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Journal of Economic Theory 203 (2022) 105468

JOURNAL OF Economic Theory

www.elsevier.com/locate/jet

The measurement of the value of a language ☆

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Received 8 February 2021; final version received 17 April 2022; accepted 18 April 2022
Available online 28 April 2022

Abstract

We address the problem of assessing the value of a language. We consider a stylized model of multilingual societies in which we introduce axioms formalizing the principles of impartiality, monotonicity, invariance and consistency. We show that the combination of these axioms characterizes a family of communicative benefit functions which assign a value to each language in the society. The functions within the family involve a two-step procedure. First, they identify the groups of agents that can communicate in each language. Second, each group is assigned an aggregate (size-dependent) value, which is evenly divided among the languages in which the group can communicate. Our novel approach could be useful in a wide range of empirical applications and policy decisions.

JEL classification: C72; D62; D63; Z13

Keywords: Value of a language; Communicative benefits; Measure; Axioms; Characterization

☆ We thank Marzena Rostek (editor of this journal), the anonymous associate editor and two anonymous referees for helpful comments and suggestions. The first author acknowledges the financial support from the Spanish Government through grant PGC2018-093542-B-I00 funded by MCIN/AEI/10.13039/501100011033 and by “ERDF A way of making Europe”. The second author acknowledges the Spanish Government through grant PID2020-115011GB-I00, funded by MCIN/AEI/10.13039/501100011033. Funding for publishing: Universidad Pablo de Olavide/CBUA.

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https://doi.org/10.1016/j.jet.2022.105468

1. Introduction

The latest version of the Ethnologue database (www.ethnologue.com) contains more than seven thousand distinct languages spoken all over the world. As there are only a few hundred nations, it follows that a large number of them, if not most, are multilingual. The distribution of linguistic skills in multi-lingual societies is crucial to explain opportunities and challenges, both at the individual and societal level. Moreover, the linguistic landscape is not static and could be altered by the presence of economic and cultural incentives to acquire languages in addition to one's mother tongue.

Economists and social scientists alike have long been concerned with studying the impact of acquiring foreign languages on economic outcomes (see, for instance, Ginsburgh and Weber (2020) and the literature cited therein). From the individual perspective, each agent must evaluate the benefits of learning other languages and weigh them against the cost of language acquisition. Proficiency in languages has important consequences on earnings. Job opportunities are more often open to applicants who speak several languages, though not all languages are identical in that respect. Another important aspect of language acquisition for individuals is the challenges of migration. Once in the new country, or even prior to that, the migrant might have to learn (or at least improve the knowledge of) the local language to get a job, and thus be faced with a learning decision. The importance of linguistic skills for migrants' labor-markets is confirmed by the literature on patterns of language acquisition by immigrants in various countries.[1]

Linguistic policies in multilingual societies, such as the selection of official languages (e.g., Pool, 1991) or the choice of the language of school instruction (e.g., Ginsburgh and Weber, 2020), are of great importance and might have profound economic and societal implications. A decision on which official documents, collective goods or public services are offered in each language, and the subsidization (partial or full) of the acquisition of some languages may impact the patterns of language acquisition and, consequently, the economic development of the society.

Both the individual decision of learning a new language and the implementation of a linguistic policy require a proper cost-benefit analysis, which relies on the measurement of the benefits associated to a given language (or set of languages).

The formal approach to the benefits of language learning was initially developed in the seminal paper by Selten and Pool (1991), which subsumes both private monetary rewards and 'pure communicative' benefits of exposure and access to different cultures. The more people an individual can speak with, the more advantageous the learning of other languages may seem. As Lazear (1999) points out, "the incentives are greater for each individual to learn the majority language when only a few persons in the country speak his or her native language." The benefits could be related to expanded employment opportunities and higher monetary returns, but being immersed into a different culture and gaining unfiltered access to its history, arts, and literature in the original language could be viewed as important by some. To make the approach

[1] Dustmann and Fabbri (2003) showed that, in the UK, language proficiency has a positive effect on employment probabilities, and lack of English fluency leads to earning losses. Bleakley and Chin (2004) found a significant positive effect of English proficiency on wages among adults who immigrated to the United States as children. Ginsburgh and Prieto-Rodriguez (2007) showed that a second language (in most cases, English) raises wages in the range of five to fifteen percent in Austria, Finland, France, Germany, Greece, Italy, Portugal and Spain. Albouy (2008) found substantial wage differentials and, therefore, incentives, for a French-speaking Canadian to learn English, while the reverse is not true. For a survey of this growing brand of the literature, the reader is referred to Chiswick and Miller (2014).

in Selten and Pool (1991) operational, Church and King (1993) examined a model where the communicative benefits are simply represented by the number of people an individual can communicate with.[2]

The growing attention to the nature of the economic trade-off between benefits and costs of language acquisition, and the intuitive appeal of the notion of communicative benefits, leave open the search for fundamentals on which this notion can be built on. The purpose of this paper is to axiomatically analyze the measurement of the value of a language, interpreted as evaluating the communicative benefits it yields. We acknowledge that a language might be valuable for many reasons and we certainly do not pretend to capture all of them. Our aim is to focus on its role to permit interactions and economic activity, thus leaving aside its "pure consumption value", or its diversity value. Consequently, we approach the problem in a purely objective fashion and without reference to individual subjective views about languages. This will become clear in the axioms considered in our model.

Our analysis can be done from the individual perspective (i.e., how much benefit an agent obtains from knowing each language) or from the social one (i.e., how much benefit society as a whole obtains from the knowledge of each language). We adopt this second approach in our main analysis, but we will discuss later how the results can be adapted to the individual learning decisions.

In our model, a linguistic landscape of a society is described by a matrix with dichotomous entries, depending on whether the corresponding agent (row) speaks the corresponding language (column) or not. The aim is to derive the communicative benefits of each of the languages in society. Instead of assuming a specific functional form, as it is done in the existing literature, we approach the problem by introducing several axioms that formalize appealing principles from a normative perspective. Our first two axioms refer to the principle of *impartiality*, one of the most basic principles in the theory of justice (e.g., Moreno-Ternero and Roemer, 2006). A *monotonicity* axiom and an *invariance* axiom reflect how the communicative benefits should react to certain changes of the linguistic landscape. We also introduce a *consistency* axiom, another notion with a long tradition of use in normative economics (e.g., Thomson, 2012).[3] Our main result states that the combination of these five axioms characterizes a family of communicative benefit functions, assigning to each language its value by means of a two-step procedure: First, it identifies the groups of agents that can communicate in that language. Second, each group is assigned an aggregate communicative size-dependent value, which is evenly divided among the languages of communication of this group.

We believe our work could be useful in a variety of policy implications. For instance, in a multilingual society, public authorities might be interested in promoting multilingualism upon subsidizing the acquisition of one (or some) specific language(s). Which should be the chosen language(s)? We believe this decision could be driven by the communicative benefit functions derived in this paper, which allows us to rank all the existing languages, as well as to evaluate linguistic policies, as we shall show in Section 4. In a cost-benefit framework, one could actually use the cost per unit of (communicative) benefit gained, akin to what the so-called cost-per-QALY-gained concept conveys in the economic evaluation of health care programs (e.g., Neumann et al., 2014).[4]

[2] See also Ginsburgh et al. (2007), Gabszewicz et al. (2011) and Athanasiou et al. (2016).

[3] When formally introducing each of the axioms in Section 3, we shall discuss in detail the value judgments they formalize, as well as some of the implications they convey.

[4] See Hougaard et al. (2013) for an axiomatic characterization of QALYs as a measure of health outcomes.

Moreover, our analysis has potential empirical applications. Our family of communicative benefit functions could be used, for instance, to measure the value of different languages in the (pre and post-Brexit) European Union (e.g., Ginsburgh et al., 2017), the choice of official languages (e.g., in South Africa, see Ginsburgh and Weber, 2011), as well as in multilingual countries where the linguistic policies were linked to economic development (Easterly and Levine, 1997) or devastating conflicts (Castañeda-Dower et al., 2017). Our measures might also be relevant to study the welfare effect of language barriers in communication (e.g., Giovannoni and Xiong, 2019) or the effects of communicative benefits in models of language dynamics (e.g., Abrams and Strogatz, 2003).

The rest of the paper is organized as follows. In Section 2, we introduce the notation and definitions. In Section 3, we present our axioms and the characterization result. In Section 4, we develop some applications of our result. Section 5 offers some additional insights from our result and, finally, Section 6 concludes. The proofs are relegated to the Appendix A.

2. Notation and definitions

Let $\mathcal{N}$ be the universal set of agents and $\mathcal{L}$ be the universal set of languages. Both sets can be finite or infinite. A particular situation is a triple (N, L, A), where $N \subseteq \mathcal{N}$ is a finite set of agents, $L \subseteq \mathcal{L}$ is a finite set of languages, and A is a $0-1$ matrix that summarizes the multilingual reality of the society N over the set of languages L. Formally, $a_{il} = 1$ if agent $i \in N$ speaks language $l \in L$, and $a_{il} = 0$ otherwise. We thus assume that there is no distinction between speaking a language well or not, or between native and non-native languages. Let $\mathcal{S}$ be the set of possible situations. We define, for each situation $(N, L, A) \in \mathcal{S}$, the set of speakers of a given language $l \in L$ by $N_A(l)$, i.e., $N_A(l) = \{i \in N : a_{il} = 1\}$. Then, a set of agents M can communicate with a given language l if $M \subseteq N_A(l)$. We denote the collection of all coalitions that can communicate with languages in L by $N_A(L)$, i.e., $N_A(L) = \{M \subseteq N :$ there exists $l \in L$ for which $M \subseteq N_A(l)\}$. We also define, for each situation $(N, L, A) \in \mathcal{S}$, the indicator function that describes if group $M \subseteq N$ can communicate in language $l \in L$ or not as $d^l_A(M)$, i.e., $d^l_A(M) = 1$ if $M \subseteq N_A(l)$, and $d^l_A(M) = 0$ otherwise. We denote the number of languages in which group $M \subseteq N$ can communicate by $d^L_A(M)$, i.e., $d^L_A(M) = \sum_{l \in L} d^l_A(M)$.

Given a pair of situations $(N, L, A), (N, L', A') \in \mathcal{S}$, with $L \cap L' = \emptyset$, we define the union of them as a new situation $(N, L \cup L', A \cup A')$ in the natural way. For each situation $(N, L, A) \in \mathcal{S}$, and for each $L' \subset L$, we denote the situation restricted to L' by $(N, L', A|_{L'})$, where $A|_{L'}$ is the resulting matrix from A after dismissing all the columns from $L \setminus L'$.

Given a situation $(N, L, A) \in \mathcal{S}$, we define a communicative benefit function $\phi_{(N,L,A)} : L \to \mathbb{R}_+$ that associates, for each language $l \in L$, a non-negative real number indicating the communicative benefits of this language in this situation.[5] We define $\phi \equiv \bigcup_{(N,L,A) \in \mathcal{S}} \phi_{(N,L,A)}$. For normalizing purposes, we assume that there exists a situation $(N, L, A) \in \mathcal{S}$ and a language $l \in L$ such that $\phi_{(N,L,A)}(l) = 0$. Let $\Phi : \mathcal{S} \to \mathbb{R}_+$ be such that $\Phi(N, L, A) = \sum_{l \in L} \phi_{(N,L,A)}(l)$ for each $(N, L, A) \in \mathcal{S}$. This function Φ indicates the total communicative benefits of all languages in a society.[6]

5 We denote by $\mathbb{N}$ the set of natural numbers, by $\mathbb{R}$ the set of real numbers, and by $\mathbb{R}_+$ the set of non-negative real numbers.

6 Note that we assume an unweighted aggregation of each language's communicative benefits, thus reflecting an impartiality judgment to be properly formalized next.

3. Axioms and characterization

Our goal is to derive communicative benefit functions axiomatically. For that matter, we impose some axioms that we find compelling.

Our first axiom, *Anonymity*, is a standard formalization of the principle of impartiality, which refers to the fact that the identity of each agent should not matter in the evaluation. To define it formally, let $\Pi^{\mathcal{N}}$ be the class of bijections from $\mathcal{N}$ into itself. For each $(N, L, A) \in \mathcal{S}$, and each $\pi \in \Pi^{\mathcal{N}}$, let $\pi(N, L, A) = (\pi(N), L, (a_{\pi(i)l})_{(i,l)\in N\times L})$, where $\pi(N) = \{i \in \mathcal{N} : \pi^{-1}(i) \in N\}$.

***Anonymity*:** For each $(N, L, A) \in \mathcal{S}$, each $\pi \in \Pi^{\mathcal{N}}$ and each $l \in L$, $\phi_{(N,L,A)}(l) = \phi_{\pi(N,L,A)}(l)$.

Anonymity requires that the name of each of the agents (an ethically irrelevant information) should be excluded from the evaluation process. The next axiom, *Equal Treatment of Equal Languages*, establishes a similar idea for languages expressed in very weak terms.

***Equal Treatment of Equal Languages*:** For each $(N, L, A) \in \mathcal{S}$ and each pair $l, l' \in L$ such that $a_{il} = a_{il'}$ for each $i \in N$, $\phi_{(N,L,A)}(l) = \phi_{(N,L,A)}(l')$.

Equal Treatment of Equal Languages implies that if we have a situation in which two languages have the same set of speakers, then their communicative benefits are the same. Observe that this property is weaker than the classic Neutrality property, which says that a permutation of languages permutes analogously the languages' communicative benefits.[7]

The previous two axioms prevent asymmetric treatment of equal (according to the model) agents and languages. There might be convincing reasons to do otherwise. For instance, one may want to advocate placing a higher value on languages spoken, say, by native people if they have suffered past injustices. This is, obviously, an interesting issue, but needs to be addressed with a more general model (including dynamic aspects) than the one we present here.[8]

To introduce the next axiom, we define the following concept of communicational inclusion of situations: for each pair $(N, L, A), (N, L', A') \in \mathcal{S}$, we say that $(N, L, A) \subseteq (N, L', A')$ whenever for each $l \in L$, there exists $l' \in L'$ such that $a'_{il'} = 1$ for each $i \in N$ with $a_{il} = 1$.[9] Then, the axiom of *Communicational Inclusion Monotonicity* says the following:

***Communicational Inclusion Monotonicity*:** For each pair $(N, L, A), (N, L', A') \in \mathcal{S}$ such that $(N, L, A) \subseteq (N, L', A')$, $\Phi(N, L', A') \geq \Phi(N, L, A)$.

The *Communicational Inclusion Monotonicity* axiom conveys two ideas of weak monotonicity (adding languages is weakly good for the total communicative benefits of society, and adding speakers to a language is also weakly good for the same purpose) and an idea of supermodularity (it is not worse for the total communicative benefits of society to have a common language than a set of languages whose union of speakers coincides with the speakers of the common language). The following example illustrates this axiom.

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7 The formal definition of Neutrality is: For each $(N, L, A) \in \mathcal{S}$, each possible bijection μ from $\mathcal{L}$ to itself, and each $l \in L$, $\phi_{(N,L,A)}(l) = \phi_{\mu(N,L,A)}(\mu(l))$, where $\mu(N, L, A) = (N, \mu(L), (a_{i\mu(l)})_{(i,l)\in N\times L})$, with $\mu(L) = \{l \in \mathcal{L} : \mu^{-1}(l) \in L\}$. Note that, although Neutrality implies *Equal Treatment of Equal Languages*, the opposite is not true.

8 We briefly outline in Section 6 a possible extension of our model in this direction.

9 Note that it is possible that $(N, L, A) \subseteq (N, L', A')$ with $L \nsubseteq L'$ and even with $L' \subset L$.

Example 1. Let $N = \{1, 2, 3, 4, 5, 6\}$, $L = \{a, b, c\}$, $L^2 = \{a, b, c, d\}$, $L^3 = \{e, c\}$, and

$$A = \begin{array}{c} \\ 1\\2\\3\\4\\5\\6 \end{array}\begin{array}{c} \begin{matrix} a & b & c \end{matrix} \\ \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \end{array} \quad A^1 = \begin{array}{c} \\ 1\\2\\3\\4\\5\\6 \end{array}\begin{array}{c} \begin{matrix} a & b & c \end{matrix} \\ \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \end{array}$$

$$A^2 = \begin{array}{c} \\ 1\\2\\3\\4\\5\\6 \end{array}\begin{array}{c} \begin{matrix} a & b & c & d \end{matrix} \\ \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{array} \quad A^3 = \begin{array}{c} \\ 1\\2\\3\\4\\5\\6 \end{array}\begin{array}{c} \begin{matrix} e & c \end{matrix} \\ \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \end{array}.$$

Note that the only difference between situation (N, L, A) and situation (N, L, A^1) is that agent 1 speaks language c in the latter, but not in the former. Thus, $(N, L, A) \subseteq (N, L, A^1)$ and, by *Communicational Inclusion Monotonicity*, $\Phi(N, L, A^1) \geq \Phi(N, L, A)$. Similarly, the only difference between situation (N, L, A) and situation (N, L^2, A^2) is that a new language d is added in the latter. Thus, $(N, L, A) \subseteq (N, L^2, A^2)$ and, by *Communicational Inclusion Monotonicity*, $\Phi(N, L^2, A^2) \geq \Phi(N, L, A)$. Finally, the only difference between situation (N, L, A) and situation (N, L^3, A^3) is that languages a and b are replaced by language e, and it turns out that all agents speaking one of the former languages (a and b) in the first situation speak the new language e in the last situation. Thus, $(N, L, A) \subseteq (N, L^3, A^3)$ and, by *Communicational Inclusion Monotonicity*, $\Phi(N, L^3, A^3) \geq \Phi(N, L, A)$.

Communicational Inclusion Monotonicity has important and non-trivial implications. For instance, given a society N, if everyone can speak a common language, then this is the first-best scenario according to the total communicative benefits of the society and it does not matter who else can speak any other language. Formally, if $(N, L, A) \in \mathcal{S}$ is such that there exists $l \in L$ with $a_{il} = 1$ for all $i \in N$, then $(N, L', A') \subseteq (N, L, A)$ for any $(N, L', A') \in \mathcal{S}$ and, thus, by *Communicational Inclusion Monotonicity*, $\Phi(N, L, A) \geq \Phi(N, L', A')$. Somewhat related, suppose we want to compare a situation in which there exists one common language with another in which this common language coexists with other languages. In that context, by the above argument, *Communicational Inclusion Monotonicity* implies that the total communicative benefits of both situations are exactly the same (note that the axiom can be applied in both directions). In other words, what matters is whether agents can communicate or not. The number of languages in which that occurs is irrelevant.

The next axiom is called *Irrelevance of Non Speakers*. It says that the communicative benefits of a new language in a situation should not depend on the agents that do not speak that language.

***Irrelevance of Non Speakers*:** For each trio $(N, L, A), (N, L, A'), (N, \{l\}, A'') \in \mathcal{S}$, with $l \notin L$, such that $a_{ik} = a'_{ik}$ for each $i \in N_{A''}(l)$ and each $k \in L$,

$$\phi_{(N,L\cup\{l\},A\cup A'')}(l) = \phi_{(N,L\cup\{l\},A'\cup A'')}(l).$$

This axiom is reminiscent of the classical axiom of Independence of Irrelevant Alternatives in social choice theory (e.g., Arrow, 1951). It excludes the possibility that information about agents who do not speak a language influence the value of such a language, as Independence of Irrelevant Alternatives excludes information about other alternatives when deciding the social ranking about a pair of alternatives. The following example illustrates this axiom.

Example 2. Let $N = \{1, 2, 3, 4, 5, 6\}$, $L = \{a, b, c\}$, and

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}\begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \end{array} \quad A' = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}\begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \end{array} \quad A'' = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}\begin{array}{c} l \\ \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} \end{array}$$

$$A \cup A'' = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}\begin{array}{c} \begin{array}{cccc} a & b & c & l \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{array} \quad A' \cup A'' = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}\begin{array}{c} \begin{array}{cccc} a & b & c & l \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{array}.$$

Let $l \notin L$. Then, the situation $(N, \{l\}, A'')$ reflects that, except for agents 1 and 3, all other agents in N speak language l. Observe that all those agents (2, 4, 5 and 6) have the same language proficiency (of languages a, b and c) in situations (N, L, A) and (N, L, A'). Thus, by *Irrelevance of Non Speakers*, $\phi_{(N,L\cup\{l\},A\cup A'')}(l) = \phi_{(N,L\cup\{l\},A'\cup A'')}(l)$.

The final axiom is called *Null Agent Consistency*. It says that the addition of an agent that does not speak any language has no impact on the total communicative benefits of the situation. This is the only axiom of our set of properties that refers to populations of different sizes.

***Null Agent Consistency*:** For each pair $(N, L, A), (N \cup \{i\}, L, A') \in \mathcal{S}$ such that $i \notin N$, and for each $l \in L$, $a'_{il} = 0$ and $a'_{jl} = a_{jl}$ for each $j \in N$,

$$\Phi(N \cup \{i\}, L, A') = \Phi(N, L, A).$$

This axiom implies that the communicative benefit function should take into account the agents that speak the language, but not the remaining ones. Thus, *Null Agent Consistency* is compatible with functions that are based, for example, on the absolute number of speakers of each language. It is incompatible though with functions that include relative measures, such as, for instance, the proportion of individuals speaking each language in society.

We now describe a specific family of communicative benefit functions. Each of them assigns a value to each language l in a situation (N, L, A) by means of the following procedure. First, it focuses only on the groups of agents that can communicate between them in this language: these are the subgroups M such that $M \subseteq N_A(l)$. Second, each group M has a total communicative value that depends on its size, $\omega(|M|)$, and this value is divided evenly between the languages in which the group can communicate. Then, the communicative benefits of language l for the subgroup M are $\omega(|M|) \cdot \frac{d_A^l(M)}{d_A^L(M)}$. Finally, the total communicative benefits of language l are the sum of these values for all coalitions that can communicate in this language.[10]

Definition 1. A communicative benefit function ϕ belongs to the class $\mathcal{F}$ if there exists a mapping $\omega : \mathbb{N} \to \mathbb{R}_+$ such that

$$\phi_{(N,L,A)}(l) = \sum_{M \subseteq N,\, M \neq \emptyset} \left(\omega(|M|) \cdot \frac{d_A^l(M)}{d_A^L(M)} \right),$$

for each $(N, L, A) \in \mathcal{S}$ and each $l \in L$.

The members of this class of communicative benefit functions differ on the weights of each subgroup size, represented by the mapping ω, for which the unique restriction is that they should be non–negative. For each mapping ω, we shall denote by ϕ^ω the communicative benefit function within class $\mathcal{F}$ associated with ω. Observe that if $\phi = \phi^\omega$, then

$$\Phi^\omega(N, L, A) = \sum_{l \in L} \phi^\omega_{(N,L,A)}(l) = \sum_{M \subseteq N_A(L),\, M \neq \emptyset} \omega(|M|),$$

for each $(N, L, A) \in \mathcal{S}$.

The main result of the paper states that the set of axioms introduced above characterizes the class of communicative benefit functions $\mathcal{F}$.

Theorem 1. *A communicative benefit function ϕ satisfies Anonymity, Equal Treatment of Equal Languages, Communicational Inclusion Monotonicity, Irrelevance of Non Speakers and Null Agent Consistency if and only if it belongs to the class $\mathcal{F}$.*

Theorem 1 is a tight result, as the next proposition states.

Proposition 1. *Anonymity, Equal Treatment of Equal Languages, Communicational Inclusion Monotonicity, Irrelevance of Non Speakers and Null Agent Consistency are independent axioms.*

As we have mentioned in the Introduction, we have decided to develop the model measuring the social benefits of each language, but a similar approach can be done to measure the benefits each agent obtains from each set of languages. This would imply the construction of a function $\varphi_{(N,L,A)}$ that associates for each agent $i \in N$ the communicative benefits this agent obtains from

[10] As $d_A^l(M) = 0$ for the languages in which the group M cannot communicate, we can express the total value of language l by the formula at Definition 1. Note also that, if $d_A^l(M) = d_A^L(M) = 0$, we replace $\frac{d_A^l(M)}{d_A^L(M)}$ by 0.

situation (N, L, A). A family of these functions, dubbed $\mathcal{F}^\star$, sharing the spirit of our family $\mathcal{F}$ characterized above, would be the following:

$$\varphi^{\omega}_{(N,L,A)}(i) = \sum_{i \in M \subseteq N_A(L)} \frac{\omega(|M|)}{|M|},$$

for each $(N, L, A) \in \mathcal{S}$ and each $i \in N$.

In words, first the groups in which i can communicate are identified (formally, $M \subseteq N_A(L)$ such that $i \in M$). Second, the total communicative value of each of these groups (defined by $\omega(|M|)$ as in Definition 1) is divided evenly among its members. It can be shown (with similar arguments to the ones used at the proof of Theorem 1) that a suitable adaptation of the axioms presented above would also characterize this family of individual communicative benefit functions.[11]

To conclude with this section, we provide an example to illustrate how to calculate the communicative benefits of languages and agents with the functions from families $\mathcal{F}$ and $\mathcal{F}^\star$.

Example 3. Let $N = \{1, 2, 3, 4, 5\}$, $L = \{a, b, c\}$, and

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \end{array} \quad A' = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \end{array}.$$

The following tables collect the communicative benefits that the functions from $\mathcal{F}$ and $\mathcal{F}^\star$ yield for each agent $i \in N$ and each language $l \in L$, at situations (N, L, A) and (N, L, A'), depending on the weight function ω.

l	$\phi^{\omega}_{(N,L,A)}(l)$	$\phi^{\omega}_{(N,L,A')}(l)$
a	$\frac{11}{6}\omega(1) + 4\omega(2) + \frac{7}{2}\omega(3) + \omega(4)$	$\frac{5}{6}\omega(1) + \frac{1}{2}\omega(2)$
b	$\frac{11}{6}\omega(1) + 4\omega(2) + \frac{7}{2}\omega(3) + \omega(4)$	$\frac{11}{6}\omega(1) + \frac{5}{2}\omega(2) + \omega(3)$
c	$\frac{4}{3}\omega(1) + 2\omega(2) + \omega(3)$	$\frac{7}{3}\omega(1) + 5\omega(2) + 4\omega(3) + \omega(4)$

i	$\varphi^{\omega}_{(N,L,A)}(i)$	$\varphi^{\omega}_{(N,L,A')}(i)$
1	$\omega(1) + 2\omega(2) + \frac{4}{3}\omega(3) + \frac{1}{4}\omega(4)$	$\omega(1) + \omega(2) + \frac{1}{3}\omega(3)$
2	$\omega(1) + 2\omega(2) + 2\omega(3) + \frac{1}{2}\omega(4)$	$\omega(1) + 2\omega(2) + \frac{4}{3}\omega(3) + \frac{1}{4}\omega(4)$
3	$\omega(1) + 2\omega(2) + \frac{4}{3}\omega(3) + \frac{1}{4}\omega(4)$	$\omega(1) + \frac{3}{2}\omega(2) + \omega(3) + \frac{1}{4}\omega(4)$
4	$\omega(1) + 2\omega(2) + \frac{5}{3}\omega(3) + \frac{1}{2}\omega(4)$	$\omega(1) + \frac{3}{2}\omega(2) + \omega(3) + \frac{1}{4}\omega(4)$
5	$\omega(1) + 2\omega(2) + \frac{5}{3}\omega(3) + \frac{1}{2}\omega(4)$	$\omega(1) + 2\omega(2) + \frac{4}{3}\omega(3) + \frac{1}{4}\omega(4)$

[11] Observe that $\sum_{i \in N} \varphi^{\omega}_{(N,L,A)}(i) = \sum_{l \in L} \phi^{\omega}_{(N,L,A)}(l) = \Phi^{\omega}(N, L, A)$, for each ω.

Consequently, the total values of situations (N, L, A) and (N, L, A') are

$$\Phi^\omega(N, L, A) = \sum_{l\in L} \phi^\omega_{(N,L,A)}(l) = \sum_{i\in N} \varphi^\omega_{(N,L,A)}(i) = 5\omega(1) + 10\omega(2) + 8\omega(3) + 2\omega(4),$$

and

$$\Phi^\omega(N, L, A') = \sum_{l\in L} \phi^\omega_{(N,L,A')}(l) = \sum_{i\in N} \varphi^\omega_{(N,L,A')}(i) = 5\omega(1) + 8\omega(2) + 5\omega(3) + \omega(4),$$

respectively.

We could move beyond the previous example to apply our measures to survey data. For instance, the so-called Special Eurobarometer 386 provides information about the citizens' attitudes towards foreign languages and multilingualism within the European Union (EU). It looks at the ways in which Europeans learn and use foreign languages. From here, we could potentially estimate the situation (in the parlance of our model) associated to the EU, and obtain the communicative benefits of each language and each agent therein, using a member from our family.

4. Applications

In this section, we aim to illustrate how our novel approach could be useful in a wide range of empirical applications and policy decisions.

We first present a simple model to analyze linguistic policies in dual language societies.[12] More precisely, we explore subsidizing language education in such societies. We show that, in such a setting, some interesting results (with non-trivial policy implications) are robust to considering the whole family $\mathcal{F}$ of communicative benefit functions.

We then move to scrutinize two seminal models within the existing literature on the economics of language: the selection of official languages and individual decisions for language acquisition. In both cases, (individual or government) decisions are modeled upon describing the costs and benefits of each of the existing alternatives. And the proposed benefit functions try to reflect the communicative benefits (for the individual, or the whole society) each of the alternatives yield. Typically, as we shall show, those functions are specific members of our family. We analyze whether results are robust to changes of the communicative benefit function within our family or rather depend on its particular choice. We show that, for both models, some of the conclusions that can be obtained are contingent on the selection of communicative benefit functions within the family.

4.1. Subsidizing language education

Consider a society partitioned into two non-empty disjoint linguistic groups. Denote by 1 and 2 the two languages and by N_1 and N_2 the two corresponding sets of agents speaking each (and only) one of those languages.[13] For each $i \in \{1, 2\}$, let n_i denote the size of group N_i. Thus, communication only exists within groups, but not between them. In order to improve this situation, the central government might consider two possible policies:

[12] We thank an anonymous referee for suggesting this specific application.

[13] The non-existence of bilingual agents can be dismissed and the ensuing arguments would just be easily extended.

(a) subsidizing some agents of each group to learn the other language,
(b) subsidizing some agents of both groups to learn a *neutral* third language.

To appeal to a real-life case, one may think of a dual language country, such as Canada, where some regions are English-speaking and others are French-speaking. The Canadian government could then subsidize learning French in the English-speaking provinces and English in the French-speaking provinces, or rather subsidizing learning a *neutral* third language in both provinces.[14] We will call the first option a *bilingual policy* and the second option a *monolingual policy*. Formally, a *bilingual policy* is a pair (n_{12}, n_{21}) such that n_{ij} denotes the number of subsidies allocated to group N_i to learn language j. Similarly, a *monolingual policy* is a pair (m_1, m_2) such that, for each $k \in \{1, 2\}$, m_k denotes the number of subsidies allocated to group N_k to learn the neutral language.[15]

In what follows, we show how our family of communicative benefit functions can help the government deal with this policy choice. To do that, we introduce first the concept of *policy dominance*. We say that policy S *weakly dominates* policy T, which we write as $S \succsim T$, if there is no member of $\mathcal{F}$ assigning a strictly higher increase in the total communicative benefits of the society with the application of T than with the application of S.[16] We say that policy S *strictly dominates* policy T, which we write as $S \succ T$, if $S \succsim T$ but not $T \succsim S$.

As the following example illustrates, $\succsim$ only induces a partial ordering.

Example 4. Let $n_1 = n_2 = 10$. Let S be the bilingual policy defined by $n_{12} = n_{21} = 5$, and let T denote the monolingual policy defined by $m_1 = m_2 = 8$. Let $\phi^\omega \in \mathcal{F}$ be such that $\omega(2) = 1$ and $\omega(i) = 0$ for each $i \neq 2$. Finally, let $\phi^{\omega'} \in \mathcal{F}$ be such that $\omega'(16) = 1$ and $\omega'(i) = 0$ for each $i \neq 16$. Then, according to ϕ^ω, S increases the total communicative benefits by 75, whereas T increases the total communicative benefits by 64. However, according to $\phi^{\omega'}$, S leaves total communicative benefits unchanged, whereas T increases the total communicative benefits by 1. Thus, neither $S \succsim T$ nor $T \succsim S$.

As shown next, in spite of not having a complete ordering of policies, we are able to deduce some interesting policy comparisons.

First, two almost trivial comparisons. On the one hand, a bilingual policy weakly dominates all other bilingual policies with weakly less subsidies in each dimension. And this domination becomes strict when there are strictly more subsidies in the former policy, provided not all agents from any group are subsidized in the latter policy. Formally,

FACT 0a: *If $n'_{12} \leq n_{12}$ and $n'_{21} \leq n_{21}$, then $(n_{12}, n_{21}) \succsim (n'_{12}, n'_{21})$. If, additionally, at least one of the inequalities is strict, and neither $n'_{12} = n_1$ nor $n'_{21} = n_2$, then $(n_{12}, n_{21}) \succ (n'_{12}, n'_{21})$.*

[14] Another somewhat related case is the so-called 3 ± 1 language policy, successfully implemented in India in the 1960s. English and Hindi shared the status of All-Union languages. The third language was the language of the state in which each Indian was living. This meant that some (those living in states with Hindi as official language) only had to learn $3 - 1 = 2$ languages, whereas others (those whose mother tongue was neither the state language nor one of the All-Union languages) had to learn $3 + 1 = 4$ languages (see Laitin, 1989). Similar policies were also implemented later in other countries such as Nigeria, Senegal or Kazakhstan.

[15] We assume subsidies always guarantee learning the language, which naturally implies $n_{12} \leq n_1$, $n_{21} \leq n_2$, $m_1 \leq n_1$ and $m_2 \leq n_2$.

[16] This concept of policy dominance may depend on the society (n_1, n_2) in which policies are applied. However, for ease of exposition, we shall avoid this dependence in the notation.

Similarly, a monolingual policy weakly dominates all other monolingual policies with less subsidies in each dimension. Again, this domination becomes strict when there are strictly more subsidies in the former policy, provided the number of subsidies in each group in that policy is strictly positive. Formally,

FACT 0b: *If $m_1' \leq m_1$ and $m_2' \leq m_2$, then $(m_1, m_2) \succsim (m_1', m_2')$. If, additionally, at least one of the inequalities is strict, and $m_1 m_2 > 0$, then $(m_1, m_2) \succ (m_1', m_2')$.*

The previous (monotonicity) comparisons are not extremely interesting because more subsidies in both dimensions render the subsequent policy more expensive. Thus, we focus on more interesting comparisons. First, suppose that the government has decided to adopt a bilingual policy and that extra funding becomes available to subsidize one more agent (to learn the language of the other group). Then, as the next fact states, this agent should be picked from the minoritarian group.

FACT 1: $(n_{12} + 1, n_{21}) \succsim (n_{12}, n_{21} + 1) \Leftrightarrow n_1 \leq n_2$.

Fact 1 establishes that, with bilingual policies, it is unambiguously better, from the communicative benefits perspective, to subsidize an agent from the minoritarian group rather than an agent from the majoritarian group. The argument is simple: learning the language of the other group allows to create communication between the chosen agent and all the agents from the other group. Thus, the bigger the other group is, the higher the added communicative benefits. As stated next, it is possible to establish a similar result for monolingual policies.

FACT 2: $(m_1 + 1, m_2) \succsim (m_1, m_2 + 1) \Leftrightarrow m_1 \leq m_2$.

Fact 2 establishes that, with monolingual policies, it is unambiguously better, from the communicative benefits perspective, to evenly subsidize the learning of a new language to the two groups. Observe that the sizes of the two original groups are irrelevant for this fact. Again, the argument is simple: learning the neutral language allows an agent to have new communication channels, provided this neutral language is also learnt by agents from the other group; thus, it is better to evenly subsidize the learning of this new language in both groups, rather than concentrating subsidies on only one group.

Although our family of communicative benefit functions only cares about the efficiency of the communication among agents, thus ignoring equity aspects, both Facts 1 and 2 convey an equity flavor in the distribution of the subsidies. On the one hand, as just mentioned, Fact 2 suggests an equal distribution of the subsidies across groups. On the other hand, Fact 1 goes one step further defending to prioritize the subsidies to the agents from the minoritarian group.[17]

The next fact illustrates that our family not only serves to unambiguously compare policies from the same type, but also policies from both types. For instance, if a bilingual and a monolingual policy have the same number of overall subsidies, then the former weakly dominates the latter. And this domination becomes strict if the policies do not subsidize all agents.

[17] Nevertheless, as will be discussed later, the minoritarian group might oppose that policy because it might threaten its native culture (eventually making it disappear in the long run).

FACT 3: *If $m_1 + m_2 = n_{12} + n_{21}$, then $(n_{12}, n_{21}) \succsim (m_1, m_2)$. If, additionally, $0 < m_1 + m_2 < n_1 + n_2$, then $(n_{12}, n_{21}) \succ (m_1, m_2)$.*

Fact 3 establishes that it is always better to implement a bilingual policy than a monolingual policy with the same number of learners. The underlying argument is the following: with a monolingual policy, any learner of the neutral language can only communicate with the members from the other group that also learned this neutral language. However, if the agent has learnt the language of the other group, which is the case with a bilingual policy, then she can also communicate with the remaining members of the other group. Therefore, if the cost of subsidizing the learning of a new language is the same across both policies, our family of communicative benefit functions clearly favors bilingual policies. Note that Fact 2 had established that the best way to implement a monolingual policy is to divide evenly the learners of the new language between the two original groups. Fact 3 shows that, even in that case, it is always better to implement any bilingual policy with the same number of learners.

Nevertheless, it is always possible to add enough subsidies to the monolingual policy to revert the above-mentioned domination.[18] The minimal number of additional subsidies to reach that goal is what we call the *monolingual premium*. More precisely, for each bilingual policy (n_{12}, n_{21}) and group sizes (n_1, n_2), we define $k((n_{12}, n_{21}), (n_1, n_2))$ as the minimal number for which there exists a monolingual policy (m_1, m_2), with $m_1 + m_2 = k((n_{12}, n_{21}), (n_1, n_2))$, such that (n_{12}, n_{21}) does not strictly dominate (m_1, m_2) in a society with group sizes (n_1, n_2). Thus, $l((n_{12}, n_{21}), (n_1, n_2)) = k((n_{12}, n_{21}), (n_1, n_2)) - (n_{12} + n_{21})$ is the number of extra learners of the neutral language in a monolingual policy that are needed to compensate with respect to the bilingual policy (n_{12}, n_{21}) in a society with group sizes (n_1, n_2). We refer to $l((n_{12}, n_{21}), (n_1, n_2))$ as the *monolingual premium*.[19] The next fact establishes the monotonicity of the monolingual premium with respect to the size of each group.

FACT 4: *If $n_{12} > 0$ and $n_{21} = 0$, then $l((n_{12}, n_{21}), (n_1, n_2))$ is strictly increasing on n_2 and is constant on n_1. If $n_{21} > 0$ and $n_{12} = 0$, then $l((n_{12}, n_{21}), (n_1, n_2))$ is strictly increasing on n_1 and constant on n_2. If $n_{12} > 0$ and $n_{21} > 0$, then $l((n_{12}, n_{21}), (n_1, n_2))$ is strictly increasing on n_1 and n_2.*

We explain the first of the three statements (the other two are along the same lines). Whereas Fact 3 established that subsidizing a certain number of agents from group 1 to learn the neutral language is worse than subsidizing them to learn the language of group 2, Fact 4 establishes that this loss (associated to a monolingual policy) becomes larger as group 2 becomes larger, but remains constant with the size of group 1. The reason for the former is that learning the language of the other group contributes with more efficiency gains when the size of the other group is larger and, therefore, more learners of the neutral language are needed to compensate that the government would opt for a monolingual policy. The reason for the latter is that learning the language of the other group does not increase the communication options with the members of your own group.

[18] For instance, the monolingual policy (m_1, m_2) such that $m_1 = n_1$ and $m_2 = n_2$ is not strictly dominated by any bilingual policy (n_{12}, n_{21}).

[19] A similar analysis can be done if $k((n_{12}, n_{21}), (n_1, n_2))$ is defined as the minimal number such that there exists a monolingual policy (m_1, m_2) with $m_1 + m_2 = k((n_{12}, n_{21}), (n_1, n_2))$ and $(m_1, m_2) \succ (n_{12}, n_{21})$, in a society with group sizes (n_1, n_2).

To conclude, we deduce an interesting feature from all the previous facts, which builds onto the following two items:

- By Fact 3, bilingual policies dominate monolingual policies with the same number of subsidies. Also, by Fact 1, fixing a number of subsidies, the optimal bilingual policy in terms of communicative benefits is to concentrate these subsidies on the minoritarian group. Then, it can be deduced that, from the communicative benefits perspective, and assuming that all subsidies are equally expensive, the optimal solution is clear: select the bilingual policy in which agents from the minoritarian group learn the language from the majoritarian group. Formally, if $n_1 \leq n_2$, the optimal policy with c subsidies is the bilingual policy $(n_{12}, n_{21}) = (c, 0)$.
- By the first statement of Fact 4, $l((c,0),(n_1,n_2)) < l((c,0),(n_1,n_2+1))$ and $l((c,0),(n_1,n_2+1)) = l((c,0),(n_1-1,n_2+1))$. Thus, $l((c,0),(n_1,n_2)) < l((c,0),(n_1-1,n_2+1))$. This result holds for any group sizes and, in particular, when $n_1 \leq n_2$.

Joining the two items above, we can conclude that the extra cost for adopting a monolingual policy with respect to the optimal bilingual policy is higher when polarization is lower.[20] The reason behind this conclusion is the following: as the optimal bilingual policy only subsidizes agents from the minoritarian group, its impact (in terms of communicative benefits) is bigger when the majoritarian group is bigger and is not affected by the size of the minoritarian group. Furthermore, by Fact 2, the impact of the monolingual policies is not affected by the sizes of the groups. It thus becomes clear that a monolingual policy with more subsidies is necessary to compensate the bigger impact of an optimal bilingual policy when polarization is lower.

This conclusion is somewhat related to what we mentioned in Footnote 17. We argued there that a bilingual policy could create political conflicts because the minoritarian group can feel that its native culture is discriminated and that, eventually, might disappear. And the more similar the sizes of both groups, the higher the risk for political conflict because both groups would feel more unfair the intentions of imposing the other group's language as the prioritarian one. Therefore, the decision of opting for a monolingual policy to avoid political conflicts would be a more attractive decision for highly polarized societies. However, as we have seen, apart from this political reason, there is an economic one to hope to find more monolingual policies in highly polarized societies: the monolingual premium is lower when polarization is higher.

4.2. *Selecting official languages*

The choice of official languages is another natural application of our analysis. Governments, firms, associations, and international organizations decide the languages that they require or permit to be used in official businesses. This is a natural political question that calls for an economic evaluation of the available options. Pool (1991), in what can be considered as the seminal paper to deal with this issue, suggests to formalize the benefits of each option as the number of agents that speak at least one of the languages deemed as official. Ginsburgh et al. (2005) endorse the same idea to determine optimal sets of official languages in the European Union.[21] This corre-

[20] Observe that, in our context, (n_1, n_2) is a more polarized society than (n_1-1, n_2+1) when $n_1 \leq n_2$.

[21] To be precise, both papers consider translation costs and adoption costs (or benefits expressed in negative numbers). The latter depend on the languages spoken by each agent and the set of official languages. On aggregate terms, these benefits can just be expressed as the number of agents that speak at least one of the official languages (under the assumption that adoption is homogeneous across languages).

sponds to a particular communicative benefit function within our family $\mathcal{F}$; namely, ϕ^ω such that $\omega(1) = 1$ and $\omega(i) = 0$, for each $i \neq 1$.

The above assumption conveys an important (albeit questionable) value judgment: the only interactions that matter with respect to the official languages are the ones generated by single agents. This is meaningful, for instance, when the main concern is to guarantee the individual access to official documents. Nevertheless, other important concerns could also be considered. For instance, many official meetings require, by law, that they are carried out in one of the official languages. Closer to home, think for example of university department meetings in many countries. Regulation at the university determines the official languages, and any department member has the right to veto the choice of another language to carry out an official department meeting. This implies that the selection of the appropriate communicative benefit function should also yield a positive weight to groups of size greater than 1. Our family $\mathcal{F}$ yields many options to do so. As we show in the next example, a change of the communicative benefit function, from the one adopted by Pool (1991) and Ginsburgh et al. (2005) to another within the family $\mathcal{F}$, could change the selection of the official languages in some instances.

Example 5. Let $N = \{1, 2, 3, 4, 5, 6, 7\}$, $L = \{a, b, c\}$, and

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{array}.$$

If the cost of selecting more than one official language is very high compared with the cost of selecting only one official language, the decision is straightforward (not only with the specific communicative benefit function in Pool (1991) and Ginsburgh et al. (2005), but also with any other member within the family $\mathcal{F}$): c should be the unique official language.[22] However, if the cost of selecting two official languages is similar to the cost of selecting only one official language, the situation changes. To wit, if one adopts the specific communicative benefit function in Pool (1991) and Ginsburgh et al. (2005), a and b should be the official languages. Nevertheless, with other members of the family $\mathcal{F}$ (more precisely, those ϕ^ω for which $\omega(4)+3\omega(3)+2\omega(2) > \omega(1)$), then c should be one of the official languages.[23]

4.3. Individual decisions for language acquisition

We now turn to individual decisions for language acquisition. Lazear (1999) introduced an influential model of culture and language, in which those decisions were instrumental to analyze

[22] Note that $\Phi^\omega(N, \{a\}, A|_{\{a\}}) = \Phi^\omega(N, \{b\}, A|_{\{b\}}) = 3\omega(1) + 3\omega(2) + \omega(3)$, whereas $\Phi^\omega(N, \{c\}, A|_{\{c\}}) = 4\omega(1) + 6\omega(2) + 4\omega(3) + \omega(4)$.

[23] Note that $\Phi^\omega(N, \{a, b\}, A|_{\{a,b\}}) = 6\omega(1) + 6\omega(2) + 2\omega(3)$, whereas $\Phi^\omega(N, \{a, c\}, A|_{\{a,c\}}) = \Phi^\omega(N, \{b, c\}, A|_{\{b,c\}}) = 5\omega(1) + 8\omega(2) + 5\omega(3) + \omega(4)$.

important issues such as optimal patterns of assimilation, culture adoption or immigration policies. In his model, individual decisions for language acquisition were based on the benefits side on the number of new individuals you can communicate with. This is also the case in the model introduced by Church and King (1993).[24] This modeling assumption is a specific case within our family $\mathcal{F}^\star$; namely, the one associated with ω such that $\omega(2)=1$ and $\omega(i)=0$ for each $i \neq 2$. Other cases could thus be considered and, as shown next, they would have different implications.

Example 6. Let $N = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $L = \{a, b, c\}$, and

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix} \end{array}.$$

Suppose that agent 4, who speaks language b, faces the option of learning language a or c. If the cost of learning each is the same, and we endorse Lazear's communicative benefit function, the decision is straightforward: agent 4 should learn language a. Nevertheless, with other members of the family $\mathcal{F}^\star$ (more precisely, those for which $\frac{1}{5}\omega(5) + \frac{3}{4}\omega(4) + \frac{2}{3}\omega(3) > \frac{1}{2}\omega(2)$), agent 4 should learn language c instead.[25]

5. Further insights

Our axioms characterize a whole family of functions, giving freedom to consider arbitrary weights for group sizes. For instance, the communicative benefit function assigning a value of zero to all languages in all situations belongs to the family $\mathcal{F}$. This implies that our axioms are rather weak, to the extent that they cannot discard a trivial function such as the one just suggested. Nevertheless, they are sufficiently strong to describe a stylized family with well-defined structure, as Theorem 1 states, and with interesting applications, as the ones described in Section 4. If one wants to discard the trivial constant function just mentioned, it is necessary to slightly strengthen one of the axioms of the result. More specifically, note that our axiom of *Communicational Inclusion Monotonicity* is defined with a weak inequality, although there would be some comparisons embedded in this axiom in which it would be very natural to assume a strict inequality: for instance, when $L' = L \cup \{l\}$ and the new language l is spoken only by agents that do not speak any other language of L. Strengthening this axiom requiring a strict inequality in some cases (like those ones) would be enough to rule out the trivial constant function mentioned

24 More recently, Armstrong (2015) has considered a different model, which postulates that the command of a non-native language may provide employers with a verifiable signal of the skills of an employee. In this setting, the individual benefits associated to learning the non-native language are the extra wage employers pay to workers with that skill.

25 By learning a, agent 4 would increase her communicative benefits by $\frac{1}{4}\omega(4) + \omega(3) + \frac{3}{2}\omega(2)$, whereas, by learning c, she would increase them by $\frac{1}{5}\omega(5) + \omega(4) + \frac{5}{3}\omega(3) + \omega(2)$.

above. One might be interested into being more accurate and shrink the family to a unique or just several functions, which could be obtained adding axioms with implications on the weights. For instance, one could consider the basic axiom stating that a canonical society with a unique agent offers null communicative benefits.[26] This would amount to imposing the condition $\omega(1) = 0$ in the definition of our family (thus, shrinking it). Likewise, one could also consider additional axioms that would end up reflecting into the monotonicity of the ω mapping.

An implication of our family that deserves attention can be illustrated with the following comparison of situations, both concerning the society $N = \{1, 2, 3, 4, 5, 6\}$. First, consider the situation (N, L, A), with $L = \{l, a, b, c\}$, such that three individuals (say agents 1, 2 and 3) speak language l and the other three all speak their own language and are unable to communicate with anyone else. Second, consider another situation (N, L, A') such that language l has no speakers and each of the other three languages has two different speakers, thus everyone can talk with one other person. That is,

$$A = \begin{array}{c} \\ 1\\2\\3\\4\\5\\6 \end{array}\begin{array}{c} \begin{array}{cccc} l & a & b & c \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array} \qquad A' = \begin{array}{c} \\ 1\\2\\3\\4\\5\\6 \end{array}\begin{array}{c} \begin{array}{cccc} l & a & b & c \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}.$$

If we compute the communicative benefits of each agent with a member of family $\mathcal{F}^\star$, we obtain that $\varphi^\omega_{(N,L,A')}(i) = \frac{1}{2}\omega(2) + \omega(1)$ for each $i \in N$, whereas $\varphi^\omega_{(N,L,A)}(j) = \frac{1}{3}\omega(3) + \omega(2) + \omega(1)$ for each $j \in \{1, 2, 3\}$ and $\varphi^\omega_{(N,L,A)}(k) = \omega(1)$ for each $k \in \{4, 5, 6\}$. Thus, although the distribution of the communicative benefits is more unequal in (N, L, A) than in (N, L, A'), the sum of communicative benefits of all agents is higher in the former situation. This implies a trade-off from the social point of view. Our family opts in this trade-off for a *utilitarian* perspective, as Φ^ω is constructed by adding the communicative benefits φ^ω of all individuals. Thus, our family states that situation (N, L, A) is weakly better-off (in the sense of giving more communicative benefits to the society) than situation (N, L, A'). Alternative perspectives, making use of alternative aggregation procedures (for instance, a *Rawlsian* one) might also be applied and could well generate the opposite ranking between situations (N, L, A) and (N, L, A').

A plausible question arising from our work is to check whether agents endorse our family to measure the value of languages in their decisions. To answer this question we would need to test the empirical content of our axioms or, more directly, our family of communicative benefit functions. This question is related to the theory of revealed preference, which has a very long and distinguished tradition in economics (e.g., Chambers and Echenique, 2016). It is based on the idea that agents' preferences are revealed with their choices. In our context, a data point is a situation, and as this is a much more complicated object than a choice set, the observability problem is more severe than it would be in a standard decision theoretic setting. Nevertheless, we could potentially address this issue (at least, partially) in some applications, such as those presented in the previous section. For instance, consider the case of selecting official languages within the EU. Although languages from countries joining the EU become official languages, three (English,

[26] Formally, for each $(N, L, A) \in \mathcal{S}$ such that $|N| = 1$, $\Phi(N, L, A) = 0$.

French and German) have been selected as 'procedural' languages.[27] This implicitly says that they have reached a higher net value than other combinations. More precisely, for each subset of official languages K, $\Phi^\omega(N, \{E, F, G\}, A|_{\{E,F,G\}}) - C(\{E, F, G\}) \geq \Phi^\omega(N, K, A|_K) - C(K)$, where C is a function that reflects the translation costs of implementing each set of languages as 'procedural'. As the matrix A can be inferred from the Special Eurobarometer mentioned in Section 3, we can compute from this condition some restrictions to the weights ω. Similar conditions could be obtained upon scrutinizing individual decisions regarding learning languages, which are also questioned in the Special Eurobarometer.

We also acknowledge that our model implicitly assumes that a society N is "closed", in the sense that there are no individuals outside N interacting with those of N (either because N is the entire world, or because the agents in N do not interact with those in the rest of the world). We obtain the communicative benefits a language (or a set of languages) yields to this society under that assumption. To address the communicative benefits of languages for an open society, one should consider the knowledge of each language by the agents outside the society. Then, denoting N as the entire world, we should construct, for each society $S \subseteq N$, each set of languages L and each matrix A describing the knowledge of languages, a measure $\varphi_{(N,L,A)}(S)$ that evaluates the communicative benefits this open society S obtains from this situation of the entire world. Observe that this analysis has been already done for the specific case in which S is a singleton. More precisely, in Section 3, we have computed the communicative benefits each individual i obtains in each situation (N, L, A), thanks to $\varphi_{(N,L,A)}(i)$. This approach could be extended to non-singleton sets.

While our model is distinctive and generates an innovative and novel result, we would like to point out some links to classical contributions in the literature. First, the communicative benefit function we derive for each language is reminiscent to the Shapley value for cooperative games (e.g., Shapley, 1953). More precisely, for each situation (N, L, A) and each mapping $\omega : \mathbb{N} \to \mathbb{R}_+$, we can describe a TU-game (L, v^ω), where v^ω is the characteristic function that assigns to each subset of languages $K \subseteq L$ the amount

$$v^\omega(K) = \sum_{M \subseteq N_A(K), M \neq \emptyset} \omega(|M|),$$

to be interpreted as the communicative benefits K generates within the situation (N, L, A), according to ω. The Shapley value of the TU-game (L, v^ω) yields precisely $\phi^\omega_{(N,L,A)}$.[28] Shapley (1953) characterized his value relying on an *additivity* axiom, a *symmetry* axiom, and an axiom requiring that the sum of the awards to *non-dummy* players equals the value of the grand coalition (Axiom 2 in Shapley, 1953). The last axiom can be decomposed into two axioms: *efficiency* and *dummy player*. Our characterization of the family $\mathcal{F}$ involves two symmetry axioms (*Anonymity* and *Equal Treatment of Equal Languages*) and a dummy axiom (*Null Agent Consistency*). Additivity cannot be mimicked in our model, but its role in Shapley's proof is similar to the one played here by *Irrelevance of Non Speakers*: whereas Shapley's original proof uses the symmetry and dummy axioms to address "canonical" games, and the additivity axiom to extend them

27 Almost all documents are written in these languages, and then translated to the other official languages (e.g., Ginsburgh et al., 2017).

28 Analogously, we could describe a TU-game (N, v^ω), for agents instead of languages, whose Shapley value rationalizes the φ^ω function (instead of the ϕ^ω function).

to general games, we address "nested" situations[29] with *Anonymity* and *Null Agent Consistency* (as well as *Communicational Inclusion Monotonicity*) and then use *Irrelevance of Non Speakers* to move from nested situations to general ones. Thus, one might say that, in some sense, we align with the tradition of deriving *Shapley value functions* for a diverse range of problems, including airport problems (e.g., Littlechild and Owen, 1973), telecommunication problems (e.g., van den Nouweland et al., 1996), museum-pass problems (e.g., Ginsburgh and Zang, 2003) or broadcasting problems (e.g., Bergantiños and Moreno-Ternero, 2020).

Second, the input of our problem coincides with that of *approval voting* (e.g., Brams and Fishburn, 1978). In that case, each voter casts her vote for as many candidates as she approves. The votes are then added by candidate, and the winner is the one who gets the largest number of approvals. All other candidates can also be ranked, according to the number of approvals they obtain. An alternative aggregation procedure, akin to the one we obtain here, is *Cumulative Voting* (e.g., Glasser, 1959; Sawyer and MacRae, 1962), which allows voters to distribute points among candidates in any arbitrary way.[30] An interesting case is the one in which every agent is endowed with a fixed number of points that are evenly divided among all candidates for whom she approves. In our setting, if we interpret that each group of agents (instead of only individuals alone) approve the languages in which they can communicate, our family of communicative benefit functions coincides with the *Cumulative Voting* score of each language (when all coalitions of the same size receive the same number of points).

6. Conclusion

We have explored in this paper the axiomatic approach to the problem of measuring communicative benefits of languages. We show that the combination of intuitive axioms reflecting the celebrated principles of impartiality, monotonicity, invariance and consistency characterizes a family of two-stage communicative benefit functions. First, the groups of agents that can communicate in a given language are identified. Second, each member of the family associates to each group a value that depends on its size, and this value is divided evenly among the languages in which the group can communicate. As such, our work fully aligns with the tradition of axiomatic work that can be traced back to the seminal contributions of Nash (1950), Arrow (1951) and Shapley (1953). The aim is to provide a list of requirements (axioms) formalizing ethical or operational principles that a rule should satisfy. The ideal is to derive the set of rules that fulfill such axioms. During the ensuing seven decades countless authors have applied the axiomatic approach to a variety of problems and measures ranging from conventional concepts such as taxation (e.g., Young, 1988), income inequality (e.g., Bossert, 1990), or claims problems (e.g., Ju et al., 2007) to more sophisticated ones (and somewhat unconventional) such as polarization (e.g., Esteban and Ray, 1994) or resilience (e.g., Asheim et al., 2020).

As mentioned at the Introduction, there is by now an established field dealing with the economics of language (e.g., Ginsburgh and Weber, 2020). There is consensus that Marschak (1965), who explicitly introduced economic concepts such as costs and benefits into linguistic analysis, should be credited for the origin of the field as a separate discipline. Linguist Joseph Greenberg suggested possible connections with economics a decade earlier (Greenberg, 1956), but it was

[29] Nested situations are those in which there exists a language that is spoken by everyone that speaks another language. The formal definition is in the Appendix A, where we include the proof of our theorem.

[30] Both Approval Voting and Cumulative Voting can be seen as members of a family of voting procedures dubbed as *Size Approval Voting*, characterized by Alcalde-Unzu and Vorsatz (2009).

probably not until the 90's that a large number of articles connected language, economics, and business (we already referred to some of them earlier in the paper). Rubinstein (2000) collected some influential essays briefly addressing some issues between economic theory and the study of language, such as the evolutionary development of the meaning of words, the distinct targets of binary relations, or the rhetoric of game theory. We largely depart from this existing literature by studying a theoretical framework in which we provide normative foundations for the measurement of communicative benefits. This was a pending issue in the field and we aimed to fill the gap with our work.

Finally, our work can be extended in plausible ways.

One refers to the case in which intermediate levels of language proficiency related to *partial learning* (e.g., Blume, 2000) are considered. Recently, Brock et al. (2021) have extended the dichotomous language acquisition framework to allow for the option of a limited language acquisition at a lower cost than that of full learning. Augmenting their analysis to provide an axiomatic support for the language acquisition model with partial learning would represent an interesting avenue for future research.

Another refers to the case in which linguistic distances (e.g., Dyen et al., 1992) might play a role in determining the communication of a group of agents.

Yet another is to deal with linguistic justice as part of rectificatory justice. This would require a dynamic extension of our model to be able to deal with several periods and the linguistic reality in each of them. A possibility would be to consider two periods with the same set of agents, $(N, L_1, A_1), (N, L_2, A_2)$, such that the value of each language l at L_2 in the second period should depend also on the individual values the speakers of l in period 2 obtained in period 1 from the entire set of languages. These lines are left for further research.

Appendix A

A.1. Proof of Theorem 1

We first need two lemmata providing several implications of the axioms in the statement of the theorem. To present the first lemma, we need to introduce some notation. We say that a situation is *nested* if there exists a language that is spoken by all agents that speak any other language. Formally, $(N, L, A) \in \mathcal{S}$ is nested if there exists $l \in L$ with $a_{il} = 1$ for each $i \in N$ such that $a_{il'} = 1$ for some $l' \in L$.[31] We denote the set of *nested situations* as $\mathcal{S}^* \subseteq \mathcal{S}$. If $(N, L, A) \in \mathcal{S}^*$, we denote $l^*_{(N,L,A)}$ a language that is spoken by the maximal set of agents in this situation.[32] Then, the first lemma states that, under *Anonymity*, *Communicational Inclusion Monotonicity* and *Null Agent Consistency*, the total communicative benefits of all languages in any nested situation only depend on the number of speakers of the maximal language.

Lemma 1. *Let ϕ be a communicative benefit function that satisfies Anonymity, Communicational Inclusion Monotonicity and Null Agent Consistency. Then, there exists a mapping* $\Omega : \mathbb{N} \cup \{0\} \to \mathbb{R}_+$ *such that, for each* $(N, L, A) \in \mathcal{S}^*$,

$$\Phi(N, L, A) = \Omega(|N_A(l^*_{(N,L,A)})|).$$

[31] This is reminiscent of the concept of nested graphs (e.g., König et al., 2014; Joshi et al., 2020).

[32] Observe that $l^*_{(N,L,A)}$ may not be unique.

Proof. Let $x \in \mathbb{N} \cup \{0\}$ and $(N, \{l\}, A) \in \mathcal{S}$ be such that $|N_A(l)| = x$. Let $\Omega(x) = \Phi(N, \{l\}, A)$.

We first show that for each $(\hat{N}, \{\hat{l}\}, \hat{A}) \in \mathcal{S}$ such that $|\hat{N}_{\hat{A}}(\hat{l})| = x$, $\Phi(\hat{N}, \{\hat{l}\}, \hat{A}) = \Omega(x)$. To do so, we distinguish two cases:

Case 1: $|N| = |\hat{N}|$.

Let $\pi \in \Pi^{\mathcal{N}}$ be such that $\pi(N_A(l)) = \hat{N}_{\hat{A}}(\hat{l})$ and $\pi(N) = \hat{N}$. Then, $\pi(N, \{l\}, A) = (\hat{N}, \{l\}, (a_{\pi(i)l})_{i\in N})$. By *Anonymity*, $\phi_{(N,\{l\},A)}(l) = \phi_{\pi(N,\{l\},A)}(l)$. Therefore, $\Phi(N, \{l\}, A) = \Phi(\hat{N}, \{l\}, (a_{\pi(i)l})_{i\in N})$. Observe that $(\hat{N}, \{l\}, (a_{\pi(i)l})_{i\in N}) \subseteq (\hat{N}, \{\hat{l}\}, \hat{A})$ and $(\hat{N}, \{\hat{l}\}, \hat{A}) \subseteq (\hat{N}, \{l\}, (a_{\pi(i)l})_{i\in N})$. Then, we can apply *Communicational Inclusion Monotonicity* twice to obtain that $\Phi(\hat{N}, \{l\}, (a_{\pi(i)l})_{i\in N}) = \Phi(\hat{N}, \{\hat{l}\}, \hat{A})$. Thus, $\Phi(N, \{l\}, A) = \Phi(\hat{N}, \{\hat{l}\}, \hat{A})$.

Case 2: $|N| \neq |\hat{N}|$.

Assume, without loss of generality, that $|N| > |\hat{N}|$. Let $(\bar{N}, \{\hat{l}\}, \bar{A}) \in \mathcal{S}$ be such that $\hat{N} \subset \bar{N}$, $|\bar{N}| = |N|$, $\bar{a}_{i\hat{l}} = 0$ for each $i \in \bar{N} \setminus \hat{N}$, and $\bar{a}_{j\hat{l}} = \hat{a}_{j\hat{l}}$ for each $j \in \hat{N}$. As $|N_A(l)| = |\bar{N}_{\bar{A}}(\hat{l})|$ and $|\bar{N}| = |N|$, we can deduce from Case 1 that $\Phi(N, \{l\}, A) = \Phi(\bar{N}, \{\hat{l}\}, \bar{A})$. Therefore, by iterated application of *Null Agent Consistency*, $\Phi(\hat{N}, \{\hat{l}\}, \hat{A}) = \Phi(\bar{N}, \{\hat{l}\}, \bar{A})$ and, thus, $\Phi(N, \{l\}, A) = \Phi(\hat{N}, \{\hat{l}\}, \hat{A})$.

Second, let $(\tilde{N}, \tilde{L}, \tilde{A}) \in \mathcal{S}^*$ be such that $|\tilde{N}_{\tilde{A}}(\tilde{l}^*_{(\tilde{N},\tilde{L},\tilde{A})})| = x$. By the analysis above, we know that $\Phi(\tilde{N}, \{\tilde{l}^*_{(\tilde{N},\tilde{L},\tilde{A})}\}, \tilde{A}|_{\{\tilde{l}^*_{(\tilde{N},\tilde{L},\tilde{A})}\}}) = \Omega(x)$. As $(\tilde{N}, \tilde{L}, \tilde{A}) \subseteq (\tilde{N}, \{\tilde{l}^*_{(\tilde{N},\tilde{L},\tilde{A})}\}, \tilde{A}|_{\{\tilde{l}^*_{(\tilde{N},\tilde{L},\tilde{A})}\}})$ and $(\tilde{N}, \{\tilde{l}^*_{(\tilde{N},\tilde{L},\tilde{A})}\}, \tilde{A}|_{\{\tilde{l}^*_{(\tilde{N},\tilde{L},\tilde{A})}\}}) \subseteq (\tilde{N}, \tilde{L}, \tilde{A})$, we can apply *Communicational Inclusion Monotonicity* twice to obtain that $\Phi(\tilde{N}, \tilde{L}, \tilde{A}) = \Phi(\tilde{N}, \{\tilde{l}^*_{(\tilde{N},\tilde{L},\tilde{A})}\}, \tilde{A}|_{\{\tilde{l}^*_{(\tilde{N},\tilde{L},\tilde{A})}\}})$. Thus, $\Phi(\tilde{N}, \tilde{L}, \tilde{A}) = \Omega(x)$. ■

The second lemma states that, under *Irrelevance of Non Speakers*, the communicative benefits of any language in any situation coincide with the communicative benefits that this language has in a particular nested situation in which it is a maximal language.

Lemma 2. *Let ϕ be a communicative benefit function that satisfies Irrelevance of Non Speakers. For each $(N, L, A) \in \mathcal{S}$, and each $l \in L$, let A' be such that, for each $l' \in L$,*

$$a'_{il'} = \begin{cases} a_{il'} & \text{if } i \in N_A(l) \\ 0 & \text{otherwise.} \end{cases}$$

Then,

$$\phi_{(N,L,A)}(l) = \phi_{(N,L,A')}(l).$$

The proof of Lemma 2 is obtained by applying *Irrelevance of Non Speakers* to the situations $(N, L \setminus \{l\}, A|_{L\setminus\{l\}})$, $(N, L \setminus \{l\}, A'|_{L\setminus\{l\}})$ and $(N, \{l\}, A|_{\{l\}})$.

With Lemmas 1 and 2, we can now proceed to prove Theorem 1.

It is straightforward to check that all communicative benefit functions within class $\mathcal{F}$ satisfy the axioms in the statement. Conversely, let ϕ be a communicative benefit function that satisfies these axioms. By Lemma 1, there exists a mapping $\Omega : \mathbb{N} \cup \{0\} \to \mathbb{R}_+$ such that for each $(N, L, A) \in \mathcal{S}^*$, $\Phi(N, L, A) = \Omega(|N_A(l^*_{(N,L,A)})|)$. We now construct iteratively from Ω the mapping $\hat{\omega} : \mathbb{N} \cup \{0\} \to \mathbb{R}$ as follows:

$$\hat{\omega}(x) = \begin{cases} \Omega(x) & \text{if } x = 0 \\ \Omega(x) - \sum_{y=0}^{y=x-1} \left[\binom{x}{y} \cdot \hat{\omega}(y) \right] & \text{otherwise.} \end{cases}$$

Observe that then, for each $x \in \mathbb{N}$,

$$\Omega(x) = \sum_{y=0}^{y=x} \left[\binom{x}{y} \cdot \hat{\omega}(y) \right].$$

Note that there are two differences between $\hat{\omega}$ and the ω mapping introduced in the definition of the communicative benefit functions within class $\mathcal{F}$. On the one hand, the domain of $\hat{\omega}$ does not include only the natural numbers, but also 0. On the other hand, it is not clear with this construction whether the range of $\hat{\omega}$ includes only non–negative real numbers, as it occurs with ω.

The rest of the proof is decomposed into four steps. The first one establishes that the communicative benefits of a language can be computed applying a similar formula to the one from Definition 1, but with two differences: on the one hand, it calculates the communicative value of each group M by $\hat{\omega}(|M|)$ instead of $\omega(|M|)$ and, on the other hand, it also considers that the empty group could have a value $\hat{\omega}(0)$. The two intermediate steps are dedicated to prove that the range of $\hat{\omega}$ is $\mathbb{R}_+$ and that $\hat{\omega}(0) = 0$. Finally, the last step wraps up the previous three steps to finish the proof.

Step 1: *For each $(N, L, A) \in \mathcal{S}$ and each $l \in L$,*

$$\phi_{(N,L,A)}(l) = \sum_{M \subseteq N} \left(\hat{\omega}(|M|) \cdot \frac{d_A^l(M)}{d_A^L(M)} \right).$$

Let $(N, L, A) \in \mathcal{S}$ and $l \in L$. Assume that we have already proved that

$$\phi_{(\hat{N},\hat{L},\hat{A})}(\hat{l}) = \sum_{M \subseteq \hat{N}} \left(\hat{\omega}(|M|) \cdot \frac{d_{\hat{A}}^{\hat{l}}(M)}{d_{\hat{A}}^{\hat{L}}(M)} \right) \text{ for each } (\hat{N}, \hat{L}, \hat{A}) \in \mathcal{S} \text{ and each } \hat{l} \in \hat{L} \text{ with}$$

$$|\hat{N}_{\hat{A}}(\hat{l})| < |N_A(l)|,$$

and we are going to prove that $\phi_{(N,L,A)}(l)$ follows the formula of the statement of Step 1.[33]

Let A' be such that, for each $l' \in L$,

$$a'_{il'} = \begin{cases} a_{il'} & \text{for each } i \in N_A(l) \\ 0 & \text{otherwise.} \end{cases}$$

By Lemma 2, $\phi_{(N,L,A)}(l) = \phi_{(N,L,A')}(l)$. Thus, it suffices to show that

$$\phi_{(N,L,A')}(l) = \sum_{M \subseteq N} \left(\hat{\omega}(|M|) \cdot \frac{d_A^l(M)}{d_A^L(M)} \right).$$

Observe that $(N, L, A') \in \mathcal{S}^*$ and that $l^*_{(N,L,A')} = l$. Then, we can apply Lemma 1 to obtain

[33] In order to avoid redundancy, we do not explicitly provide the proof of the base case (i.e., when $|N_A(l)| = 0$) because its proof is analogous to the upcoming one.

$$\Phi(N, L, A') = \Omega(|N_{A'}(l)|) = \sum_{y=0}^{y=|N_{A'}(l)|} \left[\binom{|N_{A'}(l)|}{y} \cdot \hat{\omega}(y) \right].$$

Note that $\Phi(N, L, A') = \sum_{\hat{l}:N_{A'}(\hat{l}) \subset N_{A'}(l)} \phi_{(N,L,A')}(\hat{l}) + \sum_{\hat{l}:N_{A'}(\hat{l}) = N_{A'}(l)} \phi_{(N,L,A')}(\hat{l})$ and, thus,

$$\sum_{\hat{l}:N_{A'}(\hat{l}) = N_{A'}(l)} \phi_{(N,L,A')}(\hat{l}) = \sum_{y=0}^{y=|N_{A'}(l)|} \left[\binom{|N_{A'}(l)|}{y} \cdot \hat{\omega}(y) \right] - \sum_{\hat{l}:N_{A'}(\hat{l}) \subset N_{A'}(l)} \phi_{(N,L,A')}(\hat{l}). \tag{1}$$

Let $\bar{l} \in L$ be such that $N_{A'}(\bar{l}) \subset N_{A'}(l)$. By the induction hypothesis,

$$\phi_{(N,L,A')}(\bar{l}) = \sum_{M \subseteq N} \left(\hat{\omega}(|M|) \cdot \frac{d_{A'}^{\bar{l}}(M)}{d_{A'}^{L}(M)} \right).$$

As $d_{A'}^{\bar{l}}(M) = 0$ for each $M \not\subset N_{A'}(l)$, it follows that

$$\phi_{(N,L,A')}(\bar{l}) = \sum_{M \subset N_{A'}(l)} \left(\hat{\omega}(|M|) \cdot \frac{d_{A'}^{\bar{l}}(M)}{d_{A'}^{L}(M)} \right).$$

Observe now that, for each $\hat{l} \in L$ such that $N_{A'}(\hat{l}) = N_{A'}(l)$, $d_{A'}^{\hat{l}}(M) = 1$ for each $M \subset N_{A'}(l)$. Let $\alpha_l = |\{\hat{l} \in L : N_{A'}(\hat{l}) = N_{A'}(l)\}|$. Thus,

$$\sum_{\bar{l}:N_{A'}(\bar{l}) \subset N_{A'}(l)} \phi_{(N,L,A')}(\bar{l}) = \sum_{M \subset N_{A'}(l)} \left[\hat{\omega}(|M|) \cdot \left(\sum_{l' \in L} \frac{d_{A'}^{l'}(M) - \alpha_l}{d_{A'}^{L}(M)} \right) \right].$$

Then,

$$\sum_{\bar{l}:N_{A'}(\bar{l}) \subset N_{A'}(l)} \phi_{(N,L,A')}(\bar{l}) = \sum_{M \subset N_{A'}(l)} \left[\hat{\omega}(|M|) \cdot \left(1 - \frac{\alpha_l}{d_{A'}^{L}(M)} \right) \right].$$

Note that $\binom{|N_{A'}(l)|}{y} = |\{M \subseteq N_{A'}(l) : |M| = y\}|$ for each $y \in \{0, \ldots, |N_{A'}(l)|\}$. It follows by (1) that

$$\sum_{\hat{l}:N_{A'}(\hat{l}) = N_{A'}(l)} \phi_{(N,L,A')}(\hat{l}) = \sum_{M \subseteq N_{A'}(l)} \left(\hat{\omega}(|M|) \cdot \frac{\alpha_l}{d_{A'}^{L}(M)} \right).$$

By *Equal Treatment of Equal Languages*, $\alpha_l \cdot \phi_{(N,L,A')}(l) = \sum_{\hat{l}:N_{A'}(\hat{l}) = N_{A'}(l)} \phi_{(N,L,A')}(\hat{l})$. Therefore,

$$\phi_{(N,L,A')}(l) = \sum_{M \subseteq N_{A'}(l)} \left(\hat{\omega}(|M|) \cdot \frac{1}{d_{A'}^{L}(M)} \right).$$

As $d^l_{A'}(M) = 1$ for each $M \subseteq N_{A'}(l)$, and $d^l_{A'}(M) = 0$ for each $M \nsubseteq N_{A'}(l)$, we have

$$\phi_{(N,L,A')}(l) = \sum_{M \subseteq N} \left(\hat{\omega}(|M|) \cdot \frac{d^l_{A'}(M)}{d^L_{A'}(M)} \right).$$

Finally, observe that, by construction of A', for each $M \subseteq N$:

- $d^l_{A'}(M) = 1$ if and only if $d^l_A(M) = 1$,
- If $d^l_{A'}(M) = 1$, then $d^L_{A'}(M) = d^L_A(M)$.

Therefore, for each $M \subseteq N$,

$$\frac{d^l_{A'}(M)}{d^L_{A'}(M)} = \frac{d^l_A(M)}{d^L_A(M)}.$$

Consequently,

$$\phi_{(N,L,A')}(l) = \sum_{M \subseteq N} \left(\hat{\omega}(|M|) \cdot \frac{d^l_A(M)}{d^L_A(M)} \right),$$

as desired.

Step 2: $\hat{\omega}(x) \geq 0$ *for each* $x \in \mathbb{N} \cup \{0\}$.

Let $x \in \mathbb{N} \cup \{0\}$. Let $(N, \{l\}, A) \in \mathcal{S}$ be such that $|N_A(l)| = x$. By Lemma 1, $\Phi(N, \{l\}, A) = \Omega(x)$. As there is only one language in this situation, $\Phi(N, \{l\}, A) = \phi_{(N,\{l\},A)}(l)$ and, thus, $\phi_{(N,\{l\},A)}(l) = \Omega(x)$.

If $x = 0$, then $\Omega(x) = \hat{\omega}(x)$ and, thus, $\phi_{(N,\{l\},A)}(l) = \hat{\omega}(x)$. As, by definition, the range of ϕ is $\mathbb{R}_+$, we obtain that $\hat{\omega}(0) \in \mathbb{R}_+$.

If $x \geq 1$, then $\Omega(x) = \sum_{y=0}^{y=x} \left[\binom{x}{y} \cdot \hat{\omega}(y) \right]$. Thus, $\Phi(N, \{l\}, A) = \sum_{y=0}^{y=x} \left[\binom{x}{y} \cdot \hat{\omega}(y) \right]$. Let $(N, L, A') \in \mathcal{S}$ be such that

- $|L| = x$,
- $|N_{A'}(\bar{l})| = x - 1$ for each $\bar{l} \in L$,
- $N_{A'}(\bar{l}) \neq N_{A'}(\bar{l}')$ for each pair $\bar{l}, \bar{l}' \in L$, and
- $\bigcup_{\bar{l} \in L} N_{A'}(\bar{l}) = N_A(l)$.

By Step 1,

$$\phi_{(N,L,A')}(\bar{l}) = \sum_{M \subseteq N} \left(\hat{\omega}(|M|) \cdot \frac{d^{\bar{l}}_{A'}(M)}{d^L_{A'}(M)} \right) \text{ for each } \bar{l} \in L.$$

As, for each $M \not\subset N_A(l)$, $d^{\bar{l}}_{A'}(M) = 0$ for each $\bar{l} \in L$, we have

$$\phi_{(N,L,A')}(\bar{l}) = \sum_{M \subset N_A(l)} \left(\hat{\omega}(|M|) \cdot \frac{d^{\bar{l}}_{A'}(M)}{d^L_{A'}(M)} \right) \text{ for each } \bar{l} \in L.$$

Thus,

$$\Phi(N, L, A') = \sum_{\bar{l} \in L} \phi_{(N,L,A')}(\bar{l}) = \sum_{\bar{l} \in L} \sum_{M \subset N_A(l)} \left(\hat{\omega}(|M|) \cdot \frac{d^{\bar{l}}_{A'}(M)}{d^{L}_{A'}(M)} \right)$$
$$= \sum_{M \subset N_A(l)} \hat{\omega}(|M|) \cdot \left(\sum_{\bar{l} \in L} \frac{d^{\bar{l}}_{A'}(M)}{d^{L}_{A'}(M)} \right).$$

Now, for each $M \subset N_A(l)$, there is $\bar{l} \in L$ such that $M \subseteq N_{A'}(\bar{l})$. Then, $\left(\sum_{\bar{l} \in L} \frac{d^{\bar{l}}_{A'}(M)}{d^{L}_{A'}(M)} \right) = 1$ for each $M \subset N_A(l)$. Therefore,

$$\Phi(N, L, A') = \sum_{M \subset N_A(l)} \hat{\omega}(|M|).$$

Finally, observe that there are exactly $\binom{x}{|M|}$ subsets of $N_A(l)$ with size $|M|$. Therefore,

$$\Phi(N, L, A') = \sum_{y=0}^{y=x-1} \left[\binom{x}{y} \cdot \hat{\omega}(y) \right].$$

We have then deduced that $\Phi(N, \{l\}, A) = \Phi(N, L, A') + \hat{\omega}(x)$. As $(N, L, A') \subseteq (N, \{l\}, A)$, it follows, by *Communicational Inclusion Monotonicity*, that $\Phi(N, \{l\}, A) \geq \Phi(N, L, A')$. Therefore, $\hat{\omega}(x) \geq 0$, as desired.

Step 3: $\hat{\omega}(0) = 0$.

One of the assumptions of the model is that there exists $(\bar{N}, \bar{L}, \bar{A}) \in \mathcal{S}$ and a language $\bar{l} \in L$ such that $\phi_{(\bar{N},\bar{L},\bar{A})}(\bar{l}) = 0$. Then, by Step 1,

$$\phi_{(\bar{N},\bar{L},\bar{A})}(\bar{l}) = 0 = \sum_{M \subseteq \bar{N}} \left(\hat{\omega}(|M|) \cdot \frac{d^{\bar{l}}_{\bar{A}}(M)}{d^{\bar{L}}_{\bar{A}}(M)} \right).$$

As, by Step 2, $\hat{\omega}(x) \geq 0$ for each $x \in \mathbb{N} \cup \{0\}$, a necessary and sufficient condition for this equality to hold is that $\hat{\omega}(0) = \ldots = \hat{\omega}(|\bar{N}_{\bar{A}}(\bar{l})|) = 0$, which concludes the proof.

Step 4: *Conclusion.*

By Step 1, we have that for each $(N, L, A) \in \mathcal{S}$ and each $l \in L$,

$$\phi_{(N,L,A)}(l) = \sum_{M \subseteq N} \left(\hat{\omega}(|M|) \cdot \frac{d^{l}_{A}(M)}{d^{L}_{A}(M)} \right).$$

By Step 3, $\hat{\omega}(0) = 0$ and, therefore, we have that for each $(N, L, A) \in \mathcal{S}$ and each $l \in L$,

$$\phi_{(N,L,A)}(l) = \sum_{M \subseteq N, M \neq \emptyset} \left(\hat{\omega}(|M|) \cdot \frac{d^{l}_{A}(M)}{d^{L}_{A}(M)} \right).$$

Let ω be the restriction of $\hat{\omega}$ to the domain $\mathbb{N}$. That is, $\omega : \mathbb{N} \to \mathbb{R}$ is such that $\omega(x) = \hat{\omega}(x)$ for all $x \in \mathbb{N}$. By Step 2, the range of ω is $\mathbb{R}_+$. Therefore, we have shown that there exists a mapping $\omega : \mathbb{N} \to \mathbb{R}_+$ such that for each $(N, L, A) \in \mathcal{S}$ and each $l \in L$,

$$\phi_{(N,L,A)}(l) = \sum_{M \subseteq N, M \neq \emptyset} \left(\omega(|M|) \cdot \frac{d_A^l(M)}{d_A^L(M)} \right).$$

Thus, ϕ belongs to class $\mathcal{F}$.

A.2. Proof of Proposition 1

Consider the following communicative benefit functions:

- Let v^1 be a communicative benefit function that behaves similarly to one from class $\mathcal{F}$, but only taking into account the coalitions to which one particular agent $j \in \mathcal{N}$ belongs to. Formally, consider a mapping $\xi : 2^{\mathcal{N}} \to \mathbb{R}_+$ such that $\xi(M) = 1$ if $j \in M$ and 0 otherwise, and a mapping $\omega : \mathbb{N} \to \mathbb{R}_+$. Then, v^1 is such that, for each $(N, L, A) \in \mathcal{S}$ and each $l \in L$,

$$v^1_{(N,L,A)}(l) = \sum_{M \subseteq N} \left(\omega(|M|) \cdot \xi(M) \cdot \frac{d_A^l(M)}{d_A^L(M)} \right).$$

  Then, v^1 satisfies *Equal Treatment of Equal Languages*, *Communicational Inclusion Monotonicity*, *Irrelevance of Non Speakers* and *Null Agent Consistency*, but not *Anonymity*.

- Let v^2 be a communicative benefit function that considers the groups of agents that can communicate in each language and the value of each group, that depends on its size, is divided among the languages in which the group can communicate, but unevenly. Formally, consider a particular language $\bar{l} \in \mathcal{L}$ and a mapping $\delta : \mathcal{L} \to \mathbb{R}_+$ such that $\delta(\bar{l}) = 2$ and $\delta(l') = 1$ for each $l' \in \mathcal{L} \setminus \{\bar{l}\}$. Consider also a mapping $\omega : \mathbb{N} \to \mathbb{R}_+$. Then, v^2 is such that, for each $(N, L, A) \in \mathcal{S}$ and each $l \in L$,

$$v^2_{(N,L,A)}(l) = \sum_{M \subseteq N} \left(\omega(|M|) \cdot \frac{d_A^l(M) \cdot \delta(l)}{\sum_{l' \in L} \left(d_A^{l'}(M) \cdot \delta(l') \right)} \right).$$

  Then, v^2 satisfies *Anonymity*, *Communicational Inclusion Monotonicity*, *Irrelevance of Non Speakers* and *Null Agent Consistency*, but not *Equal Treatment of Equal Languages*.

- Let v^3 be the communicative benefit function that yields for each language a value equal to its number of speakers. Formally, for each $(N, L, A) \in \mathcal{S}$ and each $l \in L$,

$$v^3_{(N,L,A)}(l) = |N_A(l)|.$$

  Then, v^3 satisfies *Anonymity*, *Equal Treatment of Equal Languages*, *Irrelevance of Non Speakers* and *Null Agent Consistency*, but not *Communicational Inclusion Monotonicity*.

- Let v^4 be the communicative benefit function arising from normalizing a member within the class $\mathcal{F}$. Formally, consider a communicative benefit function ϕ within the class $\mathcal{F}$, and let v^4 be such that, for each $(N, L, A) \in \mathcal{S}$ and each $l \in L$,

$$v^4_{(N,L,A)}(l) = \frac{\phi_{(N,L,A)}(l)}{\Phi(N, L, A)}.$$

Then, v^4 satisfies *Anonymity*, *Equal Treatment of Equal Languages*, *Communicational Inclusion Monotonicity* and *Null Agent Consistency*, but not *Irrelevance of Non Speakers*.

- Let v^5 be the communicative benefit function arising from combining two members within the class $\mathcal{F}$, depending on whether the number of agents in the situation is odd or even. Formally, consider two communicative benefit functions ϕ, ϕ' within the class $\mathcal{F}$, and let v^5 be such that, for each $(N, L, A) \in \mathcal{S}$ and each $l \in L$,

$$v^5_{(N,L,A)}(l) = \begin{cases} \phi_{(N,L,A)}(l) & \text{if } |N| \text{ is odd.} \\ \phi'_{(N,L,A)}(l) & \text{if } |N| \text{ is even.} \end{cases}$$

Then, v^5 satisfies *Anonymity*, *Equal Treatment of Equal Languages*, *Communicational Inclusion Monotonicity* and *Irrelevance of Non Speakers*, but not *Null Agent Consistency*.

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