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Facultad de Ciencias Económicas y Empresariales

TRABAJO FIN DE GRADO EN ECONOMÍA

BUSINESS CYCLE AND MONETARY ANALYSIS IN A NEW KEYNESIAN MODEL

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Module: Economic analysis

Abstract

Macroeconomic models should be designed to adequately explain the effects of different shocks affecting the economy, such as the sovereign debt crisis, the Covid-19 pandemic, or the war in Ukraine which led to an increase in inflation. Moreover, it is interesting to know how these models can be used by consumers, producers, and public institutions to analyze and deal with the effects of these shocks. To this end, throughout this paper we described a model based on the optimizing behavior of individuals, firms, the Central Bank, and the government, considering a general-equilibrium model, but adjusted to resemble reality, through monopolistic competition and price stickiness. Besides, the different model parameters are calibrated in line with existing papers. Two types of monetary policy rules are introduced to compare the Central Bank's instruments to stabilize output, and inflation fluctuations around its steady state values. I find that Taylor's rule outperforms the money-growth rule on stabilizing inflation and output because it renders a lower volatility in the deviations from their long-term values.

Key words: Ney Keynesian model, Business Cycle Analysis, Impulse-response functions, Taylor rule, Money-growth rule**.**

Resumen

Los modelos macroeconómicos deben ser diseñados para explicar adecuadamente los efectos de diferentes shocks que afectan a la economía, como la crisis de la deuda soberana, la pandemia del Covid-19 o la guerra de Ucrania que provocó un aumento de la inflación. Además, es interesante saber cómo pueden utilizar estos modelos los consumidores, los productores y las instituciones públicas para analizar y solventar los efectos de estos impactos. Para ello, a lo largo de este trabajo describimos un modelo basado en el comportamiento optimizador de los individuos, las empresas, el banco central y el gobierno, considerando un modelo de equilibrio general, pero ajustado para asemejarse más a la realidad, a través de competencia monopolística y rigidez de precios. También se han calibrado los diferentes parámetros del modelo en línea con trabajos realizados. Se introducen dos tipos de reglas de política monetaria con el fin de comparar los diferentes instrumentos del Banco Central para estabilizar las fluctuaciones de producción e inflación entorno a sus valores de estado estacionario. Encuentro que la regla de Taylor supera a la regla de crecimiento del dinero en la estabilización de la inflación y la producción, porque produce una menor volatilidad en las desviaciones de sus valores a largo plazo.

Palabras clave: Nuevo modelo Keynesiano, Análisis de Ciclo Económico, Funciones de impulso-respuesta, Regla de Taylor, Regla del crecimiento monetario.

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Table of Abbreviations

Abbreviation	**Meaning**
AR(1)	Autoregressive Model of order 1
BCA	Business Cycle Analysis
CB	Central Bank
CRRA	Constant Relative Risk Aversion
DSGE	Dynamic Stochastic General Equilibrium
ECB	European Central Bank
GDP	Gross Domestic Product
IRF	Impulse-response functions
NK	Ney Keynesian
RBC	Real Business Cycle

1. INTRODUCTION

Macroeconomics is a broad field of study in which different socio-economic phenomena can be analyzed and modelled. It covers, for example, the behavior of individuals and firms' social welfare, the impact of monetary and fiscal policies and unexpected shocks such as changes in trade, technology, and environmental conditions.

There exist models and assumptions which are being considered in the development of the model being introduced in this paper. The purpose is to elaborate a model that resembles reality to analyze the impact and effect of different macroeconomic shocks. The Real Business Cycle (RBC) model Kydland & Prescott (1982), Cooley & Hansen (1989), households and firms behave rationally and operate in markets under perfect competition and flexible prices. Small changes can be incorporated to transform this general equilibrium model into a more realistic one. Firstly, by introducing monopolistic competition as in Dixit & Stiglitz (1977), so firms are able to make profits during each economic period. Secondly, substituting flexible by sticky prices a la Calvo (1983), so certain firms will adjust the optimal price while others will remain operating under the same prices as in previous period. By doing so, the endogenous variables of the model will react not only to the exogenous technology shocks but also to monetary and fiscal policies carried out by the public sector, having an impact on real variables such as consumption, output, or employment.

The New Keynesian (NK) model is developed as a consequence of having both, monopolistic competition, and sticky prices in an RBC model. Rotemberg & Woodford (1997) first described the general characteristics of this NK model based on the seminar papers presented: monopolistic competition by Dixit and Stiglitz, and Calvo's model of price rigidity, that describes the optimal prices equation. Moreover, there are also some previous models that explain price adjustments which incorporate elements such as the "shopping time technology" King & Wolman (1996), to introduce the medium-of-exchange role of money. These two seminar papers introduce a model that explains reality in a more complete way.

The remainder of this paper is organized as follows. Section 2 includes the model description. Section 3 provides a baseline calibration for the parameters considered in the model. Section 4 includes a Business Cycle Analysis (BCA) with impulse-response exercises, as well as second moment statistics calculations. Section 5 concludes the paper.

2. MODEL DESCRIPTION

The purpose of this chapter is to describe the different elements of the NK model that is going to be used for the later analysis of impulse-response shocks, and the computation of second moment statistics.

The elements of the model could be divided into two parts: the private and public sectors. On the one hand, the private sector is made up of households and firms. Their behavior will be analyzed within a monopolistic competitive market and the equations and terms describing their behavior will be developed and explained in detail. On the other hand, the public sector consists of the Central Bank (CB), and the government. Their behavior will also be described and analyzed with the corresponding functions.

2.1. Households

The final aim of the representative household is to maximize its welfare. For that reason, to measure the level of the individual's satisfaction it is necessary to develop a utility function that will represent her preferences.

In this case, the utility function that describes the representative household behavior features is the Constant Relative Risk Aversion (CRRA). Assuming this form of utility function, there exists the possibility of introducing money, as compared to the canonical Keynesian model, which does not incorporate this term, making a difference. The fact of not introducing this aspect, as in the canonical model, means that the quantity of money circulating in the economy would have no effect on the decision making of households. Everything would happen independently from the evolution of money: individuals do not demand money, and the monetary policy of the CB is ignored. Nevertheless, if the quantity of money is included in the model, is possible to use it as a monetary policy instrument. For that reason, the utility function in this model incorporates money, so households decide how much money want to hold in each period.

2.1.1. Utility function and budget constraint development

The utility function of the representative household depends on three different variables: consumption, the quantity of money in real terms, and hours of labor, as represented in the following way:

$$(1)\ U_t(c_t, m_t, n_t) = \frac{Ct^{1-\sigma}}{1-\sigma} + \psi_m \frac{{m_t}^{1-\gamma}}{1-\gamma} - \psi_m \frac{{n_t}^{1+\kappa}}{1+\kappa}$$

This specification of the utility function satisfies the property of a constant intertemporal elasticity of substitution. To give an interpretation, an explanation procedure needs to be done for each of the three elements, starting with consumption and applying the same methodology for the other two terms (equations needed for the development of the function can be seen in Appendix A). The approach to analyze the utility, is to focus on the constant elasticity of the marginal utility of consumption with respect to consumption (function 2, Appendix A).

Firstly, consider two properties that are usually assumed for the household preferences:

- The marginal utility of consumption is always positive. This means that when there is an increase in the quantity consumed, the individual is happier and gains utility.
- At the same time, the marginal utility of consumption is decreasing on consumption. In other words, the more the household consumes, the smaller the increase in utility associated with that unit of consumption is.

There is an additional condition that is satisfied under the CRRA form. The elasticity of the marginal utility of consumption is constant, and in a mathematical way is represented as follows (after simplifying and using functions from Appendix A):

$$(2)\ \mathrm{E} = \frac{\partial \mathrm{UCt}/\mathrm{U}Ct}{\partial Ct/Ct} = \frac{\partial \mathrm{UCt}}{\partial Ct}\frac{Ct}{\mathrm{U}Ct} = -\sigma C_t^{-\sigma-1}\frac{C_t}{C_t^{-\sigma}} = -\sigma$$

where the σ parameter is a constant elasticity.

The second element of the utility function is money. The role of money in the economy is being the medium of exchange, facilitating consumers when making their transactions, therefore, not having any intrinsic value. People demand money to make their purchases, so there is not the problem of a barter economy. For that reason, money should be incorporated in a transaction cost function, decreasing it, as having money facilitates households' life when conducting transactions. Nevertheless, money demand and holding liquid money has transaction costs associated in the form of bonds, as individuals can save in this way so generating interest rates. These two forms are competing in the market.

Lastly, the third component corresponds to the amount of labor that individuals supply to firms in exchange for income. Individuals will have to decide the number of hours they want to work, as well as their free time.

With these three components, the representative household utility function in this model is developed so it is possible to measure the level of satisfaction. Considering that individuals are looking for maximizing their utility, the function that must be derived for solving this optimization problem is:

$$(3) \quad \sum_{j=0}^{\infty} \boldsymbol{\beta} \cdot \left(\frac{C_{t+j}{}^{1-\sigma}}{1-\sigma} + \psi_{\mathrm{m}} \frac{m_{t+j}{}^{1-\gamma}}{1-\gamma} - \psi_{\mathrm{m}} \frac{n_{t+j}{}^{1+\kappa}}{1+\kappa}\right)$$

where $\sigma, \gamma, \kappa > 0$, are parameters that will be given in the calibration (see Table 2). Also, $\beta = \frac{1}{1+\rho} < 1, with\ \rho > 0$, is a discount factor incorporated in the sum of utilities equation because when households move away in time, the level of significance decreases, so giving a lower weight. If for instance the current period is t, the representative households will not only care about its consumption, money balances and labor supply in period t, but also the following periods ($_{t+1,\, t+2,\, t+3\ldots}$). Everything happening from $_{t+j}$ matters, where $_j$ takes values from 0 till ∞.

Households will try to maximize their utility as presented in equation (3). However, there are certain aspects that will restrict how much they are going to be able to do so. For that reason, a budget constraint as a limitation of their expenditure is introduced. This equation can be written in nominal terms as well as in real terms. Firstly, the nominal budget constraint is characterized by taking the following form:

$$(4)\ W_t n_t + D_t - Tax_t = P_t C_t + (1 + Rt)^{-1} B_{t+1} - B_t + M_t - M_{t-1}$$

where W_t, is the nominal salary per hour worked, n_t, are the hours the representative household works, D_t, is the dividend a household receives by being the owner of the representative firm, and Tax_t, is the tax element which corresponds to the fixed amount that individuals pay each time period to the government, so can finance its expenditures. These components determine the left-hand side of the budget constraint, which is the labor income ($W_t n_t$), and dividend payment (D_t), both increasing household income, and taxes (Tax_t), decreasing it. As this model does not consider physical capital, having just one production factor, labor, there is not capital rent as source of income.

On the right-hand side of the budget constraint, there are two components. First, total consumption that is the price of each good multiplied by the quantity consumed ($P_t C_t$). The second component correspond to the private savings that includes the possibility of portfolio choice: individuals can use their savings either for net purchases of bonds, or to increase their money holdings. Why incorporating the purchase of bonds in this specific way? Households in the present period t, must decide about the redemption value of those bonds that are purchasing today but will be liquidated next period (B_{t+1}).

Once introduced the households budget constraint in nominal terms, this equation can be easily transformed into real terms by simply dividing all the terms by P_t. While the left-hand side is straightforward, in the right-hand side of the budget constraint there are two terms that are not expressed in the same period. Some changes are needed (see Appendix B) so that to achieve the final real budget constraint that will be introduced in the model.

(5) $w_t n_t + d_t - tax_t = c_t + (1+R_t)^{-1} b_{t+1}(1+\pi_{t+1}) - b_t + m_t - m_{t-1}(1+\pi_t)^{-1}$

where, $\pi = \frac{P_t}{P_{t-1}} - 1$ is the rate of inflation from period $_{t-1}$ to $_t$. All the elements included in this equation are presented in real terms, but the purpose is the same as explained above. The next step is considering both, the utility function, and the budget constraint, to solve the optimization problem for the representative household.

2.1.2. Household optimization problem

Once developed the utility function and the budget constraint, the optimization problem can be solved. Households are looking for maximizing their utility when making their consumption and saving decisions, and tax payment. Therefore, it consists of solving a problem that maximizes the utility of the representative household (3) subject to the real budget constraint (5):

(6) $Max \sum_{j=0}^{\infty} \beta^j \left(\frac{{c_{t+j}}^{1-\sigma}}{1-\sigma} + \psi_m \frac{{m_{t+j}}^{1-\gamma}}{1-\gamma} - \psi_m \frac{{n_{t+j}}^{1+\kappa}}{1+\kappa}\right)$

s.to. $w_{t+j}\, n_{t+j} + d_{t+j} - tax_{t+j} - c_{t+j} - (1+r_{t+j})^{-1} b_{t+1+j} + b_{t+j} - m_{t+j} + m_{t+j}(1+\pi_{t+j})^{-1} = 0$

where $_j$=0,1,2, …

Moreover, it is important to know the choice variables for the representative household in the current period t, which are: consumption (c_t), labor supply (n_t), if decide to accumulate more bonds (b_{t+j}), or more liquidity (m_t).

To solve this problem, there exists the possibility of applying Lagrange, by combining the objective function plus the Lagrange multiplier, that multiply this function in period $_t$, period $_{t+1}$, etc. The Lagrange function will look as:

(7) $$L = \sum_{j=0}^{\infty} \beta^{j} E_t \left(\frac{c_{t+j}{}^{1-\sigma}}{1-\sigma} + \psi_m \frac{m_{t+j}{}^{1-\gamma}}{1-\gamma} - \psi_n \frac{n_{t+j}{}^{1+\kappa}}{1+\kappa}\right) + \lambda_t \left(w_{t+j} n_{t+j} + d_{t+j} - tax_{t+j} - c_{t+j} - (1 + r_{t+j})^{-1} b_{t+1+j} + b_{t+j} - m_{t+j} + (1 + \pi_{t+j})^{-1} m_{t-1+j}\right) + \beta E_t \lambda_{t+1} \left(w_{t+j} n_{t+j} + d_{t+j} - tax_{t+j} - c_{t+j} - (1 + r_{t+j})^{-1} b_{t+1+j} + b_{t+j} - m_{t+j} + (1 + \pi_{t+j})^{-1} m_{t-1-j}\right) + \cdots$$

where λ_t is the Lagrange multiplier associated to the budget constraint in period $_t$. The Lagrange function introduces the rational expectation operator, the expectation in period $_t$ but that comes after, as in this objective function there are some unknown elements. Besides, it has infinite terms represented by the sum factor, but after the derivation it will only be affected by the current period. Then, β^j, is a discount factor as when moving towards the future, the restriction matters less than in period $_t$.

The way of solving this system is taking first-order derivative of the Lagrange function with respect to each of the corresponding choice variables and make it equal to 0. As there are four choice variables, there are also four first-order conditions, which ends up being the optimality conditions. Additionally, the budget restriction must be satisfied so the Lagrange derivative with respect to the Lagrange multiplier must be 0, too. Now, the procedure is to analyze each of the choice variables one by one.

Firstly, the consumption function that determines the evolution of consumption over time. It is also known as the Euler equation which states how much each household consumes and therefore saves, each period. Where does this equation come from? (See steps in Appendix C). If applying the definition of the Lagrange multiplier, that tells the marginal utility of consumption, the exact interpretation given in this model is the shadow value of a unit of income. In other words, if households were to take that unit of income and transform it into consumption, is the gain that extra unit of consumption have, the marginal utility of consumption. Then, if taking the first-order condition of bonds, telling how individuals manage their savings, it can be rewritten by substituting the Lagrange multiplier for the marginal utility of consumption.

After taking derivatives with respect the choice variables, and rearranging terms, the optimal function can be developed. The result is the Euler equation, determining the optimal consumption of individuals over time. It is represented in the following way:

$$(8)\ \frac{C_t{}^{-\sigma}}{1+r_t} = \beta E_t(c_{t+1})^{-\sigma}$$

The optimal choice, when the representative household maximizes her utility, is when both sides yield the same number. On the one hand, the left-hand side of the equation tells the marginal satisfaction of present consumption. One unit consumed today will increase the happiness by its marginal utility, but also having an opportunity cost associated (real interest rate of the bond). For that reason, it is divided by $1 + r_t$ as if the individual consumes today, it is not saving in the form of bonds for next period. In the right-hand side it is observable the welfare improvement of future consumption in terms of marginal utility. However, as future consumption is not as relevant as the present one, it is penalized with the β parameter. Therefore, having the expected value of the marginal utility of future consumption. Only when this equality is satisfied, households are being rational and maximizing their utility.

If for instance, the left-hand side delivers a higher value, not being in equilibrium, the household will prefer consuming more today than in the future. Current consumption will increase, the number of bond holdings will decrease, with less savings and lower future consumption until restoring the equality condition.

The second-choice variable is the labor supply. By taking the first-order condition of n_t and substituting by the Lagrange multiplier, λ_t, (see steps in Appendix C), the following optimal labor supply function is obtained:

$$(9)\ \psi_n n_t{}^{\kappa} = c_t{}^{-\sigma} w_t$$

The left-hand side tells the household satisfaction if deciding not to work, being equal to the labor disutility, that will increase if the individual does not supply one unit of labor, so having an extra unit of leisure. On the right-hand side, the value of one unit of time at work, the real wage multiplied by the marginal utility of consumption. To decide in a rational way, there should not exist improvement options. If the left-hand side value is greater, the individual would prefer working less hours, n_t will be lower and the value will go decreasing until reaching an equality. On the contrary, if the right-hand side value is larger, households would prefer working extra hours as the profit from working will be greater than the benefit from not working.

Lastly, the money demand function derived from the first-order conditions (see intermediate steps in Appendix C):

$$\psi_m m_t^{-\gamma} = c_t^{-\sigma} \frac{R_t}{1+R_t} \quad (10)$$

This optimal condition relates the marginal profit to the marginal cost of money. In the left-hand side it can be observed the marginal profit of money because of the satisfaction it gives to individuals when they hold it. An increase in money holdings will increase the utility of households. In the right-hand side, the marginal cost associated with having liquidity. There is not a direct effect, but money has an opportunity cost in terms of portfolio choice. When comparing money and bonds, there is a yield spread, which is the nominal interest rate in the present value. Then, the opportunity cost of money is an income that households will not hold because of having money and not bonds. If having it, it would give individuals a satisfaction equal to that profitability multiplied by the value given to that unit in terms of utility, that is why it is multiplied by the marginal utility of consumption, to transform the income loss in terms of utility. If the left-hand side value would be higher, households would demand more money within their portfolio choice. If it would be lower, individuals would demand more bonds, until reaching the equality.

Overall, money demand depends positively on consumption and negatively on the opportunity cost, nominal interest rate of bonds.

2.2. Firms

In this subsection, the firm optimizing problem is being solved.

2.2.1. Deviations from classical framework

This firm also behaves in a rational way, looking for maximizing profits in a market that operates in monopolistic competition by Dixit & Stiglitz (1977). This is the main deviation from the classical general equilibrium model in which firms operate in perfect competition with flexible prices. This classical paradigm resulted in a model behavior far from reality, as after a demand shock, everything could be quickly updated through price adjustments, while the real variables such as consumption, employment, and output were not affected. These models known as business cycles (RBC) only depend on technology shocks Kydland & Prescott (1982). Nevertheless, since the monetary policy was not optimally analyzed, some assumptions were changed so to make models more in line with reality. That is the reason why considering the representative firm operating within a monopolistic competition framework, instead of under perfect competition.

The remarkable aspect is that firms are no longer price takers but price setters, so, in comparison with the classical model, are going to make positive profits in equilibrium at the end of each period. Due to the price being higher than the marginal cost of production, being the ratio larger than 1, there exists a mark-up.

The second important aspect is price stickiness. Firms not only fix the price and make a profit after the operating year, but within this industry there are also price rigidities. Prices will not be fully adjusted within a period but will move slowly. To introduce this new aspect in the model, the Calvo (1983) model is followed. Calvo establishes a fix probability of firms not being able to set the optimal price within a given period. This probability (η) goes from 0 to 1 and brings these two limit cases:

- When $\eta = 0$, there are fully flexible prices.
- When $\eta = 1$, the probability of not changing prices is one, so the price will remain the same as in previous period, having fixed prices.

When having monopolistic competition, firms face a demand constraint when setting the price that maximizes their profits. However, the competition term means that households will observe the firm price as well as the aggregate price, so allowing them to make comparisons and substitutions, translated into higher or lower sales for companies. If firms set a higher price than competitors in relative terms, they will sell less than when setting a lower price. This demand constraint is represented in the following way:

$$y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} y_t \qquad (11)$$

where $\theta > 1$, is the Dixit-Stiglitz constant elasticity of substitution across consumption goods. Also, $y_t(i)$, represents the real output produced by firm $_i$, and $P_t(i)$, the optimal price set by firm $_i$. y_t, is the real output level in the economy and P_t, the aggregate price level. This equation indicates that what the firm produces depend on the relative prices and the aggregate output level. The value of θ is a measure of elasticity and tells how much the firm production decreases if the relative price set to the aggregate price level increases in 1%. Therefore, the mark-up under flexible prices is indicated as $\frac{\theta}{\theta-1}$.

Once explained the two aspects that deviates this model from the classical optimizing behavior, under RBC, it is time to proceed with the optimization problem.

2.2.2. Firm optimization problem

The representative firm wants to maximize intertemporal profits. Therefore, to solve this optimization problem, it must be considered a weighted sum of profits for the current period and the expected profits of future periods, as represented in the next equation:

$$(12) \qquad Max\ E_t \sum_{j=0}^{\infty} \beta^j \eta^j \left(\frac{P_t(i)Y_{t+j}(i)}{P_{t+j}} - w_{t+j}n_{t+j}(i)\right)$$

It is a conditional sum, where the η^j parameter is included indicating the probability that in two periods from now the firm remains with the same price. For that reason, the equation considers future periods by raising to the power of $_j$. This parameter is known as Calvo probability of sticky prices, and its calibration can be seen in Table 2.

Besides, two terms can be observed in equation (12). The first one corresponds to the real income, which is the price of the firm multiplied by output, divided by the aggregate price of all the firms operating in the market. The second is the associated cost, which in this model is the work demanded by the representative firm.

For solving the optimization problem, instead of using the Lagrange multiplier, the equation can be replaced and rewrited, as $Y_{t+j}(i)$ is included in the optimal equation. The firm determines $P_t(i)$, while considering the Dixit and Stiglinz demand constraint (11), which must be satisfied. In this way, considering that when firms set a relative high price, they will have to produce less as the quantity sold will decrease. Also, for simplification purposes, only one first-order condition will be conducted, $P_t(i)$. Nevertheless, this variable can be active next period due to the Calvo probability, determining if this firm is able or not to adjust the optimal price the following period.

The substitution process can be seen in Appendix D, in which the price ratio is observed twice: the price that firms choose at a moment in time and the aggregate price that goes changing, because firms will determine $P_t(i)$, the price that will maximize their profits in the current period. Then, with the Dixit and Stiglinz demand constraint, firms will find the amount of output they can produce in each period. Once this production quantity is determined, by applying the production technology equation (13), firms will know exactly how much labor they need to produce that amount of goods. The production technology is:

(13) $$Y_t(i) = e^{z_t} n_t(i);$$

where z_t, is the technology shock with zero mean. This is a reduced form as production only depends linearly on labor and on the technology shock. It represents the quantity of labor multiplied by the exponential of the technological shock.

After introducing these equations, the optimization problem can be solved by taking the first-order condition of equation (12) with respect to the price set by the representative firm (the next steps computed can be seen in Appendix E).

The resulted equation 24) explains the relationships between firms' profits and the optimal price. It is a negative relationship as $\theta > 1$, so being inversely related. If the representative firm decides to set a higher optimal price, although per each unit sold it will earn more money, the real effect is negative as the quantity of output sold decreases. Households can compare market prices and substitute according to the relative price.

When considering the amount of labor supplied by households, n_t, it is also affected by the optimum price. For that reason, the derivative of the second part of the optimization equation, with respect to the price, must be taken. If prices change, the quantity of output sold and produced will change, so the number of workers hired considering the technological shock, too. To do so, it has been decomposed into two equations. The change in prices affects production in a negative way, higher prices less production, and the change in output affects the labor demand in a positive way, less output less workers hired. The steps taken to derive the final equation can be checked in Appendix F, until arriving to the following equation.

(14) $$E_t \sum_{j=0}^{\infty} \beta^j \eta^j \left[(1-\theta)\left(\frac{P_t(i)}{P_t}\right)^{-\theta} \frac{Y_{t+j}}{P_{t+j}} + \theta mc_{t+j} \left(\frac{P_t(i)}{P_{t+j}}\right)^{-\theta-1} \frac{Y_{t+j}}{P_{t+j}}\right] = 0$$

The first part of the equation, without including the next periods would be if $\eta = 0$. However, the following periods must be added when $_j$ takes a value different from 0. Firm's prices may remain, but the aggregate price level changes. Besides, the rational expectation operator, E_t must be included, too.

To analyze equation (14), two different scenarios can be considered. Firstly, considering a particular case, when the Calvo's probability is $\eta = 0$, as if operating under flexible prices. When this happens, firms can always adjust their optimal price. Although being an extreme case, it is interesting to analyze it as leads the economy to a fully flexible price scenario. By doing so, the standing equation is quite simplified as the only terms remaining are the first two ones:

(15) $$(1-\theta)\left(\frac{P_t(i)}{P_t}\right)^{-\theta}\frac{Y_t}{P_t} = mc_t\,(-\theta)\left(\frac{P_t(i)}{P_t}\right)^{-\theta-1}\frac{Y_t}{P_t}$$

By rearranging terms: $1-\theta = -\theta mc_t\left(\frac{P_t(i)}{P_t}\right)^{-1}$; $(1-\theta) = -\theta mc_t\frac{P_t}{P_t(i)}$; $\frac{P_t(i)}{P_t} = \frac{-\theta}{1-\theta}mc_t$

And the final equation takes the shape of:

(16) $$P_t(i) = \frac{\theta}{\theta-1}P_t\,mc_t$$

This equation is the optimal price under flexible prices, when the Calvo probability is zero. This would be the case when firms can set the optimal price every period without worrying about the following periods price as they will be able to change it. The optimal price depends on θ multiplied by the nominal marginal cost. As θ > 1, the ratio θ/θ-1, which is the mark-up, will always be positive. If firms behave in an optimal way and under monopolistic competition, the pricing decision will be taken by looking to the marginal cost and applying a mark-up. Therefore, selling their products with a positive differential over the marginal cost. For that reason, the driving force and key variable to determine the optimal price of each period is the nominal marginal cost. When the marginal cost increases, prices also increase, and the other way around. This makes sense as if the marginal cost increases, is due to an increase in production costs, so decreasing the willingness of firms to produce. Then, to still being profitable, firms need to increase the final price. If firms observe that the marginal cost is decreasing, profits will increase by decreasing the price. As in this way, firms can produce more and taking advantage of the marginal cost, so making higher profits.

However, this is just a special scenario that does not reflect the reality that companies face while doing business. Now, by analyzing the situation under sticky prices, when $\eta > 0$, the proximity to firms' reality will increase.

Equation (14) must be equal to 0, and depends on the value of j, so that every element goes changing over time. The next steps and the reorganization of terms can be seen in Appendix G. The final optimal price equation for the representative firm is the following one:

$$(17) \qquad P_t(i) = \frac{E_t \sum_{j=0}^{\infty} \beta^j y^j \cdot mc_{t+j} \cdot (P_{t+j})^{\theta} Y_{t+j}}{-1 \quad E_t \sum_{j=0}^{\infty} \beta^j y^j \cdot (P_{t+j})^{\theta-1} \cdot Y_{t+j}}$$

The first element represents the mark-up that firms apply over the marginal cost to set the price for its goods. By developing this equation, the rule of following the marginal cost as a drive for setting optimal prices is observable. However, this rule must be forward looking as firms care not only about the present but also the future marginal costs, anticipating to what can happen in period one, two and the following. Why? because it is probable that the price that firms set today will remain in future periods due to Calvo's probability. For that reason, firms should consider their expectations about future values so to set the price in an optimal way at the moment.

2.2.3. Aggregate price function

Once the optimum price equation, with numerator and denominator with infinite sum of terms is determined, it must be combined with another equation to know how all prices are aggregated. This will tell how the optimal price is set by the fraction of firms that do not have this random Calvo probability, and therefore, are able to set the optimal price. The aggregation scheme is the following one:

$$(18) \qquad P_t = [(1-\eta)P_t(i) + \eta P_{t-1}]$$

As observed in equation (18), the determination of the aggregate price depends on two relative average weights. The first one, related to the Calvo probability that explains the price rigidity of firms between periods. Those firms that have received that probability will keep their price as it was in the previous period. However, those companies that have received 1-η, will be able to adjust their prices to the optimum. Nevertheless, as operating in monopolistic competition, the way of moving from individual prices to weighted prices, is not done with a single weighted average. Due to the price stickiness, it is separated into two: those prices that remain the same as in previous period, P_{t-1}, and those that are optimally adjusted by firms, $P_t(i)$. However, all companies will set the same optimum price as all firms have the same technology, have the same demand curve, and have the same final aim of maximizing their profits. Moreover, an indexation rule could be considered for those firms that are not able to adjust the optimum price so can update it according to the inflation level.

2.3. Public sector

In this section, the public sector behavior, CB, and Government, is described and incorporated into the model. They are the ones in charge of designing economic policies or introducing shocks into the economy, so generating business cycles. So, which is a representative behavior of CBs and Governments in an economy? There is a way of representing both in a simple way.

2.3.1. Central Bank

Firstly, the CB that designs monetary policies and uses the interest rates as policy instrument, considers three aspects when taking decisions:

1) Stabilizing inflation at some target value (for example, 2% per year).
2) Economic growth stability to avoid shocks, meaning that Gross Domestic Product (GDP) go in line with its capacity. In this way, big booms or recessions can be avoided.
3) Financial stability so to have little volatility. It is not desired that the interest rates suffer a sudden increase or fall but go smoothly over time.

With these three aspects the monetary policy rules of CBs that will be applied in this model ca be derived. It will depend on the CB which aspect is more relevant, so they give a higher weight to that factor. For instance, some may prefer achieving inflation stability before economic growth stability, or the other way around.

To develop an equation which explains CBs behavior, the Taylor (1993) rule is introduced in the following form, which corresponds to the first model equation:

1. $R_t - R = \mu_R(R_{t-1} - R) + (1 - \mu_R)[\mu_\pi(\pi_t - \pi) + \mu_y\hat{y}_t] + \chi_t$

where μ_R, is the smoothing coefficient, μ_π, takes a value larger than 1, μ_y, a value higher than 0, and χ_t, is a monetary policy shock.

Regarding the nominal interest rate, it can be observed that is determined with deviations from the long run. There are two relative weights which determine the first both components of this equation; the one from previous period, the lagged interest rate, and the one that tells the macroeconomic stabilization.

Considering the lagged interest rate part of the equation, it is observable the introduction of the smoothing interest rate (μ_R). If it is too high, the CB works in a cautious way regarding the financial stability and therefore, there are not relevant changes in nominal interest rates. If on the other hand, it is too little, the CB is not very concerned about the growth or decline of interest rates that could lead to a significant volatility, so considering other aspect as more relevant when deciding the monetary policies.

Regarding the macroeconomic stabilization part of the equation, CBs are worried about inflation and output. These two stabilization coefficients (μ_π, μ_y) are positive because if there is increasing inflation or increasing output growth, within this model, the interest rate policy will be contractionary. CBs will have to increase interest rates so to achieve the equality in both sides of the equation.

The last component, χ_t, is an interest rate shock.

In addition to the Taylor rule, which is an interest rate based monetary rule, a money-growth rule is introduced for the later business cycle analysis explained in section 4. This new monetary policy is represented by the following linear equation

2. $g_{Mt} - g_M = \phi_{gM}(g_{Mt-1} - g_M) + (1 - \phi_{gM})[-(\phi_\pi)(\pi_t - \pi) - (\phi_y)\hat{y}] + \chi_t{}^{gM}$

where g_{Mt}, is the nominal money growth rate that is assumed to be controlled by the CB, and ϕ_{gM} is the smoothing coefficient. Moreover, ϕ_π and ϕ_y are two stabilization coefficients, which unlike the Taylor rule, they take negative signs. For instance, when there is inflation, the money growth should decrease, so interest rate increases, having negative impact on demand, production, and employment, as they will fall. Therefore, households' salaries will drop, decreasing the marginal costs. Overall, optimum prices set by firms will be lower, leading to a decrease in inflation. Finally, $\chi_t{}^{gM}$ is an exogenous variable, being an additional monetary policy shock.

Under a money-growth rule, establishing the real money definition depending on money growth and inflation, is needed. Considering the real money balances definition, $m_t = \frac{M_t}{P_t}$, its lag value, applying logs and making the difference, it gives the following model equation:

(19) $\hat{m} - \hat{m}_{-1} = (g_{Mt} - g_M) - (\pi_t - \pi)$

These two different monetary rules will be applied in section 4 for comparing interest-based and money-growth policies that can be applied by CB's.

2.3.2. Government

The last participant considered in this model is the government, which for simplicity purposes collects taxes in the form of lump-sum; not considering the specific circumstances of firms or households when taxing them, but just collecting a fixed amount from every individual. Therefore, the government finances its public expenditure through the collection of lump-sum taxes, with debt by issuing bonds or by involving the CB and issuing currency by the creation of new money. The resulting equation which includes these three components is known as the government budget constraint and is represented in the following way:

$$(20) \qquad g^{e^{g_t}} = tax_t + (1 + r_t)^{-1} b_{t+1} - b_t + m_t - (1 + \pi_t)^{-1} m_{t-1}$$

This equation can be substituted into the households' real budget constraint (5), as well as replacing the representative firm's dividends with the dividends of those firms operating in monopolistic competition markets. The resulting function is the overall resources constraint, which is also known as the goods market clearing condition. It states that everything produced in the economy must be equal to what is demanded in equilibrium, represented by the next equation (21):

$$(21) \qquad y = c + g^{e^{g_t}}$$

With this equation exists the possibility of analyzing what would happen to an economy when there is a fiscal shock because of a fiscal policy implemented by the government.

However, this is not the final equation that will be introduced in the model as the variable needed is $\hat{y}_t$. To achieve so, the variable is transformed by applying log fluctuations (see Appendix H) until achieving the third equation of the model:

$$3. \quad \hat{y}_t = \left(\frac{c}{y}\right) \hat{c}_t + \left(\frac{g}{y}\right) \hat{g}_t$$

It represents the private and public expenditure with their corresponding weights regarding the amount of production that will go to private and public consumption.

2.4. Set of linearized equations

Even though having developed the optimal equations for the private sector, there is a further step that needs to be done before incorporating them into the desired dynamic model. These functions must be transformed into linear expressions, which can be done by taking logarithms and using fair approximations.

By introducing a system of three equations, the optimum price evolution $P_t(i)$ presented above (17), the aggregate price level behavior P_t, and the inflation evolution π_t, can be explained. This system can be reduced to one equation to find how the inflation rate evolves in the economy. However, as it is a non-linear system of equations, a previous transformation is needed. The procedure adopted is known as log-linearization, which consists of linear transformations in natural logarithms. By doing so, a non-linear function becomes linear in logarithms. The three basic definitions in Appendix I can be applied.

The purpose of linearizing the equations is to solve the system, for which there must be as many linear equations as endogenous variables in the model. Regarding the variables included, there are three different types, explained in the following Table 1:

Table 1 Model variables per type and explanation

	Type of variable	Model variables	Explanation
	Endogenous variables	$\hat{n}_t$;	Labor
		$\hat{c}_t$;	Consumption
		$\hat{m}_t$;	Real Money
		$\pi_t - \pi$;	Inflation rate
		$\hat{mc}_t$;	Real marginal cost
		$\hat{w}_t$;	Real wages
		$r_t - r$;	Real interest rate
		$R_t - R$;	Nominal interest rate
		$\hat{y}_t$;	Output
		f_{n_t}	Marginal productivity of labor
		g_{M_t}	Nominal money growth
State variables	Exogenous variables	z_t	Technology shock – AR(1)
		χ_t	Monetary shock – White noise
		g_t	Government spending shock - AR(1)
		r_t	Inflation shock - AR(1)
	Predetermined variables	R_{t-1}	Lagged nominal interest rate
		$g_{M_{t-1}}$	Lagged nominal money growth
		$\hat{m}_{t-1}$	Lagged real money

Exogenous and predetermined variables together are called state variables as represent the state of the economy, the information package that households receive when they must make decisions. For instance, how are the technological shocks or the government decision behavior.

For solving the system of equations, the log-linearization must take place first, starting with the households' functions. Firstly, the labor supply optimal function (9), which is log-linearized by taking logs in both sides of the equation (see Appendix J)

By defining new variables known as the "hat" variables (e.g., $\hat{y} = log(y_t) - log(y) \cong \frac{y_t - y}{y}$), which are log-deviations from the steady state value, it ends up in the following linear equation, which is the fourth model equation:

4. $\kappa \hat{n} = \hat{w}_t - \sigma \hat{c}_t$

This linear equation corresponds to the labor supply. The same procedure must be repeated for each of the model endogenous variables, to develop the equations one by one. Next, the consumption and real money demand functions, fifth and sixth equations:

5. $\hat{c}_t = E_t \hat{c}_{t+1} - \frac{1}{\sigma}(r_t - r)$
6. $\hat{m}_t = \frac{\sigma}{\gamma} \hat{c}_t - \frac{1}{\gamma R}(R_t - R)$

Once introduced these three equations which explain the behavior of the representative household, the firm exercise can be finished, too. This can be done by taking the optimal price equation resulting from applying the first-order condition on the pricing decisions of firms operating in monopolistic competition and facing price rigidity. The equation (17) must be log-linearized (see steps in Appendix K) until reaching the seventh linear equation of the model, in which it has been included a last term known as indexation rule:

7. $\pi_t - \pi = \beta E_t(\pi_{t+1} - \pi) + \frac{(1-\beta y)(1-y)}{y} \hat{mc}_t + (1 - \beta \rho_c) r_t$

After deriving the optimal price and the aggregate price, a combination between both must be done so to find a relationship between the inflation rate and the relative prices, given by equation (50)). This inflation relationship can be positive or negative. If the optimal price is above the aggregate price level, inflation increases, greater in comparison to the steady state value, therefore, the optimal price pulls. If instead the optimal price is below the aggregate price level, firms are decreasing their prices, so inflation decreases.

This lineal equation (7.) is known as the New Keynesian Philips Curve, which determines the inflation evolution, depending on:

- The expectations on future inflation (forward looking). The price rigidity only allows some firms to adjust the price of its differentiated good. If they adjust them, that price may remain for a long period of time, so they have to foresee and take decisions in the present with respect to future periods.
- The marginal cost: log fluctuations with respect to its steady state value. When that value is positive, inflation in the short-run will be above the long-run level.
- The indexation rule incorporates an exogenous shock. When firms are not able to adjust the optimal price, instead of leaving it as in last period, the price is adjusted by looking at previous period's inflation, so prices are updated accordingly.

Now, the marginal cost, $mc_t = \frac{w_t}{f_{n_t}}$, the real wage divided by the marginal productivity of labor. For this last element, the production function (13) is needed first. As mentioned above, it only has one input factor, labor, and follows a linear relationship. Besides, there is a technological shock that follows an AR(1) process. These technology shock innovations are white noise, generated by a normal distribution function with mean 0 and constant variance. Moreover, there is a term related to previous period as innovation does not disappear from one period to the next, usually being quite high. It takes the exponential form because in case of being a technology shock at the expected 0 value, the exponential of 0 will take the value equal to 1, being not relevant in the production function that would only depend on the labor input. The resulting production function when taking logarithms and doing the difference with respect the steady state (see steps in Appendix L), is the following:

8. $\hat{y}_t = z_t + \hat{n}_t$

Then, the marginal productivity of labor, calculated by taking the derivation of the output produced with respect to labor; $f_{n_t} = \frac{\partial y_t}{\partial n_t} = e^{z_t}$, determines the ninth-model equation.

9. $\hat{f}_{n_t} = z_t$

In this case, due to the production function form, it results in the technological shock. It is considered linear and not decreasing with labor for simplifying purposes. However, it would be more realistic if having the decreasing component, as the more hours an individual works, the lower productivity.

Coming back to the marginal cost linear equation, after taking logs (see Appendix M) the final function corresponding to the marginal cost, which depends positively on salary and negatively on productivity, is the following:

10. $\hat{mc}_t = \hat{w}_t - z_t$

Moreover, the fisher equation definition (see Appendix B) which relates the real and nominal interest rates must be also introduced after log-linearizing it:

11. $r_t - r = (R_t - R) - E_t(\pi_{t+1} - \pi)$

This definition explains the relationship between the real interest rate with respect its steady state value, with the nominal interest rate, by considering the expected inflation.

Once these log-linearized equations have been introduced, the model functions for the private sector are all developed and interpreted.

These eleven log-linearized functions explain the eleven endogenous variables of the model. Nevertheless, an additional equation is needed to explain the behavior of the predetermined variable, the lagged nominal interest rate, representing the persistence of the four exogenous variables.

12. $E_t(R_{t-1} - R)_{+1} = R_t - R$

With this last equation, the model can be used for conducting BCA, simulations, or impulse-response exercises.

3. BASELINE CALIBRATION

Table 2 provides a baseline quarterly calibration for the parameters of the model.

Table 2 Baseline calibration of model parameters

Baseline calibration of model parameters	
Risk aversion coefficient	$\sigma = 1.39$
Labor utility curvature	$\kappa = 2$
Intertemporal preference rate	$\rho = 0.005$
Discount factor	$\beta = 0.995$
Calvo probability of sticky prices	$\eta = 0.75$

Dixit-Stiglitz elasticity	$\theta = 6$
Taylor-type monetary policy rule	$\mu_{\pi} = 1.5; \mu_{y} = \frac{0.5}{4} = 0.125$
Interest-rate smoothing coefficient	$\mu_R = 0.79$
Money demand elasticity	$\gamma = 5$
Nominal interest rate in steady state	$R = 0.0125$
Money growth monetary policy rule	$\phi_{\pi} = -1.5; \phi_{y} = -0.125$
Money growth smoothing coefficient	$\phi_{Gm} = 0.79$

Regarding the risk aversion coefficient, according to the empirical evidence reported by Smets & Wouters (2007), I set $\sigma = 1.39$. This value is derived from the estimation exercise done in a Dynamic Stochastic General Equilibrium (DSGE) model with US macro-economic quarterly data, for which σ takes the value that is going to be used and incorporated in this model.

Then, κ, determines the curvature of labor disutility. It indicates the relationship between the labor supply and the real wage, determining how much extra time workers will be willing to work when there is an increase in real wage. Indeed, $1/\kappa$ determines this elasticity of labor supply to the real wage. Papers such as Card (1994), and Altonji (1986), find that this labor supply elasticity is rather a small number what indicates that households respond very little in the labor supply to changes in real wages. For that reason, taking $\kappa = 2$, so that the labor supply Frisch elasticity is $\frac{1}{\kappa} = 0.50$.

As for the intertemporal preference rate, a value of $\rho = 0.005$ is considered as in equilibrium, it is equal to the real interest rate. For that reason, a rational value for this parameter is 0.5% as means that in steady state, the annualized real interest rate would be 0.5% per quarter (2% per year). Therefore, the quarterly discount factor is $\beta = \frac{1}{1+\rho} = \frac{1}{1.005} = 0.995$.

Price rigidity is defined by the Calvo probability of non-optimal pricing, $\eta = 0.75$, so as firms to have an average frequency of 25% of setting the optimal price, as observed in the paper published by Erceg, Henderson, & Levin (2000). As observations are quarterly,

$\frac{1}{1-y} = \frac{1}{0.25} = 4$, every 4 quarters, on average, firms set the optimal price (once a year), while during the other three quarters the price remains unchanged. Nevertheless, as the model includes an indexation rule, the optimum price will be adjusted according to that rule by looking at the past period's inflation so to move prices accordingly.

According to the Dixit-Stiglitz elasticity, it takes the value $\theta = 6$, so $\frac{\theta}{\theta-1} = \frac{6}{5} = 1.2$, giving a 20% mark-up in steady state, which is a reasonable value for this parameter.

The Taylor-type monetary policy rule is implemented with the original coefficients suggested by Taylor (1993) which take the following values: $\mu_\pi = 1.5$ and $\mu_y = \frac{0.5}{4} = 0.125$. The latter is divided by four because in Taylor's paper uses annual data. Moreover, the rule incorporates an interest-rate smoothing coefficient as in Clarida, Gali, & Gertler (2000), and takes the estimated value $\mu_R = 0.79$, using US economic data from the Volcker and Greenspan terms in as Fed governors. Additionally, I have taken a money-growth rule with the same quantitative response coefficients that I use in the interest-rate rule, to do the monetary policy comparison exercise. However, the two coefficients enter the equation with positive sign, therefore the inflation and output coefficients must be of negative sign. I set the following values $\phi_\pi = -1.5$ for the reaction of money growth to inflation deviations, and $\phi_y = -0.125$ for the response to log-fluctuations of output, which I think are reasonable. Regarding the money growth smoothing coefficient, I also use the same one as under the Taylor rule, $\phi_{Gm} = 0.79$.

With respect to the money demand, I set $\gamma = 5$, which implies an elasticity of money demand with respect to the interest rate at -0.2%, which is a reasonable value. Besides, the nominal interest rate in steady state, if the real interest rate is 2% and assuming $\pi = 3\%$ per year in steady state is: $R \cong r + \pi = 0.005 + 0.0075 = 0.0125$ per quarter (5% per year).

Finally, it is assumed that the public expenditure is 30% of GDP in steady state, $\frac{g}{y} = 0.30$, which is a reasonable percentage (private expenditure, $\frac{c}{y} = 0.70$, 70% in steady state).

Regarding the time unit, quarter are considered as most macroeconomic series are published in quarterly series. Therefore, in this model quarterly decision behavior is considered to allow possible comparisons of the model series.

4. BUSINESS CYCLE ANALYSIS

The aim of this chapter is to present two different exercises by applying the model presented above, using the two different monetary policy rules, and by using MATLAB as a software. First, by applying the Taylor rule in an impulse-response exercise with technology and inflation shocks in an economy coming from steady state. Besides, second moment statistics are computed. Second, the same two exercises are presented by replacing Taylor's rule by a money-growth rule, so to compare results.

To carry out these two exercises, the services of Klein (2000), are requested. In addition to Paul Klein's code ("solvek.m"), two other files are needed to be able to run this routine: "qzswitch" and "reorder". Applying the characteristics described in McCallum (1998) paper for solving linear models with rational expectations based on Paul Klein's code, four matrices must be defined in MATLAB. By doing so, it is possible to ensure that all model equations are fulfilled at the same time, as well as to see how each variable evolves without considering rational expectations. In other words, endogenous variables being dependent of observable variables and not the rational expectation that is unobservable.

This leads to the Klein's (2000) solution form (22) which relates the vector of endogenous variables (Y_t) to the vector of state variables: predetermined (K_t), and shocks (U_t).

$$(22) \qquad \begin{aligned} Y_t &= M1K_t + M2U_t \\ K_{t+1} &= M3K_t + M4U_t \end{aligned}$$

Moreover, the following period is included by introducing matrices M3 and M4, since the predetermined variable is chosen in the current period. Furthermore, five inputs are needed (matrices A, B, C and R, and the number of predetermined variables), so with the previous equations the output related to that input can be delivered.

Finally, persistence of exogenous shocks must be considered by introducing their expected next-period values:

$$(23) \qquad E_tU_{t+1} = RU_t$$

where $U_t = [z_t \chi_t g_t r_t]'$ is the column vector of the exogenous variables. Three out of the four exogenous variables show persistence, representing their inertia observed in the matrix diagonal. Production technology, government spending and inflation shocks show persistence while the monetary policy shock not, as it is a white noise so being the expected value 0.

4.1. Impulse-response functions

First, considering the calibration in Table 2, an impulse-response exercise is done. The shock introduced in the model is a one-time innovation, taking value 1 in period 1, and then value 0 for the following periods, so being observable at the time of the shock. Nevertheless, the innovation will have an impact on successive periods as the information from previous quarters is considered (see (23)).

3.1.1. Technology shock

Firstly, a 1% positive technology shock, represented by an Autoregressive Model of order 1 [AR(1)] is analyzed. 40 quarters ahead are considered as some effects could be observed within the next ten years after the shock because of the AR(1) process and due to its high persistence, as 95% of what happens will remain in the economy (see Table 3).

Figure 1 displays impulse-response functions following a 1% technology shock with the two different monetary policy rules. The Taylor Rule as well as a money-growth rule, which is incorporated for comparative purposes. Under this policy, the CB instead of adjusting straightforward the interest rates according to the economic situation, fixes the quantity of money that will later lead to an interest rate.

It can be observed that the fact of applying different monetary rules, does not influence the overall effect, which is expansionary, as output in the current period rises by around 0.44%, almost half the size of the shock. In subsequent quarters the GDP goes decreasing until achieving its steady state value. However, due to the high persistence of this innovation it does not come back to zero within the ten years studied.

Moreover, due to the technological progress, productivity increases so firms develop more efficient processes, reducing its production costs. By achieving higher productivity, firms will demand less labor, hiring less workers therefore reducing the real marginal cost. At the same time, real wage falls to restore equilibrium in the labor supply curve. Those firms that got the Calvo's probability to adjust the price in the current period will be the only ones taking advantage of the decrease in marginal costs, as the others will lose market share. The firms adjusting the optimal price will decrease it by applying a mark-up over the lower marginal cost. By doing so, a decrease in inflation will take place.

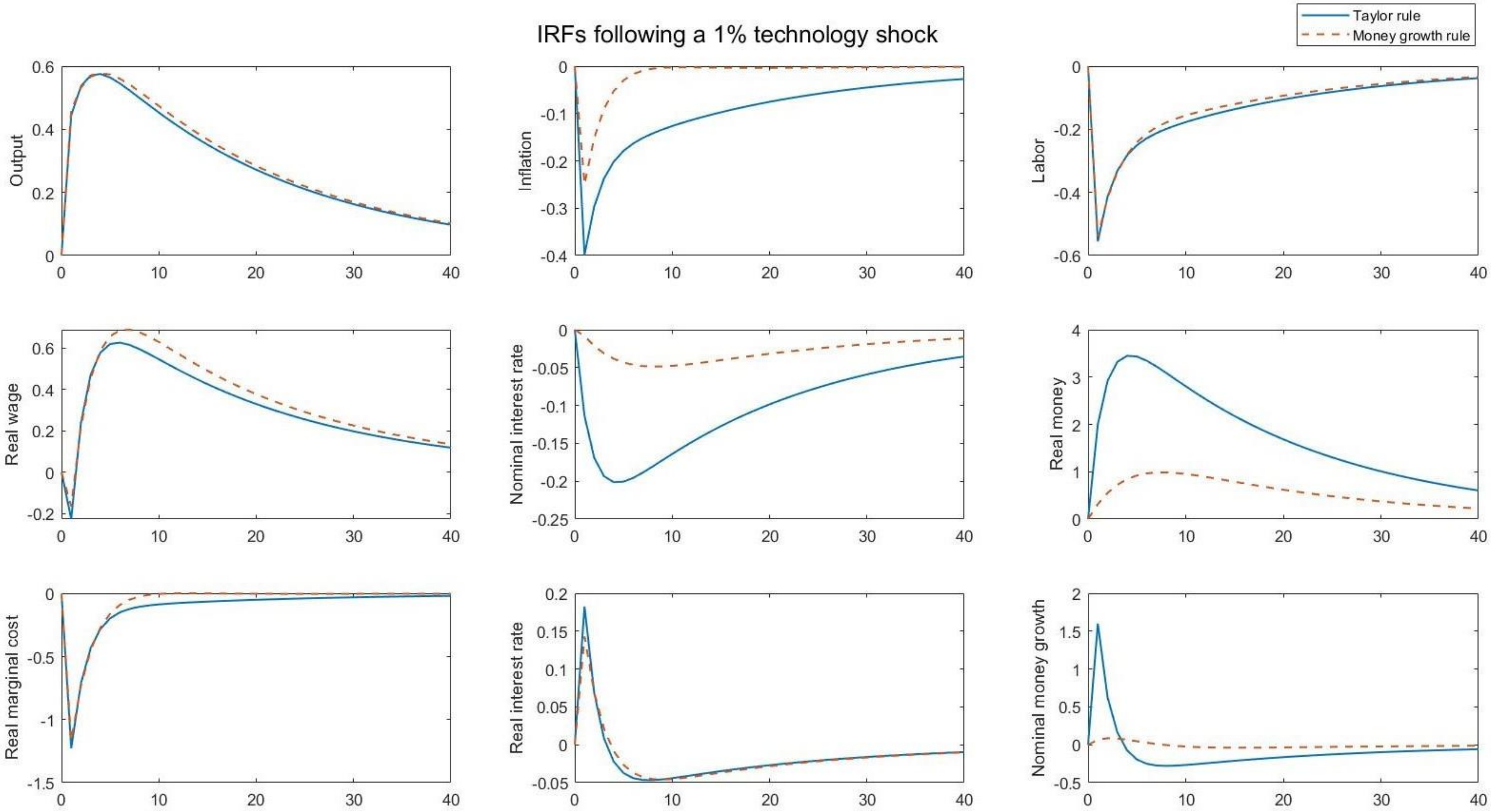

Figure 1 IRFs 1% Production Technology Shock

From the point of view of households, they will demand more money because of the expansionary effect of the technology shock. Money demand rises while the CB lowers the nominal interest rates to stabilize inflation and aggregate output fluctuations. As seen in Figure 1, these two variables are more volatile when the CB uses the interest rate as policy instrument, as the values further separate from the target value 0. Also, due to the persistence of the technology shock, it takes more than 40 quarters (10 years) to fully return to the long-run equilibrium values.

As this innovation directly affects the firm's productivity and the role of the CB is just adjusting the policy instruments to stabilize inflation and output, the graphical representation of the endogenous variables applying different monetary policy rules does not differ that much. Nevertheless, some interesting facts can be highlighted.

First, real wage behaves similarly under the two monetary policy rules. The reason is that real model variables are not greatly affected by the type of monetary rule but behave in a similar way. However, the nominal variables are impacted, so observing different behaviors. For instance, the nominal interest rate decreases when there is a technology shock. This is due to the decrease in inflation. As the main CB objective is to stabilize inflation, it conducts a contractionary monetary policy so to respond to lower inflation. Besides, slight differences can be seen in other variables related to monetary policies, such as real money, and nominal money growth.

Finally, what I would highlight from this impulse-response exercise is that it seems that the money-growth rule performs better than the Taylor rule. The CB is concerned with stabilizing output as well as inflation around their steady state values. In this case, when looking at Figure 1, would mean being the closest to 0 as possible. Visually, the variability of output fluctuations in both series is mostly the same, while inflation is clearly lower with the money-growth rule. There is a drop of around forty basic points for Taylor rule, and approximately twenty-two for money-growth rule, for which in quarter eight is already close to 0. Therefore, the latest has a higher capacity to stabilize inflation following a technology shock.

3.1.2. Inflation shock

Next, a 1% positive inflation shock, also generated by an AR(1) process, is introduced from the price indexation rule applied by those firms that cannot set the optimal price.

The effect will be observed many quarters ahead due to its persistence, measured at the 85% autocorrelation (see Table 3). Figure 2 represents the different model variables considering both monetary policy rules during 20 quarters after the inflation shock.

Firstly, unlike the shock analyzed previously, this one is contractionary. The CB applies the Taylor-type rule and rises the nominal interest rate, following an expansionary monetary policy, in response to the observed higher inflation. The real interest rate will also go up. Households prefer saving so consumption and expenditure will decrease. As a component of aggregate demand, through the transmission mechanism, output will decrease below its steady state value. As represented in Figure 2, it suffers a sudden decrease at the time of the shock, but then goes increasing, getting closer to the target value 0. Specifically, output in the current period decreases by 0.35%. Moreover, the fact that households' demand less goods, directly impact firm's sales. Firms will cut production so less labor will be needed. The rise in prices reduces the real wage, and the decrease in labor demand, drops the marginal cost of firms. As before, those firms that got Calvo's probability will lower the optimum price. In this way, inflation can be managed and stabilized, so coming back to its steady state value.

Households, apart from preferring saving and reducing their consumption, they also prefer holding bonds to money. The decrease in money demand is aligned with a higher nominal interest rate, monetary policy applied by the CB to stabilize output and inflation around their long-term values.

When changing the monetary policy of the CB, the overall contractionary effect of the inflation shock can be observed, too. The economy declines below its steady state, in the current period, output decreases by 0.34%. Under the money-growth rule, the CB instead of applying an expansionary policy cutting the interest rates, will adjust the quantity of money in circulation by reducing the nominal money growth. By doing so, both the nominal and real quantities increase, what will lead to an increase in the nominal interest rate in the short run, while the real interest rate will move downwards due to inflation expectations.

As with the technology innovation, there are some variables that are greatly affected by the change in monetary policy rule, mostly the instruments that are used by the CB, such as the nominal interest rate, or nominal money growth, while others are not significantly affected, such as output, that fluctuates in a similar way.

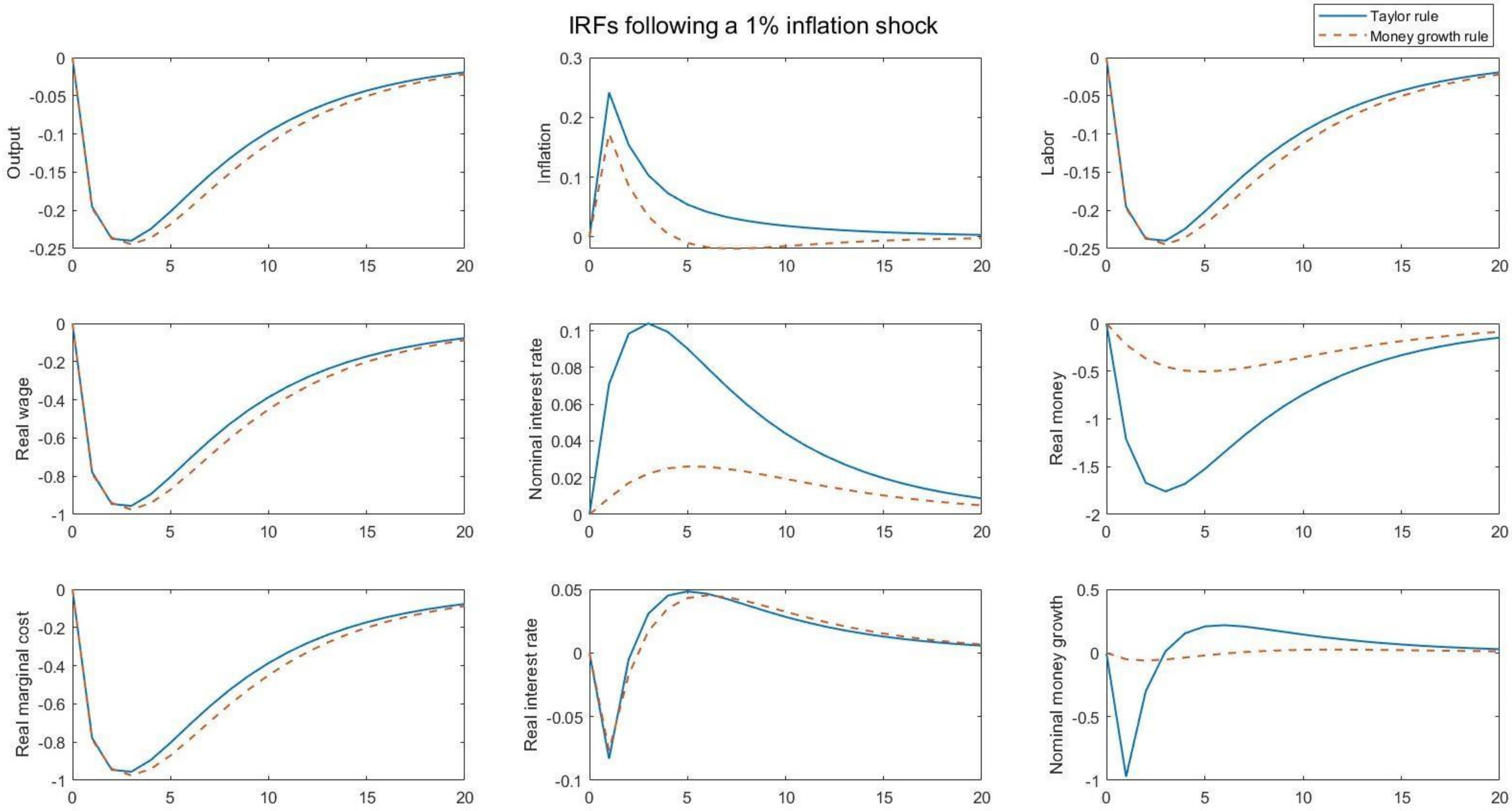

Figure 2 IRFs 1% Inflation Shock

The most remarkable aspect of this impulse-response exercise is related with inflation, macroeconomic variable for which the CB is most concerned with. The nominal money-growth rule shows the best inflation stabilization capacity for both, the technology, and the inflation shocks. In the case of output fluctuations, both monetary policy rules show a very similar IRF. However, the Taylor rule manages to dampen the innovation shock effect a little more.

Even though the money-growth rule seems to react better against both shocks, the CB cannot determine the amount of money circulating in the economy. This fact has to be considered when calibrating the model so to obtain results closer to reality. For that reason, a higher standard deviation is given when solving the model with a money-growth rule than with the Taylor rule. Thus, taking into account the fact that the CB can only control the monetary base through balance sheet operations.

Appendix N includes impulse response functions following a government spending shock, and Appendix O, of a monetary policy shock (either interest-rate shock to the Taylor rule, or nominal money shock to the money-growth rule).

4.2. Second moment statistics

This section aims to present the different sources of variability. Specifically, three dimensions can be analyzed: volatility, cyclicality, and persistence.

For doing the BCA, it is important to specify the characteristics of the exogenous part of the model since it is the one that determines the magnitude of the fluctuations. I have done the calibration of the exogenous processes presented in Table 3. With it, I have simulated the model for 10,000 iterations and 200 quarters of observations, to bring convergence in the statistical results.

Table 3 Exogenous variables with corresponding autocorrelation coefficients and standard deviations

Exogenous variable	Autocorrelation coefficient	Standard deviation
Production technology	$r_z = 0.95$	$steps_z = 0.5$
Government spending	$r_g = 0.80$	$steps_g = 1.3$
Monetary policy	-	$steps_{chi} = 0.2$; $steps_{chi} = 0.6$
Inflation	$r_i = 0.85$	$steps_i = 0.6$

Note: In this table the corresponding autocorrelation coefficients and the standard deviation of the innovation shocks can be seen

For computing the second moment statistics, I will use a different standard deviation for the monetary policy regarding the chosen rule. While applying the Taylor Rule, I consider that 0.2 is a reasonable value as this policy states that there must be a fit, so the deviation should be the smallest possible. However, if changing the monetary policy to a money-growth rule, I will consider $steps_{chi} = 0.6$. The reason is that the CB cannot determine the value of the amount of circulating money in the economy because it can only control, through balance sheet operations, the monetary base. Therefore, it should be considered the variance of the monetary shock of a money-growth rule to be higher than that of the monetary shock of an interest rate rule. Thus, in the calibration of the elements characterizing the exogenous variables (shocks), I have given a standard deviation for the money-growth rule shock that is three times that of the Taylor rule.

For each of the analyzed dimensions, three second-moment statistics are examined: the standard deviation for assessing the variability, cross-correlations with respect to GDP fluctuations, for cyclicality, and autocorrelations for persistence. These coefficients are calculated for all endogenous variables by artificially series randomly generated by the model solution form. A MATLAB command called “randn” is used, which generates random numbers arising from a normal distribution with mean 0 and variance 1. For that reason, each time that the program is run, different results and figures are displayed. To get the results, the solution form presented in equation (22) is applied.

In Table 4, I report the results under the Taylor’s rule. Then, in Table 5, the same statistics are presented but after applying the money-growth rule. Despite reporting them for all endogenous variables, only the most relevant ones related to the monetary policy, and the two important macroeconomic variables, are highlighted.

Table 4 Second moment statistics applying Taylor’s Rule

SECOND MOMENT STATISTICS			
	Standard Deviation	**Cross correlation with $\hat{y}$**	**Autocorrelation**
Output, $\hat{y}$	1.104	1	0.892
Consumption, $\hat{c}$	1.513	0.819	0.917

Labor supply, $\hat{n}$	0.783	-0.049	0.769
Real wages, $\hat{w}$	2.112	0.776	0.788
Nominal interest rate, $\hat{R}$	0.397	-0.842	0.891
Real interest rate, $\hat{r}$	0.320	-0.319	0.556
Inflation, $\hat{\pi}$	0.463	-0.512	0.717
Marginal cost, $\widehat{mc}$	1.998	0.250	0.718
Real money, $\hat{m}$	6.778	0.841	0.893
Nominal money growth, g_M	2.991	0.082	-0.100

Note: This table shows the statistical values under the Taylor rule of the three second moment statistics for the endogenous model variables.

Table 5 Second moment statistics applying Money-growth rule

SECOND MOMENT STATISTICS			
	Standard Deviation	**Cross correlation with** $\hat{y}$	**Autocorrelation**
Output, $\hat{y}$	1.211	1	0.873
Consumption, $\hat{c}$	1.658	0.850	0.897
Labor supply, $\hat{n}$	0.887	0.149	0.758
Real wages, $\hat{w}$	2.755	0.806	0.778
Nominal interest rate, $\hat{R}$	0.098	-0.721	0.984
Real interest rate, $\hat{r}$	0.361	-0.468	0.671
Inflation, $\hat{\pi}$	0.534	0.272	0.631
Marginal cost, $\widehat{mc}$	2.616	0.436	0.730
Real money, $\hat{m}$	1.978	0.773	0.983
Nominal money growth, g_M	0.601	0.399	0.605

Note: This table shows the statistical values under the Money-growth rule of the three second moment statistics for the endogenous model variables.

The standard deviation shows information regarding the volatility of each of the variables. Due to the random process, each innovation is adjusted by its calibrated standard deviation. To compute these numbers, first the standard deviations and then the mean for each variable is computed in MATLAB. The corresponding coefficients values can be seen in Tables 4 and 5.

Based on the preceding subchapter, the money-growth rule led to better results in terms of stabilizing the main variables considered by the CB. Therefore, I have taken into account the increase of the standard deviation to achieve more realistic results when computing the second moment statistics.

Focusing on the relevant macroeconomic variables, output and inflation, differences can be observed. The Taylor rule leads to a lower standard deviation of output, as well as inflation in comparison to the money-growth rule, having fluctuations of lower magnitude. However, the change to a money-growth rule, makes the standard deviation of the nominal interest rate to be much lower, decreasing from around 0.40 to 0.10, so having less volatility than when using the Taylor rule. Nevertheless, the main objectives of CBs regarding macroeconomic stabilization are usually either to stabilize inflation, or to control output fluctuations. By looking to these figures, the Taylor rule could achieve both. However, if considering a lower standard deviation for the monetary shock, each of the monetary rules could only control one of these variables' volatilities. Thus, if aiming to control inflation, could better apply the money-growth rule, which also implies a lower volatility of the nominal interest rate, important when looking at financial markets.

Additionally, another interesting fact is the great change that occurs in the standard deviations of money and nominal money-growth, when changing the monetary policy. When the CB applies the Taylor rule as policy instrument, the volatility of these two variables increases a lot. This could be a problem as in the real world when there is a sudden significant change, can lead to financial and banking instability, asset bubbles, debt crisis, etc.

Summing up, the Taylor rule seems to perform better than the money-growth rule for stabilizing the model endogenous variables, helping them to return to their steady state values.

The cross-correlation with respect to output fluctuations and one endogenous variable, measures the variable cyclicality. In this way, it can be measured how procyclical, countercyclical, or acyclical a variable is with respect to output. To compute the figures presented in Tables 4 and 5, first, the linear correlation coefficients of each variable with respect to output are computed. However, what really matters is the mean of those values as could be the case that they take different positive or negative values during the 10,000 iterations. By computing the mean across columns, I obtained the cross correlation with respect to output fluctuations for each variable.

Firstly, regardless of the monetary policy rule applied by the CB, consumption, real wages, and real money are characterized by being highly procyclical variables as the cross-correlation coefficient is positive and close to one. On the other hand, the nominal interest rate presents just the opposite relationship, as is quite countercyclical with respect to the economic cycle. This means that it behaves regardless of what is happening in the economy. One of the reasons could be the fact that is the instrumental variable used by the CB under the Taylor rule, used to stabilize output and inflation when external shocks hit the economy.

Then, when applying a money-growth rule, the values of labor supply and inflation switch from being negative to positive, so from countercyclical to procyclical. However, the values are near to 0, so not reacting very significantly to changes in the economy.

According to the inflation shock, it may be that in market-data this correlation is positive, as happens when changing from the Taylor rule to the money-growth rule. However, when considering the Taylor Rule and the calibration given, it may be negative due to the persistence of the technology shock. Production increases while its costs decrease due to technological improvements and productivity gains. Marginal cost falls so those firms that can adjust the optimal price will lower it, thus decreasing inflation. Therefore, output is increasing while inflation decreases, when in the data both would rise. It can be said that the technology shock is a key driver of the business cycle, which leads the cross correlation of output and inflation to be negative.

Finally, the nominal money-growth variable appears to be more procyclical when the CB uses the money-growth rule. This makes sense as this instrumental variable is only used when applying this monetary policy rule.

Lastly, to analyze the variables' persistence, autocorrelations of order 1 with respect to the immediate previous period are computed. The series from period 1 to period $_{t-1}$, and from period 2 to period $_t$ are linearly regressed. Nevertheless, the value needed for analyzing the persistence is the average autocorrelation, as each time the code is run, different values are delivered. When doing so, a number between minus 1 and 1 results. The closer to one, the more persistent the variable is.

Tables 4 and 5 show the different values applying the two monetary policy rules. Regardless the CB's rule, macroeconomic variables such as output, and consumption are quite persistent. What happens in one period determines to a large extent the behavior of the following ones. One possible reason may be the innovation shocks and their persistence. Besides, households do not consider the CB's policy when making consumption or savings decisions, so that when an external shock hits the economy, these variables behavior might not be linked to the instrument used by the CB. However, the persistence is generally lower when applying a money-growth rule, what could be linked to the uncertainty of CBs to control the monetary base. Additionally, inflation takes a value around 0.5 although its persistence decreases when the CB applies a money-growth rule, too. This could indicate that the series itself explains what happens in different periods, but also other factors can affect its fluctuations.

The monetary variables, nominal and real interest rates, and real money, are quite persistent, too. However, the switch from the Taylor rule, increases it even further. Finally, since the CB applying the Taylor rule uses the interest rate as instrument for adjusting economic variables, it justifies that the nominal money-growth series take a negative value, close to 0. Nevertheless, when applying a money-growth rule, the series becomes quite persistent.

Overall, it is observable that in general all endogenous variables present a strong autocorrelation since the values are positive and close to 1.

To conclude, Table 6 presents the standard deviations of selected variables to evaluate the stabilizing performance of alternative monetary policy rules. Considering the CB's preferences, it may prefer certain variables to take smaller values.

Table 6 Stabilizing performance of alternative monetary policy rules

	$std(\pi)$	$std(y)$	$std(R)$	$std(g^M)$

Taylor Rule	0.463	1.104	0.397	2.991
Money-growth rule	0.534	1.211	0.098	0.601

Note: In this table the standard deviations of the main macroeconomic variables, inflation, and output, and the two instrumental variables, nominal interest rate and nominal money-growth, are presented.

Taylor rule performs slightly better than the money-growth rule for the inflation stabilization around its target rate. The standard deviation takes a smaller value, having lower volatility. Therefore, if the CB is very oriented to fight against inflation, it will apply this monetary policy as delivers a lower standard deviation.

Moreover, Taylor rule is a bit superior to the money-growth rule to reduce the severity of a recession. It is so, because of the lower standard deviation value, achieving a faster recovery towards the target value 0. The CB should apply this monetary policy if its main target is to avoid sudden booms or recessions that could impact other macroeconomic variables such as employment.

These two conclusions show how a policy based on interest rates can help the economy to recover faster from an external shock, and without deviating that much from its steady state values.

The last two, nominal interest rate and nominal money growth, are instrumental variables. It is interesting to have a closer look to their volatility as they can affect financial markets. In comparison with the previous analyzed variables, larger differences in variables are observed when comparing the monetary policy rules. Taylor rule performs worse than money-growth rule for both, nominal interest rate and money growth stabilization. Particularly, the standard deviation is four, and five times bigger when applying the Taylor rule. These large fluctuations can lead to changes in individuals' expectations and their actions or decisions in financial markets. For that reason, the CB should consider this fact when applying one of these instruments to make market corrections.

5. CONCLUSIONS

The purpose of this project is to present a macroeconomic model that describes the behavior of both, the private and public sectors in the economy. This is done through the analysis of the optimizing behavior of both, households, and firms, as well as the policy interventions of the CB and the government, and how they interact with each other. A model has been developed by introducing utility functions, solving optimality problems, and using log-linearized equations.

Based on existing literature, a calibration for the model parameters is done. Moreover, the use of several MATLAB codes has helped to solve the system of equations. Once this is done, different economic exercises can be carried out to understand how macroeconomic studies are done, which are then used and applied in real life situations by organizations such as private firms or even the CB.

Furthermore, during the analysis, the value of the parameters can be adjusted to see possible changes in the results. In this case, it has been analyzed how the change in the CB's monetary policy can affect the overall economy. Doing a BCA in which impulse-response exercises, with one-time innovations, as well as the computation of the second moment statistics.

The money-growth rule outperforms Taylor rule on stabilizing inflation following both, a technology shock, and an inflation shock. Nevertheless, the fact that the CB can only control the monetary base, is enough for recommending the Taylor rule rather than the money-growth rule to return to steady state values in an economy suffering from these shocks. Additionally, the Taylor rule leads to lower output and inflation volatility than the money-growth rule because of the smaller standard deviation values.

Moreover, I have realized that by applying a single monetary rule, CBs can hardly fulfil several objectives at the same time. One rule may drive to output stability by avoiding booms or recessions that may alter the overall state of society. Others might be more effective in controlling inflation volatility. However, both macroeconomic variables cannot be fully controlled by applying a single monetary rule. Therefore, the CB must clearly set its main objectives to fulfil them in the best possible way, while trying to control other important variables for economic stability.

In the current context where there is a positive inflation shock, the economy may turn into a recession. Nevertheless, this recession could be milder if the CB applies the Taylor rule instead of a money-growth rule. This is not only in terms of output, variable that mainly concerns the CB, the reason why is included in the monetary policy rules, but also employment and real wage, which would also fall less with the implementation of the Taylor rule. The recent evolution of the EURIBOR indicator is a signal of future announcements of higher interest rates by the European Central Bank (ECB). When there is an inflation shock, the reaction of CB's should be increasing nominal interest rates. The 12-month EURIBOR has moved up from -0.5% during the Covid-19 pandemic to around 0.25% when observing the inflation shock during the last months. Therefore, market participants are anticipating that the ECB monetary policy, aimed at increasing the interest rate, might use the Taylor rule to respond to this inflation episode. The Federal Reserve Bank already announced (on May 5th, 2022) a 50 basis points increase in the official discount rate for commercial banks.

To sum up, developing models that can explain how the economy may evolve or how different external shocks might affect specific variables is an interesting and necessary tool to anticipate adverse situations, or mitigate their effects by using the correct policy responses.

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APPENDICES

Appendix A: Utility function development

Real money demand:

1) $m_t = \frac{Mt}{Pt}$

Marginal Utility of Consumption:

2) $U_{Ct} = \frac{\partial U_{c_t}}{\partial c_t} > 0$

Elasticity of consumption:

3) $E = \frac{\partial U c_t / \partial c_t}{\partial c_t / c_t}$

Represent the marginal utility of consumption. It is always positive, but the higher the consumption, the lower utility.

4) $Uc_t = \frac{(1-\sigma) C_t^{1-\sigma-1}}{(1-\sigma)} = c_t^{-\sigma}$

Taking derivatives:

5) $\partial Uc_t = -\sigma c_t^{-\sigma-1}$

Appendix B: Household budget constraint transformation

Households budget constraint expressed in real terms. Transformation from nominal to real:

6) $w_t n_t + d_t - Tax_t = c_t + (1 + r_t)^{-1} \frac{B_{t+1}}{P_t} - b_t + m_t - \frac{M_{t+1}}{P_t},$

These price levels in the denominator are the problem, therefore we should multiply both terms with the corresponding ratios as follows:

7) $w_t n_t + d_t - tax_t = c_t + (1 + r_t)^{-1} \frac{B_{t+1}}{P_t + 1} \frac{P_{t+1}}{P_t} - b_t + m_t - \frac{M_{t+1}}{P_t - 1} \frac{P_{t-1}}{P_t},$

Then, the inflation rate must be incorporated, as well as the fisher equation which are defined as:

8) $\pi_t = \frac{P_t - P_{t-1}}{p_{t-1}} = \frac{P_t}{P_{t-1}} - 1$

9) $\pi_{t+1} = \frac{P_{t+1}-P_t}{p_t} = \frac{P_{t+1}}{P_t} - 1$

Fisher equation: which related the nominal and real interest rates by introducing the inflation expectation in period t in the following period. By applying these steps, the real budget constraint is developed.

$1 + r_t = \frac{1+R_t}{1+E_t\pi_{t+1}}$;

Appendix C: First-order conditions

C.1. The first-order condition: Consumption.

10) $\frac{\partial Lt}{\partial c_t} = 0 = (1-\sigma)\frac{(C_t)^{1-\sigma-1}}{1-\sigma} + \lambda_t(-1) = C_t^{-\sigma} - \lambda_t = 0 \rightarrow \lambda_t = C_t^{-\sigma}$

C.2. The second first-order condition: Labor Supply.

11) $\frac{\partial Lt}{\partial n_t} = 0 = -\psi_n (1+\kappa)\frac{(n_t)^k}{1+k} + \lambda_t w_t = -\psi_n n_t^{\kappa} + \lambda_t w_t$

C.3. The third first-order condition: Number of Bonds households hold and that will be reimburse in the following period.

12) $\frac{\partial Lt}{\partial b_{t+1}} = 0 = -\lambda_t (1+r_t)^{-1} + \beta E_t \lambda_{t+1}$;

b_{t+1} is not in the objective function, but it appears in two budget constraints: the one this period when the representative household buy bonds, and the next period that is when it sells the bonds and gets the profits.

C.4. The fourth first-order condition: Real Money Balances that individuals desire.

13) $\frac{\partial Lt}{\partial m_t} = 0 = \psi_m m_t^{-\gamma} - \lambda_t + \beta E_t \lambda_{t+1} (1+\pi_{t+1})^{-1}$

Firstly, we substitute the first order condition of the bond.

14) $\psi_m m_t^{-\gamma} = \lambda_t - \beta E_t \lambda_{t+1}(1+\pi_{t+1})^{-1}$

Substitute by: $\beta E_t \lambda_{t+1} = \lambda_t (1 + r_{t+j})^{-1}$

15) $\psi_m m_t^{-\gamma} = \lambda_t - \lambda_t (1+r_{t+j})^{-1}(1+\pi_{t+1})^{-1}$

Then, by applying the Fisher condition, the nominal interest rate appears.

16) $(1+R_t)^{-1} = (1+r_t)^{-1}(1+E_t\pi_{t+1})^{-1}$

17) $\psi_m m_t^{-\gamma} = \lambda_t - \lambda_t(1 + R_t)^{-1}$

18) $\psi_m m_t^{-\gamma} = \lambda_t (1 - \frac{1}{1+R_t})^{-1} = \lambda_t \frac{R_t}{1+R_t}$

As: $\lambda_t = c_t^{-\sigma}$

Appendix D: Firm demand constraint

19) $y_{t+j}(i) = (\frac{P_t(i)}{P_{t+j}})^{-} y_{t+j}$

20) $Max\ E_t \sum_{j=0}^{\infty} \beta^j \eta^j (\frac{P_t(i) Y_{t+j}(i)}{P_{t+j}} - w_{t+j} n_{t+j}(i))$

By substituting equation, the first equation into the second one:

21) $Max\ E_t \sum_{j=0}^{\infty} \beta^j \eta^j (\frac{P_t(i)}{P_{t+j}} (\frac{P_t(i)}{P_{t+j}})^{-} Y_{t+j} - w_{t+j} n_{t+j}(i));$

$$Max\ E_t \sum_{j=0}^{\infty} \beta^j \eta^j ((\frac{P_t(i)}{P_{t+j}})^{1-} Y_{t+j} - w_{t+j} \cdot n_{t+j}(i))$$

Appendix E: Firms' maximization procedure

22) $Max\ E_t \sum_{j=0}^{\infty} \beta^j \eta^j ((\frac{P_t(i)}{P_{t+j}})^{1-} y_{t+j} - w_{t+j} n_{t+j}(i))$

23) $\frac{\partial Profit}{\partial P_t(i)} = 0$

24) $(1 - \theta)(\frac{P_{t+j}(i)}{P_t})^{1--1} \frac{1}{P_t} y_t - w_t \frac{\partial n_t(i)}{\partial y_t(i)} \frac{\partial y_t(i)}{\partial P_t(i)} = 0$

Appendix F: Derivatives

25) $\frac{\partial n_t(i)}{\partial y_t(i)} : 1 = f_{n_t}(i)$; this term represents the inverse of the marginal productivity of employment.

By dividing the real wage by the marginal utility of labor, the resulting equation is the real marginal cost:

26) $\frac{w_t}{f_{n_t}(i)} = mc_t$

The real total cost is the nominal cost divided by the aggregate price, as there are no more costs for firms than labor:

$$27)\ TC_t = \frac{W_t n_t}{P_t} = w_t n_t$$

By taking derivative of the real total cost with respect to production, as within the model, production and employment are related. This results in the inverse of the marginal productivity, as production only depends on employment:

$$28)\ \frac{\partial TC_t}{\partial y_t} = w_t \frac{\partial n_t}{\partial y_t} = \frac{w_t}{fn_t};\ \text{as } y_t = fn_t$$

Finally, taking derivative of firms' production with respect to the aggregate price level, the Dixit and Stiglitz demand constraint.

$$29)\ \frac{\partial Y_t(i)}{\partial P_t(i)} = -\theta \left(\frac{P_t(i)}{P_t}\right)^{-\theta-1} \frac{1}{P_t} y_t + \cdots;\ \text{consecutive periods.}$$

$$30)\ \frac{\partial Profit_t}{\partial P_t(i)} = (1-\theta)\left(\frac{P_t(i)}{P_t}\right)^{-\theta} \frac{y_t}{P_t} - mc_t(-\theta)\left(\frac{P_t(i)}{P_t}\right)^{-\theta-1} \frac{y_t}{P_t} + \beta\eta E_t \Big[(1-\theta)\left(\frac{P_t(i)}{P_t}\right)^{-\theta} \frac{y_{t+j}}{P_{t+j}} - mc_{t+j}(-\theta)\left(\frac{P_t(i)}{P_{t+j}}\right)^{-\theta-1} \frac{y_{t+j}}{P_{t+j}}\Big] + \cdots = 0$$

Appendix G: Optimal price transformation

To continue with the transformation both sides of the equation can be multiplied by $(P_t(i))$.

$$31)\ (P_t(i))E_t \sum_{j=0}^{\infty} \beta^j \eta^j \Big[(1-\theta)\left(\frac{P_t(i)}{P_t}\right)^{-\theta} \frac{y_{t+j}}{P_{t+j}} + \theta mc_{t+j}\left(\frac{P_t(i)}{P_{t+j}}\right)^{-\theta-1} \frac{y_{t+j}}{P_{t+j}}\Big] = 0(P_t(i));$$

$$E_t \sum_{j=0}^{\infty} \beta^j \eta^j \big[(1-\theta)(P_{t+j})^{-1} y_{t+j} + \theta mc_{t+j}(P_t(i))^{-1}(P_{t+j})\, y_{t+j}\big] = 0;$$

$$E_t \sum_{j=0}^{\infty} \beta^j \eta^j (1-\theta)(P_{t+j})^{-1} y_{t+j} = E_t \sum_{j=0}^{\infty} \beta^j \eta^j (-\theta) mc_{t+j} (P_t(i))^{-1} \cdot (P_{t+j})\, y_{t+j}$$

Until arriving to the final equation:

$$32)\quad P_t(i) = \frac{E_t \sum_{j=0}^{\infty} \beta^j y^j mc_{t+j} (P_{t+j})^{\theta} y_{t+j}}{-1\ E_t \sum_{j=0}^{\infty} \beta^j y^j (P_{t+j})^{\theta-1} y_{t+j}}$$

Appendix H: Log-linearization of the output equation

Log transformation/fluctuations of the output equation:

33) $Y_t = C_t + ge^{g_t}$

34) $Y = C + g$

35) $\frac{y_t - y}{y} = \frac{c_t - c}{c} + \frac{ge^{g_t} - g}{g}$

36) $\hat{y}_t = \frac{c}{y}\frac{c_t - c}{c} + \frac{g}{y}\frac{g_t - g}{g}$

Appendix I: Logs basic properties.

37) log (A*B) = log (A) + log (B)

38) log (A/B) = log (A) - log (B)

39) log A^B = B * log (A)

Appendix J: Log-linearization of the Labor Supply Function

Taking logs in both sides of the equation and using the basic properties:

40) $log\psi_n + logn_t^k = logc_t^{-\sigma} + logw_t;$

41) $log\psi_n + \kappa logn_t = -\sigma logc_t + logw_t$

Good and useful approximation: Log fluctuations with respect to steady state. In dynamic macroeconomic models, such as this Keynesian model, economists are concerned about Business Cycles.

42) $logx_t - logx \approx \frac{x_t - x}{x};$

To apply the approximation rule, compute first the stationary state:

43) $log\psi_n + \kappa logn = -\sigma logc + logw$

By subtracting both equations, results in a good approximation:

44) $(log\psi_n + \kappa logn_t) - (log\psi_n + \kappa logn) = (-\sigma logc_t + logw_t) - (-\sigma logc + logw)$

Defining new variables:

45) $logn_t - logn = \hat{n}$

46) $logw_t - logw = \hat{w}_t$

47) $logc_t - logc = \hat{c}_t$

Appendix K: Log-linearization of the optimal price equation.

Taking the optimum price equation:

48) $\hat{p}_t(i) = (1 - \beta\eta) \sum_{j=0}^{\infty} \beta^j \eta^j E_t(\widehat{mc}_{t+j} + \hat{p}_{t+j})$

The interpretation is similar to the one relating price with marginal cost. If firms could set the optimal price, they would look at the marginal cost and apply the mark-up, selling each unit at a premium. If the mark-up is 30%, selling each unit at a price 30% higher than the marginal cost. In the equation is observable the real marginal cost and the aggregate price, adding both as they are logarithms. However, if they were not, they would be multiplying each other, being the nominal marginal cost, and therefore prices reacting to it.

The thing to highlight is that not only matters the marginal cost observed in period t, but also the following expectations about future marginal costs. As a firm, they may not be able to change and adjust the price due to the Calvo's probability. If the company foresees that in the future it will face higher marginal cost, as it is not certain that it will be able to adjust the optimal price, part of the increase is applied today.

Considering the aggregate price level equation, add and subtract the same term to rearrange:

P_t: $\hat{p}_t = (1 - \eta)\hat{p}_t(i) + \eta\hat{p}_{t-1} \rightarrow \hat{p}_t = (1 - \eta)\hat{p}_t(i) + \eta\hat{p}_{t-1} - (1 - \eta)\hat{p}_t + (1 - \eta)\hat{p}_t$;

$\eta\hat{p}_t = (1 - \eta)(\hat{p}_t(i) - \hat{p}_t) + \eta\hat{p}_{t-1}$

49) $\eta(\hat{p}_t - \hat{p}_{t-1}) = (1 - \eta)(\hat{p}_t(i) - \hat{p}_t)$

By log-linearizing, the non-linear part has been removed and then, the aggregate price is a weighted average of the optimal price and the previous period's price, using the Calvo's probability as it measures price rigidity. To the optimal price it is given one minus the Calvo's probability because those companies are the ones that will be able to update the prices in that period.

- If the probability of Calvo is 0, the aggregate price would be equal to the optimal price.
- If the probability of Calvo is 1, it would be equal to the aggregate price of the previous period.

Then, the deviation of the inflation rate from its steady state is equal to the logarithmic difference of prices in period t with respect to the previous period.

$\pi_t - \pi = \hat{p}_t - \hat{p}_{t-1}$;

50) $\pi_t - \pi = \frac{1-y}{y}(\hat{p}_t(i) - \hat{p}_t)$

Combining previous two equations the desired equation results:

51) $\pi_t - \pi = \beta E_t(\pi_t - \pi) + \frac{(1-\beta y)(1-y)}{y}\widehat{mc}_t$

Appendix L: Log-linearization of the production function

Production function: $Y_t(i) = e^{z_t} n_t(i)$

Technological shock: $z_t = \rho_z z_{t-1} + \varepsilon_t^z$; where ε_t^z is a white noise. $\varepsilon_t^z \approx N(0, \sigma^2{}_{sz})$

By applying logs in both sides of the equation:

52) $logy_t = z_t + logn_t$

Steady state function:

53) $logy = 0 + logn$

Taking differences, until reaching the lineal equation:

54) $logy_t - logy = z_t + logn_t - logn$;

Appendix M: Log-linearization of the marginal cost function

By applying logs in both sides of the equation:

55) $logmc_t = logw_t - log\mathfrak{f}_{n_t}$

Steady state function:

56) $logmc = logw - log\mathfrak{f}n$

Taking differences, until reaching the lineal equation:

57) $logmc_t - logmc = logw_t - logw - log\mathfrak{f}_{n_t} + log\mathfrak{f}n$

Appendix N: IRFs 1% Government Spending Shock

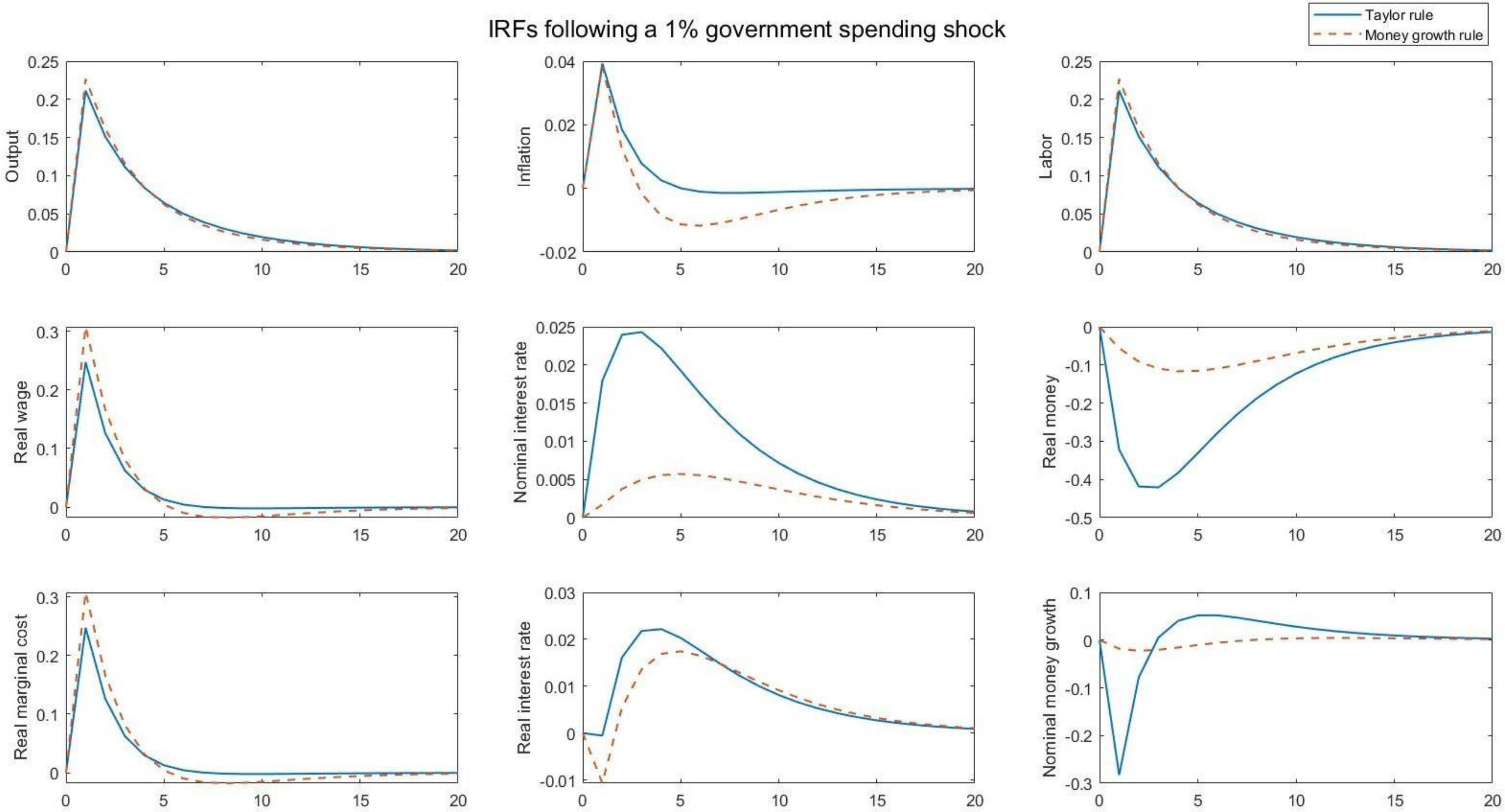

Figure 3 IRFs 1% Government Spending Shock

Appendix O: IRFs 1% Monetary Policy Shock

With a money-growth rule, the positive shock is expansionary: it creates an increase in output, a decrease in the real interest rate, and an increase in inflation. If changing the rule to an interest rate one, a positive monetary shock will be contractionary. Both can be seen in Figure 4.

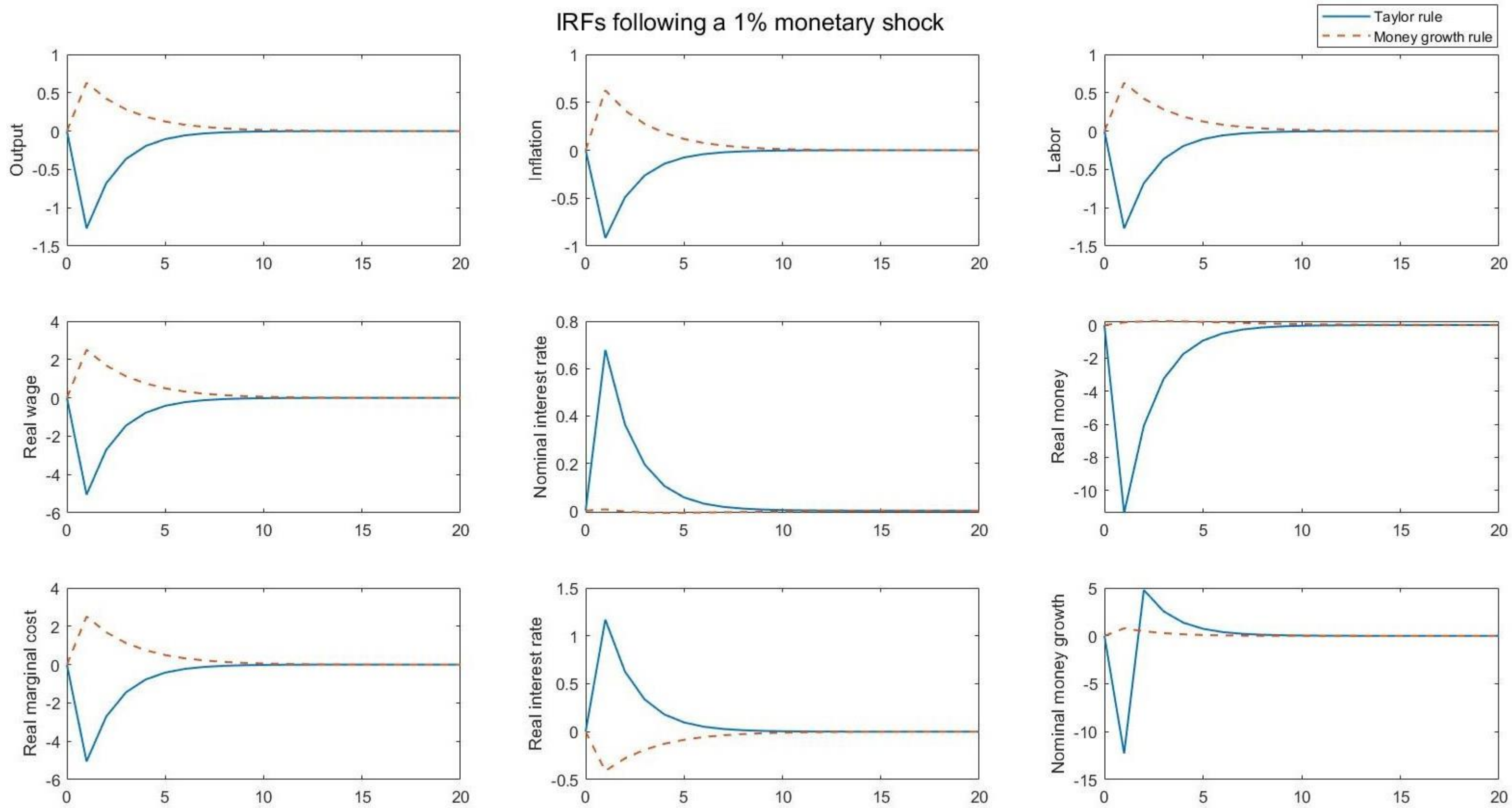

Figure 4 IRFs 1% Monetary Policy Shock