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Evaluating Noise in Passive Beam Forming Networks for Multibeam Applications

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***Abstract*— In this work, we identify that the analysis in terms of noise of multibeam passive beam forming networks requires a proper formulation, in order not to overestimate the noise generated by the network. We show that the inherent spatial diversity offered by this type of networks results in an improvement of the SNR, similar to the one obtained by digital architectures, with a penalty due to the insertion loss. To illustrate this phenomena, two generalized equivalent circuits are proposed are a numerical example is presented.**

***Index Terms*— Beam forming, noise factor, passive circuits, phased arrays, signal-to-noise ratio.**

I. INTRODUCTION

NOISE, defined as unpredictable, statistic perturbations affecting signals that contain information [1] has been a recurrent subject of study since the appearance of telecommunications due to its detrimental effects in extracting the signal's information at receiver systems. Shannon-Hartley's theorem [2] establishes that the capacity of a communication channel with a bandwidth B_W has a direct relation with the available signal and noise powers at the receiver. Such relation is known as Signal-to-Noise Ratio (SNR). At microwave frequencies, where other noise contributions with power spectral densities inversely proportional to frequency [3] can be neglected, noise power is often described as a thermal noise given by the agitation of charge carriers [4], [5] which has a nearly constant power spectral frequency (white noise) and follows a Gaussian amplitude distribution, with mean μ=0 and variance σ^2. For a fixed bandwidth B_W, the thermal noise power is computed as:

$$N_{\text{thermal}} = \mathrm{k_B} B_{\mathrm{W}} T_{\text{eq}} = \mathrm{k_B} B_{\mathrm{W}} \mathrm{T_0} N_{\text{eq}}, \tag{1}$$

where $\mathrm{k_B}$ is Boltzmann's constant ($1.38\cdot10^{-23}$ J·K^{-1}) and T_{eq} is the equivalent noise temperature of the system, which, analogously, can be expressed as an equivalent noise power coefficient, N_{eq}, multiplying the reference temperature $\mathrm{T_0}$.

When assuming thermodynamic equilibrium (N_{eq}=1), this noise temperature is related to the physical temperature of the system by the noise factor (F), which is a figure of merit that essentially accounts for the degradation of the SNR due to a component (for instance, an amplifier). This figure of merit is completely independent from the signal or noise power present at the input port. Equivalently, it can be expressed also in terms of the noise added (N_{added}) by a two-port component with signal gain, G, when connected to a noise source delivering a power N_{ref} at the reference temperature, $\mathrm{T_0}$:

$$F = \frac{\mathrm{SNR_{in}}}{\mathrm{SNR_{out}}} = 1 + \frac{T_{\mathrm{e}}}{\mathrm{T_0}} = 1 + \frac{N_{\text{added}}}{N_{\text{ref}} \cdot G}. \tag{3}$$

In the remainder of this work, the physical temperature will be assumed the reference temperature ($\mathrm{T_0}$=290K). At this point, any two-port network can be described by two main parameters: the noise factor, F, and the signal gain, G. For instance, it is well-known that the noise factor of a matched attenuator at reference temperature $\mathrm{T_0}$ corresponds to its loss (F=L=1/G) [6]–[9]. This is a direct consequence of the application of Bosma's Theorem [10], which allows us to compute the noise power generated intrinsically in any passive multiport network with known scattering matrix $\boldsymbol{S}$. The theorem establishes that:

$$\boldsymbol{C} = \overline{\boldsymbol{c}\boldsymbol{c}^*} = \mathrm{k_B} B_{\mathrm{W}} \mathrm{T_0} (\mathbf{I} - \boldsymbol{S}\boldsymbol{S}^*) \tag{4}$$

where the superscript (*) represents the conjugate transpose, ($\overline{}$) denotes a time average and $\mathbf{I}$ is the identity matrix. Here, $\boldsymbol{C}$ is the intrinsic noise matrix, containing in its diagonal the intrinsically generated, outgoing noise power at each port, and the correlated products between such noises in the off-diagonal terms. In fact, it can be checked (see [11]-[12] for further insight), that the time average of the product of two signals is related to their correlation and covariance, so that:

$$\overline{xy} = \mathrm{cov}(x, y) + \mu_x \mu_y = \rho_{xy}(\sigma_x \sigma_y) + \mu_x \mu_y \tag{5}$$

Where $0<|\rho_{xy}|<1$ is the correlation coefficient between both signals and σ_i and μ_i are, respectively, the standard deviation and the average (mean) of the signal. In the case of white Gaussian noise (thermal noise), it is widely known that its mean is zero and that its power over a fixed bandwidth is given by its variance, thus simplifying (5).

Bosma's theorem has been used widely together with the Noise Wave Theory (NWT) – a typical approach consisting of treating the noise as an infinite series of noise waves [13], [14] – to analyze any passive multiport microwave network with known scattering matrix in terms of noise. In this work, the main goal is to analyze passive beam forming networks (BFN)

[1]This paragraph of the first footnote will contain the date on which you submitted your paper for review. "This work was supported in part by the Spanish Ministry of Science and Innovation under Grant FPU18/00013".

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that feature multibeam operation, such as [15]–[18]. This analysis is performed following the formulation of the NWT. This goal is motivated by the ambiguity found when analyzing their performance in terms of noise and the general misconception that every passive component will severely deteriorate the SNR in the system. Whereas it is undeniable that losses in the network will incur in the generation of noise – a direct consequence of (4) – the division and recombination of signal and noise power within the multiple paths in this type of networks provide a spatial diversity that can be harnessed to enhance the SNR with respect to the input ports. Such analysis is considered essential to properly design and model hybrid passive-analog/active-digital architectures [19].

II. Noise Analysis in Multibeam BFNs

Let $\boldsymbol{S}$ be the scattering matrix of a P-port passive, multibeam microwave network, where P=M+N. The first M ports are the input ports (in a reception scenario, the antenna ports) and the last N ports are the output ports (beam ports in this case).

Let a be a P×1 array containing the amplitude and phase of the incoming waves at each port of the network (although only the first M elements may be considered different to zero). Since the network is considered LTI (linear, time-invariant), at each port 'i', the wave a_i can be expressed as a linear combination of a signal a_i^{s} and noise contribution a_i^{n} that are independent and hence uncorrelated. Let b be a P×1 array containing the amplitude and phase of the outgoing waves at each port of the network. Similarly, the wave b_i can be expressed as a linear combination of a signal b_i^{s} and noise contribution b_i^{n}. Thus:

$$a = a^{\mathrm{s}} + a^{\mathrm{n}}; \qquad b = b^{\mathrm{s}} + b^{\mathrm{n}}. \tag{6}$$

It is worth noting that, whereas signal and noise are independent and hence uncorrelated to each other, the different signal contributions may be correlated among themselves, and so could be the noise contributions as well. Let c be a P×1 array containing the noise generated intrinsically by the network outgoing at each port. Now, a relation between the incoming and outgoing waves is established by:

$$b = \boldsymbol{S}a + c, \tag{7}$$

where the time dependency of each term has been obviated for clarity. In addition, the power outgoing each port can be retrieved from the diagonal elements of the matrix:

$$\boldsymbol{B} = \overline{bb^*} = \boldsymbol{SAS}^* + \boldsymbol{C} = \boldsymbol{S}\overline{aa^*}\boldsymbol{S}^* + \overline{cc^*}, \tag{8}$$

which can also be expressed as two independent matrices:

$$\boldsymbol{B}^{\mathbf{s}} = \overline{b^{\mathrm{s}}b^{\mathrm{s}*}} = \boldsymbol{SA}^{\mathbf{s}}\boldsymbol{S}^*, \tag{9a}$$

$$\boldsymbol{B}^{\mathbf{n}} = \overline{b^{\mathrm{n}}b^{\mathrm{n}*}} = \boldsymbol{SA}^{\mathbf{n}}\boldsymbol{S}^* + \mathrm{k_B} B_W T_0 (\mathbf{I} - \boldsymbol{SS}^*). \tag{9b}$$

A. Determination of SNR and noise factor

At this point, the ratio of the SNR at an input port, '$\mathrm{SNR_{in}}$', and the SNR at an output port, '$\mathrm{SNR_{out}}$' could be expressed in terms of the matrices $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{S}$. We now define an equivalent noise factor matrix, F_{eq}:

$$\boldsymbol{F}_{\mathbf{eq}} = \frac{\mathrm{SNR_{in}}}{\mathrm{SNR_{out}}} = \frac{(diag(\boldsymbol{A}^{\mathbf{s}}) \oslash diag(\boldsymbol{A}^{\mathbf{n}}))}{(diag(\boldsymbol{B}^{\mathbf{s}}) \oslash diag(\boldsymbol{B}^{\mathbf{n}}))^{\mathrm{T}}} \tag{10}$$

Where $\oslash$ represents the element-wise division of two vectors. This matrix allows us to assess the SNR enhancement or deterioration in network for any pair of ports. Without loss of generality, it could be checked that, after developing (9) further, the (i,j)-th element of (10) would be computed as:

$$F_{\mathrm{eq,ji}} = \frac{A_{\mathrm{i,i}}^{\mathrm{s}}/A_{\mathrm{i,i}}^{\mathrm{n}}}{B_{\mathrm{j,j}}^{\mathrm{s}}/B_{\mathrm{j,j}}^{\mathrm{n}}} =$$

$$\frac{\left(|\sigma_{\mathrm{i}}^{\mathrm{s}}|^2 + \overline{a_i^{\mathrm{s}}}^2\right)}{N_{\mathrm{i}}} \left[\frac{\sum_{\mathrm{p=1}}^{\mathrm{P}}\left[\sum_{\mathrm{m=1}}^{\mathrm{P}} S_{\mathrm{jm}} \rho_{\mathrm{mp}}^{\mathrm{n}} \left(\sqrt{N_{\mathrm{m}} N_{\mathrm{p}}}\right)\right] S_{\mathrm{jp}}^* + \left(1 - \sum_{\mathrm{m=1}}^{\mathrm{P}} S_{\mathrm{jm}} S_{\mathrm{jm}}^*\right)}{\sum_{\mathrm{p=1}}^{\mathrm{P}}\left[\sum_{\mathrm{m=1}}^{\mathrm{P}} S_{\mathrm{jm}} \left(\rho_{\mathrm{mp}}^{\mathrm{s}} (\sigma_{\mathrm{m}}^{\mathrm{s}} \sigma_{\mathrm{p}}^{\mathrm{s}}) + \overline{a_{\mathrm{m}}^{\mathrm{s}} \cdot a_{\mathrm{p}}^{\mathrm{s}}}\right)\right] S_{\mathrm{jp}}^*}\right]. \tag{11}$$

Here, the power associated to each signal and noise contribution is represented following (5). In addition, it can be seen that both correlation coefficients ρ^{s} and ρ^{n} are included to account for any possible correlation among incoming signals (a typical scenario in multibeam networks), as well as among the incoming noise contributions (The reader might check on [20]–[23] for more information regarding noise correlation in antenna arrays). In addition, N_{m}, N_{p} are the equivalent noise power coefficients, defined in (2). The $\mathrm{k_B} B_W \mathrm{T_0}$ factor is simplified from both numerator and denominator.

This expression can be further developed if a pure definition of the noise factor is sought (namely, that $N_{\mathrm{i}}=1$ ∀i and $\rho_{\mathrm{xy}}^{\mathrm{n}}=0$ ∀x ≠ y). However, the resulting expression would still depend on the inputs of the system. Hence, when computing the noise factor between the input port i and the output port j, only a single incoming signal shall be considered, arriving to:

$$F_{\mathrm{ji}} = \frac{A_{\mathrm{i,i}}^{\mathrm{s}}/A_{\mathrm{,ii}}^{\mathrm{n}}}{B_{j,j}^{\mathrm{s}}/B_{\mathrm{j,j}}^{\mathrm{n}}} = \frac{\sum_{\mathrm{p=1}}^{P} N_p |S_{\mathrm{jp}}|^2}{N_i |S_{\mathrm{ji}}|^2} + \frac{1 - \sum_{\mathrm{p=1}}^{P} |S_{\mathrm{jp}}|^2}{N_i |S_{\mathrm{ji}}|^2} \xrightarrow{N_i = N_p = 1} \frac{1}{|S_{\mathrm{ji}}|^2} \tag{12}$$

Here the signal power has been simplified from both numerator and denominator. The first term in (12) accounts for the scattering of every noise contribution across the network, whereas the second term accounts for the noise generated intrinsically. If we now substitute $N_{\mathrm{p}}=N_{\mathrm{i}}=1$ (pure definition of the noise factor), it is observed that F_{ji} corresponds to the loss experienced by the signal travelling between them, as in an attenuator. This is the typical approach when considering a passive network (i.e., F equals losses). Nevertheless, the fact that several signal and noise contributions might be combined at the output 'j' port must not be forgotten. Hence, despite presenting a noise factor $F_{\mathrm{ji}}>1$ between one input and one output, this does not necessarily mean that the SNR will be deteriorated, because the network is intrinsically offering spatial diversity. This can be better understood with a simple numerical example.

B. Example: Quadrature hybrid

This sample scenario is depicted in Fig. 1 together with its scattering matrix, with P=4 and M=N=2. The ports labelling is customized with respect to the typical notation to be consistent with the designation of M inputs and N outputs. Every port is considered matched, with ideal isolation between the ports. In addition, the transmission parameter has been expressed in terms of the canonical division factor (1:2) and an efficiency $0<\eta\leq 1$ that accounts for the ohmic and dielectric loss along each path. For simplicity, a balanced network is assumed. A quadrature hybrid can be thought of as the simplest topology

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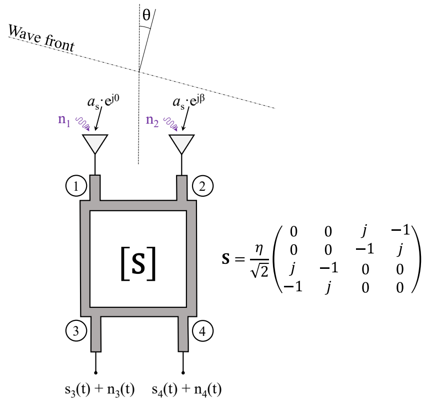

Fig. 1. Example scenario concerning the reception of a tilted wave front by a phased array fed by a quadrature hybrid.

(2×2) of a Butler Matrix, defining two mirrored beams with respect to boresight, generating a relative phase between the antenna ports of ±90º depending on the port being used. Here, we analyze the reciprocal reception case. An incoming plane wave is assumed to arrive with an inclination θ with respect to boresight, generating a phase difference β between the signals excited at each antenna, a_i^{s}. The total power captured by each antenna, will be proportional to the antenna's effective area (gain). For simplicity, every antenna is considered equal, and the excitations are assumed sinusoids with amplitude a_s, with zero mean and a variance (average power) of $a_s^2/2$. In addition, each antenna captures a noise contribution, a_i^{n}, with an average noise power of $k_B B_W T_0 N_i$, so that:

$$\mathrm{SNR_{in}} = \mathrm{a_s^2}/(2\mathrm{k_B}B_\mathrm{W}T_0N_\mathrm{i}) \tag{13}$$

Let us now compute the $\mathrm{SNR_{out}}$ at port 3, equivalent to that of port 4 due to the symmetry of the network. The intrinsically generated noise outgoing each port, N_{added}, depends exclusively on the loss incurred in the network and is independent from the division factor (1:2). Applying (4):

$$N_{\mathrm{added}} = \mathrm{k_B}B_\mathrm{W}T_0(1-\eta^2) \tag{14}$$

This power is added to the scattered noise contributions present at the output, so that the total noise power at port 3 is:

$$B_{3,3}^{\mathrm{n}} = \mathrm{k_B}B_\mathrm{W}T_0\big[N_1|S_{31}|^2 + N_2|S_{32}|^2 + 2\rho_{21}^{\mathrm{n}}|S_{31}||S_{32}|\sqrt{N_1N_2} + (1-\eta^2)\big] \tag{15}$$

The correlation between the noises captured by each antenna at this point is yet unknown and is represented by the correlation coefficient ρ_{21}^{n}. The signal power outgoing this port can be computed straightforwardly, since the correlation between signals is defined by the cosine of their relative phase difference (which is also affected by the phase introduced by each S_{ij}):

$$B_{3,3}^{\mathrm{s}} = \frac{\mathrm{a_s^2}}{2}\left(|S_{31}|^2 + |S_{32}|^2 + 2|S_{31}||S_{32}|\cos\left(\beta - \frac{\pi}{2}\right)\right) \tag{16}$$

The ratio of SNR at the input/output ports (equivalent noise factor) is computed as in (11), assuming $N_1=N_2$ for simplicity:

$$F_{\mathrm{eq},31} = \frac{\eta^2N_1(1+\rho_{21}^{\mathrm{n}})+(1-\eta^2)}{\eta^2N_1(1+\cos(\beta-\pi/2))}, \tag{17}$$

In a plausible scenario – in which the signals arrive from the direction of maximum radiation (namely, β=π/2) and where the noise contributions are independent ($\rho_{21}^{\mathrm{n}} = 0$) – this factor can be lower than unity (the SNR is enhanced) due to the spatial diversity inherent to the network:

$$F_{\mathrm{eq},31} = \frac{\eta^2(N_1-1)+1}{2\eta^2N_1} = \frac{1}{2} - \frac{1}{2N_1} + \frac{1}{2\eta^2N_1} \xrightarrow{N_1=1} \frac{1}{2\eta^2} \tag{18}$$

However, if an adequate expression, consistent with the pure noise factor definition, is desired, (15) and (16) are reduced to:

$$B_{3,3}^{\mathrm{n}} = \mathrm{k_B}B_\mathrm{W}T_0; \qquad B_{3,3}^{\mathrm{s}} = \frac{\mathrm{a_s^2}}{2}\cdot\frac{\eta^2}{2} \tag{19}$$

So that the actual noise factor between ports 1 and 3 is:

$$F_{31} = \frac{2}{\eta^2} = \frac{1}{|S_{\mathrm{ji}}|^2} \tag{20}$$

The fundamental difference between (20) and (17), besides the simplifications made in between, is that the later takes into account the spatial diversity offered intrinsically by division and recombination taking place inside the network, whereas (20) treats individually the contribution coming from each input port. As such, if we consider (18) for N_1=1, the equivalent noise factor (ratio between SNRs) is 4 times (M^2) lower than that in (20), thanks to the coherent combination of two signals.

C. Equivalent representation

At this point, the equivalent circuit in Fig. 2 could be considered. After the division from each input to two channels (a process which generates noise), each channel is affected by an attenuation and a phase delay which are related to the corresponding S_{ij} parameter between each pair of ports. Then, the combination of signal and noise contributions is performed with an ideal power combiner (i.e., a lossless Wilkinson), which does not introduce noise. By using this representation, the same results as in previous section obtained in either way – (17), (18) and (20), depending on the N_i and number of signals considered.

One last remark worth mentioning is that the attenuator blocks used in this representation lack a physical meaning, since their attenuation, $L=1/(2\cdot\eta^2)$, is lower than unity for values of η close to 1. Nevertheless, they do not need to be realizable since this is an equivalent representation. In terms of attenuation, these blocks compensate the additional attenuation introduced the power divider and combiner (1/N and 1/M, respectively). Otherwise, since L already accounts for the S_{ij} transmission parameter, both the signal and noise captured by the antennas would be attenuated excessively. In terms of noise,

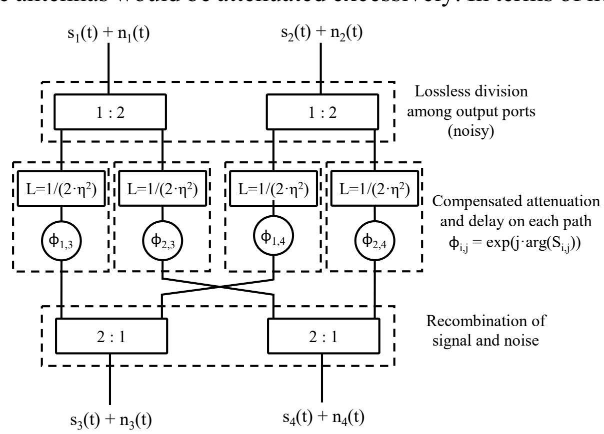

Fig. 2. Equivalent circuit of the quadrature hybrid accounting for division and recombination within the network.

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the power divider at the input generates an excess of thermal noise power, which is compensated by the L block. This could also be understood as a correlation factor of $\rho_{xy} = -1$ between both noises, so that the noise generated by the L block interferes destructively with the noise excess generated by the divider.

III. Generalized Model and Discussion

The scheme in Fig. 2 is now generalized to the case of a passive multibeam forming network with M inputs (antennas) and N outputs (beam ports) in Fig. 3, consisting of M·N interconnected channels. Both, signal and noise, arriving at an antenna are first divided among N channels (introducing noise in this division). In each channel, the incoming signal and noise, as well as the noise generated in the power divider, are compensated with an attenuation/gain that depends on the corresponding S_{ij} transmission parameter between each pair of ports, as well as on the number of ports $[L=1/(M\cdot N\cdot|S_{ij}|^2)]$. In addition, each signal and noise contribution then experience a delay (established by the corresponding S_{ij}). Finally, the M channels dedicated for each output port are combined without the generation of additional noise. This last combining block is responsible for taking into account every possible correlation between the different signals or the different noises captured by the antennas (the noises generated internally within each chain are naturally uncorrelated). In terms of individual pairs of input/output port, we could follow the path in between to check that the gain across this path corresponds to the specific transmission parameter:

$$G_{j,i} = 1/N \cdot MN|S_{ij}|^2 \cdot 1/M = |S_{ij}|^2 \qquad (21)$$

In the same manner, the noise factor of each chain is obtained:

$$F_{j,i} = N + \frac{1/MN|S_{ij}|^2 - 1}{1/N} + \frac{M}{MN|S_{ij}|^2/N} = \frac{1}{|S_{ij}|^2} \qquad (22)$$

which agrees with that obtained in (20) for the sample scenario. Thus, the equivalent circuit proposed in Fig. 3 successfully models the behavior of an arbitrary multibeam network. Nevertheless, the traditional representations of a reception scenario with an antenna array, such as those proposed by Lee [24] or Gatti [25], consist of analyzing the system as M parallel channels contributing to an output port.

In such works, when every channel is equal (same gain and noise factor), the noise factor of the system corresponds to that of an individual channel. However, special care must be taken when using those representations for a multibeam network. In this case, assuming the noise factor in (22) for every channel would incur in the computation of an excess in noise power of $(M-1)\cdot k_B B_W T_0$. Therefore, a correction must be introduced.

This is addressed by dividing the noise factor of the chain by the number of antennas contributing to each port, as shown in Fig. 4. This representation is equivalent to that in Fig. 3, and both satisfy all previous calculations in *Sections II* and *III*. Here, each output (beam) port is considered individually as the combination of M channels, just as in [24] and [25]. However, the fundamental difference is that each channel has a noise factor of $1/(M|S_{ji}|^2)$, instead of $1/|S_{ji}|^2$, which is typically considered for every passive component.

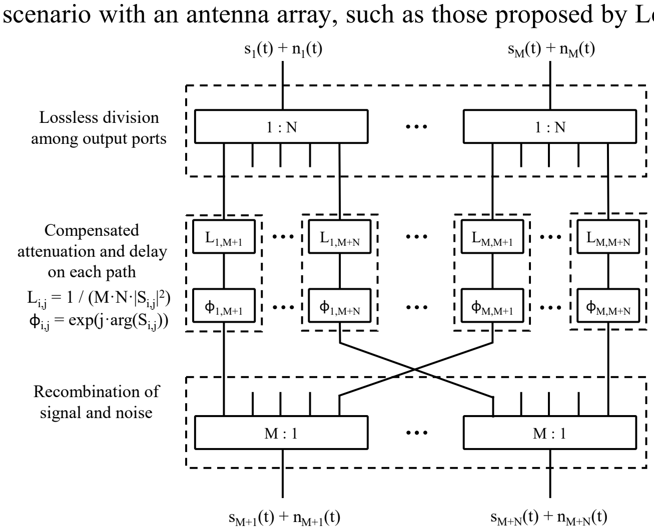

Fig. 3. Generalized model (equivalent circuit) of a passive multibeam BFN accounting for the division and recombination occurring inside the network.

IV. Conclusion

Namely, it can be concluded that any multibeam passive beam forming network can be represented as the combination of M·N passive channels, where each channel generates $(1/M)^{th}$ of the thermal noise generated by the network (the number of antennas contributing to a single port). The noise factor of each channel is then $(1/M)^{th}$ of the noise factor of the system. Here, we aimed to demonstrate that, despite being passive (and hence lossy), the spatial diversity – inherent to multibeam networks – allows us to potentially achieve SNR enhancements similar to those achieved with digital architectures, at the expense of a reduced SNR enhancement due to the unavoidable insertion loss of the network. This is contrary to the common misbelief that every passive component deteriorates the SNR with respect to their inputs. Consequently, using passive BFN as an alternative to digital phased arrays should not impose a significant SNR deterioration, provided that the insertion loss in the network is low.

This analysis could be of great use in the development of hybrid architectures, in which passive BFN are combined with digital architectures, allowing the designer to relax the requirements of the digital processing units while maintaining their performance in terms of SNR.

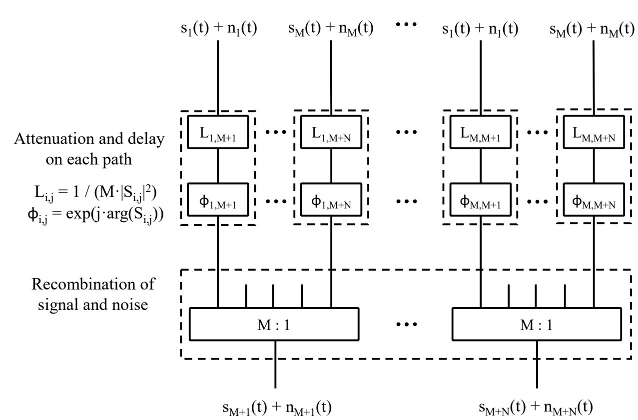

Fig. 4. Generalized model (equivalent circuit) of a passive multibeam BFN as the combination of M independent channels for each of the N output ports.

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