An evolutionary model of prenatal and postnatal discrimination against females

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Abstract

Discrimination against born and unborn females is a well-documented phenomenon in countries such as India, China, Taiwan or Korea. Empirical studies support both additive and substitutive relationships between prenatal and postnatal discriminatory practices against females. We introduce a theoretical evolutionary model that endogenizes the preference for sons in a society, and consequently, can explain why one type of relationship or the other emerges in a society.

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1. Introduction

Sex-based discrimination against female children has led to an alarming decline in the number of young females in many countries, but mainly in the Asia-Pacific region. Since Rosenzweig and Schultz (1982) found that female children receive less educational and health endowments in some communities than their male counterparts, a wide range of studies have confirmed the practice of postnatal discrimination against young females (Bhalotra, 2010; Echavarri and Husillos, 2016; Oster, 2009; Qian, 2008). Females also experience discrimination even before birth through the practice of sex-selective abortion of female fetuses (Echavarri and Ezcurra, 2010; Kim, 2005; Lin et al., 2014). The impact of prenatal and postnatal discrimination has modified the overall population's statistics. According to data from the United Nations Population Division (2018) –compiled by the World Bank–, the 2016 worldwide ratio of male births to female births (*sex ratio at birth*) was 1.5 percent higher than in 1962. Further, this 2016 ratio is 1.07, which is greater than the biologically expected figure of 1.05. This sharp increase in the sex ratio at birth has been mainly observed in Asia. For instance, this ratio has increased by 4.5 percent in the eastern Asia-Pacific region and 3.5 percent in South Asia. The increase in the sex ratio at birth has been of particular concern in China, where it stands at 7.6 percent. However, this phenomenon is not confined to developing countries, as the sex ratio at birth in upper-middle-income countries has increased by 3.3 percent between 1962 and 2016.

The prenatal and postnatal dimensions of discrimination are intricately related. However, controversial empirical evidence exists regarding the nature of this relationship. On the one hand, Goodkind's (1996) pioneering study shows how, in China during the late 1970s and the 1980s, there was a decrease in postnatal discrimination as well as a substantial increase in prenatal discrimination derived from access to sex-selective abortion technologies (this is called a *substitutive* relationship between prenatal and postnatal discrimination). On the other hand, Goodkind (1996) finds an increase in both postnatal and prenatal discrimination in India during almost the same period (this is called an *additive* relationship between prenatal and postnatal discrimination). Recent literature has documented both types of relationships (see, for instance, Lin et al., 2014, and Echavarri and Husillos, 2016, for substitutive relationships, and Nandi, 2015, for a non-substitutive relationship).

Lin et al. (2014) provide the only formal model to our knowledge that captures the relationship between prenatal and postnatal discrimination.[2] This model can only explain the substitutive relationship between both discriminatory practices with the following transmission channel: the diffusion of prenatal sex-detection technologies leads couples who would have otherwise neglected a girl after birth to terminate a female pregnancy.

The model developed by Lin et al. (2014) is based on the assumption of exogenous –and therefore, static– preferences. However, previous literature shows that dropping this assumption uncovers the coexistence of multiple equilibria in most models and the conjunction of additional transmission mechanisms. Two different evolutionary approaches have been proposed to examine the changes in gender-related preferences. One modeling avenue considers preference changes as the result of a Bayesian learning process about *true* preferences. Following this approach, Fernandez (2013) explains the geographical diversity in gender inequality in labor markets. The existence of true preferences is a controversial assumption in some scenarios, as it is the case of preferences for sons over daughters. The second modeling avenue departs from the true-preferences assumption and extends evolutionary biological models of natural selection to account for cultural transmission (see the pioneering model by biologists Cavalli-Sforza and

[2] Other models have focused on only one type of discrimination. See, for instance, Bhaskar (2011) and Kim (2005) for models regarding prenatal discrimination, and Rosenzweig and Schultz (1982) for a model regarding postnatal discrimination.

Feldman (1981)). Following this route, Hiller and Baudin (2016) develop a model *à la* Bisin and Verdier (2001) to explain the diversity in gender roles in different societies, and Fogarty and Feldman (2011) model the evolutionary dynamics of the perceived value of daughters in the society. Specifically, in Fogarty and Feldman's (2011) model, sex-biased preferences in the society might spread or erode depending on cultural traits, while these preferences would simultaneously affect cultural traits. This paper follows Fogarty and Feldman's (2011) explanation for the evolution of societal sex-biased preferences: we consider a variable called *societal son preference* that reflects the perceived value of sons over daughters in the society, and we use the levels of prenatal and postnatal discrimination as the cultural traits.

Boyd and Richerson (1985), and thereafter Bowles (1998), model cultural transmission processes using the replicator dynamics, which provides a more intuitive and explicit mechanism to capture the influence of past values of one variable on its present value as the result of the natural selection process. We follow this approach and propose a system of replicator dynamics equations that include both the impact of societal son preference on the evolution of the behavioral discriminatory practices and the impact of these practices on the evolution of societal son preference.

Therefore, the key aspect of our model is the endogenous determination of societal son preference, which depends on the diffusion of discrimination against born and unborn females. Consequently, the diffusion of prenatal sex-detection technologies has two effects on postnatal discrimination against females. On the one hand, there is the same transmission channel of Lin et al. (2014) that favors the emergence of a substitutive relationship. On the other hand, a new force favors an additive relationship: the increase in prenatal sex discrimination could strengthen the societal son preference, which would increase the predisposition to discriminate against daughters. The strength of each force will determine the relationship that emerges in each society. By doing so, our paper contributes to the literature by presenting a simple theoretical evolutionary model that characterizes the context for the emergence of each type of relationship between prenatal and postnatal discrimination.

Understanding the dynamics of discrimination has substantial policy implications. For instance, consider a society that has experienced a decrease in postnatal discrimination. Our model would help to determine: (*i*) if this decrease is a result of an erosion in the societal son preference, or (*ii*) if societal son preference remains invariant, and the observed decrease in postnatal discrimination is caused by the substitution of postnatal by prenatal discrimination.

Additionally, our findings show that there are scenarios in which the rise of sex-selective abortions might legitimize discriminatory practices against females, thus increasing the societal son preference, which results in further discrimination in the postnatal period as well. These results become relevant with the diffusion of inexpensive technologies to detect the sex of the fetus (*e.g.*, amniocentesis or chorionic villus sampling ultrasound) that make prenatal discrimination a feasible option in many communities.

The remainder of this paper is organized as follows. Section 2 introduces the evolutionary model and the solution concept that we use: institutionalized social states. Section 3 studies the impact of the access to prenatal sex-detection technologies on the institutionalized social states. Section 4 presents some concluding remarks, while the appendices include complementary results and proofs.

2. The model

Our model assumes for simplicity that two disjoint groups of couples or families exist in each period t: those who carry a female pregnancy, S^t, and those who have a young daughter, N^t.

There are societies in which couples in S^t have no access to modern technologies to detect the sex of the fetus such as amniocentesis, chronic villus sampling, or ultrasound testing; therefore, these couples cannot pursue sex-selective abortions. If there is access to such technologies in the society, couples in S^t must choose whether to allow a full-term pregnancy. Couples in N^t must choose whether to provide the same survival resources to daughters as they would have provided to sons. Let p^t be the proportion of couples in S^t that choose to practice sex-selective abortions, and q^t be the proportion of couples in N^t that choose to allocate survival resources to their daughters' detriment.

The following subsections present how couples in S^t and N^t make these decisions.

2.1. Decision on discriminating against unborn females

Our model assumes that couples who have a female pregnancy in each period t tend to imitate the actions of the previous generation (*i.e.*, the decisions of couples in S^{t-1}). Therefore, we consider that each couple in S^t has a status-quo option, which is the choice selected by a corresponding matching couple in S^{t-1}. If no further behavioral assumptions apply –namely, it is not possible to deviate from the status-quo option– we would set $p^t = p^{t-1}$. However, we allow for the possibility that social interactions cause couples to question their status-quo options. Specifically, we assume that each couple in S^t interacts with another couple in S^t. If both have the same status-quo option, no scope for questioning exists, and both couples will choose their status-quo options. However, when the two couples differ in their status-quo options, each couple acknowledges that acting otherwise is possible, and evaluates the available alternatives using a particular evaluation rule.[3] The idea behind this decision-making process is that people have two cognitive systems, the automatic and the reasoning system, such

[3]This way of modeling social interaction is consistent with previous literature (see, for instance, Bowles, 1998).

that they follow the automatic system to select their status-quo options unless social interaction renders the reasoning system salient (see Kahneman, 2003, for a support of this type of modeling behavior).

We assume for simplicity that the evaluation rule is the same for all couples, and subsequently, all couples that question their status-quo options make the same decision.[4] This evaluation rule of each couple in S^t is represented by a function $d_p : [0,1] \times \{0,1\} \to \mathbb{R}$ that depends on the diffusion of sex-selective abortions in society in the previous period, $p^{t-1} \in [0,1]$, and on the intensity of *societal son preference* (*i.e.*, the social reference point for the value of having a son over a daughter) in that period, $v^t \in \{0,1\}$.[5] This function d_p captures the difference between the payoffs associated with either aborting a female pregnancy or choosing not to do so. If $d_p(p^{t-1}, v^t) = 0$, then the payoffs of avoiding a female birth are the same as allowing the full-term pregnancy, while positive (respectively, negative) values describe situations in which allowing the full-term pregnancy is more (respectively, less) costly than avoiding the female birth. We assume that $d_p(\cdot)$ is an increasing function of v^t. Meanwhile, no constraint is imposed on the effect of p^{t-1} on $d_p(\cdot)$, because there are arguments to support both directions.[6]

We use this evaluation rule d_p to model the decisions of couples that use the cognitive system based on reasoning in each period t by the following simple

[4] Section 4 presents a discussion on dropping this assumption.

[5] We model societal son preference as a binary variable v, where 0 denotes a moderate societal son preference and 1 a high societal son preference. Section 2.3 will describe the evolution of this variable over time. It is possible to extend the model by defining v as a continuous variable, but the results do not qualitatively change.

[6] Notice that we have assumed that d_p does not depend on q^{t-1} because we consider that the influence of postnatal discrimination, if it exists, is mediated by the societal son preference rather than directly influencing discrimination against the female fetus. All in all, its inclusion would not substantially affect the results.

decision rule:

$$D_p(p^{t-1}, v^t) = \begin{cases} 1 & \text{if } \; d_p(p^{t-1}, v^t) > 0, \\ 0 & \text{if } \; d_p(p^{t-1}, v^t) = 0, \\ -1 & \text{if } \; d_p(p^{t-1}, v^t) < 0, \end{cases}$$

where 1 means that these couples choose to practice prenatal discrimination, -1 indicates that none of these couples discriminate, and 0 means that these couples continue with their respective status-quo options.

As a result of the two cognitive systems and this decision rule, when couples have access to sex-detection technologies, the proportion that practice sex-selective abortions in each period evolves over time following a type of differential equation called (discrete-time) "replicator equation". Then, the evolution of prenatal discrimination against females is driven by:

$$p^t(p^{t-1}, v^t) = \begin{cases} 0 & \text{if } \; A^t = 0, \\ p^{t-1} + p^{t-1} \cdot (1 - p^{t-1}) \cdot \beta \cdot D_p(p^{t-1}, v^t) & \text{otherwise}, \end{cases} \tag{1}$$

where $\beta > 0$ is a technical parameter that is small enough to avoid overshooting in the replicator dynamic[7]; and $A \in \{0, 1\}$ indicates whether couples have access to sex-detection technologies (0 means no access, 1 means access).[8]

Equation (1) explains the evolution of prenatal discrimination. On the one hand, access to prenatal sex-detection technologies is a condition that is necessary, but not sufficient, for the practice of discrimination against the female fetus. Once the technology is available in society ($A^t = 1$), the evolution of prenatal

[7]For further technical arguments regarding this assumption, see Weibull (1995), pp. 123-129.

[8]This version of the replicator equation slightly differs from the classical one. The difference is that in the classical definition $D_p(\cdot)$ would be substituted by $d_p(\cdot)$. That is, our version implies that the speed in which society increases or decreases the value of p depends on the sign of the difference of the payoffs between discriminating and not. In contrast, in the classical definition this speed depends on the size of the difference of payoffs. Adopting one version or the other does not change our results, as we are only interested in the stationary values of the variable and not in the speed of convergence to them.

discrimination against females is driven by a replicator equation, in which the change in the proportion of couples that practice sex-selective abortions, $(p^t - p^{t-1})$, depends on the number of couples that question their status-quo options and these couples' decisions. Given our assumptions, the number of couples that question their status-quo options is $p^{t-1} \cdot (1 - p^{t-1})$, which is the number of matching couples with different status-quo options, and their ultimate decisions are defined by $D_p(p^{t-1}, v^t)$.

2.2. Decision on discriminating against young girls

Our model assumes that couples who have a young female in each period t must choose whether to provide the same survival resources to these daughters that they would have provided to sons. We also assume here that these couples tend to imitate the decisions of the previous generation; therefore, we consider that each couple in N^t has a status-quo option, which is the choice that a corresponding matching couple in N^{t-1} selected. We also allow here for the possibility of questioning the status-quo by interacting with other couples with different status-quo options. We model this decision in the same terms as with the discrimination against unborn females: each couple in N^t interacts with another couple in N^t, and they only evaluate the different possibilities when they differ in their status-quo options.

We also assume here that the evaluation rule is the same for all couples, and therefore, so are the decisions of all couples in N^t that use the reasoning system. This evaluation rule is a function $d_q : [0,1]^2 \times \{0,1\} \to \mathbb{R}$, which measures the relative payoffs of allocating family resources against young girls. If $d_q(p^{t-1}, q^{t-1}, v^t) = 0$, then the payoffs of allocating survival resources against daughters are the same as the payoffs of allocating the resources equally among children, while positive (respectively, negative) values reflect higher (respectively, lower) payoffs from this discrimination. We naturally assume that $d_q(\cdot)$ is an increasing function of v^t, and also that it is a decreasing function of p^{t-1}, as prenatal discrimination against females directly influences the cost of raising girls by decreasing the number of those who are unwanted (Lin et al., 2014).

Then, we model the decisions of couples that use the cognitive system based on reasoning in each period t with the following decision rule:

$$D_q(p^{t-1}, q^{t-1}, v^t) = \begin{cases} 1 & \text{if} \quad d_q(p^{t-1}, q^{t-1}, v^t) > 0, \\ 0 & \text{if} \quad d_q(p^{t-1}, q^{t-1}, v^t) = 0, \\ -1 & \text{if} \quad d_q(p^{t-1}, q^{t-1}, v^t) < 0, \end{cases}$$

where 1, -1 and 0 have parallel meanings as in D_p.

Consequently, the proportion of couples that practice postnatal discrimination against females in each period evolves over time following a replicator equation, specified as follows:

$$q^t(p^{t-1}, q^{t-1}, v^t) = q^{t-1} + q^{t-1} \cdot (1 - q^{t-1}) \cdot \beta \cdot D_q(p^{t-1}, q^{t-1}, v^t). \qquad (2)$$

Equation (2) explains the evolution of postnatal discrimination and has a very similar structure to the second part of Equation (1). The change in the proportion of couples that discriminate against daughters ($q^t - q^{t-1}$) depends on the proportion of couples that question their status-quo options, $q^{t-1} \cdot (1 - q^{t-1})$, as well as these couples' decisions, $D_q(p^{t-1}, q^{t-1}, v^t)$.

2.3. Evolution of societal son preference

We have modeled couples' decisions to discriminate against females, both before and after birth, as a combination of two cognitive systems: an automatic and a reasoning one. While the automatic system leads to a replication of the status-quo option, the reasoning system leads to an evaluation of the alternatives, and this evaluation depends on the intensity of societal son preference. We now introduce the dynamics of societal son preference in our model.

Fogarty and Feldman's (2011) pioneering work models the evolution of societal son preference as a variable that affects and is affected by some *cultural* *traits*.[9] Following this approach, we represent the evolution of societal son preference as a function that depends on the proportion of couples that discriminate against female fetuses (variable p) and on the proportion of couples that discriminate against young girls (variable q). Specifically, we consider a function $d_v : [0,1]^2 \to \mathbb{R}$ such that $d_v(p^{t-1}, q^{t-1})$ captures the effect of the prenatal and postnatal discrimination against females on societal son preference. We assume that d_v is increasing in p^{t-1} and q^{t-1}, and that the evolution of societal son preference over time follows the next equation:

$$v^t(p^{t-1}, q^{t-1}) = \begin{cases} 1 & \text{if} \quad d_v(p^{t-1}, q^{t-1}) > 0, \\ 0 & \text{otherwise}, \end{cases} \tag{3}$$

where, as mentioned in footnote 5, 1 means a high societal son preference and 0 a moderate societal son preference.

2.4. Evolutionary dynamics of the system

We define a social state as a triple (p, q, v) that describes the level of prenatal discrimination, the level of postnatal discrimination, and the intensity of societal son preference. These three variables are interrelated, as shown by the two intertwined replicator equations and the evolution of societal son preference. Specifically, Equation (3) illustrates how the evolution of societal son preference is influenced by prenatal and postnatal discriminatory practices, and Equations (1) and (2) indicate how societal son preference influences individuals' decisions regarding their discrimination against females, and how the diffusion of each discriminatory practice affects the other, whether directly or indirectly.

We examine the evolutionary dynamics of the system by introducing additional notation and definitions. For each variable [*i.e.*, prenatal discrimination p, post-

[9] As mentioned in the introduction, the authors do not use the term societal son preference, but rather 'perceived value of a daughter' in the population.

natal discrimination q, and societal son preference v], we define a set of functions that capture the direction and intensity of change.

Definition 1. Consider any $\hat{p}, \hat{q} \in [0,1]$ and $\hat{v} \in \{0,1\}$.

- We represent the direction and intensity of change of prenatal discrimination with function $\gamma_p^{\hat{v}} : [0,1] \to [-1,1]$ such that for each $p^t \in [0,1]$,

$$\gamma_p^{\hat{v}}(p^t) = p^{t+1}(p^t, \hat{v}) - p^t.$$

- We represent the direction and intensity of change of postnatal discrimination with function $\gamma_q^{(\hat{p},\hat{v})} : [0,1] \to [-1,1]$ such that for each $q^t \in [0,1]$,

$$\gamma_q^{(\hat{p},\hat{v})}(q^t) = q^{t+1}(\hat{p}, q^t, \hat{v}) - q^t.$$

- We represent the direction and intensity of change of societal son preference with function $\gamma_v^{(\hat{p},\hat{q})} : \{0,1\} \to \{-1,0,1\}$ such that for each $v^t \in \{0,1\}$,

$$\gamma_v^{(\hat{p},\hat{q})}(v^t) = v^{t+1}(\hat{p}, \hat{q}) - v^t.$$

The functions described in Definition 1 provide the change in each variable between two consecutive periods at each value of the variable, given some fixed values for the other variables. For example, function $\gamma_q^{(\hat{p},\hat{v})}$ captures the difference between q^{t+1} and q^t, for each possible value of postnatal discrimination at time t, q^t, when the other variables are fixed at $\hat{p}$ and $\hat{v}$. That is, $\gamma_q^{(\hat{p},\hat{v})}$ identifies the dynamics introduced in Equation (2). Thus, function $\gamma_q^{(\hat{p},\hat{v})}$ could also be expressed as $\gamma_q^{(\hat{p},\hat{v})}(q^t) = q^t \cdot (1 - q^t) \cdot \beta \cdot D_q(\hat{p}, q^t, \hat{v})$. The same logic applies to function $\gamma_p^{\hat{v}}$ such that it corresponds to the dynamics of prenatal discrimination as described in Equation (1).

We now use Definition 1 to introduce the notion of stationarity.

Definition 2. Consider any $\hat{p}, \hat{q} \in [0,1]$ and $\hat{v} \in \{0,1\}$.

- A value p^* is a stationary state of variable p given $\hat{v}$ if $\gamma_p^{\hat{v}}(p^*) = 0$.
- A value q^* is a stationary state of variable q given $\hat{p}$ and $\hat{v}$ if $\gamma_q^{(\hat{p},\hat{v})}(q^*) = 0$.
- A value v^* is a stationary state of variable v given $\hat{p}$ and $\hat{q}$ if $\gamma_v^{(\hat{p},\hat{q})}(v^*) = 0$.

Stationarity identifies the states of a variable that are stable in the sense that if the variable takes this value at a given period, it would not change immediately. Then, stationarity is a necessary condition to yield good predictions of endogenous variables in a dynamic model. As a social state in our model includes three endogenous variables, we extend the stationarity condition to social states requiring that all variables take stationary values simultaneously.

Definition 3. A social state (p^*, q^*, v^*) is stationary if each of the variables take simultaneously stationary values given the values of the other variables: $\gamma_p^{v^*}(p^*) = \gamma_q^{(p^*,v^*)}(q^*) = \gamma_v^{(p^*,q^*)}(v^*) = 0$.

It is noteworthy that if p^* and q^* are stationary values, then $(p^*, q^*,\ v^t(p^*, q^*))$ is the unique stationary social state in which prenatal discrimination is p^* and postnatal discrimination is q^*. Then, the existence of stationary social states is guaranteed in our model, given that the corner values 0 and 1 are always stationary for p and q, independently of the values of the other variables.[10] However, we are not interested in all stationary social states as a prediction of our model, but only in a subclass of them. This is because a small perturbation on a stationary social state (modeled as an arbitrarily small proportion of couples behaving differently from what is prescribed by the social state) might foster population dynamics, and consequently, the society would arrive at a different stationary social state. We introduce an additional condition to prevent this phenomenon.

[10] Observe that whenever $A^t = 0$, p is exogenous and equal to 0, and then, this is the unique stationary value of prenatal discrimination.

Definition 4. Consider any stationary social state (p^*, q^*, v^*).

- p^* is self-correcting whenever

$$\begin{cases} \frac{\partial d_p(p^*,v^*)}{\partial p} < 0 & \text{if } p^* \in (0,1). \\ \nexists\, \epsilon^* > 0 \text{ such that } d_p(p=\epsilon, v^*) \geq 0 \text{ for all } \epsilon < \epsilon^* & \text{if } p^* = 0. \\ \nexists\, \epsilon^* > 0 \text{ such that } d_p(p=1-\epsilon, v^*) \leq 0 \text{ for all } \epsilon < \epsilon^* & \text{if } p^* = 1. \end{cases}$$

- q^* is self-correcting whenever

$$\begin{cases} \frac{\partial d_q(p^*,q^*,v^*)}{\partial q} < 0 & \text{if } q^* \in (0,1). \\ \nexists\, \epsilon^* > 0 \text{ such that } d_q(p^*, q=\epsilon, v^*) \geq 0 \text{ for all } \epsilon < \epsilon^* & \text{if } q^* = 0. \\ \nexists\, \epsilon^* > 0 \text{ such that } d_q(p^*, q=1-\epsilon, v^*) \leq 0 \text{ for all } \epsilon < \epsilon^* & \text{if } q^* = 1. \end{cases}$$

Essentially, a stationary value of a variable is also self-correcting if it survives an arbitrarily small shock to this value. To accomplish this, Definition 4 states that when an arbitrarily small proportion of the population drifts from the stationary social state, the dynamics of the system lead society to return to its original stationary value. Definition 4 differentiates between interior and corner stationary values. Regarding interior stationary values, self-correctness corresponds to a negative derivative of the corresponding evaluation function, which captures the idea that the population would have the propensity to decrease (respectively, increase) a discriminatory practice in response to a shock that has increased (respectively, decreased) this behavior. In corner stationary values, the derivative of the evaluation function is not defined, and subsequently, we opt to define self-correctness using a discrete change definition.

We then use this additional property to define our solution concept. The definition differs depending on the access to sex-selective abortion technologies.

Definition 5. A social state (p^*, q^*, v^*) is institutionalized with $A^t = 0$ if it is stationary, and q^* is also self-correcting. A social state (p^*, q^*, v^*) is institutionalized with $A^t = 1$ if it is stationary, and p^* and q^* are also self-correcting.

Observe that when there is no access to prenatal discrimination, variable p is exogenous in the model, and then, it does not make sense to require that is self-correcting.

2.5. Strategy to identify the relationship between prenatal and postnatal discrimination

The objective of this model is to identify the nature of the relationship between prenatal and postnatal discrimination in each society. We accomplish this by comparing two social states: the institutionalized social state in which the society is in absence of access to prenatal discrimination, and the institutionalized social state at which the society will arrive when has access to prenatal sex-detection technologies. Specifically, we model the period t in which prenatal sex-detection technologies appear as a twofold effect: (i) a change from $A^{t-1} = 0$ to $A^t = 1$, and (ii) an arbitrarily small shock on variable p such that an arbitrarily small proportion of the population begins to practice sex-selective abortions (*i.e.*, $p^{t-1} = 0$ and $p^t = \epsilon$, with ϵ arbitrarily small).[11] If the new institutionalized social state at which the society arrives is such that prenatal discrimination takes a strictly positive value and postnatal discrimination decreases relative to its value in period $t-1$, we can conclude that this society experiences a *substitutive* relationship between prenatal and postnatal discrimination. In contrast, if the new institutionalized social state is such that prenatal discrimination takes a strictly positive value, but postnatal discrimination increases with the shock as compared to its value in period $t-1$, then we have an *additive* relationship between the two components of female discrimination. This analysis helps us to discern how discriminatory practices will evolve in societies in which sex-selective

[11]For simplicity of exposition, we assume that the society was at an institutionalized social state in $t-1$. We characterize the institutionalized social states when there is no access to sex-selective abortions in Appendix A.

abortion technologies are spreading.[12]

It is necessary that institutionalized social states exist to perform this analysis. However, the existence of institutionalized social states is not guaranteed for every specification of functions d_p, d_q, and d_v. Fortunately, the existence of institutionalized social states is guaranteed under the condition that the replicator equations have continuous first partial derivatives, as derived from the Picard-Lindelöf theorem (see Weibull, 1995, for further discussion). This condition is satisfied in our model if, for example, the functions d_p, d_q, and d_v are linear. We then opt to assume linear specifications of these functions, and solve the model for these cases. Although it is a restrictive assumption, this guarantees a solution for our questions, and simultaneously, it already includes an ample range of specifications. Table 1 displays the linear specifications of functions d_p, d_q, and d_v.

Table 1: Linear specifications of the functions

	prenatal discrimination $i = p$	postnatal discrimination $i = q$	societal son preference $i = v$
$d_i(\cdot)$	$d_p^c + d_p^p \cdot p^{t-1} + d_p^v \cdot v^t$	$d_q^c + d_q^p \cdot p^{t-1} + d_q^q \cdot q^{t-1} + d_q^v \cdot v^t$	$d_v^c + d_v^p \cdot p^{t-1} + d_v^q \cdot q^{t-1}$

A linear specification of a function requires a parameter for the constant term and one extra parameter for each of the variables that determine the function. As the evaluation rule d_p depends on p^{t-1} and v^t, the assumption of a linear specification requires the introduction of three parameters. We denote such parameters by d_p^c (the constant term), d_p^p (the marginal effect of p on d_p), and d_p^v (the marginal effect of v on d_p). Similarly, the linear specification of the evaluation rule d_q

[12] A dual analysis can be performed to analyze a ban on releasing information to parents regarding the sex of the fetus: the ban can be modeled as a period t in which a change occurs from $A^{t-1} = 1$ to $A^t = 0$ and a shock on variable p appears such that $p^t = 0$.

depends on four parameters, denoted by d_q^c (the constant term), d_q^p (the marginal effect of p on d_q), d_q^q (the marginal effect of q on d_q), and d_q^v (the marginal effect of v on d_q). Finally, the linear specification of d_v depends on three parameters: d_v^c (the constant term), d_v^p (the marginal effect of p on d_v), and d_v^q (the marginal effect of q on d_v).

The assumptions made in Sections 2.1 to 2.3 are that d_p is increasing in v^t, d_q is decreasing in p^{t-1} and increasing in v^t, and d_v is increasing in both p^{t-1} and q^{t-1}. Given our linear specifications, this is equivalent to assume that parameters d_p^v, d_q^v, d_v^p and d_v^q are positive and parameter d_q^p is negative.

The assumption that d_p, d_q, and d_v are linear also implies that starting from a particular noninstitutionalized social state, the dynamics would lead to a unique institutionalized social state. This is important because it guarantees that there is only one possible social state at which the society could arrive after the shock in p that represents the emergence of sex-selective abortions. We denote by $(p, q, v)_{(q^*, v^*)}$ this unique prediction of the institutionalized social state that arises with the access to sex-selective abortions in a society that was at period $t-1$ when $A^{t-1} = 0$ in state $(0, q^*, v^*)$.[13] Therefore, the linear specifications facilitate our strategy of identifying the relationship between prenatal and postnatal discrimination.

3. Results

The institutionalized social state that arises after the shock that models the access to sex-selective abortion technologies, $(p, q, v)_{(q^*, v^*)}$, and the original institutionalized social state in period $t-1$, $(0, q^*, v^*)$, can be: (*i*) the same, $(p, q, v)_{(q^*, v^*)} = (0, q^*, v^*)$; (*ii*) different, because the shock produces an increase

[13] Observe that the social state $(p, q, v)_{(q^*, v^*)}$ is not attained at period t immediately after the shock, but at some subsequent period because the dynamics of the model need time to arrive at the new institutionalized social state.

in p and a decrease in q, $p > 0$ and $q < q^*$ (*i.e.*, a substitutive relationship between prenatal and postnatal discrimination); or (*iii*) different, because the shock produces an increase both in p and q, $p > 0$ and $q > q^*$ (*i.e.*, an additive relationship between prenatal and postnatal discrimination).[14] In what follows, we present three propositions to explain which of these cases emerges in each society. The first proposition characterizes societies in which the access to sex-detection technologies does not change the institutionalized social state.

Proposition 1. *Let the social state $(0, q^*, v^*)$ be the institutionalized social state in which the society is when $A^{t-1} = 0$. Suppose that, in period t, $A^t = 1$ and a shock on prenatal discrimination leads it to $p^t = \epsilon$, with ϵ arbitrarily small. Then, the institutionalized social state at which the society will end after this shock, denoted by $(p, q, v)_{(q^*, v^*)}$, will be equal to the initial institutionalized social state $(0, q^*, v^*)$ if, and only if,*

$$\begin{cases} \left[d_p^c + d_p^v \leq 0 \text{ and, in case of equality, } d_p^p < 0\right] & \text{whenever } v^* = 1. \\ \\ \left[d_p^c \leq 0 \text{ and, in case of equality, } d_p^p < 0\right] & \text{whenever } v^* = 0. \end{cases}$$

Proposition 1 implies that the conditions for having no change in the institutionalized social state with the shock are weaker if there was a moderate societal son preference before the shock ($v^* = 0$) than if this societal son preference was high ($v^* = 1$). This is because $d_p^v > 0$ by assumption. To obtain insights regarding the conditions of Proposition 1, we present a compelling example of a society satisfying them and show why the shock does not change the institutionalized social state.

[14]It is also possible that $p > 0$ and $q = q^*$, but we omit this possibility in the main text for gaining simplicity. Nevertheless, we discuss in footnote 21 the conditions under which this case occurs.

Example 1: Let a society be such that $d_p^c = -0.4$, $d_q^c = 0.5$, $d_p^p = 2.5$, $d_q^p = d_q^q = -1$, $d_v^p = d_v^q = 0.2$, $d_v^c = -0.14$, $d_p^v = 0.2$ and $d_q^v = 0.3$. This society satisfies both $d_p^c < 0$ and $d_p^c + d_p^v < 0$, and thus, Proposition 1 predicts that the institutionalized social state will not change with the shock, regardless of the level of societal son preference existing before the shock. Let us analyze why this occurs. First, using Propositions 4 to 6 in Appendix A, we obtain that $(0, 0.5, 0)$ and $(0, 0.8, 1)$ are the two plausible institutionalized social states before the introduction of sex-selective abortion technologies (*i.e.*, $A^{t-1} = 0$) in this society. Suppose that the society was at the institutionalized social state $(0, 0.5, 0)$ in period $t - 1$ when $A^{t-1} = 0$.[15] Then, the shock causes that prenatal discrimination changes from 0 in period $t - 1$ to ϵ in period t. This implies that some couples in S^{t+1} (to be precise, $\epsilon \cdot (1 - \epsilon)$) question their respective status-quo options in period $t + 1$ and evaluate the alternatives according to the evaluation rule d_p. Observe that $d_p(p^t, v^{t+1}) = d_p(\epsilon, 0) = -0.4 + 2.5 \cdot \epsilon + 0.2 \cdot 0$. Given that ϵ is arbitrarily small, $d_p(\epsilon, 0)$ is negative, and therefore, $D_p(p^t, v^{t+1}) = D_p(\epsilon, 0) = -1$; *i.e.*, the couples of S^{t+1} that question their status-quo options choose not to practice prenatal discrimination in period $t + 1$.[16] This behavioral change reduces the level of prenatal discrimination from ϵ in period t to an even lower value in period $t + 1$. The process continues with subsequent reductions in p during the following periods until no couple discriminates prenatally, and therefore, returning society to the original institutionalized social state. Appendix C displays a graphical representation of the dynamics that this society follows with the shock. $\square$

Proposition 1 leaves open the question of which type of relationship (additive or substitutive) would emerge when the conditions of the proposition are not satis-

[15] A similar analysis can be done if the society coordinated before the access to sex-selective abortions at the institutionalized social state $(0, 0.8, 1)$.

[16] This reasoning assumes that $v^{t+1} = 0$. This is guaranteed because $d_v(p^t, q^t) = d_v(\epsilon, 0.5) = -0.14 + 0.2 \cdot \epsilon + 0.2 \cdot 0.5$, and this expression is strictly negative since ϵ is arbitrarily small.

fied, and therefore, the institutionalized social state at which the society arrives after the shock differs from the original one. We answer this question with the following proposition, which characterizes the type of relationship that occurs in each case. Notice that the proposition only analyzes the societies with an interior value of postnatal discrimination in the initial institutionalized social state. This is because a shock cannot produce an additive (respectively, substitutive) relationship if postnatal discrimination equals the corner value 1 (respectively, 0) in the initial institutionalized social state, as postnatal discrimination cannot further increase (respectively, decrease). In this regard, and as can be seen from Proposition 5 in Appendix A, $d_q^q < 0$ is a necessary condition for the existence of at least one institutionalized social state $(0, q, v)$, with $q \in (0, 1)$, when $A^{t-1} = 0$. Therefore, a negative value of d_q^q is an implicit assumption in the proposition.

Proposition 2. *Let social state $(0, q^*, v^*)$, with $q^* \in (0, 1)$ be the institutionalized social state in which the society is when $A^{t-1} = 0$. Suppose that, in period t, $A^t = 1$ and a shock on prenatal discrimination leads it to $p^t = \epsilon$, with ϵ arbitrarily small. Then, the institutionalized social state at which the society will end after this shock, denoted by $(p, q, v)_{(q^*, v^*)}$, is such that $q > q^*$ if, and only if, $v^* = 0$, $v = 1$ and $d_q^v > |d_q^p| \cdot p$.*

We can obtain some conclusions from Proposition 2 for the cases in which the conditions of Proposition 1 are not met. The first conclusion is that a sufficient condition to obtain a substitutive relationship is the absence of an increase in societal son preference. That is, if either the society already had high societal son preference before the shock or the society had moderate societal son preference and the shock is not sufficient to change this to a high one, the increase in prenatal discrimination will be accompanied by a decrease in postnatal discrimination. Observe that this condition of non-increase of societal son preference with the shock includes the case of invariant preferences. Then, we can explain why the main theoretical result of Lin et al. (2014), in which exogenous preferences are assumed, is a substitutive relationship. We present an example of a society satisfying this sufficient condition to show why the shock produces a substitutive relationship.

Example 2: Let a society be that of Example 1, except with parameters d_p^c and d_p^p that are now equal to 0.4 and to -2.5, respectively. It can be computed, using Propositions 4 to 6 in Appendix A, that the possible institutionalized social states before the introduction of sex-selective abortion technologies (*i.e.*, $A^{t-1} = 0$) in this society are the same as in Example 1: $(0, 0.5, 0)$ and $(0, 0.8, 1)$. Suppose that when $A^{t-1} = 0$, the society was at the institutionalized social state $(0, 0.8, 1)$. Observe that this society is such that $d_p^c + d_p^v > 0$, and then, by Proposition 1, $(p, q, v)_{(0.8,1)} \neq (0, 0.8, 1)$. Note also that since societal son preference already had a high value before the shock, the prediction of Proposition 2 is that $p > 0$ and $q < 0.8$. Let us see why this is the case. The fact that q is an stationary value of postnatal discrimination given p and v (because $(p, q, v)_{(0.8,1)}$ is an institutionalized social state) implies, if q is interior (*i.e.*, $q \in (0, 1)$), that $D_q(p, q, v) = 0$ (see Equation (2)). Therefore, $d_q(p, q, v) = 0.5 - p - q + 0.3v = 0$. As $v \in \{0, 1\}$ and $p \geq 0$, we obtain $q \leq 0.8$. Given that $q = 0.8$ implies that $p = 0$ and $v = 1$, and this contradicts that $(p, q, v)_{(0.8,1)} \neq (0, 0.8, 1)$, we conclude that $q < 0.8$. Thus, we have obtained a substitutive relationship.[17] Appendix C graphically represents the dynamics of how this society evolves from the original institutionalized social state to the final one. □

A second conclusion from Proposition 2 is that an increase in societal son preference from $v^* = 0$ to $v = 1$ with the shock is not sufficient to produce an additive relationship. To understand why, observe that, with this increase in societal son

[17] This reasoning is incomplete because we have not rejected the possibility that $q = 1$. However, this case is not possible, as can be observed from the proof of Proposition 2.

preference, there are two effects on the evaluation rule d_q with the shock. On the one hand, the increase in societal son preference tends to increase d_q because $d_q^v > 0$. On the other hand, the increase in p produced by the shock tends to decrease d_q because $d_q^p < 0$. Then, to obtain an additive relationship, it is also necessary that the impact of the change of v on the evaluation rule d_q is greater than the impact of the change of p on d_q, and this occurs when $d_q^v > |d_q^p| \cdot p$. We now present examples of two societies in which the societal son preference has increased from $v^* = 0$ to $v = 1$ with the shock. In Example 3, the additional condition of $d_q^v > |d_q^p| \cdot p$ is not satisfied, and therefore, the increase in societal son preference does not result in an additive relationship. The opposite occurs in Example 4.

Example 3: Let a society be that of Example 2, except with parameters d_v^p and d_p^v that are both now equal to 0.5. It can be computed, using Propositions 4 to 6 in Appendix A, that the possible institutionalized social states before the introduction of sex-selective abortion technologies (*i.e.*, $A^{t-1} = 0$) in this society are the same as in Examples 1 and 2: $(0, 0.5, 0)$ and $(0, 0.8, 1)$. Suppose that when $A^{t-1} = 0$, the society was at the institutionalized social state $(0, 0.5, 0)$. Since $d_p^c > 0$, we know by Proposition 1 that $(p, q, v)_{(0.5,0)} \neq (0, 0.5, 0)$. Let us suppose that the condition of an increase in societal son preference is satisfied in this society, and thus $v = 1$.[18] Proceeding as in Example 2, the fact that q is an stationary value of postnatal discrimination given p and $v = 1$ (because $(p, q, 1)_{(0.5,0)}$ is an institutionalized social state) implies, if q is interior, that $d_q(p, q, 1) = 0.5 - p - q + 0.3 \cdot 1 = 0$. Operating, we obtain $q = 0.8 - p$. Thus, q is greater or less than 0.5 depending on whether p is less or greater than 0.3. Since p is also a stationary value of prenatal discrimination given $v = 1$, we have that $d_p(p, 1) = 0.4 - 2.5p + 0.5 \cdot 1 = 0$ if $p \in (0, 1)$. Therefore, $p = 0.36 > 0.3$,

[18] The increase in societal son preference indeed occurs, as can be observed from the dynamics that this society follows after the shock, which are represented in Appendix C.

and consequently, $q < 0.5$. As $d_q^v = 0.3 < |-1| \cdot 0.36 = |d_q^p| \cdot p$, we have found a substitutive relationship in this society. Appendix C graphically represents the dynamics of how this society evolves from the original institutionalized social state to the final one.[19] $\square$

Example 4: Let a society be that of Example 3, except with parameter d_p^v that is now equal to 0.2. As in all preceding examples, and as can be checked with Propositions 4 to 6 in Appendix A, the possible institutionalized social states before the introduction of sex-selective abortion technologies (*i.e.*, $A^{t-1} = 0$) in this society are $(0, 0.5, 0)$ and $(0, 0.8, 1)$. Suppose, as in Example 3, that the society when $A^{t-1} = 0$ was at the institutionalized social state $(p^*, q^*, v^*) = (0, 0.5, 0)$. The same reasoning as in Example 3 leads us to show that the society arrives at a new institutionalized social state $(p, q, 1)_{(0.5,0)} \neq (0, 0.5, 0)$ such that if q is interior, then $q = 0.8 - p$. Then, again, q is greater or less than 0.5 depending on whether p is less or greater than 0.3. However, the calculus with prenatal discrimination is different now: as p is also a stationary value of prenatal discrimination given $v = 1$, we have that $d_p(p, 1) = 0.2 - 2.5p + 0.5 \cdot 1 = 0$ if $p \in (0, 1)$. Therefore, $p = 0.28 < 0.3$, and thus, $q > 0.5$. Since $d_q^v = 0.3 > |-1| \cdot 0.28 = |d_q^p| \cdot p$, we have found an additive relationship in this society.[20] Appendix C graphically represents how this society evolves from the original institutionalized social state to the final one. $\square$

An important remark concerning Proposition 2 is that, although it states conditions that provide an intuition of why a substitutive or an additive relationship

[19] As in Example 2, the reasoning is incomplete as to why a substitutive relationship appears because we have not rejected the possibility that q and/or p take corner values in the final institutionalized social state. Appendix C illustrates the dynamics of how this society evolves, which shows that both p and q are interior.

[20] A similar caveat with the corner values of the variables than in the previous examples apply here.

emerges in each society, these conditions cannot be easily checked *ex ante* with the parameters of the model. First, the condition that societal son preference increases with the shock from $v^* = 0$ to $v = 1$ cannot be directly tested knowing the structure of the evaluation rules d_p, d_q and d_v. Second, the condition $[d_q^v > |d_q^p| \cdot p]$ requires knowledge of the value of the prenatal discrimination in the institutionalized social state after the shock and again this cannot be easily deduced from the parameters of the model. Therefore, we are going to present in Proposition 3 a set of conditions based only on the primitives of the model that characterizes the emergence of an additive relationship. However, Proposition 3 has the cost of a more complex statement than Proposition 2. First, we need to introduce some values for the variables that will appear in the result:

$$\hat{p} = \begin{cases} 1 & \text{if } \ d_p^p \geq 0 \\ \min\{1, \frac{d_p^c}{|d_p^p|}\} & \text{if } \ d_p^p < 0. \end{cases}$$

$$\hat{q} = \max\{0, \frac{d_q^c + d_q^p \cdot \hat{p}}{|d_q^q|}\}.$$

$$\bar{p} = \begin{cases} 1 & \text{if } \ d_p^p \geq 0 \\ \min\{1, \frac{d_p^c + d_p^v}{|d_p^p|}\} & \text{if } \ d_p^p < 0. \end{cases}$$

The proof of the proposition shows that $\hat{p}$ and $\hat{q}$ are the values that prenatal and postnatal discrimination would attain after the shock if societal son preference were exogenous and equal to a moderate value ($v^* = v = 0$). Similarly, $\bar{p}$ corresponds to the value that prenatal discrimination will take after the shock in our model, in which societal son preference is endogenous and increases from a moderate value before the shock ($v^* = 0$) to a high value after the shock ($v = 1$). We are now ready to introduce the proposition.

Proposition 3. *Let a society be such that there exists an institutionalized social state* $(0, q^*, 0)$*, with* $q^* \in (0, 1)$*, when* $A^{t-1} = 0$*. Suppose that, in period* t*,* $A^t = 1$

and a shock on prenatal discrimination leads it to $p^t = \epsilon$, with ϵ arbitrarily small. Then, the following two statements are equivalent:

- *If the society is at $(0, q^*, 0)$ when $A^{t-1} = 0$, the institutionalized social state at which the society will end after the shock, denoted by $(p, q, v)_{(q^*,0)}$, is such that $q > q^*$.*
- *The following conditions hold:*
  (i) $d_p^c \geq 0$ and, in case of equality, $d_p^p \geq 0$.
  (ii) $d_v^c + d_v^p \cdot \hat{p} + d_v^q \cdot \hat{q} > 0$
  (iii) $d_q^v > |d_q^p| \cdot \bar{p}$.

Conditions (i), (ii), and (iii) characterize the emergence of an additive relationship, and can be related with the conditions in the previous propositions. First, condition (i) is related to Proposition 1 that characterizes the conditions of societies in which the shock does not change the institutionalized social state. As an additive relationship requires a change from the original institutionalized social state with this shock, condition (i) is exactly the counterpart of the condition of Proposition 1 referred to a moderate societal son preference. Second, condition (ii) refers to the condition of Proposition 2 that the dynamics of the system has to increase the societal son preference from 0 to 1 as a result of the shock. The proof of Proposition 3 shows that this increase occurs if, and only if, $d_v(\hat{p}, \hat{q}) > 0$; *i.e.*, if the dynamics of societal son preference implies a high value for this variable when the society is at the levels of prenatal and postnatal discrimination to which the society would have arrived in the fictitious situation of exogenous preferences. Finally, condition (iii) is exactly the second condition of Proposition 2, in which we have included the actual value that prenatal discrimination takes after the shock, $\bar{p}$.[21]

[21] If condition (i) is not satisfied, then Proposition 1 indicates that $(p, q, v)_{(q^*,v^*)} = (0, q^*, v^*)$. If conditions (i) and (ii) are satisfied, but $d_q^v = |d_q^p| \cdot \bar{p}$, we would have that $p > 0$ and $q = q^*$.

4. Concluding remarks

We have presented an evolutionary model to study the relationship between prenatal and postnatal discrimination against females. This model assumes that families decide to practice each type of discrimination using a combination of an automatic system, which involves following the actions of other families in the society, and a reasoning system, which evaluates the payoffs of each alternative. We present a set of propositions that allows us to recognize which type of relationship emerges in each society after a shock that models the access to sex-selective abortions. First, if the society satisfies the condition of Proposition 1, then the access to sex-selective abortions will change neither the prenatal nor postnatal discrimination against females. Second, if the society does not satisfy the condition of Proposition 1, then the access to sex-selective abortions will be used by some families to discriminate prenatally. To identify the effect on postnatal discrimination in these cases, Propositions 2 and 3 characterize the societies in which the relationship is substitutive (*i.e.*, the increase in prenatal discrimination is accompanied by a decrease in postnatal discrimination), and those in which it is additive (*i.e.*, the increase in prenatal discrimination comes with an increase in postnatal discrimination).

The model has been solved for linear specifications of the evaluation functions that guarantee the existence of institutionalized social states. The results of the paper do not qualitatively change if the model is solved for other structures of the evaluation functions, such as convex or concave functions.

These evaluation functions depend on some parameters for which we have imposed three types of restrictions: (i) a higher societal son preference tends to increase the payoffs of each discriminatory practice (*i.e.*, $d_p^v > 0$ and $d_q^v > 0$); (ii)

In the remaining cases in which not all conditions are satisfied, we would have a substitutive relationship: $p > 0$ and $q < q^*$.

the societal son preference is endogenous and the diffusion of each of the discriminatory practices tends to increase this societal son preference (*i.e.*, $d_v^p > 0$ and $d_v^q > 0$); and (*iii*) the practice of sex-selective abortions tends to decrease the postnatal discrimination against females (*i.e.*, $d_q^p < 0$). It is interesting to remark two issues about these restrictions.

On the one hand, the first two types of restrictions together imply that when the sex ratio increases, the payoffs of practicing each type of discrimination tend to increase: the diffusion of each discriminatory practice tends to increase the societal son preference (see (*ii*)), and this potential increase tends to increase the payoffs of each discriminatory practice (see (*i*)). This is natural in most contexts, but it is also possible that if the sex ratio becomes extremely high, the relationship would become inverse: an additional increase in the sex ratio could increase the market value of brides in the marriage market (especially in monogamous societies), decreasing in turn the discrimination against female fetuses and children (as the market value of young girls increases). Our model can be adapted to study this phenomenon by substituting the positive linear effect of v on p and q by inverted U-shaped effects.

On the other hand, one implicit assumption of the model is that the decision functions, D_p and D_q, and therefore, the preferences are the same for all couples. This assumption of working with representative couples is done for the sake of exposition because it is possible to define also a similar model with heterogeneous agents and arrive at similar conclusions. One possible direction to incorporate heterogeneous agents in a simple manner is to include heterogeneity in the decision rules. This could be done by defining the decision rule of a couple $i \in S_t$ as $D_p^i(p^{t-1}, v^t) = 1$ if $d_p(p^{t-1}, v^t) > x_i$, 0 if $d_p(p^{t-1}, v^t) = x_i$, and -1 otherwise. Similarly, the decision rule of a couple $i \in N_t$ would be defined as $D_q^i(p^{t-1}, q^{t-1}, v^t) = 1$ if $d_q(p^{t-1}, q^{t-1}, v^t) > x_i$, 0 if $d_q(p^{t-1}, q^{t-1}, v^t) = x_i$, and -1 otherwise. In this way, couples with higher (respectively, lower) values of x_i

would be less (respectively, more) predisposed to prefer sons over daughters, and then, less (respectively, more) inclined to select discriminatory practices. This simple generalization allows to include heterogeneity of decisions in the same context among couples, and thus, heterogeneity of preferences. Assuming that the values of x_i across S_t and N_t follow a normal distribution with mean normalized at 0, the expected dynamics of the system in this more general model would be exactly the same as in the model we have analyzed in the paper.[22]

All in all, adopting the structures of the evaluation functions of the paper or any of these alternative specifications, the model explains the different forces that affect the prenatal and postnatal dimensions of the missing women phenomenon. The underlying argument is that each dimension of discrimination itself affects and can sustain both discriminatory practices. This enforcement result provides important implications for policy interventions: combined public policies that affect simultaneously both dimensions of discrimination are more likely to successfully fight inequality than the implementation of sequential public policies.

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[22] Another way of including heterogeneity is to allow variation in the evaluation functions: each couple $i \in S^t$ (respectively, $i \in N^t$) has an evaluation function d_p^i (respectively, d_q^i) with specific weights $d_p^{c,i}$, $d_p^{p,i}$ and $d_p^{v,i}$ (respectively, $d_q^{c,i}$, $d_q^{p,i}$, $d_q^{q,i}$ and $d_q^{v,i}$). If these parameters are normally distributed, this extension also produces the same dynamics as with the basic model.

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Appendix A: The institutionalized social states before the access to sex-detection technologies

This section characterizes the institutionalized social states that arise before the access to prenatal sex-detection technologies. We divide the characterization in three propositions. The first proposition characterizes the conditions under which we have corner institutionalized social states.

Proposition 4. *Consider a society without access to prenatal sex-detection technologies ($A^{t-1} = 0$). Then,*

- $(0,0,0)$ *is an institutionalized social state if, and only if,* $d_v^c \leq 0$ *and* $[d_q^c \leq 0$ *and, in case of equality,* $d_q^q < 0]$.
- $(0,0,1)$ *is an institutionalized social state if, and only if,* $d_v^c > 0$ *and* $[d_q^c + d_q^v \leq 0$ *and, in case of equality,* $d_q^q < 0]$.
- $(0,1,0)$ *is an institutionalized social state if, and only if,* $d_v^c + d_v^q \leq 0$ *and* $[d_q^c + d_q^q \geq 0$ *and, in case of equality,* $d_q^q < 0]$.
- $(0,1,1)$ *is an institutionalized social state if, and only if,* $d_v^c + d_v^q > 0$ *and* $[d_q^c + d_q^q + d_q^v \geq 0$ *and, in case of equality,* $d_q^q < 0]$.

Proof. We only show that the social state $(0,0,0)$ is an institutionalized social state if, and only if, $d_v^c \leq 0$ and $[d_q^c \leq 0$ and, in case of equality, $d_q^q < 0]$ (we omit the other cases since the proofs are similar). Observe first that, given the assumption that $A^{t-1} = 0$, the no access to prenatal sex-detection technologies implies that no couple will discriminate against unborn females. Thus, $p = 0$ in any social state, institutionalized or not.

Suppose that $d_v^c \leq 0$ and $[d_q^c \leq 0$ and, in case of equality, $d_q^q < 0]$. We have to show that $(0,0,0)$ is an institutionalized social state. First, we have that $\gamma_q^{(p,v)}(0) = 0$ for any values of p and v, and then, $q = 0$ is stationary for any values of the other variables. Second, we have that $d_v(0,0) = d_v^c \leq 0$. Then, if $p = 0$ and $q = 0$ are stationary, $v = 0$ is also stationary. Thus, $(0,0,0)$ is a stationary social state. Finally, we have to show that $q = 0$ is also self-correcting. This means that there should not be $\epsilon^* > 0$ such that $d_q(0,\epsilon,0) \geq 0$ for all $\epsilon < \epsilon^*$. Observe that $d_q(0,\epsilon,0) = d_q^c + d_q^q \cdot \epsilon$. If $d_q^c < 0$, we have that $d_q(0,\epsilon,0) < 0$ for all $\epsilon < \frac{|d_q^c|}{|d_q^q|}$. If, however, $d_q^c = 0$ and $d_q^q < 0$, we have that $d_q(0,\epsilon,0) < 0$ for all $\epsilon > 0$. Thus, $q = 0$ is self-correcting, and therefore, $(0,0,0)$ is an institutionalized social state when $A^{t-1} = 0$.

Suppose now that $d_v^c > 0$. Then, we show that $v = 0$ is not stationary given $p = 0$ and $q = 0$. Observe that $d_v(0,0) = d_v^c > 0$. Then, if $p = 0$ and $q = 0$ are stationary, the stationary value of variable v is 1.

Suppose now that $d_q^c > 0$. Then, although $(0,0,0)$ is a stationary social state, we show that $q = 0$ is not self-correcting. In this case, since $d_q(0,\epsilon,0) = d_q^c + d_q^q \cdot \epsilon$, we have that $d_q(0,\epsilon,0) > 0$ for all $\epsilon < \frac{d_q^c}{|d_q^q|}$, and then, $q = 0$ is not self-correcting.

Finally, suppose that $d_q^c = 0$, but $d_q^q \geq 0$. Then, as in the previous paragraph, although $(0,0,0)$ is a stationary social state, we show that $q = 0$ is not self-correcting. In this case, we obtain that $d_q(0,\epsilon,0) = d_q^q \cdot \epsilon \geq 0$ for all $\epsilon > 0$. Then, $q = 0$ is not self-correcting. $\square$

The following proposition presents a condition under which the only plausible institutionalized social states are the corner social states.

Proposition 5. *Consider a society without access to prenatal sex-detection technologies ($A^{t-1} = 0$) and $d_q^q \geq 0$. Then, a social state (p,q,v) can be an institutionalized social state only if $p = 0$ and $q \in \{0,1\}$.*

Proof. Given the assumption that $A^{t-1} = 0$, the no access to prenatal sex-detection technologies implies that no couple will discriminate against unborn females. Thus, $p = 0$ in any state, institutionalized or not. We now show that a social state $(0, q, v)$ in which $q \in (0, 1)$ cannot be an institutionalized social state whenever $A^{t-1} = 0$ and $d_q^q \geq 0$. Suppose by contradiction that in one of these societies there exists an institutionalized social state $(0, q^*, v^*)$ such that $q^* \in (0, 1)$. This requires that q^* is self-correcting. By definition, $q^* \in (0, 1)$ is self-correcting if $\frac{\partial d_q(\cdot)}{\partial q}$ evaluated at the social state $(0, q^*, v^*)$ is negative. Given the linear specification assumed for the evaluation function d_q (see Table 1), we have that $\frac{\partial d_q(\cdot)}{\partial q} = d_q^q$. However, we have assumed that $d_q^q \geq 0$, and therefore, $\frac{\partial d_q(\cdot)}{\partial q}$ in any social state is non-negative. Then, we have a contradiction. $\square$

Proposition 5 shows that if the marginal effect of postnatal discrimination on the evaluation rule d_q is non-negative, all institutionalized social states when $A^{t-1} = 0$ satisfy that all couples in N_t behave in the same way. The following proposition studies the remaining cases in which interior values for q can occur in institutionalized social states when $A^{t-1} = 0$.

Proposition 6. *Consider a society without access to prenatal sex-detection technologies ($A^{t-1} = 0$) and $d_q^q < 0$. Then:*

- $(0, x, 1)$, *with* $x \in (0, 1)$, *is an institutionalized social state if, and only if,* $x = -\frac{d_q^c + d_q^v}{d_q^q}$ *and* $d_v^c + d_v^q \cdot x > 0$.
- $(0, y, 0)$, *with* $y \in (0, 1)$, *is an institutionalized social state if, and only if,* $y = -\frac{d_q^c}{d_q^q}$ *and* $d_v^c + d_v^q \cdot y \leq 0$.

Proof. We only show that the social state $(0, x, 0)$, with $x \in (0, 1)$ is an institutionalized social state if, and only if, $x = -\frac{d_q^c + d_q^v}{d_q^q}$ and $d_v^c + d_v^q \cdot x > 0$ (we omit the other case since the proof is similar). First, given the assumption that $A^{t-1} = 0$, we obviously have that $p = 0$ in all social states.

Suppose that $(0, x, 1)$, with $x \in (0, 1)$, is an institutionalized social state. Since x is a stationary value of postnatal discrimination given $p = 0$ and $v = 1$, we have that $D_q(0, x, 1) = 0$, and therefore, $d_q(0, x, 1) = 0$. Then, $d_q^c + d_q^q \cdot x + d_q^v = 0$, and isolating x in this expression, we obtain that $x = -\frac{d_q^c + d_q^v}{d_q^q}$, as desired. Similarly, since $v = 1$ is stationary given the stationary values of $p = 0$ and $q = x$, we have that $d_v(0, x) > 0$. Then, $d_v^c + d_v^q \cdot x > 0$.

Suppose now by contradiction that $(0, x, 1)$, with $x \in (0, 1)$ and $x \neq -\frac{d_q^c + d_q^v}{d_q^q}$, is an institutionalized social state. Then, x is stationary given $p = 0$ and $v = 1$, and since $x \in (0, 1)$, we should have that $D_q(0, x, 1) = 0$, and therefore, $d_q(0, x, 1) = 0$. It can be easily seen that $d_q^c + d_q^q \cdot x + d_q^v \neq 0$ for all values $x \neq -\frac{d_q^c + d_q^v}{d_q^q}$ and we have a contradiction.

Finally, suppose by contradiction that $(0, x, 1)$, with $x = -\frac{d_q^c + d_q^v}{d_q^q} \in (0, 1)$, is an institutionalized social state, but $d_v^c + d_v^q \cdot x \leq 0$. Then, $d_v(0, x) = d_v^c + d_v^q \cdot x \leq 0$. Then, if $p = 0$ and $q = x$ are stationary, the stationary value of variable v is 0 and this contradicts that $(0, x, 1)$ is an institutionalized social state. $\square$

Proposition 6 characterizes the level of postnatal discrimination in interior institutionalized social states for each value of societal son preference. Specifically, the value of postnatal discrimination that corresponds to the institutionalized social state for a high value of societal son preference is not lower than the corresponding value of the institutionalized social state for a moderate societal son preference.

Appendix B: Proofs of the results of the main text

Proof of Proposition 1

We only do the proof for the case of $v^* = 1$ (the case of $v^* = 0$ is similar and thus omitted).

Suppose first that $d_p^c + d_p^v \leq 0$ and, in case of equality, $d_p^p < 0$. We have to show that $(p, q, v)_{(q^*,1)} = (0, q^*, 1)$ or, what is the same, that the shock that represents the access to sex-selective abortion technologies does not change the institutionalized social state. It is then enough to see that 0 is a self-correcting value of prenatal discrimination given $v = 1$. This means that there should not be $\epsilon^* > 0$ such that $d_p(\epsilon, 1) \geq 0$ for all $\epsilon < \epsilon^*$. Observe that $d_p(\epsilon, 1) = d_p^c + d_p^p \cdot \epsilon + d_p^v$. If $d_p^c + d_p^v < 0$, we have that $d_p(\epsilon, 1) < 0$ for all $\epsilon < \frac{|d_p^c + d_p^v|}{|d_p^p|}$. If, however, $d_p^c + d_p^v = 0$ and $d_p^p < 0$, we have that $d_p(\epsilon, 1) < 0$ for all $\epsilon > 0$. Thus, $p = 0$ is self-correcting, and therefore, $(p, q, v)_{(q^*,1)} = (0, q^*, 1)$.

Suppose now that $d_p^c + d_p^v > 0$. Then, we show that $p = 0$ is not self-correcting given $v = 1$. Since $d_p(\epsilon, 1) = d_p^c + d_p^p \cdot \epsilon + d_p^v$, we obtain that $d_p(\epsilon, 1) > 0$ for all $\epsilon < \frac{d_p^c + d_p^v}{|d_p^p|}$. Then, $p = 0$ is not self-correcting.

Finally, suppose that $d_p^c + d_p^v = 0$, but $d_p^p \geq 0$. Then, $d_p(\epsilon, 1) = d_p^p \cdot \epsilon \geq 0$ for all $\epsilon > 0$, and again we obtain that $p = 0$ is not self-correcting given $v = 1$.

Proof of Proposition 2

Suppose that $(0, q^*, v^*)$, with $q^* \in (0, 1)$, is the institutionalized social state in which the society is when $A^{t-1} = 0$ and let $(p, q, v)_{(q^*,v^*)}$ be the institutionalized social state at which the society will end after the shock that opens the access to sex-selective abortions. Observe that, by Proposition 5, the fact that $q^* \in (0, 1)$ implies that $d_q^q < 0$.

Step 1: We show that if $v \leq v^*$, then $q \leq q^*$.

Observe first that we know that $q^* \in (0,1)$ is a stationary value of postnatal discrimination given the values of 0 for prenatal discrimination and v^* for societal son preference, and then, $d_q(0, q^*, v^*) = 0$. Therefore, $d_q^c + d_q^q \cdot q^* + d_q^v \cdot v^* = 0$. If $q \in (0,1)$, we can obtain similarly that, since q is a stationary value of postnatal discrimination given the values of p for prenatal discrimination and v for societal son preference, $d_q(p, q, v) = 0$, and thus, $d_q^c + d_q^p \cdot p + d_q^q \cdot q + d_q^v \cdot v = 0$. Then, $d_q^c + d_q^q \cdot q^* + d_q^v \cdot v^* = d_q^c + d_q^p \cdot p + d_q^q \cdot q + d_q^v \cdot v$. Thus, $d_q^v \cdot (v^* - v) = d_q^p \cdot p + d_q^q \cdot (q - q^*)$. Since $d_q^v > 0$ and $v \leq v^*$, the LHS of the equation is non-negative. Similarly, since $d_q^p < 0$, $d_q^q < 0$ and $p \geq 0$, it is necessary that $q \leq q^*$ for the RHS to be also non-negative. Then, $q \leq q^*$. It only remains to be shown that q cannot be 1. Suppose otherwise that $q = 1$. Then, we can obtain, since 1 is a self-correcting value of postnatal discrimination in the social state $(p, 1, v)$, that there is no $\epsilon^* > 0$ such that $d_q(p, 1 - \epsilon, v) \leq 0$ for all $\epsilon < \epsilon^*$. Consider ϵ^* such that $1 - \epsilon^* > q^*$, which exists because $q^* \in (0,1)$. Then, we have that $d_q(p, 1 - \epsilon, v) > 0$ for some $\epsilon < \epsilon^*$, and thus, $d_q^c + d_q^p \cdot p + d_q^q \cdot (1 - \epsilon) + d_q^v \cdot v > 0$. Then, $d_q^c + d_q^p \cdot p + d_q^q \cdot (1 - \epsilon) + d_q^v \cdot v > d_q^c + d_q^q \cdot q^* + d_q^v \cdot v^*$. Thus, $d_q^p \cdot p + d_q^q \cdot [(1 - \epsilon) - q^*] > d_q^v \cdot (v^* - v)$. Since $d_q^p < 0$, $d_q^q < 0$, $p \geq 0$ and $(1 - \epsilon) > q^*$, the LHS of the equation is strictly negative. Similarly, since $d_q^v > 0$ and $v \leq v^*$, the RHS of the equation is non-negative. This is a contradiction.

Step 2: We show that whenever $v > v^*$ and $q \in (0,1)$, then $[q > q^*$ if, and only if, $d_q^v > |d_q^p| \cdot p]$.

Suppose that $q \in (0,1)$ and $v > v^*$, and thus, $v^* = 0$ and $v = 1$. Since $q^* \in (0,1)$ is a stationary value of postnatal discrimination given the values of 0 for prenatal discrimination and 0 for societal son preference, we have that $d_q(0, q^*, 0) = 0$, and thus, $d_q^c + d_q^q \cdot q^* = 0$. Similarly, since $q \in (0,1)$ and q is a stationary value of postnatal discrimination given the values of p for prenatal discrimination and 1 for societal son preference, $d_q(p, q, 1) = 0$, and thus, $d_q^c + d_q^p \cdot p + d_q^q \cdot q + d_q^v = 0$. Then, $d_q^c + d_q^q \cdot q^* = d_q^c + d_q^p \cdot p + d_q^q \cdot q + d_q^v$. Thus, $d_q^q \cdot (q^* - q) = d_q^p \cdot p + d_q^v$. Since we already know that $d_q^q < 0$, $d_q^p < 0$ and $d_q^v > 0$, we obtain that $q > q^*$ if, and only if, $d_q^v > |d_q^p| \cdot p$.

Step 3: We show that whenever $v > v^*$ and $q \in \{0, 1\}$, then $[q > q^*$ if, and only if, $d_q^v > |d_q^p| \cdot p]$.

Suppose that $q \in \{0, 1\}$ and $v > v^*$, and thus, $v^* = 0$ and $v = 1$. Observe first that we know that $q^* \in (0, 1)$ is a stationary value of postnatal discrimination given the values of 0 for prenatal discrimination and 0 for societal son preference, and then, $d_q(0, q^*, 0) = 0$. Therefore, $d_q^c + d_q^q \cdot q^* = 0$. If $q = 1$, we can obtain, since 1 is a self-correcting value of postnatal discrimination in the social state $(p, 1, 1)$, that there is no $\epsilon^* > 0$ such that $d_q(p, 1 - \epsilon, 1) \leq 0$ for all $\epsilon < \epsilon^*$. Consider ϵ^* such that $1 - \epsilon^* > q^*$, which exists because $q^* \in (0, 1)$. Then, we have that $d_q(p, 1 - \epsilon, 1) > 0$ for some $\epsilon < \epsilon^*$, and thus, $d_q^c + d_q^p \cdot p + d_q^q \cdot (1 - \epsilon) + d_q^v > 0$. Then, $d_q^c + d_q^p \cdot p + d_q^q \cdot (1 - \epsilon) + d_q^v > d_q^c + d_q^q \cdot q^*$. Thus, $d_q^p \cdot p + d_q^v > d_q^q \cdot [q^* - (1 - \epsilon)]$. Since $d_q^q < 0$ and $q^* < (1 - \epsilon)$, the RHS of the equation is strictly positive. Then, we need that $d_q^p \cdot p + d_q^v > 0$. Since $d_q^p < 0$, $d_q^v > 0$ and $p \geq 0$, this implies that $d_q^v > |d_q^p| \cdot p$, as desired.

A similar analysis can be done with $q = 0$ to show that, in that case, $d_q^v < |d_q^p| \cdot p$. This concludes the proof of Step 3. Observe also that the union of the three steps prove the proposition.

Proof of Proposition 3

Let a society be such that there exists an institutionalized social state when $A^{t-1} = 0$, $(0, q^*, 0)$, with $q^* \in (0, 1)$, at which the society is at period $t - 1$. Observe that, by Proposition 5, the fact that $q^* \in (0, 1)$ implies that $d_q^q < 0$. Note that, by Proposition 1, the condition $[d_p^c \leq 0$ and, in case of equality, $d_p^p < 0]$ is necessary and sufficient to obtain that $(p, q, v)_{(q^*, 0)} = (0, q^*, 0)$. Then, since condition (i) is the negation of this condition, it is obvious that condition

(i) should belong to the set of necessary and sufficient conditions to obtain that $q > q^*$. Then, we assume from now on that condition (i) is satisfied.

We know by Proposition 2 that one of the set of necessary and sufficient conditions to obtain an additive relationship is that societal son preference should increase from $v^* = 0$ to $v = 1$. We now show that this condition is equivalent to condition (ii). Suppose first by contradiction that condition (ii) is satisfied, but societal son preference does not change. Then, after the shock that leads prenatal discrimination from 0 to ϵ, the dynamics of prenatal and postnatal discrimination will arrive at a social state with stationary values for these variables (given a value of 0 for societal son preference). We denote these values by $\hat{p}$ and $\hat{q}$. Knowing that $\hat{p} > 0$ by condition (i), we proceed to compute these values.

If $\hat{p} \in (0,1)$, it will satisfy that $d_p(\hat{p}, 0) = 0$. Then, $d_p^c + d_p^p \cdot \hat{p} = 0$, and therefore, $\hat{p} = -\frac{d_p^c}{d_p^p}$. Since $d_p^c \geq 0$ by condition (i), having an interior value for $\hat{p}$ requires that $d_p^p < 0$, and then, $\hat{p} = \frac{d_p^c}{|d_p^p|}$. If $d_p^p \geq 0$ or $\frac{d_p^c}{|d_p^p|} \geq 1$, then $d_p(p, 0) > 0$ for all $p \in (0, 1]$, and therefore, the stationary value at which prenatal discrimination will arrive would be $\hat{p} = 1$.

If $\hat{q} \in (0,1)$, it will satisfy that $d_q(\hat{p}, \hat{q}, 0) = 0$. Then, $d_q^c + d_q^p \cdot \hat{p} + d_q^q \cdot \hat{q} = 0$, and therefore, $\hat{q} = -\frac{d_q^c + d_q^p \cdot \hat{p}}{d_q^q}$. Since $d_q^q < 0$ by assumption, having an interior value for $\hat{q}$ requires that $\hat{q} = \frac{d_q^c + d_q^p \cdot \hat{p}}{|d_q^q|}$. If $d_q^c + d_q^p \cdot \hat{p} < 0$, we would have that $d_q(\hat{p}, q, 0) < 0$ for all $q \in [0, 1]$, and therefore, the stationary value at which postnatal discrimination will arrive would be $\hat{q} = 0$.

Then, we have that $\hat{p}$ is a stationary value of prenatal discrimination given a value of 0 for societal son preference, and $\hat{q}$ is a stationary value of postnatal discrimination given the values $\hat{p}$ for prenatal discrimination and 0 for societal son preference. Additionally, since we have assumed that societal son preference remains in 0, we have that 0 is a stationary value of v given $\hat{p}$ and $\hat{q}$. That is, it should be that $d_v(\hat{p}, \hat{q}) \leq 0$ or, what is the same, that $d_v^c + d_v^p \cdot \hat{p} + d_v^q \cdot \hat{q} \leq 0$. This contradicts condition (ii) of the proposition. It can be proved in a similar way that, if condition (ii) is not satisfied ($i.e.$, $d_v^c + d_v^p \cdot \hat{p} + d_v^q \cdot \hat{q} \leq 0$), then societal son preference remains in a moderate value in the institutionalized social state after the shock. That is, condition (ii) belongs to the set of necessary and sufficient conditions to obtain an additive relationship.

We have that conditions (i) and (ii) ensures that we pass from a moderate societal son preference to a high one. Finally, by Proposition 2, the last condition that should belong to the set of necessary and sufficient conditions is that d_q^v should be greater than the product of the absolute value of d_q^p and the level of prenatal discrimination at which we will arrive. Thus, we only need to compute this value, that we denote by $\bar{p}$.

If $\bar{p} \in (0,1)$, it will satisfy that $d_p(\bar{p}, 1) = 0$. Then, $d_p^c + d_p^p \cdot \bar{p} + d_p^v = 0$, and therefore, $\bar{p} = -\frac{d_p^c + d_p^v}{d_p^p}$. Since $d_p^c \geq 0$ by condition (i) and $d_p^v > 0$ by assumption, having an interior value for $\bar{p}$ requires that $d_p^p < 0$, and then, $\bar{p} = \frac{d_p^c + d_p^v}{|d_p^p|}$. If $d_p^p \geq 0$ or $\frac{d_p^c + d_p^v}{|d_p^p|} \geq 1$, we would have that $d_p(p, 1) > 0$ for all $p \in [0,1]$, and therefore, the stationary value at which prenatal discrimination will arrive would be $\bar{p} = 1$. This concludes the proof of the proposition.

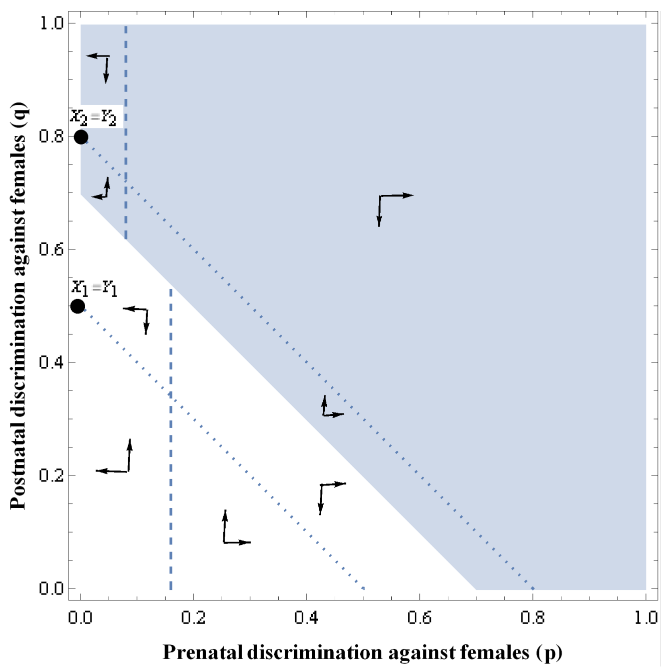
Appendix C: Dynamics of the societies of the examples

This appendix graphically represents the dynamics of the societies of the examples.

Dynamics of Example 1

This society can be represented with Figure 1.

Figure 1:



It is noteworthy to explain here the construction of the five ingredients that comprise this figure. First, we represent a square of side 1, $[0,1] \times [0,1]$, in which each axis measures one type of discrimination: the horizontal axis measures the proportion of couples that practice prenatal discrimination against females (p); and the vertical axis measures the proportion of couples that practice postnatal discrimination against females (q).

Second, we divide the square into two areas: the shadowed area includes the points (p,q) such that $d_v(p,q) > 0$, and the white area includes the points (p,q)

such that $d_v(p,q) \leq 0$. Then, the shadowed (respectively, white) area represents the combinations of prenatal and postnatal discrimination such that, if any of them occur in period t, there would be a high (respectively, moderate) societal son preference in period $t+1$, $v^{t+1}=1$ (respectively, $v^{t+1}=0$). Observe that the frontier between these two areas consists of points (p,q) such that $d_v(p,q)=0$. Then, this division is the line $d_v^p \cdot p + d_v^q \cdot q = -d_v^c$ or, equivalently, $q = -\frac{d_v^c + d_v^p \cdot p}{d_v^q}$. As we have assumed that d_v^p and d_v^q are positive, this line has negative slope and the points situated in the upper-right region belong to the shadowed area, while those in the lower-left region belong to the white area. In the society of this example, the line that separates the two areas is given by $q = 0.7 - p$.

Third, we include a dashed line representing all interior points in which the value of p is stationary given the values of the other variables (the value of q represented by the vertical coordinate of the point, and the value of v represented by the color of the area in which the point is located). Then, the dashed line includes the interior points (p,q) such that $d_p(p,v)=0$. This function changes depending on the value of v, and then, the representation of the dashed line is different in the shadowed and in the white area: this line in the shadowed area is $p = -\frac{d_p^c + d_p^v}{d_p^p}$, while in the white area is $p = -\frac{d_p^c}{d_p^p}$. In other words, the dashed line splits into two sections, one in each area of the square, and both are vertical. In our example, we obtain that the dashed line is $p = 0.08$ in the shadowed area and $p = 0.16$ in the white area. Similarly, we include a dotted line representing all interior points in which the value of q is stationary given the values of the other variables (the value of p represented by the horizontal coordinate of the point, and the value of v represented by the color of the area in which the point is located). Then, the dotted line includes the interior points (p,q) such that $d_q(p,q,v)=0$. As for the dashed line, the representation of the dotted line is different in the shadowed and in the white area: this line in the shadowed area is $q = -\frac{d_q^c + d_q^p \cdot p + d_q^v}{d_q^q}$, while in the white area is $q = -\frac{d_q^c + d_q^p \cdot p}{d_q^q}$. Then, in this example, the dotted line is given by

$q = 0.8 - p$ in the shadowed area, and by $q = 0.5 - p$ in the white area.

Fourth, the dashed and the dotted lines define some points that are relevant for the analysis. On the one hand, the intersections between the dashed and dotted lines represent the interior stationary social states when there is access to sex-selective abortions. To see why, observe that (*i*) since these points belong to both the dashed and dotted lines, they represent combinations of values of p and q that are stationary; and (*ii*) the value of v (represented by the color of the area in which each of these points is located) is stationary given the values of the other variables, since variable v can only change when the values of p and/or q change and we already know that the values of p and q at these points are stationary.[23] On the other hand, the intersections between the dotted line and the vertical axis are the interior stationary social states when there is no access to prenatal discriminatory practices. This is because these points are the social states in which $p = 0$ (observe that this is satisfied in all points of the vertical axis, and that $A^{t-1} = 0$ implies that $p = 0$), q is stationary because these points belong to the dotted line, and the value of v (represented by the color of the area in which each of these points is located) is stationary since p and q at these points are stationary.[24]

Recall that not all stationary social states are necessarily institutionalized social states, as it is necessary to check self-correctness. The last ingredient in the graphical representation helps us to examine self-correctness: we represent arrows that indicate the direction of the dynamics of the system from out-of-stationary

[23] Note that, since 0 and 1 are always stationary values for both p and q, there exist other stationary social states: the four corners of the square, the intersections between the dashed line and the horizontal sides of the square, and the intersections between the dotted line and the vertical sides of the square.

[24] Similar to the previous footnote, since 0 and 1 are always stationary values for q, there exist two other stationary social states when there is no access to sex-selective abortions: the two corners of the square in the vertical axis.

social states. To see how we represent these arrows, observe that each section of the dashed line separates the points in which $d_p(p, v) > 0$ from those in which $d_p(p, v) < 0$. The areas in which $d_p(p, v) > 0$ (respectively, $d_p(p, v) < 0$) represent the social states in which the dynamics of the system will increase (respectively, decrease) the value of p, and subsequently, we represent an arrow pointing to the right (respectively, left) in these areas. Similarly, each section of the dotted line separates the points in which $d_q(p, q, v) > 0$ from those in which $d_q(p, q, v) < 0$. The areas in which $d_q(p, q, v) > 0$ (respectively, $d_q(p, q, v) < 0$) represent the social states in which the dynamics of the system will increase (respectively, decrease) the value of q, and subsequently, we represent an arrow pointing upward (respectively, downward) in these areas. It can be checked that when $d_p^p < 0$, the horizontal arrows point to the dashed line, while when $d_p^p > 0$, they point in the opposite direction. The same can be said about d_q^q and the relationship between vertical arrows and the dotted line: when $d_q^q < 0$, the vertical arrows point to the dotted line, while the opposite occurs when $d_q^q > 0$. Since in our example $d_p^p > 0$ and $d_q^q < 0$, all vertical arrows point to the dotted line, while the horizontal ones point in the opposite direction of the dashed line. So defined, the arrows help to check self-correctness of the stationary social states: roughly speaking, if the arrows in the neighborhood of a point signal the point, then this point is self-correcting.

With the ingredients of the figure at hand, we can now identify the institutionalized social states for the society of this example before the introduction of sex-selective abortion technologies (*i.e.*, $A^{t-1} = 0$). First, observe that with $A^{t-1} = 0$, there exist four stationary social states: two corners, $(0, 0, 0)$ and $(0, 1, 1)$, as the corner values 0 and 1 are always stationary for q; and two interior social states, $(0, 0.5, 0)$ and $(0, 0.8, 1)$, which correspond to the intersections between the dashed line and the vertical axis. To check which ones are also self-correcting, it is necessary to check the vertical arrows in the neighborhood of each of these points.[25] In this case, the vertical arrows in the neighborhood of each of the corner stationary social states, $(0,0,0)$ and $(0,1,1)$, fail to signal the corresponding point, and then, these social states are not institutionalized social states. To see why, consider for example the social state $(0,0,0)$ and assume that a shock causes a few couples to behave differently than what is described by this social state concerning the postnatal discrimination against females; *i.e.*, we move to social state $(0,\epsilon,0)$, with $\epsilon > 0$ arbitrarily small. Then, the dynamics of the system, which are represented by the vertical arrows in the graph, would lead the society to additional increases of variable q in such a way that the society will arrive at a social state that differs from $(0,0,0)$. In contrast, the vertical arrows in the neighborhood of the interior stationary social states in this case, $(0,0.5,0)$ and $(0,0.8,1)$, signal the respective point, and then, these social states satisfy self-correctness and are institutionalized social states. To see why, consider for example $(0,0.5,0)$ and assume a shock by which a few couples begin to behave differently from what is described by this social state regarding postnatal discrimination; *i.e.*, we move to social state $(0,0.5+\epsilon,0)$, with ϵ positive or negative, but with absolute value arbitrarily small. Then, the dynamics of the system, which are represented by the vertical arrows in the graph, would lead society to return to the social state $(0,0.5,0)$. The computation of the exact values of the variables in the institutionalized social states before the access to sex-selective abortions for any society can be done using Propositions 4 to 6 in Appendix A.

Finally, we analyze how the access to sex-selective abortion technologies impacts this particular society. Suppose that when $A^{t-1} = 0$, the society was at the institutionalized social state $(0,0.5,0)$, a point that is denoted by X_1 in Figure 1. Remember that we model the access to sex-selective abortions as a shock by which $A^t = 1$ for the first time and a small number of couples start to practice sex-

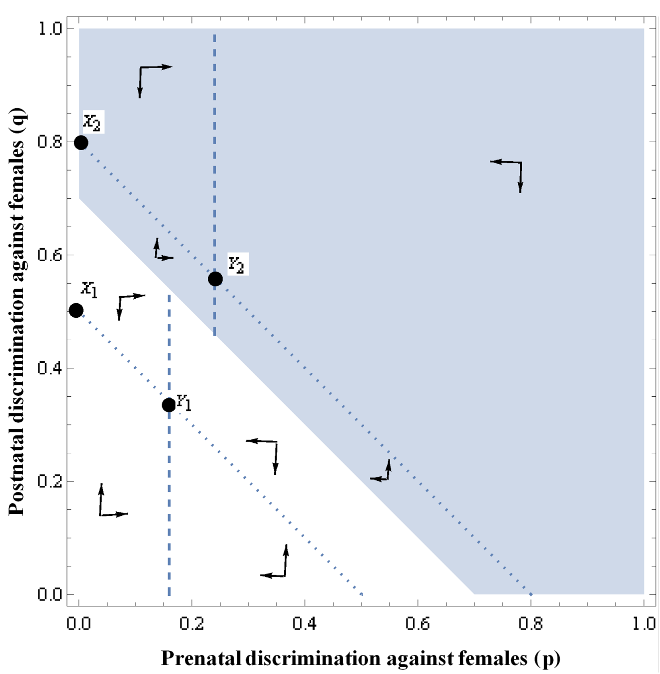
[25]Observe that, when $A^{t-1} = 0$, we have to check only the vertical arrows, since p is invariant in 0 and perturbations can only occur in q.

selective abortions; *i.e.*, we move to social state $(\epsilon, 0.5, 0)$, with $\epsilon > 0$ arbitrarily small. Then, the horizontal arrow in the graph shows that the dynamics of the system will force a return to the original social state $(0, 0.5, 0)$, denoted by Y_1 in the figure. This is exactly what Proposition 1 states, because we are in a society in which $d_p^c = -0.4 < 0$.[26]

Dynamics of Example 2

Figure 2 represents the society of this example in similar terms as Figure 1 does with Example 1.

Figure 2:



[26]A similar analysis can be done if the society was before the access to sex-selective abortions at the institutionalized social state $(0, 0.8, 1)$, denoted by X_2 in the figure. As $d_p^c + d_p^v = -0.2 < 0$, the society would remain in this social state after the access to sex-selective abortions. This can be also observed with the horizontal arrow of the figure that, in the neighborhood of the point representing this social state, signals back to the point, and then, the dynamics force to end in Y_2, which coincides with X_2.

Observe that the equation of the frontier between the shadowed and white areas ($q = -\frac{d_v^c + d_v^p \cdot p}{d_v^q}$) does not depend on any of the parameters that have changed from Example 1. Subsequently, this frontier is given here also by $q = 0.7 - p$. Similarly, as the equations of the dotted line in the shadowed area ($q = -\frac{d_q^c + d_q^p \cdot p + d_q^v}{d_q^q}$) and in the white area ($q = -\frac{d_q^c + d_q^p \cdot p}{d_q^q}$) depend neither on d_p^c nor on d_p^p, the dotted line is the same as in Example 1: $q = 0.8 - p$ in the shadowed area and $q = 0.5 - p$ in the white area. However, the change in parameters d_p^c and d_p^p changes the dashed line relative to Example 1, and it now equals $p = 0.16$ in the white area and $p = 0.24$ in the shadowed area. Similarly, as d_p^p shifts from a positive to a negative value, the horizontal arrows shift relative to Example 1. As we are going to see, this affects the impact of the access to sex-selective abortion technologies.

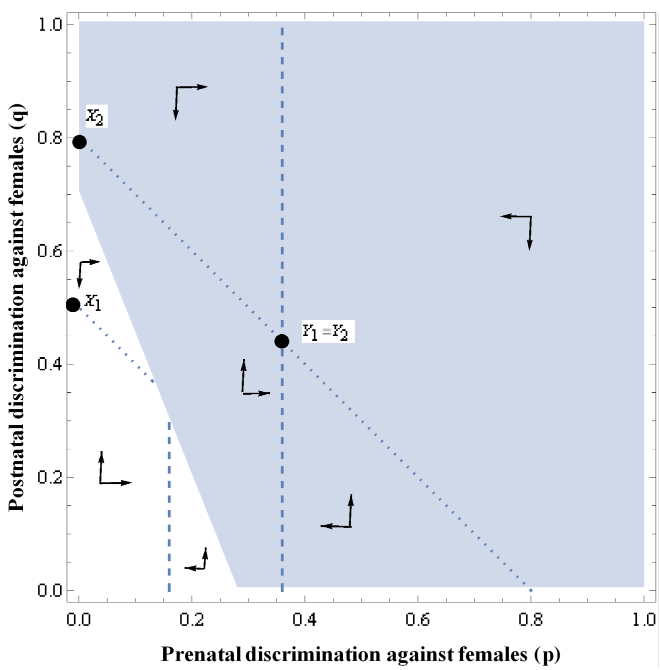
Observe first that, since the stationary social states with $A^{t-1} = 0$ only depend on the dotted line and the partition between the white and the shadowed areas, there exist the same four stationary social states than in Example 1: $(0, 0, 0)$, $(0, 1, 1)$, $(0, 0.5, 0)$ and $(0, 0.8, 1)$. To check self-correctness of these stationary social states, we should proceed analyzing the vertical arrows in the neighborhood of these points. As these remain invariant, we can infer that the unique institutionalized social states are the same as in Example 1: $(0, 0.5, 0)$ and $(0, 0.8, 1)$, denoted by X_1 and X_2 in Figure 2.

Finally, we analyze how the access to sex-selective abortion technologies impacts this particular society. Suppose that when $A^{t-1} = 0$, the society was at the institutionalized social state $(0, 0.5, 0)$. The shock then leads the society to social state $(\epsilon, 0.5, 0)$, with $\epsilon > 0$ arbitrarily small. Then, the arrow points to the right, and thus, the society would not return to the original institutionalized social state. It can be seen following the arrows that the dynamics of the system leads the society to point Y_1 in the figure. Observe that Y_1 is stationary (it is the intersection of the dotted and dashed lines in the white area) and is also self-correcting, since all arrows in its neighborhood point back to Y_1. The access to sex-selective abortion technologies then leads the society from the social state X_1 to the new institutionalized social state Y_1. As Y_1 corresponds to the social state $(0.16, 0.34, 0)$, the society in the example experiences a decrease in postnatal discrimination following an increase in prenatal discrimination. This is a clear example of a substitutive relationship between the two types of discrimination, and is exactly what is stated by Proposition 2, since the value of societal son preference has not changed from X_1 to Y_1.[27]

Dynamics of Example 3

Figure 3 represents the society of this example.

Figure 3:



[27] A similar analysis can be done if the society coordinated at the institutionalized social state $(0, 0.8, 1)$ before the access to sex-selective abortions. The society then would move, with the access to sex-selective abortions, to the social state $(0.24, 0.56, 1)$, denoted by Y_2 in the figure. Again, it appears a substitutive relationship, as Proposition 2 predicts for any situation in which the society already had a high societal son preference before sex-selective abortions emerged.

Observe first that, with the change in parameters, the dotted line is the same as in the previous examples. However, the frontier between the shadowed and white areas changes: since it corresponds to $q = -\frac{d_v^c + d_v^p \cdot p}{d_v^q}$, the increase of d_v^p from 0.2 to 0.5 has increased the absolute value of the slope of this frontier, maintaining invariant the vertical intercept. Then, the equation of this frontier between the two areas is now $q = 0.7 - 2.5p$. The change of d_p^v modifies the dashed line in the shadowed area, such that it is now $p = 0.36$, while the dashed line in the white area is not affected and remains $p = 0.16$. Given that d_p^p and d_q^q retain the same values as in Example 2, the arrows maintain the same direction.

Although the partition between the shadowed and white areas of the square has changed, the vertical intercept of the frontier between these areas has not changed, and thus, all points of the vertical axis belong to the same area as in the previous examples. Since the stationary social states with $A^{t-1} = 0$ only depend on the dotted line and the partition between the shadowed and white areas in the vertical axis, we have the same four stationary social states as in the previous examples: $(0, 0, 0)$, $(0, 1, 1)$, $(0, 0.5, 0)$ and $(0, 0.8, 1)$. Similarly, the vertical arrows are the same as in Example 2, and thus, we again have that the institutionalized social states before the access to sex-selective abortions are $(0, 0.5, 0)$ and $(0, 0.8, 1)$, denoted by X_1 and X_2 in Figure 3.

We now analyze how the access to sex-selective abortion technologies impacts this particular society. Suppose that the society was at the institutionalized social state $(0, 0.5, 0)$ when $A^{t-1} = 0$. Then, as in the previous examples, the shock causes the society to move to social state $(\epsilon, 0.5, 0)$, with $\epsilon > 0$ arbitrarily small. Then, the horizontal arrow points to the right, and consequently, the society does not return to the original institutionalized social state. This movement following the arrows causes that the society arrives to the shadowed area, in which the direction of the vertical arrow changes to point upward. The arrows in the figure reveal that this movement will continue until the society arrives to the institutionalized social state denoted by Y_1, which corresponds to the social state $(0.4, 0.4, 1)$. Then, this society has experienced a decrease in postnatal discrimination following an increase in prenatal discrimination, constituting another case of a substitutive relationship between the two types of discrimination.[28] Observe that this is the result predicted by Proposition 2 since, although the societal son preference has increased with the shock from a moderate value to a high one, it is not satisfied that $d_q^v = 0.3$ is greater than $|d_q^p| \cdot p = |-1| \cdot 0.4$. We can obtain the same conclusion with Proposition 3: since condition (*iii*) is not satisfied, we do not find an additive relationship between prenatal and postnatal discrimination.

Finally, in comparing this society with that of Example 2, it is noteworthy that, although they both began in the same institutionalized social state $(0, 0.5, 0)$, the substitutive relationship has been of lower intensity in this case than in Example 2. This is because we have here two effects on q with the adoption of sex-selective abortions. First, the increase in p derived from the shock leads to a decrease in q. Second, the increase in p has also increased v in this society, leading to an increase in q. In the society of Example 2, the first effect was already present, but not the second, as societal son preference remained invariant with the shock.

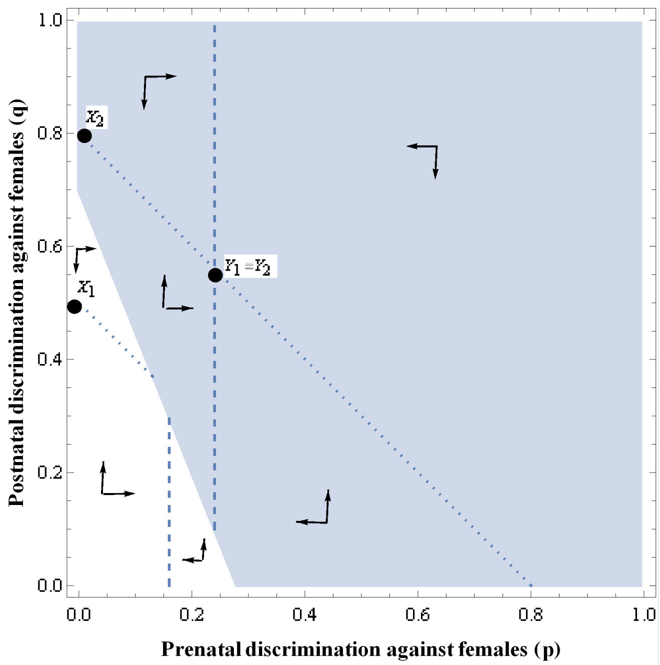
Dynamics of Example 4

As in the previous examples, Figure 4 represents this society.

Note that the unique change in the representation of the society relative to Example 3 is the dashed line in the shadowed area that is now $p = 0.24$. It is then easy to see that the institutionalized social states before the access to sex-selective

[28]A similar analysis can be done if the society coordinated at the institutionalized social state $(0, 0.8, 1)$ before the access to sex-selective abortions. The society then would move, with the access to sex-selective abortions, to the same social state than if it had started in X_1, $(0.4, 0.4, 1)$, denoted by Y_2 in the figure. Again, it appears a substitutive relationship, as Proposition 2 predicts for any situation in which the society already had a high societal son preference before the emergence of sex-selective abortions.

Figure 4:



abortions are the same as in previous examples: $(0, 0.5, 0)$ and $(0, 0.8, 1)$, denoted again by X_1 and X_2 in Figure 4.

We now analyze how the access to sex-selective abortion technologies impacts this particular society, assuming that the society when $A^{t-1} = 0$ was at the institutionalized social state $(0, 0.5, 0)$. As in the previous examples, the shock provokes that the society moves to social state $(\epsilon, 0.5, 0)$, with $\epsilon > 0$ arbitrarily small. Since the horizontal arrow points to the right, the dynamics do not return society to the original institutionalized social state. In contrast, following the arrows we obtain that the dynamics of the society lead to the new institutionalized social state Y_1, which represents the social state $(0.24, 0.56, 1)$. In this case, we have an additive relationship between prenatal and postnatal discrimination. Note that this is the result that Proposition 2 predicts, since we have a society that has increased societal son preference from $v = 0$ to $v = 1$ with the access to sex-selective abortions and we also have that $d_q^v = 0.3$ is greater than $|d_q^p| \cdot p = |-1| \cdot 0.24$. We can obtain the same conclusion with Proposition 3: since all conditions of the proposition are satisfied, we have an additive relationship.[29]

In comparison with Example 3, we also have here the same two effects on q with the adoption of sex-selective abortions, but the decrease of d_p^v has increased the size of the effect that tends to increase q in such a way that it now dominates the other effect.

[29] If the society coordinated before the access to sex-selective abortions at the institutionalized social state $(0, 0.8, 1)$, denoted by X_2 in the figure, we would have that the society would move, with the access to sex-selective abortions, to the same social state than if it had started in X_1, $(0.24, 0.56, 1)$. So, in this case we have a substitutive relationship, as Proposition 2 predicts for any society that already had a high societal son preference before the emergence of sex-selective abortions.