Noise in Coherently Radiating Periodic Structures Beam Forming Networks

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***Abstract* — Following the noise wave theory, beam forming networks based on Coherently Radiating Periodic Structures (CORPS-BFN) are analysed and proven to be capable of enhancing the Signal to Noise Ratio of the system by analogically multiplexing the signal and noise contributions present at every input port. The geometry of the network determines the maximum enhancement achievable, which is demonstrated to be independent from insertion losses. These findings are supported by a mathematical approach, as well as with experimental data.**

***Keywords* — noise wave theory, CORPS, signal to noise ratio, noise factor reduction, beam forming, network, passive**

I. Introduction

Beam Forming Networks based on Coherently Radiating Periodic Structures [1], (CORPS-BFN) are versatile, passive beam forming networks that allow multiplexing several independent sources sharing a common antenna array. In addition, they have been proved suitable for beam steering applications [2]. These type of networks are receiving increasing attention due to this versatility. In fact, they have been chosen recently to feed the OLAF instrument for an ESA project concerning a Synthetic Aperture Radar (SAR) for Earth observation [3]–[5]. For such application, the Noise Equivalent Sigma Zero (NESZ), concerning the sensibility and resolution of the system, is a driving parameter [6]. Consequently, characterizing this type of networks in terms of noise was needed. As a consequence, the goal of this work is to analyse CORPS-BFN by means of the Noise Wave Theory (NWT), to define an equivalent noise figure (F) that takes into account the signal mulplexing within the network, extending the analysis not only to SAR applications, but also to generic transmission and reception scenarios.

II. Noise Wave Theory

Signal-to-Noise ratio (SNR or S/N) is a key parameter to both wired and wireless systems, as it is directly related to their performance. For instance, the capacity of a communications system, as postulated in Shannon-Hartley's Theorem (1), increases for higher SNRs, due to the possibility to employ more bandwidth-efficient modulations.

$$C = B \cdot \log_2\left(1 + \frac{S}{N}\right) \tag{1}$$

This figure of merit, however, can be compromised not only due to the noise captured from the channel, but also due to the system (in either transmitter or receiver chains), being affected especially by thermal noise (due to the physical temperature of the system) and the active components in the chain. The noise factor (*F*) of a component or a transmitter/receiver chain accounts for this intrinsic degradation, and it is defined as the relation between the SNR available at an input port and the SNR at an output port. NWT, described in [7]–[9], is a powerful tool for studying the behaviour of noise in microwave multiport networks. Essentially, the NWT starts by considering incoming and outgoing noise waves, *a* and *b* respectively, and an N-port microwave network (also called multiport), which is characterized by two matrices *S* and C_s, namely the scattering matrix and the intrinsic noise matrix. The former describes the ratio of voltage amplitudes between an outgoing wave at port 'i' and an incoming wave at port 'j'. The latter accounts for the noise generated within the network. If every port in the network is considered matched, so that no reflections take place, the outgoing noise power (*N*) matrix can be expressed as:

$$N = SAS^{\dagger} + C_s \tag{2}$$

Where † is the Hermitian conjugate and:

$$A = \overline{aa^{\dagger}}; N = \overline{bb^{\dagger}};\ C_s = \overline{cc^{\dagger}} \tag{3}$$

Furthermore, if thermodynamic equilibrium at a physical temperature T_0 is assumed, Bosma's Theorem [7] can be applied, obtaining (4), where *I* is the identity matrix.

$$C_s = \overline{cc^{\dagger}} = k_B T_0 B(I - SS^{\dagger}) \tag{4}$$

The analysis can be simplified by making some reasonable assumptions. First, it could be assumed that the incoming noise sources are uncorrelated, so that the off-diagonal terms in (2) and (4) are neglected. Secondly, the incoming noise power at port 'j' could be expressed in terms of an equivalent noise temperature, $T_{e,j} = A_{n,j} \cdot T_0$, so that the diagonal elements of *A* are:

$$A_{jj} = k_B T_0 A_{n,j} B \tag{5}$$

Last, (6) is derived for any general case of passive multiport by substituting (3)-(5) in (2), where *B* has been normalized.

$$N\}_{ii} = k_B T_0 \left(\sum_{j=1}^{N} |s_{ij}|^2 A_{n,j}\right) + k_B T_0 \left(1 - \sum_{j=1}^{N} |s_{ij}|^2\right)$$

$$N\}_{ii} = k_B T_0 \left(1 + \sum_{j=1}^{N} \left(|s_{ij}|^2 (A_{n,j} - 1)\right)\right) \tag{6}$$

This expression shows that, when $A_{n,j}$ equals unity – no additional noise contributions rather than the intrinsic thermal noise are present – the outgoing power corresponds to just that thermal noise power, which is consistent with the condition of thermodynamic equilibrium. On the other hand, when $A_{n,j} >> 1$ – the intrinsic thermal noise can be neglected with respect to the incoming noise – then (6) reduces to (7):

$$N\}_{ii} = k_B T_0 \sum_{j=1}^{N} \left(\left| s_{ij} \right|^2 A_{n,j} \right) \quad (7)$$

III. CORPS-BFN

The beam forming networks subject of this study are the CORPS-BFN, which, as introduced earlier, offer a high degree of versatility. These networks consist of several layers of the so-called Split (S) and Recombine (R) nodes, which are essentially power combiner/dividers (PCDs). Fig. 1 depicts an example of CORPS-BFN with 3-port PCDs. It is worth noting that the implementation of a CORPS-BFN is not limited to that of 3-port PCDs. For instance, the network to be employed in the SAR instrument consist of 4-port PCDs [4], [5]. One key advantage of CORPS-BFN is that every S/R node is constituted by the same passive PCD, simplifying its design. Moreover, this fact allows that every path across the network is electrically equivalent (hence coherent), controlling the phase of any signal.

For a proper operation of the network, it is crucial that the PCDs present isolation between their outputs, so that the signals propagate 'vertically' (towards higher layers) and the inputs of the network remain isolated and independent. Consequently, Wilkinson [10] or Gysel [11] PCDs have been employed traditionally. Moreover. it can be checked that, by employing PCDs with a division factor of 1:D, a signal coming from an input port will reach up to (D-1)+L output ports. An example of propagation across the network is illustrated in Fig. 2. Here, each input port presents a signal with a normalized amplitude, represented by a different colour each. The signals are considered to have the same relative phase. The coloured factors at each output port correspond to the respective S_{ij} scattering parameter. It can be observed how at the interface between each layer an amplitude of 1 is recovered after addition of the different signal contributions.

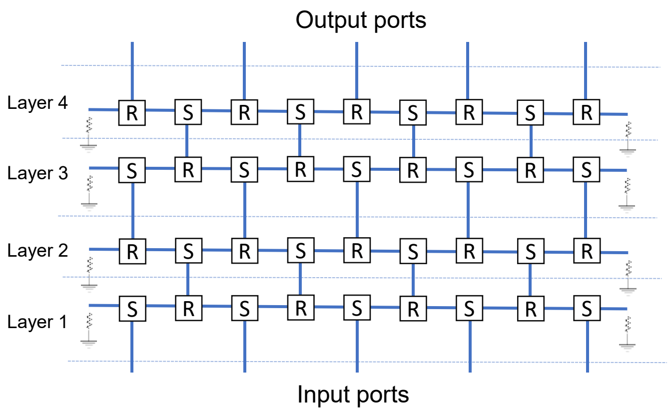

Fig. 1. Sketch of generic CORPS-BFN composed by 3-port power combiner/dividers.

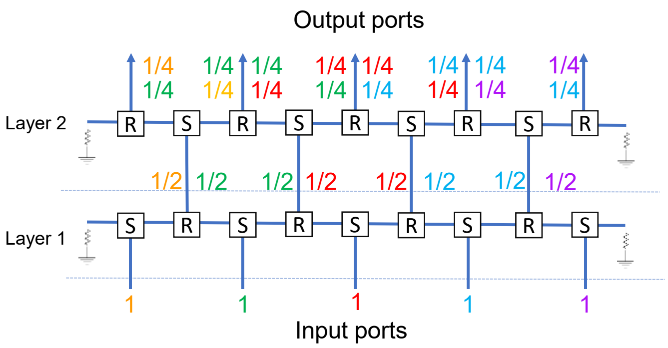

Fig. 2. Illustration of signal propagation across the network. Different colours have been selected for distinguishing among signals from different sources. The signals are considered to be in phase.

On the other hand, it is also observed that the unbalanced operation of the nodes located at the edges incur in power loss, decreasing the efficiency of the network. Previous studies [12] determined that such loss increases with the number of layers, and proposed modifications to enhance their efficiency. Nevertheless, for a fixed number of layers, it can be stated that the loss become less significant for a larger number of inputs, since the losses at the edges are fixed, whereas the input power increases with the number of input ports.

IV. Noise in CORPS-BFN

In the previous section, it was seen that a unitary amplitude is recovered at an output port 'i' located far from the edges in a lossless scenario. This can be expressed by the following property:

$$\sum_{j=1}^{M} s_{\mathrm{ij}} = 1 \quad (8)$$

Where M<N is the number of input ports that can reach a specific output port 'i'. It is important to note that the input and output ports are respectively isolated and that every port is matched. If we now consider that each input port presents a signal amplitude $A_{\mathrm{s,j}}$ and an incoming noise with equivalent noise temperature of $A_{\mathrm{n,j}} \cdot T_0$, the calculation of the SNR at an input and output port is straightforward with knowledge of the scattering matrix *S*. Furthermore, we can simplify it by normalizing the port impedance, which is considered equal for every port. As a result, the incoming power at each port, P_{j}, could be expressed as $A_{\mathrm{s,j}}^2$. The following expressions describe the SNR present at a generic input port 'j' and output port 'i'.

$$SNR_{\mathrm{in,j}} = \frac{A_{\mathrm{s,j}}^2}{k_{\mathrm{B}} T_0 A_{\mathrm{n,j}}} \quad (9)$$

$$SNR_{\mathrm{out,i}} = \frac{\left(\sum_{j=1}^{M} s_{\mathrm{ij}} \cdot A_{\mathrm{s,j}} \right)^2}{N_{\mathrm{ii}}} \quad (10)$$

Substituting (6) in (10) we derive:

$$SNR_{\mathrm{out,i}} = \frac{\left(\sum_{j=1}^{N} s_{\mathrm{ij}} \cdot A_{\mathrm{s,j}} \right)^2}{k_{\mathrm{B}} T_0 \left(1 + \sum_{j=1}^{N} \left(\left| s_{\mathrm{ij}} \right|^2 \left(A_{n,\mathrm{j}} - 1 \right) \right) \right)} \quad (11)$$

Now, we could define an equivalent noise factor for a pair of input/output port as the ratio of their SNRs:

$$F_{\mathrm{ij}} = \frac{SNR_{\mathrm{in,j}}}{SNR_{\mathrm{out,i}}} = \frac{A_{\mathrm{s,j}}^2 \cdot \cancel{k_B T_0}\left(1+\sum_{j=1}^{N}(|s_{\mathrm{ij}}|^2(A_{n,\mathrm{j}}-1))\right)}{\left(\sum_{j=1}^{M} s_{\mathrm{ij}} \cdot A_{\mathrm{s,j}}\right)^2 \cdot \cancel{k_B T_0} A_{\mathrm{n,j}}} \tag{12}$$

Which must be analysed in terms of $A_{n,j}$ and the number of signals to be combined at the output port 'i'. Furthermore, it is worth noting how the CORPS-BFN is symmetrical, not only horizontally but vertically. In addition, every S/R node consist of identical passive PCD. Therefore, this analysis is reciprocal and valid for transmission and reception modes.

In this work, we will analyse the scenario of single and multiple incoming signals, whereas an equivalent incoming noise power will be considered for every port, regardless the amount of input signals..

A. *Single input signal*

In this case, no coherent combination of signals takes place within the network. As a result, the numerator in (11) denotes a power loss, due to the scattering of the signal among different ports. A similar effect is found to take place in other passive beam forming networks employed for transmission scenarios, such as Butler matrices [13] (BM) or Rotman Lenses [14] (RL), where the power from an input is distributed to the output ports (with an specific phase distribution). As for the relevance of $A_{\mathrm{n,j}}$, two special cases can be differentiated:

$$F_{\mathrm{ij}} = \frac{A_{\mathrm{s,j}}^2}{(s_{\mathrm{ij}} \cdot A_{\mathrm{s,j}})^2} = F_{\mathrm{ij},att} \; ; \quad A_{\mathrm{n,j}} = 1 \tag{13}$$

$$F_{\mathrm{ij}} = \frac{A_{\mathrm{s,j}}^2 \cdot \left(\sum_{j=1}^{N}(|s_{\mathrm{ij}}|^2(A_{n,\mathrm{j}}))\right)}{(s_{\mathrm{ij}} \cdot A_{\mathrm{s,j}})^2 \cdot A_{\mathrm{n,j}}} \; ; \quad A_{\mathrm{n,j}} \gg 1 \tag{14}$$

The first case corresponds to the model of an attenuator, in which the noise factor is equivalent to the insertion (including scattering) losses. The second case can be further simplified as:

$$F_{\mathrm{ij}} = \frac{\mathrm{SNR}_{\mathrm{in,j}}}{SNR_{\mathrm{out,i}}} = F_{\mathrm{ij},att} \cdot \sum_{j=1}^{N} |s_{\mathrm{ij}}|^2 \tag{15}$$

Where the sum has been kept because there might be noise contributions at every other port despite having just one signal contribution. In any case, it can be observed that it will always be lower than unity since every s_{ij}<1. As a result, the BFN scatters the incoming noise contributions (except for the intrinsic thermal noise) and the overall noise factor of the system is lower than the one expected from an attenuator (the SNR is improved with respect to the attenuator model).

B. *Multiple input signals*

In this case, the network multiplexes M incoming signals among the output ports. Analogously to the previous case, (12) can be assessed differently depending on the significance of $A_{\mathrm{n,j}}$.

$$F_{\mathrm{ij}} = \frac{A_{\mathrm{s,j}}^2}{\left(\sum_{j=1}^{M} s_{\mathrm{ij}} \cdot A_{\mathrm{s,j}}\right)^2} \; ; \quad A_{\mathrm{n,j}} = 1 \tag{16}$$

$$F_{\mathrm{ij}} = \frac{A_{\mathrm{s,j}}^2 \cdot \left(\sum_{j=1}^{N}(|s_{\mathrm{ij}}|^2(A_{n,\mathrm{j}}))\right)}{\left(\sum_{j=1}^{M} s_{\mathrm{ij}} \cdot A_{\mathrm{s,j}}\right)^2 \cdot A_{\mathrm{n,j}}} ; \quad A_{\mathrm{n,j}} \gg 1 \tag{17}$$

As for (16), it can be checked that, in a lossless scenario, the denominator would be unitary, resulting in an unchanged SNR value. Otherwise, the noise factor would correspond to the insertion loss of the overall network minus the gain of combining M signals. On the other hand, when assuming significantly large noise contributions, (17) can be reformulated if every $A_{\mathrm{n,j}}$ and $A_{\mathrm{s,j}}$ are respectively considered equal, resulting in (18), which is observed independent from both incoming noise and signals.

$$F_{\mathrm{ij}} = \frac{\left(\sum_{j=1}^{N}(|s_{\mathrm{ij}}|^2)\right)}{\left(\sum_{j=1}^{M} s_{\mathrm{ij}}\right)^2}. \tag{18}$$

Given the passive character of the network, $|s_{ij}|<1$, it can be demonstrated that (18) is lower than unity. Therefore, the CORPS-BFN allows enhancing the SNR by the coherent division and recombination of the incoming signals and the scattering of the uncorrelated noise contributions. The maximum achievable enhancement will depend on the network's topology (which defines the *S*-matrix), and will be penalized by any phase shift $\Delta\varphi$ among the incoming signals, as $\cos(\Delta\varphi/2)$ terms will be introduced at the R-nodes.

V. Experimental Validation

In order to validate the presented calculations, an experiment was run under laboratory conditions. A prototype similar to the one presented in [4] consisting of 4-port PCD with coaxial ports was fabricated. A circular network was built upon connection of 6 cells, so that no matched resistors were needed. The goal was to model the central part of a large CORPS-BFN, in which no scattering losses take place. Fig. 3 (left) shows a picture of the fabricated network. The measured S-matrix magnitude at 1.25 GHz (centre frequency) was:

$$S_{\mathrm{network}} = \begin{pmatrix} 0 & 0 & 0 & 0.3 & 0.3 & 0.3 \\ 0 & 0 & 0 & 0.3 & 0.3 & 0.3 \\ 0 & 0 & 0 & 0.3 & 0.3 & 0.3 \\ 0.3 & 0.3 & 0.3 & 0 & 0 & 0 \\ 0.3 & 0.3 & 0.3 & 0 & 0 & 0 \\ 0.3 & 0.3 & 0.3 & 0 & 0 & 0 \end{pmatrix} \tag{19}$$

According to (18), the expected SNR enhancement would be 3 (4.77 dB) when having significant noise contributions at the inputs, which is due to the coherent combination of three signals (in spite of the insertion loss of the network). This scenario is depicted in Fig. 3 (right). However, we were limited in generating three coherent signals and controlling their phase over every cable connection. Therefore, we opted for working with a single signal and demonstrating (15).

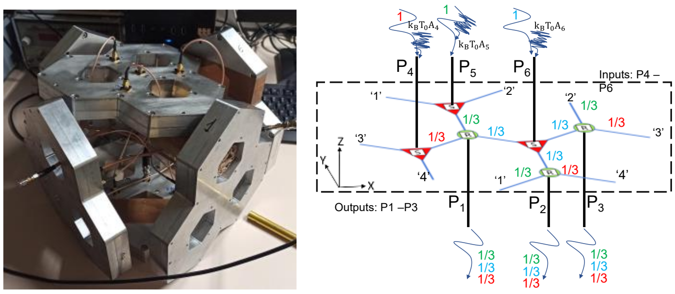

Fig. 3. Prototype of the circular CORPS-BFN (left). Sketch of the network for a lossless combination of three coherent signals.

Table 1. Summarized results of the experiment.

	SNR NGC OFF (dB)	SNR NGC ON (dB)	SNR degradation (dB)
Splitter output	56.3	49.3	7
C-BFN output	46.3	43.2	3.1
SNR degradation (dB)	10 (IL)	6.1	**Difference** 3.9

For this purpose, three 'noise generating chains' (NGCs) were built, consisting of a noise generator and two power amplifiers each. The goal of the NGCs was to generate high-power, uncorrelated noise contributions. Then, a splitter was used to combine the generated noise with the 1.25 GHz signal coming from a signal generator, connecting its output to the corresponding input of the CORPS-BFN (Port 2). Port 5 was connected to a spectrum analyser (SA), whereas the other ports were loaded with matched resistors.

The process to evaluate (13) consisted of measuring the SNR at the output of the splitter (after combining the noise and the signal) and at the output of the CORPS-BFN (connecting the splitter to the input of the network) in two cases: with the NGC ON (DC bias turned on) and NGC OFF (bias off). These results are summarized in Table 1. The calculation was normalized to 1 Hz. The noise power was taken as the average noise background in the SA. The signal power was taken straight from the centre frequency.

Overall, Table 1 shows that turning on the NGC degrades the SNR in a factor of 7 dB. However, this same noise contribution when travelling through the CORPS-BFN causes only a degradation of about 3 dB in the SNR. Hence, an improvement of almost 4 dB has been obtained compared to the attenuator model. The difference in SNR for the NGC OFF case between the two scenarios is due to the scattering (and insertion) losses within the network. In addition, a scenario with M signals being coherently combined would enhance this SNR due to the intrinsic gain of spatial diversity

It is worth noting that the applied noise contribution was higher than the intrinsic noise in the system (turning the NGC ON increases the background noise level in 7dB) but that such increase might not be large enough to completely neglect the thermal noise generated within the network (namely, the insertion loss). This explains that the obtained SNR enhancement is around 4dB instead of the theoretical 4.77dB computed with (15).

VI. Conclusion

In this work, CORPS-BFN have been analysed under the Noise Wave Theory to demonstrate they capability of enhancing the SNR of signals affected by significant contributions of noise. The proposed model has been validated by performing an experiment under laboratory conditions. An improvement of 4dB against the maximum 4.77dB obtained by the mathematical formulation has been achieved and the deviation from the ideal value has been discussed.

This SNR enhancement, together with the specific properties of CORPS, such as its versatility and their simple design (with every node constituted by the same power combiner/divider) make them interesting candidate BFNs for future array systems operating in both transmission and reception modes.

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