Industrial Engineering, Informatics and Telecommunications College

Mathematical study of catenaries and their application in the real assembly of a 20 kV overhead electrical network. Regulatory analysis.

Final Degree Project

Engineering degree in Industrial Technologies

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MATHEMATICAL STUDY OF CATENARIES AND REGULATORY ANALYSIS
Saturnino Sola Giménez

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I hope I have made the most of this opportunity.

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#### Abstract

In the present study, hyperbolic mathematics will be applied to obtain the exact equations that define a catenary. This will seek to demonstrate how, based on calculation tools, there is the possibility of analyzing the characteristics and mathematical properties of a power line without the need to resort to approximations that can lead to significant errors.

The application and compliance with the High Voltage power lines Regulations will be studied in depth. Among other topics, the application of the equation of change of conditions or the study of minimum safety distances in different situations will be discussed. The development of calculation programs for solving problems related to the design of overhead power lines, complements the theoretical analysis of the study. The report concludes with the presentation of a real application on a support tower, going in depth in the analysis of the regulatory requirements.


## Resumen

En el presente trabajo se va a aplicar la matemática hiperbólica para la obtención de las ecuaciones exactas que definen una catenaria. Se buscará demostrar cómo, a partir de herramientas de cálculo, existe la posibilidad de analizar las características y propiedades matemáticas de una línea aérea sin necesidad de recurrir a aproximaciones que puedan llevar a errores significantes.

Se profundizará en la aplicación y cumplimiento del Reglamento de líneas aéreas de Alta Tensión. Entre otros temas se tratará la aplicación de la ecuación de cambio de condiciones o el estudio de distancias mínimas de seguridad en diferentes situaciones. La elaboración de programas de cálculo para la resolución de problemas relacionados con el diseño de líneas aéreas eléctricas, complementa el análisis teórico del estudio. El trabajo concluye con la exposición de una aplicación real sobre una torre de apoyo profundizando en el análisis de las prescripciones reglamentarias.

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## Keyword list

- Hyperbolic trigonometry
- Catenary
- Electric overhead power lines
- Equation of change of conditions
- Overhead power lines crossovers


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## 1. Objectives and justification

With the intention of drawing conclusions in relation to current regulations on the matter, this study requires first of all a mathematical study that later allows to analyze the results obtained and evaluate them. Today the electrical world is expanding and the need for a large infrastructure is undeniable. Examples such as the electrification of transport show the growing dependence on electricity transport from generation plants to places of consumption.

The most widely used means of electrical transport is through high-voltage overhead power lines. It is not easy to appreciate a landscape that does not contain a power line crossing it. That is why studying the coexistence of these structures with their environment and knowing the mathematics behind any power line will be essential to provide future solutions in the most cohesive way. Throughout the drafting of the study, various topics will be discussed that will help to plan the design of power lines and to program essential calculation tools for the construction of any line in different conditions and environments. As a summary, the report will seek to collect the set of writings, calculations and conclusions that reflect the magnitude and relevance of the design of electric overhead lines.

Related to the objectives of the technical study, it is worth mentioning the Sustainable Development Goals (SDGs) related to the area studied. Among the different objectives, all of them included in the 2030 Agenda, this report shares similarities with three of them:

- SDG7. Ensure access to affordable, reliable, sustainable and modern energy for all.
- SDG9. Build resilient infrastructure, promote inclusive and sustainable industrialization and foster innovation.
- SDG15. Protect, restore and promote the sustainable use of terrestrial ecosystems, sustainably manage forests, combat desertification and halt and reverse land degradation, and halt biodiversity loss.

By way of justification, in the case of the first objective mentioned, it will be the mathematical and technical study of the power lines that will allow us to know the possibilities of construction of new structures that guarantee the supply of energy in any location. Regarding the second objective, it is hoped to be able to adopt sufficient knowledge that will help to propose solutions and alternatives to current problems in the future, promoting sustainability and innovation. Finally, mention will also be made of the security measures to be taken, especially in the case of high-voltage overhead lines, to guarantee the preservation of the birdlife through the necessary equipment and protections.

As a whole, this report will seek to meet future requests and needs regarding the construction of overhead lines from design to compliance with regulations. With this, the different emerging problems raised by the regulations will be addressed, responding to the needs of the study.

## 2. Standards and references

Regarding the legal framework, it must be adopted the current regulations regarding the power lines that are being studied. In this case, Royal Decree 223/2008, of February 15, which approves the Regulation on technical conditions and safety guarantees in high voltage power lines and its complementary technical instructions ITC-LAT 01 to 09 ("BOE" 03/19/08). Special mention deserves, as will be specified throughout the report, the complementary technical instruction ITC-LAT 07 which includes the specifications of overhead lines with bare conductors.

With regard to bare conductors, the different "UNE" standards accepted by the Spanish Association for Standardization will be used. As will be introduced throughout the study, the UNE-EN 50182 standard should be highlighted, which presents the properties of conductors for overhead power lines of round wires cabled in concentric layers. As a reference throughout the study, when not specified, 47-AL1/8ST1A (old designation LA56) will be taken as universal conductor due to its widespread use in the electrical world of transportation.

## 3. Introduction to hyperbolic trigonometry

The triangle is a basic figure in the study of mathematics. Plane trigonometry is a branch of mathematics, whose etymological meaning comes from the Greek terms trigōnos 'triangle' and metron 'measure' ('the measurement of triangles'). Although in the history of mathematics the applications of trigonometry are based on the right-angled triangle, the scope of trigonometry is much more than that. While planar trigonometry is already established in popular knowledge, the hyperbolic application arises not being as well known. A less studied dimension due to its complex appearance. What is really interesting is that what is called "hyperbolic trigonometry", without strictly following the definition of trigonometry, presents an operational similarity to the equations of plane trigonometry that justify its baptism. Subsequently, his surname "hyperbolic" is justified by its growth following the exponential functions based on " $e$ ".
in different external annexes.
The elementary functions that we consider transcendent to be used throughout the report are listed below. On the other hand, certain justifications and deductions will be reflected

$y_{6}{ }^{2}-y_{4}{ }^{2}=1 \quad \cosh ^{2} x-\sinh ^{2} x=1 \quad$ [equation 4]

## GRAPHS:


3.1 Inverse functions


| PLANE | TRIGONOMETRY | EXPONENTIAL FUNCTION | HYPERB | LIC TRIGONOMETRY |
| :---: | :---: | :---: | :---: | :---: |
| $y=\sin x$ | Inverse function $x=\sin y$ <br> ' $y$ ' is the angle (arc) whose sine is ' $x$ ' $y=\arcsin x$ | $y_{4}=\frac{2}{2} \quad{ }_{\text {We clear }}\left(\begin{array}{c} x=\frac{2}{2} \end{array}\right.$ | $y_{4}=\sinh x$ | ' $y$ ' is the value (argument) whose hyperbolic sine is ' $x$ ' |
|  | [graph 9] | value of ' $y$ ' by giving <br> values to ' $x$ '$\quad y=$ ?? |  | [graph 11] |
| $y=\cos x$ | $\xrightarrow{\text { Inversef function }} x=\cos y$ | $y_{6}=\frac{e^{x}+e^{-x}}{2} \xrightarrow{\text { Inverse function }} x=\frac{e^{y}+e^{-y}}{2}$ | $y_{6}=\cosh x$ | $\xrightarrow[\begin{array}{c} \text { 'y' is the value (argument) } \\ \text { whose hyperbolic cosine is ' } x^{\prime} \end{array}]{\text { Inverunction }} \quad x=\cosh y$ |
|  | ' $y$ ' is the angle (arc) whose cosine is ' $x$ ' $y=\arccos x$ | It is not easy to get the |  |  |
|  | [graph 10] | value of '' by giving <br> values to ' $x$$\quad y_{y}=$ ?? |  |  |

## GRAPHS:

FUNCTIONS


Graph 1


Graph 2


Graph 6


INVERSE FUNCTIONS


Graph 9


Graph 10


Differentiation of the inverse functions of the sine and the hyperbolic sine:

| PLANE TRIGONOMETRY | EXPONENTIAL FUNCTION | HYPERBBOLIC TRIGONOMETRY |
| :---: | :---: | :---: |
| $\begin{aligned} & \substack{y=\sin x \\ \text { We derive } \\ y^{\prime}=\cos x} \\ & \text { We derive }\left\{\begin{array}{l} \sin y=x \\ \cos y \cdot y^{\prime}=1 \end{array}\right. \\ & y^{\prime}=\frac{1}{\cos y}=\frac{1}{\sqrt{1-\sin ^{2} y}=? ?}=\frac{1}{\sqrt{1-x^{2}}} \end{aligned}$ |  | $\begin{aligned} & \begin{array}{c} y_{4}=\sinh x \\ \text { We derive } \\ y_{4^{\prime}}=\cosh x \end{array} \xrightarrow{\text { Inversefunction }} y=\operatorname{argsinh} x \\ & y^{\prime}=? ? \\ & \text { We derive } \end{aligned} \begin{aligned} & \sinh y=x \\ & \cosh y \cdot y^{\prime}=1 \end{aligned} y_{\text {We transform it }} \begin{aligned} & \text { ( }=\frac{1}{\cosh y}=\frac{1}{\sqrt{1+\sinh ^{2} y}}=\frac{1}{\sqrt{1+x^{2}}} \end{aligned}$ |
| We proceed to study the general case in which ' $x$ ' becomes $f(x)$. If we start from the function ... $y^{\prime}=\frac{f^{\prime}(x)}{\cos y}=\frac{f^{\prime}(x)}{\sqrt{1-\sin ^{2} y}}=\frac{f^{\prime}(x)}{\sqrt{1-f(x)^{2}}}$ |  | We proceed to study the general case in which ' $x$ ' becomes $f(x)$. If we start from the function ... $y^{\prime}=\frac{f^{\prime}(x)}{\cosh y}=\frac{f^{\prime}(x)}{\sqrt{1+\sinh ^{2} y}}=\frac{f^{\prime}(x)}{\sqrt{1+f(x)^{2}}}$ |
| $y^{\prime}=\frac{f^{\prime}(x)}{\sqrt{1-f(x)^{2}}} \quad$ [equation 5] |  | $y^{\prime}=\frac{f^{\prime}(x)}{\sqrt{1+f(x)^{2}}}$ <br> [equation 6] |

Differentiation of inverse cosine and hyperbolic cosine functions:

| PLANE TRIGONOMETRY | EXPONENTIAL FUNCTION | HYPERBOLIC TRIGONOMETRY |
| :---: | :---: | :---: |
|  |  |  |
| We proceed to study the general case in which ' $x$ ' becomes $f(x)$. If we start from the function ... $\begin{aligned} & \text { We derive }\left\{\begin{array}{c} y=\arccos f(x) \\ \cos y=f(x) \\ -\sin y \cdot y^{\prime}=f^{\prime}(x) \end{array}\right. \\ & y^{\prime}=\frac{-f^{\prime}(x)}{\sin y}=\frac{-f^{\prime}(x)}{\sqrt{1-\cos ^{2} y}}=\frac{-f^{\prime}(x)}{\sqrt{1-f(x)^{2}}} \end{aligned}$ |  | We proceed to study the general case in which ' $x$ ' becomes $f(x)$. If we start from the function ... $\begin{array}{r} y=\operatorname{argcosh} f(x) \\ \text { We derive }\left(\begin{array}{c} \cosh y=f(x) \\ \sinh y \cdot y^{\prime}=f^{\prime}(x) \end{array}\right. \\ y^{\prime}=\frac{f^{\prime}(x)}{\sinh y}=\frac{f^{\prime}(x)}{\sqrt{-1+\cosh ^{2} y}}=\frac{f^{\prime}(x)}{\sqrt{-1+f(x)^{2}}} \end{array}$ |
| $y^{\prime}=\frac{-f^{\prime}(x)}{\sqrt{1-f(x)^{2}}} \quad$ [equation 7] |  | $y^{\prime}=\frac{f^{\prime}(x)}{\sqrt{-1+f(x)^{2}}}$ <br> [equation 8 ] |


| ANOTHER UTILITY OF PLANE AND HYPERBOLIC TRIGONOMETRIES |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| In the resolution of differential equations, terms arise under a square root and depending on the signs we can make a change of vari $f(x)=\sin t$ of $f(x)=\sinh t$ eliminating, with ease, the square root. |  |  |  |  |
| PLANE TRIGONOMETR |  | EXPONENTIAL FUNCTION | HYPERBOLIC TRIGONOMETRY |  |
| $\sin ^{2} x+\cos ^{2} x=1$ | [equation 1] |  | $\cosh ^{2} x-\sinh ^{2} x=1$ | [equation 4] |
| $y=\arcsin f(x) \rightarrow y^{\prime}=\frac{f^{\prime}(x)}{\sqrt{1-f(x)^{2}}}$ | [equation 5] |  | $y=\operatorname{argsinh} f(x) \rightarrow y^{\prime}=\frac{f^{\prime}(x)}{\sqrt{1+f(x)^{2}}}$ | [equation 6] |
| $y=\arccos f(x) \rightarrow y^{\prime}=\frac{-f^{\prime}(x)}{\sqrt{1-f(x)^{2}}}$ | [equation 7] |  | $y=\operatorname{argcosh} f(x) \rightarrow y^{\prime}=\frac{f^{\prime}(x)}{\sqrt{-1+f(x)^{2}}}$ | [equation 8 ] |

[^0]
## 4. Catenaries

### 4.1 Definition

Catenary: Conductor in balance working under traction and under the action of his own weight.


Calling ' $w$ ' the weight of the conductor/unit of length.


$$
\begin{aligned}
& \sin \alpha=\frac{w \cdot l}{T}=\frac{d y}{d l} \\
& \cos \alpha=\frac{T_{0}}{T}=\frac{d x}{d l} \\
& \tan \alpha=\frac{w \cdot l}{T_{0}}=\frac{d y}{d x}
\end{aligned}
$$

In the figure the points $A$ and $B$ represent the supports of the power line whereas $C$ and $D$ are the lowest point and a generic point of the catenary, respectively.

The partial piece $\overline{C D}$ is then isolated to study the forces acting on it: $T$ and $T_{0}$ as clamping forces and $w \cdot l$ the weight of the piece $\overline{C D}$.

Mentioned piece is in equilibrium as well. Proof of this is that the sum of forces is equal to zero. Then a system of three differential equations is obtained. The first two of the Sine and Cosine work the ' l ' as a function of ' y ' $(l=f(y)$ ) and ' x ' $(l=f(x)$ ), unlike the third equation, the Tangent, which works the ' $y$ ' as a function of the ' x ' $(y=f(x)$ ) (Cartesian coordinates with which it is customary to work)

However, in this last differential equation, 'l' (as it is unknown) is a function of 'x' and 'y' ( $l=f(x, y)$ ) so it cannot be easily solved until an expression of 'l' has been obtained in function of ' $x$ ' or ' $y$ '.

The expression of $l=f(x)$ is calculated using the equation of $\cos \alpha$.
$1^{\text {st }}$ Step: $\cos \alpha=\frac{T_{0}}{T}=\frac{d x}{d l}$

$$
\begin{aligned}
& \frac{T_{0}}{T}=\frac{d x}{d l} \\
& \frac{T_{0}}{\sqrt{(w \cdot l)^{2}+T_{0}^{2}}}=\frac{d x}{d l} \\
& \frac{T_{0}}{\sqrt{(w \cdot l)^{2}+T_{0}^{2}}} \cdot d l=d x
\end{aligned}
$$

A differential equation of separate variables is obtained It is solved by integrating both members of the equality

$$
\int_{0}^{l} \frac{T_{0}}{\sqrt{(w \cdot l)^{2}+T_{0}^{2}}} \cdot d l=\int_{0}^{x} d x
$$

$1^{\text {st }}$ member

$$
\begin{gathered}
\int_{0}^{l} \frac{T_{0}}{\sqrt{(w \cdot l)^{2}+T_{0}^{2}}} \cdot d l=\frac{T_{0}}{w} \cdot \int_{0}^{l} \frac{w / T_{0}}{\sqrt{\frac{(w \cdot l)^{2}+T_{0}^{2}}{T_{0}^{2}}}} \cdot d l= \\
=\frac{T_{0}}{w} \cdot \int_{0}^{l} \frac{w / T_{0}}{\sqrt{1+\left(\frac{w \cdot l}{T_{0}}\right)^{2}}} \cdot d l=\left\{\begin{array}{c}
\text { Page 12 } \\
\text { [equation } 6]
\end{array}\right\}=\frac{T_{0}}{w} \cdot\left[\arg \sinh \left(\frac{w \cdot l}{T_{0}}\right)\right]_{0}^{l}= \\
=\frac{T_{0}}{w} \cdot\left[\arg \sinh \left(\frac{w \cdot l}{T_{0}}\right)-\arg \sinh 0\right]=\frac{T_{0}}{w} \cdot \arg \sinh \left(\frac{w \cdot l}{T_{0}}\right)
\end{gathered}
$$

$2^{\text {nd }}$ member

$$
\int_{0}^{x} d x=[x]_{0}^{x}=x-0=x
$$

Once each member is resolved, they are matched again

$$
\begin{aligned}
& \frac{T_{0}}{w} \cdot \arg \sinh \left(\frac{w \cdot l}{T_{0}}\right)=x \\
& \arg \sinh \left(\frac{w \cdot l}{T_{0}}\right)=\frac{w}{T_{0}} \cdot x \\
& \frac{w \cdot l}{T_{0}}=\sinh \left(\frac{w}{T_{0}} \cdot x\right) \\
& \quad l=\frac{T_{0}}{w} \cdot \sinh \left(\frac{w}{T_{0}} \cdot x\right) \quad \text { [equation 10] }
\end{aligned}
$$

REMARK: Calculating the expression for $l=f(y)$, using the expression for $\sin \alpha=\frac{d y}{d l}$, the equation obtained is (justified in external annex 5):

$$
l=\frac{T_{0}}{w} \cdot \sqrt{\left[\frac{w}{T_{0}} \cdot\left(y-c+\frac{T_{0}}{w}\right)\right]^{2}-1} \quad \text { [equation 11] }
$$

Continuing with the equation $l=f(x)$ [equation 10], the equation of the $\tan \alpha$ is solved by substituting the value obtained from ' l '.
$2^{\text {nd }}$ Step: $\tan \alpha=\frac{w \cdot l}{T_{0}}=\frac{d y}{d x}$

$$
\begin{gathered}
\frac{w \cdot l}{T_{0}}=\frac{d y}{d x} \\
\frac{w}{T_{0}} \cdot \frac{T_{0}}{w} \cdot \sinh \left(\frac{w}{T_{0}} \cdot x\right)=\frac{d y}{d x} \\
\sinh \left(\frac{w}{T_{0}} \cdot x\right) \cdot d x=d y
\end{gathered}
$$

A differential equation of separate variables is obtained It is solved by integrating both members of the equality

$$
\int_{0}^{x} \sinh \left(\frac{w}{T_{0}} \cdot x\right) \cdot d x=\int_{c}^{y} d y
$$

$1^{\text {st }}$ member

$$
\begin{gathered}
\int_{0}^{x} \sinh \left(\frac{w}{T_{0}} \cdot x\right) \cdot d x=\frac{T_{0}}{w} \cdot \int_{0}^{x} \frac{w}{T_{0}} \cdot \sinh \left(\frac{w}{T_{0}} \cdot x\right) \cdot d x= \\
=\frac{T_{0}}{w} \cdot\left[\cosh \left(\frac{w}{T_{0}} \cdot x\right)\right]_{0}^{x}=\frac{T_{0}}{w} \cdot\left[\cosh \left(\frac{w}{T_{0}} \cdot x\right)-\cosh 0\right]=\frac{T_{0}}{w} \cdot\left[\cosh \left(\frac{w}{T_{0}} \cdot x\right)-1\right]
\end{gathered}
$$

$2^{\text {nd }}$ member

$$
\int_{c}^{y} d y=[y]_{c}^{y}=y-c
$$

Once each member is resolved, they are matched again

$$
\begin{aligned}
& \frac{T_{0}}{w} \cdot\left[\cosh \left(\frac{w}{T_{o}} \cdot x\right)-1\right]=y-c \\
& y=\frac{T_{0}}{w} \cdot\left[\cosh \left(\frac{w}{T_{o}} \cdot x\right)-1\right]+c \quad[\text { equation 12] }
\end{aligned}
$$

Iff $\frac{T_{0}}{w}=c$ [equation 13]
This value of ' $c$ ' is called the "catenary parameter"

$$
\begin{aligned}
& y=c \cdot\left[\cosh \left(\frac{1}{c} \cdot x\right)-1\right]+c \\
& y=c \cdot\left[\cosh \left(\frac{x}{c}\right)-1\right]+c \\
& y=c \cdot \cosh \left(\frac{x}{c}\right)-c+c \\
& \quad y=c \cdot \cosh \left(\frac{x}{c}\right) \quad \text { [equation 14] }
\end{aligned}
$$

4.2 Compilation of equations and new expressions
To start working with real hypothetical catenary problems, the following equations obtained in the previous study are collected:
[equation 9]
[equation 10]
[equation 11]
[equation 12]

$y=\frac{T_{0}}{w} \cdot\left[\cosh \left(\frac{w \cdot x}{T_{0}}\right)-1\right]+c$
Using the catenary parameter 'c', the equations obtained are transformed into simpler ones.

Another meaningful expression can also be obtained.
If the [equation 11] is replaced in the [equation 9]:

And if it is applied, on the latter, the [equation 13] $\left\{\frac{T_{0}}{w}=c\right\}$ :

[^1]$T=w \cdot y$

### 4.3 Numerical management of the catenary

Three situations are foreseen that will be analyzed under their respective numerical example:

- Situation 1. Two supports at equal height, knowing the span and the sag.
- Situation 2. Shortening of conductor length

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- Situation 3. Cable subjected to maximum mechanical stress Page 25

All transcendent equations are solved with a calculator.

## Starting data:

The cable used in high voltage lines, formerly called LA56 and currently 47-AL1/8-ST1A (UNE 21018 ACSR), has the following standard values:

- Mass/unit length $=188.8 \mathrm{~kg} / \mathrm{km}$
- Tensile strength $=16.29 \mathrm{kN}$


### 4.3.1 Exercise 1. Calculation of the parameter " $c^{\prime \prime}, T_{\max }, T_{\min }$ and L knowing the span and sag

The exercise that is going to be considered below deals with a real case in which is desired to ensure that, either is wanted to raise two of the posts (fixed points) of a high voltage line in height, or else, is desired to check that with the arrival of winter the cold will not subject the cable to such a contraction as to break it, the maximum strain that the cable supports never exceeds the breaking stress.

To do this, a cable is assumed to be laid between 2 fixed points, at the same height, $\mathbf{1 5 0} \mathbf{~ m}$ apart, and which have a $\mathbf{2 0} \mathbf{~ m}$ sag as shown in the following drawing:


Firstly we are going to translate the starting data for its correct handling.

$$
\begin{gathered}
\boldsymbol{w}=\frac{F_{w}}{l}=\frac{m}{l} \cdot g=\frac{188.8 \mathrm{~kg}}{\mathrm{~km}} \cdot 9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}=1850.24 \frac{\mathrm{~N}}{\mathrm{~km}} \cdot \frac{1 \mathrm{~km}}{1000 \mathrm{~m}} \cdot \frac{1 \mathrm{daN}}{10 \mathrm{~N}}=\mathbf{0 . 1 8 5 0 2 4} \frac{\boldsymbol{d} \boldsymbol{a} \boldsymbol{N}}{\boldsymbol{m}} \\
T_{\max }=16.29 \mathrm{kN} \cdot \frac{100 \mathrm{daN}}{1 \mathrm{kN}}=1629 \mathrm{daN}
\end{gathered}
$$

By adopting a safety factor of $\mathbf{3}$, it must be guaranteed that the conductor is never subjected to a stress greater than the following value:

$$
\boldsymbol{T}_{\max / 3}=1629 \mathrm{daN} / 3=\mathbf{5 4 3} \mathbf{d a N}
$$

## Analysis of this hypothesis:

To find the stress to which the current cable is subjected, we begin by obtaining the value of the catenary parameter " c ".

Taking into account the relationship between the height " y ", the sag " f " and the catenary parameter "c" ...

$$
y_{\max }=a_{y}=b_{y}=c+f=c+20
$$

And from the [equation 14], the value of " c " is found

$$
\begin{aligned}
& y=\mathrm{c} \cdot \cosh \left(\frac{x}{c}\right) \\
& c+20=c \cdot \cosh \left(\frac{x}{c}\right) \\
& \mathrm{c}=143.840 \\
& \sum x=b_{x}=75
\end{aligned}
$$

Known " $c$ " and " f " we can obtain the value of the maximum height " $y_{\max }$ " (referred to the abscissa axis of the catenary)

$$
y_{\max }=a_{y}=b_{y}=c+20=143.84+20=163.84 m
$$

We continue working with the rest of the equations
[equation 13]

$$
T_{0}=w \cdot c=0.185 \cdot 143.84=26.614 \mathrm{daN}
$$

[equation 19] $|T|=w \cdot y_{\max }=w \cdot a_{y}=w \cdot b_{y}=0.185 \cdot 163.84=30.314 \mathrm{daN}<\mathbf{5 4 3} \mathbf{d a N}$
[equation 16]

$$
\begin{gathered}
L / 2=l=\mathrm{c} \cdot \sinh \left(\frac{x}{c}\right)=143.84 \cdot \sinh \left(\frac{75}{143.84}\right)=78.444 \mathrm{~m} \\
L=2 l=2 \cdot 78.444=156.89 \mathrm{~m}
\end{gathered}
$$

It is found that the cable is currently subjected to a maximum stress lower than that of breaking.

In addition to the minimum stress " $T_{0}$ " and the maximum stress " $T$ ", the length of the conductor in the reference span has been determined.

### 4.3.2 Exercise 2. Variation of mechanical stress due to the conductor reduction

Now we are proposed to study the stresses in the event that, choosing the same cable (exercise 1 ), its length decreases by $\mathbf{1}$ meter:


It is defined a new length:

$$
\begin{gathered}
L^{\prime}=L-1=156.89-1=155.89 \mathrm{~m} \\
l^{\prime}=\frac{L^{\prime}}{2}=\frac{155.89}{2}=77.945 \mathrm{~m}
\end{gathered}
$$

From the [equation 16], the value of " $\boldsymbol{c}^{\prime}$ " is found. This equation has a double solution. Then, the calculation from the $2^{\text {nd }}$ solution will be also recorded.

$$
\text { [equation 16] } \begin{array}{cc}
l^{\prime}=\mathrm{c}^{\prime} \cdot \sinh \left(\frac{x}{c^{\prime}}\right) \\
77.945=\mathrm{c}^{\prime} \cdot \sinh \left(\frac{75}{c^{\prime}}\right)
\end{array}
$$

| $1^{\text {st }}$ solution | $2^{\text {nd }}$ solution |
| :---: | :---: |
| $c_{1}{ }^{\prime}=155.42$ | $c_{2}{ }^{\prime}=-155.42$ |

The value of " $y_{\max }$ " is obtained from the [equation 14]

$$
\begin{gathered}
y_{1}{ }_{\text {max }}=\mathrm{c}^{\prime} \cdot \cosh \left(\frac{x}{c^{\prime}}\right)=155.42 \cdot \cosh \left(\frac{75}{155.42}\right)= \\
=173.87 \mathrm{~m}={a_{y 1}}^{\prime}={b_{y 1}}^{\prime}
\end{gathered}
$$

Hence the value of the sag " $f$ "

$$
\begin{gathered}
y_{1}{ }^{\prime} \text { max }=a_{y 1}{ }^{\prime}=b_{y 1}{ }^{\prime}=c_{1}{ }^{\prime}+f_{1}{ }^{\prime} \\
f_{1}^{\prime}=y_{1}{ }^{\prime} \text { max }-c_{1}{ }^{\prime}=173.87-155.42=18.45 \mathrm{~m}
\end{gathered}
$$

We continue working with the rest of the equations [equation 13] $T_{0_{1}}^{\prime}=w \cdot c_{1}^{\prime}=0.185 \cdot 155.42=28.757 \mathrm{daN}$
[equation 19] $\left|T_{1}{ }^{\prime}\right|=w \cdot y_{1}{ }^{\prime}{ }_{\text {max }}=w \cdot a_{y 1}^{\prime}=w \cdot b_{y 1}^{\prime}=$

$$
=0.185 \cdot 173.87=32.17 d a N<543 \text { daN }
$$

It is verified that if the cable contracts by 1 meter, it will be subjected also to a maximum stress lower than that of breaking.

$$
\begin{gathered}
y_{2}{ }^{\prime}{ }_{\text {máx }}=a_{y 2}{ }^{\prime}=b_{y 2}{ }^{\prime}= \\
=-173.87 \mathrm{~m} \\
f_{2}{ }^{\prime}=-18.45 \mathrm{~m} \\
T_{0_{2}}{ }^{\prime}=-28.757 \mathrm{daN} \\
\left|T_{2}{ }^{\prime}\right|=-32.17 \mathrm{daN}
\end{gathered}
$$

This $2^{\text {nd }}$ solution of " $C$ " leads us to a situation not applicable to cables but to rigid elements that are subjected to compression.

### 4.3.3 Exercise 3. Conductor behavior at the maximum stress admissible

What would happen if the maximum stress reached the limit value of 543 daN?


Using the [equation 19] the new value of the height is obtained

$$
\begin{gathered}
\left|T^{\prime \prime}\right|=w \cdot y^{\prime \prime}{ }_{\text {max }} \\
543=0.185 \cdot y^{\prime \prime} \text { max }^{\prime} \\
y^{\prime \prime}{ }_{\text {max }}=a_{y}^{\prime \prime}=b_{y}^{\prime \prime}=2934.754 \mathrm{~m}
\end{gathered}
$$

From the [equation 14], the value of " $\boldsymbol{c}$ " " is found. This equation has a double solution. Then, the development from the $2^{\text {nd }}$ solution will be also recorded.

$$
\begin{gathered}
\text { [equation 14] } y_{1}{ }^{\prime \prime}=\mathrm{c}_{1}{ }^{\prime \prime} \cdot \cosh \left(\frac{x}{\mathrm{c}_{1}^{\prime \prime}}\right) \\
2934.754=\mathrm{c}_{1}{ }^{\prime \prime} \cdot \cosh \left(\frac{75}{\mathrm{c}_{1}{ }^{\prime \prime}}\right)
\end{gathered}
$$

$\underline{1^{\text {st }} \text { solution }}$

$$
c_{1}^{\prime \prime}=2933.795
$$

Hence the value of the sag " $f$ "

$$
\begin{gathered}
y_{1}{ }^{\prime \prime}{ }_{\max }=a_{y 1}{ }^{\prime \prime}=b_{y 1}{ }^{\prime \prime}=c_{1}{ }^{\prime \prime}+f_{1}{ }^{\prime \prime} \\
f_{1}{ }^{\prime \prime}=y_{1}{ }^{\prime \prime}{ }_{\text {max }}-c_{1}{ }^{\prime \prime}=2934.754-2933.795=0.959 \mathrm{~m}
\end{gathered}
$$

The current length of the conductor is calculated
[equation 17]

$$
L_{1}{ }^{\prime \prime} / 2=l_{1}{ }^{\prime \prime}=\sqrt{y_{1}{ }^{\prime \prime 2}-c_{1}{ }^{\prime \prime 2}}=
$$

$$
=\sqrt{2934.754^{2}-2933.795^{2}}=75.008 \mathrm{~m}
$$

$$
L_{1}{ }^{\prime \prime}=2 l_{1}{ }^{\prime \prime}=2 \cdot 75.008=150.16 \mathrm{~m}
$$

And the value of the stress at the lowest point would be
[equation 13] $T_{0_{1}}{ }^{\prime \prime}=w \cdot c_{1}{ }^{\prime \prime}=0.185 \cdot 2933.795=$

$$
=542.823 d a N
$$

Under these characteristics, it is observed that the cable is subjected to high stresses and the length of cable used is almost equal to the distance between the fixed points. Consequently, the sag obtained is very small.
$2^{\text {nd }}$ solution

$$
c_{2}{ }^{\prime \prime}=12.132
$$

$$
f_{2}^{\prime \prime}=2922.622 \mathrm{~m}
$$

$$
\begin{aligned}
& l_{2}{ }^{\prime \prime}=2934.729 \mathrm{~m} \\
& L_{2}{ }^{\prime \prime}=5869.459 \mathrm{~m}
\end{aligned}
$$

$$
T_{0_{2}}^{\prime \prime}=2.245 \mathrm{daN}
$$

This $2^{\text {nd }}$ solution of " $c$ " reflects that the maximum stress " $T$ " can be reached at point B, almost only by the action of the conductor's own weight. (Situation that is not useful to us)

### 4.3.4 Graphic summary of exercises 1,2 and 3



### 4.4 Determination of dimensions, sags, stresses and lengths

### 4.4.1 Supporting towers at equal height



As has already been deduced, the general equation of a catenary is:

$$
y=c \cdot \cosh \left(\frac{x}{c}\right) \quad \text { [equation 14] }
$$

It starts from the premise of guaranteeing the balance of the moments supported by support B:

$$
\sum M_{B}=0=-T_{0} \cdot f+\int_{0}^{x} w \cdot d l \cdot\left(b_{x}-x\right)
$$

Such support is also in balance. Solving the equality, the solution to the differential equation is obtained by deducing the sag " f " as a function of ' x ' $(f=f(x)$ ):

$$
\begin{aligned}
& \boldsymbol{T}_{\mathbf{0}} \cdot \boldsymbol{f}=\int_{0}^{b_{x}} w \cdot\left(b_{x}-x\right) \cdot d l=\int_{0}^{b_{x}} w \cdot\left(b_{x}-x\right) \cdot \sqrt{d x^{2}+d y^{2}} \cdot \frac{d x}{d x}= \\
& =\int_{0}^{b_{x}} w \cdot\left(b_{x}-x\right) \cdot \sqrt{1+\left(\frac{d y}{d x}\right)^{2}} \cdot d x=\left\{\begin{array}{c}
y=c \cdot \cosh \left(\frac{x}{c}\right) \\
\frac{d y}{d x}=y^{\prime}=c \cdot \frac{1}{c} \sinh \left(\frac{x}{c}\right)
\end{array}\right\}= \\
& =\int_{0}^{b_{x}} w \cdot\left(b_{x}-x\right) \cdot \sqrt{1+\sinh ^{2}\left(\frac{x}{c}\right)} \cdot d x=\left\{\begin{array}{c}
\text { Page } 7 \\
{[\text { equation 4] }}
\end{array}\right\}= \\
& =w \cdot \int_{0}^{b_{x}}\left(b_{x}-x\right) \cdot \cosh \left(\frac{x}{c}\right) \cdot d x=w \cdot\left(\int_{0}^{b_{x}} b_{x} \cdot \cosh \left(\frac{x}{c}\right) \cdot d x-\int_{0}^{b_{x}} x \cdot \cosh \left(\frac{x}{c}\right) \cdot d x\right)= \\
& =\left\{\begin{array}{c}
x=u \quad \rightarrow \quad d x=d u \\
\cosh \left(\frac{x}{c}\right) d x=d v \rightarrow v=c \cdot \sinh \left(\frac{x}{c}\right)
\end{array}\right\}=
\end{aligned}
$$

$$
\begin{aligned}
& =w \cdot\left(b_{x} \cdot c \cdot \int_{0}^{b_{x} \cosh \left(\frac{x}{c}\right)} \frac{c}{c} \cdot d x-\left[x \cdot c \cdot \sinh \left(\frac{x}{c}\right)-\int_{0}^{b_{x}} c \cdot \sinh \left(\frac{x}{c}\right) \cdot d x\right]_{0}^{b_{x}}\right) \\
& =w \cdot\left(b_{x} \cdot c \cdot\left[\sinh \left(\frac{x}{c}\right)\right]_{0}^{b_{x}}-\left[x \cdot c \cdot \sinh \left(\frac{x}{c}\right)-\int_{0}^{b_{x}} c \cdot \frac{c}{c} \cdot \sinh \left(\frac{x}{c}\right) \cdot d x\right]_{0}^{b_{x}}\right)= \\
& =w \cdot\left(b_{x} \cdot c \cdot \sinh \left(\frac{b_{x}}{c}\right)-\left[x \cdot c \cdot \sinh \left(\frac{x}{c}\right)-c^{2} \cdot \cosh \left(\frac{x}{c}\right)\right]_{0}^{b_{x}}\right)= \\
& =w \cdot b_{x} \cdot c \cdot \sinh \left(\frac{b_{x}}{c}\right)-w \cdot b_{x} \cdot c \cdot \sinh \left(\frac{b_{x}}{c}\right)+w \cdot c^{2} \cdot \cosh \left(\frac{b_{x}}{c}\right)+ \\
& +w \cdot 0 \cdot c \cdot \sinh \left(\frac{0}{c}\right)-w \cdot c^{2} \cdot \cosh \left(\frac{0}{c}\right)=w \cdot c^{2} \cdot \cosh \left(\frac{b_{x}}{c}\right)-w \cdot c^{2}= \\
& =w \cdot c^{2} \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-\mathbf{1}\right) \\
& T_{0} \cdot f=w \cdot c^{2} \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-1\right) \\
& f=\frac{w}{T_{0}} \cdot c^{2} \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-1\right) \\
& \left\{\frac{T_{0}}{w}=c \quad \text { [equation 13] }\right\} \\
& f=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-1\right) \quad \text { [equation 20] }
\end{aligned}
$$

NOTE: With this equation the solution on page 22 would have been obtained also, because when dealing with two supports at the same altitude the parameter $b_{x}$ is already known.

### 4.4.2 Supporting towers at different heights



$$
\begin{aligned}
& \text { Of the [equation 14] } \begin{aligned}
y & =c \cdot \cosh \left(\frac{x}{c}\right) \\
a_{y} & =c \cdot \cosh \left(\frac{a_{x}}{c}\right) \\
b_{y}-a_{y}=d \quad \text { Is obtained } \quad \Rightarrow \quad b_{y} & =c \cdot \cosh \left(\frac{b_{x}}{c}\right) \\
b_{y}-a_{y} & =c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-\cosh \left(\frac{a_{x}}{c}\right)\right) \\
d & =c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-\cosh \left(\frac{a_{x}}{c}\right)\right) \quad \text { [equation 21] }
\end{aligned}
\end{aligned}
$$

$$
b_{x}-a_{x}=\overline{A B}
$$

$$
a_{x}=b_{x}-\overline{A B} \quad \Rightarrow \quad d=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-\cosh \left(\frac{b_{x}-\overline{A B}}{c}\right)\right) \text { [equation 22] }
$$

NOTE: It is now unknown $b_{x}$, in addition to " $c$ ". The [equation 22] is an equation with two unknowns.
Firstly it will be proposed to determine " $c$ ", for which $T_{0}$ will be estimated:

$$
\left.\frac{T_{0}}{w}=c \quad \text { [equation } 13\right]
$$

## Process:

1. Is adopted ${ }^{\prime} T_{0}{ }^{\prime}$
2. It is calculated ' $c$ ' by the [eq. 13]
3. Known ' $d$ ' and $\overline{A B}$, are determined ' $b_{x}{ }^{\prime} y^{\prime} a_{x}{ }^{\prime}$ and ${ }^{\prime} b_{y}{ }^{\prime} y^{\prime} a_{y}{ }^{\prime}$ with it by the [eq. 22 and 14]
4. Sags are determined ${ }^{\prime} f_{A}{ }^{\prime}$ and ${ }^{\prime} f_{B}{ }^{\prime}$ by [eq. 20] or by geometry
5. ${ }^{\prime} T_{A}{ }^{\prime}$ and ${ }^{\prime} T_{B}{ }^{\prime}$ can already be calculated by the [eq. 19]
6. ' $l_{C B}$ ' and ' $l_{C A}$ ' are determined by the [eq. 16 and 17] and with it, total ' $L$ ' is obtained

### 4.4.2.1 Case 1. Lowest point of the catenary between supporting towers

The point ' $C$ ' or lower point of the catenary is in the interval $\overline{A B}$.

$$
\text { (0, } \overline{A B}=150 \mathrm{~m}
$$

Starting data:

$$
\begin{array}{ll}
w=0.185 \mathrm{daN} / \mathrm{m} & \text { LA56 conductor } \\
\overline{A B}=150 \mathrm{~m} & \text { Span } \\
d=20 \mathrm{~m} & \text { Height difference } \\
T_{\max }=543 \mathrm{daN} & \text { With safety factor }=3
\end{array}
$$

1. For this case it is assumed $T_{0}=80 \mathrm{daN}$

| Step | Equation |  | Result |
| :---: | :---: | :---: | :---: |
| 2. | [equation 13] | $c=\frac{T_{0}}{w}$ | $c=432.376 \mathrm{~m}$ |
| 3. | [equation 22] | $d=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-\cosh \left(\frac{b_{x}-\overline{A B}}{c}\right)\right)$ | $b_{x}=132.195 \mathrm{~m}$ |
|  | geometry | $a_{x}=b_{x}-\overline{A B}$ | $a_{x}=-17.805 m$ |
|  | [equation 14] | $y=c \cdot \cosh \left(\frac{x}{c}\right)$ |  |
|  |  | $a_{y}=c \cdot \cosh \left(\frac{a_{x}}{c}\right)$ | $a_{y}=432.74 \mathrm{~m}$ |
|  |  | $b_{y}=c \cdot \cosh \left(\frac{b_{x}}{c}\right)$ | $b_{y}=452.74 \mathrm{~m}$ |
|  | geometry | $b_{y}=a_{y}+d$ |  |
| 4. | [equation 20] | $f_{A}=c \cdot\left(\cosh \left(\frac{a_{x}}{c}\right)-1\right)$ | $f_{A}=0.367 \mathrm{~m}$ |
|  | geometry | $f_{A}=a_{y}-c$ |  |
|  | [equation 20] | $f_{B}=c \cdot\left(\cosh \left(\frac{b_{\chi}}{c}\right)-1\right)$ | $f_{B}=20.376 \mathrm{~m}$ |
|  | geometry | $f_{B}=b_{y}-c$ |  |
| 5. | [equation 19] | $T=w \cdot y$ |  |
|  |  | $T_{A}=w \cdot a_{y}$ | $T_{A}=80.068 \mathrm{daN}$ |
|  |  | $T_{B}=w \cdot b_{y}$ | $T_{B}=83.768 \mathrm{daN}$ |
| 6. | [equation 16] | $l=c \cdot \sinh \left(\frac{x}{c}\right)$ |  |
|  | [equation 17] | $l=\sqrt{y^{2}-c^{2}}$ |  |
|  | [equation 16] | $l_{C A}=c \cdot \sinh \left(\frac{a_{x}}{c}\right)$ | $l_{C A}=17.810 \mathrm{~m}$ |
|  | [equation 17] | $l_{C A}=\sqrt{a_{y}{ }^{2}-c^{2}}$ |  |
|  | [equation 16] | $l_{C B}=c \cdot \sinh \left(\frac{b_{x}}{c}\right)$ | $l_{C B}=134.264 \mathrm{~m}$ |
|  | [equation 17] | $l_{C B}=\sqrt{b_{y}{ }^{2}-c^{2}}$ |  |
|  | geometry | $L=l_{C B}+l_{C A}$ | $L=152.074 \mathrm{~m}$ |

### 4.4.2.2 Case 2. Lowest point of the catenary outside the supporting towers

 The point ' C ' or lower point of the catenary is outside the interval $\overline{\boldsymbol{A B}}$.

Starting data:

$$
\begin{array}{ll}
w=0.185 \mathrm{daN} / \mathrm{m} & \text { LA56 conductor } \\
\overline{A B}=150 \mathrm{~m} & \text { Span } \\
d=20 \mathrm{~m} & \text { Height difference } \\
T_{\max }=543 \mathrm{daN} & \text { With safety factor }=3
\end{array}
$$

1. For this case it is assumed $T_{0}=200$ daN

| Step | Equation |  | Result |
| :---: | :---: | :---: | :---: |
| 2. | [equation 13] | $c=\frac{T_{0}}{w}$ | $c=1081.081 \mathrm{~m}$ |
| 3. | [equation 22] | $d=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-\cosh \left(\frac{b_{x}-\overline{A B}}{c}\right)\right)$ | $b_{x}=218.605 \mathrm{~m}$ |
|  | geometry | $a_{x}=b_{x}-\overline{A B}$ | $a_{x}=68.606 \mathrm{~m}$ |
|  | [equation 14] | $y=c \cdot \cosh \left(\frac{x}{c}\right)$ |  |
|  |  | $a_{y}=c \cdot \cosh \left(\frac{a_{x}}{c}\right)$ | $a_{y}=1083.258 \mathrm{~m}$ |
|  |  | $b_{y}=c \cdot \cosh \left(\frac{b_{x}}{c}\right)$ | $b_{y}=1103.258 \mathrm{~m}$ |
|  | geometry | $b_{y}=a_{y}+d$ |  |
| 4. | [equation 20] | $f_{A}=c \cdot\left(\cosh \left(\frac{a_{x}}{c}\right)-1\right)$ | $f_{A}=2.177 \mathrm{~m}$ |
|  | geometry | $f_{A}=a_{y}-c$ |  |
|  | [equation 20] | $f_{B}=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-1\right)$ | $f_{B}=22.177 \mathrm{~m}$ |
|  | geometry | $f_{B}=b_{y}-c$ |  |
| 5. | [equation 19] | $T=w \cdot y$ |  |
|  |  | $T_{A}=w \cdot a_{y}$ | $T_{A}=200.403 \mathrm{daN}$ |
|  |  | $T_{B}=w \cdot b_{y}$ | $T_{B}=204.103 \mathrm{daN}$ |
| 6. | [equation 16] | $l=c \cdot \sinh \left(\frac{x}{c}\right)$ |  |
|  | [equation 17] | $l=\sqrt{y^{2}-c^{2}}$ |  |
|  | [equation 16] | $l_{C A}=c \cdot \sinh \left(\frac{a_{x}}{c}\right)$ | $l_{C A}=68.652 \mathrm{~m}$ |
|  | [equation 17] | $l_{C A}=\sqrt{a_{y}{ }^{2}-c^{2}}$ |  |
|  | [equation 16] | $l_{C B}=c \cdot \sinh \left(\frac{b_{x}}{c}\right)$ | $l_{C B}=220.098 \mathrm{~m}$ |
|  | [equation 17] | $l_{C B}=\sqrt{b_{y}{ }^{2}-c^{2}}$ |  |
|  | geometry | $L=l_{C B}-l_{C A}$ | $L=151.447 \mathrm{~m}$ |

## 5. Equation of change of conditions

A catenary has been defined as the mathematical equation that presents a conductor in equilibrium under the action of its own weight. As it has also been introduced and analyzed in section 4 of this study, overhead line conductors are hung between supports in such a way that a traction stress is produced to be borne by the conductor that must not exceed a pre-set limit.

The mechanical calculation prioritises to know the sags and stresses of the conductor in all possible conditions of temperature, ice and/or wind. Throughout the previous sections, the necessary equations have been deduced to determine the parameters that we now want to study. The peculiarity of the change of conditions equation is its ability to modify the parameters of the catenary taking into account the overload effects caused by temperature, wind and/or ice. Depending on the location of the power line analyzed, the overloads considered will be more or less relevant, but in any case the instructions set by the regulations must be respected.

When designing different supports, it is very important to know the mechanical stress in the conductor. Tensile stresses below the maximum admissible stresses that could cause a support to collapse must be guaranteed. On the other hand, it is important to be able to calculate the maximum sags of the span. Depending on their magnitude; the height of the supports and their location must be selected guaranteeing a minimum distance to the surroundings of the overhead line (this will be analysed in greater depth in topic number 6). Therefore, it is necessary to emphasise the need for an exact calculation method that allows us to accurately obtain the mechanical stresses in any span and its maximum sags in order to guarantee safety and comply with current regulations.

## - Variation of length by mechanical stress:

The conductor will be subjected to a tensile stress that will cause an elongation proportional to the stress it supports and which is defined by Hooke's Law:

$$
\left.\Delta L=\frac{1}{E} \cdot \frac{T}{S} \cdot L[m] \quad \text { [equation } 23\right]
$$

Where:
$L[m]$; is the initial length of the conductor without being subjected to stress and at room temperature.
$S\left[\mathrm{~mm}^{2}\right]$; is the conductor section.
$T$ [daN]; It is the mechanical stress to which the conductor is subjected.
$E\left[d a N / m^{2}\right]$; is the modulus of elasticity or Young's modulus of the conductor.
Different values have been found for the modulus of elasticity and it is that, in the case of a conductor, it increases with the stresses to which it is subjected. For this reason, initial modulus of elasticity and the final modulus of elasticity, which is the one that is reached after time, are considered.

The calculations will be made with the final modulus of elasticity that is included in the UNE 21018: 1980 standard and which is normally the one provided by the manufacturers. When applying the equation "Hooke's Law", it can be seen that a decrease in " $E$ " it will involve an increase in the length of the conductor. As a consequence, the sag will increase and the stresses will decrease.

## - Length variation due to thermal expansion:

The variation in temperatures will cause the contraction or elongation of the conductor, experiencing a variation in length that will be defined by the expression:

$$
\Delta L=L \cdot \alpha \cdot \Delta \theta[m] \quad \text { [equation 24] }
$$

Where:
$L[m]$; is the initial length of the conductor without being subjected to stress and at room temperature.
$\alpha$ [ ${ }^{\circ} \mathrm{C}^{-1}$ ]; is the coefficient of linear expansion of the conductor.
$\Delta \theta\left[{ }^{\circ} \mathrm{C}\right]$; is the increase in temperature.

## - Total elongation:

Consequently, a conductor subjected to the action of its own weight, to the uniform overloads to which it is subjected (wind and/or ice), to temperature variations $(\Delta \theta)$ and to the laying strain, it experiences a variation in length ( $\Delta L$ ) equal to:

$$
\Delta L=\frac{T \cdot L}{E \cdot S}+L \cdot \alpha \cdot \Delta \theta[m] \quad[\text { equation } 25]
$$

Being:
$L[m]$; the length of the conductor without stress, without overloads and at room temperature.

Note: It is known that, at ordinary temperatures, the modulus of elasticity "E" does not vary with temperature and that the coefficient of thermal expansion " $\alpha$ " does not depend practically on tensile stresses. Therefore, the study will consider that the parameters "E" and " $\alpha$ " will be constant and the values indicated in the UNE 21018: 1980 and UNE-EN 50182 standards will be adopted. (It must be warned the existence of an error in the values of " $\alpha$ " as it is not specified the need to multiply it by the coefficient $10^{-6}$ )

For the practical cases that will be studied in this work, it will be considered that the standardized conductor to be used will be the conductor with a steel core and aluminium sheathing "LA56". The values that current regulations establish for the different parameters of said conductor are:

| Bibliography <br> LA56 | $\begin{aligned} & \text { Standard UNE } \\ & 21018 \end{aligned}$ | Line designer's guide | Electrician Technician School |
| :---: | :---: | :---: | :---: |
| Final modulus of elasticity (E) | $8100 \mathrm{kp} / \mathrm{mm}^{2}$ | $8500 \mathrm{kp} / \mathrm{mm}^{2}$ | $7500 \mathrm{kp} / \mathrm{mm}^{2}$ |
| Coefficient of linear expansion ( $\alpha$ ) | 19.1 $10^{10}{ }^{-6}{ }^{\circ} \mathrm{C}^{-1}$ | $18.8 \cdot 10^{-6}{ }^{\circ} \mathrm{C}^{-1}$ | $19.5 \cdot 10^{-6}{ }^{\circ} \mathrm{C}^{-1}$ |
| Breaking load ( $T_{\text {max }}$ ) | 1670 kp | 1673 kp | -- |

Once an idea of the mechanical characteristics has been collected, it is decided to take as data for the resolution of the different practical cases those indicated in the UNE 21018:1980 standard. With the intention of collecting and knowing all the variables that characterize the LA56 conductor, the following table extracted from the UNE 21018: $\mathbf{1 9 8 0}$ standard is attached:


It is time to analyze and define the process for determining the values of stresses and sags that will be observed in the construction of overhead power lines according to environmental conditions and the spacing between supports. Its determination will be conditioned by the requirements required by the Regulations for High Voltage Overhead Power Lines in force.

### 5.1. Normative

The prescriptions set by the Regulation on technical conditions and safety guarantees of high voltage power lines and its complementary technical instructions ITC-LAT 01 to 09 will be followed. In the case of the change of conditions equation, special attention will be paid to the Complementary Technical Instruction 7 (ITC-LAT 07). As this rule communicates:

> "The provisions contained in this instruction refer to the technical prescriptions that high-voltage overhead power lines with bare conductors must comply with, understood as those of three-phase alternating current at 50 Hz frequency, whose effective nominal voltage between phases is higher than $1 \mathrm{kV} . "$

Regarding the characteristics and dimensions of the conductors, it is established that:
"The conductors can be made up of round or trapezoidal-shaped wires made of aluminium or aluminium alloy and can contain, to reinforce them, galvanized steel wires or aluminium-coated steel wires. The conductors must comply with the UNE-EN 50182 Standard and will be of one of the following types:
a) Homogeneous aluminium conductors (AL1).
b) Homogeneous aluminium alloy conductors (ALx).
c) Aluminium or aluminium alloy composite (bimetallic) conductors reinforced with galvanized steel (AL1/STyz or ALx/STyz).
d) Composite conductors (bimetallic) of aluminium or aluminium alloy reinforced with aluminiumcoated steel (AL1/SAyz or ALx/SAyz).
e) Aluminium composite (bimetallic) conductors reinforced with aluminium alloy (AL1/ALx).

For the characteristics of the conductors described in a), b) and c), the tables of the UNE-EN 50182 Standard will be taken as reference, for those described in d) UNE 21018, as long as they are not included in the UNE-EN standard 50182 and for those described in e) the IEC 61089 standard."

On the other hand, with regard to the deduction and determination of the equation of change of conditions, the mechanical calculations are studied. In it we can find the consideration of loads and overloads to which the constituent components of the line will be subjected. Permanent loads due to the own weight of the elements, wind forces on overhead line components and overloads caused by ice can be highlighted.

When dealing with the equation of change of conditions, it is wanted to analyze the behaviour of the conductor before the variation of the environmental characteristics. Therefore, the loads and overloads to be considered in the first place will be those subjected to the conductors.

Regarding permanent loads, the conductor's own weight will be considered. Its value is included in the UNE-EN 50182 standard. Regarding the wind forces on the conductors, the considerations imposed by Complementary Technical Instruction 7 (ITC- LAT 07) are:
"A minimum reference wind speed of $120 \mathrm{~km} / \mathrm{h}(33.3 \mathrm{~m} / \mathrm{s})$ will be considered, except in special category lines, where a minimum wind speed of $140 \mathrm{~km} / \mathrm{h}(38.89 \mathrm{~m} / \mathrm{s})$ will be considered. The horizontal wind will be assumed, acting perpendicular to the surfaces on which it has an impact.

The action of the wind, depending on its speed $V_{V}$ in $\mathrm{km} / \mathrm{h}$, gives rise to the forces that are indicated below on the conductors.

Wind pressure on the conductors causes forces transverse to the direction of the power line, as well as increasing stresses on the conductors. Considering the adjacent spans, the force of the wind on an alignment support will be, for each conductor of the bundle:

$$
F_{c}=q \cdot d \cdot \frac{a_{1}+a_{2}}{2}[\text { daN }] \quad[\text { equation } 26]
$$

being:
$d[m]$; conductor diameter .
$a_{1}, a_{2}[m]$;lengths of adjacent spans, in meters. The semi sum of $a_{1}$ and $a_{2}$ is the wind gap $a_{v}$.
$q$ [daN $\left./ \mathrm{m}^{2}\right]$; wind pressure:

$$
\begin{array}{ll}
q=60 \cdot\left(\frac{V_{V}}{120}\right)^{2}\left[\frac{d a N}{m^{2}}\right] & \text { for conductors with } d \leq 16 \mathrm{~mm} \\
q=50 \cdot\left(\frac{V_{V}}{120}\right)^{2}\left[\frac{d a N}{m^{2}}\right] & \text { for conductors with } d>16 \mathrm{~mm}
\end{array}
$$

In the case of combined ice and wind overloads, the diameter including the thickness of the ice sleeve must be considered, for which it is advisable to consider a specific volumetric ice weight of $750 \mathrm{daN} / \mathrm{m}^{3}$.

The total force of the wind on the bundled conductors will be defined as the sum of the forces on each of the conductors, without taking into account possible screen effects between conductors, not even in the case of bundles of phase conductors."

In the case of overloads caused by ice, it should be remembered that the country is classified into three zones according to the Regulation:

- Zone A: The one located at an altitude lower than 500 meters above sea level.
- Zone B: The one located at an altitude between 500 and 1,000 meters above sea level.
- Zone C: The one located at an altitude higher than 1,000 meters above sea level.

Following what is established by current regulations, the overloads according to the area in which the power lines are located, will be the following:
"- Zone A: Any overload caused by ice will not be taken into account.

- Zone B: Conductors and ground cables will be considered subject to overloading by an ice sleeve with a value of: $0,18 \cdot \sqrt{d}$ daN per linear meter, where $d$ is the diameter of the conductor or ground cable in millimeters.
- Zone C: The conductors and ground cables will be considered subject to overloading by an ice sleeve of the following value: $0,36 \cdot \sqrt{d}$ daN per linear meter, where $d$ is the diameter of the conductor or ground cable in millimeters. For altitudes above 1500 meters, the designer must establish the ice overloads through relevant studies, not being able to consider ice overload lower than that indicated above.

The values of the overloads to be considered for each zone may be increased, if the particular specifications of the distribution or transport companies responsible for the service so establish."

Once the requirements regarding the loads and overloads to which the conductors may be subjected have been known and studied, the mathematical process that allows the determination of the values of stresses and sags of any practical case can be developed. To do this, we will begin by determining and knowing the maximum admissible traction force of the conductors. According to the Regulation:
"The maximum traction of the conductors and ground cables will not be greater than their breaking load, minimum divided by 2.5, if they are cabled conductors, or divided by 3, if they are one-wire conductors."

Throughout the study, different overload hypotheses for conductors can be considered depending on the area in which the practical case is developed ( $\mathrm{A}, \mathrm{B}$ or C ). The conditions of the hypotheses that limit the maximum admissible traction and required by the Regulation are set out in the following table:

| ZONE A |  |  |  |
| :---: | :---: | :---: | :---: |
| Hypothesis | Temperature ( ${ }^{\circ} \mathrm{C}$ ) | Wind overload | Ice overload |
| Maximum wind traction | -5 | Minimum 120 or $140 \mathrm{~km} / \mathrm{h}$ depending on the line voltage | Does not apply |
| ZONE B |  |  |  |
| Hypothesis | Temperature ( ${ }^{\circ} \mathrm{C}$ ) | Wind overload | Ice overload |
| Maximum wind traction | -10 | Minimum 120 or 140 km/h depending on the line voltage | Does not apply |
| Maximum ice traction | -15 | Does not apply | According to Zone A, B or C |
| Maximum ice + wind traction (1) | -15 | Minimum 60 km/h | According to Zone A, B or C |
| ZONE C |  |  |  |
| Hypothesis | Temperature ( ${ }^{\circ} \mathrm{C}$ ) | Wind overload | Ice overload |
| Maximum wind traction | -15 | Minimum 120 or 140 km/h depending on the line voltage | Does not apply |
| Maximum ice traction | -20 | Does not apply | According to Zone A, B or C |
| Maximum ice + wind traction (1) | -20 | Minimum 60 km/h | According to Zone A, B or C |

(1) The hypothesis of maximum ice + wind traction is applied to special category lines and to all those lines that the particular rule of the electricity company so establishes or when the designer considers that the line may be subject to the aforementioned combined load.

The Regulation also specifies that:
"In the case that in the area crossed by the line it is feared the appearance of exceptional wind speeds, the conductors and ground cables will be considered, at the temperature of $-5^{\circ} \mathrm{C}$ in zone $A,-10^{\circ} \mathrm{C}$ in zone $B$ and $-15^{\circ} \mathrm{C}$ in zone $C$, subject to its own weight and a wind overload corresponding to a speed greater than $120 \mathrm{~km} / \mathrm{h}$ or $140 \mathrm{~km} / \mathrm{h}$. The value of the exceptional wind speed will be set by the designer or in accordance with the particular specifications of the electricity company, depending on the speeds recorded at the meteorological stations closest to the area where the line runs.

In the hypothesis of maximum wind traction, a wind speed of $140 \mathrm{~km} / \mathrm{h}$ will be considered for all special category lines, even if they have voltages lower than 220 kV ."

It is time to propose a real practical exercise for which the regulatory stresses and sags will be obtained at different temperatures.

We proceed with the mechanical calculation:

- The study will begin by establishing which is the most restrictive scenario that must be evaluated for which the maximum tensile stress that the conductor will withstand can be determined considering the corresponding safety factor. Following the requirements established by the current Regulation, the starting point is considering a safety factor of 3 .
- Throughout the study of different possible scenarios, with regard to stress analysis, we can represent the resulting forces of each of the following cases:

| Cases | Representation |
| :---: | :---: |
| Conductor in balance under <br> the action of his own <br> weight | $\downarrow w_{w}=w_{T}$ |
| Conductor in balance under <br> the action of his own <br> weight and wind |  |
| Conductor in balance under <br> the action of his own <br> weight and ice |  |
| Conductor in balance under <br> the action of his own <br> weight, wind and ice | $\downarrow_{w_{w}}$ |

### 5.2. Supporting towers at equal height

## Steps:

1. Knowing $T_{\text {máx }}$, it is calculated 'y' by the equation: ( $y \equiv$ "relative" height $)$

$$
T=w \cdot y \quad[e q .19]
$$

2. 'C' is calculated by the equation:
( $c \equiv$ catenary parameter $)$

$$
y=c \cdot \cosh \left(\frac{x}{c}\right)
$$

3. It is determined ' $f$ ' by the [eq. 20] or by geometry:

$$
(f \equiv \operatorname{sag})
$$

$$
f=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-1\right) \quad \text { [eq. 20] }
$$

4. Determine 'l': length of the section from the lowest point of the catenary to one of the supports under the conditions set by the regulations with any of the following equations:

$$
\begin{array}{ll}
l=c \cdot \sinh \left(\frac{x}{c}\right) \\
l=\sqrt{y^{2}-c^{2}} & \text { [eq. 16] 17] }
\end{array}
$$

Apart from 'l', we can get 'L': total length of the conductor once it is laid.
5. The initial length of the conductor before being laid ' $L_{0}$ ' is calculated from the equation:

$$
\Delta L=L-L_{0}=\frac{1}{E} \cdot \frac{T}{S} \cdot l_{0} \quad[e q .23]
$$

This length will be the one that would really have to be purchased if we are at the temperature that the regulation sets for the maximum admissible stress.
6. The following equation will be used:

$$
\Delta L=l_{0} \cdot \alpha \cdot \Delta \theta \quad[e q .24]
$$

In order to calculate the stress values and sags for other temperature conditions, once the maximum admissible traction required by the Regulation is obtained. With the [eq. 24] initial lengths ' $l_{0}$ ' can be known before conductor laying for any temperature value.
7. Once the temperature to be studied has been chosen, it is time to determine the value of the stress in the supports due to the conductors. For this, it will be necessary to work by iteration considering different traction values. It must be chosen an initial stress value ' $T_{0}$ ':
7.1 With a chosen value ' $T_{i}$ ', it is obtained from the [eq. 23] the elongation of the conductor ' $\Delta L^{\prime}$ due to the assumed stress $\left(T_{i}\right)$ :

$$
\Delta L=\frac{1}{E} \cdot \frac{T_{i}}{S} \cdot l_{0} \quad[e q .23]
$$

7.2 From the elongation, the total length ' $L$ ' of the conductor over the entire span and half of its value 'l' can be obtained.
7.3 ' $c$ ' is calculated by the equation:

$$
l=c \cdot \sinh \left(\frac{x}{c}\right) \quad \text { [eq. 16] }
$$

7.4 It is calculated 'y' by the equation:

$$
y=c \cdot \cosh \left(\frac{x}{c}\right)
$$

7.5 It is calculated ' $T_{j}$ 'by the equation:

$$
T_{j}=w \cdot y \quad[e q .19]
$$

7.6 The assumed and the finally obtained stress values ' $T_{i}$ ' and ' $T_{j}$ 'are compared. If obtained a large difference between their values, a new value ' $T_{i}$ ' will be assumed again that is within the range marked by the two values already obtained previously. It is time to go back to step 7.1 until a small error is obtained between the assumed and the obtained traction forces.
8. Once the real value of stress that the conductor will withstand has been established with the iteration method, it can be obtained by [eq. 20] or by geometry the sag 'f' that will present the laying:

$$
f=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-1\right) \quad \text { [eq. 20] }
$$

9. By repeating the entire process for different environmental conditions of temperature, wind and ice, it is possible to obtain all the values for stresses and sags that will have to be considered in order to comply with the requirements established by the current Regulation.

### 5.2.1. Exercise 1. Power line in zone "A"

There are two supports at the same height with a span between them of 140 m . The power lines are located at an altitude lower than $\mathbf{5 0 0}$ meters above sea level. The conductor to be used will be LA56 according to the specifications contained in the UNE-EN 50182 standard:

- By adopting a safety factor of 3, it must be guaranteed that the conductor is never subjected to a stress greater than the following value:

$$
\begin{gathered}
T_{\max }=16.29 \mathrm{kN} \cdot \frac{100 \mathrm{daN}}{1 \mathrm{kN}}=1629 \mathrm{daN} \\
\boldsymbol{T}_{\boldsymbol{\operatorname { m a x }} / 3}=1629 \mathrm{daN} / 3=\mathbf{5 4 3} \mathbf{d a N}
\end{gathered}
$$

- Knowing the maximum stress value to which the conductors could be subjected, the loads and overloads that the conductor will present due to its own weight and the action of the wind must be defined. The permanent load due to the conductor itself is calculated as:

$$
\boldsymbol{w}_{\boldsymbol{w}}=\frac{F_{w}}{l}=\frac{\mathrm{m}}{l} \cdot g=\frac{188.8 \mathrm{~kg}}{\mathrm{~km}} \cdot 9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}=1850.24 \frac{\mathrm{~N}}{\mathrm{~km}} \cdot \frac{1 \mathrm{~km}}{1000 \mathrm{~m}} \cdot \frac{1 \mathrm{daN}}{10 \mathrm{~N}}=\mathbf{0 . 1 8 5 0 2 4} \frac{\boldsymbol{d a N}}{\boldsymbol{m}}
$$

- For the forces caused by the wind, the equation indicated in the ITC-LAT 07 will be applied, which in this report is called [equation 26]:

$$
F_{c}[d a N]=q \cdot d \cdot \frac{a_{1}+a_{2}}{2}=60 \cdot\left(\frac{V_{V}}{120}\right)^{2} \cdot d \cdot \frac{a_{1}+a_{2}}{2}
$$

Where:

$$
\begin{gathered}
V_{V}=120 \frac{\mathrm{~km}}{\mathrm{~h}} \\
d=9.45 \mathrm{~mm} \cdot \frac{1 \mathrm{~m}}{1,000 \mathrm{~mm}}=9.45 \cdot 10^{-3} \mathrm{~m} \\
\text { Vano }=\frac{a_{1}+a_{2}}{2}=140 \mathrm{~m} \\
\boldsymbol{w}_{\text {wind }}\left[\frac{\mathrm{daN}}{\mathrm{~m}}\right]=60 \cdot\left(\frac{120}{120}\right)^{2} \cdot 9.45 \cdot 10^{-3}=\mathbf{0 . 5 6 7} \frac{\mathrm{daN}}{\mathrm{~m}}
\end{gathered}
$$

- The conditions of the hypotheses that limit the maximum admissible traction and that the Regulation requires for lines in Zone A, it is recalled:

| ZONE A |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Hypothesis | Temperature <br> $\left({ }^{\circ} \mathrm{C}\right)$ | Wind overload | Ice overload | Representation |  |
| Maximum <br> wind traction | -5 | Minimum 120 or <br> $140 \mathrm{~km} / \mathrm{h}$ <br> depending on the <br> line voltage | Does not <br> apply | ${ }^{w_{w}} \checkmark_{w_{T}}^{w_{\text {wind }}}$ |  |

- Based on this information, the case will be studied with the following starting data:


| DATA | Case 1 |
| :---: | :---: |
| Span | $140 \mathrm{~m}\left(x=\frac{140}{2}=70 \mathrm{~m}\right)$ |
| Maximum admissible stress $(T)$ | 543 daN |
| Conductor weight $\left(w_{w}\right)$ | $0.185024 \mathrm{daN} / \mathrm{m}$ |
| Wind Force $\left(w_{\text {wind }}\right)$ | $0.567 \mathrm{daN} / \mathrm{m}$ |
| Resulting force $\left(w_{T}\right)$ | $w_{T}={\sqrt{w_{w}{ }^{2}+w_{\text {wind }}{ }^{2}}=0.596 \mathrm{daN} / \mathrm{m}}^{\text {Modulus of elasticity }(E)}$ |
| Coefficient of linear expansion $(\alpha)$ | $19.1 \cdot 10^{-6}{ }^{\circ} \mathrm{C}^{-1}$ |
| Conductor section $(S)$ | $54.6 \mathrm{~mm}^{2}$ |

- The previously explained and detailed process is followed for the assumption of being at a temperature of $-5^{\circ} \mathrm{C}$ with a wind overload of $120 \mathrm{~km} / \mathrm{h}$ (requirement set by the Regulations for power lines located in Zone A):

| Process | Equation |  | Result |
| :---: | :---: | :--- | :---: |
| 1. | [equation 19] | $y=\frac{T}{w_{T}}$ | $y=910.44 \mathrm{~m}$ |
|  | [equation 14] | $y=c \cdot \cosh \left(\frac{x}{c}\right)$ | $c=907.74 \mathrm{~m}$ |
| 3. | [equation 20] | $f=c \cdot\left(\cosh \left(\frac{x}{c}\right)-1\right)$ | $f=2.700 \mathrm{~m}$ |
|  | geometry | $f=y-c$ |  |
| 4. | [equation 16] | $l=c \cdot \sinh \left(\frac{x}{c}\right)$ |  |
|  | [equation 17] | $l=\sqrt{y^{2}-c^{2}}$ | $L=140.139 \mathrm{~m}$ |
|  | geometry | $L=2 l$ | $L_{0}=139.967 \mathrm{~m}$ |
| 5. | [equation 23] | $L-L_{0}=\frac{1}{E} \cdot \frac{T}{S} \cdot L_{0}$ | $\Delta L=0.172 \mathrm{~m}$ |
|  |  | $\Delta L=L-L_{0}$ |  |

## APPLICATION FOR CHANGE OF CONDITIONS

## Case 1.1:

- Assuming the case in which we want to obtain the stress and sag values for other environmental conditions while complying with the requirements set by the regulation, the proposed procedure is continued. A temperature of $15^{\circ} \mathrm{C}$ and absence of wind will be chosen as new environmental conditions. For this, the data of the new hypothesis that is summarized in the following table will have to be considered:

| ZONE A |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Hypothesis | Temperature <br> $\left({ }^{\circ} \mathrm{C}\right)$ | Wind Overload | Ice overload | Representation |  |
| Traction | 15 | Does not apply | Does not apply | $\downarrow w_{w}=w_{T}$ |  |

- The resultant force to which the conductor is subjected changes with respect to the first assumption. Therefore, the change in temperature must be taken into account:

| Study at a temperature of $15^{\circ} \mathrm{C}$ and absence of wind |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6. | [equation 24] | $\Delta L=L_{0} \cdot \alpha \cdot \Delta \theta$ |  |  |  | $L_{0}{ }^{15^{\circ} \mathrm{C}}=140.020 \mathrm{~m}$ |
|  |  | $\begin{aligned} & \left.\left.\left.L_{0}\right]^{T_{f}}=L_{0}\right]^{T_{0}}+\Delta L\right]_{T_{0}}^{T_{f}}= \\ & \left.\left.=L_{0}\right]^{T_{0}} \cdot(1+\alpha \cdot \Delta \theta]_{T_{0}}^{T_{f}}\right) \end{aligned}$ |  |  | $\begin{aligned} & \boldsymbol{T}_{0}=-5^{\circ} \mathrm{C} \\ & \boldsymbol{T}_{\boldsymbol{f}}=15^{\circ} \mathrm{C} \end{aligned}$ |  |
| For different values of $T_{i}$ we proceed with the equations: |  |  |  |  |  |  |
| 7. | [equation 23] | 7.1 | $\Delta L=L-L$ | $\frac{1}{E}$. | $\cdots \cdot L_{0}$ | $l_{i}$ |
|  |  | 7.2 | $l=\frac{L_{0}+\frac{1}{E}}{2}$ |  |  |  |
|  | [equation 16] | 7.3 | $l=c \cdot \sinh$ |  |  | $c_{i}$ |
|  | [equation 14] | 7.4 | $y=c \cdot \cosh$ |  |  | $y_{i}$ |
|  | [equation 19] | 7.5 | $T=w_{T} \cdot y$ |  | $0.19 \frac{d a N}{m}$ | $T_{j}$ |
| 8. | [equation 20] | $f=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-1\right)$ |  |  |  | $f$ |

- The following table shows an example of iteration to determine the stress to which the conductor will be subjected under the environmental conditions established in case 1.1. Throughout the rows, the results are displayed for each of the calculated parameters, while the number of iterations is appreciated by columns:

| Steps | Obtaining the parameter T per iteration |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{7 . 1}$ | $T_{i}[\mathrm{daN}]$ | $T_{0}=200$ | $T_{1}=210$ | $T_{2}=211$ | $T_{3}=211.5$ | $T_{4}=211.9$ |  |
| 7.2 | $l[\mathrm{~m}]$ | 70.0419 | 70.0435 | 70.0436 | 70.0437 | 70.0438 |  |
| 7.3 | $c[\mathrm{~m}]$ | 1168.606 | 1147.125 | 1145.041 | 1144.004 | 1143.176 |  |
| 7.4 | $y[\mathrm{~m}]$ | 1170.703 | 1149.261 | 1147.182 | 1146.146 | 1145.319 |  |
| 7.5 | $T_{j}[\mathrm{daN}]$ | 216.580 | 212.613 | 212.228 | 212.037 | 211.884 |  |
| $\mathbf{7 . 6}$ | $\epsilon=\left\|T_{i}-T_{j}\right\|$ | 16.580 | 2.613 | 1.229 | 0.537 | 0.015 |  |
| $\mathbf{8}$ | $f[\mathrm{~m}]$ | ------ | ------ | ------ | ------ | 2.144 |  |

## Case 1.2:

- After working on the case in which we assume an ambient temperature of $15^{\circ} \mathrm{C}$ without considering any other influence, we can consider adding to this example the consideration of a possible disturbance caused by the appearance of wind. Therefore, a temperature of $15^{\circ} \mathrm{C}$ is maintained, but the consideration of loading caused by the wind is added. For this, the data of the new hypothesis that is summarized in the following table will have to be considered:

| ZONE A |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Hypothesis | Temperature <br> $\left({ }^{\circ} \mathrm{C}\right)$ | Wind overload | Ice overload | Representation |  |
| Traction plus <br> wind action | 15 | Minimum 120 or <br> 140 km/h <br> depending on the <br> line voltage | Does not <br> apply | $w_{w} W_{w_{T}}$ |  |

- The resultant force to which the conductor is subjected changes with respect to the previous assumption. Therefore, the following must be taken into account in the calculation:

| Study at a temperature of $15^{\circ} \mathrm{C}$ and presence of wind |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6. | [equation 24] | $\Delta L=L_{0} \cdot \alpha \cdot \Delta \theta$ |  |  |  | $L_{0}{ }^{15^{\circ} \mathrm{C}}=140.020 \mathrm{~m}$ |
|  |  | $\begin{aligned} & \left.\left.\left.L_{0}\right]^{T_{f}}=L_{0}\right]^{T_{0}}+\Delta L\right]_{T_{0}}^{T_{f}}= \\ & \left.\left.=L_{0}\right]^{T_{0}} \cdot(1+\alpha \cdot \Delta \theta]_{T_{0}}^{T_{f}}\right) \end{aligned}$ |  |  | $\begin{array}{\|l} \hline \boldsymbol{T}_{\mathbf{0}}=-\mathbf{5}^{\circ} \mathrm{C} \\ \hline \boldsymbol{T}_{\boldsymbol{f}}=\mathbf{1 5} 5^{\circ} \mathrm{C} \\ \hline \end{array}$ |  |
| For different values of $T_{i}$ we proceed with the equations: |  |  |  |  |  |  |
| 7. | [equation 23] | 7.1 | $\Delta L=L-L_{0}=\frac{1}{E} \cdot \frac{T}{S} \cdot L_{0}$ |  |  | $l_{i}$ |
|  |  | 7.2 | $l=\frac{L_{0}+\frac{1}{E} \cdot \frac{T}{S} \cdot L_{0}}{2}$ |  |  |  |
|  | [equation 16] | 7.3 | $l=c \cdot \sinh \left(\frac{x}{c}\right)$ |  |  | $c_{i}$ |
|  | [equation 14] | 7.4 | $y=c \cdot \cosh \left(\frac{x}{c}\right)$ |  |  | $y_{i}$ |
|  | [equation 19] | 7.5 | $T=w_{T} \cdot y$ |  | 0.596 $\frac{\mathrm{daN}}{\mathrm{m}}$ | $T_{j}$ |
| 8. | [equation 20] | $f=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-1\right)$ |  |  |  | F |

- The following table shows an example of iteration to determine the stress to which the conductor will be subjected by the environmental conditions of case 1.2:

| Steps | Obtaining the parameter T per iteration |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{7 . 1}$ | $T_{i}[\mathrm{daN}]$ | $T_{0}=250$ | $T_{1}=450$ | $T_{2}=470$ | $T_{3}=480$ | $T_{4}=485$ |  |
| 7.2 | $l[\mathrm{~m}]$ | 70.0498 | 70.0814 | 70.0846 | 70.0862 | 70.0870 |  |
| 7.3 | $c[\mathrm{~m}]$ | 1071.707 | 837.952 | 822.131 | 814.549 | 810.836 |  |
| 7.4 | $y[\mathrm{~m}]$ | 1073.994 | 840.877 | 825.112 | 817.558 | 813.859 |  |
| 7.5 | $T_{j}[\mathrm{daN}]$ | 640.549 | 501.514 | 492.112 | 487.606 | 485.399 |  |
| $\mathbf{7 . 6}$ | $\epsilon=\left\|T_{i}-T_{j}\right\|$ | 390.549 | 51.514 | 22.112 | 7.606 | 0.399 |  |
| $\boldsymbol{8}$ | $f[\mathrm{~m}]$ | ------ | ------ | ------ | ------ | 3.024 |  |

### 5.2.2. Exercise 2. Power line in zone "B"

There are two supports at the same height with a span between them of 140 m . The power lines are located at an altitude between $\mathbf{5 0 0}$ and $\mathbf{1 0 0 0}$ meters above sea level. The conductor to be used will be LA56 according to the specifications contained in the UNE-EN 50182 standard:

- As in Case 1 the following parameters are known:

$$
\begin{gathered}
\boldsymbol{T}_{\max / 3}=1629 \mathrm{daN} / 3=\mathbf{5 4 3} \mathbf{d a N} \\
\boldsymbol{w}_{\boldsymbol{w}}=0.185024 \frac{d a \mathrm{~N}}{\mathrm{~m}} \\
\boldsymbol{w}_{\boldsymbol{w i n d} \boldsymbol{d}}=0.567 \frac{\mathrm{daN}}{\mathrm{~m}}
\end{gathered}
$$

- For the forces due to ice, the area in which our laying is located must be known. In this case, in which the altitude above sea level is between 500 and 1000 meters, we are in the so-called Zone B. In the case of conductors, the overload of an ice sleeve of value must be considered:

$$
w_{i}\left[\frac{d a N}{m}\right]=0,18 \cdot \sqrt{d}
$$

Where:

$$
d=9.45 \mathrm{~mm}
$$

$$
w_{i}=0.18 \cdot \sqrt{9.45 \mathrm{~mm}}=0.553 \frac{d a N}{m}
$$

- The conditions of the hypotheses that limit the maximum admissible traction and that the Regulation requires for power lines in Zone B, it is recalled that they are:

| ZONE B |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Hypothesis | Temperature <br> $\left({ }^{\circ} \mathrm{C}\right)$ | Wind overload | Ice overload | Representation |
| Maximum wind <br> traction | -10 | Minimum 120 or <br> 140 km/h <br> depending on the <br> line voltage | Does not <br> apply | $w_{w}$ |
| Maximum ice <br> traction | -15 | Does not apply | According to <br> Zone A, B or <br> C | $\downarrow w_{w_{w}}$ |
| Maximum ice + <br> wind traction <br> (1) | -15 | Minimum 60 km/h | $w^{\prime}$ <br> According to <br> Zone A, B or <br> C | $w_{w_{w}}$ |

(1) The maximum ice + wind traction hypothesis applies to special category lines and to all those lines that the particular rule of the electricity company so establishes or when the designer considers that the line may be subject to the aforementioned combined load.

- At case 2 as a special category line is not studied, the maximum traction due to ice overload will be considered as the most unfavorable case (second assumption of the table for Zone B).
- Based on this information, the case will be studied with the following starting data:


| DATA | $\underline{\text { Case 2 }}$ |
| :---: | :---: |
| Span | $140 \mathrm{~m}\left(x=\frac{140}{2}=70 \mathrm{~m}\right)$ |
| Maximum admissible stress $(T)$ | 543 daN |
| Conductor weight $\left(w_{w}\right)$ | $0.185024 \mathrm{daN} / \mathrm{m}$ |
| Ice overload $\left(w_{i}\right)$ | $0.553 \mathrm{daN} / \mathrm{m}$ |
| Resulting force $\left(w_{T}\right)$ | $w_{T}=w_{w}+w_{i}=0.738 \mathrm{daN} / \mathrm{m}$ |
| Modulus of elasticity $(E)$ | $8100 \mathrm{~N} / \mathrm{mm}^{2}$ |
| Coefficient of linear expansion $(\alpha)$ | $19.1 \cdot 10^{-6}{ }^{\circ} \mathrm{C}^{-1}$ |
| Conductor section $(S)$ | $54.6 \mathrm{~mm}^{2}$ |

- The indicated and detailed process is followed by the suppose of being at a temperature of $-15^{\circ} \mathrm{C}$ with an ice overload (requirement set by the Regulations for power lines located in Zone B):

| Process |  | Equation | Result |
| :---: | :---: | :---: | :---: |
| 1. | [equation 19] | $y=\frac{T}{w_{T}}$ | $y=735.414 m$ |
| 2. | [equation 14] | $y=c \cdot \cosh \left(\frac{x}{c}\right)$ | $c=732.065 \mathrm{~m}$ |
| 3. | [equation 20] | $f=c \cdot\left(\cosh \left(\frac{x}{c}\right)-1\right)$ | $f=3.349 \mathrm{~m}$ |
|  | geometry | $f=y-c$ |  |
| 4. | [equation 16] | $l=c \cdot \sinh \left(\frac{x}{c}\right)$ | $l=70.107 \mathrm{~m}$ |
|  | [equation 17] | $l=\sqrt{y^{2}-c^{2}}$ |  |
|  | geometry | $L=2 l$ | $L=140.213 \mathrm{~m}$ |
| 5. | [equation 23] | $L-L_{0}=\frac{1}{E} \cdot \frac{T}{S} \cdot L_{0}$ | $L_{0}=140.041 \mathrm{~m}$ |
|  |  | $\Delta L=L-L_{0}$ | $\Delta L=0.172 \mathrm{~m}$ |

## APPLICATION FOR CHANGE OF CONDITIONS

## Case 2.1:

- Assuming the case in which we want to obtain the stress and sag values for other environmental conditions while complying with the requirements set by the regulation, the proposed procedure is continued. A temperature of $15^{\circ} \mathrm{C}$ and wind overload will be chosen as new environmental conditions. For this, the data of the new hypothesis that is summarized in the following table will have to be considered:

| ZONE B |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Hypothesis | Temperature <br> $\left({ }^{\circ} \mathrm{C}\right)$ | Wind Overload | Ice overload | Representation |
| Traction plus <br> wind action | 15 | Minimum 120 or <br> 140 km $/ \mathrm{h}$ | Does not <br> apply | $w_{w} \downarrow$ depending on the $_{\text {line voltage }}$ |

- The resultant force to which the conductor is subjected changes with respect to the previous assumption. Therefore, the following must be taken into account in the calculation:

| Study at a temperature of $15^{\circ} \mathrm{C}$ and presence of wind |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6. | [equation 24] | $\Delta L=L_{0} \cdot \alpha \cdot \Delta \theta$ |  |  |  | $L_{0}{ }^{15{ }^{\circ} \mathrm{C}}=140.122 \mathrm{~m}$ |
|  |  | $\begin{aligned} & \left.\left.\left.L_{0}\right]^{T_{f}}=L_{0}\right]^{T_{0}}+\Delta L\right]_{T_{0}}^{T_{f}}= \\ & \left.\left.=L_{0}\right]^{T_{0}} \cdot(1+\alpha \cdot \Delta \theta]_{T_{0}}^{T_{f}}\right) \end{aligned}$ |  |  | $\begin{aligned} & \hline T_{0}=-15^{\circ} \mathrm{C} \\ & \hline T_{f}=\mathbf{1 5}{ }^{\circ} \mathrm{C} \\ & \hline \end{aligned}$ |  |
| For different values of $T_{i}$ we proceed with the equations: |  |  |  |  |  |  |
| 7. | [equation 23] | 7.1 | $\Delta L=L-L_{0}=\frac{1}{E} \cdot \frac{T}{S} \cdot L_{0}$ |  |  | $l_{i}$ |
|  |  | 7.2 | $l=\frac{L_{0}+\frac{1}{E} \cdot \frac{T}{S} \cdot L_{0}}{2}$ |  |  |  |
|  | [equation 16] | 7.3 | $l=c \cdot \sinh \left(\frac{x}{c}\right)$ |  |  | $c_{i}$ |
|  | [equation 14] | 7.4 | $y=c \cdot \cosh \left(\frac{x}{c}\right)$ |  |  | $y_{i}$ |
|  | [equation 19] | 7.5 | $T=w_{T} \cdot y$ |  | $0.596 \frac{\mathrm{daN}}{\mathrm{m}}$ | $T_{j}$ |
| 8. | [equation 20] | $f=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-1\right)$ |  |  |  | $f$ |

- The following table shows an example of iteration to determine the stress to which the conductor will be subjected under the environmental conditions of case 2.1:

| Steps | Obtaining the parameter Tper iteration |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7.1 | $T_{i}[\mathrm{daN}]$ | $T_{0}=200$ | $T_{1}=340$ | $T_{2}=380$ | $T_{3}=400$ | $T_{4}=405$ |  |
| 7.2 | $l[\mathrm{~m}]$ | 70.0926 | 70.1147 | 70.1211 | 70.1242 | 70.1250 |  |
| 7.3 | $c[\mathrm{~m}]$ | 786.069 | 706.051 | 687.335 | 678.519 | 676.367 |  |
| $\mathbf{7 . 4}$ | $y[\mathrm{~m}]$ | 789.188 | 709.524 | 690.903 | 682.133 | 679.993 |  |
| 7.5 | $T_{j}[\mathrm{daN}]$ | 470.692 | 423.178 | 412.072 | 406.841 | 405.565 |  |
| 7.6 | $\epsilon=\left\|T_{i}-T_{j}\right\|$ | 270.692 | 83.178 | 32.072 | 6.841 | 0.565 |  |
| $\boldsymbol{8}$ | $f[\mathrm{~m}]$ | ------ | ------ | ------ | ------ | 3.627 |  |

### 5.2.3. Excel programming

Once the process for the determination of stresses and sags of different overhead lines have been deduced and presented, the need arises to design a mathematical and computer program that allows and facilitates the calculations automatically. The fact that different iterative procedures are required both in the resolution of the parameter " c " of the catenary, as well as the stress to which the cable will be subjected, makes data handling difficult due to its costly procedure. On the other hand, using calculation tools that allow their programming, it will be easier to obtain results for an infinity of cases that may arise.

In this case, the Excel application and calculation tool will be used. It is intended to design and program the different steps and methods presented theoretically to finally obtain results based on the characteristics of the power line to be analyzed. For this reason, text cells will be enabled where to insert the initial data so that the program returns the values of stress and sags according to the marked conditions.

Firstly, the program must know is the type of conductor that will be used in the construction of the power line. Depending on the conductor, there will be different characteristics of the same that will determine the working conditions and effort of the whole line. The parameters that are set according to the conductor used will be: conductor section ( $\mathrm{mm}^{2}$ ), maximum tensile strength $(\mathrm{kN})$, conductor diameter ( mm ), thermal coefficient of expansion $\left({ }^{\circ} \mathrm{C}^{-1}\right)$, modulus of elasticity $\left(\mathrm{N} / \mathrm{mm}^{2}\right)$ and mass per unit of length $(\mathrm{kg} / \mathrm{km})$. As in the practical cases carried out, the assignment of names for each of the variables would be presented as follows:

| Data | Variable name in EXCEL |
| :---: | :---: |
| Span | Vano |
| Breaking strain $\left(T_{\max }\right)$ | $T r$ |
| Conductor weight $\left(w_{w}\right)$ | $w p$ |
| Wind Force $\left(w_{\text {wind }}\right)$ | $w v$ |
| Ice overload $\left(w_{i}\right)$ | $w h$ |
| Resulting force $\left(w_{T}\right)$ | $w p ;$ wpv; wph or $w p v h$ |
| Modulus of elasticity $(E)$ | $E$ |
| Coefficient of linear expansion $(\alpha)$ | Alpha |
| Conductor section $(S)$ | $S$ |
| Conductor diameter $(D)$ | $D$ |
| Mass per unit length $(m / l)$ | $m$ |
| Security coefficient | $C f$ |

From these data, the program will have the necessary information to consider the characteristics of the conductor. Subsequently, it will be requested to determine the span of the line in meters. Within the calculation and design of a line, one of the parameters that will be modified the most is the distance between the supports due to the needs of the terrain or technical conditions. That is why this value will be decisive in the calculation of the stresses to which the conductor will be subjected.

Among other variables chosen throughout the process according to technical criteria justified above, parameters such as the safety coefficient to be used can be established, which can be modified depending on the conditions and evaluations that are estimated in the construction of a specific line. In the case of this study, it has been considered that the coefficient used will be 3, while the regulation allows a value of 2.5 . When planning and designing a program, the option of modifying this variable will be possible.

As already explained, it is necessary to refer to the regulations to know the conditions that must be considered depending on the area in which the power line is going to be built. This also means that for different cases, the resulting weight to be considered for the conductor, being able to add an overload due to wind and/or ice, varies in different geographical areas. As examples, wind overload considered depends on the diameter of the conductor and in the case of a ice sleeve thickness it depends on the height above sea level. The functions that have been used for its definition are:

| Cases | Representation | EXCEL |  |
| :---: | :---: | :---: | :---: |
|  |  | Cell | Function |
| Conductor in balance under the action of his own weight | $\downarrow w_{w}=w_{T}$ | wp | $=m * 9.8 / 1000 / 10$ |
| Overload of wind | $\xrightarrow{w_{\text {wind }}}$ | wv | $\begin{gathered} =I F\left(D<=16 ; 60^{*} D / 1000^{*}(V V / 120)^{\wedge} 2 ;\right. \\ \left.50^{*} D / 1000^{*}(V V / 120)^{\wedge} 2\right) \end{gathered}$ |
| Conductor in balance under the action of his own weight and wind | $w_{w} \downarrow \underbrace{w_{\text {wind }}}_{w_{T}}$ | wpv | $=S Q R T\left(w p^{\wedge} 2+w v^{\wedge} 2\right)$ |
| Overload of ice | $\downarrow w_{i}$ | wh | $\begin{aligned} & =I F\left(Z O N E=" A " ; 0 ; I F\left(Z O N E=" B^{\prime \prime} ;\right.\right. \\ & 0.18 * S Q R T(D) ; 0.36 * S Q R T(D)) \end{aligned}$ |
| Conductor in balance under the action of his own weight and ice | $\downarrow \begin{aligned} & \downarrow w_{w} \\ & w_{i} \\ & w_{T} \end{aligned}$ | wph | $=w p+w h$ |
| Conductor in balance under the action of his own weight, wind and ice |  | wpvh | $=S Q R T\left(w v^{\wedge} 2+(w p+w h)^{\wedge} 2\right)$ |

Once the aforementioned has been introduced in the Excel template, it is time to program the limit operating conditions set by the Regulation depending on the characteristics of the line. To do this, the program must ask the user what information is available regarding the geographical area, characteristics of the line or the load hypothesis. These variables will be named in the program as:

| Data | Variable name in EXCEL | Values user can take |
| :---: | :---: | :---: |
| Geographical area | ZONE | A, B or C |
| Special category | Cat.Esp | Yes or No |
| Traction hypothesis | $H i p$ | TMwind or TMice |
| Wind speed | $V_{V}$ | 120 or $140 \mathrm{~km} / \mathrm{h}$ |
| Wind speed Cat.Esp | Vcat.esp | $V \geq 60 \mathrm{~km} / \mathrm{h}$ |

Once these characteristics have been established, their translation must be programmed to the values that the Regulation establishes with respect to these considerations. From this, the program must know the variables of temperature, wind overload and ice overload limits that must be guaranteed for the mechanical strain of the conductor by applying the established safety coefficient to it. The functions that will specify these parameters will be:

| Parameters | EXCEL |  |  |
| :---: | :---: | :---: | :---: |
|  | Cell | Function |  |

With all the necessary data already registered, we proceed with the method deduced and explained above. From this point on, the Excel Solver function is introduced to determine by iterative calculation the parameter " $c$ " of the catenary from the equation 14. As the span length and the parameter " $y$ " are available, but it is not a linear equation, it is essential to use calculation methods that approximate the required solution.

If the functions used for the first part are available, where is wanted to obtain the length of the conductor without being subjected to stress and under the conditions determined by the regulations, the following steps are possible:

| Steps | Equation | EXCEL |  |
| :---: | :---: | :---: | :---: |
|  |  | Cell | Function |

Once the parameters of the catenary have been calculated according to the limit conditions established by the Regulation, it is time to apply the equations of change of conditions according to the needs when buying material or building any overhead line. To do this, there are cells to incorporate the new information into the program. It will be necessary to determine a new working temperature and determine whether or not the existence of overloads due to wind or ice. For this step, the following cells are available:

| Data | Variable name in EXCEL | Values user can take |
| :---: | :---: | :---: |
| New Temperature | Temp.new | Insert value |
| New Condition Wind | V.new | Yes or No |
| New Condition Ice | H.new | Yes or No |

With the new conditions introduced, it is time to proceed with the iterative method that allows obtaining the stress at which the conductor should be laid when building the overhead line according to the established and chosen characteristics. For this, the last steps of the method to be used have been programmed. For the iterative calculation, it is necessary to repeat the same operation as many times as necessary to finally obtain a very small error between the solution and the initially assumed value. For this reason, the "macro" design tool will be used to facilitate the automation of the program and speed up the calculations. Within this programming the following operations and functions will be used:

| Steps |  | Equation | EXCEL |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Cell | Function |
| 6. | 6.1 |  | [equation 24] | lcb0.new | $=l c b 0 *\left(1+\right.$ Alpha* $10^{\wedge}-6^{*}($ Temp.new-Temp $)$ ) |
|  | 6.2 | geometry | Lab0.new | =lcb0.new*2 |
| For different values of $T_{i}$ we proceed with the equations: |  |  |  |  |
| 7. | 7.1 <br> 7.2 | [equation 23] | lcb.new (i) | =lcb0.new + Ti*lcb0.new/(E*S) |
|  | 7.3 | [equation 16] | c.new (i) | =c.new*SINEH(x/c.new)-Icb.new(i) |
|  | 7.4 | [equation 14] | y.new (i) | =c.new(i)* $\operatorname{COSH}(x / \mathrm{c} . n e w(i))$ |
|  | 7.5 | [equation 19] | T.new (i) | $\begin{gathered} \text { =IF(AND(V.new="No";'H.new="No");y.new(i)*wp; } \\ \text { IF(AND(V.new="Yes";H.new="No"); y.new(i)*wpv; } \\ \text { IF(AND(V.new="Yes";H.new="Yes"); y.new(i)*wpvh; } \\ \text { IF(AND(V.new="No";H.new="Yes"); y.new(i)*wph)))) } \end{gathered}$ |

It can be seen that, as in previous steps, IF conditional functions have been used that allow proposing different operations according to the conditions required in the application of the change of conditions. In the case of the seventh step, it is necessary to calculate the stress value to which the conductor should be subjected and for this it is necessary to choose as the total weight of the cable the one established according to the new working conditions. Knowing these variables, the program will be the one who chooses the resulting weight concerned that, multiplying by the parameter " $y$ " of the catenary, it is possible to determine the mechanical stress of the conductor in both supports.

Applying the iterative method to different stress values, new results are obtained which present errors with respect to the one initially considered. By calculating the error in each of the cases, a stress value close to the actual stress value can be estimated and taken as a reference. Knowing this value, it is possible to work in a domain closer to the reference value in order to adjust and calibrate the final stress by iteration. Using the "XLOOKUP" style functions, this reasoning can be implemented in the Excel sheet so that the program is able to automatically extract the final results with the least possible error.

To define this type of function, we must first set the value wanted to be found, then define the range where expected to find that value, and finally specify from which column is wanted to export the cell related to the one searched. In the case of this programming, the solution aims to show the conductor length parameters ( $l_{c b}$ and $l_{c b 0}$ ) from one of the supports to the lowest point of the catenary both without being subjected to strain and while being laid, the parameter of the catenary $(c)$, the height of the catenary in the support $(y)$, the mechanical strain of the conductor ( $T$ ) and its sag $(f)$. In order to be able to export these values from the calculation tables and thus obtain the desired results, the following programming commands have been designed:


For the design of "macros" in Excel, the "Programmer" tool has been used. Among its functions is the possibility of recording a macro by steps from which it has been possible to write the code and commands necessary for the total execution of the program. The structure of the designed macro is as follows:

## MACRO PROGRAMMING:

(1) 'Shortcut: CTRL + t
(2) Range ("c.new (1)"). Select

ActiveCell.FormulaR1C1 = "500"
(3) Range ("c.new (1)"). Select

Selection.AutoFill Destination: = Range ("Column.c.new"), Type: = xlFillDefault
Range ("Column.c.new"). Select
(4) SolverOk SetCell: = "Ec.c (i)", MaxMinVal: $=3$, ValueOf: $=0$, ByChange: $=$ "c.new (i)", Engine _
: = 1, EngineDesc: = "GRG Nonlinear" ""
SolverOk SetCell: = "Ec.c (i)", MaxMinVal: = 3, ValueOf: = 0, ByChange: = "c.new (i)", Engine _ : = 1, EngineDesc: = "GRG Nonlinear"
SolverSolve True
(5) End Sub

In programming, the different commands used must be explained. First of all, a shortcut via the keyboard is defined for the execution of the program. In the second step it is observed that it is necessary to start by assuming an initial value of the parameter " $c$ " of the catenary for the iterations. In this case, a value of 500 was chosen, which is established for the first iteration. Since the spreadsheet understands numerous identical iterations, this same value is "dragged" along the column that collects the " $c$ " values from the different iterations.

Once the starting values necessary for the correct resolution have been set, the "Solver" function is programmed to be executed for each of the iterations. In a generic way, the fourth step of the macro shows how the characteristics of this process are established. In the first place, the resolution objective is established, which in this case corresponds to the equation 16. This is entered in such a way that it can be equalized to a null value, therefore it is then notified with the number " 3 " that the equation presents said disposition. The cell where is intended to track the iteration and obtain the final result is specified below, in this case being the cell corresponding to the "c.new" parameter that is obtained. Finally, the resolution method used is indicated, in this case it is the "GRG Nonlinear".

As can be seen, the code for the use of the "Solver" tool has been presented with reference to generic cells with the help of the " $i$ " parameter. This means that the macro must include as many lines as necessary to execute the iterative process for all the chosen stress values and therefore for each of the parameters of " c " that can be determined.

With all this, an automatic program is structured that allows to obtain in a dynamic and agile way the values of stresses and sags that are required in the construction of any overhead power line subject to the specific conditions and characteristics of each case and scenario.

### 5.3. Supporting towers at different heights

In this case, the procedure and reasoning for the determination of the maximum stresses and the calculation of the sags marked by the regulations changes. The starting data required in previous case is not able to provide all the information necessary for the design of the catenary with the supports at different heights. The essential values that must be set are the span, the difference in height between the supports and the cable to be used.

The biggest difference and difficulty found with respect to a laying with supports at the same height is that it is not known where the lowest point of the catenary is located. Therefore, by way of iteration, stress values should be assumed at the low point of the catenary, which is defined from the equation 13. Once the assumption is made, we will proceed to calculate, with the equations deduced in previous chapters, the stresses that will be obtained in the supports. It will be from these results that the limit operating point marked by the maximum admissible strain will be determined.

As already mentioned in the previous study of supports at different heights, it is evident that the highest support will be the one that supports the maximum stress while the lower support will not reach these values. Setting the requirements considered by the Regulation and studying the different parameters that define the catenary, the following method of deduction of stresses in the supports, length and sags of any line is proposed:

## Steps:

1. Determine the value of the mechanical stress at the lowest point of the catenary. For this, it will be necessary to work by iteration considering different stress values. Choose an initial stress value ' $T_{0}{ }^{\prime}$ wanted:
1.1 With a chosen stress value ' $T_{0_{i}}{ }^{\prime}$ ', it is obtained from the [eq. 13] the parameter ' $c$ ' of the catenary according to the assumed stress ( $T_{0_{i}}$ ):

$$
c=\frac{T_{0}}{w} \quad \text { [eq. 13] }
$$

1.2 Knowing the difference in heights between the different supports, the distances from both supports to the lowest point of the catenary are calculated by the equations:

$$
\begin{gathered}
d=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-\cosh \left(\frac{b_{x}-\overline{A B}}{c}\right)\right) \\
a_{x}=b_{x}-\overline{A B} \quad \text { [geometry] 22] }
\end{gathered}
$$

1.3 The parameters ' $y$ ' are calculated for each of the supports by the equation:

$$
y=c \cdot \cosh \left(\frac{x}{c}\right) \quad \text { [eq. 14] } \quad \begin{cases}a_{y}=c \cdot \cosh \left(\frac{a_{x}}{c}\right) & {[\text { support } A]} \\ b_{y}=c \cdot \cosh \left(\frac{b_{x}}{c}\right) & {[\text { support } B]}\end{cases}
$$

1.4 It is calculated ' $T_{i}$ ' in each of the supports from the equation:

$$
T_{i}=w \cdot y \quad\left[\text { eq. 19] } \quad \left\{\begin{array}{ll}
T_{A}=w \cdot a_{y} & {[\text { support A] }} \\
T_{B}=w \cdot b_{y} & {[\text { support B] }}
\end{array}\right.\right.
$$

1.5 The stress values' $T_{i}$ ' are compared with the maximum allowed according to the regulatory requirements. The iteration should continue assuming new values of ' $T_{0}{ }^{\prime}$ until reaching the maximum value of ${ }^{\prime} T_{i}{ }^{\prime}$ admitted in the supports. To do this, it is needed to go back to step 1.1 until a small error is obtained between the final stress obtained and the maximum established. In this first step we are able to locate the lowest point of the catenary.
2. Once the lowest point of the catenary is defined, the sag values' $f$ ' are determined by the [eq. 20] regarding both supports:

$$
f=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-1\right) \quad \text { [eq. 20] } \quad \begin{cases}f_{A}=c \cdot\left(\cosh \left(\frac{a_{x}}{c}\right)-1\right) & {[\text { support } A]} \\ f_{B}=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-1\right) & {[\text { support } B]}\end{cases}
$$

3. Determine 'l': length of the sections from the lowest point of the catenary to both supports under the conditions set by the regulation with any of the following equations:

$$
\begin{aligned}
& l=c \cdot \sinh \left(\frac{x}{c}\right) \quad \text { [eq. 16] } \quad \begin{cases}l_{C A}=c \cdot \sinh \left(\frac{a_{x}}{c}\right) & \text { [support A] } \\
l_{C B}=c \cdot \sinh \left(\frac{b_{x}}{c}\right) & \text { [support B] }\end{cases} \\
& l=\sqrt{y^{2}-c^{2}} \quad \text { [eq.17] } \quad \begin{cases}l_{C A}=\sqrt{a_{y}{ }^{2}-c^{2}} & \text { [support } A] \\
l_{C B}=\sqrt{b_{y}{ }^{2}-c^{2}} & \text { [support } B]\end{cases}
\end{aligned}
$$

4. Apart from 'l', 'L' can be obtained: total length of the conductor once it is laid, taking into account the two possible assumptions. It is possible that the lowest point of the catenary is between both supports, in which case the total length of the conductor will be the sum of the previously calculated lengths. On the contrary, if the lowest point of the catenary is outside the interval $\overline{A B}$, the total length will be equal to the difference between those previously obtained.

$$
\left\{\begin{array}{c}
\text { If the lowest point of the catenary is in } \overline{A B} \rightarrow L=l_{C B}+l_{C A} \\
\text { If the lowest point of the catenary is outside } \overline{A B} \rightarrow L=l_{C B}-l_{C A}
\end{array}\right.
$$

5. The initial length of each section of conductor ' $l_{0}$ ' before being laid is calculated from the equation:

$$
\Delta l=l-l_{0}=\frac{1}{E} \cdot \frac{T}{S} \cdot l_{0} \quad[\text { eq.23] }
$$

This length will be the one that would really have to be purchased if we are at the temperature that the regulation sets for the maximum admissible stress in the upper support. For each section, the stress supported by each tower ' $T$ ' will be different.
6. The following equation will be used:

$$
\Delta l=l_{0} \cdot \alpha \cdot \Delta \theta \quad[e q .24]
$$

In order to calculate the stress values and sags for other temperature conditions, the study of the case in which the maximum admissible traction required by the Regulation is obtained. With the [eq. 24] initial lengths ' $l_{0}$ ' can be known before conductor laying for any temperature value.
7. Once the temperature to be studied has been chosen, it is time to determine the value of the stresses in the supports due to the laying of the conductors and possible overloads. For this, it will be necessary to work by iteration considering different stress values. It must be chosen an initial stress value ' $T_{0}{ }^{\prime}$ :
7.1 With a chosen value, ' $T_{0}{ }_{j}$ ' it is obtained from the [eq. 13$]$ the parameter ' $c$ ' of the catenary according to the assumed stress ( $T_{0_{j}}$ ):

$$
c=\frac{T_{0}}{w} \quad \text { [eq.13] }
$$

7.2 Knowing the difference in heights between the different supports, the distances from both supports to the lowest point of the catenary are calculated by the equations:

$$
\begin{gather*}
d=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-\cosh \left(\frac{b_{x}-\overline{A B}}{c}\right)\right)  \tag{eq.22}\\
a_{x}=b_{x}-\overline{A B} \quad \text { [geometry] }
\end{gather*}
$$

7.3 The parameters ' $y$ ' are calculated for each of the supports by the equation:

$$
y=c \cdot \cosh \left(\frac{x}{c}\right) \quad \text { [eq. 14] } \quad \begin{cases}a_{y}=c \cdot \cosh \left(\frac{a_{x}}{c}\right) & {[\text { support A] }} \\ b_{y}=c \cdot \cosh \left(\frac{b_{x}}{c}\right) & {[\text { support } B]}\end{cases}
$$

7.4 It is calculated ' $T_{j}^{\prime}$ ' in each of the supports by the equation:

$$
T_{j}=w \cdot y \quad\left[\text { eq. 19] } \quad \left\{\begin{array}{ll}
T_{A}=w \cdot a_{y} & {[\text { support } A]} \\
T_{B}=w \cdot b_{y} & {[\text { support } B]}
\end{array}\right.\right.
$$

7.5 Sag values' $f$ ' are determined by the [eq. 20] regarding the two supports:

$$
f=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-1\right) \quad \text { [eq. 20] } \quad \begin{cases}f_{A}=c \cdot\left(\cosh \left(\frac{a_{x}}{c}\right)-1\right) & {[\text { support } A]} \\ f_{B}=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-1\right) & {[\text { support } B]}\end{cases}
$$

7.6 Determine 'I': length of the sections from the lowest point of the catenary to both supports under the conditions set in the study of change of conditions:

$$
l=\mathrm{c} \cdot \sinh \left(\frac{x}{c}\right) \quad \text { [eq. 16] } \quad \begin{cases}l_{C A}=c \cdot \sinh \left(\frac{a_{x}}{c}\right) & {[\text { support } A]} \\ l_{C B}=c \cdot \sinh \left(\frac{b_{x}}{c}\right) & {[\text { support } B]}\end{cases}
$$

7.7 It is obtained from [eq. 23] the elongation of the conductor ' $\Delta l^{\prime}$ due to the stress calculated in step $7.4\left(T_{j}\right)$ :

$$
\Delta l=\frac{1}{E} \cdot \frac{T_{j}}{S} \cdot l_{0} \quad[e q .23]
$$

7.8 The maximum stress values are compared by the Regulation ' $T_{\text {máx }}$ ' and the one finally obtained ' $T_{j}$ '. If obtained a large difference between their values, a new value ' $T_{0_{j}}$ ' will be assumed again being within the range marked by the two values already obtained previously. It is time to go back to step 7.1 until a small error is obtained between the maximum set stress and that obtained.
8. Once the real value of stress that the conductor will withstand has been established with the iteration method, the total length of conductor necessary for its placement and the projection of the sag can be determined in the event of lateral forces according to the parameters of temperature, wind and ice defined by changing conditions. For this calculation the same reasoning is followed as in step 4 of the process.
9. By repeating the entire process for different environmental conditions of temperature, wind and ice, it is possible to obtain all the values for stresses and sags that will have to be considered in order to comply with the requirements established by the current Regulation.

### 5.3.1. Exercise 3 . Power line in zone " A "

There are two supports at a height difference of $\mathbf{4 m}$ with a span between them of 120 m . The power lines are located at an altitude lower than $\mathbf{5 0 0}$ meters above sea level. The conductor to be used will be LA56 according to the specifications contained in the UNE-EN 50182 standard:

- The conditions of the hypotheses that limit the maximum admissible traction and that the Regulation requires for lines in Zone $A$, it is recalled:

| ZONE A |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Hypothesis | Temperature <br> $\left({ }^{\circ} \mathrm{C}\right)$ | Wind overload | Ice overload | Representation |  |
| Maximum <br> wind traction | -5 | Minimum 120 or <br> $140 \mathrm{~km} / \mathrm{h}$ <br> depending on the <br> line voltage | Does not <br> apply | $w_{w} \checkmark_{W_{T}}^{w_{\text {wind }}}$ |  |

- Adopting the same considerations regarding the conductor to be used that have been analyzed in the case of supports at the same height, the case will be studied with the following starting data:


| DATA | Case 1 |
| :---: | :---: |
| Span | $120 \mathrm{~m}\left(x=\frac{120}{2}=60 \mathrm{~m}\right)$ |
| Height difference between supports (d) | 4 m |
| Maximum admissible stress $(T)$ | 543 daN |
| Conductor weight $\left(w_{w}\right)$ | $0.185024 \mathrm{daN} / \mathrm{m}$ |
| Wind Force $\left(w_{\text {wind }}\right)$ | $0.567 \mathrm{daN} / \mathrm{m}$ |
| Resulting force $\left(w_{T}\right)$ | $w_{T}=\sqrt{w_{w}{ }^{2}+w_{\text {wind }}{ }^{2}}=0.596 \mathrm{daN} / \mathrm{m}$ |
| Modulus of elasticity $(E)$ | $8100 \mathrm{~N} / \mathrm{mm}^{2}$ |
| Coefficient oflinear expansion $(\alpha)$ | $19.1 \cdot 10^{-6}{ }^{\circ} \mathrm{C}^{-1}$ |
| Conductor section $(S)$ | $54.6 \mathrm{~mm}^{2}$ |

- The previously explained and detailed process is followed for the assumption of being at a temperature of $-5^{\circ} \mathrm{C}$ with a wind overload of $120 \mathrm{~km} / \mathrm{h}$ (requirement set by the Regulations for power lines located in Zone A):

| Process | Equation |  |  |  | Result |
| :---: | :---: | :---: | :---: | :---: | :---: |
| For different values of $T_{0}$ we proceed with the equations: |  |  |  |  |  |
| 1. | [equation 13] | 1.1 | $c=\frac{T_{0}}{w_{T}}$ |  | $c_{i}$ |
|  | [equation 22] | 1.2 | $d=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-\cosh \left(\frac{b_{x}-\overline{A B}}{c}\right)\right)$ |  | $b_{x_{i}}$ |
|  | geometry |  | $a_{x}=b_{x}-\overline{A B}$ |  | $a_{x_{i}}$ |
|  | [equation 14] | 1.3 | $b_{y}=c \cdot \cosh \left(\frac{b_{x}}{c}\right)$ |  | $b_{y_{i}}$ |
|  |  |  | $a_{y}=c \cdot \cosh \left(\frac{a_{x}}{c}\right)$ |  | $a_{y_{i}}$ |
|  | [equation 19] | 1.4 | $T_{B}=w_{T} \cdot b_{y}$ | $w_{T}=0.19 \frac{d a N}{m}$ | $T_{B i}$ |
|  |  |  | $T_{A}=w_{T} \cdot a_{y}$ |  | $T_{A i}$ |
| Once the stress values are obtained in both supports, the following information is available: |  |  |  |  |  |
| $b_{x}=90.169 \mathrm{~m}$ |  |  |  | $T_{B}=542.979 \mathrm{daN}$ |  |
| $a_{x}=-29.831 \mathrm{~m}$ |  |  |  | $T_{A}=540.593 \mathrm{daN}$ |  |
| Proceeding with the rest of the steps to follow: |  |  |  |  |  |
| 2. | [equation 20] | $f_{B}=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-1\right)$ |  |  | $f_{B}=4.491 \mathrm{~m}$ |
|  |  | $f_{A}=c \cdot\left(\cosh \left(\frac{a_{x}}{c}\right)-1\right)$ |  |  | $f_{A}=0.491 \mathrm{~m}$ |
| 3. | [equation 16] | $l_{B}=c \cdot \sinh \left(\frac{b_{x}}{c}\right)$ |  |  | $l_{B}=90.318 \mathrm{~m}$ |
|  |  | $l_{A}=c \cdot \sinh \left(\frac{a_{x}}{c}\right)$ |  |  | $l_{A}=29.836 \mathrm{~m}$ |
| 4. | geometry | $L=l_{B} \pm l_{A}$ |  |  | $L=120.154 \mathrm{~m}$ |
| 5. | [equation 23] | $l-l_{0}=\frac{1}{E} \cdot \frac{T}{S} \cdot l_{0}$ |  |  | $l_{0_{B}}=90.207 \mathrm{~m}$ |
|  |  |  |  |  | $l_{0 A}=29.800 \mathrm{~m}$ |
|  |  | $\Delta l=l-l_{0}$ |  |  | $\Delta l_{B}=0.111 \mathrm{~m}$ |
|  |  |  |  |  | $\Delta l_{A}=0.036 \mathrm{~m}$ |

- With all this, it is time to study possible assumptions regarding variations in operating conditions that may lead to different values of stresses and sags, always within the margin that the regulation requires and establishes.


## APPLICATION FOR CHANGE OF CONDITIONS

## Case 3.1:

- Assuming the case in which we want to obtain the stress and sag values for other environmental conditions while complying with the requirements set by the regulation, the proposed procedure is continued. A temperature of $15^{\circ} \mathrm{C}$ and absence of wind will be chosen as new environmental conditions. For this, the data of the new hypothesis that is summarized in the following table will have to be considered:

| ZONE A |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Hypothesis | Temperature <br> $\left({ }^{\circ} \mathrm{C}\right)$ | Wind overload | Ice overload | Representation |  |
| Traction | 15 | Does not apply | Does not apply | $\downarrow w_{W}=w_{T}$ |  |

- The resultant force to which the conductor is subjected changes with respect to the first assumption. Therefore, the change in temperature must be taken into account:

| Study at a temperature of $15^{\circ} \mathrm{C}$ and absence of wind |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 6. | [equation 24] | $\Delta l=l_{0} \cdot \alpha \cdot \Delta \theta$ |  |  |  |
|  |  | $\begin{aligned} & \left.\left.\left.l_{0}\right]^{T_{f}}=l_{0}\right]^{T_{0}}+\Delta l\right]_{T_{0}}^{T_{f}}= \\ & \left.\left.=l_{0}\right]^{T_{0}} \cdot(1+\alpha \cdot \Delta \theta]_{T_{0}}^{T_{f}}\right) \end{aligned}$ |  | $T_{0}=-5^{\circ} \mathrm{C}$ | $l_{0}{ }_{0}^{15{ }^{\circ} \mathrm{C}}=90.242 \mathrm{~m}$ |
|  |  |  |  | $\mathrm{T}_{\boldsymbol{f}}=15^{\circ} \mathrm{C}$ | $l_{0}{ }_{0}^{15{ }^{\circ} \mathrm{C}}=29.811 \mathrm{~m}$ |
| For different values of $T_{0_{j}}$ we proceed with the equations: |  |  |  |  |  |
| 7. | [equation 13] | 7.1 | $c=\frac{T_{0}}{w_{T}}$ |  | $c_{j}$ |
|  | [equation 22] | 7.2 | $d=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-\cosh \left(\frac{b_{x}-\overline{A B}}{c}\right)\right)$ |  | $b_{x_{j}}$ |
|  | geometry |  | $a_{x}=b_{x}-\overline{A B}$ |  | $a_{x_{j}}$ |
|  | [equation 14] | 7.3 | $b_{y}=c \cdot \cosh \left(\frac{b_{x}}{c}\right)$ |  | $b_{y_{j}}$ |
|  |  |  | $a_{y}=c \cdot \cosh \left(\frac{a_{x}}{c}\right)$ |  | $a_{y_{j}}$ |
|  | [equation 19] | 7.4 | $T_{B}=w_{T} \cdot b_{y}$ | ${ }_{T}=0.19 \frac{d a N}{m}$ | $T_{B j}$ |
|  |  |  | $T_{A}=w_{T} \cdot a_{y}$ |  | $T_{A_{j}}$ |
|  | [equation 20] | 7.5 | $f_{B}=c \cdot\left(\cosh \left(\frac{b_{\chi}}{c}\right)-1\right)$ |  | $f_{B j}$ |
|  |  |  | $f_{A}=c \cdot\left(\cosh \left(\frac{a_{x}}{c}\right)-1\right)$ |  | $f_{A_{j}}$ |
|  | [equation 16] | 7.6 | $l_{B}=c \cdot \sinh \left(\frac{b_{x}}{c}\right)$ |  | $l_{B_{j}}$ |
|  |  |  | $l_{A}=c \cdot \sinh \left(\frac{a_{\chi}}{c}\right)$ |  | $l_{A_{j}}$ |
|  | [equation 23] | 7.7 | $l-l_{0}=\frac{1}{E} \cdot \frac{T}{S} \cdot l_{0}$ |  | $l_{0_{B j}}$ |
|  |  |  |  |  | $l_{0_{A j}}$ |
| 8. | geometry | $L=l_{B} \pm l_{A}$ |  |  | $L$ |

- The following table shows an example of iteration to determine the stress to which the conductor will be subjected to the environmental conditions established in case 1.1. Throughout the rows, the results are displayed for each of the calculated parameters, while the number of iterations is appreciated by columns:

| Steps | Obtaining the parameter $T$ per iteration |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7.0 | $T_{0_{j}}[$ daN $]$ | $T_{0_{1}}=200$ | $T_{0_{2}}=150$ | $T_{0_{3}}=170$ | $T_{04}=168$ | $T_{0_{5}}=167.6$ |
| 7.1 | $c_{j}[m]$ | 1080.941 | 810.706 | 918.800 | 907.990 | 905.828 |
| 7.2 | $b_{x_{j}}[m]$ | 96.006 | 86.994 | 90.599 | 90.239 | 90.167 |
|  | $a_{x_{j}}[m]$ | -23.994 | -33.006 | -29.401 | -29.761 | -29.833 |
| 7.3 | $b_{y_{j}}[m]$ | 1085.207 | 815.378 | 923.270 | 912.478 | 910.320 |
|  | $a_{y_{j}}[m]$ | 1081.207 | 811.378 | 919.270 | 908.478 | 906.320 |
| 7.4 | $T_{B j}[$ daN] | 200.789 | 150.864 | 170.827 | 168.830 | 168.431 |
|  | $T_{A_{j}}[$ daN $]$ | 200.049 | 150.124 | 170.087 | 168.090 | 167.691 |
| 7.5 | $f_{B j}[m]$ | 4.266 | 4.672 | 4.470 | 4.488 | 4.491 |
|  | $f_{A_{j}}[m]$ | 0.266 | 0.672 | 0.470 | 0.488 | 0.491 |
| 7.6 | $l_{B j}[m]$ | 96.132 | 87.161 | 90.746 | 90.387 | 90.316 |
|  | $l_{A_{j}}[m]$ | 23.996 | 33.015 | 29.406 | 29.767 | 29.839 |
| 7.7 | $l_{0_{B j}}[\mathrm{~m}]$ | 96.089 | 87.131 | 90.711 | 90.353 | 90.281 |
|  | $l_{0_{A_{j}}}[\mathrm{~m}]$ | 23.985 | 33.004 | 29.394 | 29.755 | 29.827 |
| 7.8 | $\epsilon=\left\|T_{\text {máx }}-T_{j}\right\|$ | 11.674 | 6.303 | 0.886 | 0.167 | 0.056 |
| 8 | $L[m]$ | ---- | ------- | ------- | ------- | 120.154 |

With these results it is possible to obtain all the necessary data in the case of wanting to build a span equal to 120 meters whose supports have a difference in height of 4 meters and located in the area classified as " $A$ ". It is time to carry out a similar study for another possible scenario considering the case of wanting the presence of possible wind overloads to be considered in the change of conditions.

## Case 3.2:

- We will consider adding to the previous example the consideration of a possible disturbance caused by the appearance of wind. Therefore, a temperature of $15^{\circ} \mathrm{C}$ is maintained, but the consideration of loading caused by the wind is added. The new data is:

| ZONE A |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Hypothesis | Temperature <br> $\left({ }^{\circ} \mathrm{C}\right)$ | Wind overload | Ice overload | Representation |  |
| Traction plus <br> wind action | 15 | Minimum 120 or <br> 140 km/h <br> depending on the <br> line voltage | Does not <br> apply | $w_{W}$ |  |$w_{w_{T}}$.

- The resultant force to which the conductor is subjected changes with respect to the previous assumption. Therefore, the following must be taken into account in the calculation:

| Study at a temperature of 15 ${ }^{\circ} \mathrm{C}$ and presence of wind |  |  |  |  |
| :---: | :---: | :--- | :--- | :--- |
| 6. | $\Delta l=l_{0} \cdot \alpha \cdot \Delta \theta$ |  |  |  |
|  |  | $\begin{array}{l}\left.\left.\left.l_{0}\right]^{T_{f}}=l_{0}\right]^{T_{0}}+\Delta l\right]_{T_{0}}^{T_{f}}= \\ \\ \end{array}$ | $\left.l_{0}\right]^{T_{0}} \cdot(1+\alpha \cdot \Delta \theta]_{T_{0}}^{T_{f}}$ |  |$)$

- For different values of ' $T_{0}$ ' we proceed with the equations and the procedure previously presented. The following table presents the results of the iteration to determine the stress to which the conductor will be subjected to the environmental conditions established in case 3.2:

| Steps | Obtaining the parameter $T$ per iteration |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7.0 | $T_{0 j}[$ daN $]$ | $T_{0_{1}}=250$ | $T_{0_{2}}=450$ | $T_{03}=550$ | $T_{04}=540$ | $T_{05}=540.1$ |
| 7.1 | $c_{j}[m]$ | 419.164 | 754.495 | 888.628 | 905.395 | 905.562 |
| 7.2 | $b_{\chi_{j}}[m]$ | 73.922 | 85.119 | 89.593 | 90.152 | 90.158 |
|  | $a_{x_{j}}[m]$ | -46.078 | -34.881 | -30.407 | -29.848 | -29.842 |
| 7.3 | $b_{y_{j}}[m]$ | 425.699 | 759.302 | 893.148 | 909.887 | 910.054 |
|  | $a_{y_{j}}[m]$ | 421.699 | 755.302 | 889.148 | 905.887 | 906.054 |
| 7.4 | $T_{B j}$ [daN] | 253.898 | 452.867 | 532.696 | 542.679 | 542.779 |
|  | $T_{A_{j}}$ [daN] | 251.512 | 450.481 | 530.310 | 540.293 | 540.393 |
| 7.5 | $f_{B_{j}}[m]$ | 6.535 | 4.806 | 4.520 | 4.492 | 4.492 |
|  | $f_{A_{j}}[m]$ | 2.535 | 0.806 | 0.520 | 0.492 | 0.492 |
| 7.6 | $l_{B_{j}}[m]$ | 74.306 | 85.299 | 89.745 | 90.301 | 90.307 |
|  | $l_{A_{j}}[\mathrm{~m}]$ | 46.171 | 34.894 | 30.413 | 29.853 | 29.848 |
| 7.7 | $l_{0_{B j}}[\mathrm{~m}]$ | 74.263 | 85.212 | 89.637 | 90.191 | 90.196 |
|  | $l_{0_{A_{j}}}[\mathrm{~m}]$ | 46.145 | 34.858 | 30.377 | 29.817 | 29.811 |
| 7.8 | $\epsilon=\left\|T_{\text {máx }}-T_{j}\right\|$ | 32.312 | 10.076 | 1.170 | 0.056 | 0.046 |
| 8 | $L[m]$ | ---- | ------- | ------- | ------- | 120.154 |

### 5.3.2. Exercise 4. Power line in zone "B"

There are two supports with a height difference of $\mathbf{4 m}$ and a span between them of 120 m . The power lines are located at an altitude between $\mathbf{5 0 0}$ and $\mathbf{1 0 0 0}$ meters above sea level. The conductor to be used will be LA56 according to the specifications contained in the UNE-EN 50182 standard:

- The conditions of the hypotheses that limit the maximum admissible traction, which are required by the Regulations for lines in Zone B and which will be used will be:

| ZONE B |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Hypothesis | Temperature <br> $\left({ }^{\circ} \mathrm{C}\right)$ | Wind overload | Ice overload | Representation |  |
| Maximum ice <br> traction | -15 | Does not apply | According to <br> Zone A, B or <br> C | $w_{w}$ |  |
| $w_{i}$ |  |  |  |  |  |

- Based on this information, the case will be studied with the following starting data:


| DATA | $\underline{\text { Case 2 }}$ |
| :---: | :---: |
| Span | $120 \mathrm{~m}\left(x=\frac{120}{2}=70 \mathrm{~m}\right)$ |
| Height difference between supports $(d)$ | 4 m |
| Maximum admissible stress $(T)$ | 543 daN |
| Conductor weight $\left(w_{w}\right)$ | $0.185024 \mathrm{daN} / \mathrm{m}$ |
| Ice overload $\left(w_{i}\right)$ | $0.553 \mathrm{daN} / \mathrm{m}$ |
| Resulting force $\left(w_{T}\right)$ | $w_{T}=w_{w}+w_{i}=0.738 \mathrm{daN} / \mathrm{m}$ |
| Modulus of elasticity $(E)$ | $8100 \mathrm{~N} / \mathrm{mm}^{2}$ |
| Coefficient of linear expansion $(\alpha)$ | $19.1 \cdot 10^{-6}{ }^{\circ} \mathrm{C}^{-1}$ |
| Conductor section $(S)$ | $54.6 \mathrm{~mm}^{2}$ |

- The indicated and detailed process is followed by the suppose of being at a temperature of $-15^{\circ} \mathrm{C}$ with an ice overload (requirement set by the Regulations for power lines located in Zone B):

| Process | Equation |  |  |  | Result |
| :---: | :---: | :---: | :---: | :---: | :---: |
| For different values of we proceed with the equations: $T_{0}$ |  |  |  |  |  |
| 1. | [equation 13] | 1.1 | $c=\frac{T_{0}}{w_{T}}$ |  | $c_{i}$ |
|  | [equation 22] | 1.2 | $d=c \cdot(\cosh$ | $\left.-\cosh \left(\frac{b_{x}-\overline{A B}}{c}\right)\right)$ | $b_{x_{i}}$ |
|  | geometry |  | $a_{x}=b_{x}-\overline{A B}$ |  | $a_{x_{i}}$ |
|  | [equation 14] | 1.3 | $b_{y}=c \cdot \cosh$ |  | $b_{y_{i}}$ |
|  | Lequation |  | $a_{y}=c \cdot \cosh$ |  | $a_{y_{i}}$ |
|  |  |  | $T_{B}=w_{T} \cdot b_{y}$ |  | $T_{B i}$ |
|  |  |  | $T_{A}=w_{T} \cdot a_{y}$ | $m$ | $T_{A i}$ |

Once the stress values are obtained in both supports, the following information is available:

| $b_{\chi}=84.319 \mathrm{~m}$ |  |  | $T_{B}=542.997 \mathrm{daN}$ |
| :---: | :---: | :---: | :---: |
| $a_{x}=-35.681 \mathrm{~m}$ |  |  | $T_{A}=540.043 \mathrm{daN}$ |
| Proceeding with the rest of the steps to follow: |  |  |  |
| 2. | [equation 20] | $f_{B}=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-1\right)$ | $f_{B}=4.872 \mathrm{~m}$ |
|  |  | $f_{A}=c \cdot\left(\cosh \left(\frac{a_{x}}{c}\right)-1\right)$ | $f_{A}=0.872 \mathrm{~m}$ |
| 3. | [equation 16] | $l_{B}=c \cdot \sinh \left(\frac{b_{x}}{c}\right)$ | $l_{B}=84.507 \mathrm{~m}$ |
|  |  | $l_{A}=c \cdot \sinh \left(\frac{a_{x}}{c}\right)$ | $l_{A}=35.695 \mathrm{~m}$ |
| 4. | geometry | $L=l_{B} \pm l_{A}$ | $L=120.154 \mathrm{~m}$ |
| 5. | [equation 23] | $l-l_{0}=\frac{1}{E} \cdot \frac{T}{S} \cdot l_{0}$ | $l_{0_{B}}=84.403 \mathrm{~m}$ |
|  |  |  | $l_{0 A}=35.651 \mathrm{~m}$ |
|  |  | $\Delta l=l-l_{0}$ | $\Delta l_{B}=0.104 \mathrm{~m}$ |
|  |  |  | $\Delta l_{A}=0.044 \mathrm{~m}$ |

- With all this, it is time to proceed studying possible assumptions regarding variations in operating conditions that may lead to different values of stresses and sags, always within the margin that the regulation requires and establishes.


## APPLICATION FOR CHANGE OF CONDITIONS

## Case 4.1:

- It is decided to choose as new environmental conditions a temperature of $15^{\circ} \mathrm{C}$ and wind overload. For this, the data of the new hypothesis that is summarized in the following table will have to be considered:

| ZONE B |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Hypothesis | Temperature <br> $\left({ }^{\circ} \mathrm{C}\right)$ | Wind overload | Ice overload | Representation |  |
| Traction plus <br> wind action | 15 | Minimum 120 or <br> $140 \mathrm{~km} / \mathrm{h}$ <br> depending on the <br> line voltage | Does not <br> apply | $w_{W}$ |  |$w_{w_{T}}$

- The resultant force to which the conductor is subjected changes with respect to the previous assumption. Therefore, the following must be taken into account in the calculation:

| Study at a temperature of $15^{\circ} \mathrm{C}$ and presence of wind |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 6. | [equation 24] | $\Delta l=l_{0} \cdot \alpha \cdot \Delta \theta$ |  |  |
|  |  | $\left.\left.l_{0}\right]^{T_{f}}=l_{0}\right]^{T_{0}}+\Delta l l_{T_{0}}^{T_{f}}=$ | $\boldsymbol{T}_{\mathbf{0}}=-5^{\circ} \mathrm{C}$ | $l_{0}{ }_{0}^{15{ }^{\circ} \mathrm{C}}=84.452 \mathrm{~m}$ |
|  |  | $\left.\left.=l_{0}\right]^{T_{0}} \cdot(1+\alpha \cdot \Delta \theta]_{T_{0}}^{T_{f}}\right)$ | $T_{f}=15^{\circ} \mathrm{C}$ | $l_{0}{ }_{0}^{15{ }^{\circ} \mathrm{C}}=35.672 \mathrm{~m}$ |

- For different values of $T_{0}$ we proceed with the necessary equations for case 4.1:

| Steps | Obtaining the parameter $T$ per iteration |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7.0 | $T_{0_{j}}[$ daN $]$ | $T_{0_{1}}=250$ | $T_{0_{2}}=500$ | $T_{0_{3}}=400$ | $T_{04}=440$ | $T_{05}=435.5$ |
| 7.1 | $c_{j}[m]$ | 419.164 | 838.328 | 670.663 | 737.729 | 730.184 |
| 7.2 | $b_{x_{j}}[m]$ | 73.922 | 87.915 | 82.322 | 84.559 | 84.308 |
|  | $a_{x_{j}}[m]$ | -46.078 | -32.085 | -37.678 | -35.441 | -35.692 |
| 7.3 | $b_{y_{j}}[m]$ | 425.699 | 842.942 | 675.721 | 742.580 | 735.056 |
|  | $a_{y_{j}}[m]$ | 421.699 | 838.942 | 671.721 | 738.580 | 731.056 |
| 7.4 | $T_{B_{j}}$ [daN] | 253.898 | 502.752 | 403.017 | 442.894 | 438.406 |
|  | $T_{A_{j}}$ [daN] | 251.512 | 500.366 | 400.631 | 440.508 | 436.020 |
| 7.5 | $f_{B_{j}}[m]$ | 6.535 | 4.614 | 5.059 | 4.851 | 4.873 |
|  | $f_{A_{j}}[m]$ | 2.535 | 0.614 | 1.059 | 0.851 | 0.873 |
| 7.6 | $l_{B_{j}}[\mathrm{~m}]$ | 74.306 | 88.077 | 82.528 | 84.745 | 84.495 |
|  | $l_{A_{j}}[m]$ | 46.171 | 32.093 | 37.698 | 35.454 | 35.707 |
| 7.7 | $l_{0_{B j}}[\mathrm{~m}]$ | 74.263 | 87.976 | 82.453 | 84.660 | 84.411 |
|  | $l_{0_{A_{j}}}[\mathrm{~m}]$ | 46.145 | 32.056 | 37.664 | 35.419 | 35.671 |
| 7.8 | $\epsilon=\left\|T_{\text {max }}-T_{j}\right\|$ | 20.661 | 7.140 | 3.991 | 0.461 | 0.040 |
| 8 | $L[m]$ | ------- | ------- | ------- | ------- | 120.202 |

### 5.3.3. Excel programming

Once the process for the determination of stresses and sags of different overhead lines have been deduced and presented, and specifically with supports at different heights, the need arises to design a mathematical program that allows and facilitates calculations automatically as well as the one designed for supports at the same height. The substantial difference that both programs will present will be the need to introduce additional data into the program regarding the difference in heights between supports. This second program, which is intended to be presented below, will be able to solve both the problems at different heights and at the same heights, the difference in height being zero in the latter case. Using calculation tools that allow programming, will facilitate obtaining results for an infinity of cases that may arise.

In this case, the Excel application and calculation tool will be used again. It is intended to redesign and reprogram the different steps and methods presented theoretically to finally obtain results based on the characteristics of the power line to be analyzed. As can be seen, the methods proposed for the different possible scenarios differ in small approaches that must be readjusted to obtain the final program. As has been suggested, text cells will also be enabled to insert the initial data so that the program will ultimately return the stress and sags values according to the marked conditions.

The first thing the program must know is the type of conductor that will be used in the construction of the power line. Depending on the conductor, there will be different characteristics of the same that will determine the working conditions and effort of the whole line. For this, in the final program there will be a multiple selection cell where user can access the choice of different conductors with their respective characteristics. The parameters that are set according to the conductor used will be the same presented in the programming for supports at the same height. The assignment of names for each of the variables would be as follows, adding the variable "Difference in heights between supports" for the final program:

| Data | Variable name in EXCEL |
| :---: | :---: |
| Span | Vano |
| Height difference between supports $(d)$ | dif. height |
| Breaking strain $\left(T_{\max }\right)$ | $T r$ |
| Conductor weight $\left(w_{w}\right)$ | $w p$ |
| Wind Force $\left(w_{\text {wind }}\right)$ | $w v$ |
| Ice Overload $\left(w_{i}\right)$ | $w h$ |
| Resulting force $\left(w_{T}\right)$ | wp; wpv; wph or wpvh |
| Modulus of elasticity $(E)$ | $E$ |
| Coefficient of linear expansion $(\alpha)$ | $S$ |
| Conductor section $(S)$ | $D$ |
| Conductor diameter $(D)$ | $m$ |
| Mass per unit length $(m / l)$ | $C f$ |
| Security coefficient |  |

The importance in choosing the span and safety factor can be highlighted as they will be especially relevant in the subsequent study. Likewise, the difference in heights between supports will be a determining factor due to its influence on the calculation of the lowest point of the catenary and the mechanical stresses on the supports. It is recalled that in the case of this work it has been considered that the coefficient used is 3 , while the regulation requires a minimum value of 2.5 . If necessary, this variable can be modified within the program.

As already explained, it is necessary to refer to the regulations to know the conditions that must be considered depending on the area in which the power line is going to be built. This also means that for different cases, the resulting weight to be considered for the conductor, being able to add an overload due to wind and/or ice, varies in different geographical areas. As examples, wind overload depends on the diameter of the conductor whereas in the case of ice sleeve thickness it depends on the height above sea level. The functions that had been proposed in the first program and that will continue to be used in the final program are:

| Cases | Representation | EXCEL |  |
| :---: | :---: | :---: | :---: |
| Conductor in <br> balance under the <br> action of his own <br> weight |  | Cell | Function |

To program the limit study conditions established by the Regulation depending on the characteristics of the line, the starting point is the same conditions and operations marked for the previous program presented. To do this, the program must ask the user what information is available regarding the geographical area (Zone A, B or C), characteristics of the line (whether or not it is a special category line) or the hypothesis load and their corresponding magnitudes to consider (wind, ice ...). These variables will be named in the program in the same way as in the programming carried out for supports at the same height (see attached tables).

Once the initial data necessary to proceed with the calculations has been established, its application must be programmed in terms of magnitude that allow the program to decide between different study values. Like the first program proposed, the new calculation tool must know the variables of temperature, wind overload and ice overload limits that must be guaranteed. It should be noted that these conditions will be totally independent from those that may be required in the application of the equation of change of conditions that is desired later. This differentiation should be meticulously careful not to mix information throughout the study. The functions that will specify these parameters will be equivalent to those presented in the case of supports at the same height:

| Parameters | EXCEL |  |
| :---: | :---: | :---: |
|  | Cell | Function |
| Temperature | Temp |  |
| Wind overload | V | =IF(ZONE="A";Vv;IF(AND(ZONE="B";Hip="T.M.wind";Cat.Esp="No"); <br> Vv;IF(AND(ZONE="B";Hip="T.M.ice";Cat.Esp="No");0; <br> IF(AND(ZONE="B";Cat.Esp="Yes");Vcat.esp;IF(AND(ZONE="C"; <br> Hip="T.M.wind";Cat.Esp="No");Vv;IF(AND(ZONE="C";Hip="T.M.ice"; <br> Cat.Esp="No");0;IF(AND(ZONE="C";Cat.Esp="Yes");Vcat.esp) $)$ ) $)$ ) ) |
| Ice overload | H | =IF(ZONE="A";O;IF(AND(ZONE="B";Hip="T.M.wind";Cat.Esp="No"); 0;IF(AND(ZONE="B";Hip="T.M.ice";Cat.Esp="No");0,18*SQRT(D); IF(AND(ZONE="B";Cat.Esp="Yes"); $0,18 * S Q R T(D) ;$ IF(AND(ZONE="C"; Hip="T.M.wind";Cat.Esp="No");0;IF(AND(ZONE="C"; Hip="T.M.ice";Cat.Esp="No");0,36*SQRT(D); IF(AND(ZONE="C";Cat.Esp="Yes");0,36*SQRT(D)(J)ננ)ננ) |

With all the necessary data already recorded, we proceed with the method deduced and explained above, for which cases 3 and 4 have been presented. The Solver function of Excel will be used to determine, by iterative calculation, the unknown variables in equations non-linear for which this tool is essential. Additionally, it is worth highlighting the application and use of the programming of "macros" in Excel to carry out the successive iterative calculations whose code will be presented later and which will be in charge of executing the set of steps that will be presented below.

Through conditional functions that allow the study of different scenarios depending on the meteorological or geographical conditions, it is possible to program the calculation based on the previous information that is known at the time of carrying out the analysis. If the functions used for the first part of the proposed method are available, in which it is desired to obtain the length of the conductor without being subjected to stress and under the conditions determined by the regulations, the following steps are possible:

| Steps |  | Equation | EXCEL |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Cell | Function |
| 1. | 1.1 |  | [equation 13] | c0_i | ```=IF(ZONE="A";TO_i/wpv;IF(AND(OR(ZONE="B";ZONE="C"); Hip="T.M.wind";Cat.Esp="No"); TO_i/wpv; IF(AND(OR(ZONE="B";ZONE="C");Hip="T.M.ice"; Cat.Esp="No"); TO_i/wph;IF(AND(OR(ZONE="B";ZONE="C") ;Cat.Esp="Yes"); T0_i/wpvh) J))``` |
|  | 1.2 | [equation 22] | bxO_i | $=c O_{-} i^{*}\left(\operatorname{COSH}\left(b x O_{-} i / c O_{-} i\right)-\operatorname{COSH}\left(\left(b x O_{-} i-S p a n\right) / c O_{-} i\right)\right]$-dif.height |
|  |  | geometry | axO_i | $=b x O_{-} i$-Span |
|  | 1.3 | [equation 14] | byO_i |  |
|  |  |  | ayO_i | =co_i* ${ }^{*} \mathrm{COSH}\left(a x O_{-} \mathrm{i} / \mathrm{co}-\mathrm{i}\right)$ |
|  | 1.4 | [equation 19] | TbO_i | ```\(=I F\left(Z O N E=" A " ; b y O_{-} i^{*}\right.\) wpv;IF(AND(OR(ZONE="B";ZONE="C"); Hip="T.M.wind";Cat.Esp="No"); byO_i*wpv; IF(AND(OR(ZONE="B";ZONE="C");Hip="T.M.ice"; Cat.Esp="No"); byO_i*wph;IF(AND(OR(ZONE="B";ZONE="C"); Cat.Esp="Yes"): by0 \(i^{*}\) wpuh) 1 )``` |
|  |  |  | TaO_i |  |

In this first iterative process, the first data must be extracted from the catenary under initial conditions. For this, the following search functions will be used based on the errors obtained from the stresses in the supports with respect to the maximum allowed:

| Parameters | EXCEL |  |
| :---: | :---: | :---: |
|  | Cell | Function |
| Distance in the X axis from support B to the lowest point of the catenary | bx0 | =XLOOKUP(MIN(ABS(Column.ERROR));Column.ERROR;Column.bxo_i) |
| Distance in the X axis from support A to the lowest point of the catenary | $a \times 0$ | =XLOOKUP(MIN(ABS(Column.ERROR));Column.ERROR;Column.ax0_i) |
| Mechanical stress of the conductor at support B | Tb0 | =XLOOKUP(MIN(ABS(Column.ERROR));Column.ERROR;Column.Tbo_i) |
| Mechanical stress of the conductor at support A | Ta0 | =XLOOKUP(MIN(ABS(Column.ERROR));Column.ERROR;Column.TaO_i) |

Once these initial data are available, the rest of the parameters that will define the catenary can be calculated in the limit conditions set by the regulation:

| Steps | Equation | EXCEL |  |
| :---: | :---: | :---: | :---: |
|  |  | Cell | Function |
| 2. | [equation 20] | fbo | $=c 0^{*}(\operatorname{Cosh}(b \times 0 / c 0)-1)$ |
|  |  | fa0 | $=c 0^{*}(\operatorname{Cosh}(a x 0 / c 0)-1)$ |
| 3. | [equation 16] | lcb | =c0*SINEH $(\mathrm{bxO} / \mathrm{c} 0)$ |
|  |  | Ica | $=c 0 * S I N E H(A B S(a x 0) / c 0)$ |
| 4. | geometry | Lab | $=I F(a x 0>0 ; 1 c b-I c a ; I F(a x 0<=0 ; 1 c b+l c a))$ |
| 5. | [equation 23] | lcbo | $=l c b /\left(1+T b 0 /\left(S^{*} E\right)\right)$ |
|  |  | Ica0 | = Ica/(1+Ta0/(S*E) |
|  | geometry | Var.lcb | $=l c b-l c b 0$ |
|  |  | Var.lca | $=1 c a-l c a 0$ |

From this point, it is time to apply the equations of change of conditions according to the needs when buying material or building any power line. For this, there are cells to notify the new information known on the conditions of study of the electrical power line. The changes can be reflected in temperature variations or new considerations of overloads caused by wind and/or ice. For this step, as in the first Excel program presented, the following cells are available:

| Data | Variable name in EXCEL | Values user can take |
| :---: | :---: | :---: |
| New Temperature | Temp.new | Insert value |
| New Condition Wind | V.new | Yes or No |
| New Condition Ice | H.new | Yes or No |

With the new conditions introduced, it is time to proceed with a new iterative method that allows obtaining the stress at which the conductor should be laid when building the overhead line according to the new considerations. The "macro" design tool will be used again to facilitate program automation and speed up calculations. The steps that will be carried out and their equations that will be monitored within the programming of the "macros" are presented below:

| Steps |  | Equation | EXCEL |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Cell | Function |
| 6. |  |  | [equation 24] | lcbo.new | $=l c b 0 *(1+$ Alpha*10^-6*(Temp.new-Temp) $)$ |
|  |  | Ica0.new |  | $=I c a 0 *\left(1+\right.$ Alpha* $10^{\wedge}-6^{*}($ Temp.new-Temp) $)$ |
| For different values of $T_{0_{j}}$ we proceed with the equations: |  |  |  |  |
| 7. | 7.1 | [equation 13] | c.new (i) | =IF(AND(V.new="No";H.new="No");T0_i/wp; IF(AND(V.new="Yes";H.new="No"); TO_i/wpv; IF(AND(V.new="Yes";H.new="Yes"); T0_i/wpvh; IF(AND(V.new="No";H.new="Yes"); TO_i/wph)) )) |
|  | 7.2 | [equation 22] | bx.new (i) | $=$ c.new $(i)^{*}(\operatorname{COSH}(b x . n e w(i) /$ c.new $(i))$ $\operatorname{COSH}(($ bx.new(i)-Span)/ c.new(i)))-dif.height |
|  |  | geometry | ax.new (i) | $=$ bx.new(i)-Span |
|  | 7.3 | [equation 14] | by.new (i) | $=c . n e w(i) * \operatorname{COSH}(b x . n e w(i) / c . n e w(i))$ |
|  |  |  | ay.new (i) | $=c . n e w(i) * \operatorname{COSH}(a x . n e w(i) / c . n e w(i))$ |
|  | 7.4 | [equation 19] | Tb.new (i) | =IF(AND(V.new="No";H.new="No");by.new(i)*wp; IF(AND(V.new="Yes";H.new="No");by.new(i)*wpv; IF(AND (V.new="Yes";H.new="Yes");by.new(i)*wpvh; $\operatorname{IF}($ AND (V.new="No";H.new="Yes");by.new(i)*wph)))) |
|  |  |  | Ta.new (i) | =IF(AND(V.new="No";H.new="No");ay.new(i)*wp; IF(AND(V.new="Yes";H.new="No");ay.new(i)*wpv; IF(AND (V.new="Yes";H.new="Yes");ay.new(i)*wpvh; $\operatorname{IF}($ AND (V.new="No";H.new="Yes");ay.new(i)*wph) $)$ ) |
|  | 7.5 | [equation 20] | fb.new (i) | $=c . n e w(i) *$ ( $\operatorname{COSH}(b x . n e w(i) / c . n e w(i))-1)$ |
|  |  |  | fa.new (i) |  |
|  | 7.6 | [equation 16] | Icb.new (i) | =c.new(i)*SINEH(bx.new(i)/c.new(i)) |
|  |  |  | Ica.new (i) | $=c . n e w(i) * S I N E H(A B S(a x . n e w(i)) / c . n e w(i)) ~$ |
|  | 7.7 | [equation 23] | Icbo.new (i) | $=l c b . n e w(i) /\left(1+\right.$ Tb.new $\left.(i) /\left(S^{*} E\right)\right)$ |
|  |  |  | Ica0.new (i) | $=$ Icanew $(i) /\left(1+\right.$ Ta.new $\left.(i) /\left(S^{*} E\right)\right)$ |

It can be seen that, as in previous steps, IF conditional functions have been used that allow proposing different operations according to the conditions required in the application of the change of conditions. In the case of the seventh step, it is necessary to calculate the stress value to which the conductor should be subjected and for this it is necessary to choose as the total weight of the cable the one established according to the new working conditions. Knowing these variables, the program will be the one who chooses the resulting weight concerned which, multiplying by the parameters " $y$ " of the catenary, it is possible to determine the mechanical stresses of the conductor in each of the supports.

Applying the iterative method to different stress values, new traction results are obtained in the supports which present errors with respect to the one initially considered. By calculating the error in each of the cases, a stress value close to the actual stress value can be estimated and taken as a reference. Taking this value, it is possible to work in a domain closer to the reference value to adjust and calibrate the final stress by iteration. Using the search functions that Excel presents, such as "XLOOKUP", it is possible to implement this reasoning in the program so that it is able to extract the final results that present the least error.

To define this type of function, we must first set the value wanted to find, then define the range where expected to find that value, and finally specify from which column is wanted to export the cell related to the one searched. In the case of this programming, the solution aims to show the conductor length parameters from each of the supports to the lowest point of the catenary both without being subjected to strain ( $l_{c b 0}$ and $l_{c a 0}$ ) and also when being lying ( $l_{c b}$ and $l_{c a}$ ), the parameter of the catenary ( $c$ ), the height of the catenary at the supports ( $b_{y}$ and $a_{y}$ ), the distances from the supports to the lowest point of the catenary ( $b_{x}$ and $a_{x}$ ), the mechanical stress of the conductor in each of the supports ( $T_{b}$ and $T_{a}$ ) and their respective sags ( $f_{b}$ and $f_{a}$ ). In order to export these values from the calculation tables and thus obtain the desired results, the following commands have been designed within certain output cells:

| Parameters | EXCEL |  |
| :---: | :---: | :---: |
|  | Cell | Function |
| CB conductor <br> length without <br> being stressed | Icbo.def | $=$ =XLOOKUP(MIN(ABS(Column.ERROR));Column.ERROR;Column.lcbo.new) |
| AC conductor <br> length without <br> being stressed | Ica0.def | $=$ =XLOOKUP(MIN(ABS(Column.ERROR));Column.ERROR;Column.Ica0.new) |
| CB conductor <br> length stressed | Icb.def | $=$ =XLOOKUP(MIN(ABS(Column.ERROR));Column.ERROR;Column.lcb.new) |
| AC conductor <br> length stressed | Ica.def | $=$ =XLOOKUP(MIN(ABS(Column.ERROR));Column.ERROR;Column.lca.new) |
| Catenary <br> parameter | c.def | $=$ =XLOOKUP(MIN(ABS(Column.ERROR));Column.ERROR;Column.c.new) |
| Height of the <br> catenary at <br> support B | by.def | $=$ =XLOOKUP(MIN(ABS(Column.ERROR);Column.ERROR;Column.by.new) |
| Catenary <br> height at <br> support A | ay.def | $=$ =XLOOKUP(MIN(ABS(Column.ERROR));Column.ERROR;Column.ay.new) |



For the design of "macros" in Excel, the "Programmer" tool has been used. Among its functions is the possibility of recording a macro by steps from which it has been possible to write the code and commands necessary for the total execution of the program. In summary, the structure of the macro designed for the second program presented is the following:

## MACRO PROGRAMMING:

(1) ' Shortcut: CTRL+d

Range("bx0(1)").Select
ActiveCell.FormulaR1C1 = "20"
(3) Range("bx0(1)").Select

Selection.AutoFill Destination:=Range("Column.bx0"), Type:=xlFillDefault
Range("Column.bx0").Select

SolverOk SetCell:="Ec.Sol.bx(i)", MaxMinVal:=3, ValueOf:=0, ByChange:="bx0(i)", Engine _ :=1, EngineDesc:="GRG Nonlinear"""
SolverOk SetCell:="Ec.Sol.bx(i)", MaxMinVal:=3, ValueOf:=0, ByChange:="bx0(i)", Engine _ :=1, EngineDesc:="GRG Nonlinear"
SolverSolve True
(5) Range("bx.new(1)").Select

$$
\begin{align*}
& \text { ActiveCell.FormulaR1C1 = "20" } \\
& \text { Range("bx.new(1)").Select } \\
& \text { Selection.AutoFill Destination:=Range("Column.bx.new"), Type:=xlFillDefault } \\
& \text { Range("Column.bx.new").Select } \\
& \\
& \text { SolverOk SetCell:="Ec.Sol.bx.new(i)", MaxMinVal:=3, ValueOf:=0, ByChange:="bx.new(i)", } \\
& \text { Engine _:=1, EngineDesc:="GRG Nonlinear""" } \\
& \text { SolverOk SetCell:="Ec.Sol.bx.new(i)", MaxMinVal:=3, ValueOf:=0, ByChange:="bx.new(i)", } \\
& \text { Engine_:=1, EngineDesc:="GRG Nonlinear" }  \tag{8}\\
& \text { SolverSolve True }
\end{align*}
$$

```
End Sub
```

As was done in the explanation of the first proposed program, it will be explained what the code used in the programming of the presented "macro" consists of. The start and staging of its execution is carried out through the "ctrl" key plus the letter "d". To do this, user have previously entered the starting data from which is desired to obtain results. In the case of this second program, which includes both the study of power lines with supports at the same height and at different heights, the calculations of initial parameters according to the conditions of the Regulation and subsequent ones related to the application of the change in conditions, have an independent iterative method.

These can be seen differentiated in programming between steps 2-4 and 5-7. The first of them, based on the data known and established by the regulations, solves for different stress values at the lowest point of the catenary $\left(T_{0}\right)$ the equation that allows determining the horizontal distance from the same point to support $\mathrm{B}\left(b_{x}\right)$. It can be seen how to obtain these results, in the second and third step of the macro the iterative process is prepared to start with an initial value of 20 . This is done to avoid starting with a null value that can lead to errors due to indeterminacies when solving the equations concerned.

As previously mentioned, the "Solver" function is used to solve these equations, which must be programmed within the code concerned to be executed. In a generic way, the fourth step of the macro shows how the characteristics of this process are established. First, the resolution objective is established, which in this case corresponds to equation 22 . This equation is entered in the resolution cell in such a way that it is equal to zero. In order for the "macro" to know that the equation has this structure, in the "MaxMinVal" command the number " 3 " is entered, which is required for its correct resolution.

Next, the cell where it is intended to track the iteration and obtain the final result is specified next, being in this case the cell corresponding to the parameter "bx0" that is obtained. Finally, the resolution method used is indicated, in this case it is the "GRG Nonlinear". These lines are closed with the execution of the "Solver" with the help of the "True" command that allows accepting possible pop-up confirmation windows after the resolution.

With these first commands it is possible to conclude the first part of the process presented. At this point, the program already has the necessary data to determine the characteristics of the catenary under the conditions set by the regulations. The following steps presented in the "macro" correspond to the second part and application of the change of conditions that will be discussed now.

Similar to steps 2-4, another iterative process is presented along steps 5-7 that takes the new information provided as a reference in this case. To do this, the user must introduce the new conditions of temperature, wind and ice.

It is known that in this method iteration is once again required. For this reason, a code equivalent to the one previously explained is proposed in which it once again is equation 22 the one to be solved. The difference that is found with respect to the previous case is that the variable to be determined at this time becomes "bx.new". This qualification is important to emphasize since it differentiates the results obtained from the requirements set by the Regulation from those obtained according to the change in conditions proposed. Finally, by correctly marking the parameters that configure the "Solver" tool to solve the non-linear equations, the programming is concluded with the "End Sub" command.

As can be seen, the code that allows the "Solver" tool to be used in a generic way for different cells has been definitively presented. With this, it should be clear that in the real macro constructed, as many lines as necessary to execute the iterative process must be included for all the chosen stress values and therefore for each of the parameters of " $c$ " that can be determined.

With all this, an automatic program is structured that allows to obtain in a dynamic and agile way the values of stresses and sags that are required in the construction of any overhead line subject to the specific conditions and characteristics of each case and scenario, even if supports are at the same height or at a different height.

## 6. Minimum safety distances



Along the different overhead lines of an electrical system, there may be cases in which a minimum safety distance must be ensured with respect to other systems that are around it. Examples of these can be the distances that the power lines present with respect to a road on which vehicles circulate, rivers through which boats pass or simply crossovers between different overhead lines, among others.

In this section we will consider the resolution and calculation of distances between a catenary, defined by the location of its supports and its corresponding state according to current regulations, and different elements of its environment for which it will be necessary to keep a certain minimum security distance.

First, we must know the basic prescriptions regarding electrical isolation distances to avoid accidents. Extracted from the Complementary Technical Instruction ITC-LAT 07, which refers to transmission lines that use bare conductors, it says:
"In lines it is necessary to distinguish between internal and external distances. The internal distances are only given to design a line with an acceptable capacity to withstand surges. The external distances are used to determine the safety distances between the live conductors and the objects below or in the vicinity of the line. The objective of the external distances is to avoid the damage of the electric shocks to the general public, to the people who work in the vicinity of the power line and to the people who work in its maintenance.

When it is not specified that the distance is "horizontal" or "vertical", the smallest distance between the parts with tension and the object considered will be taken, taking into account in the case of wind load the deviation of the conductors and the chain of insulators."

According to the regulations, three types of electrical distances are defined:

| Parameter | Definition |
| :---: | :---: |
| $D_{\text {el }}$ | Minimum specified isolation distance in air, to prevent a disruptive discharge <br> between phase conductors and objects at ground potential in slow or fast <br> front overvoltages. $D_{\text {el }}$ it can be both internal, when distance from the <br> conductor to the tower structure is considered, and external, when a distance <br> from the conductor to an obstacle is considered. |
| $D_{p p}$ | Specified minimum airborne isolation distance to prevent a disruptive <br> discharge between phase conductors during fast or slow front overvoltages. <br> $D_{p p}$ is an internal distance. |
| $a_{s o m}$ | Minimum value of the discharge distance of the insulator string, defined as the <br> shortest distance in a straight line between live parts and grounded <br> parts. |

To determine the internal and external distances, the following considerations will be taken into account:
"A) The electrical distance, $D_{e l}$, prevents electrical discharges between live parts and objects at ground potential, under normal operating conditions of the network. Normal conditions include latching operations, lightning strikes and surges resulting from network faults.
b) The electrical distance, $D_{p p}$, prevents electrical discharges between phases during maneuvers and lightning surges.
c) It is necessary to add to the external distance, $D_{\text {el }}$, an additional isolation distance, $D_{\text {add }}$, so that in the minimum safety distances to the ground, to power lines, to wooded areas, etc. make sure that people or objects do not come closer than $D_{e l}$ to the power line.
d) The probability of discharge through the minimum internal distance, $a_{\text {som }}$, must always be greater than discharge through an external object or person. Thus, for very long insulator strings, the risk of shock must be greater over the internal distance $a_{\text {som }}$ than to external objects or people. For this reason, the minimum external safety distances $\left(D_{\text {add }}+D_{e l}\right)$ must always be greater than 1.1 times $a_{\text {som. }}$."

The values of $D_{e l}$ and $D_{p p}$ are defined according to the highest voltage of the line $\boldsymbol{U}_{S}$ and are included in table 15 of the ITC-LAT 07 attached below:

Table 15. Electrical isolation distances to avoid shock

| Highest grid voltage <br> $U_{s}(\mathrm{kV})$ | $D_{e l}(\mathrm{~m})$ | $D_{p p}(\mathrm{~m})$ |
| :---: | :---: | :---: |
| 3.6 | 0.08 | 0.10 |
| 7.2 | 0.09 | 0.10 |
| 12 | 0.12 | 0.15 |
| 17.5 | 0.16 | 0.20 |
| 24 | 0.22 | 0.25 |
| 30 | 0.27 | 0.33 |
| 36 | 0.35 | 0.40 |
| 52 | 0.60 | 0.70 |
| 72.5 | 0.70 | 0.80 |
| 123 | 1.00 | 1.15 |
| 145 | 1.20 | 1.40 |
| 170 | 1.30 | 1.50 |
| 245 | 1.70 | 2.00 |
| 420 | 2.80 | 3.20 |

"In certain situations, such as crossovers and parallels with other lines or with communication routes or over urban areas, and in order to reduce the probability of an accident by increasing the safety of the line, in addition to the above general considerations, other special requirements must be met.

It will not be necessary to adopt special provisions at crossovers and parallels with non-navigable waterways, bridle paths, paths, sidewalks, streams and non-built fences, unless the latter may require an increase in the height of the conductors."

### 6.1. Internal distances

The first distances to be covered in any power line are the distances in support. These minimum safety distances are internal distances used solely to ensure the ability to withstand surges. In section 5.4 of the complementary technical instruction analyzed we find different cases:

## "Distances between conductors

The distance between the phase conductors of the same circuit or different circuits must be such that there is no risk of a short circuit between phases, bearing in mind the effects of the oscillations of the conductors due to the wind and the detachment of the snow accumulated on them.

For this purpose, the minimum separation between phase conductors shall be determined by the following formula:

$$
D=K \sqrt{F+L}+K^{\prime} D_{p p}
$$

in which:
$D=$ Separation between phase conductors of the same circuit or different circuits in meters.
$K=$ Coefficient that depends on the oscillation of the conductors with the wind, which will be taken from table 16.
$K^{\prime}=$ Coefficient that depends on the nominal voltage of the line $K^{\prime}=0.85$ for special category lines and $K^{\prime}=0.75$ for the rest of the lines.
$F=$ Maximum sag in meters, for the wind, temperature and ice hypotheses.
$L=$ Length in meters of the suspension chain. In the case of conductors fixed to the support by mooring chains or rigid insulators $L=0$.
$D_{p p}=$ Specified minimum overhead distance, to prevent a disruptive discharge between phase conductors during slow or fast front overvoltages. The values of $D_{p p}$ are indicated in table 15, depending on the highest voltage of the line.

This formula is applicable in the case of distances between conductors in the same span of the line. To calculate the distance between these and those deriving from the same support, the indications of the UNE-EN 50341-1 standard should be followed, maintaining the distance at least $D_{p p .}$.

The coefficient K is determined following the conditions studied in the section dedicated to the study of the equation of the change of conditions according to overloads by weight, wind and/or ice. The calculation of the angle of oscillation is recalled in the regulations:
"The values of the tangents of the oscillation angle of the conductors are given, for each load case, by the quotient of the wind overload divided by the own weight plus the ice overload if applicable according to area, per linear meter of conductor, the first being determined for a wind speed of 120 km/h."

Depending on the angle of oscillation and according to the nominal voltage of the line, the coefficients $K$ are established, which are included in section 5.4.1 of ITC-LAT 07 in table 16:

Table 16. K coefficient as a function of the oscillation angle

| Oscillation angle | K values |  |
| :---: | :---: | :---: |
|  | Lines with a nominal <br> voltage greater than 30 kV | Lines of nominal voltage <br> equal to or less than 30 kV |
| Greater than $65^{\circ}$ | 0.7 | 0.65 |
| Between $40^{\circ}$ and $65^{\circ}$ | 0.65 | 0.6 |
| Less than $40^{\circ}$ | 0.6 | 0.55 |

Other considerations of interest regarding the determination of the distances between conductors that are specified in section 5.4.1 are shown below:
"Among the preventive measures to avoid the phenomenon of galloping of conductors are the use of spacers between phases, or the installation of special accessories in the line (for example eccentric weights, dampers for the wind, devices for torsional control, pendulums for detuning, aerodynamic controllers etc.).

In areas where particularly important ice formations can be foreseen on the conductors, the risk of inadmissible approaches between them will be analyzed with special care.

The proposed formula corresponds to the same conductors and with the same sag. In the case of different conductors or with different sags, the separation between the conductors will be determined with the same formula and the greater $K$ coefficient and the greatest sag $\boldsymbol{F}$ of the two conductors. In the case of adopting smaller separations, the values used must be duly justified.

The separation between conductors and ground cables will be determined analogously to the separations between conductors, in accordance with all the previous paragraphs."

In the same way that it is necessary to guarantee a minimum distance between conductors, the regulations require other considerations for the distances that are kept between the different grounded elements and the conductors themselves. The most relevant clarifications are included in section 5.4.2 (ITC-LAT 07) as follows:

## "Distances between conductors and grounded parts

The minimum separation between the conductors and their accessories in tension and the supports shall not be less than $D_{e l}$, with a minimum of 0.2 m .

The values of $D_{e l}$ are indicated in section 5.2, depending on the highest voltage of the line.
In the case of suspension chains, the conductors and the insulator chain deflected under the action of half the wind pressure corresponding to a wind speed of $120 \mathrm{~km} / \mathrm{h}$ will be considered. For these purposes, the mechanical stress of the conductor subjected to the action of half the wind pressure corresponding to a wind speed of $120 \mathrm{~km} / \mathrm{h}$ and a temperature of $-5{ }^{\circ} \mathrm{C}$ for zone $A,-10{ }^{\circ} \mathrm{C}$ for zone $B$ and $-15{ }^{\circ} \mathrm{C}$ for zone $C$.

The application of the reference parameters is independent of the line category, with $120 \mathrm{~km} / \mathrm{h}$ wind speed for all lines. In the case of bridges, the conductors and the deviated insulator chain will be considered under the action of half the wind pressure corresponding to a $120 \mathrm{~km} / \mathrm{h}$ wind speed.

The counterweights will not be used on a whole line repeatedly, although they may be used exceptionally to reduce the deviation of a suspension chain, in which case the designer will justify the values of the deviations and distances to the support."

### 6.2. External distances

### 6.2.1. Approach to calculation of distances in crossovers

As shown in the sketch presented in page 79, it is intended to obtain the minimum distance between two crossing catenaries. Due to its apparent complication, it will be preferable to begin by devising the resolution of a similar problem taking two lines as functions. Taking the Cartesian system as reference axes, it would be easy to solve the problem mathematically by simply knowing two points belonging to each of the lines. With these data, it would be possible to deduce the equations of each one of the lines that would allow to solve the three-dimensional problem.

To do this, we are going to start by listing the steps to follow to determine the minimum distance between two simple crossing lines. Once the minimum distance between lines has been obtained, it will also be possible to calculate which points, on each of the lines, certify said separation. As a schematic for the proposed problem, it can be drawn:


Considering a three-dimensional analysis, leaving aside the mathematical dimension, the steps that could be proposed would be:

1. An auxiliary line " $t$ " is drawn parallel to line " $r$ " that intersects line " $s$ ".
2. From the auxiliary line " $t$ " and the line " $s$ " the plane " $\alpha$ " that contains them is drawn.
3. Through a point " $x$ " of the line " $r$ ", a line " $u$ " is drawn perpendicular to the plane " $\alpha$ ".
4. Find the intersection between the plane " $\alpha$ " and the line $" u$ ", resulting in a point " $y$ ".
5. The magnitude between the points " $x$ " and " $y$ " can be calculated giving as a solution the minimum distance between the two crossing lines.

If it is wanted to know the exact coordinates of the two points that present this separation:
6. We proceed by drawing a new plane " $\beta$ " between lines " $r$ " and " $u$ ".
7. The intersection between the planes " $\alpha$ " and " $\beta$ " would result in a new line " v ".
8. The point of intersection " 1 " between the lines " v " and " s " will be one of the points to be determined (specifically, it is the point belonging to the line "s").
9. To finally obtain its "counterpart" but on the line " $r$ ", it would be enough to draw a new line " $w$ " perpendicular to the plane " $\alpha$ " through the point " 1 " obtained in the previous step, whose intersection with the line " $r$ " will give the desired point " 2 " as a solution.

Once the problem and its resolution have been imagined, it would be time to transfer its execution to a mathematical approach that solves the different equations to find the final solution. Observing the case of two crossing lines, it is now possible to think about whether it would be possible to propose a similar solution for the case in which a simple catenary is involved. Through its equations, deduced at the beginning of the study, it would be possible to model it.

The complication that the geometry of the catenaries presents, or even the parabolas in the case of resorting to mathematical simplifications, is that it is not so easy to create and manage the geometric space of the points on the surface that contains two catenaries crossing. In other words, while with two crossing lines we can draw a plane that contains them, in the case of two catenaries, it is not so easy to draw a surface that includes them.

Therefore, there would be no possibility of obtaining the minimum distance between two crossing catenaries, as there is no reference such as a "plane" on which to draw perpendicular lines ... Due to the difficulty of implementing a procedure similar to that presented in the previous page for two lines, it is necessary to propose another solution to our problem.

Adopting the Cartesian system as a reference in our study, we are going to start by proposing the calculation of the distance between two points in space from their coordinates. The diagram that represents this simple problem can be seen below:


$$
\overline{A B}=\sqrt{\left(b_{x}-a_{x}\right)^{2}+\left(b_{y}-a_{y}\right)^{2}+\left(b_{z}-a_{z}\right)^{2}}
$$

It can be seen that, once the Cartesian coordinates of different points are known, it is possible to obtain the distance between two points through a very simple calculation. That is why a resolution based on the measurement of the distance between different points from which to finally extract, among all the measurements, the smallest magnitude is going to be designed.

The resolution procedures will be implemented in different spreadsheets, each one attending to different situations. Studying the current regulations, the different cases of interest that must be examined due to the importance of guaranteeing sufficient safety to avoid accidents derived from electric shocks will be analyzed.

### 6.2.2. Proposed resolution method

As data for the problem, the program will require the Cartesian coordinates of the elements to be considered in the study. In the case of overhead lines, the points of the support towers will be specified, while for roads, lands or rivers it will be sufficient to know the limit points that are in the same plane that contains the catenary. In other words, the limits of the second element analyzed that, in plan view, cut the path of the power line, will be taken as representative points.

In either case, the resolution procedure will be based on obtaining different points from the mathematical functions that represent the two elements considered. Between the different points that are chosen, it will be possible to measure the distances by carrying out all the possible combinations between the points of the two elements. The minimum value obtained from all measurements will constitute the minimum distance between the two elements.

In order to obtain the final result with this approach in just one step, it would be necessary to have a program capable of extracting infinite points from each element and then calculating all the distances between them and extracting the one with the lowest value. This operating capacity would be very expensive for a program, so a more efficient strategy must be devised when choosing the points that will be involved in the study.


As a step-by-step explanation, the previous illustrations show the proposed resolution in a schematic way. It is considered the most complete and complicated case to be analyzed in this report. This consists of two overhead lines which are represented by two catenaries with their respective mathematical equations. Once the calculation program for this case has been designed, it will be easier to propose the resolution of simpler systems.

Ultimately, the proposed program tries to solve the problem by choosing a finite number of points and, by analyzing the distances between them, it will be able to converge to a final solution as the precision of the study increases. Initially, only the location of the supports in space (XZ plane) and their height ( Y axis) are known. The distance indicated as "DM" is the solution to our problem, that is, the minimum distance between the two elements considered.

From this point, it can be observed how the resolution goes through choosing different points of both catenaries and determining the two points that present the least distance in the first approximation $\left(D M_{1}\right)$. With these two points the procedure is repeated but in a smaller range increasing the precision in the calculations and results. The procedure ends after different iterations after which the most exact solution is obtained through the selection process.

Following the steps reflected in the figures on the previous page, we can see how the distance is obtained from the first iteration (step 2) $D M_{1}$ while in the case of the second iteration (step 3) the smallest distance calculated is $D M_{2}$. Finally, in the last iteration process (step 4), the minimum distance $D M_{3}$ is obtained, which will turn out to be the solution to the problem and, therefore, will be called as $D M$.

Taking into account that the illustrations shown are a simple sketch of the proposed resolution, it should be clarified that the number of points evaluated by the program will be much higher, even being able to be modified. With this, it is possible to obtain a greater accuracy of the study, guaranteeing the correct resolution of the problem and providing the program with great flexibility when it comes to providing greater or lower precision depending on the case considered. It will be convenient to carry out a cost-precision balance to determine the ideal number of points in each of the possible situations.

With all this, the method of solving this type of problem is introduced based on the most peculiar and complicated case, such as the crossover between two catenaries whose equations present an operation not as simple as that found in functions of lines, parabolas ...

The next step that must be taken is to carry out the programming that is going to obtain the minimum distances in different scenarios and that allows finally to draw conclusions according to the regulatory prescriptions in terms of safety. That is why each case presented in the regulations will have a personalized spreadsheet according to the details that concern it.

### 6.2.3. Excel programming

As previously introduced, one of the objectives of studying the regulations is to be able to design a program that allows us to calculate the minimum distances between different elements, guaranteeing the necessary security levels. Following the cases shown in the regulations, all will consist of a power line (first element) which we will try to represent from the equations deduced in the first part of the study and another system (second element) that in each situation can be modeled according to more convenient model.

The starting data must be the Cartesian coordinates of points that represent the supports of the overhead line (element 1) or the limits of the systems (element 2) located in the vicinity of the overhead line. In the first place, we will study how to work with catenaries in programming. In this case, it will be very useful to know which of the supports is the highest in order to easily make use of the program devised at the beginning of the study. This will be very useful to define the different parameters of the catenary. To do this, we will start by calling the "A" support the one with the lowest height, while the support " B " will be represented by the highest support of the two. The coordinates chosen for each support are obtained from the following formulas:

| Parameters | Formulas |
| :---: | :---: |
| $y 1 A$ | $=I F(y 1 A .0=y 1 B .0 ; y 1 A .0 ; M I N(y 1 A .0: y 1 B .0))$ |
| $x 1 A$ | $=$ IF $(y 1 A .0=y 1 B .0 ; x 1 A .0 ; X L 00 K U P(y 1 A ;(y 1 A .0: y 1 B 0) ;(x 1 A .0: x 1 B 0)))$ |
| $z 1 A$ | $=$ IF(y1A.0 $=y 1 B .0 ; z 1 A .0 ; X L 00 K U P(y 1 A ;(y 1 A .0: y 1 B 0) ;(z 1 A .0: z 1 B 0)))$ |
| $y 1 B$ | $=$ IF(y1A.0 $=y 1 B .0 ; y 1 B .0 ; M A X(y 1 A .0: y 1 B .0))$ |
| $x 1 B$ | $=$ IF(y1A.0 $=y 1 B .0 ; x 1 B .0 ; X L 00 K U P(y 1 B ;(y 1 A .0: y 1 B 0) ;(x 1 A .0: x 1 B 0)))$ |
| $z 1 B$ | $=$ IF(y1A.0 $=y 1 B .0 ; z 1 B .0 ; X L 00 K U P(y 1 B ;(y 1 A .0: y 1 B 0) ;(z 1 A .0: z 1 B 0)))$ |

When working with a three-dimensional system, it is important to remember the importance of the position of the supports in space due to its relevance when obtaining the final results. Taking what has been said into account, as surely the catenary is not in the ideal XY plane, it is necessary to propose a method prepared to solve the problem independently of the threedimensional coordinates of the points that represent the supports.

Bearing in mind that the $Y$ axis will represent the heights, we are interested in defining the projection of the catenary in the XZ plane (a straight line). For this we must define the equation of the line that joins the points ( $\mathrm{x} 1 \mathrm{~A}, \mathrm{z} 1 \mathrm{~A}$ ) and ( $\mathrm{x} 1 \mathrm{~B}, \mathrm{z} 1 \mathrm{~B}$ ) (projections of the supports A and B in the $X Z$ plane). From elementary mathematics it is known that the equation of the line that passes through two points can be written in the form:

$$
\begin{gathered}
\frac{x-x_{1}}{x_{2}-x_{1}}=\frac{z-z_{1}}{z_{2}-z_{1}} \\
\left(x-x_{1}\right)\left(z_{2}-z_{1}\right)=\left(z-z_{1}\right)\left(x_{2}-x_{1}\right) \\
x\left(z_{2}-z_{1}\right)-x_{1}\left(z_{2}-z_{1}\right)=z\left(x_{2}-x_{1}\right)-z_{1}\left(x_{2}-x_{1}\right) \\
\boldsymbol{x}\left(z_{2}-z_{1}\right)=\boldsymbol{z}\left(x_{2}-x_{1}\right)-z_{1}\left(x_{2}-x_{1}\right)+x_{1}\left(z_{2}-z_{1}\right) \\
\boldsymbol{x}=\frac{\boldsymbol{z}\left(x_{2}-x_{1}\right)-z_{1}\left(x_{2}-x_{1}\right)+x_{1}\left(z_{2}-z_{1}\right)}{z_{2}-z_{1}}
\end{gathered}
$$

Finally obtaining:

$$
\boldsymbol{x}=\frac{x_{2}-x_{1}}{z_{2}-z_{1}} \boldsymbol{z}-\frac{z_{1}\left(x_{2}-x_{1}\right)}{z_{2}-z_{1}}+x_{1} \equiv m \mathbf{z}+n
$$

Equivalently, it can also be expressed:

$$
\mathbf{z}=\frac{z_{2}-z_{1}}{x_{2}-x_{1}} \boldsymbol{x}-\frac{x_{1}\left(z_{2}-z_{1}\right)}{x_{2}-x_{1}}+z_{1} \equiv m^{\prime} \boldsymbol{x}+n^{\prime}
$$

By introducing these equations in our program, we will be able to know the slope and the $y$ intercept of the line:

| Parameters | Formulas |
| :---: | :---: |
| m.x1 | =IF(z1A=z1B;"Inf"; $x 1 B-x 1 A) /(z 1 B-z 1 A))$ |
| n.x1 | =IF(z1A=z1B;"Inf";x1A-z1A*(x1B-x1A)/(z1B-z1A) |
| m.z1 | =IF(x1A=x1B;"Inf";(z1B-z1A)/(x1B-x1A)) |
| n.z1 | =IF(x1A=x1B;"Inf";z1A-x1A*(z1B-z1A)/(x1B-x1A)) |

In the same way that the distance between two points in three-dimensional space can be calculated, the distance between the two projections of the supports A and B on the XZ plane can be calculated that reveal the span of the catenary through the relationship:


$$
\text { Vano } \equiv \overline{A B}=\sqrt{\left(b_{x}-a_{x}\right)^{2}+\left(b_{z}-a_{z}\right)^{2}}
$$

In Excel language and according to the data of our problem, the formula will be collected as:

| Parameter | Formula |
| :---: | :---: |
| Span | $=$ SQRT $\left((x 1 B-x 1 A)^{\wedge} 2+(z 1 B-z 1 A)^{\wedge} 2\right)$ |

Another piece of information that will be necessary to define the equation of the catenary is the difference in heights "d" between the two supports A and B. For this, it will be sufficient to perform the subtraction between the coordinates "y" of both supports. Once the span and the difference in height between supports are known, the first program presented in the report will be used to determine the parameter " $c$ " of the catenary in addition to the location of the lowest point of the catenary with respect to the supports (c, ax0 and bx0). From this data collection, the maximum sags can be obtained with respect to the supports:

| Parameters | Formulas |
| :---: | :---: |
| fb0.L1 | $=\mathrm{c} 0 . \mathrm{L} 1 *(\mathrm{COSH}(\mathrm{b} 0 . \mathrm{L} 1 / \mathrm{c} 0 . \mathrm{L} 1)-1)$ |
| fa0.L1 | $=\mathrm{c} 0 . \mathrm{L} 1 *(\mathrm{COSH}(\mathrm{a} 0 . \mathrm{L} 1 / \mathrm{c} 0 . \mathrm{L} 1)-1)$ |

These last two values will be useful to obtain all the Y coordinates with respect to the same reference. This is to solve a problem that arises in the preparation of this study. This consists in realizing that the characteristic parameters of two catenaries (c) do not have the same reference line, so the values " $y$ ", which can be calculated from the characteristic equation of the catenary $y=c \cdot \cosh (x / c)$, are not prepared to be compared either between different catenaries.

This difference is evidenced in the following drawing for two points " I " and " J ", each one located on a different power line:


Once the need to take the base of the supports (the ground) as a reference is justified, it will be useful to initially calculate the distances " s " that will represent the vertical distance from the lowest point of the catenary to the ground (chosen reference). For this, the calculation will be very simple from the heights of the supports and the values of the sags in both supports of the overhead line:

| Parameters | Formulas |
| :---: | :---: |
| s0.L1 | =y1B-fb0.L1 |
|  | $=y 1 A-f a 0 . \mathrm{L} 1$ |

From this characteristic value of the line, the height with respect to the ground of any point of the catenary can be determined by knowing its sag (vertical distance from the chosen point to the lowest point of the catenary) and adding its value to the one of the parameter "s". This calculation will be developed later.

Once all the necessary data have been entered to carry out the calculations of minimum distances, different points of the catenary must be selected, in the case of power lines, or of the equation of the line if it is other element such as roads, rivers... Focusing on the catenary equation, we begin by selecting different "X" points that, from the line that represents the projection of the catenary in the XZ plane, the "Z" coordinates can be easily calculated.

As long as the supports have "X" coordinates of different values, no problem will arise, but the possibility that both coordinates are equal must be taken into account. In this case, the slope of the line $z=m x+n$ would be infinite, so it would not be possible to deduce the coordinates on the " $Z$ " axis for the value of " X ".

For this, different conditional functions "IF" will be programmed that allow to start assigning the coordinates in the most convenient axis ( X or Z ). In this way, in the event that two of the " X " or " Z " components of the two supports are equal, the equation of the line used will be the one whose slope is not infinite and which allows obtaining all the desired coordinates.

The first point of the series will be fixed by the smallest coordinate of the two supports:

| Parameters | Formulas |
| :---: | :---: |
| x_0 | =IF(x1A=x1B;m.x1*z_0+n.x1;MIN(x1A;x1B) |
| z_0 | =IF(x1A=x1B;MIN(z1A;z1B);m.z1*x_0+n.z1) |

Starting from the point of origin, already known, the points that will participate in the study must be defined, taking into account the warnings mentioned on the previous page. In each measurement an amount of twenty points will be chosen. The first approximation will have as limits the representative points of the supports. To do this, one twentieth of the distance between equivalent coordinates is taken as the separation between points:

| Parameters | Formulas |
| :---: | :---: |
| x_i | =IF(x1A=x1B;m.x1*z_i+n.x1;x_i-1+ABS(x1B-x1A)/20) |
| Z_i | =IF(x1A=x1B;z_i-1+ABS(z1B-z1A)/20;m.z1*x_i+n.z1) |

For each "X" coordinate already defined, its position is calculated with respect to the lowest point of the catenary on the axis fixed by the line in the XZ plane that joins the projections of the supports. This value will allow us to calculate the sag of each point, that is, the vertical distance from each chosen point to the lowest point of the catenary. The formulas used would be:

| Parameters | Formulas |
| :---: | :---: |
| ax_i | $=$ SQRT $\left(\left(x \_i-x 1 A\right)^{\wedge} 2+\left(z_{-} i-z 1 A\right)^{\wedge} 2\right)+a 0 . L 1$ |
| f_i | $=c 0 . L 1 *\left(C O S H\left(a x \_i / c 0 . L 1\right)-1\right)$ |

The last step to obtain all the coordinates of the chosen points is to determine the " Y " coordinate. To do this, the sags of each of the points have already been calculated, therefore, taking into account the reference distance "s" defined above, the "Y" coordinates of the different points are obtained as follows:

| Parameter | Formula |
| :---: | :---: |
| $y_{-} \mathrm{i}$ | $=$ s0.L1+f_i |

By repeating this sequence of steps for each of the points, the twenty points belonging to the catenary with their respective coordinates are obtained. From these, the distances will be calculated with respect to the chosen points of another catenary or other element. As will be justified in the cases that will be discussed later according to the regulations, unlike the cases in which the second element is about another power line and, therefore, is also defined from the equation of the catenary, the rest of the elements will be represented by lines.

In the case of treating the second element as another catenary, the procedure for selecting points will be strictly identical to that proposed for the first element (power line). In the case of resorting to the equation of the line to define other elements, the procedure is simplified as it is a simpler geometry and does not require such complex equations.

From the two points on the ground that cross the overhead line (element 1), it is possible to obtain the equation of the line. To facilitate and resemble this procedure to the previous one, the idea of first calculating the equation of the line projected in the XZ plane will be reused.

The procedure to follow in these cases will be discussed now.

Modifying only the data regarding the case of the overhead line, the formulas would be:

| Parameters | Formulas |
| :---: | :---: |
| m.x2 | =IF(z2A=z2B;"Inf";(x2B-x2A)/(z2B-z2A)) |
| n.x2 | =IF(z2A=z2B;"Inf";x2A-z2A*(x2B-x2A)/(z2B-z2A)) |
| m.z2 | =IF(x2A=x2B;"Inf";(z2B-z2A)/(x2B-x2A)) |
| n.z2 | =IF(x2A=x2B;"Inf";z2A-x2A*(z2B-z2A)/(x2B-x2A)) |

Taking as a reference the procedure followed in the case of overhead lines, it is time to select different points on the "X" or "Z" axis that, from the line that represents the projection of the catenary in the XZ plane, it can be easily calculated the "Z" or "X" coordinates, respectively. In this way, it is intended to give a solution to the appearance of equations with infinite slopes that make it impossible to determine the coordinates necessary to continue with the study.

For this reason, different "IF" conditional functions will also be programmed that allow us to start assigning the coordinates on the most convenient axis ( X or Z ). The first point of the series will be fixed by the smallest coordinate of the two known points:

| Parameters | Formulas |
| :---: | :---: |
| $\mathrm{x}_{-} 0$ | $=\mathrm{IF}\left(\mathrm{x} 2 \mathrm{~A}=\mathrm{x} 2 \mathrm{~B} ; \mathrm{m} \cdot \mathrm{x} 2^{*} \mathrm{z}_{\_} 0+\mathrm{n} \cdot \mathrm{x} 2 ; \mathrm{MIN}(\mathrm{x} 2 \mathrm{~A} ; \mathrm{x} 2 \mathrm{~B})\right)$ |
| $\mathrm{z}_{-} 0$ | $=\mathrm{IF}\left(\mathrm{x} 2 \mathrm{~A}=\mathrm{x} 2 \mathrm{~B} ; \mathrm{MIN}(\mathrm{z} 2 \mathrm{~A} ; \mathrm{z2B}) ; \mathrm{m} . \mathrm{z} 2^{*} \mathrm{x}_{-} 0+\mathrm{n} . \mathrm{z} 2\right)$ |

For each measurement, as in the case of the first element, the number of points chosen will be a total of twenty. Remember that one twentieth of the distance between equivalent coordinates is taken as the separation between points:

| Parameters | Formulas |
| :---: | :---: |
| x_i | =IF(x2A=x2B;m.x2*z_i+n.x2;x_i-1+ABS(x2B-x2A)/20) |
| Z_i | =IF(x2A=x2B;z_i-1+ABS(z2B-z2A)/20;m.z2*x_i+n.z2) |

When it comes to determining the " Y " coordinates, the lines that represent the projections in the XY or YZ planes will be defined, thus adopting the same reasoning performed when obtaining the equation of the line projected on the XZ plane in previous steps of the study. The necessary equations would be:

$$
\frac{x-x_{1}}{x_{2}-x_{1}}=\frac{y-y_{1}}{y_{2}-y_{1}}=\frac{z-z_{1}}{z_{2}-z_{1}}
$$

Behind the operational developing are obtained:

$$
\begin{aligned}
& \boldsymbol{y}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \boldsymbol{x}-\frac{x_{1}\left(y_{2}-y_{1}\right)}{x_{2}-x_{1}}+y_{1} \equiv m \boldsymbol{x}+n \\
& \boldsymbol{y}=\frac{y_{2}-y_{1}}{z_{2}-z_{1}} \boldsymbol{z}-\frac{z_{1}\left(y_{2}-y_{1}\right)}{z_{2}-z_{1}}+y_{1} \equiv m^{\prime} \mathbf{z}+n^{\prime}
\end{aligned}
$$

The formulas implemented in Excel to obtain the slope and the $y$-intersect of the lines projected on the XY or YZ planes would be:

| Parameters | Formulas |
| :---: | :---: |
| m.y1 | =IF(x2A=x2B;"Inf";(y2B-y2A)/(x2B-x2A)) |
| n.y1 | =IF(x2A=x2B;"Inf";-x2A*(y2B-y2A)/(x2B-x2A)+y2A) |
| m.y2 | =IF(z2A=z2B;"Inf"; ${ }^{\text {(y2B-y2A }}$ )/(z2B-z2A) $)$ |
| n.y2 | =IF(z2A=z2B;"Inf";-z2A*(y2B-y2A)/(z2B-z2A)+y2A) |

With the equations of the known lines, it is now possible to go to the last step, which consists of obtaining all the " Y " coordinates. The proposed formula is:

| Parameter | Formula |
| :---: | :---: |
| y_i |  |

With all this, the first twenty points of each element would be obtained which, together with the points extracted from the first element, would already be available to calculate the distances between the two systems. Arranged in such a way that a table is obtained and it relates all the possible combinations between the different points of the two defined elements, the distances are calculated from the formula:

| Parameter | Formula |
| :---: | :---: |
| $\mathrm{d}_{-} \mathrm{ij}$ | $=$ SQRT((x-j-x_i $\left.)^{\wedge} 2+\left(y_{-} j-y_{-} \mathrm{i}\right)^{\wedge} 2+\left(z_{-}-z_{-} i\right)^{\wedge} 2\right)$ |

Where the components " i " and " j " refer to the different points of the first and second element, respectively. From all the measurements carried out, we are interested in extracting the minimum. After this first iteration, the minimum distance will be useful as a reference for the following iterations in which we will want to approximate the study range to converge to the absolute minimum solution. To extract the location of the points that present a smaller distance in the first iteration, the XLOOKUP function is used.

From this moment on, the selection of points to be evaluated will have a different criterion. Taking as reference points in each element those extracted from the XLOOKUP functions, ten new points will be taken towards each "side". It should be noted that in order to adequately advance in solving the problem, the distance between these new points must be less than that chosen in the previous iteration, thus enclosing the domain of the study. For the selection of new points, the following formulas will be used:

| Parameters | Formulas |
| :---: | :---: |
| x_i (left) | $\begin{gathered} =\mathrm{IF}\left(\mathrm{x} 1 \mathrm{~A}=\mathrm{x} 1 \mathrm{~B} ; \mathrm{m} . \mathrm{x} 1^{*} \mathrm{z}_{-} \mathrm{i}+\mathrm{n} . \mathrm{x} 1 ; \mathrm{IF}\left(\mathrm{x}-\mathrm{i}+1<=\mathrm{MIN}(\mathrm{x} 1 \mathrm{~A} ; \mathrm{x} 1 \mathrm{~B}) ; \mathrm{MIN}(\mathrm{x} 1 \mathrm{~A} ; \mathrm{x} 1 \mathrm{~B}) ; \mathrm{x}_{-} \mathrm{i}\right.\right. \\ +1-\mathrm{ABS}(\mathrm{x} 1 \mathrm{~B}-\mathrm{x} 1 \mathrm{~A}) / 80)) \end{gathered}$ |
| z_i (left) | $\begin{gathered} =\mathrm{IF}\left(\mathrm{x} 1 \mathrm{~A}=\mathrm{x} 1 \mathrm{~B} ; \mathrm{IF}\left(\mathrm{z}_{-} \mathrm{i}+1<=\mathrm{MIN}(\mathrm{z} 1 \mathrm{~A} ; \mathrm{z} 1 \mathrm{~B}) ; \mathrm{MIN}(\mathrm{z} 1 \mathrm{~A} ; \mathrm{z} 1 \mathrm{~B}) ; \mathrm{z}_{-} \mathrm{i}+1-\mathrm{ABS}(\mathrm{z} 1 \mathrm{~B}-\right.\right. \\ \left.\mathrm{z} 1 \mathrm{~A}) / 80) ; \mathrm{m} . \mathrm{z} 1^{*} \mathrm{x} \_\mathrm{i}+\mathrm{n} . \mathrm{z} 1\right) \end{gathered}$ |
| x_i (right) | $\begin{gathered} \hline=\mathrm{IF}\left(\mathrm{x} 1 \mathrm{~A}=\mathrm{x} 1 \mathrm{~B} ; \mathrm{m} . \mathrm{x}^{*} \mathrm{z}_{-} \mathrm{i}+\mathrm{n} . \mathrm{x} 1 ; \mathrm{IF}\left(\mathrm{x} \_\mathrm{i}-1>=\mathrm{MAX}(\mathrm{x} 1 \mathrm{~A} ; \mathrm{x} 1 \mathrm{~B}) ; \mathrm{MAX}(\mathrm{x} 1 \mathrm{~A} ; \mathrm{x} 1 \mathrm{~B}) ; \mathrm{x} \_\mathrm{i}-\right.\right. \\ 1+\mathrm{ABS}(\mathrm{x} 1 \mathrm{~B}-\mathrm{x} 1 \mathrm{~A}) / 80)) \end{gathered}$ |
| z_i (right) | $\begin{gathered} =\mathrm{IF}\left(\mathrm{x} 1 \mathrm{~A}=\mathrm{x} 1 \mathrm{~B} ; \mathrm{IF}\left(\mathrm{z}_{-} \mathrm{i}-1>=\mathrm{MAX}(\mathrm{z} 1 \mathrm{~A} ; \mathrm{z1B}) ; \mathrm{MAX}(\mathrm{z1A} ; \mathrm{z1B}) ; \mathrm{z} \mathrm{i} \mathrm{i} 1+\mathrm{ABS}(\mathrm{z1B}-\right.\right. \\ \left.\mathrm{z} 1 \mathrm{~A}) / 80) ; \mathrm{m} . \mathrm{z} 1^{*} \mathrm{x}-\mathrm{i}+\mathrm{n} . \mathrm{z1}\right) \end{gathered}$ |

With all the " X " and " Z " coordinates of the new points already calculated, it would only remain to obtain the " Y " coordinates from the equations previously presented. This process of selecting new points is perfectly applicable to the two elements that are under study. As a result of the process, forty new points are obtained, twenty belonging to each element, each time closer. Considering all the possible combinations between them, the distances are calculated.

From now on, all iterations have an identical resolution mechanism. The only qualification to remember, as the iterations progress, is the obligation to reduce the distance between selected points, thus increasing the precision of the analysis. At the end of the number of iterations raised (the designed program shows a total of four iteration processes), the minimum distance obtained in the last step will be the final result of the problem. This value will be the one that must later be evaluated with respect to the regulations to check whether the planned installation meets the requirements set in the regulatory framework.

### 6.2.4. Cases and problems

### 6.2.4.1. Distances to the terrain, pathways, paths and non-navigable waterways

As a first study, section 5.5 of ITC-LAT 07 will be analyzed. The elements to consider are:

|  | Element 1 | Element 2 |
| :---: | :---: | :---: |
| Description | Power line | Terrain, pathway, path or <br> non-navigable waterways |
| Representation | Catenary equation | Line equation |

Now it is necessary to know the regulations regarding pathways, lands, paths and nonnavigable waterways that will allow us to know if the minimum distance obtained by the program, according to the data of the problem, is sufficient to be able to finally build said power line on the analyzed terrain. The regulation states:
"The height of the supports will be that necessary so that the conductors, with their maximum vertical sag according to the temperature $\left(\mathbf{5 0}^{\circ} \mathrm{C}\right)$ and ice hypotheses, are located above any point on the ground, path, sidewalk or non-navigable water surfaces, at a minimum height of:

$$
D_{a d d}+D_{e l}=5.3+D_{e l} \text { in meters, }
$$

with a minimum of $\mathbf{6}$ meters. However, in places that are difficult to access, the above distances may be reduced by one meter.

The values of $D_{e l}$ are indicated in section 5.2, depending on the highest voltage of the line.
When the lines cross fenced cattle farms or agricultural farms, the minimum height will be 7 meters, in order to avoid accidents due to water projection or the circulation of agricultural machinery, trucks and other vehicles.

In the hypothesis of the calculation of maximum sags under the action of the wind on the conductors, the previous minimum distance may be reduced by one meter, considering in this case the conductor with the deviation produced by the wind.

Between the position of the conductors with their maximum vertical sag, and the position of the conductors with their sag and deviation corresponding to the wind hypothesis a) of section 3.2.3, the safety distances to the ground will be determined by the envelope curve of the distance circles drawn at each intermediate position of the conductors, with a radius interpolated between the distance corresponding to the vertical position and that corresponding to the position of maximum linear deviation of the deviation angle.

In the case of the high voltage line that supports optical fiber cables, as these dielectrics $D_{\text {el }}$ will be considered zero, their minimum distance to the ground and non-navigable waterways will be 6 meters, and can be reduced by 1 meter in difficult-to-access areas. In fenced cattle farms or farms the minimum height to the ground will be 7 meters."

From this fragment of regulations, the necessary considerations can be extracted to verify, after calculating the minimum vertical distance between an overhead line and one of the specified areas, if it meets the minimum requirements. This evaluation is already programmed in the resolution sheet in such a way that it will be deducted directly once the distance calculation is obtained.

Knowing the treatment that will be given to each element in the study, the prepared Excel spreadsheet is used to solve the problem. Below is a sketch showing the two elements to be analyzed and a hypothetical distance between them as an example:


### 6.2.4.1.1. Exercise 1. Distance between power line and pathway

Exercise 1: Calculate the minimum distance between an overhead line located in zone B and knowing the coordinates of its supports with respect to a pathway that crosses the line below, knowing its points that cross the vertical plane that contains the power line. The overhead line has a maximum voltage of $\mathbf{1 2 3} \mathbf{~ k V}$ and does not support optical fiber cables. The pathway is not difficult to access and does not cross any livestock or agricultural exploitation:

The data given for the problem are the coordinates of the different points that in turn represent the supports in the case of the overhead line or the ends of the terrain, pathway... in the case of the second element

| DATA |  | X coordinate | Y coordinate | Z coordinate |
| :---: | :---: | :---: | :---: | :---: |
| Element 1 | Support A | -30 | 40 | 0 |
|  | Support B | 70 | 44 | 0 |
| Element 2 | Point A | 10 | 30 | 0 |
|  | Point B | 30 | 30 | 0 |

## Problem resolution:

The first data that is deduced from the power line are:

| Steps | Equation |  | Result |
| :---: | :---: | :--- | :---: |
| 1. | geometry | Span $=\sqrt{\left(x_{b}-x_{a}\right)^{2}+\left(z_{b}-z_{a}\right)^{2}}$ | Span $=100 \mathrm{~m}$ |
| 2. | geometry | $d=\left\|y_{b}-y_{a}\right\|$ | $d=4 \mathrm{~m}$ |

Using the Excel program designed in the first part of this study which allows to calculate the characteristic parameter of the catenary among other values according to the conditions required by the weight, temperature, wind and ice regulations, it is intended to obtain more information about the catenary. The data obtained are:

| Steps |  | Equation | Result |
| :---: | :---: | :---: | :---: |
| 3. | [equation 13] | $c=\frac{T_{0}}{w_{T}}$ | $c=731.256 \mathrm{~m}$ |
|  | [equation 22] | $d=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-\cosh \left(\frac{b_{x}-\overline{A B}}{c}\right)\right)$ | $b_{x}=79.220 \mathrm{~m}$ |
|  | geometry | $a_{x}=b_{x}-\overline{A B}$ | $a_{x}=-20.780 \mathrm{~m}$ |
| We proceed with the rest of the steps to follow in the original program: |  |  |  |
| 4. | [equation 20] | $f_{B}=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-1\right)$ | $f_{B}=4.295 \mathrm{~m}$ |
|  |  | $f_{A}=c \cdot\left(\cosh \left(\frac{a_{x}}{c}\right)-1\right)$ | $f_{A}=0.295 \mathrm{~m}$ |
| 5. | geometry | $s_{B}=y_{b}-f_{B}$ | $s_{B}=39.705 \mathrm{~m}$ |
|  |  | $s_{A}=y_{a}-f_{A}$ | $s_{A}=39.705 \mathrm{~m}$ |

In relation to the second element studied, which in this problem is a path, the program is capable to work directly with its two already defined limit points and extract the necessary points for the calculation. In this way and after performing the different iterations in the program, the minimum distance is obtained between both elements:

| Steps | Equation | Result |
| :---: | :---: | :---: |
| $\mathbf{6}$. | Iterations | $M D=9.954 \mathrm{~m}$ |

Now it is necessary to check if this distance obtained complies with the requirements of the regulations as far as pathways are concerned. The problem statement stated that the pathway is not difficult to access and that it does not cross any agricultural or livestock farm. A piece of information that is also required to determine the value of $D_{e l}$ is to know that the maximum voltage of the line is 123 kV . With all this, the program, based on the data shown below, performs the following evaluations:

$$
\left.\left.\begin{array}{c}
D_{\text {add }}=5.3 \mathrm{~m} \\
\left.\begin{array}{c}
U_{s}=123 \mathrm{kV} \\
\text { Optical fiber: } \mathrm{No}
\end{array}\right\} D_{e l}=1 \mathrm{~m} \\
\begin{array}{c}
\text { Difficult accessibility }
\end{array} \rightarrow D_{\text {add }}+D_{e l}=6.3 \mathrm{~m} \\
\text { Livestock } / \text { Agricultural } \rightarrow N o
\end{array}\right\} \text { Lim }_{\text {min }}=6 \mathrm{~m}, \begin{array}{c} 
\\
D_{\text {min }}=\operatorname{Max}\left(D_{\text {add }}+D_{e l} ; \operatorname{Lim}_{\text {min }}\right)=6.3 \mathrm{~m} \\
M D=9.954 \mathrm{~m}
\end{array}\right\}
$$

The final verdict is:

$$
M D>D_{\min } \rightarrow \text { CORRECT }
$$

What can finally be observed is that the distance obtained as a result of the study is greater than the minimum required according to the conditions set by the regulations. This statement means that there would be no problem in building the overhead line crossing the pathway taking as supports the points set at the beginning of the problem.

### 6.2.4.2. Distances to other electrical or telecommunication power lines

The second proposed study deals with the case included in section 5.6 of the ITC-LAT 07 where the two elements to be analyzed are:

|  | Element 1 | Element 2 |
| :---: | :---: | :---: |
| Description | Power line | Power line |
| Representation | Catenary equation | Catenary equation |

Now the regulations regarding distances between overhead power lines will be presented. It will allow to evaluate if the minimum distance obtained by the program for the case studied is enough. The regulation begins by considering that there is a crossing between the two power lines:
"The owner of the line to be crossed must send, at the request of the entity that is going to make the crossing, as soon as possible, the basic data of the line for example the type and section of the conductor, voltage, etc.), in order to perform the calculations and avoid errors due to lack of information.

The special prescriptions defined in section 5.3 apply, being modified as follows:
Condition a): In lines with a nominal voltage higher than 30 kV , the existence of a connection per conductor in the crossing span can be admitted.

Condition b): Wooden supports can be used as long as they are fixed to the ground using metal or concrete stringers.

Condition c): Compliance is exempt.
At the crossings of overhead power lines, the highest voltage will be located higher and, in the case of the same voltage; the one that is installed later. In any case, whenever it is necessary to raise the pre-existing line, the owner of the new line will be responsible for modifying the line already installed.

It will be ensured that the crossing is made in the proximity of one of the supports of the highest line, but the distance between the conductors of the lower line and the closest parts of the supports of the upper line should not be less than:

$$
D_{a d d}+D_{e l}=1.5+D_{e l} \text { in meters, }
$$

with a minimum of:

- 2 meters for power lines up to 45 kV .
- 3 meters for power lines of voltage higher than 45 kV and up to 66 kV .
- 4 meters for power lines of voltage higher than 66 kV and up to 132 kV
- 5 meters for power lines of voltage higher than 132 kV and up to 220 kV .
- 7 meters for power lines of voltage higher than 220 kV and up to 400 kV .
and considering the conductors of the same in their position of maximum deviation, under the action of the wind hypothesis a) of section 3.2.3. The values of $D_{\text {el }}$ are indicated in section 5.2 depending on the highest voltage of the lower line.

The minimum vertical distance between the phase conductors of both lines in the most unfavourable conditions, shall not be less than:

$$
\boldsymbol{D}_{a d d}+\boldsymbol{D}_{\boldsymbol{p} \boldsymbol{p}} \text { in meters. }
$$

For the additional insulation distance, $D_{\text {add }}$, the values in table 17 will apply:

Table 17. Additional isolation distances $D_{\text {add }}$ to other overhead power lines or overhead telecommunication lines

| NOMINAL NETWORK <br> VOLTAGE (kV) | For distances from the <br> support of the upper <br> line to the crossing <br> point $\leq 25 \mathrm{~m}$ | For distance from <br> the support of the <br> upper line to the <br> crossing point <br> $>25 \mathrm{~m}$ |
| :---: | :---: | :---: |
|  | 1.8 | 2.5 |
| From 3 to 30 | add |  |
| 45 or 66 | 2.5 |  |
| $110,132,150$ | 3 |  |
| 220 | 3.5 |  |
| 400 | 4 |  |

The values of $D_{p p}$ are indicated in section 5.2, depending on the highest voltage of the line.
To determine $\boldsymbol{D}_{\text {add }}$, in table 17, the nominal voltage of the network corresponding to the line with the lowest voltage will be used. To determine $\boldsymbol{D}_{\text {pp }}$, in table 15, the nominal voltage of the network corresponding to the line with the highest voltage will be used.

Regardless of the crossing point of both lines, the minimum vertical distance between the phase conductors of both lines, or between the phase conductors of the upper electrical line and the guard cables of the lower electrical line, if they exist, it will be verified considering:
a) The phase conductors of the upper power line in the most unfavourable conditions of maximum sag established in the line project.
b) The phase conductors or the guard cables of the lower electrical line without any overload at the minimum temperature according to the zone ( $-5^{\circ} \mathrm{C}$ in zone $A,-15^{\circ} \mathrm{C}$ in zone $B$ and $-20^{\circ} \mathrm{C}$ in zone C).

In general, when the crossing point of both lines is near the centre of the span of the lower line, the possible deviation of the phase conductors due to the action of the wind will be taken into account.

As indicated in section 5.2, the minimum external safety distances $D_{a d d}+D_{e l}$ must always be greater than 1.1 times $a_{\text {som }}$, the discharge distance of the insulator chain, defined as the shortest distance in a straight line, between the live parts and the grounded parts.

When the resultant of the conductor's efforts in any of the crossing supports of the lower line has an ascending vertical component, due precautions will be taken so that the conductors, insulators or supports do not detach.

Line crossovers may be made, without the upper line meeting the special requirements indicated in section 5.3 at the crossing, if the lower line is protected at the crossing by a bundle of steel cables, located between the two, with sufficient mechanical resistance to withstand the fall of the conductors of the upper line; in the event that they break or detach.

The protective steel cables shall be made of galvanized steel and shall be grounded under the conditions prescribed in section 7.

The bundle of protection cables shall have a length on the lower line equal to at least one and a half times the horizontal projection of the separation between the end conductors of the upper line, in the direction of the lower line. Said bundle of protection cables may be placed on the same or different supports of the lower line but, in any case, the supports that support it in its buried part will be metal or concrete.

In this case, the minimum vertical distances between the conductors of the upper and lower line and the bundle of protection cables will be $1.5 \times D_{\text {el }}$, with a minimum of 0.75 meters, for the respective voltages of the lines concerned.

The competent body of the Administration may exceptionally authorize, previous justification, that two lines that cross are fixed on the same support. In this case, in said support and in the conductors of the upper line, the reinforced safety prescriptions determined in section 5.3 will be met.

In these cases in which, due to unique circumstances, it is necessary for the line with the lowest voltage to cross over the one with the highest voltage, it will be necessary to obtain express authorization, bearing in mind all the prescriptions and criteria set forth in section 5.3."

Knowing the mathematical analysis to consider in the case of both elements, the prepared Excel spreadsheet is used to solve the problem. As an example, the following exercise will be solved, the sketch of which can be represented visually as follows:


### 6.2.4.2.1. Exercise 2. Distance between two power lines

Exercise 2.1: Calculate the minimum distance between two overhead lines located in zone B and knowing the coordinates of their supports. The higher voltage overhead line has a nominal voltage of $\mathbf{4 2 0} \mathbf{~ k V}$ while the lower voltage overhead line is $\mathbf{1 5 0} \mathbf{~ k V}$ :

The data given for the problem are the coordinates of the different points that in turn represent the supports of both overhead lines:

| DATA |  | X coordinate | Y coordinate | Z coordinate |
| :---: | :---: | :---: | :---: | :---: |
| Element 1 | Support A | 40 | 70 | -10 |
|  | Support B | 40 | 75 | 100 |
| Element 2 | Point A | -20 | 65 | 30 |
|  | Point B | 100 | 70 | 30 |

## Problem resolution:

The first data that is deduced from power line $\mathbf{1}$ are:

| Steps | Equation |  | Result |
| :---: | :---: | :--- | :---: |
| $\mathbf{1 .}$ | geometry | Span $=\sqrt{\left(x_{b}-x_{a}\right)^{2}+\left(z_{b}-z_{a}\right)^{2}}$ | Span $=110 \mathrm{~m}$ |
| $\mathbf{2 .}$ | geometry | $d=\left\|y_{b}-y_{a}\right\|$ | $d=5 \mathrm{~m}$ |

Making use of the Excel program designed that allows us to calculate the characteristic parameter of the catenary among other values according to the conditions required by the weight, wind and ice regulations, the data obtained is:

| Steps |  | Equation | Result |
| :---: | :---: | :---: | :---: |
| 3. | [equation 13] | $c=\frac{T_{0}}{w_{T}}$ | $c=577.763 \mathrm{~m}$ |
|  | [equation 22] | $d=c \cdot\left(\cosh \left(\frac{b_{\chi}}{c}\right)-\cosh \left(\frac{b_{x}-\overline{A B}}{c}\right)\right)$ | $b_{x}=81.213 \mathrm{~m}$ |
|  | geometry | $a_{x}=b_{x}-\overline{A B}$ | $a_{x}=-28.787 \mathrm{~m}$ |
| We proceed with the rest of the steps to follow in the original program: |  |  |  |
| 4. | [equation 20] | $f_{B}=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-1\right)$ | $f_{B}=5.717 \mathrm{~m}$ |
|  |  | $f_{A}=c \cdot\left(\cosh \left(\frac{a_{x}}{c}\right)-1\right)$ | $f_{A}=0.717 \mathrm{~m}$ |
| 5. | geometry | $s_{B}=y_{b}-f_{B}$ | $s_{B}=69.283 \mathrm{~m}$ |
|  |  | $s_{A}=y_{a}-f_{A}$ | $s_{A}=69.283 \mathrm{~m}$ |

In relation to the second element studied, which in this problem is about another power line, the procedure is repeated:
The initial data that are deduced from power line $\mathbf{2}$ are:

| Steps | Equation |  | Result |
| :---: | :---: | :--- | :---: |
| $\mathbf{1}$. | geometry | Span $=\sqrt{\left(x_{b}-x_{a}\right)^{2}+\left(z_{b}-z_{a}\right)^{2}}$ | Span $=120 \mathrm{~m}$ |
| $\mathbf{2}$. | geometry | $d=\left\|y_{b}-y_{a}\right\|$ | $d=5 \mathrm{~m}$ |

Calculated the characteristic parameter of the catenary according to the conditions required by the weight, wind and ice regulations, the values are obtained:

| Steps | Equation |  |  |
| :---: | :---: | :--- | :---: |
| 3. | [equation 13] | $c=\frac{T_{0}}{w_{T}}$ | $c=729.635 \mathrm{~m}$ |
|  | [equation 22] | $d=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-\cosh \left(\frac{b_{x}-\overline{A B}}{c}\right)\right)$ | $b_{x}=90.358 \mathrm{~m}$ |
|  | geometry | $a_{x}=b_{x}-\overline{A B}$ | $a_{x}=-29.642 \mathrm{~m}$ |


| We proceed with the rest of the steps to follow in the original program: |  |  |  |
| :---: | :---: | :--- | :--- |
| 4. | [equation 20] | $f_{B}=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-1\right)$ | $f_{B}=5.602 \mathrm{~m}$ |
|  |  | $f_{A}=c \cdot\left(\cosh \left(\frac{a_{x}}{c}\right)-1\right)$ | $f_{A}=0.602 \mathrm{~m}$ |
|  |  | $s_{B}=y_{b}-f_{B}$ | $s_{B}=64.398 \mathrm{~m}$ |
|  |  | $s_{A}=y_{a}-f_{A}$ | $s_{A}=64.398 \mathrm{~m}$ |

In this way and after performing the different iterations in the program, the minimum distance between both elements is obtained:

| Steps | Equation | Result |
| :---: | :---: | :---: |
| $\mathbf{6}$. | Iterations | $M D=4.358 \mathrm{~m}$ |

Now it is necessary to analyze if this distance complies with the regulatory requirements regarding crossovers of overhead lines. An important piece of information to determine the value of $D_{e l}$ is that the maximum nominal voltage of the lines is known to be 420 kV . With all this, the program, based on the aforementioned considerations, makes the following calculations:

$$
\left.\left.\left.\begin{array}{c}
D_{\text {add }}=1.5 \mathrm{~m} \\
U_{s}=420 \mathrm{kV} \rightarrow D_{e l}=2.8 \mathrm{~m}
\end{array}\right\} \begin{array}{c}
D_{\text {add }}+D_{e l}=4.3 \mathrm{~m} \\
\operatorname{Lim}_{\min }=7 \mathrm{~m}
\end{array}\right\} \begin{array}{c}
D_{\text {min }}=\operatorname{Max}\left(D_{\text {add }}+D_{e l} ; \operatorname{Lim}_{\text {min }}\right)=7 \mathrm{~m} \\
M D=4.358 \mathrm{~m}
\end{array}\right\}
$$

The final verdict is:

$$
M D<D_{\min } \rightarrow E R R O R
$$

The distance obtained as a result of the study is not enough to guarantee safety since it is less than the minimum required. Therefore, these two power lines cannot be arranged according to the data provided in the problem statement. To find a solution to this problem, the same case will be rethought, but reducing the height of the supports of line number 2 by five meters each to see if in this way the minimum distance between lines would be guaranteed.

Exercise 2.2: Calculate the minimum distance between two overhead lines located in zone B and knowing the coordinates of their supports. The higher voltage overhead line has a nominal voltage of $\mathbf{4 2 0} \mathbf{~ k V}$ while the lower voltage overhead line is $\mathbf{1 5 0} \mathbf{~ k V}$.

The data given for the problem are the coordinates of the different points which in turn represent the supports of both overhead lines. Regarding exercise 2.1, solved previously, the height of the supports on line 2 is modified:

| DATA |  | X coordinate | Y coordinate | Z coordinate |
| :---: | :---: | :---: | :---: | :---: |
| Element 1 | Support A | 40 | 70 | -10 |
|  | Support B | 40 | 75 | 100 |
| Element 2 | Point A | -20 | 60 | 30 |
|  | Point B | 100 | 65 | 30 |

## Problem resolution:

The first data that are known about power line $\mathbf{1}$ are:

| Steps | Equation |  | Result |
| :---: | :---: | :--- | :---: |
| $\mathbf{1 .}$ | geometry | Span $=\sqrt{\left(x_{b}-x_{a}\right)^{2}+\left(z_{b}-z_{a}\right)^{2}}$ | Span $=110 \mathrm{~m}$ |
| $\mathbf{2 .}$ | geometry | $d=\left\|y_{b}-y_{a}\right\|$ | $d=5 \mathrm{~m}$ |

Using the Excel program to calculate the characteristic parameter of the catenary among others values according to the conditions required by the weight, wind and ice regulations, the results obtained are:

| Steps |  | Equation | Result |
| :---: | :---: | :---: | :---: |
| 3. | [equation 13] | $c=\frac{T_{0}}{w_{T}}$ | $c=577.763 \mathrm{~m}$ |
|  | [equation 22] | $d=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-\cosh \left(\frac{b_{x}-\overline{A B}}{c}\right)\right)$ | $b_{x}=81.213 \mathrm{~m}$ |
|  | geometry | $a_{x}=b_{x}-\overline{A B}$ | $a_{x}=-28.787 m$ |
| We proceed with the rest of the steps to follow in the original program: |  |  |  |
| 4. | [equation 20] | $f_{B}=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-1\right)$ | $f_{B}=5.717 \mathrm{~m}$ |
|  |  | $f_{A}=c \cdot\left(\cosh \left(\frac{a_{x}}{c}\right)-1\right)$ | $f_{A}=0.717 \mathrm{~m}$ |
| 5. | geometry | $s_{B}=y_{b}-f_{B}$ | $s_{B}=69.283 \mathrm{~m}$ |
|  |  | $s_{A}=y_{a}-f_{A}$ | $s_{A}=69.283 \mathrm{~m}$ |

In relation to the second element studied, that this problem is about another power line, the process is repeated:

The initial data deduced from power line $\mathbf{2}$ are:

| Steps | Equation |  | Result |
| :---: | :---: | :--- | :---: |
| $\mathbf{1}$. | geometry | Span $=\sqrt{\left(x_{b}-x_{a}\right)^{2}+\left(z_{b}-z_{a}\right)^{2}}$ | Span $=120 \mathrm{~m}$ |
| $\mathbf{2 .}$ | geometry | $d=\left\|y_{b}-y_{a}\right\|$ | $d=5 \mathrm{~m}$ |

Calculating the parameter characteristic of the catenary according to the conditions required by the weight, wind and ice regulations, the data obtained is:

| Steps | Equation |  |  |
| :---: | :---: | :--- | :---: |
| 3. | [equation 13] | $c=\frac{T_{0}}{w_{T}}$ | $c=729.635 \mathrm{~m}$ |
|  | [equation 22] | $d=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-\cosh \left(\frac{b_{x}-\overline{A B}}{c}\right)\right)$ | $b_{x}=90.358 \mathrm{~m}$ |
|  | geometry | $a_{x}=b_{x}-\overline{A B}$ | $a_{x}=-29.642 \mathrm{~m}$ |


| We proceed with the rest of the steps to follow in the original program: |  |  |  |
| :---: | :---: | :--- | :--- |
| 4. | [equation 20] | $f_{B}=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-1\right)$ | $f_{B}=5.602 \mathrm{~m}$ |
|  |  | $f_{A}=c \cdot\left(\cosh \left(\frac{a_{x}}{c}\right)-1\right)$ | $f_{A}=0.602 \mathrm{~m}$ |
|  |  | $s_{B}=y_{b}-f_{B}$ | $s_{B}=59.398 \mathrm{~m}$ |
|  |  | $s_{A}=y_{a}-f_{A}$ | $s_{A}=59.398 \mathrm{~m}$ |

In this way and after performing the different iterations in the program, the minimum distance obtained between both elements is:

| Steps | Equation | Result |
| :---: | :---: | :---: |
| $\mathbf{6}$. | Iterations | $M D=9.352 \mathrm{~m}$ |

Now it is necessary to analyze if this distance complies with the requirements of the regulations regarding crossovers of overhead lines. An important piece of information to determine the value of $D_{e l}$ is that the maximum voltage of the lines is known to be 420 kV . With all this, the program, based on the data shown below, draws the following conclusions:

$$
\left.\left.\left.\begin{array}{c}
D_{\text {add }}=1.5 \mathrm{~m} \\
U_{\text {s.max }}=420 \mathrm{kV} \rightarrow D_{e l}=2.8 \mathrm{~m}
\end{array}\right\} \begin{array}{c}
D_{\text {add }}+D_{e l}=4.3 \mathrm{~m} \\
\operatorname{Lim}_{\min }=7 \mathrm{~m}
\end{array}\right\} \begin{array}{c}
D_{\text {min }}=\operatorname{Max}\left(D_{\text {add }}+D_{e l} ; \operatorname{Lim}_{\min }\right)=7 \mathrm{~m} \\
M D=9.352 \mathrm{~m}
\end{array}\right\}
$$

The final verdict is:

$$
M D>D_{\min } \rightarrow \text { CORRECT }
$$

In this way, a favourable result is achieved that guarantees safety according to requirements marked by regulations. In this new assumption, the overhead lines could be built by placing the supports at the indicated points and in this way the problem initially found in exercise 2.1 would be solved.

Although unlike the crossovers between overhead lines for which two example exercises have been proposed, in the case of telecommunication lines, only the considerations that the regulation collects will be exposed as there is not enough data on the characteristics of the conductors used:
"Telecommunication lines will be considered as low voltage power lines and their crossing will therefore be subject to the prescriptions of this section.

## Crosses with telecommunication lines of dielectric cables.

For telecommunication lines that use dielectric telecommunication cables (for example optical fiber cables) to calculate the minimum vertical distance to the phase conductors of the power line, an electrical distance, $D_{\text {el }}$, corresponding to the highest voltage of the power line, will be taken. high voltage. This distance will be determined under the same assumptions a) and b) established in this section 5.6.1.

The minimum distance between the support of the telecommunication line and the phase conductors will be:

$$
D_{a d d}+D_{e l}=1.5+D_{e l} \text { in meters, }
$$

with a minimum of

- 2 meters for power lines up to 45 kV
- 3 meters for power lines of voltage higher than 45 kV and up to 66 kV
- 4 meters for power lines of voltage higher than 66 kV and up to 132 kV
- 5 meters for power lines of voltage higher than 132 kV and up to 220 kV
- 7 meters for power lines of voltage higher than 220 kV and up to 400 kV
and considering the line conductors in their position of maximum deviation, under the action of the wind hypothesis a) of section 3.2.3. The values of $D_{\text {el }}$ are indicated in section 5.2 depending on the highest voltage of the electric line.

The minimum distance between the support of the high voltage line and the telecommunication conductors will be 2 meters. Regardless of the previous distances, for maintenance work or in general for any work with electrical risk, the established distances will be respected and the corresponding safety measures will be taken by virtue of the applicable legislation on the prevention of occupational risks."

In the case of parallels between lines, the following points should be taken into account:
"It is understood that parallelism exists when two or more neighbouring lines follow substantially the same direction, even if they are not rigorously parallel.

Whenever possible, avoid the construction of parallel electricity transmission or distribution lines, at distances less than 1.5 times the height of the highest support, between the traces of the closest conductors. Access areas to generating plants and transformer stations are excepted from the previous recommendation.

In any case, between the contiguous conductors of the parallel lines, there should not be a separation less than that prescribed in section 5.4.1, considering the values $K, K^{\prime}, L, F$ and $D_{p p}$ of the line with the highest voltage.

The laying of lines of different voltage on common supports will be allowed when they are of the same characteristics in order to the class of current and frequency, except in the case of transport and telecommunication lines or maneuvering of the same company and provided that the latter are affected exclusively at the service of the former. The highest line will preferably be the one with the highest tension, and the supports will be high enough so that the separations between the conductors of both lines and, between them and that one, are those that are generally required and so that the distance to the ground of the lowest conductor, in the most unfavourable conditions, is that established in section 5.5.

Lines on common supports will be considered as having a tension equal to that of the highest, for the purposes of exploitation, conservation and security in relation to people and goods. The insulation of the lower voltage line shall not be less than the corresponding grounding of the higher voltage line.

The parallelism of high voltage power lines with telecommunication lines will be avoided whenever possible, and when this is not possible, a minimum distance equal to 1.5 times the height of the highest support shall be kept between the traces of the closest conductors of one line and the other."

## 7. Annexes

| ANNEX 1: |  |
| :---: | :---: |
| Real assembly of a 20kV overhead electrical network |  |
| Assembly experiences | Real experiences <br> Physical contact with materials <br> Photo report |

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### 7.1. Annex 1. Real assembly of a 20 kV overhead electrical network

### 7.1.0. Introduction

Being able to witness the assembly of a real overhead branch observing the support lifting, the conductors laying, the safety measures adopted in the works, or the placement of the birdlife protection, etc., thanks to the opportunity that the Beloki Electrical Assemblies Company has given me and the attention of its workers transmitting the information of all their actions, have allowed me to "touch the ground" landing in a discipline that I only knew theoretically.

In this section I am going to describe the elements that I have been able to manipulate and have even seen assembled.

### 7.1.1. Starting point

### 7.1.1.1. Location map and data of the installation

The company Iberdrola Distribución Eléctrica, SAU, with NIF: A95075578, and registered office: Plaza Euskadi, no 5, Bilbao, is the owner of the 13.2 kV High Voltage (MV) electrical line called "Alloz-Salinas" .

From the aforementioned line and its support N. 0053, a branch will be made that will feed a 50 kVA outdoor transformation centre, on a C3000 type metal lattice post and 12 meters high. The purpose of this new supply is to feed a farm, which currently had a totally insufficient autonomous electricity supply, which is located on smallholding 480, of polygon 2 , in the area "peña del molino", in the municipality of Salinas de Oro, Navarra.

The Iberdrola Company, in its technical-economic conditions provided to the property, requires, among other requirements, the authorization of access from the public highway to the electrical meter module that will be located in the vicinity of the support that holds the transformer and also is required that the entire installation is prepared for a future voltage change to 20 kV and an insulation level of 24 kV . Figure 1 is attached:

## Google Maps Salinas de Oro



Figure 1. Aerial photo from Google Maps

### 7.1.1.2. Excavations of holes and trenches. Filling

As a first phase for the installation of the metal tower, it is necessary to make a hole where the base of the metal structure and its foundation will be housed. The sidewalk covering the foundation "wire mesh" will be built on it. The pipelines will be carried out underground and will have access to them through a register located on the sidewalk and for which access the key that opens it will be necessary.

In the image Fig. 2 we can see the hole that we can define as parallelepipedic and whose dimensions are: base $0.97 m \times 0.97 m, 2.32 m$ height. Figure 3 shows the formwork that will delimit the sidewalk and shows certain channels through which the grounding cables of the low voltage neutral conductor, the high voltage fittings, the grounding of the autovalves and the supply itself, in low voltage, to the farm.


The lower part of the tower structure is introduced into the hole, by means of the corresponding crane (photo Fig. 4), and the grooving tubes and the neutral grounding conductor will run through the trench as indicated in Figure 5. An extra tube will be provided for possible new pipes.


Figure 4


Figure 5

Photographs 6 and 7 show the foundation and the pipes as well as the cabin for the centralization of electrical meters. The dry concrete slab, 20 cm thick, which acts as a perimeter sidewalk around the entire contour of the overhead transformer substation, measuring 1.6 meters on each side, it will have a mesh connected to the protection ground. For the support foundation have been used $2,41 \mathrm{~m}^{3}$ of concrete with a resistance of $250 \mathrm{daN} / \mathrm{m}^{3}$.



Figure 7

Figure 6

The trench has a depth of 0.8 meters and in it, at the bottom, the PTC 110 tubes and the earth conducting wires are placed, covered with a light layer of concrete HM-20, filled with earth of about 10 cm thick, a plastic tape "Live cables. Danger" and the total fill.

The Complementary Technical Instruction ITC-LAT 07 specifies that foundations whose stability is entrusted to horizontal reactions must also meet the requirement that the angle of rotation of the foundation does not have a tangent greater than 0.01 (this prescription requires a specific calculation, the study of which is reflected in the Electrical Engineering magazine March-April 1964 carried out by the engineer Tadeo Maciejewski).

The support we are analysing, given that it supports unstressed conductors and the type of terrain is totally compact, does not present any risk of inclination and I do not include its analysis and development in our study, but I cite it as an external annex to the report, drawing attention that a "small" legislative phrase hides an important technological background, possible content of another Final Degree Study.

### 7.1.1.3. Lattice metallic tower

Image 8 reflects the complete assembly of the upper part of our support which, by means of the corresponding crane, will be lifted to its anchorage with the lower part that has been cemented.


Figure 8
The support will have a smooth surface up to a height of 2.5 m , using galvanized metal sheets with registers for checking and measuring the ground connection.

### 7.1.2. Overhead transformation centre

### 7.1.2.1. Machine. Transformer

The transformer to be installed (figures 9 and 10) presents the nameplate that is reflected in photograph 11 and in which can be highlighted the possibility to be connected to a 13,200 V three-phase network and in the future to a $20,000 \mathrm{~V}$ network. Our transformer has a Dyn11 connection that tells us that it has its primary in delta, its secondary in star with neutral and that the secondary compound voltages are $30^{\circ}$ behind the primary compound voltages.


Figure 10

Figure 9


Figure 11

|  | Values |
| :---: | :---: |
| Power | 50 kVA |
| Primary voltage | $20,000-13,200 \mathrm{~V}$ |
| Regulation | $20,000+2.5 \%+5 \%+7.5 \%+10 \%$ |
| Secondary voltage | $400-231 \mathrm{~V}$ |
| Insulation level | 24 kV |
| Short circuit voltage | $4 \%$ |
| Refrigerant | Mineral oil |
| Rules | NI-72.30.03 three-phase oil-immersed transformer <br> for low voltage distribution - pole type |

It has regulation taps that will have to be manipulated when the day of its start-up arrives to achieve the desired voltage on the farm (see annex 2). At the top of the transformer there are 2 switches, under 2 orange caps (figures 12 and 13). One, has the mission of enabling the reception bivoltage of 13.2 kV or 20 kV and the other switch allows modifying the number of loops of the high voltage winding, enabling the regulation $+2.5 \%,+5 \%,+7.5 \%,+10 \%$.


The short-circuit voltage is $4 \%$, so we are being told that the nominal current should be multiplied by 25 to anticipate the possible short-circuit current:

$$
\text { If, } \begin{array}{ccc}
U_{s c}=4 \% V_{1} & -\cdots-------- & I_{n} \\
U=100 \% V_{1} & -\cdots-\cdots---- & I_{s c} \\
& & \\
I_{s c}=25 \cdot I_{n} &
\end{array}
$$

Our support, which we remember is $\mathrm{C}-3000 \mathrm{H} 12$, allows us to place the transformer's high voltage terminals 8 meters above ground level, higher than the minimum required, which is 6 meters.

The transformer has circuit separation, how could it be otherwise, oil-cooled and prepared for being placed in the outdoors.

### 7.1.2.2. Accessories

### 7.1.2.2.1. Transformer bracket

Figures 14 and 15 show the support on which the transformer will rest, to which it will be screwed to prevent any movement. Said support has two mainstays that give it the necessary robustness.


Figure 14


Figure 15

### 7.1.2.2.2. Protection of birdlife

For the connections in the transformer sockets, insulating capsules will be used that enclose the metallic parts in tension, avoiding contact between points with different electrical potential. These capsules contribute to the protection of birdlife (Photos 16 and 17).


### 7.1.2.2.3. Autovalves

Attached to the transformer itself, three $15 \mathrm{kV}-10 \mathrm{kA}$ type HMX autovalves-lightning arresters will be installed. When the voltage of a conductor, with respect to earth, presents a potential greater than 15 kV , these are "activated" and allow the discharge to ground of the overcurrent, preventing the overvoltage from affecting the transformer windings.

The autovalve supports consist of metal parts that, screwed to the transformer casing, allow them to be supported and secured (Photos 18 and 19).


The connection of the autovalves to the terminals of the HV transformer will be made by means of a bare copper conductor of $50 \mathrm{~mm}^{2}$ section, type C-50 (NI-54.10.01), having terminals type TA-50C (NI-58.49.02). The conductors will be covered by a flexible preformed cover of the CUP-16-F/30 type.

The connection of the autovalves to ground will be made by means of a $50 \mathrm{~mm}^{2}$ section bare copper conductor, type C-50 (NI-54.10.01).

### 7.1.2.2.4. Danger of death sign post

The support will have a sign indicating "High Voltage. Danger of death" and that they will be located at a visible and legible height from the ground.

### 7.1.2.3. XS fuses

### 7.1.2.3.1. Clamping bracket

Image 20 reflects the crosspiece in which the three fuses will be attached.


Figure 20

### 7.1.2.3.2. XS fuse holder and XS fuse

The illustrations Fig. 21, 22, 23, 24, 25, 26 and 27 show the fastening structure of the fuses with an insulating coating between their junction to the tower crosspiece and the fastening of the fuse capsule.


Figure 21


Figure 23


Figure 24


Figure 22


Figure 25


Figure 26


Figure 27

Inside the fuse capsule is the conductive "material" that, in the event of overcurrents higher than its nominal intensity, would melt and due to the mechanical tension of a spring, which is loaded manually, it will open the circuit, the spring will be released and will cause the fall of the fuse holder, easily visualizing its opening.

### 7.1.2.3.3. XS fuses driver

For the manual connection and/or disconnection of the "XS" fuses, there is an insulating pole (photo 28) that allows us to manoeuvre, remotely, with total safety (image 29).


### 7.1.2.3.4. Protection of birdlife

They also have protective covers to protect the birdlife, making it impossible to access electrically dangerous points (Photos 30 and 31).


### 7.1.2.4. Retention of bare aluminium conductors

- Current designation: 47-AL1/8-ST1A (formerly LA56)


### 7.1.2.4.1. GA-1 clamp

The element that retains the conductor, called the tie clamp, is visualized in figures 32 and 33.


Figure 32


Figure 33

### 7.1.2.4.2. RY clamp

Photograph 34 shows us the RY clamp that, on the one hand, is attached to the GA- 1 clamp, across a through hole, and on the other it has a housing for a ball joint.


Figure 34

### 7.1.2.4.3. Insulator bar

Formerly, standardized insulator strings were used that are being replaced in new installations by the bar photographed in image 35 , which has one ball joint such that it will be crimped with the RY clamp and the other end is a fork that is tied to the crosspiece of the corresponding support. This element is the one that isolates the points of tension of the distribution line and the metallic structure of the tower.


Figure 35

### 7.1.2.4.4. Protection of birdlife

In this section we have a flexible and rollable tube that we can see in figure 36 and whose mission is to avoid the support of the birdlife on the insulator bar.


Figure 36
A plastic folder is added to the fastening system of the GA-1 clamp that protects the birdlife (this assembly is photographed in Fig. 37, 38, 39 and 40).


Figure 37


Figure 39


Figure 38


Figure 40

### 7.1.2.5. Clamping and terminals for bare aluminium conductors

- Current designation: 47-AL1/8-ST1A (formerly LA56)


### 7.1.2.5.1. Introduction (Hydraulic press)

The joints between conductors and different components must be made in an efficient way that does not allow their separation. For this, a hydraulic press is used that has numerous dies for tightening. Attached photographs Fig. 41, 42, 43 and 44.


Figure 41


Figure 42


Figure 43


Figure 44

### 7.1.2.5.2. Terminals and protection of birdlife

In our case we have opted for a hexagonal-shaped die that allows us to apply, to the terminal in the photograph Fig. 45, with its two screws in the blade (Fig. 46), inserting the conductor (Fig. 47), the press (Fig. 48) and obtain the union of the conductor with the terminal in a totally satisfactory way (Fig. 49).


Figure 45


Figure 46


Figure 47


Figure 48


Figure 49

These terminals, once the leftovers have been removed, together with the conductor, will be covered with the corresponding protection cover for the protection of birdlife.

Depending on the covers used, it may be necessary to introduce these protections before or after making the joints permanently. For this reason, there are different designs of covers prepared for both cases. In this work we have tubular covers, preformed covers and roll-up tube covers.

### 7.1.3. Branch assembly on HV

### 7.1.3.1. Hoisting of the upper part of the metal lattice tower

On the ground the "upper part", "central body" and "fittings" are assembled.
Thanks to the arrangement of a crane truck and a truck with a "basket", the different elements are lifted, positioned and anchored: I present the photographs Fig. 50, 51, 52, 53, $54,55,56,57$ and 58 . Now it is appreciated the importance of the correct placement of the tower base and the care that is needed for the union of the base with the rest of the tower. The use of adjustable straps and the expertise of the professionals allow me to see how the apparent failure seen in photos 54 and 55 turns into the success reflected in figure 57 and that allows the "aspiring professional" to screw the pieces. (see image 58).


Figure 50


Figure 51


Figure 52


Figure 53


Figure 54


Figure 56


Figure 55


Figure 57


Figure 58

### 7.1.3.2. Hoisting of the transformer

The images Fig. 59, 60 and 61 show the lifting of the transformer.


### 7.1.3.3. Wiring and installation of components

The illustrations Fig. 62 and 63 show that the protections have been placed, the conductors have been fastened, the lower part of our tower has been closed, etc. and we are waiting for the connection to the company's transport line.


Figure 62


Figure 63

### 7.1.4. High voltage handling. Power line branch of electric Company

I have been lucky enough to be able to observe this section in-situ. The work has been carried out by the workers of EIFFAGE (subsidiary "EDS") who with a special crane, with an insulating arm, and two baskets are allowed to work on the HV lines as if they were birds. Once the impossibility of electric leakage to earth through the insulating arm of the crane ( 66 kV ) has been ensured, they are protected from the voltage existing between two active wires through insulating covers of approximately 3 meters of length. First, they isolate the active wire closest to the crane access and then the rest. The company's existing line had no protection for birdlife and as a new supply has been requested, the company takes advantage of this and charges the applicants for the regulatory update of the support. We observe how they are replacing the current insulators with modern bars, how they are protecting the different elements and with exceptional skill they place the crosspiece and the corresponding disconnectors so that our line can be connected (the disconnectors remain in open position until our line passes the mandatory tests) (the photographs shown are Fig. 64, 65, 66, 67, 68, 69 y 70).


Figure 64


Figure 66


Figure 65


Figure 67


Figure 68


Figure 69


Figure 70

To mechanically hold, tighten and relax mechanically the conductors for the renewal of the insulators, they use some "tractel" that are reflected in the photographs F71 and F72.


Figure 71


Figure 72

To carry out the union of the new derivation to the conductors of the existing line and, of course, without interruption of the electrical supply, they use a tripping clamping system that connects the conductors, making their mechanical disconnection impossible and guaranteeing their electrical connection. Starting from some cartridges with gunpowder, a "gun" is used that manages to introduce the central piece between the conductors with the pressure necessary to fix them. The junction points are subsequently covered with suitable protective folders, guaranteeing their isolation (figures 73, 74 and 75 are attached, which, thanks to the attention of the EIFFAGE workers, I have been able to obtain).


Figure 73


Figure 74


Figure 75

Once the work is finished, all the insulation elements placed, provisionally, at the beginning of the day are removed. With all this, the installation remains in a position to proceed with its connection to the new branch, but it remains at the expense of the approval of the operating and safety conditions by the authorized external company (photograph F76 indicates the prohibition of handling the disconnector and in the Fig. 77 I present an overview of the installation "almost" finished).


Figure 76

The workers are already leaving, as seen in figure 78, but this snapshot of the truck allows us to analyze that in addition to the first section of insulating tube (long white tube), there is a second, shorter insulating tube, which serves as a second protection and that safeguards the possibility that some metallic part of the arm could be in contact with the tower (I was able to observe the different types of conductors that this installation company uses, in their day to day, such as LA56, LA78 or the LA110 (Fig. 79)).


Figure 78


Figure 79

### 7.1.5. Protection of birdlife in overhead conductors

Apart from the polymeric covers that serve as security for the birdlife, there is another concern and that is the risk that the birds will not see the lines, for this reason, black bands are placed giving the catenaries visibility (photographs are attached Fig 80, 81 and 82). It is a red plastic cube that allows the conductors to be hooked to remain fixed and not be able to detach in the event of possible gusts of wind or other atmospheric phenomena (the holes in the upper part have the function of facilitating their placement with the help of two screwdrivers that by inserting them and crossing between them, it is possible to force the red parallelepiped to open and to be able to insert the conductor into its housing).


Figure 80


Figure 82

Figure 81

The covers used for the protection of birdlife by the company EIFFAGE can be seen in the photographs Fig. 83 and 84.


Figure 83


Figure 84

### 7.1.6. Verifications and inspections

### 7.1.6.1. Technical report of the pre-commissioning verifications

The Complementary Technical Instruction ITC-LAT 03 requires that the company that has carried out the assembly is classified, at least, with the LAT1 category: for high voltage overhead or underground lines up to 30 kV .

As an authorized installation company, they must, among others issues:

- Execute assembly
- Check that the executed line passes the regulatory tests and trials
- Issue the installation certificates
- Attend the inspections carried out by the control body
- Keep up to date a record of the installations carried out

Below, I reflect the technical report of the installation company removing certain private data:

| TECHNICAL REPORT ON THE VERIFICATIONS PRIOR TO COMMISSIONING |  |  |
| :---: | :---: | :---: |
| Relating to: | Expte Code: SAT-10913 |  |
|  | Installation location: | Polygon 2, plot 480 ground floor <br> Salinas de Oro - 31175 <br> Cups: ES0021000036914588CE |
|  | Owner of the installation: | Mr. Jo\&\& Gir\&\&\&\& Bar\&\&\&\&\& DNI number 72 \&\&\&\&\&\& X <br> C / del No\&\&\&, 2 <br> 31175 - Salinas de Oro (Navarra) |
|  | Project written by: Technical direction carried out by: | Xabier Apostua Iraizoz <br> DNI number 72 \&\&\&\&\&\& H <br> Xabier Apostua Iraizoz <br> DNI number 72 \&\&\&\&\&\& H |
|  | Inspection carried out by: | SGS Inspecciones Regulamentarias SA C / Aizoain, 10- Office 32-33, 2nd Floor 31013 Ansoain (Navarra) |
|  | Installing Company: | Montajes Eléctricos Beloki, SLCIF B31615677 <br> Polígono Mendikur, no 19 31170 Orkoien (Navarra) <br> RII No. 15-B-D12-00051259 |

```
Technical characteristics of the installation: 13.2-20 kV HV line
50 kVA overhead transformation center (Transformer
protocol: no 19292-4)
Low voltage supply 3x400 / 230 V
Air Conductor 47-AL 1/8-ST1A (LA56)
Span length: 15m
Distance between bare conductors:
At all times greater than 0.25m
Distance between bare conductor and ground:
At all times greater than 0.22m
Protection exhaustive birdlife
Fitting ground resistance: 2.71\Omega
Neutral earth resistance: 2.79\Omega
Values certified by SGS Reglementary Inspections SA:
Step voltage: 0V
Permissible: 1070 V
Contact voltage: 0 V
Permissible: 107 V
```


## ENABLED INSTALLER CERTIFICATION

| Name: | Alberto | Surnames: | Beloki Arrondo | DNI | 18 \&\&\&\&\&\& Z |
| :--- | :--- | :--- | :--- | :--- | :--- |

The installation has been executed in accordance with the project, with the prescriptions of the regulation on technical conditions and safety guarantees in high voltage electrical installations and its complementary technical instructions, approved by Royal Decree 337/2014, of May 9 and under the Technical Direction of Xabier Apostua Iraizoz

### 7.1.6.2. Documentation and commissioning of HV power lines

The Complementary Technical Instruction ITC-LAT 04 indicates that the installation must be executed according to the project that must be drawn up and signed by a competent qualified technician, who will be directly responsible for its adaptation to the regulatory provisions and, where appropriate, to the particular specifications approved to the transport and distribution company to which it connects.

### 7.1.6.3. Measurements made by "ACO"

The Complementary Technical Instruction ITC-LAT 05 requires that all lines must be subject to a verification prior to commissioning and a periodic inspection, at least every three years. For lines with nominal voltage less than or equal to 30 kV , which is our case, the periodic inspection can be replaced by a periodic verification.

The property, the engineer drafting the project and director of the work and the installation company have incorporated a control body to make the appropriate measurements and I have been able to observe, and even participate, in the tests that the technician of the company SGS Regulatory Inspections SA has carried out and that I summarize below:

- Companion line tower grounding measurement
- Measurement of the grounding of the transformer substation
- Measurement of the grounding of the neutral of the low voltage line
- Measurements of step and touch voltages in the support of the company
- Measurements of step and touch voltages in the support of the transformer substation

To measure the grounding, the tellurometer in image 85 is used, which is accompanied by two auxiliary stakes.


Figure 85
For the measurement of step voltages, two electrodes are used, each weighing 25 kp , which simulate the action of a person's leg and the distance at which they are placed is one meter. To carry out the measurements, a current modulator is used, with Bluetooth, which together with the application on the mobile phone collects the data to be obtained. Ultimately, its function consists of injecting transient currents that allow the step voltages to be measured. Formerly, when these machines did not exist, DC generators were used that presented a certain danger by supplying the intensity permanently. Currently, even coming into contact with the electrodes in the measurement itself, there is no risk whatsoever.


Figure 87

Figure 86


Figure 88


Figure 90


Figure 89


Figure 91

Several measurements were made, among which I highlight several assumptions:

- A person with both feet off the paved sidewalk and both radially aligned to the tower. Photograph 92.


Figure 92

- One person with one foot out and one foot on the paved surface. Photos 93 and 94.


Figure 93


Figure 94

- One person with both feet inside the foundation area. Photograph 95.


Figure 95

- The person with both feet together and touching the sheet metal with the hand. Photos 96 and 97.


Figure 97

Figure 96

The results obtained were null, so it is foreseeable that in the event of atmospheric discharges or high amperage faults (values much higher than 50A) the step and contact voltages will be negligible.

As a clarification I indicate that the neutral grounding has a sectioning box that duly protected will allow its opening, by professional personnel, and will allow the appropriate measures in future revisions (Fig. 98). Regarding the ground fittings, there is no sectioning box, but there is a detachable screw and nut (Fig. 99).


Figure 98


Figure 99

### 7.1.7. Energy measurement

The standardized centralization of meters, housed in a concrete solid, is called CPM2-D / ER4-M, according to NI42.72.00 and will house the meter that will measure the energy, both active and reactive, and the power demanded in the six hourly periods. According to company regulations, the equipment and, of course, the main power line are protected by high breaking power fuses ( 120 kA ) of 80 A , in BUC type bases. The line that will feed the farm has been built with RZ $0.6 / 1 \mathrm{kV}$ conductors of $3\left(1 \times 50 \mathrm{~mm}^{2} \mathrm{Cu}\right) \mathrm{F}+1\left(1 \times 50 \mathrm{~mm}^{2} \mathrm{Cu}\right) \mathrm{N}$ (Photos 100 and 101).


Figure 100


Figure 101

When carrying out the measurement in low voltage and being the supply in high voltage, we must know that a monthly over-measurement of $6 \mathrm{kWh} / \mathrm{kVA}$ of transformer and a surcharge of $4 \%$ of the energy term will be billed, in order to compensate for the losses of the transformer.

### 7.1.8. LA56 conductor properties. Essays

I have been able to verify that a LA56 conductor meter really weighs $185 \mathrm{gp} / \mathrm{m}$ and that is what the different tables and manufacturers indicate in their catalogs (illustrations 102 and 103).


Figure 102


Figure 103

I was able to unravel one end to see the six aluminium wires and the steel core (Fig. 104). Finally, I have "subjected" the LA56 conductor to the tensile test and I have no doubt that it supports 1670 daN (Fig. 105).


Figure 104


Figure 105

## ANNEX 2:

Commissioning of the regulation taps

### 7.2. Anexo 2. Commissioning of the regulation taps

The installation company Montajes Eléctricos Beloki, S.L. before proceeding to the delivery of the installation to the property, verifies that the regulation taps of the chosen transformer supply the appropriate voltages to the farm.

The recorder that was placed was active from 26/03/2021 (13h 47') to 28/03/2021 (23h 21 ') and made a total of 3455 measurements. For each minute, average, minimum and maximum values of each variable are shown according to the measuring device but, in reality, the three values refer to the effective value of the wave. Technically we would say that this is the arithmetic mean of the effective values detected, the minimum value of the effective values detected and the maximum value of the effective values detected, respectively. The true maximum value of a sine wave is $\sqrt{2}$ times the effective value of the wave and the true mean value would be 0 .

Among the documents that complement this report, in the Excel sheet "Annex 2. Record of low voltages (Salinas)" are shown the measurements of:

- The three simple voltages (voltages between active and neutral wire): $V_{1}, V_{2}$ and $V_{3}$
- The three compound voltages (tensions between two active wires): $V_{12}, V_{23}$ and $V_{31}$
- Frequency

Reviewing the measurements, all are correct so it is not necessary to modify the regulation taps of the transformer.

NOTE: The regulation taps are always handled without tension.

| ANNEX 3: |  |
| :---: | :---: |
| Additional cases and problems on calculating crossover distances |  |
| Crossovers | Normative |
|  | Resolution of exercises |

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### 7.3. Annex 3. Additional cases and problems on calculating crossover distances

### 7.3.1. Distances to roads

As a third study, we will analyze the case presented in section 5.7 of the ITC-LAT 07:

|  | Element 1 | Element 2 |
| :---: | :---: | :---: |
| Description | Power line | Road |
| Representation | Catenary equation | Line equation |

Now it is necessary to know the specifications of the regulations regarding roads which, as in the previous cases, will allow us to know if the minimum distance obtained after the calculations is enough to be able to build said overhead line on the road involved. In this matter, the regulations sustain:
"For the installation of the supports, both in the case of crossover and in the case of parallelism, the following considerations will be taken into account:
a) For the State Highway Network, the installation of supports will preferably be carried out behind the building limit line and at a distance from the outer edge of the road that is higher than one and a half times its height. The building limit line is the one located 50 meters away on motorways, expressways and expressways, and 25 meters away on the rest of the roads of the State Highway Network from the outer edge of the road.
b) For the roads that do not belong to the State Highway Network, the installation of the supports must comply with the current regulations of each autonomous community applicable for this purpose.
c) Regardless of whether or not the road belongs to the State Road Network, for the placement of supports within the area affected by the road, the appropriate authorization will be requested from the competent bodies of the Administration. For the State Highway Network, the affected area comprises a distance of 100 meters from the outer edge of the grading in the case of highways, expressways and expressways, and 50 meters for the rest of the roads of the State Highway Network. State.
d) In exceptional topographic circumstances, and prior technical justification and approval of the competent body of the Administration, the placement of supports may be allowed at distances less than those fixed."

In reference to crossovers between power lines and highways:
"The special prescriptions defined in section 5.3 are applicable and are modified as follows:
Condition a): Regarding the crossover with local and neighbourhood highways, the existence of one connection per conductor in the crossover span is admitted for lines with a nominal voltage greater than 30 kV .

The minimum distance of conductors on the grade of the road will be:

$$
D_{\text {add }}+D_{e l} \text { in meters, }
$$

with a minimum distance of 7 meters. The values of $D_{e l}$ are indicated in section 5.2 depending on the highest voltage of the line.

Where: $D_{\text {add }}=7.5$ for special category lines.

$$
D_{\text {add }}=6.3 \text { for lines of the rest of the categories. }
$$

In the case of high voltage lines that support optical fiber cables, as these are dielectric, $D_{e l}$ will be considered zero and the minimum distance between these optical fiber cables and the grade of the road will be 7 m ."

These considerations will be analyzed in the final conclusions of the proposed problem.

Knowing the mathematical modeling of each element in the study, the prepared Excel spreadsheet is used to solve the problem. As an example, the following exercise will be solved, the sketch of which can be represented easily and in a similar way to the first case studied:


### 7.3.1.1. Exercise 3. Distance between power line and road

Exercise 3: Calculate the minimum distance between an overhead line located in zone A and knowing the coordinates of its supports with respect to a road that crosses below the line, knowing its points that cross the vertical plane that contains the overhead line. It has a maximum voltage of 145 kV and supports optical fiber cables:

The data given for the problem are the coordinates of the different points that in turn represent the supports in the case of the overhead line and the ends of the road in the case of the road (second element):

| DATA |  | X coordinate | Y coordinate | Z coordinate |
| :---: | :---: | :---: | :---: | :---: |
| Element $\mathbf{1}$ | Support A | 0 | 80 | 40 |
|  | Support B | 80 | 77 | 0 |
| Element $\mathbf{2}$ | Point A | 10 | 65 | 35 |
|  | Point B | 30 | 65 | 25 |

## Problem resolution:

The first data that is deduced from the power line are:

| Steps | Equation |  | Result |
| :---: | :---: | :--- | :---: |
| $\mathbf{1}$. | geometry | Span $=\sqrt{\left(x_{b}-x_{a}\right)^{2}+\left(z_{b}-z_{a}\right)^{2}}$ | Span $=89.44 \mathrm{~m}$ |
| $\mathbf{2 .}$ | geometry | $d=\left\|y_{b}-y_{a}\right\|$ | $d=3 \mathrm{~m}$ |

Calculated the characteristic parameters of the catenary among other values according to the conditions required by the weight, wind and ice regulations, the following results are obtained:

| Steps | Equation |  | Result |
| :---: | :---: | :--- | :---: |
| 3. | [equation 13] | $c=\frac{T_{0}}{w_{T}}$ | [equation 22] |
|  | $d=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-\cosh \left(\frac{b_{x}-\overline{A B}}{c}\right)\right)$ | $b_{x}=75.140 \mathrm{~m}$ |  |
|  | geometry | $a_{x}=b_{x}-\overline{A B}$ | $a_{x}=-14.303 \mathrm{~m}$ |
| We proceed with the rest of the steps to follow in the original program: |  |  |  |
|  | [equation 20] | $f_{B}=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-1\right)$ | $f_{B}=3.113 \mathrm{~m}$ |
|  |  | $f_{A}=c \cdot\left(\cosh \left(\frac{a_{x}}{c}\right)-1\right)$ | $f_{A}=0.113 \mathrm{~m}$ |
| 5. |  | $s_{B}=y_{b}-f_{B}$ | $s_{B}=76.887 \mathrm{~m}$ |
|  |  | $s_{A}=y_{a}-f_{A}$ | $s_{A}=76.887 \mathrm{~m}$ |

In relation to the second element studied, which in this problem is a road, the program is able to work directly with its two already defined limit points. In this way and after performing the different iterations in the program, the minimum distance between both elements is obtained:

| Steps | Equation | Result |
| :---: | :---: | :---: |
| 6. | Iterations | $M D=12.828 \mathrm{~m}$ |

It is necessary to verify that the distance obtained complies with the requirements of the regulations regarding roads. The problem statement reported that the maximum line voltage is 145 kV (will serve to determine the value of $D_{e l}$ ) and supports optical fiber cables so the final conclusions are:

$$
\left.\left.\left.\begin{array}{c}
\text { No sp.cat. } \rightarrow D_{\text {add }}=6.3 \mathrm{~m} \\
\left.\begin{array}{c}
U_{s}=145 \mathrm{kV} \\
\text { Optical fiber: } Y e s
\end{array}\right\} D_{e l}=0 \mathrm{~m}
\end{array}\right\} \begin{array}{c} 
\\
D_{\text {add }}+D_{e l}=6.3 \mathrm{~m} \\
\text { Lim }_{\text {min }}=7 \mathrm{~m}
\end{array}\right\} \begin{array}{c}
D_{\text {min }}=\operatorname{Max}\left(D_{\text {add }}+D_{\text {el }} ; \operatorname{Lim}_{\text {min }}\right)=7 \mathrm{~m} \\
M D=12.828 \mathrm{~m}
\end{array}\right\}
$$

The final verdict is:

$$
M D>D_{\min } \rightarrow \text { CORRECT }
$$

What can be verified is that the distance obtained as a result of the study is greater than the minimum required according to the studied conditions of the regulations. This allows us to affirm that there would be no impediment in building the overhead line on the road taking as supports the points set in the statement of the problem.

### 7.3.2. Distances to non-electrified railways

As a fourth study, section 5.8 is dealt with in the ITC-LAT 07 with the elements:

|  | Element 1 | Element 2 |
| :---: | :---: | :---: |
| Description | Power line | Non-electrified railway |
| Representation | Catenary equation | Line equation |

The study of the regulations will lead to the conclusion of whether the construction of a line with the chosen characteristics is possible crossing a railway underneath without electrification. The rule is as follows:
"For the installation of the supports, both in the case of parallelism and in the case of crosses, the following considerations will be taken into account:
a) On both sides of the railway lines that are part of the railway network of general interest, the building limit line is established from which to the railway line any type of building, reconstruction or expansion work is prohibited.
b) The building limit line is the one located 50 meters from the outer edge of the grading measured horizontally and perpendicularly to the outer rail of the railway. The installation of supports within the area affected by the building limit line will not be authorized.
c) For the placement of supports in the protection zone of the railway lines, the appropriate authorization will be requested from the competent bodies of the Administration. The limit line of the protection zone is the one located 70 meters from the outer edge of the grading, measured horizontally and perpendicularly to the outer rail of the railway.
d) In the crossovers, the supports may not be installed at a distance from the outer edge of the grading that is less than one and a half times the height of the support.
e) In exceptional topographical circumstances, and prior technical justification and approval of the competent body of the Administration, the placement of supports at distances less than those established may be allowed."

In the case of crossovers with power lines:
"The special prescriptions defined in section 5.3 are applicable.
The minimum distance of the conductors of the power line on the heads of the rails will be the same as for crossovers with highways."

With the intention of preparing a program that solves the cases of crossover between nonelectrified railways and overhead lines, it should be noted that the same spreadsheet can be used as in the case of highways. The proposed problem is presented below.

### 7.3.2.1. Exercise 4. Distance between power line and railway without electrification

Exercise 4: Calculate the minimum distance between an overhead line located in zone B and knowing the coordinates of its supports with respect to a non-electrified railway that crosses the line below, knowing the points on it that cross the vertical plane that contains the overhead line. It has a maximum voltage of 245 kV . In this case the line does not support optical fiber cables.

The data given for the problem are the coordinates that represent the supports in the case of the overhead line and the ends of the railway in the case of the second element considered:

| DATA |  | X coordinate | Y coordinate | Z coordinate |
| :---: | :---: | :---: | :---: | :---: |
| Element 1 | Support A | 0 | 80 | 90 |
|  | Support B | 120 | 77 | 0 |
| Element 2 | Point A | 40 | 70 | 60 |
|  | Point B | 60 | 70 | 45 |

## Problem resolution:

The first data that can be deduced from the power line are:

| Steps | Equation |  | Result |
| :---: | :---: | :--- | :---: |
| $\mathbf{1 .}$ | geometry | Span $=\sqrt{\left(x_{b}-x_{a}\right)^{2}+\left(z_{b}-z_{a}\right)^{2}}$ | Span $=150 \mathrm{~m}$ |
| $\mathbf{2 .}$ | geometry | $d=\left\|y_{b}-y_{a}\right\|$ | $d=3 \mathrm{~m}$ |

Making use of the first Excel program proposed in the study, which allows us to calculate the characteristic parameter of the catenary among other values according to the conditions required by the weight, wind and ice regulations, the following results are obtained:

| Steps |  | Equation | Result |
| :---: | :---: | :---: | :---: |
| 3. | [equation 13] | $c=\frac{T_{0}}{w_{T}}$ | $c=576.141 \mathrm{~m}$ |
|  | [equation 22] | $d=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-\cosh \left(\frac{b_{x}-\overline{A B}}{c}\right)\right)$ | $b_{x}=86.490 \mathrm{~m}$ |
|  | geometry | $a_{x}=b_{x}-\overline{A B}$ | $a_{x}=-63.510 \mathrm{~m}$ |
| We proceed with the rest of the steps to follow in the original program: |  |  |  |
| 4. | [equation 20] | $f_{B}=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-1\right)$ | $f_{B}=6.504 \mathrm{~m}$ |
|  |  | $f_{A}=c \cdot\left(\cosh \left(\frac{a_{x}}{c}\right)-1\right)$ | $f_{A}=3.504 \mathrm{~m}$ |
| 5. | geometry | $s_{B}=y_{b}-f_{B}$ | $s_{B}=73.496 \mathrm{~m}$ |
|  |  | $s_{A}=y_{a}-f_{A}$ | $s_{A}=73.496 \mathrm{~m}$ |

In relation to the calculations of the road representation, the program is able to work directly with its two limit points already defined in the statement without the need for any additional calculation. Carrying out the different iterations in the program, the minimum distance between both elements is obtained:

| Steps | Equation | Result |
| :---: | :---: | :---: |
| 6. | Iterations | $M D=3.610 \mathrm{~m}$ |

It is time to check if the distance found complies with the requirements of the regulations regarding non-electrified railways.

The problem statement reported that the maximum line voltage is 245 kV , which will serve to determine the value of $D_{e l}$. Together with the data shown below, the program executes the following calculations:
$\left.\left.\left.\begin{array}{c}\left.\begin{array}{c}\text { Sp. Cat. } \rightarrow D_{\text {add }}=7.5 \mathrm{~m} \\ U_{s}=245 \mathrm{kV} \\ \text { Optical fiber: } \mathrm{No}_{0}\end{array}\right\} D_{e l}=1.7 \mathrm{~m}\end{array}\right\} \begin{array}{c} \\ D_{\text {add }}+D_{e l}=9.2 \mathrm{~m} \\ L i m_{\min }=7 \mathrm{~m}\end{array}\right\} D_{\min }=\operatorname{Max}\left(D_{a d d}+D_{e l} ; \operatorname{Lim}_{\text {min }}\right)=9.2 \mathrm{~m}\right\}$

The final verdict is:

$$
M D<D_{\min } \rightarrow E R R O R
$$

Unfortunately the distance obtained as a result does not have enough separation to meet the requirements set by the regulations. In this case it is evident that the construction of an overhead line, whose supports have the coordinates given in the problem statement, cannot be built because it does not guarantee the minimum safety distance between the overhead line and the un-electrified railway.

The solutions that should be sought in these cases is to provide the support towers with a greater height that allows increasing the distances of the conductors with respect to the nonelectrified railway or to find another location where to place the overhead line supports, complying in any case with the minimum distance between the catenary and the railway crossed.

### 7.3.3. Distances to electrified railways, trams and trolleybuses

The fifth analysis to be evaluated is based on section 5.9 of ITC-LAT 07 where:

|  | Element 1 | Element 2 |
| :---: | :---: | :---: |
| Description | Power line | Electrified railway, tram <br> and trolleybus |
| Representation | Catenary equation | Line equation |

As in the previous cases, the regulations will allow us to check if the minimum distance calculated by the methods developed, according to the proposed data, is enough to be able to build said overhead line above the electrified railway, tram or trolleybus that is going to be crossed. The fragment of regulations referring to these cases is shown below:
"For the installation of the supports, both in the case of parallelism and in the case of crossover, the indicated in section 5.8 for non-electrified railways will be followed.

At the crossover between power lines and electrified railways, trams and trolleybuses, the minimum vertical distance of the power line conductors, with their maximum vertical sag, according to the hypotheses in section 3.2.3, on the tallest conductor of all the electric, telephone and telegraphic power lines of the railway will be:

$$
D_{a d d}+D_{e l}=3.5+D_{e l} \text { in meters, }
$$

with a minimum of 4 meters. The values of $D_{\text {el }}$ are indicated in section 5.2 depending on the highest voltage of the line.

In addition, in the case of railways, trams and trolleybuses equipped with trolleys, or other electrical outlet elements that may accidentally separate from the contact line, the conductors of the electrical line must be located at such a height that, when disconnected taking into account the most unfavourable position that it may adopt, the power outlet organ is not located at a shorter distance from those defined above.

In the case of high voltage lines that support optical fiber cables, as these are dielectric, $D_{e l}$ will be zero and the minimum distance between these optical fiber cables and the highest conductor of all railway, telephone and electric power lines will be considered. telegraph will be 4 m. "

Once the conditions determined in the regulation are known, it is time to apply them to a practical case that presents a clearer conclusion in this regard. The problem statement is presented below.

### 7.3.3.1. Exercise 5. Distance between power line and tram

Exercise 5: Calculate the minimum distance between an overhead line located in zone $\mathbf{A}$ and knowing the coordinates of its supports with respect to a tram that crosses below the line, knowing its points that cross the vertical plane that contains the overhead line. It has a maximum voltage of $\mathbf{7 2 . 5} \mathbf{~ k V}$ and does not support optical fiber cables:

The coordinates of the supports in the case of the overhead line or the ends of the tram conductors in the case of the second element, are once again taken as data for the problem:

| DATA |  | X coordinate | Y coordinate | Z coordinate |
| :---: | :---: | :---: | :---: | :---: |
| Element $\mathbf{1}$ | Support A | 90 | 80 | 0 |
|  | Support B | 0 | 75 | 120 |
| Element 2 | Point A | 51 | 65 | 52 |
|  | Point B | 45 | 65 | 60 |

## Problem resolution:

The initial data that can be determined from the power line are:

| Steps | Equation |  | Result |
| :---: | :---: | :--- | :---: |
| 1. | geometry | Span $=\sqrt{\left(x_{b}-x_{a}\right)^{2}+\left(z_{b}-z_{a}\right)^{2}}$ | Span $=150 \mathrm{~m}$ |
| 2. | geometry | $d=\left\|y_{b}-y_{a}\right\|$ | $d=5 \mathrm{~m}$ |

Using the tools available to calculate the characteristic parameter of the catenary, among other values, according to the conditions required by the weight, wind and ice regulations, we have the data:

| Steps | Equation |  | Result |
| :---: | :---: | :---: | :---: |
| 3. | [equation 13] | $c=\frac{T_{0}}{w_{T}}$ | [equation 22] |
|  | $d=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-\cosh \left(\frac{b_{x}-\overline{A B}}{c}\right)\right)$ | $b_{x}=105.100 \mathrm{~m}$ |  |
|  | geometry | $a_{x}=b_{x}-\overline{A B}$ | $a_{x}=-44.900 \mathrm{~m}$ |
| We proceed with the rest of the steps to follow in the original program: |  |  |  |
|  | [equation 20] | $f_{B}=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-1\right)$ | $f_{B}=6.115 \mathrm{~m}$ |
|  |  | $f_{A}=c \cdot\left(\cosh \left(\frac{a_{x}}{c}\right)-1\right)$ | $f_{A}=1.115 \mathrm{~m}$ |
| 5. |  | $s_{B}=y_{b}-f_{B}$ | $s_{B}=73.885 \mathrm{~m}$ |
|  |  | $s_{A}=y_{a}-f_{A}$ | $s_{A}=73.885 \mathrm{~m}$ |

In relation to second element studied in this problem, which represents the location of its conductors as critical points in the event of possible contact, the program is capable of working directly with its two already defined limit points. In this way, after performing the different iterations in the spreadsheet, the minimum distance between both elements is obtained:

| Steps | Equation | Result |
| :---: | :---: | :---: |
| $\mathbf{6}$. | Iterations | $M D=9.381 \mathrm{~m}$ |

It is necessary to verify that the distance complies with the requirements of the regulations regarding electrified railways, trams or trolleybuses. The problem statement stated that the maximum line voltage is 72.5 kV which will determine the value of $D_{e l}$ and it is also known that it does not support optical fiber cables.

With all this, the program, together with the data shown below, obtains the results:

$$
\left.\left.\left.\begin{array}{c}
D_{\text {add }}=3.5 \mathrm{~m} \\
\left.\begin{array}{c}
U_{s}=72.5 \mathrm{kV} \\
\text { Optical fiber: } \mathrm{No}
\end{array}\right\} D_{e l}=0.7 \mathrm{~m}
\end{array}\right\} \begin{array}{c}
D_{\text {add }}+D_{e l}=4.2 \mathrm{~m} \\
\text { Lim }_{\min }=4 \mathrm{~m}
\end{array}\right\} \begin{array}{c}
D_{\min }=\operatorname{Max}\left(D_{\text {add }}+D_{e l} ; \operatorname{Lim}_{\min }\right)=4.2 \mathrm{~m} \\
M D=9.381 \mathrm{~m}
\end{array}\right\}
$$

The final verdict is:

$$
M D>D_{\min } \rightarrow \text { CORRECT }
$$

The value obtained as a result of the study is greater than the minimum required according to the studied conditions of the regulations. This allows us to conclude and affirm that there would be no problem in building the overhead line over the tram taking as supports the points set at the beginning of the problem.

### 7.3.4. Distances to cable railways and transport conductors

A sixth study could deal with the case of cable cars and transport cables for which the regulations present the following considerations in section 5.10 of the ITC-LAT 07:
"The special prescriptions defined in section 5.3 are applicable.
The crossover of an electrical line with cable cars or transport cables must always be carried out superiorly, except in reasonably well justified cases that are expressly authorized.

The minimum vertical distance between the conductors of the power line, with its maximum vertical sag according to the hypotheses in section 3.2.3, and the highest part of the cable car, taking into account the oscillations of the cables during normal operation and the possible elevation gain that may be achieved due to load reduction in the event of an accident will be:

$$
D_{a d d}+D_{e l}=4.5+D_{e l} \text { in meters, }
$$

with a minimum of 5 meters. The values of $D_{\text {el }}$ are indicated in section 5.2 depending on the highest voltage of the line.

The horizontal distance between the closest part of the cable car and the supports of the power line in the crossing span will be at least that obtained from the formula indicated above.

The cable car must be grounded at two points, one on each side of the crossover, in accordance with the provisions of section 7.

In the case of high voltage lines that support optical fiber cables, as these are dielectric, $D_{e l}$ will be zero and the minimum distance between these optical fiber cables and the highest part of the cable car will be considered, taking into account the oscillations of the cables of the same during normal operation and the possible elevation gain that may be achieved due to load reduction in the event of an accident will be 5 m ."

Being a case very similar to the second study analyzed of two crossing power lines, the practical study will not be covered due to lack of data related to materials and conductors used in cable cars and transport cables. Taking into account that the methodology and mathematics is the same in both scenarios, in the absence of knowledge to determine the exact characteristics and equations of the catenaries that these systems present, the example shown for two crossing power lines shall be considered equivalent to this one.

### 7.3.5. Distances to rivers and canals, navigable or floating

The seventh case proposed by the regulations is introduced in section 5.11 of the ITC-LAT 07 where the elements to consider are:

|  | Element 1 | Element 2 |
| :---: | :---: | :---: |
| Description | Power line | River and canals, navigable <br> or floating |
| Representation | Catenary equation | Line equation |

In order to know if the minimum distance deduced in the calculations is sufficient to be able to build said overhead line over the river or channel to be crossed, the regulations shown below must be known:
"For the installation of the supports, both in the case of parallelism and in the case of crossovers, the following considerations will be taken into account:
a) The installation of supports will be carried out at a distance of 25 meters and at least one and a half times the height of the supports, from the edge of the river channel corresponding to the flow of the maximum avenue. However, the placement of supports at shorter distances may be allowed if there is prior authorization from the competent administration.
b) In exceptional topographic circumstances, and prior technical justification and approval of the Administration, the placement of supports may be allowed at distances less than those established."

When analyzing crossovers with power lines:
"The special prescriptions defined in section 5.3 are applicable.
In crossovers with rivers and canals, navigable or floating, the minimum vertical distance of the conductors, with their maximum vertical sag according to the hypotheses in section 3.2.3, above the water surface for the maximum level that it can reach will be:

- Special category lines:

$$
G+D_{a d d}+D_{e l}=G+3.5+D_{e l} \text { in meters, }
$$

- Rest of lines:

$$
G+D_{a d d}+D_{e l}=G+2.3+D_{e l} \text { in meters, }
$$

where $G$ is the gauge. The values of $D_{e l}$ are indicated in section 5.2 depending on the highest voltage of the line.

In the event that there is no defined gauge, it will be considered equal to 4.7 meters.
In the case of high voltage lines that support optical fiber cables, as these are dielectric, $D_{e l}$ will be considered zero and the minimum distance of these optical fiber cables on the surface of the water for the maximum level that it can reach will be 7 m for a considered minimum gauge of 4.7 m , having to be increased by the difference between the actual gauge and 4.7 m ."

Now the conditions determined in the regulation will be applied in a practical case whose statement is introduced below.

### 7.3.5.1. Exercise 6. Distance between power line and navigable river

Exercise 6: Calculate the minimum distance between an overhead line located in zone B and knowing the coordinates of its supports with respect to a navigable river that crosses the line below, knowing its points that cross the vertical plane that contains the overhead line. It has a maximum voltage of $\mathbf{1 4 5} \mathbf{~ k V}$ and supports optical fiber cables. A maximum gauge of 5.5 meters will be assumed:

The known data to find the solution to the problem are the coordinates of the different points that represent the supports in the case of the overhead line or the ends of the river in the case of the second element considered in this exercise:

| DATA |  | X coordinate | Y coordinate | Z coordinate |
| :---: | :---: | :---: | :---: | :---: |
| Element 1 | Support A | 45 | 60 | -30 |
|  | Support B | 45 | 60 | 90 |
| Element 2 | Point A | 45 | 50 | 10 |
|  | Point B | 45 | 50 | 25 |

## Problem resolution:

The first parameters that are deduced from the power line are:

| Steps | Equation |  | Result |
| :---: | :---: | :--- | :---: |
| $\mathbf{1 .}$ | geometry | Span $=\sqrt{\left(x_{b}-x_{a}\right)^{2}+\left(z_{b}-z_{a}\right)^{2}}$ | Span $=120 \mathrm{~m}$ |
| $\mathbf{2 .}$ | geometry | $d=\left\|y_{b}-y_{a}\right\|$ | $d=0 \mathrm{~m}$ |

Using the Excel program, also used in the previous exercises, is wanted to obtain the characteristic parameter of the catenary among other values according to the conditions required by the weight, wind and ice:

| Steps |  | Equation | Result |
| :---: | :---: | :---: | :---: |
| 3. | [equation 13] | $c=\frac{T_{0}}{w_{T}}$ | $c=653.429 \mathrm{~m}$ |
|  | [equation 22] | $d=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-\cosh \left(\frac{b_{x}-\overline{A \bar{B}}}{c}\right)\right)$ | $b_{x}=60.000 \mathrm{~m}$ |
|  | geometry | $a_{x}=b_{x}-\overline{A B}$ | $a_{x}=-60.000 \mathrm{~m}$ |
| We proceed with the rest of the steps to follow in the original program: |  |  |  |
| 4. | [equation 20] | $f_{B}=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-1\right)$ | $f_{B}=2.757 \mathrm{~m}$ |
|  |  | $f_{A}=c \cdot\left(\cosh \left(\frac{a_{x}}{c}\right)-1\right)$ | $f_{A}=2.757 \mathrm{~m}$ |
| 5. | geometry | $s_{B}=y_{b}-f_{B}$ | $s_{B}=57.243 \mathrm{~m}$ |
|  |  | $s_{A}=y_{a}-f_{A}$ | $s_{A}=57.243 \mathrm{~m}$ |

In relation to the second element evaluated, which in this problem is a navigable river, the program is prepared to model it directly, knowing its two already defined limit points. In this way, after performing the different iterations in the program to obtain the minimum distance between both elements, the result is:

| Steps | Equation | Result |
| :---: | :---: | :---: |
| $\mathbf{6}$. | Iterations | $M D=7.266 \mathrm{~m}$ |

The distancing requirements set by the regulations must be met with regard to rivers or canals. The problem statement considers a maximum line voltage 145 kV (will serve to determine the value of $D_{e l}$ ) and warns that it supports optical fiber cables. It is recalled that the maximum gauge considered will be 5.5 meters, with which the final evaluation is:


The final verdict is:

$$
M D<D_{\min } \rightarrow E R R O R
$$

Unfortunately the distance obtained as a result is not enough to meet the requirements set by the regulations. In this case it is evident that the construction of an overhead line whose supports have the coordinates given in the statement, cannot be built because it cannot guarantee the minimum safe distance between the power line and the river.

As in other cases, the solutions to be found could be to provide the support towers with a higher height that allows to increase the distances of the conductors with respect to the river or to find another location to place the supports of the overhead line. In this case, the maximum allowed gauge value is very relevant since, if this is lower, the installation could be authorized by complying with the established safety requirements.

### 7.3.6. Crossover through zones

In more specific and peculiar cases, the regulations mention as follows:
"In general, for overhead power lines with bare conductors, the flight easement zone is defined as the strip of land defined by the projection on the ground of the extreme conductors, considering these and their insulator chains in the most unfavourable conditions, without considering any additional distance.

The most unfavourable conditions are to consider the conductors and their insulator chains in their position of maximum deviation, that is, subjected to the action of their own weight and a wind overload, according to section 3.1.2, for a wind speed of $120 \mathrm{~km} / \mathrm{h}$ at a temperature of $+15{ }^{\circ} \mathrm{C}$.

The application of the reference parameters in the wind hypothesis is independent of the category of the line, being for all lines, $120 \mathrm{~km} / \mathrm{h}$ of wind speed and $15^{\circ} \mathrm{C}$ of temperature."

### 7.3.6.1. Distances to forests, trees and wooded areas

Among the examples called "crossover by zones", the eighth case to be analyzed is the one collected in section 5.12.1 of the ITC-LAT 07 where there is:

|  | Element 1 | Element 2 |
| :---: | :---: | :---: |
| Description | Power line | Forests, trees and <br> woodlands |
| Representation | Catenary equation | Line equation |

In this case the regulations say:
"To avoid service interruptions and possible fires caused by the contact of branches or tree trunks with the conductors of an overhead power line, a line protection zone defined by the flight easement zone must be established through the corresponding compensation, increased by the following safety distance on both sides of said projection:

$$
D_{a d d}+D_{e l}=1.5+D_{e l} \text { in meters, }
$$

with a minimum of 2 meters. The values of $D_{e l}$ are indicated in section 5.2 depending on the highest voltage of the line.

The line protection zone will be calculated for all phase conductors of the line.
The person responsible for the operation of the line will be obliged to guarantee that the safety distance between the conductors of the line and the mass of trees within the right-of-way zone satisfies the prescriptions of this regulation, the owner of the land being obliged to allow such activities to be carried out. Likewise, it will notify the competent body of the administration the masses of trees excluded from the right-of-way zone, which could compromise the safety distances established in this regulation. It must be also ensured that the street where the line runs is kept free of any residue from cleaning, in order to prevent the generation or spread of forest fires.

In this section, the right-of-way zone refers exclusively to the line protection zone defined by the flight easement zone, increased by the safety distance defined above.

- In the event that conductors fly over trees; the safety distance will be calculated considering the conductors with their maximum vertical sag according to the hypotheses of section 3.2.3.
- For the calculation of the safety distances between the trees and the end conductors of the line, these and their insulator strings will be considered in their most unfavourable conditions described in this section.

Likewise, all those trees that constitute a danger to the conservation of the line must be cut, understanding as such those that, due to incline or accidental or caused fall, the conductors can reach in their normal position, in the hypothesis of temperature b) of section 3.2.3. This circumstance will be a function of the type and state of the tree, inclination and state of the terrain, and the location of the tree with respect to the line."

With the conditions taken from the regulation, the resolution of a practical example can be proposed that presents a clearer conclusion to be drawn in this regard.

### 7.3.6.1.1. Exercise 7. Distance between power line and wooded area

Exercise 7: Calculate the minimum distance between an overhead line located in zone $\mathbf{A}$ and knowing the coordinates of its supports with respect to a wooded area that crosses the line below, knowing its points that cross the vertical plane that contains the overhead line. It has a maximum voltage of $\mathbf{1 7 0} \mathbf{~ k V}$ :

As in the cases previously analyzed, the data given for the problem are the coordinates of the different points that represent the supports in the case of the overhead line or the ends of the wooded area in the case of the second element:

| DATA |  | X coordinate | Y coordinate | Z coordinate |
| :---: | :---: | :---: | :---: | :---: |
| Element 1 | Support A | 0 | 55 | 0 |
|  | Support B | 80 | 60 | 100 |
| Element 2 | Point A | 40 | 45 | 50 |
|  | Point B | 45 | 0 | 56.25 |

## Problem resolution:

The first data that can be deduced from the power line are:

| Steps | Equation |  | Result |
| :---: | :---: | :--- | :---: |
| $\mathbf{1 .}$ | geometry | Span $=\sqrt{\left(x_{b}-x_{a}\right)^{2}+\left(z_{b}-z_{a}\right)^{2}}$ | Span $=128.062 \mathrm{~m}$ |
| 2. | geometry | $d=\left\|y_{b}-y_{a}\right\|$ | $d=5 \mathrm{~m}$ |

Using the Excel program that allows us to calculate the characteristic parameter of the catenary among other values according to the conditions required by the weight, wind and ice regulations, the results obtained are:

| Steps | Equation |  | Result |
| :---: | :---: | :---: | :---: |
| 3. | [equation 13] | $c=\frac{T_{0}}{w_{T}}$ | [equation 22] |
|  | $d=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-\cosh \left(\frac{b_{x}-\overline{A B}}{c}\right)\right)$ | $b_{x}=99.317 \mathrm{~m}$ |  |
|  | geometry | $a_{x}=b_{x}-\overline{A B}$ | $a_{x}=-28.745 \mathrm{~m}$ |
| We proceed with the rest of the steps to follow in the original program: |  |  |  |
| 4. | [equation 20] | $f_{B}=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-1\right)$ | $f_{B}=5.457 \mathrm{~m}$ |
|  |  | $f_{A}=0.457 \mathrm{~m}$ |  |


| 5. | geometry | $s_{B}=y_{b}-f_{B}$ | $s_{B}=54.543 \mathrm{~m}$ |
| :---: | :---: | :---: | :---: |
|  |  | $s_{A}=54.543 \mathrm{~m}$ |  |

Regarding the second element studied, which in this problem is a wooded area, the program will be in charge of working directly from the two limit points already defined. In this way and after performing the different iterations that constitute the resolution method, the minimum distance between both elements is obtained:

| Steps | Equation | Result |
| :---: | :---: | :---: |
| 6. | Iterations | $M D=10.224 \mathrm{~m}$ |

Now it must be analyzed if this distance complies with the requirements of the regulations in terms of wooded areas. The problem statement reported that the maximum line voltage is 170 kV which will determine the value of $D_{e l}$. With all this, the program, based on the data shown below, makes the following calculations and conclusions:

$$
\left.\left.\left.\begin{array}{c}
D_{\text {add }}=1.5 \mathrm{~m} \\
U_{s}=170 \mathrm{kV} \rightarrow D_{e l}=1.3 \mathrm{~m}
\end{array}\right\} \begin{array}{c}
D_{\text {add }}+D_{e l}=2.8 \mathrm{~m} \\
\operatorname{Lim}_{\text {min }}=2 \mathrm{~m}
\end{array}\right\} \begin{array}{c}
D_{\min }=\operatorname{Max}\left(D_{\text {add }}+D_{e l} ; \operatorname{Lim}_{\min }\right)=2.8 \mathrm{~m} \\
M D=10.224 \mathrm{~m}
\end{array}\right\}
$$

The final verdict is:

$$
M D>D_{\min } \rightarrow \text { CORRECT }
$$

The distance obtained as a result of the study is greater than the minimum required according to the terms studied of the regulations. That is why there would be no problem in building the overhead line over the wooded area taking as supports the points set at the beginning of the problem. The installation approach is approved by guaranteeing the spacing to avoid possible accidents due to electric shocks.

### 7.3.6.2. Distances to buildings, constructions and urban areas

Another example recognized as "crossovers by zones" is the ninth case dealt with in section 5.12.2 of the technical instruction:

|  | Element 1 | Element 2 |
| :---: | :---: | :---: |
| Description | Power line | Buildings, constructions <br> and urban areas |
| Representation | Catenary equation | Line equation |

In these cases the regulation states:
"The laying of high voltage overhead power lines with bare conductors on land that is classified as urban land will be avoided, when it belongs to the territory of municipalities that have a management plan or as a population centre in municipalities that do not have such a plan. However, at the request of the owner of the facility and when technical or economic circumstances make it advisable, the competent body of the Administration may authorize the overhead laying of said lines in the aforementioned areas.

Overhead laying may be authorized of high voltage power lines with bare conductors in urban reserve areas with a legally approved general plan and in industrial areas and estates with an approved partial plan, as well as on urban land not included within the helmet of the population in municipalities that lack a management plan.

In accordance with the provisions of Royal Decree 1955/2000, of December 1, buildings and industrial facilities will not be built in the flight easement, increased by the following minimum safety distance on both sides:

$$
D_{a d d}+D_{e l}=3.3+D_{e l} \text { in meters, }
$$

with a minimum of 5 meters. The values of $D_{e l}$ are indicated in section 5.2 depending on the highest voltage of the line.

Similarly, lines will not be built above buildings and industrial facilities in the strip defined above.
However, in cases of mutual agreement between the parts, the minimum distances that must exist in the most unfavourable conditions, between the conductors of the power line and the buildings or constructions that are under it, will be the following:

- On locations accessible to people: $5.5+D_{\text {el }}$ meters, with a minimum of 6 meters.
- On locations not accessible to people: $3.3+D_{e l}$ meters, with a minimum of 4 meters.

In the most unfavourable conditions, an attempt will also be made to maintain the aforementioned distances, in horizontal projection, between the line conductors and the immediate buildings and constructions.

In the case of high voltage lines that support optical fiber cables, as these are dielectric, $D_{\text {el }}$ will be considered zero and the minimum distance between these optical fiber cables and the buildings or constructions that are under them will be 6 m over accessible locations to people and 4 m above for inaccessible locations."

Once all the considerations to be evaluated for crossovers on urban land are known, it is time to see them applied in an example in order to analyze if the regulation is complied with. The problem statement is presented now.

### 7.3.6.2.1. Exercise 8. Distance between power line and urban area

Exercise 8: Calculate the minimum distance between an overhead line located in zone B and knowing the coordinates of its supports with respect to an urban area that crosses the line below, knowing its points that cross the vertical plane that contains the overhead line. It has a maximum voltage of $\mathbf{2 4 5} \mathbf{~ k V}$ and supports optical fiber cables. There are no accessible points from the urban area to the crossing area:

The information provided to solve the problem are the coordinates of the different points that in turn represent the supports in the case of the overhead line and the limits of the urban area:

| DATA |  | X coordinate | Y coordinate | Z coordinate |
| :---: | :---: | :---: | :---: | :---: |
| Element 1 | Support A | 10 | 45 | 25 |
|  | Support B | 140 | 40 | 25 |
| Element 2 | Point A | 70 | 35 | 25 |
|  | Point B | 50 | 30 | 25 |

## Problem resolution:

The first power line data to be calculated are:

| Steps | Equation |  | Result |
| :---: | :---: | :--- | :---: |
| $\mathbf{1 .}$ | geometry | Span $=\sqrt{\left(x_{b}-x_{a}\right)^{2}+\left(z_{b}-z_{a}\right)^{2}}$ | Span $=130 \mathrm{~m}$ |
| $\mathbf{2 .}$ | geometry | $d=\left\|y_{b}-y_{a}\right\|$ | $d=5 \mathrm{~m}$ |

Using the Excel program designed to calculate the characteristic parameter of the catenary among other values according to the conditions required by the weight, wind and ice regulations, the data obtained are:

| Steps |  | Equation | Result |
| :---: | :---: | :---: | :---: |
| 3. | [equation 13] | $c=\frac{T_{0}}{w_{T}}$ | $c=576.682 \mathrm{~m}$ |
|  | [equation 22] | $d=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-\cosh \left(\frac{b_{x}-\overline{A B}}{c}\right)\right)$ | $b_{x}=87.128 \mathrm{~m}$ |
|  | geometry | $a_{x}=b_{x}-\overline{A B}$ | $a_{x}=-42.872 m$ |
| We proceed with the rest of the steps to follow in the original program: |  |  |  |
| 4. | [equation 20] | $f_{B}=c \cdot\left(\cosh \left(\frac{b_{x}}{c}\right)-1\right)$ | $f_{B}=6.594 \mathrm{~m}$ |
|  |  | $f_{A}=c \cdot\left(\cosh \left(\frac{a_{x}}{c}\right)-1\right)$ | $f_{A}=1.594 \mathrm{~m}$ |
| 5. | geometry | $s_{B}=y_{b}-f_{B}$ | $s_{B}=38.406 \mathrm{~m}$ |
|  |  | $s_{A}=y_{a}-f_{A}$ | $s_{A}=38.406 \mathrm{~m}$ |

In relation to the urban area to be evaluated, the program is able to work directly with its characterization from the points that limit its location. In this way and after performing the different iterations in the program, the minimum distance between both elements is obtained:

| Steps | Equation | Result |
| :---: | :---: | :---: |
| 6. | Iterations | $M D=4.040 \mathrm{~m}$ |

We want to know if the calculated distance meets the requirements of the regulations in terms of urban areas or buildings. The problem statement reported that the maximum line voltage is 245 kV being useful to determine the value of $D_{e l}$. It should also be noted that the power line supports optical fiber cables and that there are no access points to the crossing from the urban area analyzed:

$$
\left.\begin{array}{c}
\text { Diff.acc.: Yes } \rightarrow D_{\text {add }}=3.3 \mathrm{~m} \\
\left.\begin{array}{c}
U_{s}=170 \mathrm{kV} \\
\text { Optical fiber: Yes }
\end{array}\right\} D_{e l}=0 \mathrm{~m}
\end{array}\right\} D_{\text {add }}+D_{e l}=3.3 \mathrm{~m}, ~\left(D_{\min }=\operatorname{Max}\left(D_{\text {add }}+D_{e l} ; \operatorname{Lim}_{\min }\right)=4 \mathrm{~m}\right\}
$$

The final verdict is:

$$
M D>D_{\min } \rightarrow \text { CORRECT }
$$

The installation would obtain the authorization for its execution as far as the safety distances are concerned since the result obtained is greater than the minimum distance required according to the studied conditions of the regulations. Taking as support the points set at the beginning of the problem, there will be no problem in building the overhead line that crosses the urban area under consideration.

### 7.3.6.3. Other circumstances

To close this part of the analysis, a series of remarks are included in the regulations and also set out in cases of proximity to airports, wind farms or constructions as more peculiar cases than those collected by the ITC-LAT 07:

## "5.12.3 Proximity to airports

They are not applicable the special prescriptions defined in section 5.3.
HV overhead power lines with bare conductors that have to be built in the vicinity of airports, aerodromes, heliports and air navigation aid facilities, must comply with what is specified in the applicable legislation and provisions on the matter.

### 5.12.4 Proximity to wind farms

They are not applicable the special prescriptions defined in section 5.3.
For safety reasons of overhead power lines with bare conductors, the installation of new wind turbines is not allowed in the strip of land defined by the flight easement zone increased by the total height of the wind turbine, including the blade plus 10 m .

### 5.12.5. Proximities to constructions.

When works are carried out near overhead lines and in order to guarantee the protection of workers against electrical risks according to the applicable regulations for the prevention of occupational risks, and in particular Royal Decree 614/2001, of June 8, on provisions for the protection of the health and safety of workers against electrical risk, the promoter of the work will ensure that the signalling is carried out by means of the overhead line beaconing. The beaconing will use standardized elements and may be temporary."

## 8. Future ideas

I recognize that I have left many issues that directly or indirectly affect this study and that I present below:

- The catenary in a three-dimensional space (influence of the wind)
- Mistakes made when replacing the catenary with a parabola
- Foundation analysis
- Imbalance of tractions between spans
- Variability of the modulus of elasticity or Young's modulus
- Distance between catenaries under the influence of the wind
- Study of the resistivity of the terrain and analysis of the groundings
- Analysis of electrodynamic stresses against short circuits
- .../...

I intend, in future projects, to undertake the study of some of the electrical phenomena of overhead lines, such as:

- Capacitive effects between conductors and between conductors and ground
- Inductive reactances of the lines
- Voltage drops
- Characteristic impedance of a line


## 9. Conclusions

With all the considerations studied in this report, the application and use of hyperbolic mathematics in the determination of the characteristic equations of a catenary is justified. It can be observed how the evolution in technical means such as calculators, computers... facilitates the resolution of problems without the need to resort to the approximations that were being made in texts already written and that today could be avoided depending on the academic level of the student.

It has been possible to develop a report supported by the regulation of high voltage overhead lines, providing various calculation tools for the design of different transport lines. Different cases and situations have been evaluated, within the regulatory framework considered, together with the necessary requirements that guarantee safety in its operation.

After the mathematical study, with a more theoretical dimension, a practical case of the construction of a support tower for a branch of electricity supply from a 20 kV overhead transport line has been presented. The installation has also allowed to deepen the understanding and application of current regulations and to know the series of elements and necessary measurements that provide maximum safety to the infrastructure, reducing any risk of accident.

A challenge that this study has taken into account is to consider the possibility of programming and designing, for example, a specific academic course of catenaries, which could be taught. I have incorporated an index, which I have called "external annexes", which is not part of the study but its knowledge is essential for its understanding and which constitutes, in my opinion, the most adequate methodology.

The normative analysis and the desire to theoretically justify the prescriptions it indicates, has led me to critical considerations (v. gr. page 108) and to consider with concern chapter 8 of the present report (future ideas).

I believe that this study has given me the possibility of complementing my studies in a subject that opens, without any doubt, different possibilities for my future career.

After rereading section 8 of this report, I end by recalling the Greek philosopher SOCRATES ( $470 \div 399 \mathrm{BC}$ ), to whom is attributed the concept: "I only know that I know nothing". And, for my part, I conclude: what a nice experience it is to learn!

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[^0]:    The operational similarities between plane trigonometry and the
    exponential equations studied are evidenced, thus justifying the name of hyperbolic trigonometry and demonstrating that its knowledge and utility is easily assumed.

[^1]:    [equation 19]

