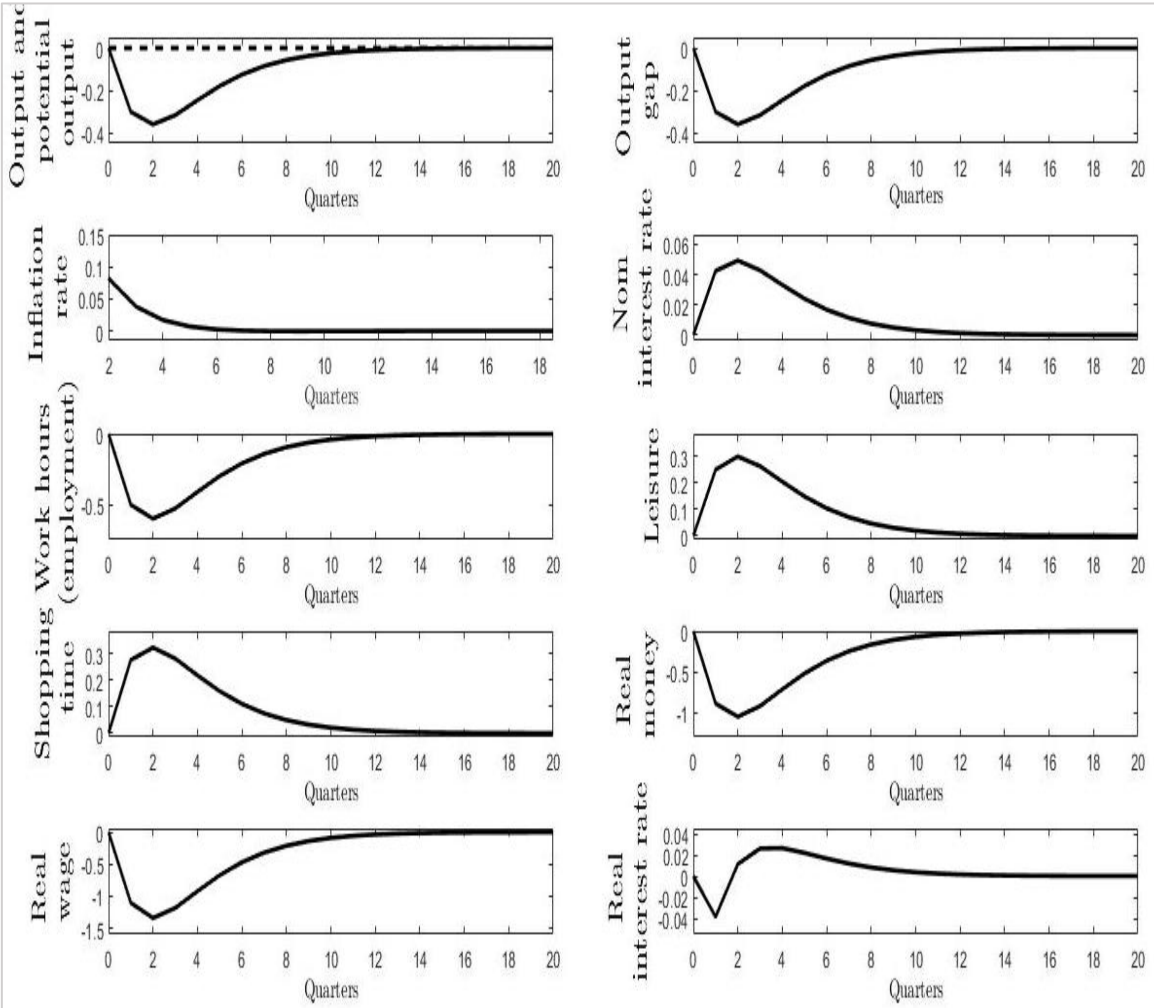

Figure 3. Impulse-response function – Inflation shock

The immediate effect is a rise in current inflation. It is important to keep in mind that these changes occur because of that portion of the companies that do not set the optimal price, and adjust their prices upwards when implementing the indexation rule.

The firms that are now setting a higher price know that they must do so at the cost of reducing their market share. These firms therefore reduce the level of output in the face of this shock. Since they are producing less in the current period than before the shock, the firms do not require the same number of workers, thus reducing their level of hiring. This reduction in demand for employment will mean that, in order to bring the labor market back into balance, the real wages of workers who have not been laid off will be reduced. The reduction in labor demand means that workers who remain in their jobs, in the first instance, will see their productivity increase. Lower wages and higher productivity will lead to lower labor costs in the first instance.

Despite the fact that the real interest rate initially fell, it will rise in the following periods, influencing the expectations of households. Due to this inter-temporal relationship, households decide to consume less in the current quarter. Regarding how they distribute their time, households have seen their working hours reduced, so they will spend more time on leisure. In order to know what happens to the time they spend on shopping, I must first analyze the effect of the shock on the level of real money balances in the economy. Real money balances are strongly reduced by two cumulative effects: the level of household consumption has fallen, and the nominal interest rate has risen instantly (again, the behavior of the Central Bank will be explained below). The reduction in real money balances is so strong that it dominates over consumption with respect to the effect that both have on the time spent on purchases of consumption goods. Although households consume less, they have much less cash, so purchases are slowed down and shopping time increases.

As previously mentioned, this shock only has an effect on those firms that do not have perfect information to set the optimal price. In a situation where there are no price rigidities, the probability that a firm will not have optimal pricing information is 0. Therefore, potential output is not affected by this inflation shock at all. This results in a decrease in the output gap identical to the increase of current output.

The Central Bank therefore sees an economy in which production has fallen (and the output gap has increased) and inflation has nevertheless risen, i.e. it is facing a case of stagflation. The tool it possesses is the nominal interest rate, but to solve both problems it would have to modify the nominal rate in opposite directions. Therefore, it first focuses on solving the problem of inflation. The Central Bank increases the nominal rate immediately, and will follow this policy during the next quarter after the shock. This decision will quickly correct the problem of inflation, but will aggravate the recession that this economy was suffering. Therefore, the initial effect of this decision on the model variables will be to aggravate their situation, increasing their difference with respect to the long term. The real interest rate, which at the time of the shock had fallen due to high inflation expectations, is corrected in the following quarters due to the effect of the increase in the nominal interest rate on it.

Once the problem of inflation is solved, the Central Bank faces the recession, carrying out a strongly expansive policy, correcting with speed the previously high nominal rates. This drop-in nominal rate will cause the variables to change completely in their direction, approaching their long-term values. With this measure, the Central Bank will have reactivated the economy after having corrected the problem of prices. With this sudden drop in the interest rate, it is important to note that inflation will strongly slow down the path towards its long-term value. In short, the Central Bank will have had to carry out first a contractive policy (rate increase) and then an expansive one (rate decrease) to solve the problem of stagflation derived from the inflation shock.

5. OPTIMAL MONETARY POLICY

5.1 Previous settings and optimal parameters

This section consists of analyzing how the Central Bank would behave if an optimality criterion is taken into account in the monetary policy. As explained before, in this model the Central Bank is concerned with the stabilization of the inflation rate and the output gap. The level of concern can be approximated by the two parameters $(\mu_{\pi}, \mu_{\tilde{y}})$ that are associated with these variables in the monetary policy equation. There are many different ways of approaching optimal monetary policies: reducing interest rate volatility, maintaining the nominal interest rate as set by some central bank regulations (e.g., keeping interannual inflation below but close to 2%), etc.

The criterion chosen in this paper is the maximization of household welfare. The Central Bank will take as theirs the household objective of intertemporal maximization. The welfare in the current period of the households is represented by the instantaneous function of utility (IUF). To ensure that utility is maximized intertemporally, it is necessary to use a second order Taylor approximation. For ease of reading, the development of the equation is explained in the appendix. The resulting equation to be maximized is as follows:

$$\sum_{j=0}^{\infty} \beta^j E\left(U\left(c_{t+j}, l_{t+j}\right)\right) = \frac{1}{1-\beta}\left(\frac{c^{1-\sigma}}{1-\sigma} + \Xi\frac{l^{1-\gamma}}{1-\gamma} + c^{-\sigma}cov(c_t, v_t) - \frac{\sigma c^{-\sigma-1}}{2}var(c_t) - \Xi\frac{\gamma l^{-\gamma-1}}{2}var(l_t)\right) \quad [41]$$

where the first two summands in the parenthesis represent the constant steady-state value of IUF $\left(\frac{c^{1-\sigma}}{1-\sigma} + \Xi\frac{l^{1-\gamma}}{1-\gamma}\right)$. Therefore, maximizing household welfare translates into minimizing the variance of leisure and consumption adjusted with the covariance between consumption and consumption shock. In other words, the Central Bank maximizes the welfare of households in the following way: it ensures that the sum of the volatility of consumption (corrected for the covariance of consumption with respect to the shock) and of leisure is minimal in the face of possible business cycles. In short, the way in which the Central Bank guarantees the maximum welfare of households is by ensuring that their situation is as stable as possible in the face of possible business cycles.

Once this equation is reached, it must be included in the model to find the parameters of the monetary policy rule that guarantee its maximum. In order to set the appropriate parameters, a restriction to these values must first be taken into account. The restriction established by Blanchard and Kahn (1980) is summarized in the Taylor principle. The Taylor principle states the need for the parameter associated with inflation to be strictly greater than 1 so that there is a single solution and there is no problem of indetermination. That means:

$$\mu_{\pi} > 1.0$$

Although the demonstration is more complex and does not concern this paper, an intuition regarding this restriction will facilitate the task. The Taylor principle states that the nominal interest rate has to vary by a value bigger than the rate of inflation. On the other hand, the equation of the real interest rate reflects the effect the nominal rate has on it and of inflation. If the nominal interest rate were to vary less than inflation, a rise in inflation would mean a fall in the real interest rate, and hence an explosion in inflation. However, if inflation were to fall, this would mean a rise in the real interest rate, which would lower inflation even further, condemning the economy to a deflationary spiral. For this not to happen, the Taylor principle must be guaranteed.

With the help of Matlab and Dynare, this maximization is included in the log-linearized dynamic model. To do this, a loop is constructed with possible parameter values, and the pair of values that guarantees the maximum value for the new included function is indicated. For the exposed and calibrated model in particular, the pair of values that maximize the welfare of households is

$$\mu_{\pi} = 1.02 \quad \mu_{\tilde{y}} = 0.01$$

In the original situation $\left(\mu_{\pi} = 1.5 , \mu_{\tilde{y}} = 0.125\right)$ it can be understood that the Central Bank was concerned about 12 times more about inflation than about the output gap. With the new values obtained from the parameters, the Central Bank's concern about inflation soars, while the issue of the output gap is barely taken into account. To maximize the welfare of households in this model, the Central Bank has to be 100 times more concerned about price developments than about the productive capacity of the economy.

Once seen the change in behavior of the Central Bank regarding its target variables, it is appropriate to add the change in the volatility of the most relevant variables. Before analyzing this change, it is important to underline that in this section one more instrument is added to the Central Bank. Besides the nominal interest rate, the Central Bank could also use the rate of growth of nominal money as a policy instrument. The log-linearized nominal growth rate of money follows the following form:

$$g_{M_t} = \widehat{M_t} - \widehat{M_{t-1}} \quad [42]$$

Knowing the definition of real balances; the relationship between these, the inflation rate and the nominal growth rate of money is as follows:

$$(\widehat{m_t} - \widehat{m_{t-1}}) = \left(g_{M_t} - g_M\right) - (\pi_t - \pi) \quad [43]$$

where growth in real money balances depends positively on nominal money growth and negatively on the rate of inflation.

5.2 Change in household welfare and economic volatility

Returning to the analysis, it is appropriate to add the effect of this change in the monetary policy parameters on the stability of the economy. To this end, table 3 includes the change in the standard deviations of several variables together with the change in the welfare of households. In addition to analyzing how the welfare of households changes, it is interesting to observe how the stability of households changes (via consumption and leisure standard deviations). It is also important to observe what happens to the volatility of inflation and the output gap, that indicate the change in the size of business cycles. Faced with the need for policy responses to stabilize aggregate fluctuations along business cycles, it is interesting to indicate how the level of aggressiveness of the Central Bank changes (via volatility of nominal interest rates and nominal money growth). Finally, as it is a monetary analysis, it is important to know what happens with the volatility of the real money balances.

Table 3. Comparison between the original situation and the optimal situation

Parameters	Initial situation	Optimal situation
Consumption standard deviation (σ_c)	2.5814	2.5429
Leisure standard deviation (σ_l)	0.7693	0.9146
Nominal interest rate standard deviation (σ_R)	0.1837	0.2085
Inflation rate standard deviation (σ_π)	0.2180	0.2894
Output gap standard deviation ($\sigma_{\tilde{y}}$)	0.8602	1.0770
Real money balances standard deviation (σ_m)	4.2511	4.4026
Nominal money growth standard deviation (σ_{g_M})	1.2488	1.1921
100xHouseholds' welfare (variable part, U_v)	-0.1788	-0.1654

The last member of the table shows the variable part of the function and represents the welfare of the households to be maximized $\left(U_v = \frac{1}{1-\beta}\left(c^{-\sigma} cov(c_t, v_t) - \frac{\sigma c^{-\sigma-1}}{2} var(c_t) - \Xi \frac{\gamma l^{-\gamma-1}}{2} var(l_t)\right)\right)$. As can be seen, the value of this part has increased (being a less negative number). The new value of the variable part, -0.0016, is the result of the Central Bank guaranteeing the maximum welfare of the households.

Volatility in consumption and nominal money growth is reduced, while that for the rest of the variables rise. The increase in the variability of inflation and the output gap are indicators that business cycles will generally be more volatile than before for these two variables. In addition, the increase in nominal interest rate volatility means that the Central Bank is generally more aggressive than before.

For a better understanding of the effect of this change in monetary policy, a graphical comparison of how the evolution of the three impulse-response functions analyzed in the previous section varies is provided. I include graphically the evolution of four variables that indicate how the situation of households is modified $\left(\widehat{c_t}, \widehat{l_t}, \widehat{n_t}, \widehat{s_t}\right)$, the evolution of the two target variables of the Central Bank $(\tilde{y}_t, \pi_t - \pi)$, the evolution of the two tools used by the Central Bank $\left(R_t - R, g_{M_t}\right)$ and as this work is a monetary analysis, I finally include the fluctuations of real money balances $(\widehat{m_t})$. In all figures, the continuous line represents the evolution of the variables for the original monetary policy parameters, while the dotted line represents the evolution for the optimal monetary policy parameters.

5.3 Comparative analysis – Technology shock

This is the shock that most amplifies its effect, as the business cycle resulting from the technological shock is now much stronger than before.

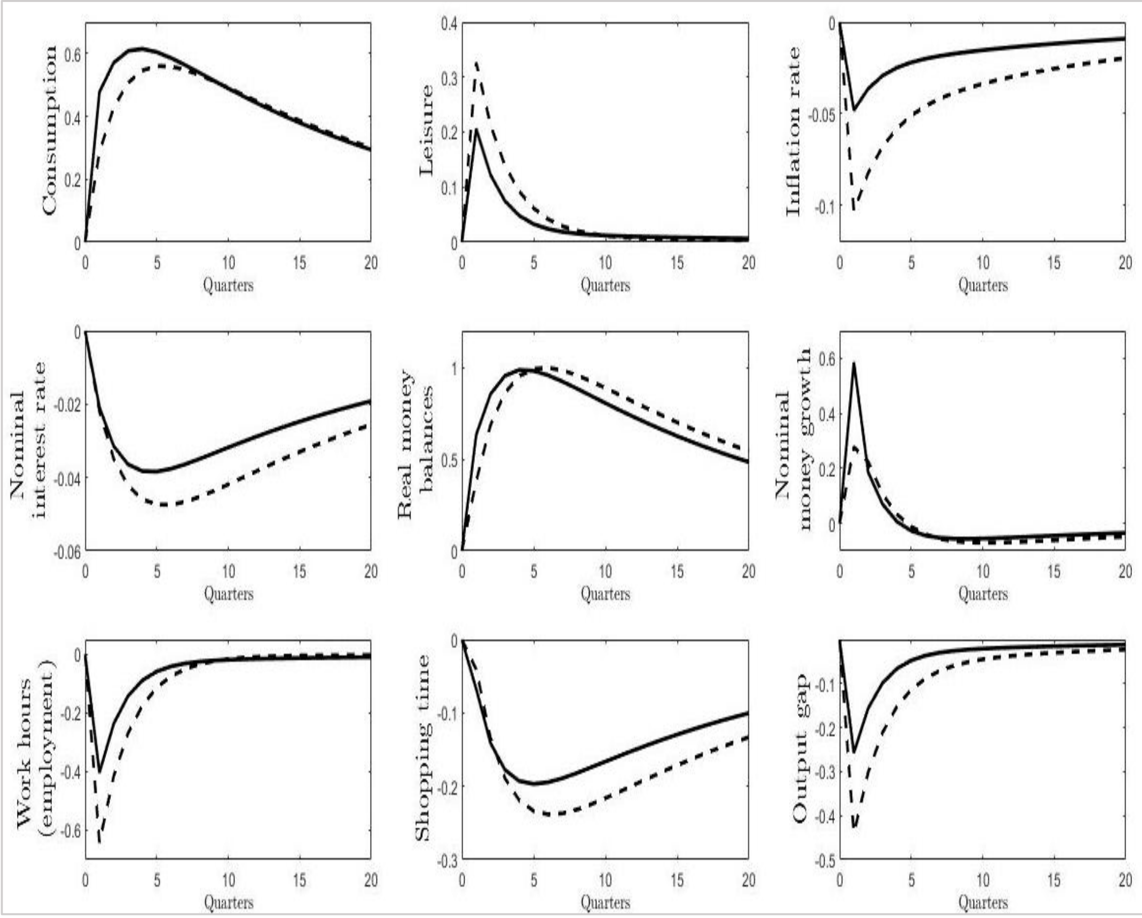

Figure 4. Comparative analysis – Technology shock

As explained in the previous section, a technological shock has the following short-term effects on households: consumption and leisure increase, and shopping time decreases (because real money balances increase, dominating this effect over the effect of increased consumption on shopping time). In the optimal situation in which the Central Bank aims at obtaining maximum household welfare, the result logically follows the same direction, but with quantitative variations. In the face of this shock, household consumption varies less than before (its standard deviation is now lower), while the change in the distribution of time will be different: leisure time increases more (its standard deviation is greater), reducing both working time and shopping time more. In fact, shopping time hardly varies in the short term, because real money balances take a new path that undergoes a transformation practically proportional to the new consumption path (varying less than before).

As leisure time increases further, the level of employment now decreases even more than in the original situation. This decrease in job hiring leads to lower wages for workers. Due to the technological shock, workers who keep their jobs have also suffered an increase in their productivity, which, together with the drop in wages, means a drop in the labor cost that workers represent for firms. This decrease in labor costs is observed by firms, which set a lower price level that guarantees their maximum profit in the face of this shock. As in this new situation the effect is stronger on the cost of labor, inflation falls even more than it did in the original situation. In the same way, the increase in productivity derived from the technological shock allows firms to produce more with fewer workers. As explained in the previous section, this technological shock has a stronger effect on the situation of non-rigidity in prices, leading to a fall in the output gap. As in this optimal situation the effects on firms are amplified, potential output will increase even more with respect to output with price rigidities, meaning an output gap in the first periods more negative than that resulting from the original situation.

In the face of this, the Central Bank manages the nominal interest rate and the growth rate of nominal money. The technological shock in this new situation brings even greater growth opportunities for this economy: inflation and the output gap have fallen more than in the original situation. Therefore, the Central Bank decides to lower rates, and even though in the first quarters it keeps them similar to the original situation, in the following quarters it sets them at even lower levels. The decision to maintain nominal rates in the first quarters is similar to the original situation, together with the lower increase in consumption, translates into a lower nominal growth of nominal money in these first periods.

In short, the technology shock with the optimized monetary policy rule has greater effects on output produced, although the behavior of these households is now more stable (less risky) in terms of consumption.

5.4 Comparative analysis – Consumption preference shock

Like the case of the technological shock, the business cycle following a consumption preference shock is magnified when the Taylor rule coefficients are optimized with respect to the original situation.

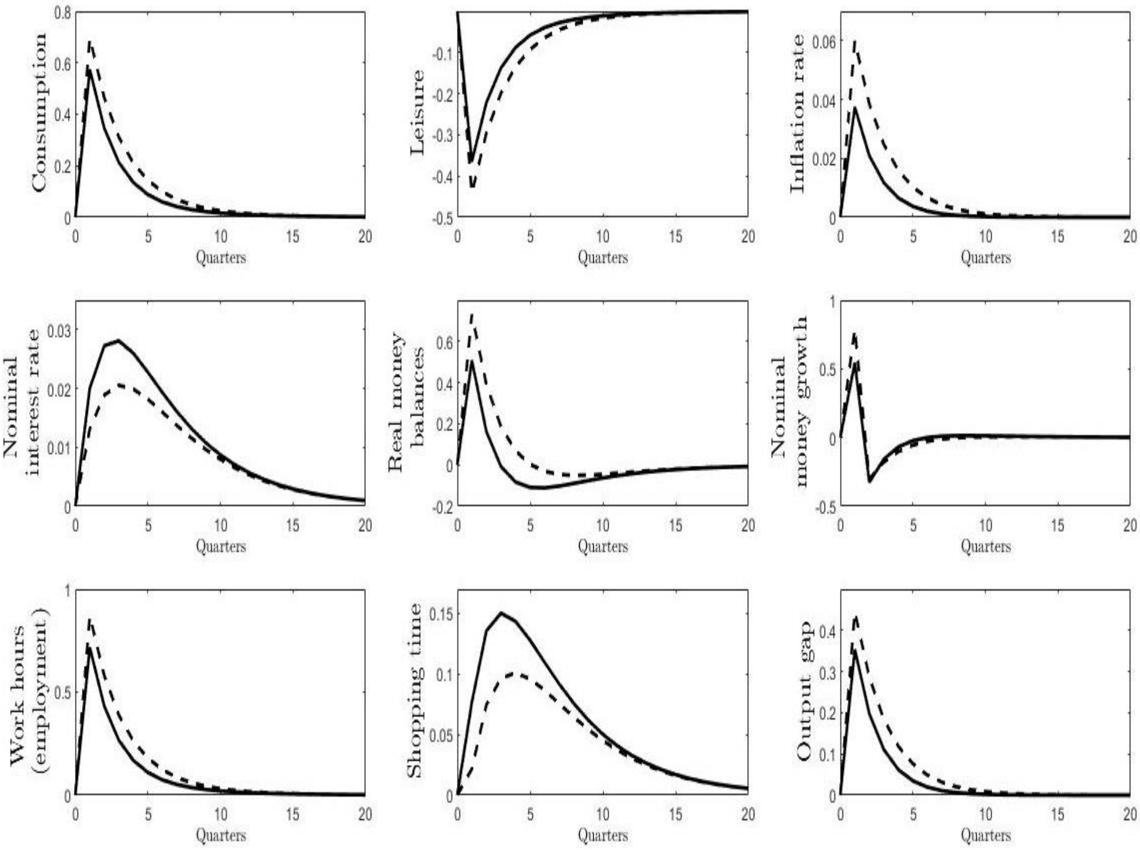

Figure 5. Comparative analysis – Consumption preference shock

In the face of a consumption preference shock, the effect on households is as follows: consumption increases, leisure time decreases, working hours increase and shopping hours increase. Faced with this shock, the effect on consumption is greater than on the level of actual balances, making the increase in the former have a greater effect on the time spent shopping than the increase in the latter. In the new situation, practically all these effects are quantitively larger, varying even more in the direction they were taking. Despite the fact that the standard deviation of consumption is lower, the covariance between it and the demand shock has increased with the maximization of welfare, causing household consumption, although it has a lower volatility than normal, to be more affected by the preference shock than before. The only effect that is softened is the time spent shopping, which now increases less than before. In other words, the effect of increased consumption on time spent shopping still outweighs the effect of increased real money balances, but to a lesser extent.

This preference shock has led to an even greater increase in aggregate demand than with the original Taylor coefficients, moving firms to produce even more than in the original situation. The increase in labor hiring means an increase in the labor cost observed by firms (as workers are paid more and are less productive). In this new situation, the labor cost increases even more than in the original case, moving firms to set even higher prices. Therefore, inflation increases in the short term more than it did in the previous case. As explained in the previous section, the situation of total price flexibility is very sensitive to the technological shock in this model, but very little sensitive to the other two shocks. Even though the potential output increases a little more than in the previous period, the output gap in this new situation is at higher levels than before.

The Central Bank therefore analyzes its target variables: inflation has increased and the output gap has widened. Therefore, it will have to carry out a monetary contraction with higher interest rates as the economy is overheating even more than before. For its part, nominal money growth is initially higher than in the original situation, although it later recovers this initial path. As now the economy suffers wider fluctuations, both the effect of the shock generated by the business cycle and the effect of the nominal interest rate that solves it are greater.

5.5 Comparative analysis – Inflation shock

The impact of the inflation shock is smaller with the optimized coefficients of the Taylor rule compared to the case with its original values (see Figure 6). In fact, unlike the other two cases, this shock now causes a less abrupt business cycle than before.

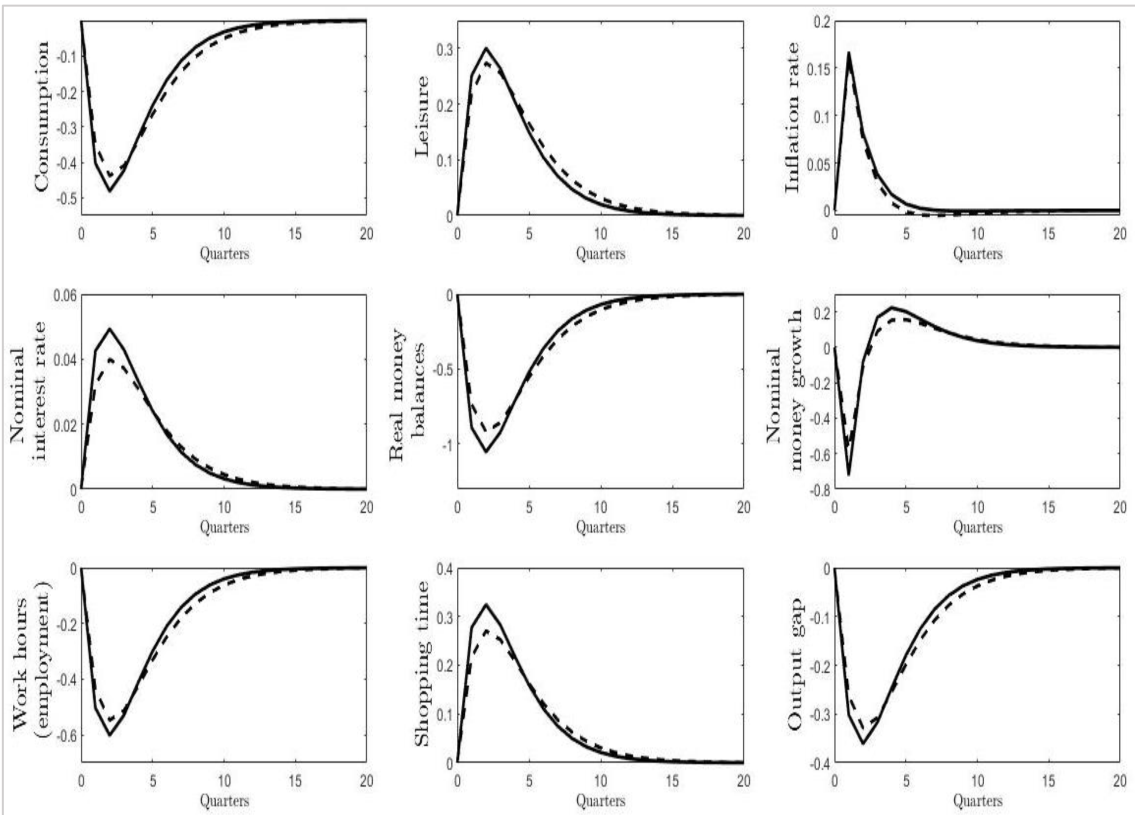

Figure 6. Comparative analysis – Inflation shock

Faced with the inflation shock, households reduce their current consumption, increase their leisure time, decrease their hours worked and increase the time spent on shopping (the effect on the fall of real money balances on shopping time dominates the effect of the fall in consumption). These effects follow the same directions with the optimized monetary policy rule, although in the first quarters more smoothly and in subsequent quarters following a similar path to the original one.

The firms, for their part, behave very similarly to the original situation, observing that the behavior of households has hardly changed. As in the original case, in the face of this inflation shock, prices will rise in the short term. In addition, the decrease in the level of contracting means that firms produce less, reducing the output gap. As can be seen, in this new situation the evolution of the output gap is similar, even slightly milder than in the original situation.

The Central Bank notices that inflation has risen and the output gap has narrowed. It raises nominal rates, solving the problem of inflation, and then lowers them to solve the problem of the output gap. As in this new situation~~,~~ the business cycle observed by the Central Bank is milder, its monetary policy will be less aggressive (it should be recalled that $\mu_\pi = 1.02$ is lower than the original value $\mu_\pi = 1.5$), raising rates less than in the initial situation. The growth of money will follow the path of the initial situation, decreasing initially and growing rapidly in the following periods, although this evolution occurs more smoothly than in the original situation.

In summary, the aggregated fluctuations following a technology shock are largely magnified, the business cycle derived from the demand shock (consumption) increases, but to a lesser extent, and the business cycle derived from the inflation shock is, to some extent, even milder.

It is very important to note that all these effects, such as the optimal parameters that maximize household welfare, are subject to a specific theoretical model, with a specific calibration and specific assumptions. Both in the previous section (business cycle analysis) and in this one (optimal monetary policy) it would be appropriate to carry out a robustness analysis. Although it is beyond the scope of this introductory research work, it would be appropriate to see what happens to the conclusions I have drawn in these sections if they are calibrated differently, if certain variables are included or removed, etc. In other words, it is essential to stress that the conclusions drawn from this work are subject to a very specific model and assumptions.

6. CONCLUSIONS

The paper essentially consists of an approach to research methods for business cycle and monetary policy analysis with a dynamic macroeconomic model. The theoretical model chosen is a New-Keynesian model with shopping time and transactions-facilitating.

The equations of the model are obtained from the rational choices of firms, households and the Central Bank. Firms maximize intertemporal profits subject to a Dixit-Stiglitz demand constraint and nominal rigidities (sticky prices) a la Calvo (1983). Households maximize their expected intertemporal utility subject to both a budget constraint and a time allocation constraint. The Central Bank follows a Taylor-style monetary policy rule (Taylor 1993). Money is included as a means of reducing transaction costs, and thus plays a role in both budgetary and time constraints, with the latter representing the time available to households. Within the time available, one of the activities of the households is shopping. The shopping time is determined by a transactions technology that depends positively on the amount of consumption and negatively on real money holdings.

All these equations represent non-linear relationships, which are very difficult to work with. Therefore, all of them have to go through a subsequent log-linearization process. In consequence, the variables of the log-linearized dynamic model represent the evolution of the variables in relation to the long term (per unit deviations with respect to the constant steady-state values). This makes it easier to work with Matlab and the Dynare extension.

For the calibration of model parameters, I set the value I consider appropriate following the estimation results of papers that use the New Keynesian methodology and looking at realistic values of the steady-state solution of the model. When log-linearizing the equations, several variables appear valued in the long term, so I have to solve the model by valuing it in the steady state and using these values as parameters added to the model.

There are three exogenous variables (shocks) included in the model: a technology shock, a consumption preference shock and an inflation shock. The inclusion of these three shocks guarantees variability in the model's economy. This means that in section 4 it is possible to analyze three impulse-response functions. This allows us to understand how the economy behaves during the first quarters in the face of each external shock: from the expansion derived from the technological shock, through the overheating of the economy derived from the consumption shock, to the recession produced by the inflation shock. In addition, in each subsection I include the mechanisms by which the Central Bank manages to bring the economy to equilibrium in the long term. A series of graphs are included to better observe the evolution of the main variables in the face of each shock.

After observing what happens with the business cycles resulting from each shock, I consider an optimum monetary policy for the Central Bank. The criterion chosen is that of maximizing the welfare of households. To this end, the optimized coefficients for the reaction of the nominal interest rate to changes in inflation or the output gap have been found at $\mu_\pi = 1.02$ and $\mu_{\tilde{y}} = 0.01$. After this, I proceed to compare (also graphically) how the business cycles analyzed in the previous section change from the original ones: from more abrupt business cycles (consumption and technology shocks) to milder (inflation shock). The optimized response of the interest rate set by the central bank turns higher following a consumption shock and lower after a technology shock. The increase in the variability of inflation and the output gap are indicators that business cycles are generally more volatile than before for these two variables. Volatility in leisure also increases, while that in consumption decreases. This optimal monetary policy increases the value of the households' welfare, with a less negative value of its variable part.

The main objective of this work is an introduction to macroeconomic research methods, from which it follows that major simplifying assumptions have been made. It is also essential to bear in mind that the analysis of both the business cycle and the optimal monetary policy are carried out for a very specific model, with very specific assumptions and calibration. Therefore, the conclusions derived from each section must be observed with great caution.

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APPENDIX

1) Derivation the Phillips curve from the FOC of the firms:

Starting from equation [40], I proceed to include how the Phillips curve would be obtained. This demonstration will be carried out without considering the inflation shock, since the inclusion of this would lengthen and obscure the process. I will first demonstrate how to arrive at the optimal price equation (including its log-linearization), and then how to obtain from this the Phillips' curve of the model.

a. Derivation of optimal price log-linearization, $\widehat{P_t}(i)$

In order to make this calculation it is important to take some previous considerations. In the steady state, I assume that the inflation rate is very close to zero, and that there is symmetry between all the firms in terms of prices and marginal labor cost $(\pi = 0.005,\ P(i) = P, \psi(i) = \psi)$. Also, if I rearrange the equation [40] without the external shock, I arrive to the following:

$$\frac{(\theta-1)}{\theta}\left(E_t \sum_{j=0}^{\infty} \beta^j \eta^j \left[\frac{P_{t+j}(i)}{P_{t+j}}\right]^{-\theta} \frac{y_{t+j}}{P_{t+j}}\right) = E_t \sum_{j=0}^{\infty} \beta^j \eta^j \left(\psi_{t+j}(i) \left[\frac{P_{t+j}(i)}{P_{t+j}}\right]^{-\theta-1} \frac{y_{t+j}}{P_{t+j}}\right) \tag{44}$$

And, if I evaluate it in the steady state, it results in the following function

$$\frac{\theta-1}{\theta}[1+\beta\eta+\beta^2\eta^2 \ldots]\frac{y}{p} = \psi[1+\beta\eta+\beta^2\eta^2 \ldots]\frac{y}{p} \tag{45}$$

which, simplifying, becomes $\frac{\theta-1}{\theta} = \psi$. In other words, the inverse of the mark-up of the monopolistically competitive companies is equal to the real marginal labor cost. As I have mentioned, it is not important to specify the firm, because in the long-term perfect symmetry is achieved.

Along with this, it is also appropriate to remember a property that I deduce from log-linearized variables. If a variable is the result of the sum of others, I can obtain its corresponding log-linearized relationship as the weighted sum of each summand with respect to the total. That is

$$a_t = b_t + c_t \leftrightarrow \widehat{a_t} = \frac{b}{a}\widehat{b_t} + \frac{c}{a}\widehat{c_t} \tag{46}$$

Therefore, I can expand the equation knowing that I work with infinite sums

$$\frac{(\theta-1)}{\theta}\left[\left(\left[\frac{P_t(i)}{P_t}\right]^{-\theta}\frac{y_t}{P_t}\right) + \beta\eta E_t\left(\left[\frac{P_{t+1}(i)}{P_{t+1}}\right]^{-\theta}\frac{y_{t+1}}{P_{t+1}}\right) + \cdots\right] = \left[\left(\psi_t(i)\left[\frac{P_t(i)}{P_t}\right]^{-\theta}\frac{y_t}{P_t}\right) + \beta\eta\, E_t\left(\psi_{t+1}(i)\left[\frac{P_{t+1}(i)}{P_{t+1}}\right]^{-\theta}\frac{y_{t+1}}{P_{t+1}}\right) + \cdots\right] \tag{47}$$

Applying the property of log-linearization that I have defined previously [46] and knowing that $\frac{1}{1-\beta\eta} = 1+\beta\eta+\beta^2\eta^2 \ldots \text{for } \beta,\eta < 1$ the equation is transformed

$$\left[\frac{\frac{(\theta-1)}{\theta}\frac{y}{p}}{\frac{(\theta-1)}{\theta}\frac{y}{p}\frac{1}{(1-\beta\eta)}}\right]\left[\left(-\theta\left(\widehat{P_t}(i)-\widehat{P_t}\right)+\widehat{y_t}-\widehat{P_t}\right)+\ \beta\eta E_t\left(-\theta\left(\widehat{P_{t+1}}(i)-\widehat{P_{t+1}}\right)+\right.\right. \quad [48]$$

$$\left.\left.\widehat{y_{t+1}}-\widehat{P_{t+1}}\right)+\cdots\right]=\left[\frac{\psi\frac{y}{p}}{\psi\frac{y}{p}\frac{1}{(1-\beta\eta)}}\right]\left[\left(-(\theta+1)\left(\widehat{P_t}(i)-\widehat{P_t}\right)+\widehat{y_t}-\widehat{P_t}+\widehat{\psi_t}(i)\right)+\right.$$

$$\left.\beta\eta E_t\left(-(\theta+1)\left(\widehat{P_{t+1}}(i)-\widehat{P_{t+1}}\right)+\widehat{y_{t+1}}-\widehat{P_{t+1}}+\widehat{\psi_{t+1}}(i)\right)+\cdots\right]$$

Applying the assumptions of the steady state explained above, grouping equal terms and simplifying

$$\left[-\widehat{P_t}+\ \beta\eta E_t\left(-\widehat{P_{t+1}}\right)+\beta^2\eta^2E_t\left(-\widehat{P_{t+2}}\right)\ldots\right]=\left[-\widehat{P_t}(i)+\widehat{\psi_t}(i)+\right. \quad [49]$$

$$\left.\beta\eta E_t\left(-\widehat{P_{t+1}}(i)+\widehat{\psi_{t+1}}(i)\right)+\beta^2\eta^2E_t\left(-\widehat{P_{t+2}}(i)+\widehat{\psi_{t+2}}(i)\right)\ldots\right]$$

which can also be written as

$$\frac{1}{1-\beta\eta}\widehat{P_t}(i)=E_t\sum_{j=0}^{\infty}\beta^j\eta^j\,\widehat{P_{t+J}}+E_t\sum_{j=0}^{\infty}\beta^j\eta^j\,\widehat{\psi_{t+J}}(i) \quad [50]$$

and clearing the optimal price I obtain the following equation

$$\widehat{P_t}(i)=(1-\beta\eta)\,E_t\ \sum_{j=0}^{\infty}\beta^j\eta^j\left[\widehat{\psi_{t+J}}(i)+\widehat{P_{t+J}}\right] \quad [51]$$

The equation I have obtained establishes the optimal price ratio according to the real marginal labor cost of firm i. As I work with a macro model, I must add the marginal labor cost. I must carry out a series of transformations, because when we find ourselves in a context of price rigidities, we do not have perfect symmetry ($\psi \neq \psi(i)$).

In this model, I work with a specific Cobb-Douglas production function. The most usual representation of the production function is that it depends on: labor, capital and technology. To simplify the procedure, in my model capital is constant and standardized to 1. In addition, technology does not intervene as an endogenous variable like labor. Here we find the third external shock of the model: the technological shock. It represents a sudden change in technology, involving an exponential increase in worker productivity.

$$y_t(i)=e^{z_t}n_t(i)^{1-\alpha} \quad [X]$$

where

$$z_t=\rho_z z_{t-1}+\varepsilon_{z_t} \quad \text{where } \varepsilon_{z_t}\sim N(0,\sigma^2_{\varepsilon_{z_t}}) \text{ and } \rho_z<1 \quad [52]$$

From the definition of labor cost, we know that

$$E_t\widehat{\psi_{t+J}}(i)-\widehat{\psi_{t+J}}=\left(E_t\widehat{w_{t+J}}-\widehat{w_{t+J}}\right)-\left(E_t\widehat{f_{n^d_{t+J}(i)}}-\widehat{f_{n^d_{t+J}}}\right) \quad [53]$$

where $\widehat{f_{n_t^d}} = \hat{y_t} - \widehat{n_t}$.

On the other hand, the production can be interpreted as a Cobb-Douglas and in this specific case as marked by Dixit and Stiglitz. Log-linearizing and clearing the workforce I get

$$\widehat{n_{t+J}^d}(i) - \widehat{n^d{}_{t+J}} = \frac{-\theta}{1-\alpha}\left[\widehat{P_t}(i) - \widehat{P_{t+J}}\right] \quad [54]$$

that relates prices to the hiring of labor. By setting a price above the general level, the firm will reduce the level of production and with it the workforce it hires.

By log-linearizing the Dixit-Stiglitz output function [35], I can obtain the difference with respect to the general level of labor productivity of the firm i

$$\widehat{f_{n_{t+J}^d(\iota)}} - \widehat{f_{n_{t+J}^d}} = \frac{\theta\alpha}{1-\alpha}\left[\widehat{P_t}(i) - \widehat{P_{t+J}}\right] \quad [55]$$

Introducing this into my initial equation [53] and assuming that wages will be the same in all firms, the relationship between the labor cost of firm i and the aggregate is

$$E_t\widehat{\psi_{t+J}}(i) = \widehat{\psi_{t+J}} - \frac{\theta\alpha}{1-\alpha}\left[\widehat{P_t}(i) - \widehat{P_{t+J}}\right] \quad [56]$$

I can introduce this relationship into our price equation [51], thus making it suitable for a macro model. After some algebra, the optimal price is cleared up as follows

$$\widehat{P_t}(i) = \frac{(1-\beta\eta)}{\left(1+\frac{\theta\alpha}{1-\alpha}\right)}\, E_t \sum_{j=0}^{\infty} \beta^j \eta^j \left[\widehat{\psi_{t+J}} + \left(1 + \frac{\theta\alpha}{1-\alpha}\right)\widehat{P_{t+J}}\right] \quad [57]$$

which maintains the positive relationship between the optimal price and expectations about the general level of prices and the general level of real marginal costs.

b. Deriving the Phillips curve from the optimal price equation

By log-linearizing the Dixit-Stiglitz aggregate price level equation with Calvo sticky-prices [38] without the shock, I have

$$\hat{P}_t - \hat{P}_{t-1} = \frac{(1-\eta)}{\eta}\left(\hat{P}_t(i) - \hat{P}_t\right) \quad [58]$$

From the properties of log-linearization and logarithms we know that

$$\hat{P}_t - \hat{P}_{t-1} = \log\left(\frac{P_t}{P}\right) - \log\left(\frac{P_{t-1}}{P}\right) = \log(P_t) - \log(P_{t-1}) \cong \frac{P_t - P_{t-1}}{P_{t-1}} = \pi_t \quad [59]$$

resulting in the following relationship between inflation and relative prices

$$\pi_t = \frac{(1-\eta)}{\eta}\left(\hat{P}_t(i) - \hat{P}_t\right) \quad [60]$$

From this equation, and using a series of equalities, I can arrive at my final equation, the Phillips curve. Firstly, I use $\widehat{P_{t+J}} = \widehat{P}_t + \sum_{k=1}^{j} \widehat{\pi_{t+k}}$ to modify the right side of the equation. After several operations, I obtain

$$\widehat{P}_t(i) - \widehat{P}_t = \frac{(1-\beta\eta)}{\left(1+\frac{\theta\alpha}{1-\alpha}\right)} E_t \sum_{j=0}^{\infty} \beta^j \eta^j \left[\widehat{\psi_{t+J}} + \left(1+\frac{\theta\alpha}{1-\alpha}\right) \sum_{k=1}^{j} \pi_{t+k}\right] \qquad [61]$$

Before advancing any further I will work on the double sum operators and simplify them. By definition of the operators

$$E_t \sum_{j=0}^{\infty} \beta^j \eta^j \left(\sum_{k=1}^{j} \pi_{t+k}\right) = \beta\eta E_t \pi_{t+1} + \beta^2\eta^2 (E_t \pi_{t+1} + E_t \pi_{t+2}) + \beta^3\eta^3 (E_t \pi_{t+1} + E_t \pi_{t+2} + E_t \pi_{t+3}) \dots \qquad [62]$$

Knowing the following property of an infinite sum (for $|\beta|$ and $|\eta|$ ϵ (0,1))

$$\frac{\beta^h\eta^h}{1-\beta\eta} = (\beta^h\eta^h + \beta^{h+1}\eta^{h+1} + \beta^{h+2}\eta^{h+2} \dots) \qquad [63]$$

And from the concrete case h = 0, I can simplify the equation to

$$E_t \sum_{j=0}^{\infty} \beta^j \eta^j \left(\sum_{k=1}^{j} \pi_{t+k}\right) = \frac{1}{1-\beta\eta} [\beta\eta E_t\pi_{t+1} + \beta^2\eta^2 E_t\pi_{t+2} + \beta^3\eta^3 E_t\pi_{t+3} \dots] \qquad [64]$$

and by applying the definition of summation

$$E_t \sum_{j=0}^{\infty} \beta^j \eta^j \left(\sum_{k=1}^{j} \pi_{t+k}\right) = \frac{1}{1-\beta\eta} \sum_{j=1}^{\infty} \beta^j \eta^j E_t \pi_{t+j} \qquad [65]$$

Which I can substitute in equation [61], and that back into [60] to clear the inflation:

$$\pi_t = \left[\frac{(1-\beta\eta)(1-\eta)}{\left(1+\frac{\theta\alpha}{1-\alpha}\right)\eta}\right] E_t \sum_{j=0}^{\infty} \beta^j \eta^j \left[\widehat{\psi_{t+J}}\right] + \frac{(1-\eta)}{\eta} E_t \sum_{j=1}^{\infty} \beta^j \eta^j \pi_{t+j} \qquad [66]$$

Since I am interested in the evolution of inflation in relation to future expectations, I establish equation [66] in terms of t+1. This equation is

$$E_t\pi_{t+1} = \left[\frac{(1-\beta\eta)(1-\eta)}{\left(1+\frac{\theta\alpha}{1-\alpha}\right)\eta}\right] E_t \sum_{j=0}^{\infty} \beta^j \eta^j \left[\widehat{\psi_{t+J+1}}\right] + \frac{(1-\eta)}{\eta} E_t \sum_{j=1}^{\infty} \beta^j \eta^j \pi_{t+j+1} \qquad [67]$$

Once this equation is found, I make the difference between current inflation in t [66] and the inflation expectation in the period t+1[67] (both in turn as differences from the long-term value). By rearranging the variables and simplifying the equation, I arrive at the Phillips curve of my Neo-Keynesian model without the shock.

$$\pi_t - \pi = (\beta E_t \pi_{t+1} - \pi) + \left[\frac{(1-\beta\eta)(1-\eta)}{\left(1+\frac{\theta\alpha}{1-\alpha}\right)\eta}\right] \widehat{\psi_t} \qquad [68]$$

2) Deriving non-linear equations in Steady State

Here I present the steady state equations used for the model

$$\frac{1}{1+r} = \beta \qquad [69]$$

$$s = \emptyset_0 + \emptyset_1 c \left(\frac{c}{m}\right)^{\emptyset_2} \qquad [70]$$

$$s_c = \emptyset_1 \emptyset_2 \left(\frac{c}{m}\right)^{\emptyset_2} \qquad [71]$$

$$s_m = -\emptyset_1 \emptyset_2 \left(\frac{c}{m}\right)^{(1+\emptyset_2)} \qquad [72]$$

$$T = s + n + l \qquad [73]$$

$$-w s_m = \frac{R}{1+R} \qquad [74]$$

$$\Xi(l)^{-\gamma} = \frac{c^{-\sigma}}{(1+w s_c)} \qquad [75]$$

$$(1+R) = (1+r) \qquad [76]$$

$$y = n^{1-\alpha} \qquad [77]$$

$$\frac{\theta-1}{\theta} = \psi \qquad [78]$$

$$\psi = \frac{w}{(1-\propto)n^{\alpha}} \qquad [79]$$

$$y = c + \delta k \qquad [80]$$

which can be used to find numerical solutions for 12 variables: y, c, l, n, s, s_c, s_m, w, ψ, r, R and m.

As we can see, there are 12 equations with 12 variables. The first equation [69] represents the intertemporal allocation of consumption in the steady state. In this particular situation, consumption itself disappears, leading to an equation that equates the objective discount factor (the interest rate, r) to the subjective discount factor (included in β).

In addition, as can be seen from the equation [76], the nominal interest rate and the real interest rate are the same in this case. This is an approximation, because I have assumed that inflation in the long term is very close to 0. Analyzing the current context, in which prices hardly vary from one year to another, this assumption does not seem unreasonable.

The last equation I have included is the over-all resources constraint, which establishes a relationship that incorporates the restriction of households and the government. It is also important to note that I have assumed in the model that capital remains constant, in order to simplify the work. To arrive at this constraint, it is necessary to make some calculations. Taking into account that capital is constant, the restriction for households is as follows

$$w_t n_t^s + r^k k + d_t + g_t = c_t + \delta k + \frac{b_{t+1}}{(1+r_t)} - b_t + m_t - \frac{m_{t-1}}{(1+\pi_t)} \quad [81]$$

d_t are the aggregate dividends

$$d_t = \sum_{i=1}^{k} \left(\frac{P_t(i)}{P_t}\right)^{1-\theta} y_t(i) - w_t \sum_{i=1}^{k} n_t(i) - r^k k \quad [82]$$

where k is the total number of firms. Simplifying the sums, I get

$$d_t = y_t - w_t n_t - r^k k \quad [83]$$

On the other hand, the government constraint is

$$g_t = \frac{b_{t+1}}{(1+r_t)} - b_t + m_t - \frac{m_{t-1}}{(1+\pi_t)} \quad [84]$$

By replacing the function of dividends and public expenditure, the over-all resources equation [80] is simplified to the following

$$y_t = c_t + r^k k$$

3) Deriving the welfare function of households:

As mentioned in the paper, to maximize the utility function I use Taylor's second-order approximations. Logically, as the order of the approximation increases, it becomes more accurate. This Taylor series basically consists of approximating a function by means of the weighted sum of diminishing order derivatives valued around a certain point, which in my case will be the value in the steady state. The general formula for Taylor's second order approximation is as follows:

$$f(x_t) = f(x) + \frac{f'(x)}{1!}(x_t - x) + \frac{f''(x)}{2!}(x_t - x)^2 \quad [85]$$

where x_t is the current value of the variable and x is the long-term value.

The function to be maximized comes from the following instantaneous utility function (IUF, [1]):

$$U(c_t, l_t) = e^{v_t} \frac{c_t^{1-\sigma}}{1-\sigma} + \epsilon \frac{l_t^{1-\gamma}}{1-\gamma}$$

To make the task easier, I separate the role in the contribution that consumption makes to households from that of leisure:

$$U(c_t, l_t) = U_t^c + U_t^l \;\; where \; U_t^c = e^{v_t} \frac{c_t^{1-\sigma}}{1-\sigma} \; and \;\; U_t^l = \Xi \frac{l_t^{1-\gamma}}{1-\gamma}$$

a) *Taylor's second order approximation for* U_t^c:

Starting from the following equation:

$$U_t^c = e^{v_t} \frac{c_t^{1-\sigma}}{1-\sigma} \qquad [86]$$

The second-order approximation becomes:

$$U_t^c \cong \frac{c^{1-\sigma}}{1-\sigma} + c^{-\sigma}(c_t - c) + \frac{c^{1-\sigma}}{1-\sigma}(e^{v_t} - e^0) - \frac{\sigma c^{-\sigma-1}}{2}(c_t - c)^2 + c^{-\sigma}(c_t - c)(e^{v_t} - e^0) \qquad [87]$$

For close to zero values, as will be the case with shock, the following approximation (3) can be applied:

$$e^{v_t} - 1 \cong v_t \qquad (3)$$

The (non-zero) unconditional expected value of the instantaneous function of consumption utility remains:

$$E(U_t^c) \cong \frac{c^{1-\sigma}}{1-\sigma} + c^{-\sigma}E(c_t - c) + \frac{c^{1-\sigma}}{1-\sigma}E(v_t) - \frac{\sigma c^{-\sigma-1}}{2}E(c_t - c)^2 + c^{-\sigma}E((c_t - c)v_t) \qquad [88]$$

Knowing that $E(c_t) = c$ and $E(v_t) = 0$, the function is as follows:

$$E(U_t^c) \cong \frac{c^{1-\sigma}}{1-\sigma} - \frac{\sigma c^{-\sigma-1}}{2} var(c_t) + c^{-\sigma} cov(c_t, v_t) \qquad [89]$$

b) *Taylor's second-order approximation to* U_t^l:

The procedure to be followed with this function is the same as that carried out with consumption, even simpler as it does not include any shock in this part, and I avoid intermediate steps. The second order approximation gives:

$$U_t^l \cong \Xi \frac{l^{1-\gamma}}{1-\gamma} + \Xi l^{-\gamma}(l_t - l) - \Xi \frac{\gamma c^{-\gamma-1}}{2}(l_t - l)^2 \qquad [90]$$

Following the above logic, the (non-zero) unconditional expected value is as follows:

$$E(U_t^l) \cong \Xi \frac{l^{1-\gamma}}{1-\gamma} - \Xi \frac{\gamma c^{-\gamma-1}}{2} var(l_t) \qquad [91]$$

c) *Obtaining the full second-order approximation*

Since the aim is to maximize the utility function of households over an infinite period, the equations obtained in the previous two sub-sections are added, taking into account the intertemporal discount rate, leaving:

$$\sum_{j=0}^{\infty} \beta^j E\left(U\left(c_{t+j}, l_{t+j}\right)\right) = \sum_{j=0}^{\infty} \beta^j \left(E(U_{t+j}^{c}) + E(U_{t+j}^{l})\right) \quad [92]$$

Knowing that the infinite sum of a constant discount rate equals the following fraction

$$\sum_{j=0}^{\infty} \beta^j = \frac{1}{1-\beta} \quad [93]$$

The utility function of household welfare [41] to be maximized is as follows:

$$\sum_{j=0}^{\infty} \beta^j E\left(U\left(c_{t+j}, l_{t+j}\right)\right) = \frac{1}{1-\beta}\left(\frac{c^{1-\sigma}}{1-\sigma} + \Xi\frac{l^{1-\gamma}}{1-\gamma} + c^{-\sigma} cov(c_t, v_t) - \frac{\sigma c^{-\sigma-1}}{2} var(c_t) - \Xi\frac{\gamma l^{-\gamma-1}}{2} var(l_t)\right)$$

where the two constant members have been grouped in the first part, and the variable components in the second part.