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Solving the stochastic team orienteering problem: comparing simheuristics with the sample average approximation method

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Abstract

The team orienteering problem (TOP) is an *NP-hard* optimization problem with an increasing number of potential applications in smart cities, humanitarian logistics, wildfire surveillance, etc. In the TOP, a fixed fleet of vehicles is employed to obtain rewards by visiting nodes in a network. All vehicles share common origin and destination locations. Since each vehicle has a limitation in time or traveling distance, not all nodes in the network can be visited. Hence, the goal is focused on the maximization of the collected reward, taking into account the aforementioned constraints. Most of the existing literature on the TOP focuses on its deterministic version, where rewards and travel times are assumed to be predefined values. This paper focuses on a more realistic TOP version, where travel times are modeled as random variables, which introduces reliability issues in the solutions due to the route-length constraint. In order to deal with these complexities, we propose a simheuristic algorithm that hybridizes biased-randomized heuristics with a variable neighborhood search and MCS. To test the quality of the solutions generated by the proposed simheuristic approach, we employ the well-known sample average approximation (SAA) method, as well as a combination model that hybridizes the metaheuristic used in the simheuristic approach with the SAA algorithm. The results show that our proposed simheuristic outperforms the SAA and the hybrid model both on the objective function values and computational time.

Keywords: team orienteering problem; random travel times; biased-randomized algorithms; simheuristics; sample average approximation

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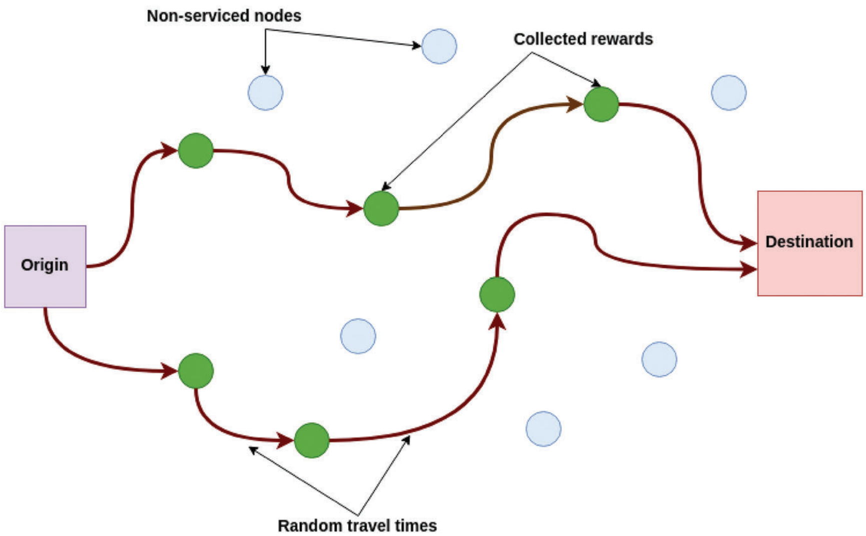

Fig. 1. Schema of the TOP with random travel times.

1. Introduction

In the team orienteering problem (TOP), a limited fleet of vehicles have to visit a subset of available nodes while traveling from the origin depot to the destination depot (Fig. 1). Visiting each location provides a reward (profit), and a threshold is typically set for the maximum length or duration of any route. The usual goal is to define the vehicle routes that maximize the total profit gathered. The classical version of the TOP assumes deterministic rewards and travel times. However, in many real-life situations these are better modeled as random variables. For instance, travel times might be subject to weather or traffic conditions, rewards might not be accurately known in advance, etc. Still, the vast majority of studies focus on the deterministic TOP, with many of them relying on exact methods, which show limited effectiveness when solving large-scale instances (Gunawan et al., 2018).

Thus, our paper focuses on the stochastic TOP in order to partially cover this gap in the scientific literature. In particular, we consider random travel times, since they are frequent in real-life scenarios. For instance, when monitoring a zone affected by a natural disaster, travel times between nodes may take longer than usual due to possible road blockages. Despite its relevance in real-life situations, the TOP with random travel times has been rarely considered so far. The introduction of random travel times, in combination with a user-defined threshold for the maximum length of any route, carries out reliability issues in any node configuration to be visited, that is, any given solution will be feasible just with a certain probability. In effect, some route failures might occur, in cases in which the random travel times tend to be higher than expected, due to a vehicle not being able to reach its final destination before the deadline provided by the threshold. In those cases, our model assumes that all collected rewards in that failed route are not consolidated at the destination node and, therefore, they are lost. Real-life applications such as aerial vehicles and self-driving electric vehicles with a limited driving range of batteries are considering the mentioned assumption since

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after losing the battery charge, all the collected rewards would be lost. Plenty of applications regarding smart cities, humanitarian logistics, wildfire surveillance, etc., are built on the considered assumptions. Because of these reliability issues, given a solution (set of planned routes), neither an exact nor a heuristic method alone can provide an accurate estimate of its expected reward. To achieve an accurate estimate the solution proposed by the optimization component has to be simulated to account for the effect of the random travel times on its reliability level and the associated reward (Faulin et al., 2008).

In this paper, we propose a simheuristic algorithm to tackle the TOP with random travel times and reliability issues. Simheuristics are flexible simulation–optimization approaches that make an iterative use of simulation during the searching process of a metaheuristic algorithm. Hence, they can provide reliability and reward estimates on any "promising" solution provided by the metaheuristic component even before the searching process is over (Gonzalez-Martin et al., 2018; Rabe et al., 2020). In the last years, simheuristics have been used to solve stochastic optimization problems in multiple fields, ranging from logistics and transportation to manufacturing (Juan et al., 2018). The optimization component of our simheuristic is based on the combination of a biased-randomized (BR) heuristics (Quintero-Araujo et al., 2017) and a variable neighborhood search (VNS) framework (Hansen and Mladenović, 2014; Gruler et al., 2020).

In order to validate the quality of the solutions provided by our simheuristic algorithm (SimBRVNS), we also use the sample average approximation (SAA) method (Shapiro et al., 2009). The application of the SAA is limited to small instances since the SAA typically relies on the solution of stochastic programming models of combinatorial problems, which are solved by CPLEX in our case. Combinatorial problems such as TOP are hard to be solved even for deterministic models using a direct solver. We apply the SAA method since (i) the results provided by the SAA-CPLEX combination are similar to the ones provided by the simheuristic, which can be applied to be compared with our approach; and (ii) they show the limitations of other methods when dealing with large-scale instances of the stochastic TOP. Finally, we apply a hybrid algorithm (SAA-BRVNS) that combines the SAA method and the optimization component of the proposed simheuristic.

The remainder of this research is structured as follows. In Section 2, we review the literature of TOP; Section 3 presents a formal definition of the deterministic TOP as an introductory and then introduces our stochastic TOP; Section 4 proposes and describes three solution approaches that have been applied to solve the model; in Section 5, the computational experiments and the final results are discussed. To conclude, suggestions for future works are presented in Section 6.

2. Literature review on the team orienteering problem

A useful literature review on the orientering problem (OP) is provided by Vansteenwegen et al. (2011) who discuss many of its variants. The authors conclude that most of the applications reported in the literature focus on activities such as the routing of technicians, city logistics, athlete recruitment, or military logistics, which require very fast and effective solving approaches. They also consider that current solving methods are too time-consuming for solving large-sized instances in practical applications. Regarding TOP, most of the existing articles assume deterministic versions of the problem. However, real life is plenty of uncertainty (e.g., varying weather conditions, unexpected obstacles on the road, etc.). In effect, whenever historical data are available for each of

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the random elements in the considered system, the best-fit probability distribution (or an empirical one) can be used to model the associated random behavior. In addition, there are just a limited number of previous works that consider stochastic versions of the TOP, which use the integration of optimization with simulation (Gosavi, 2015).

2.1. Deterministic TOP

Theoretical papers mostly involve deterministic versions of the TOP. Thus, for instance, Archetti et al. (2007) solve a TOP using two variants of a generalized tabu search algorithm and a VNS algorithm. According to their numerical experiments, the latter provides better results than the former. Similarly, Ke et al. (2008) propose an ant colony optimization algorithm to solve the TOP. The authors prove that the combination of different randomized methods can reduce computing times when searching for near-optimal solutions. Also, Vansteenwegen et al. (2009a) develop a guided local search that combines different heuristics. The authors highlight the need for including a diversification procedure to improve solutions. Vansteenwegen et al. (2009b) present an iterated local search metaheuristic to solve the TOP with time windows. They reach solutions with an average gap of 1.8% compared to some benchmarks. Souffriau et al. (2010) solve the TOP by employing a path relinking heuristic. These authors reach promising results with an average gap of 0.04% with respect to some benchmarks. Inspired by a real-life application, Tricoire et al. (2010) deal with a multiperiod TOP. They propose a VNS algorithm integrated with an exact algorithm. First, they solve the classic version of the TOP and then compare their solutions with some benchmarks. Their approach provides solutions with an average gap of 1.0%. In a similar fashion, Souffriau et al. (2013) solve the TOP by combining an iterated local search framework with a greedy randomized adaptive search procedure. The authors provide solutions with an average gap of 5.2%.

Verbeeck et al. (2014) present a time-dependent OP and propose an algorithm that combines the ideas of an ant colony optimization with a time-dependent local search algorithm. This method provides solutions with a gap of 1.4%. Vidal et al. (2016) solve a vehicle routing problem with profits, which has many similarities with a TOP. These authors propose a neighborhood search that reaches solutions with an average gap of 0.1%. They also highlight the benefits of hybrid solving approaches. Paolucci et al. (2018) introduce a hybrid problem combining a vehicle routing problem and the TOP. The objective is to determine the assignment of locations to vehicles to maximize the collected rewards. In this work, a metaheuristic approach based on a cluster-first and route-second decomposition is proposed. Then it uses a VNS, including a simulated annealing acceptance rule. Estrada-Moreno et al. (2020) introduce a biobjective TOP with a soft constraint associated with the driving range. This work investigates a BR algorithm that penalizes the cost of the routes exceeding the driving range. The BR algorithm considering penalties outperforms the results obtained by other methods for the hard-constrained TOP. In the other work, Ruiz-Meza et al. (2021) discuss the tourism industry. In this research, they consider TOP to construct the group routes with the goal of maximizing the group of travelers' preferences. They propose metaheuristics to solve the problem, and in the end, the results are compared with the available solutions of an exact method. Sankaran et al. (2022) discuss the TOP with multiple depots. In this work, an attention-based model is presented to solve the problem, and they also validate the solution approach by comparing

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the results with the ones provided by heuristics, machine learning, and exact solvers on several reconnaissance scenarios. In the end, the results show that the data generation approach presented is highly effective.

2.2. Stochastic OP and stochastic TOP

Concerning the stochastic OP, Ilhan et al. (2008) introduce uncertainties in the collected profits, while Campbell et al. (2011) solve an OP with time windows where both travel and service times are stochastic parameters. In these works a descendant VNS based on the best improvement is considered. Likewise, Papapanagiotou and Montemanni (2014) discuss the OP with stochastic travel and service time. In this work, a Monte Carlo approximation is applied to estimate the objective function. In the end, by comparing the results with the ones achieved by other analytical ways, Monte Carlo approximation is able to find better solutions. In a similar work, Papapanagiotou et al. (2015) consider the stochastic OP with probabilistic travel and service times, and the performance of different hybrid objective function evaluators are compared. The final results show that Monte Carlo sampling is working better in comparison with other evaluators. Zhang et al. (2017) analyze the probabilistic tour problem, where each location has a probability of requiring a visit. As usual, the objective is to select the nodes to be visited in order to maximize the expected profit. Also, Chou et al. (2018) present a Monte Carlo sampling technique as a part of a heuristics method for a stochastic OP. In this work, the availability of customers with a certain probability is considered as a stochastic challenge in the orienteering problem. They show that Monte Carlo sampling is a fast and efficient way to use in heuristics solvers. Also, Liao and Zheng (2018) develop an algorithm that considers tourists' behaviors to design routes for a daily tour. A simulation–optimization algorithm solves an OP under time-dependent stochastic environments. De Carolis et al. (2018) focus on an OP to solve a routing problem using underwater unmanned aerial vehicles. They consider stochastic external disturbances, which could increase the length of the route. Then, a regression model is used to perform a probabilistic prediction for defining the navigation route. In other work, Bian and Liu (2018) present an OP with stochastic travel and service times. This work considers a dynamic method where, after each customer's visit, the state of the remaining time budget is updated for reoptimizing the routing plan.

With respect to the stochastic TOP, only few articles have been published so far, most of them referring to random rewards. Thus, for example, Erdoğan and Laporte (2013) consider a TOP version in which service times are not deterministic, but they define a finite number of scenarios for random service times. Similarly, Afsar and Labadie (2013) use column generation to analyze a TOP with stochastic rewards. In a similar work, Panadero et al. (2017) discuss the stochastic TOP with random customer service times or travel times. A simheuristics algorithm consists of the metaheuristics, and Monte Carlo simulation (MCS) is applied to solve the proposed problem. Also, Bayliss et al. (2020) discuss the stochastic TOP with dynamic rewards and stochastic travel times. To solve this problem, a learnheuristic, consisting of BR heuristics and a learning module, is applied. A simulation during the search process is also applied to achieve the dynamic rewards. In the context of surveillance drones, Panadero et al. (2020b) present a simulation–optimization algorithm for solving the stochastic TOP with random travel times. The authors explore the relevance of combining heuristics with simulation to solve stochastic TOPs. However, they do not

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explore other alternative methodologies such as SAA. In recent years, Thayer and Carpin (2021) consider the stochastic TOP with random travel times. In this work, they use an adaptive algorithm based on the Markov decision process. For a given deadline and the current position of vehicles in the solution path, it should decide whether the vehicle continues its route or choose a shortcut to avoid missing the deadline. They also apply an MCS to determine how to discretize the temporal dimension in order to reduce the number of states and the number of variables in the associated optimization program. In Rabe et al. (2021), the authors discuss the hospital logistics during the COVID-19 crisis. This study considers the transportation of needed health elements as well as considering a stochastic TOP model with random travel times. Eventually, their proposed model is solved by employing a BR simheuristics algorithm. Finally, Panadero et al. (2021) address the smart cities and the recent technologies implemented in the vehicles, such as telecommunication systems, Internet-based technologies, and satellite services, to improve the efficiency of vehicles. In this work, a real-time TOP is considered due to the need for online decision making in a smart city, and an agile optimization algorithm based on fast BR heuristics and a parallel computing approach is presented as the solution approach.

3. Problem formal description

In this section, first we consider the mathematical model of the deterministic TOP. Then we extend the model to a two-stage model and present an stochastic formulation.

3.1. The deterministic TOP

In this subsection, the deterministic mathematical formulation of TOP is explained. The presented model is based on the formulation proposed by Evers et al. (2014). The network is represented by a directed graph $G = (N', A)$ consist of N' nodes, and A arc connections. The set $N' = \{0, 1, 2, \dots, n+1\}$ consists of node 0 (origin depot), node $n+1$ (destination depot), and $N = \{1, \dots, n\}$ intermediate nodes, and $A = \{(i, j)/i, j \in N', i \neq j\}$ is the set of the connection routes between the nodes. Let us assume D is the set of homogeneous vehicles, and each vehicle $d \in D$ starts its route from the origin depot, serves some of the intermediate nodes, and at the end, it goes to the destination depot. In the deterministic definition of TOP, we assume that the traveling time of each route is predefined ($t_{ij} = t_{ji} > 0$). Each vehicle starts moving on a route, and it can serve only some nodes since each vehicle has a limited time to travel (t_{max}), and before the end of its journey time must reach the end point. Serving the intermediate nodes for the first time gives rise to achieving a reward $u_i \geq 0$, and the goal is to maximize the total rewards collected by all vehicles. Note that the origin and destination depots have no associated rewards. For each arc $(i, j) \in A$ and each vehicle $d \in D$, we consider the binary variable x^d_{ij}, which is equal to 1 if vehicle d walks through edge (i, j), and takes the value 0 otherwise. Likewise, the variable y^d_i is introduced to indicate the position of node i in the tour made by vehicle d. Based on these definitions, the mathematical model of the deterministic TOP is the following:

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$$\max \sum_{d \in D} \sum_{(i,j) \in A} u_j x_{ij}^d \tag{1}$$

$$\text{s.t.} \sum_{d \in D} \sum_{i \in N'} x_{ij}^d \leq 1 \quad \forall j \in N \tag{2}$$

$$y_i^d - y_j^d + 1 \leq (1 - x_{ij}^d)|N| \quad \forall i, j \in N \quad \forall d \in D \tag{3}$$

$$\sum_{(i,j) \in A} t_{ij} x_{ij}^d \leq t_{max} \quad \forall d \in D \tag{4}$$

$$\sum_{i \in N'} x_{ij}^d = \sum_{h \in N'} x_{jh}^d \quad \forall d \in D \quad \forall j \in N \tag{5}$$

$$\sum_{j \in N} x_{0j}^d = 1 \quad \forall d \in D \tag{6}$$

$$\sum_{j \in N} x_{jn+1}^d = 1 \quad \forall d \in D \tag{7}$$

$$y_j^d \geq 0 \quad \forall j \in N \quad \forall d \in D \tag{8}$$

$$x_{ij}^d \in \{0, 1\} \quad \forall i, j \in A \quad \forall d \in D. \tag{9}$$

Equation (1) denotes the objective function to be maximized. Constraints (2) ensure that each node should be serviced at most once. Constraints (3) prevent the construction of subtours. Constraints (4) state that the total travel time of each vehicle should not be more than its threshold. Constraints (5) is a flow balance constraint, and ensure that any arrival to a node has to be compensated with a departure. Constraints (6) and (7) state that all the vehicles should come out from the original depot (node 0), and after traveling their routes, arrive to the destination depot (node $n + 1$). Finally, constraints (8) and (9) refer to the nature of y_j^d and x_{ij}^d variables.

3.2. Stochastic TOP model

In the stochastic version of the TOP, we assume that the travel times between the nodes are not deterministic, and each arc $(i, j) \in A$ is defined by a travel time, $t_{ij} = t_{ji} > 0$ that follows a best-fit probability distribution function with mean $\mathbb{E}[t_{ij}] > 0$. If a vehicle cannot complete the planned route before the deadline, it will lose all the rewards earned during its travel. We are assuming here that all the partial rewards are valid only if the vehicle reaches the destination node before the end of its travel time. The goal is to maximize the expected profits of the routes given a time distribution:

$$\max\{f(x) := \mathbb{E}[F(x, t)]\}, \tag{10}$$

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where $F(x, t)$ is the utility (or reward) function obtained from a particular tour x and travel time realizations t, and $\mathbb{E}[F(x, t)]$ represents the expected reward with respect to the distribution of t (since t_{ij} is a random variable, we define the probability distribution of t as a specified distribution function P). Also, we assume that t is well represented by a set of scenarios S with given probabilities π such that $\sum_{s\in S} \pi_s = 1$ and the probability of each scenario is equal to $\frac{1}{|S|}$. Let us introduce positive variables, r^{ds}, that compute the total rewards that vehicle $d \in D$ has reached in the scenario $s \in S$. Also let l^{ds} be binary variables that take the value 1 if vehicle $d \in D$ in scenario $s \in S$ cannot complete its route within the time frame, and takes the value 0 otherwise. Based on these definitions, the stochastic TOP is modeled as

$$\max \frac{1}{|S|} \sum_{s\in S} \sum_{d\in D} r^{ds} \tag{11}$$

Constraints (2), (3), (5), (6), (7), (8), and (9)

$$\sum_{(i,j)\in A} t^s_{ij} x^d_{ij} \leq t_{max} + Bl^{ds} \quad \forall s \in S \quad \forall\, d \in D \tag{12}$$

$$r^{ds} \leq \sum_{(i,j)\in A} u_j x^d_{ij} \quad \forall\, s \in S \quad \forall\, d \in D \tag{13}$$

$$r^{ds} \leq B(1 - l^{ds}) \quad \forall\, s \in S \quad \forall\, d \in D \tag{14}$$

$$r^{ds} \geq 0 \quad \forall\, s \in S \quad \forall\, d \in D \tag{15}$$

$$l^{ds} \in \{0, 1\} \quad \forall\, s \in S \quad \forall\, d \in D. \tag{16}$$

The objective function (11) maximizes the expected reward of the routes solution. In the stochastic TOP, the capacity constraints (12) compute the time used by vehicle d in traversing its route in a particular scenario s. If the route total time exceeds the given threshold t_{max} for a particular scenario s, the variable l^{ds} is activated and then extra time B (a large value) is allowed for such particular scenario.

Two sets of constraints are employed to model the fact that the reward for a particular route could be either the sum of the reward collected at each node visited by vehicle d, or 0, if the total time exceeds the maximum time allowed. The right-hand side of constraints (13) computes the total reward collected by vehicle d along the route. Since r^{ds} is maximized in the objective function, the optimization will favor r^{ds} to achieve the highest value. However, if the route of vehicle d in scenario s exceeds the total time, constraints (14) will be active and force the reward r^{ds} to be 0. Constraints (15) and (16) refer to the nature of the extra variables added in the stochastic TOP.

3.3. The two-stage approach

The stochastic model presented fits as a two-stage problem. The decision in the first stage is to find the routes,that is, a path or a tour, through a graph. In the second stage, the rewards under

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the different scenarios are assessed. In order to compute the rewards, the recourse action allows an extra time if the time limit constraint for each vehicle is not satisfied, and then the vehicle receives no rewards in such a particular scenario. The aim of the two-stage problem is to find a tour such that the first-stage constraints are satisfied by considering the second-stage constraints, and finally the goal is to maximize the total expected rewards for each vehicle in the tour.

4. Alternative approaches for solving the stochastic TOP

In this section, three solution approaches are presented for solving the stochastic TOP: the SAA algorithm, a simheuristic approach (Sim-BRVNS), and a hybrid model that combines the metaheuristic used in our simheuristic approach with the SAA algorithm. All these solution approaches are designed to solve optimization problems. In this paper, we consider a simulation part for the SAA method in order to generate sample observations for the travel times.

4.1. The sample average approximation method

This subsection provides the general framework of the SAA method adapted for solving the stochastic TOP. As mentioned earlier, the resulting objective function for our stochastic TOP is Equation (10). To evaluate the objective function $f(x)$, the expected value of the linear function $F(x, t)$ should be computed. There is always a potential problem in solving stochastic problems. For continuous distributions, these expectations become multivariate integrals. Computing these integrals can be approached by discretization. For discrete distributions, it requires solving a large number of linear problems for each realization of the uncertain parameter (Shapiro, 2006). To deal with this potential problem, we use the SAA method that is based on randomization by Monte Carlo sampling techniques. This method has been applied to a variety of problems (Shen et al., 2011).

The core idea of the SAA is (i) to convert the original stochastic problem into a finite number of deterministic-equivalent subproblems (e.g., by generating random values for each stochastic term in the problem); (ii) to solve each of the deterministic subproblems using standard approaches (either of exact or approximate nature); and (iii) to analyze the distribution of the solutions obtained in the previous step (e.g., using the average value of the obtained solutions as an estimate of the expected value of the stochastic problem). Note that the deterministic subproblems are generated from the random scenarios using the probability distribution function of the stochastic parameters. If some conditions of the problem are met, the method is able to solve the problem with an acceptable level of accuracy (Shapiro, 2006). Figure 2 illustrates the different steps of SAA algorithm, which are described in detail below.

In the SAA scheme, M random independent samples each of size S are introduced, that is, $(t_1^j, t_2^j, \ldots, t_S^j)$ for $j = 1, 2, \ldots, M$. The samples are generated from the probability distribution function of the travel times (P). The probability of each sample is $\frac{1}{|S|}$. The larger the size of S, the better the approximation. However, increasing this number also increases the computation time of solving the problem (Kleywegt et al., 2002).

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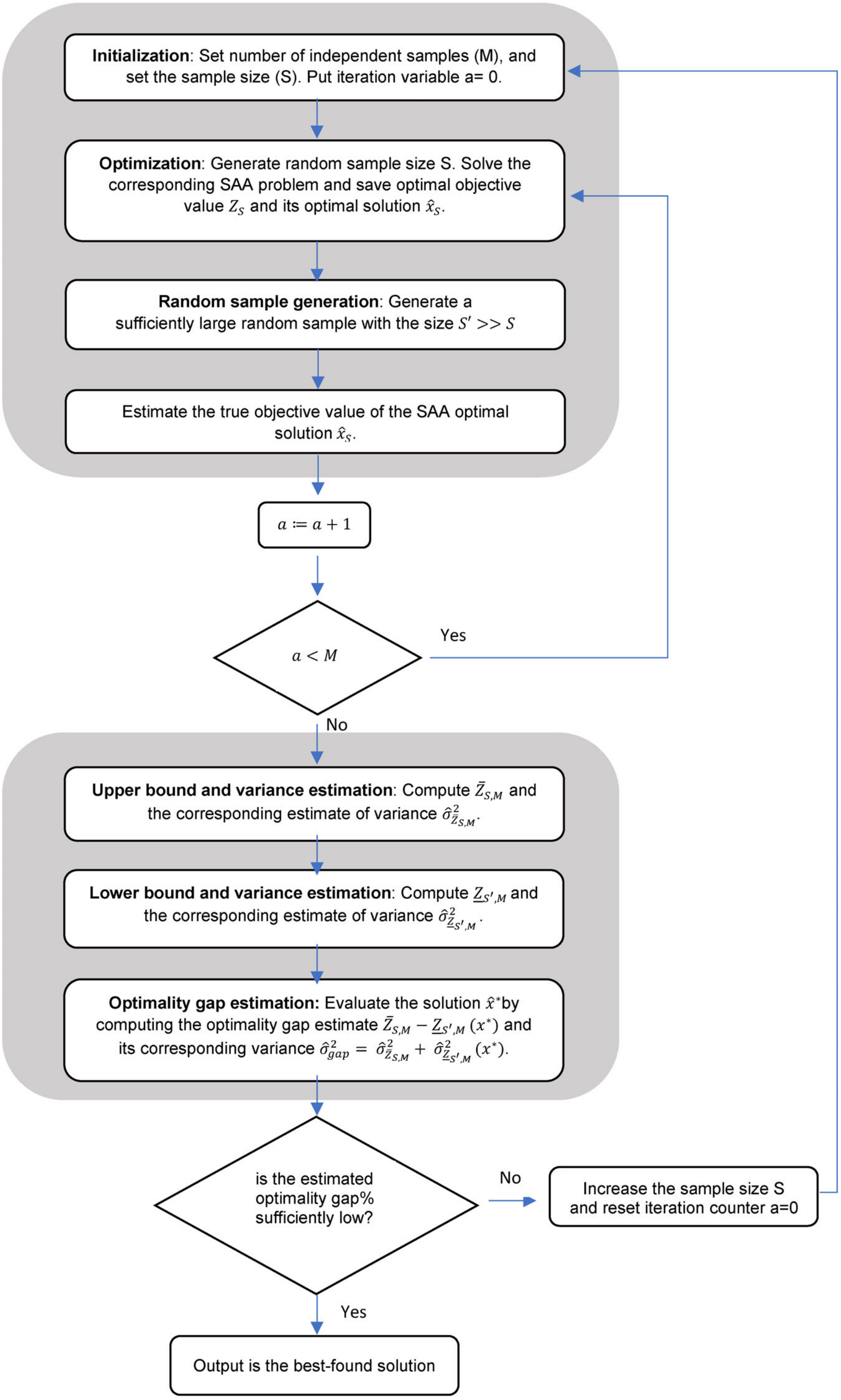

Fig. 2. Flowchart of the SAA algorithm.

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The expectation $\mathbb{E}[F(x,t)]$ can be estimated as follows:

$$Z = \frac{1}{|S|}\sum_{s\in S} F(x, t_s^j). \tag{17}$$

Equation (17) estimates the objective function for solving the SAA subproblems that should be maximized. The process of solving several subproblems with different samples is repeated M times, and each time results in a candidate solution. In every iteration, a random sample should be chosen and the corresponding subproblem is solved. Finally, the optimal solutions are $\hat{x}_S^1, \hat{x}_S^2, \ldots, \hat{x}_S^M$ and $Z_S^1, Z_S^2, \ldots, Z_S^M$ are the optimal objective functions. The average of the optimal objective functions of M subproblems is

$$\bar{Z}_{S,M} = \frac{1}{M}\sum_{m=1}^{M} Z_S^m. \tag{18}$$

It is proved that $\bar{Z}_{S,M}$ is a statistical approximation for the upper bound on the Z^* (Mak et al., 1999). For estimating the lower bound, we choose a candidate solution $\hat{x}$ (an optimal solution provided by solving the SAA problem). The value of the true objective function for $\hat{x}$ is a lower bound for the optimal objective function (Mak et al., 1999). By generating and solving $S^{'}$ independent random samples $t_1, t_2, \ldots, t_{S'}$ with a very large scenario size $S^{'} >> S$, following the same probability distribution function, the lower bound is computed as

$$\underline{Z}_{S',M}(\hat{x}) = \frac{1}{S'}\sum_{s=1}^{S'} F(\hat{x}, t^s). \tag{19}$$

Using all candidate solutions for estimating the lower bound, the solution that has the highest value is

$$\hat{x}^* \in \arg\max\{\underline{Z}_{S',M} : \hat{x} \in \{\hat{x}^1, \ldots, \hat{x}^M\}\}. \tag{20}$$

The variances of the upper and lower bounds are estimated as follows:

$$\hat{\sigma}^2_{\bar{Z}_{S,M}} = \frac{1}{M(M-1)}\sum_{m=1}^{M}(Z_S^m - \bar{Z}_{S,M})^2 \tag{21}$$

$$\hat{\sigma}^2_{\underline{Z}_{S',M}(\hat{x}^*)} = \frac{1}{S'(S'-1)}\sum_{s=1}^{S'}(F(\hat{x}^*, t) - \underline{Z}_{S',M}(\hat{x}^*))^2. \tag{22}$$

The evaluation of the optimal gap and its variance for the solution $\hat{x}^*$ are

$$\bar{Z}_{S,M} - \underline{Z}_{S',M}(\hat{x}^*) \tag{23}$$

$$\hat{\sigma}^2_{\bar{Z}_{S,M}-\underline{Z}_{S',M}(\hat{x}^*)} = \hat{\sigma}^2_{\bar{Z}_{S,M}} + \hat{\sigma}^2_{\underline{Z}_{S',M}(\hat{x}^*)}. \tag{24}$$

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The algorithm stops when the optimal gap is sufficiently low, and the achieved optimal solution is the best solution to the problem. Otherwise, a larger sample size S should be selected and the algorithm is repeated. The SAA algorithm replicates until the maximum number of iteration is reached.

4.2. Our Sim-BRVNS simheuristic algorithm

Simheuristics refer to a particular type of simulation-based optimization technique that combines simulation with heuristics or metaheuristics to solve optimization problems with stochastic components in their objective function or constraints. Simheuristics assume that high-quality solutions to deterministic versions of the optimization problems are likely to be high-quality solutions to their stochastic counterparts, at least up to a certain degree of variability in the random elements. But it does not necessarily imply that the best deterministic found solution outperforms other high-quality deterministic solutions under an uncertain scenario. Thus, simheuristics use a metaheuristic component to generate high-quality solutions for the deterministic version of the problem, and then it performs a simulation on the most promising deterministic solutions to estimate the value of the objective function in a stochastic scenario. The simulation part helps identify the uncertain character of the system as well as investigate the problem complexity by computing the statistics. Simheuristic has been successfully applied in different fields. For instance, Gonzalez-Martin et al. (2018) propose using simheuristics to solve an arc routing problem with stochastic demand. Their approach combines MCS with a BR heuristic. In another work, Panadero et al. (2020b) present a simheuristic approach to solve the stochastic TOP of unmanned aerial vehicles. A simulation incorporated into a VNS metaheuristic is considered as the simheuristic solution to this problem. Keenan et al. (2021) present a strategic oscillation simheuristic for the time-capacitated arc routing problem with stochastic demands.

In order to solve the stochastic TOP described in the previous section, a simheuristic approach is proposed. It combines a biased-randomized variable neighborhood search (BRVNS) metaheuristics to generate deterministic solutions, with MCS to deal with the stochastic nature of the problem. As discussed in Panadero et al. (2020a), the VNS offers a well-balanced combination of efficiency and relative simplicity and can be easily extended to a simheuristic. Figure 3 illustrates the flowchart of the main steps on our solution approach, based on three stages, which are described next.

- First, a feasible initial solution is generated using a constructive heuristic. Due to the particularities of the TOP problem, the heuristic has to consider the following aspects: (i) the origin and destination nodes are not necessarily the same; (ii) it is not compulsory to visit all the nodes; and (iii) the collected reward—and not just the savings in time or distance—must be also considered during the construction of the routing plan. The heuristic works by generating an initial "dummy" solution in which a route connects each location with the origin and destination depots. Afterward, these dummy routes are iteratively merged. In this phase, the heuristic tries to merge as many routes as possible, as long as the total travel time of the routes does not exceed the threshold defined for each route. In order to merge these routes, a list of potential merging arcs, sorted from higher to lower efficiency, is generated. The efficiency associated with an arc (i, j) is given by $e_{ij} = \alpha \cdot s_{ij} + (1 - \alpha) \cdot (u_i + u_j)$, where $s_{ij} = t_{i(n+1)} + t_{0(j)} - t_{ij}$ represents the time-based

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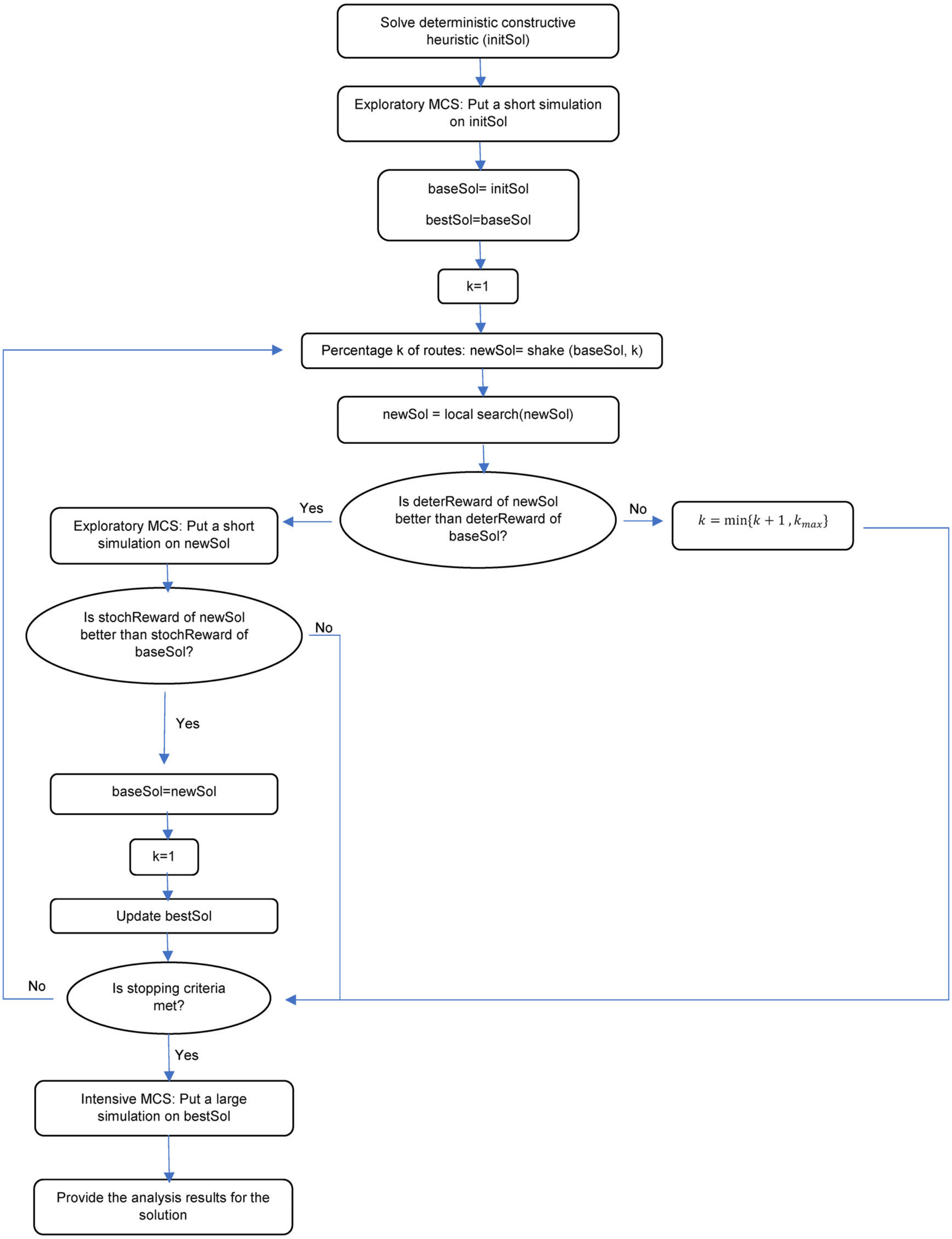

Fig. 3. Flowchart of the Sim-BRVNS algorithm.

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savings obtained with the merge, and $u_i + u_j$ is the aggregated reward. The parameter $\alpha \in (0, 1)$ depends on the heterogeneity of nodes in terms of rewards, and it needs to be empirically tuned. Thus, in a scenario with high heterogeneity, α will be close to 0. On the contrary, α will be close to 1 for homogeneous scenarios. To automatically tune α for each scenario, the constructive heuristic is executed 10 times for a range of values from 0 to 1, with α incremented by 0.1 in each new run. Once this merging process is completed, the initial solution *initSol* is generated.

- Second, the BRVNS metaheuristics improve the current solution by using MCS in iterative runs in order to search the solution space and generate a new starting point, and subsequently exploring the neighborhood of this new solution by applying a local search procedure in search of a better solution. The process is repeated until the maximum execution time (t_{max}) is met. The simulation runs are used to obtain rough estimates of the solution behavior under stochastic conditions, which allows us to generate a pool of "elite" solutions. Specifically, the process starts by shaking the base solution (*baseSol*) to generate a new solution (*newSol*). This step consists in a destruction–reconstruction procedure that randomly deletes a percentage of routes from the *baseSol*. The degree of destruction to be applied in the shaking phase depends on the value of a parameter (k), which starts at 1% and slowly increases at each iteration until it reaches the maximum level of destruction (100%). In each new iteration, the destruction level is increased by 1%. This value is reset every time the base solution is updated. To reconstruct the solution, we employ a BR version of the aforementioned constructive heuristic, transforming it into a probabilistic one. As proposed in Grasas et al. (2017), a geometric probability distribution, driven by a single parameter β_1 ($0 < \beta_1 < 1$), is employed to induce a biased (nonsymmetric) randomization effect. For experimentation purposes, after a fine-tuning process, we have set the β_1 value to 0.3. Next, the algorithm starts a local search procedure around the *newSol*, where two local searches are applied. During the first local search a 2-opt procedure (Croes, 1958) is applied to each route until it cannot be further improved. Also, for a given set of nodes, a fast-access data structure (e.g., a hash map) is employed to save the best-found-so-far route. Then, a second local search is performed. This second local search tries to improve the routes by inserting new nonserviced locations—as far as no constraint is violated. In order to that, a subset of locations (between 5% and 10% of them) are randomly removed from the new solution. Then, the deterministic insertion algorithm proposed by Dang et al. (2013) is used. We transformed this algorithm into a BR algorithm by assigning probabilities to the different nodes to be selected. Again, a geometric distribution is introduced to randomize the selection process. As before, we set the parameter of this second geometric distribution to $\beta_2 = 0.3$. When this local search process has been completed, a new solution (*newSol*) is returned. So far, this *newSol* is deterministic. In order to deal with the stochastic nature of the problem, each time that a *newSol* improves the *baseSol* in terms of deterministic reward, this *newSol* is sent through a fast simulation process to estimate its expected reward under uncertainty conditions. Moreover, this simulation process provides feedback that can be used by the metaheuristic to better guide the search. Indeed, the selection of the base and best solutions is driven by the results of the simulation process. If the stochastic reward of *newSol* outperforms the one of the *baseSol*, the latter solution is updated and the parameter k (percentage of destruction) is reset to 1%. In the same way, if the stochastic reward of the *newSol* outperforms the one of the best-found-so-far solution (*bestSol*), the latter is updated and added to the pool of elite solutions. From this stage, a reduced pool of elite solutions is obtained. Finally, if the algorithm has not reached the

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maximum computing time allowed (t_{max}), the previous steps are repeated in order to generate new elite solutions.

- Once the previous steps have been completed, before reporting the final results, an intensive MCS is carried out to assess better the solutions of the pool of elite solutions. This allows us to obtain more accurate estimates on their expected rewards. Since the number of generated solutions during the search can be large and the simulation process is time-consuming, we limit the number of MCS iterations to be executed. For our approach, the number of iterations for the exploratory and intensive MCS stages were set to 200 and 10,000, respectively.

4.3. SAA-BRVNS algorithm

To combine the SAA and BRVNS, a hybrid algorithm is proposed. In this algorithm, the fundamentals of SAA are preserved. As it is explained in Section 4.1, the SAA method consists of two steps. In the first step, the stochastic problem should be solved for a number of iterations. In each iteration, the model is solved by getting new values for the stochastic parameter produced by the probability distribution function. The output of the first step is building the best solution in the best replication. In the second step, a refinement of the first step output should be done. In this step, the problem must be solved by considering a very large number of scenarios. At the end, the solution of the first and second steps, which are called upper bound and lower bound, should be saved. The goal is to minimize the gap between these two bounds. In the hybrid algorithm, instead of using any optimization solver (CPLEX has been used in the plain SAA implementation) to solve the stochastic optimization problem, we use the presented BRVNS algorithm forcing to have a single-scenario problem (i.e., $|S| = 1$). At each iteration of the first step, the optimization problem is solved with the BRVNS heuristic instead of CPLEX. In the second step, the refinement procedure is carried out by the simulation element of the algorithm.

5. Computational experiments

This section presents the numerical instances that have been solved by three aforementioned methods to compare their efficiency and accuracy in solutions. The SAA is employed in Python in combination with the solver CPLEX in its version 12.9. The implementation is available at https://github.com/apages/SampleAverageApproxTOP. For the SAA-CPLEX, a total of 25 scenarios were considered, when the number of replications is set to be 10. Also, the total number of scenarios for evaluating the second stage of SAA is set to be 1000. The maximum computational time for solving the instances with SAA is limited to 10,000 seconds. Regarding the Sim-BRVNS, it was implemented using the Java programming language, and executed five times per instance—each one using a different seed for the pseudo-random number generator—and only the best solution across these runs is kept. The number of runs for the exploratory and intensive MCS stages were set to 200 and 10,000, respectively. Finally, in the SAA-BRVNS, 10 replications are considered for the first step, and as in the SAA-CPLEX algorithm, the number of scenarios for the first and second steps are set as 25 and 1000, respectively. The maximum computational time for solving each

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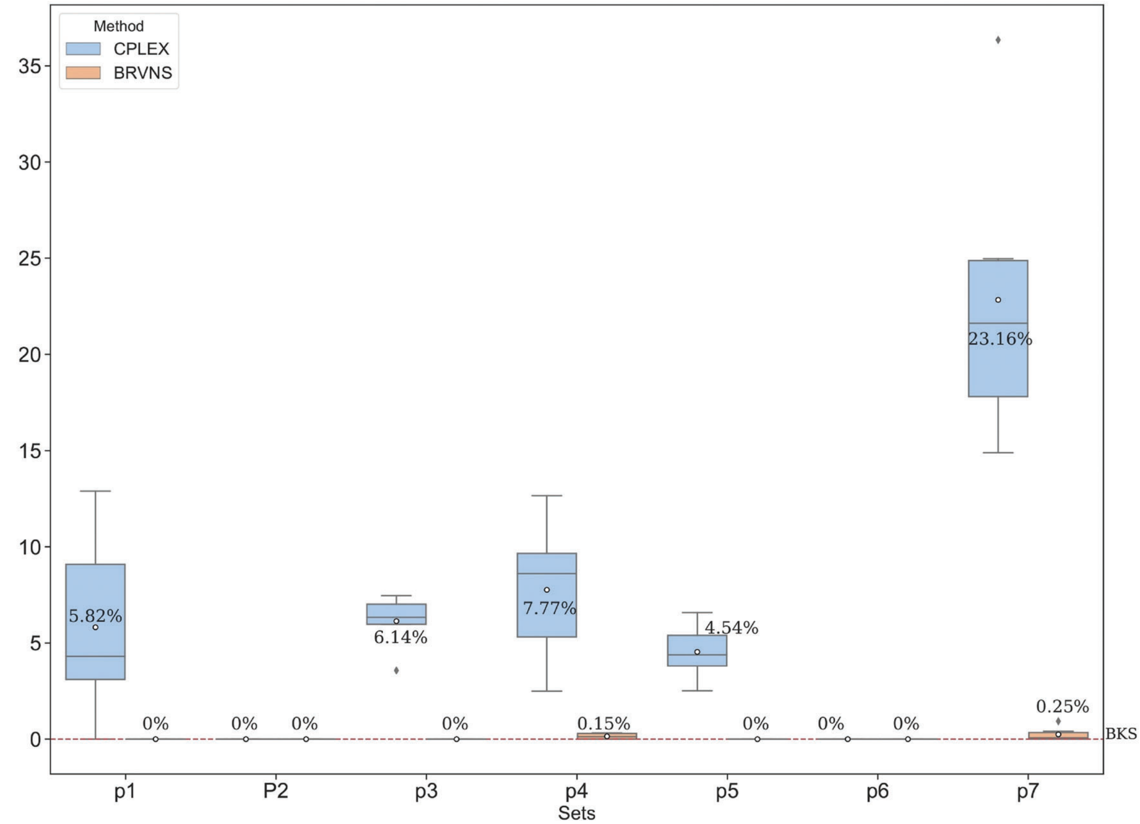

Fig. 4. Comparison of solutions obtained by CPLEX and BRVNS with BKS for deterministic TOP.

instance is 2000 seconds. All tests were run on a workstation with a multicore processor Intel Xeon E5-2630 v4 and 32 GB of RAM.

5.1. Solving the deterministic TOP

To evaluate the performance of the proposed solution approaches for the stochastic TOP, we used the classical TOP benchmark presented by Chao et al. (1996). This benchmark is widely used in the literature to test the performance of algorithms whose purpose is to solve the classical version of the TOP. The benchmark is divided into seven different subsets, which include a total of 320 instances. Each instance is characterized by the following nomenclature *pa.b.c*, where *a* represents the identifier of the subset; *b* is the number of vehicles; and finally, *c* symbolizes the maximum driving range. For our experiments, we have selected six instances from each of the seven subsets. We considered only instances with a "sufficient" driving range—one that allows us to reach the most distant node in the instance. Also, for instances sharing the same network, we selected the one with the largest number of vehicles available. Before solving the stochastic version of the TOP, we analyzed the two deterministic solution approaches (CPLEX and the BRVNS algorithm)

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performance. To compute the deterministic solutions using the BRVNS algorithm, we have executed the algorithm disabling the simulation parts (fast and intensive simulation process). Thus, this allows the algorithm to consider only the deterministic counterpart of the solution during the search space exploration. Figure 4 depicts an overview of the results. In this box plot, the vertical axis represents the gap obtained with respect to the best-known solution (BKS) reported by Vidal et al. (2016). Note that the BRVNS algorithm reaches the BKS for all of the instances in sets 1, 2, 3, 5, and 6, obtaining a 0.0% gap. Regarding the remaining classes, the BRVNS algorithm obtains average gaps of less than 0.25% in very short computing times, which proves that our BRVNS algorithm is highly competitive for the deterministic version of the TOP.

5.2. Solving the stochastic TOP

In this section, the SAA-CPLEX method, our BRVNS simheuristic algorithm, and the hybrid SAA-BRVNS are applied to solve the stochastic TOP. We used the data set proposed by Chao et al. (1996) with some modifications as follows: the deterministic travel times are used as the expected values of the random travel times, which follow a log-normal probability distribution—which constitutes a "natural" and flexible choice for modeling random variables. In a real scenario, historical data could be used to determine the most appropriate distribution for modeling the travel times. The log-normal distribution has two parameters: the location parameter, μ, and the scale parameter, σ, which relate to the expected value $E[T_{ij}]$ and the variance $\mathrm{Var}[T_{ij}]$, respectively. Equations (25) define how these parameters have been modeled. We assume that $T_{ij} \sim LogNormal(\mu_{ij}, \sigma_{ij})$, with $E[T_{ij}] = t_{ij}$ and $\mathrm{Var}[T_{ij}] = c \cdot t_{ij}$, being $c > 0$ a parameter that determines different levels of uncertainty. It is expected that the more the value of parameter c is toward zero, the more the problem will incline toward the deterministic state, and by increasing the value of c, the problem will converge to the stochastic state. We have considered three different levels of uncertainty: low ($c = 0.05$), medium ($c = 0.25$), and large ($c = 0.75$):

$$\begin{aligned} \mu_{ij} &= \ln\left(E[T_{ij}]\right) - \frac{1}{2}\ln\left(1 + \frac{\mathrm{Var}[T_{ij}]}{E[T_{ij}]^2}\right) \\ \sigma_{ij} &= \left|\sqrt{\ln\left(1 + \frac{\mathrm{Var}[T_{ij}]}{E[T_{ij}]^2}\right)}\right|. \end{aligned} \tag{25}$$

Tables 1–3 show the results obtained for the tested samples by considering three different levels of variability. The first column of these tables defines the instance, while the second column reports the total number of nodes in each set, including the origin and destination nodes. The third column (*Reward in a deterministic environment*) provides the BKS for the deterministic version of the problem ($c = 0$), while the following two columns display the expected cost obtained when the deterministic solution is evaluated under a stochastic scenario, and its associated reliability, respectively. The reliability measures the robustness of the stochastic solution. In this context, reliability computes the percentage of routes that are completed without violating the driving-range constraint in the stochastic environment. To compute the *Reward in a stochastic environment*, we have executed just the algorithm disabling the simulation part (fast simulation process), and we have applied the

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Table 1
Results of solving the stochastic TOP by SAA-CPLEX, Sim-BRVNS, and SAA-BRVNS with a low variance level ($c = 0.05$)

		Solution to the deterministic problem (deterministic solution)			SAA-CPLEX solution to stochastic problem			SIM-BRVNS solution to stochastic problem			SAA-BRVNS solution to stochastic problem			GAPS (%)					
Instance	n	Reward in a deterministic environment	Reward in a stochastic environment (a)	Reliability	Reward in a stochastic environment (b)	Reliability	Time (seconds)	Reward in a stochastic environment (c)	Reliability	Time (seconds)	Reward in a stochastic environment (d)	Reliability	Time (seconds)	(a)–(b)	(a)–(c)	(a)–(d)	(b)–(c)	(b)–(d)	(c)–(d)
p1.4.m	32	130	113.0	0.86	119.7	1.00	8294	125.9	1.00	38	96.0	0.81	1470	−6.0	−11.5	15	−5.1	19.8	23.7
p1.4.n		155	120.2	0.76	129.9	1.00	8270	135.3	1.00	9	59.7	0.45	1418	−8.1	−12.6	50.3	−4.2	54.0	55.9
p1.4.o		165	139.4	0.85	140.1	0.88	10,080	148.7	1.00	98	103.5	0.57	1502	−0.5	−6.7	25.8	−6.1	26.1	30.4
p1.4.p		175	142.3	0.84	149.3	0.93	10,084	161.7	1.00	32	107.7	0.60	1834	−4.9	−13.7	24.3	−8.3	27.9	33.4
p1.4.q		190	161.2	0.86	167.1	0.95	10,089	173.6	0.99	60	70.3	0.37	1154	−3.7	−7.7	56.4	−3.9	57.9	59.5
p1.4.r		210	159.1	0.78	179.6	1.00	8558	192.5	1.00	17	153.9	0.79	1369	−12.9	−21.0	3.3	−7.2	14.3	20.1
p2.4.f	21	105	87.5	0.71	95.3	0.86	3060	97.6	0.95	64	75.3	0.78	2183	−8.9	−11.4	14.0	−2.4	21.0	22.8
p2.4.g		105	92.1	0.91	100.9	0.76	3041	103.0	0.77	41	81.1	0.74	371	−9.5	−11.8	12.0	−2.1	19.6	21.3
p2.4.h		120	102.6	0.88	111.2	0.92	2070	112.8	0.98	7	83.0	0.84	1075	−8.4	−10.0	19.1	−1.4	25.4	26.4
p2.4.i		120	102.8	0.83	118.0	0.98	2005	119.3	0.98	97	106.2	0.94	1075	−14.8	−16.0	−3.3	−1.1	10.0	11.0
p2.4.j		120	115.3	0.95	119.2	0.80	2022	119.5	0.96	102	114.1	0.79	1085	−3.4	−3.6	1.0	−0.2	4.3	4.5
p2.4.k		180	133.4	0.79	143.8	0.88	3056	159.2	0.99	61	137.6	0.76	2085	−7.8	−19.3	−3.1	−10.7	4.4	13.6
p3.4.o	33	500	421.5	0.84	438.4	0.97	9195	472.7	0.98	6	382.3	0.76	1167	−4.0	−12.2	9.3	−7.8	12.8	19.1
p3.4.p		560	457.0	0.85	489.3	1.00	9161	513.4	1.00	10	402.3	0.66	1176	−7.1	−12.4	12.0	−4.9	17.8	21.6
p3.4.q		560	494.6	0.91	510.0	1.00	9225	555.1	1.00	11	470.1	0.83	1477	−3.1	−12.2	5.0	−8.8	7.8	15.3
p3.4.r		600	488.1	0.88	539.9	1.00	9189	562.9	1.00	15	361.2	0.67	823	−10.6	−15.3	26.0	−4.3	33.1	35.8
p3.4.s		670	501.5	0.71	560.0	1.00	9218	585.7	1.00	17	425.5	0.63	1189	−11.6	−16.8	15.2	−4.6	24.0	27.4
p3.4.t		670	599.7	0.90	626.8	0.99	9239	666.3	0.96	12	650.3	0.85	1203	−4.5	−11.1	−8.4	−6.3	−3.7	2.4
p6.3.n	64	1170	982.7	0.53	1031.7	0.93	9213	1072.1	0.90	17	747.5	0.57	1889	−5.0	−9.1	23.9	−3.9	27.5	30.3
p6.4.j		366	215.4	0.59	233.9	0.72	10,141	235.2	0.76	489	92.6	0.36	1737	−8.6	−9.2	57.0	−0.6	60.4	60.6
p6.4.k		528	313.2	0.59	322	0.8	10,149	359.3	0.67	116	127.1	0.28	1693	−2.8	−14.7	59.4	−11.6	60.5	64.6
p6.4.l		696	412.3	0.60	440.6	0.86	9212	485.1	0.92	22	297.8	0.57	2077	−6.9	−17.7	27.8	−10.1	32.4	38.6
p6.4.m		912	586.2	0.54	625.9	0.87	9208	673.9	0.89	545	330.0	0.41	1851	−6.8	−15.0	43.7	−7.7	47.3	51.0
p6.4.n		1068	734.0	0.59	781.0	0.95	9225	843.9	0.97	6	380.1	0.45	1886	−6.4	−15.0	48.2	−8.1	51.3	55.0
Average		420	319.8	0.77	340.6	0.92	7625	361.4	0.94	79	244.0	0.65	1450	−6.9%	−12.7%	22.2%	−5.5%	27.3%	31.0%

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Table 2
Results of solving the stochastic TOP by SAA-CPLEX, Sim-BRVNS, and SAA-BRVNS with a medium variance level ($c = 0.25$)

		Solution to the deterministic problem (deterministic solution)			SAA-CPLEX solution to stochastic problem			SIM-BRVNS solution to stochastic problem			SAA-BRVNS solution to stochastic problem			GAPS (%)					
Instance	n	Reward in a deterministic environment	Reward in a stochastic environment (a)	Reliability	Reward in a stochastic environment (b)	Reliability	Time (seconds)	Reward in a stochastic environment (c)	Reliability	Time (seconds)	Reward in a stochastic environment (d)	Reliability	Time (seconds)	(a)–(b)	(a)–(c)	(a)–(d)	(b)–(c)	(b)–(d)	(c)–(d)
p1.4.m	32	130	105.4	0.79	116.2	0.97	3093	119.4	0.98	11	111.7	0.91	1324	−10.2	−13.3	−6.0	−2.7	3.9	6.4
p1.4.n		155	110.3	0.71	123.6	0.99	3098	126.4	0.99	20	75.4	0.60	1341	−12.0	−14.6	31.6	−2.3	39.0	40.3
p1.4.o		165	117.0	0.71	130.7	0.93	3185	136.2	0.95	12	109.4	0.73	1347	−11.7	−16.4	6.5	−4.2	16.3	19.7
p1.4.p		175	124.1	0.72	128.1	0.96	3134	138.8	0.96	13	106.7	0.62	1349	−3.3	−11.9	14.0	−8.3	15.7	23.1
p1.4.q		190	137.2	0.74	148.1	0.99	3093	154.5	0.99	3	93.3	0.66	1352	−8.0	−12.6	32.0	−4.3	37.0	39.6
p1.4.r		210	148.6	0.72	170.9	0.95	3085	174.9	0.97	11	130.5	0.62	1366	−15.0	−17.6	12.2	−2.3	23.6	25.4
p2.4.f	21	105	74.0	0.71	87.0	0.81	2021	89.8	0.87	14	79.1	0.77	1269	−17.6	−21.5	−6.9	−3.3	9.1	11.9
p2.4.g		105	81.6	0.81	92.3	0.78	2057	94.8	0.92	42	81.0	0.75	1270	−13.2	−16.2	0.7	−2.7	12.2	14.6
p2.4.h		120	87.9	0.79	105.3	0.69	3050	108.5	0.61	19	83.0	0.84	1276	−19.7	−23.4	5.6	−3.0	21.2	23.5
p2.4.i		120	92.1	0.74	117.0	0.74	3119	118.9	0.82	9	106.2	0.94	1277	−27.0	−29.1	−15.3	−1.7	9.2	10.7
p2.4.j		120	103.9	0.85	117.8	0.82	2031	118.4	0.81	64	109.8	0.76	1283	−13.4	−13.9	−5.7	−0.5	6.8	7.3
p2.4.k		180	119.7	0.69	150.3	0.88	3057	150.4	0.92	39	137.5	0.82	1292	−25.5	−25.6	−14.9	0.0	8.5	8.6
p3.4.o	33	500	359.5	0.68	404.9	0.99	9212	428.4	0.98	15	200.4	0.47	1372	−12.6	−19.2	44.3	−5.8	50.5	53.2
p3.4.p		560	381.0	0.70	441.3	0.98	9200	467.3	0.99	25	269.2	0.50	1377	−15.8	−22.7	29.3	−5.9	39.0	42.4
p3.4.q		560	453.5	0.83	505.1	0.97	9211	510.9	0.98	7	183.6	0.43	1384	−11.4	−12.7	59.5	−1.1	63.7	64.1
p3.4.r		600	454.3	0.82	530.4	0.98	9233	543.1	0.98	53	137.2	0.20	1392	−16.7	−19.5	69.8	−2.4	74.1	74.7
p3.4.s		670	435.0	0.63	495.8	0.92	9095	545.6	0.99	52	315.8	0.53	1399	−14.0	−25.4	27.4	−10.0	36.3	42.1
p3.4.t		670	531.6	0.81	558.6	1.00	9241	602.7	0.99	33	318.3	0.44	1406	−5.1	−13.4	40.1	−7.9	43.0	47.2
p6.3.n	64	1170	698.8	0.52	861.6	0.91	9205	906.2	0.95	549	445.7	0.44	1607	−23.3	−29.7	36.2	−5.2	48.3	50.8
p6.4.j		366	205.7	0.56	206.5	0.81	10,178	206.5	0.72	268	79.1	0.22	1438	−0.4	−0.4	61.5	0.0	61.7	61.7
p6.4.k		528	296.4	0.56	297.1	0.75	10,134	299.3	0.61	40	116.0	0.27	1465	−0.2	−1.0	60.9	−0.7	61.0	61.2
p6.4.l		696	406.5	0.59	407.6	0.78	10,188	423.4	0.83	108	134.6	0.23	1503	−0.3	−4.2	66.9	−3.9	67.0	68.2
p6.4.m		912	495.2	0.54	512.8	0.82	9219	519.9	0.92	239	191.8	0.30	1549	−3.5	−5.0	61.3	−1.4	62.6	63.1
p6.4.n		1068	610.9	0.57	639.5	0.84	9220	663.3	0.86	276	174.6	0.23	1567	−4.7	−8.6	71.4	−3.7	72.7	73.7
Average		420	276.3	0.70	306.2	0.89	6140	318.6	0.90	80	157.9	0.55	1383	−11.9%	−15.7%	28.4%	−3.5%	36.8%	38.9%

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Table 3
Results of solving the stochastic TOP by SAA-CPLEX, Sim-BRVNS, and SAA-BRVNS with a high variance level ($c = 0.75$)

		Solution to the deterministic problem (deterministic solution)			SAA-CPLEX solution to stochastic problem			SIM-BRVNS solution to stochastic problem			SAA-BRVNS solution to stochastic problem			GAPS (%)					
Instance	n	Reward in a deterministic environment	Reward in a stochastic environment (a)	Reliability	Reward in a stochastic environment (b)	Reliability	Time (seconds)	Reward in a stochastic environment (c)	Reliability	Time (seconds)	Reward in a stochastic environment (d)	Reliability	Time (seconds)	(a)–(b)	(a)–(c)	(a)–(d)	(b)–(c)	(b)–(d)	(c)–(d)
p1.4.m	32	130	96.6	0.73	97.3	0.75	10,078	105.0	0.98	3	66.2	0.54	1345	−0.8	−8.8	31.5	−8.0	32.0	37.0
p1.4.n		155	104.0	0.67	108. 7	0.83	10,057	115.4	0.91	22	26.2	0.13	1351	−4.5	−10.9	74.8	−6.1	75.9	77.3
p1.4.o		165	103.0	0.63	109.3	0.81	10,085	116.2	0.95	22	45.0	0.30	1358	−6.2	−12.9	56.3	−6.3	58.8	61.3
p1.4.p		175	112.9	0.66	117.3	0.78	10,088	129.2	0.96	1	33.6	0.30	1368	−3.9	−14.5	70.2	−10.1	71.4	74.0
p1.4.q		190	129.6	0.69	130	0.79	10,090	139.9	0.95	28	48.0	0.30	1179	−0.3	−7.9	63.0	−7.6	63.1	65.7
p1.4.r		210	129.6	0.62	137.8	0.79	10,088	157.2	0.94	28	16.8	0.10	1187	−6.3	−21.3	87.0	−14.1	87.8	89.3
p2.4.f	21	105	73.1	0.70	83.8	0.68	3607	84.4	0.74	38	40.0	0.72	1072	−14.6	−15.4	45.3	−0.7	52.3	52.6
p2.4.g		105	77.3	0.78	90.0	0.75	2059	90.8	0.76	117	48.4	0.81	1069	−16.4	−17.4	37.4	−0.9	46.2	46.7
p2.4.h		120	79.8	0.75	100.1	0.84	2021	104.4	0.81	102	63.0	0.47	1075	−25.5	−30.8	21.1	−4.2	37.1	39.7
p2.4.i		120	84.7	0.69	111.1	0.72	3073	112.6	0.79	14	74.1	0.54	1085	−31.1	−32.8	12.5	−1.3	33.3	34.2
p2.4.j		120	94.1	0.77	114.2	0.77	2010	114.8	0.75	87	55.3	0.43	1089	−21.4	−22.0	41.2	−0.5	51.6	51.8
p2.4.k		180	109.9	0.62	134.1	0.71	3083	140.1	0.75	5	68.0	0.39	1097	−22.0	−27.4	38.1	−4.4	49.3	51.5
p3.4.o	33	500	314.4	0.64	354.2	0.73	9159	394.1	0.78	8	65.2	0.11	1177	−12.7	−25.4	79.3	−11.3	81.6	83.5
p3.4.p		560	352.3	0.64	369.7	0.8	9090	411.2	0.86	15	145.4	0.24	1188	−4.9	−16.7	58.7	−11.2	60.7	64.6
p3.4.q		560	412.5	0.74	419.0	0.97	9142	476.3	0.98	17	105.8	0.17	1187	−1.6	−15.5	74.4	−13.7	74.7	77.8
p3.4.r		600	419.6	0.75	478.7	0.94	9183	492.8	0.98	11	88.8	0.18	1200	−14.1	−17.5	78.8	−2.9	81.4	82.0
p3.4.s		670	396.0	0.58	501.2	0.96	9179	518.1	0.97	29	109.3	0.23	1204	−26.6	−30.8	72.4	−3.4	78.2	78.9
p3.4.t		670	475.0	0.74	531.5	0.97	9206	538.0	0.96	80	178.8	0.27	1204	−11.9	−13.3	62.4	−1.2	66.4	66.8
p6.3.n	64	1170	621.8	0.53	785.9	0.77	9199	933.7	0.78	107	350.8	0.61	2155	−26.4	−50.2	43.6	−18.8	55.4	62.4
p6.4.j		366	205.0	0.56	205.8	0.56	3056	207.8	0.61	54	78.2	0.54	1834	−0.4	−1.4	61.9	−1.0	62.0	62.4
p6.4.k		528	302.2	0.57	306	0.46	10,156	388.9	0.56	299	48.7	0.80	1876	−1.3	−28.7	83.9	−27.1	84.1	87.5
p6.4.l		696	396.7	0.58	397.9	0.54	10,155	400.4	0.68	268	84.4	0.60	1771	−0.3	−0.9	78.7	−0.6	78.8	78.9
p6.4.m		912	507.7	0.56	508.5	0.59	12,104	514.6	0.72	253	204.4	0.30	1757	−0.2	−1.4	59.7	−1.2	59.8	60.3
p6.4.n		1068	637.4	0.59	639.2	0.65	12,159	646.5	0.75	91	183.3	0.21	1821	−0.3	−1.4	71.2	−1.1	71.3	71.6
Average		420	259.8	0.66	284.6	0.76	7839	305.5	0.83	71	92.8	0.39	1360	−10.6%	−17.7%	58.5%	−6.6%	63.0%	64.9%

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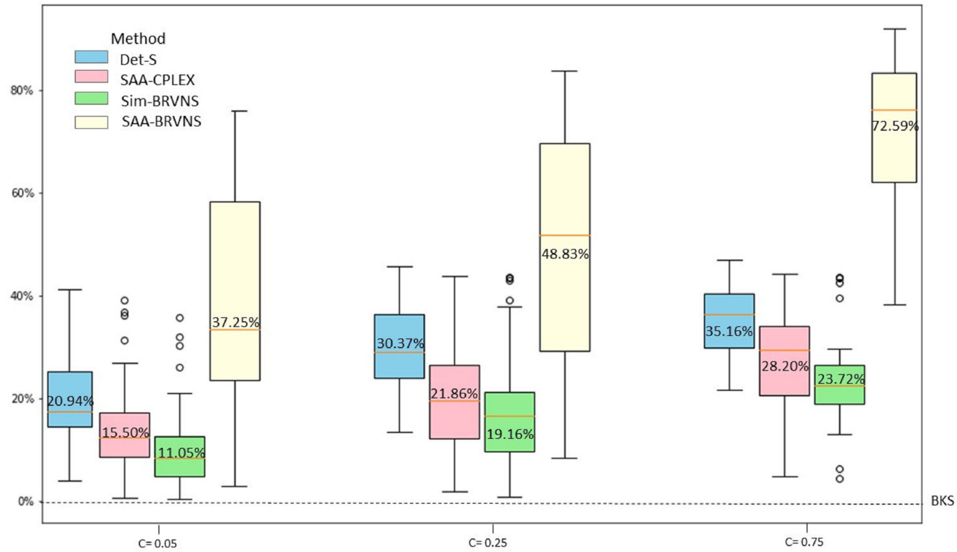

Fig. 5. Comparison of the gaps (in %) with respect to the BKS for the deterministic TOP.

"intensive" simulation process to the best deterministic solution. The following three columns give the solutions, reliability, and computing times, in seconds, for the SAA-CPLEX method. Similarly, in the next columns, the same information is reported for the proposed simheuristic algorithm and the SAA-BRVNS. Finally, the last columns show the obtained gaps of the proposed methods with respect to the stochastic cost of the deterministic solution, and also the gaps between the solutions by different methodologies. The results show that, on average, both the SAA-CPLEX and the SIM-BRVNS algorithms outperform the stochastic cost of the deterministic solution (column a) in all the tested scenarios. However, on average, the SAA-BRVNS approach does not reach the stochastic cost of the deterministic solution for any variance levels.

Figure 5 depicts a summary of the obtained results by all algorithms, where the vertical axis of the box plot represents the gap obtained with respect to the best deterministic solution (*Reward in a deterministic environment*). Note that the *Reward in a deterministic environment* ($c = 0$) value can be seen as a reference upper bound for the expected cost under stochastic conditions. In effect, as the travel time variability increases, the total achievable rewards tend to be lower since more route failures will occur.

5.3. Discussion

Regarding the proposed methods and Fig. 5, it is concluded that the solutions of our proposed simheuristics method are much better than the solutions provided by SAA-CPLEX and SAA-BRVNS in all different variance levels. As it is shown in Fig. 5, in the lowest level of variance ($c = 0.05$), our simheuristic offers an average gap of 11.05% with respect to the *Reward in a deterministic environment*, while the average gap of 15.50% and 37.25% are visible for the SAA-CPLEX

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and the SAA-BRVNS, respectively. For a higher variance level($c = 0.25$), the average gap for our simheuristic algorithm is about 19.16%, while the one provided by SAA-CPLEX and SAA-BRVNS are about 21.86% and 48.83%, and for the highest variance scenario ($c = 0.75$) the average gap of the simheuristics algorithm is about 23.72%, while the one provided by the SAA-CPLEX and SAA-BRVNS are 28.20% and 72.59%, respectively. Also note that, on average, the reliability of our simheuristic algorithm outperforms the two other solution approaches for all the tested scenarios. Additionally, the results prove that although the SAA-CPLEX method can be used to solve the stochastic TOP, the computational time employed is much greater than the total computational time of simheuristics. Finally, the results provided in the tables show that the SAA-BRVNS algorithm is not performing well enough since the SAA relies on the solution of relaxed problems of the stochastic model. The objective function value of the relaxed problem is expected to give indications of the quality of the solution when it is assessed in a stochastic environment. However, the use of the heuristic to solve the relaxed problem relies on working with an extreme relaxation, which uses a single scenario. The quality of these estimates is not reliable. Therefore, it is difficult to find promising solutions in a short computational time. It can be concluded that, however, the SAA method can provide promising solutions with a significant execution time, but combining it with BRVNS is not satisfactory.

The results allow us to state that, at least in the context of the studied problem, the Sim-BRVNS simheuristic outperforms the SAA-CPLEX and SAA-BRVNS methods, in terms of solution quality (reward), reliability, and computational time. The key differentiating factor of the simheuristics algorithm is the interaction between the simulation and optimization processes. In the simheuristics, the simulation module evaluates solutions as the optimization module generates them so that the latter can refine the searching process. In the SAA approach, the size of scenarios included at each iteration is limited by the capacity of the solver used since the *curse of dimensionality* prevents to solve more representative models with a large number of scenarios. Additionally, the results prove that although the SAA method can be used to solve small-sized instances of the stochastic TOP, its efficiency is reduced when considering large-scale instances. In contrast, simheuristic algorithms can efficiently deal with large-sized instances in short computing times. Finally, in problems with stochastic constraints, like the one discussed in this paper, stochastic programming approaches can be complemented with an *a posteriori* simulation if accurate estimates are to be obtained.

6. Conclusions and future work

In this paper, we analyzed the stochastic TOP with three powerful simulation–optimization methods. There is a large number of studies that applied metaheuristics to solve such problems. So far, however, few papers have compared stochastic programming methods with simheuristics in a detailed numerical experiment as the one presented in this paper. First, we applied the CPLEX commercial optimizer as well as our BRVNS algorithm to solve the deterministic TOP. The results have been compared with the BKSs from the literature to validate the effectiveness of our BRVNS algorithm in the deterministic scenario. Then, the stochastic version of the TOP has been addressed using a simheuristic (Sim-BRVNS) as well as two variants of the SAA method (SAA-CPLEX and SAA-BRVNS). The log-normal probability distribution has been employed to model the random

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travel times. Our results suggest that the SAA method can solve small-sized instances, but it is not able to solve the large-sized instances efficiently. Additionally, the combination of SAA and BRVNS cannot provide affordable solutions as well. On the other hand, our simheuristic algorithm can solve stochastic problems with large number of nodes in short computing times. The results also show that the simheuristic algorithm outperforms the SAA method even in the instances with small number of nodes with short computing times.

The following research lines can extend this work: (i) to develop similar comparative studies for other stochastic optimization problems, for example, vehicle routing, arc routing, scheduling, facility location, etc.; (ii) consider other stochastic programming tools apart from the SAA method; and (iii) analyze how both the SAA method and simheuristics can consider goals other than expected values (i.e., other statistics, multiobjective perspective, etc.).

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