FIRM ENTRY UNDER FINANCIAL FRICTIONS

Miguel Casares
Jean-Christophe Poutineau
D.T. 1102

Departamento de Economía
Ekonomia Saila
Firm entry under financial frictions*  

Miguel Casares†  
Universidad Pública de Navarra  

Jean-Christophe Poutineau‡  
Université de Rennes I  

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Abstract  

Introducing both endogenous firm entry and a requirement for external finance in a general-equilibrium model leads to three main results. First, the financial constraint has contractionary effects on both equity investment and the labor supply as they are inversely related to the marginal finance cost. Second, net firm creation amplifies the steady-state impact of changes in either productivity or banking efficiency due to procyclical firm entry. Third, a higher elasticity of substitution (that implies a lower mark-up) cuts the number of firms and makes aggregate output fall in steady state, opposite to standard models with constant number of firms.  

Keywords: firm creation, financial frictions, steady-state analysis.  

JEL codes: E13, E44.  

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†Departamento de Economía, Universidad Pública de Navarra, 31006, Pamplona, Spain. E-mail: mcasares@unavarra.es (Miguel Casares). Http://www.econ.unavarra.es/~mcasares/. I would like to thank Ministerio de Educación of Spain (research project ECO2008-02641) for their financial support.  
‡CREM, UMR CNRS 6211, Université de Rennes I, Rennes, France. E-mail: jean-christophe.poutineau@univ-rennes1.fr (Jean-Christophe Poutineau).
1 Introduction

Recent papers underline the key role of the creation and destruction of goods (extensive margin of activity) on output fluctuations. Empirically, Bernard, Redding, and Schott (2010) find that over a five-year period, new products (developed either by existing or new firms) represent 46.6% of GDP in the US, while the value of goods destruction represents 44% of that GDP. In contrast, the intensive margin of activity (i.e., the fluctuation in the production of existing goods) accounts for only half of these figures. The quantitative role of the creation of new goods is also highlighted by Broda and Weinstein (2010) who report that, on average, every year around 9% of US consumer spending is on purchases of new goods.

In the theoretical front, Bilbiie, Ghironi, and Melitz (2010) build a framework to study the implications of firm creation on business cycle analysis. They consider competitive firm entry when the prospective value of new firms is higher than a sunk cost representing the initial investment in new production lines. By doing so, they are able to replicate stylized facts such as the procyclicality of firm entry and profits and the countercyclicality of mark-ups. The sticky-price version of this model has been developed by Bergin and Corsetti (2008), and Lewis (2009) to conduct business cycle and monetary policy analysis.

Regarding the financial aspects of the creation of new firms, the models currently available in the literature assume that investment in new productive units is carried out by households through the accumulation of shares in their portfolio allocation. As a consequence, new firms are created as soon as the current value of future dividends is greater than the building cost of the firm. Even though such a solution is a convenient way of thinking about the creation of firms on a profitable segment of the goods market, it ignores the key role of the banking system in providing funds for firm entry. This is one of the objectives of this paper: to study how both credit availability and a variable number of firms might affect economic activity.

In our model, financial frictions are modelled through a liquidity constraint that is binding on expenditures for both consumption purchases and investment on firm creation. As a consequence, firm entry does not only depend on the sole profitability of investment but also on the
availability of loans needed to finance the new production lines. The amount of loans depends on both banking labor and the stock of collateral. Thus, financial frictions are introduced through two elements of distortion: the (contractionary) financial requirement for investment in new firms/goods and the (expansionary) collateral service of firm value. In addition, the introduction of a loan production technology will be used to examine how changes in banking efficiency can be transmitted to aggregate variables that belong to the real sector of the economy.

After describing the competitive equilibrium model in steady state and calibrating its structural parameters, the role of financial factors and firm creation is examined by comparing the baseline model to variants in which either firm entry/exit or financial frictions are dropped. The results show that the financial constraint has contractionary effects on the number of firms, output, labor, and consumption; with permanent declines that range between 1.5% and 5%. As for endogenous firm creation, we carry out simulation exercises that show how the quantitative effects of changes in productivity, banking efficiency and market power are heavily influenced by procyclical firm entry. The contribution of the dynamics of firm creation/destruction plays a very important role to explain the reaction of output to changes in productivity, banking efficiency and the elasticity of substitution (market power). By contrast, the intensive margin of output (at firm level) shows little reaction after a technology shock and opposing (counter-cyclical) reactions after changes in either banking efficiency or market power. In the model without firm entry/exit, the responses of aggregate output are completely determined by the reactions of firm-level output.

The rest of the paper is organized as follows. Section 2 introduces the model and defines its general equilibrium in steady state. Section 3 provides a numerical calibration of parameters and compares the polar cases regarding the role of firm entry and financial requirements. Section 4 carries out the quantitative analysis of the impact of changes in productivity, banking efficiency, and market profitability. Section 5 concludes.
2 A model with firm entry and financial frictions

The model describes a closed economy where production, consumption and banking activities take place. Each firm is specialized in the production of one good, and the number of firms is endogenously determined. Financial frictions are introduced through a liquidity constraint on consumption and firm creation.

2.1 Households

For any given period \( t \), the representative household allocates consumption among \( n_t \) varieties of final goods (indexed by \( \omega \)). Consumption goods are imperfectly substitutable in the household’s consumption basket. The aggregation in the basket of goods, \( c_t \), and also for the cost of living index, \( P_t^c \), are constant-elasticity combinations of \( n_t \) varieties,

\[
c_t = \left[ \int_{0}^{n_t} c_t(\omega)^{-\frac{1}{\sigma}} \, d\omega \right]^\frac{1}{1-\sigma} \quad \text{and} \quad P_t^c = \left[ \int_{0}^{n_t} P_t(\omega)^{1-\sigma} \, d\omega \right]^\frac{1}{1-\sigma},
\]

where \( \sigma > 1 \) is a constant elasticity of substitution as in Dixit and Stiglitz (1977). Defining the relative price of good \( \omega \) as \( \rho_t(\omega) = \frac{P_t(\omega)}{P_t^c} \), the optimal consumption of good \( \omega \) is

\[
c_t(\omega) = (\rho_t(\omega))^{-\sigma} c_t.
\]

Both purchases of consumption goods and investment on the creation of new firms are financially constrained as in Goodfriend and McCallum (2007), and Casares and Poutineau (2011). Thus, households demand liquidity \( tp \) cover desired spendings on current consumption and firm creation, which define the liquidity constraint as follows

\[
c_t + n_t^e v_t x_{t+1} = V \frac{L_t}{P_t^c}.
\]

On the left of (2), \( n_t^e v_t x_{t+1} \) represents the total market value of new firms because \( n_t^e \) is the number of firms created in period \( t \), while \( v_t \) is the value of firm equity and \( x_{t+1} \) is the fraction of the total capital of new firms acquired by the household. On the right of (2), \( L_t \) is the amount of nominal loans, and \( V \) is a velocity parameter that is introduced to yield a realistic ratio of
provided liquidity over loans. A Cobb-Douglas banking technology determines the amount of loan production

$$\frac{L_t}{P_t} = B(b_{t+1} + \gamma n_t v_t)\alpha (m^d_t)^{1-\alpha}, \quad (3)$$

where $B > 0$, and $0 < \alpha < 1$ are constant parameters, $b_{t+1}$ is the real value of bonds in period $t$ that serves as a collateral for loan creation, $n_t$ is the number of total existing firms, and $m^d_t$ denotes the household demand for labor at the bank. Therefore, it is assumed that the equity value of existing firms, $n_t v_t$, is accepted as a collateral to back the distribution of loans, once corrected by a penalizing parameter, $0 < \gamma < 1$, in a way that recognizes the difficulty of monitoring market value of firms relative to bonds.

Following Bilbiie et al. (2010), households hold two types of assets: shares in a mutual fund of firms and a one-period composite bond. Hence, the representative household can choose what fraction $x_{t+1}$ of the economy-wide mutual fund to own, which is currently delivering a real dividend equal to $d_t$. Alternatively, households can also buy government bonds: the amount of real bonds, $b_t$, that were subscribed in the previous period earn a real interest rate, $r^{b_t}$, and the household must decide the amount of bonds for the next period, $b_{t+1}$. Hence, the budget constraint faced by the representative household in period $t$ is

$$w_t(l^s_t + m^s_t - m^d_t) + d_t n_t x_t + g_t = c_t + v_t n_t (x_{t+1} - x_t) + v_t n^e_t x_{t+1} + (1 + r^{b_t})^{-1}b_{t+1} - b_t,$$

where there are two types of labor income: supplying $l^s_t$ hours of work to firms and working $m^s_t - m^d_t$ net hours in the bank of other households. In both cases, the household earns the hourly market-clearing real wage $w_t$. Firms pay as dividends $d_t n_t x_t$ for the equity share $n_t x_t$ owned by the household. As another source of income, the government gives households a net transfer payment $g_t$ also expressed in units of the consumption basket.

Income is spent on purchases of consumption goods, $c_t$, on a net increase of portfolio investment, $v_t n_t (x_{t+1} - x_t)$, on purchases of new firms for a total value of $v_t n^e_t x_{t+1}$, and on net purchases of government bonds, $(1 + r^{b_t})^{-1}b_{t+1} - b_t$. Putting together terms on $n_t x_t$ simplifies the household budget constraint to

$$w_t(l^s_t + m^s_t - m^d_t) + b_t + (d_t + v_t) n_t x_t + g_t = c_t + v_t (n_t + n^e_t) x_{t+1} + (1 + r^{b_t})^{-1}b_{t+1}. \quad (4)$$
Also as in Goodfriend and McCallum (2007), household preferences are defined through a log utility function, separable between consumption and leisure. Future utility is brought to the current time by applying a constant discount factor per period, $\beta$. Total time available is normalized at 1.0. Thus, the representative household maximizes

$$\sum_{j=0}^{\infty} \beta^j \left[ \phi \log c_{t+j} + (1 - \phi) \log (1 - l_{t+j}^s - m_{t+j}^s) \right] ,$$

subject to constraints (3) and (4) for the current period $t$ and all future periods. The set of first order conditions of the household is

$$\phi/c_t - \lambda_t + \xi_t = 0, \quad (c_t)$$

$$-(1 - \phi) \left( 1 - l_t^s - m_t^s \right) + \lambda_t w_t = 0, \quad (l_t^s, m_t^s)$$

$$-\lambda_t w_t - \xi_t (1-\alpha)(c_t + n_t^s v_t x_{t+1}) = 0, \quad (m_t^d)$$

$$-\lambda_t (1 + r_{t+1}^b)^{-1} + \beta \lambda_{t+1} - \xi_t \frac{\alpha(c_t + n_t^s v_t x_{t+1})}{b_{t+1} + \gamma n_t v_t} = 0, \quad (b_{t+1})$$

$$-\lambda_t v_t (n_t + n_t^s) + \beta \lambda_{t+1} (d_{t+1} + v_{t+1}) n_{t+1} + \xi_t n_t^s v_t = 0, \quad (x_{t+1})$$

where $\xi_t$ and $\lambda_t$ are Lagrange multipliers respectively associated to the financial constraint (2) and the budget constraint (4). In this model, financial frictions affect consumption, equity investment, and labor supply decisions through two main variables. First, we define, $\mu_t$, the marginal finance cost as\(^1\)

$$\mu_t = w_t \frac{\partial m_t^d}{\partial l_t^s} \frac{\partial l_t^s}{\partial c_t} = \frac{w_t m_t^d}{(1-\alpha)(c_t + n_t^s v_t x_{t+1})}, \quad (5)$$

which represents the real marginal cost of producing liquidity. Second, the marginal financial services of bonds can be measured by the increase in real income equivalent to the collateral value of bonds. Goodfriend and McCallum (2007) refer to this as the "liquidity service yield" of the bond and denote it as $LSY_t$

$$LSY_t = w_t \frac{\partial m_t^d}{\partial b_{t+1}} \frac{\partial b_{t+1}}{\partial b_{t+1}} = \frac{\alpha w_t m_t^d}{(1-\alpha)(b_{t+1} + \gamma n_t v_t)}. \quad (6)$$

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\(^1\)The technical appendix provides the partial derivatives of the loan production function used for the calculation of $\mu_t$ and $LSY_t$. 
Using the first order condition of the demand for banking labor \((m^d_t)\) and equation (5) results in \(\xi_t = -\lambda_t \mu_t\), which can be inserted in the first order condition of consumption to find how the shadow value of consumption, \(\lambda_t = \frac{\phi/c_t}{1+\mu_t}\), is determined by the marginal utility of consumption divided by one plus the marginal finance cost. This expression for \(\lambda_t\), the corresponding expression for \(\lambda_{t+1}\), the previous result \(\xi_t = -\lambda_t \mu_t\), and equation (6) of the definition of \(LSY\) are substituted in the first order condition of bonds \((b^t_{t+1})\) to reach the following equation for intertemporal allocation of consumption

\[
\frac{\phi}{c_t (1+\mu_t)} \left[ \frac{1}{1+r^b_t} - LSY_t \right] = \beta \frac{\phi}{c_{t+1} (1+\mu_{t+1})}.
\] (7)

As implied by (7), the consumption-saving decision is affected by financial factors according to two channels. First, the shadow value of consumption is corrected by the external finance cost in either the current or future periods because consumption requires additional loan production according to the financial constraint (2). Second, the liquidity service yield of bonds adds up for the total return on saving. Subsequently, households tend to hold more bonds to take advantage of both the market return and their collateral services.\(^2\)

Turning to equity investment, first order conditions \((m^d_t)\), and \((x_{t+1})\) can be combined with the definition of \(\mu_t\) given in expression (4) to obtain

\[
\lambda_t v_t (n_t + n^e_t) = \beta \lambda_{t+1} (d_{t+1} + v_{t+1}) n_{t+1} - \lambda_t \mu_t n^e_t v_t,
\]

that can be rearranged to find the intertemporal optimal portfolio condition

\[
\lambda_t v_t (n_t + n^e_t (1+\mu_t)) = \beta \lambda_{t+1} (d_{t+1} + v_{t+1}) n_{t+1}.
\] (8)

In equilibrium, the value of current portfolio investment (left-hand side of 8) is determined by the financial conditions through the \((1+\mu_t)\) increasing factor. Thus, if the economy suffers from a more severe financial constraint (higher \(\mu_t\)), the left-hand side of (8) pushes up and the household would restore equilibrium by increasing current consumption and reducing the

\(^2\)The semi-loglinear version of (7) is \(\log c_t = \log c_{t+1}/(\mu_t - \mu_{t+1}) - (r^b_t + LSY_t)\), where current consumption is inversely related to the liquidity service yield of bonds.
investment in equity shares. The financial cost of firm creation has a negative impact on equity investment.

Finally, labor effort is split up between working either in firms or at the banks. Financial frictions affect the trade-offs between consumption and leisure through the shadow value of consumption that depends on the marginal finance cost $\mu_t$. Hence, the labor supply equation can be obtained by combining the first order conditions ($l^s$, $m^s$) and ($c_t$) with the definition (5) to reach

$$\frac{1 - \phi}{1 - l^s - m^s} = w_t \frac{\phi}{c_t (1 + \mu_t)}.$$  \hspace{1cm} (9)

The marginal finance cost $\mu_t$ reduces the shadow value of one unit of consumption on the right-hand side of (9). Therefore, consumption is less desirable and leisure rises on the left-hand side of (9) with the result of a decrease in the optimal labor supply.$^3$

2.2 Firms

Firms specialize in the production of one type of good, which makes the number of goods identical to the number of firms.$^4$ In period $t$, operating firms provide final goods for consumption using the linear production function,

$$y^f_t (\omega) = Al^d_t (\omega),$$

where $l^d_t (\omega)$ is the demand of labor by firm $\omega$ and $A > 0$ is a constant productivity parameter.

Recalling the constant elasticity of substitution across goods, $\sigma$, the amount of firm-specific output, $y^f_t (\omega)$, is demand-determined in response to the relative price $\frac{P_t(\omega)}{P_t}$ and to the aggregate

$^3$Using $\log(1 + \mu_t) \simeq \mu_t$, a semi-loglinear aproximation to (9) reads $\frac{l^s}{1 - l^s - m^s} \log l^s + \frac{m^s}{1 - l^s - m^s} \log m^s = \log w_t - \log c_t - \mu_t$, where $l$ and $m$ are steady-state levels of the supply of labor to the firms and to the banking activities, respectively. Such expression identifies the marginal finance cost $\mu_t$ as a determinant of labor supply with a negative semielasticity $\frac{1 - \mu_t - m}{l}$.  

$^4$Taking a broader view, it could be said that the creation of one new good corresponds to either one additional production line in an existing firm or the creation of a single new firm.
demand for output, $y_t$, as follows

$$y^f_t (\omega) = \left( \frac{P_t (\omega)}{P^c_t} \right)^{-\sigma} y_t.$$  

Firms seek to maximize profit. In turn, the representative firm $\omega$ will choose $P_t (\omega)$ to maximize the real dividend $d_t (\omega)$,

$$d_t (\omega) = \frac{P_t (\omega)}{P^c_t} y^f_t (\omega) - w_t l^d_t (\omega),$$

that using the production technology for $l^d_t (\omega)$ and the demand curve for $y^f_t (\omega)$ becomes

$$d_t (\omega) = \left( \frac{P_t (\omega)}{P^c_t} \right)^{1-\sigma} y_t - \frac{w_t}{A} \left( \frac{P_t (\omega)}{P^c_t} \right)^{-\sigma} y_t.$$  

The optimality condition on $P_t (\omega)$ required to maximize $d_t (\omega)$ is

$$(1 - \sigma) \left( \frac{P_t (\omega)}{P^c_t} \right)^{-\sigma} y_t \frac{w_t}{A} \left( \frac{P_t (\omega)}{P^c_t} \right)^{-\sigma - 1} y_t = 0,$$

which can be simplified and expressed in terms of the relative price, $\rho_t (\omega) = \frac{P_t (\omega)}{P^c_t}$, to give

$$\rho_t (\omega) = \frac{\sigma}{\sigma - 1} \frac{w_t}{A}.$$  

(10)

Firm creation and destruction determines how the number of goods available for consumption varies from period to period. Following Bilbiie et al. (2010), it is assumed that it takes one period to build the product line (firm) that is specialized in the production of a new good. We also borrow from that paper the assumption that firm destruction is given by a constant proportion $\delta$ of all existing firms. Thus, the number of firms in period $t$, $n_t$, depends on both the number of firms in the previous period, $n_{t-1}$, and also on the number of firms that were created during the previous period, $n^c_{t-1}$, according to the dynamic equation

$$n_t = (1 - \delta) \left( n_{t-1} + n^c_{t-1} \right).$$  

(11)

The decision of investing in starting new firms is determined when the household compares the prospective value of the new firm, $v_t (\omega)$, with the marginal cost of entry in the goods market. Bilbiie et al. (2010) also assume that firms face a sunk cost of entry (as in Judd, 1985, and Romer, 1990, among others), measured as $f_e$ effective labor units. Therefore, the
entry cost is equal to $f e^w A$ in terms of baskets of consumption goods. This specification ensures that exogenous productivity shocks affect symmetrically both production of existing goods and creation of new products.\footnote{Another approach introduced by Corsetti and Begin (2008) assumes a fixed entry cost that is directly paid in terms of the consumption goods basket. Finally, Lewis (2009) assumes a congestion cost that increases with the number of competitors.} New firms enter the economy as long as the expected total profit coming from producing final goods in the future is greater than this cost.\footnote{In symmetric equilibrium, firm value is the discounted sum of all future dividends, $v_t = \sum_{j=1}^{\infty} \beta_{t+1,t+j} d_{t+j}$ where the stochastic discount factor is $\beta_{t+1,t+j} = \Pi_{k=1}^{\infty} \left( \frac{1}{1 + r_{t+k-1}} - LSY_{t+k-1} \right) \left( \frac{n_{t+k-1} n_{t+k-1} (1 + p_{t+k-1})}{n_{t+k}} \right)$.} Thus, in equilibrium the number of new firms that enter the market is determined by the no-arbitrage condition,

$$v_t(\omega) = f_e^w A$$  \hspace{1cm} (12)

### 2.3 Aggregation and general equilibrium

Since all firms share the same technology, entry costs and demand conditions, there is a complete symmetric equilibrium in which $P_t(\omega) = P_t$, $\rho_t(\omega) = \rho_t$, $y_t^f(\omega) = y_t^f$, $l_t^d(\omega) = l_t^d$, $d_t(\omega) = d$, and $v_t(\omega) = v_t$. In turn, the consumption price index is,

$$P_t^c = \left[ \int_0^{n_t} P_t(\omega)^{1-\sigma} d\omega \right]^{1/(1-\sigma)} = n_t^{1-\sigma} P_t,$$

which implies that the relative price, $\rho_t = \frac{P_t}{P_t^c}$, is tied up to the total number of firms by the "variety effect",

$$\rho_t = n_t^{\frac{1}{1-\sigma}}.$$  \hspace{1cm} (13)

Meanwhile, real aggregate output is,

$$y_t = \int_0^{n_t} \rho_t(\omega) y_t^f(\omega) d\omega = n_t \rho_t y_t^f,$$  \hspace{1cm} (14)

which indicates that aggregate output is jointly determined by the extensive margin of the number of firms, $n_t$, and the intensive margin of firm-level output, $\rho_t y_t^f$, expressed in terms of baskets of consumption goods.
The symmetric equilibrium requires that the representative household owns all the equity share, \( x_t = x_{t-1} = 1 \). In addition, the labor market-clearing condition is \( l^s_t = \int_0^{n_t} l^d_t(\omega) d\omega \), equivalent to
\[
l^s_t = n_t l^d_t,
\]
and the goods market-clearing condition is,\(^7\)
\[
y_t = c_t + v_t n^e_t,
\]
where aggregate output, \( y_t \), is spent either on purchases of baskets of consumption goods, \( c_t \), or on investment spending to acquire new firms, \( v_t n^e_t \). In summary, a competitive equilibrium is defined as a sequence of quantities
\[
\{Q_t\}_{t=0}^\infty = \left\{ y_t, y^f_t, c_t, l^s_t, l^d_t, m^s_t, m^d_t, n_t, n^e_t, x_{t+1}, b_{t+1} \right\}_{t=0}^\infty,
\]
and a sequence of real prices and returns,
\[
\{P_t\}_{t=0}^\infty = \left\{ \rho_t, w_t, r^b_t, v_t, d_t \right\}_{t=0}^\infty,
\]
that satisfy the first order conditions of the households, maximize firm dividend and keep the goods market, the labor market and the asset market in equilibrium.

### 2.4 The steady state

The general equilibrium just derived abstracts from long-run economic growth because productivity, labor and the number of firms are constant in steady state. Therefore, time subscripts might be dropped to directly indicate steady-state (constant) levels.

The dynamic equation for the evolution of the number of firms (11) brings a proportional relationship between new firms and total firms in steady state,
\[
n^e = \frac{\delta}{1-\delta} n.
\]
\(^7\)Proof available in technical appendix.
The steady-state value of the firm is obtained by rewriting (8) in steady state to cancel out the Lagrange multipliers,

$$v (n + n^e (1 + \mu)) = \beta (d + v) n,$$

and then using (17) to drop the variable that determines the number of firms $n$,

$$v \left( 1 + \frac{\delta}{1 - \delta} (1 + \mu) \right) = \beta (d + v),$$

to finally solve the expression for $v$ as follows,

$$v = \left( \frac{(1-\delta) \beta}{1-(1-\delta) \beta + \delta \mu} \right) d.$$  \hspace{1cm} (18)

Remarkably, the marginal finance cost $\mu$ erodes the value of firm equity in steady state. The need for loans to finance the creation of goods-firms explains why a higher marginal finance cost reduces the equity value.

The free entry condition (12) in steady state with symmetric equilibrium is,

$$v = f_e \frac{w}{A}.$$  \hspace{1cm} (19)

Recalling (13), the real price of individual goods in steady state is,

$$\rho = n^{\frac{1}{\alpha - 1}}.$$  \hspace{1cm} (20)

Optimal pricing implies applying a constant mark-up between the relative price, $\rho_t(\omega) = P_t(\omega) / P_t^r$, and the real marginal cost, $\frac{w}{A}$, as indicated in (10). Such expression in steady state yields,

$$\rho = \frac{\alpha}{\sigma - 1} \frac{w}{A}.$$  \hspace{1cm} (21)

The goods-market equilibrium condition (16) in steady state is,

$$y = c + n^e v.$$  \hspace{1cm} (22)

Recalling (14), the relationship between economy-wide output and firm-level output with product variety in steady state is,

$$y = n \rho y^f.$$  \hspace{1cm} (23)
The short-run equilibrium condition for asset holdings symmetry in steady state is,

\[ x = 1. \]  

(24)

The steady-state marginal finance cost \( \mu \) can be obtained by rewriting equation (5) in steady state,

\[ \mu = \frac{w_m}{(1-\alpha)(c+n^e vx)}. \]  

(25)

The firm-level linear production function in the steady-state solution of the symmetric equilibrium is

\[ y^f = Al. \]  

(26)

Using the market-clearing condition for labor, \( l^s = nl \), the labor supply equation (9) in steady state becomes,

\[ \frac{1 - \phi}{1 - nl - m} = \frac{\mu}{c(1 + \mu)}. \]  

(27)

Under complete equilibrium symmetry, the amount of firm profit is \( d_t = \rho_t y^f_t - \frac{w_t}{A} y^f_t \), which implies the following steady-state expression,

\[ d = (\rho - \frac{w}{A}) y^f. \]  

(28)

The loan production technology (3) in steady state reads,

\[ \frac{L}{P_c} = B (b + \gamma n v)^\alpha \ m^{1-\alpha}, \]  

(29)

while the financial constraint (2) determines the demand for liquidity in steady state as follows,

\[ c + n^e v x = \frac{V L}{P_c}. \]  

(30)

Finally, it is assumed, as in Goodfriend and McCallum (2007), that the stock of government bonds is at some constant proportion of output in steady state,

\[ b = \Psi y. \]  

(31)

The steady state solution of the model provides numerical values to fifteen variables: \( n, \rho, w, c, n^e, x, \mu, v, l, y, y^f, d, \frac{L}{P_c}, b, \) and \( m \), obtained by solving the above non-linear system of fifteen equations, (17)-(31), using the calibration of model parameters to be introduced next.
3 Calibration and steady-state solution

Table 1 provides the numbers chosen in the calibration of the model meant for quarterly observations.

Table 1. Baseline calibration of parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption weight</td>
<td>$\phi = 0.35$</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta = 0.995$</td>
</tr>
<tr>
<td>Productivity</td>
<td>$A = 1.0$</td>
</tr>
<tr>
<td>Death shock</td>
<td>$\delta = 0.025$</td>
</tr>
<tr>
<td>Scale parameter in entry cost</td>
<td>$f_e = 3.78$</td>
</tr>
<tr>
<td>Demand elasticity</td>
<td>$\sigma = 3.8$</td>
</tr>
<tr>
<td>Steady-state debt-output ratio</td>
<td>$\Psi = 0.51$</td>
</tr>
<tr>
<td>Labor share in loan production</td>
<td>$\alpha = 0.65$</td>
</tr>
<tr>
<td>Scale parameter in loan production</td>
<td>$B = 4.90$</td>
</tr>
<tr>
<td>Loan velocity</td>
<td>$V = 0.40$</td>
</tr>
<tr>
<td>Portfolio investment monitoring</td>
<td>$\gamma = 0.49$</td>
</tr>
</tbody>
</table>

The constant discount factor is set at $\beta = 0.995$, which leaves the detrended steady-state real interest rate at $r = 0.005$ per quarter, 2% in annualized terms. The value assigned to the consumption weight in the utility function, $\phi = 0.35$, implies that households spend one third of their time on working activities, following the standard assumption used in the real-business-cycle literature.\footnote{Since total time is normalized at 1.0, we have $l^* + m = 1/3$ in steady state.} Similarly, the scale parameter of loan production technology is set at the value $B = 4.90$ that matches the steady-state share of banking labor with the corresponding number found in recent data.\footnote{The Bureau of Economic Analysis reports that the number of people employed in banking activities is 0.84% of total private employment in May 2010. Accordingly, we have $\frac{m}{l^*+m} = 0.0084$ in the steady-state solution when $B = 4.90$.} Moreover, we chose the scale parameter of the entry cost function that results in a number of existing firms in steady-state at $n = 1$, to have it normalized against the case with constant number of firms. It implies $f_e = 3.78$. As in Bilbiie et al. (2010), the
death shock that determines firm destruction is $\delta = 0.025$, which indicates that 2.5% of firms fail every quarter, 10% in annualized terms. The parameter that determines the Dixit-Stiglitz elasticity in the demand curve is the standard value $\sigma = 3.8$, which implies a 35% mark-up in steady state.

For the banking parameters, we follow Goodfriend and McCallum (2007) to specify a share for banking labor in loan production at $\alpha = 0.65$, while the velocity parameter is $V = 0.40$ to match US data. As collateral value, the stock of bonds in steady state represents 51% of output, $\Psi = 0.51$, to match the average $b/y$ ratio found in US data over the last 40 years. Finally, the parameter that penalizes the collateral services of equity with respect to bonds is set at $\gamma = 0.49$ to have both the steady-state market return of bonds and the $LSY$ at 1% per year, i.e. $r^b = LSY = 0.0025$, as also suggested in Goodfriend and McCallum (2007).

To evaluate the role of firm entry/exit and financial constraints, we report in Table 2 the steady state solution of the model under the baseline calibration and two more variants. First, the case that ignores firm creation and destruction by dropping both the free entry condition (19) and the firm accumulation equation (17), while fixing $n = 1$ and $n^e = 0$ instead. Secondly, the model without banking elements can be reached when dropping the financial constraint (2) from the optimizing program of the representative household. The set of equations that determine the steady state solution of these variants can be found in the technical appendix.\textsuperscript{12}

\textsuperscript{10}The velocity parameter $V$ takes the value consistent with the number chosen in Goodfriend and McCallum (2007) for the particular case of lack of financial requirements for firm creation. It brings the formula $V = 0.31 (1 + vn^e/c)$; which leads to $V = 0.40$ in the baseline calibration.

\textsuperscript{11}From equations (5) and (6), $LSY = \frac{awm}{(1-\alpha)(b+\gamma n)}$ and $\frac{1}{1+r} = \frac{1}{1+\gamma} - LSY$ for the steady-state computation of $r^b$ and $LSY$.

\textsuperscript{12}For a fair comparison to the baseline model, the parameters $\gamma$ and $V$ were recalibrated in the model with no firm entry/exit to follow the criteria defined in Goodfriend and McCallum (2007). In turn, the parameters $\gamma$ and $V$ reported in Table 1 are respectively replaced for $\gamma = 0.08$ and $V = 0.31$. 

15
Table 2. Steady-state solution.

<table>
<thead>
<tr>
<th></th>
<th>Baseline model</th>
<th>No firm entry/exit</th>
<th>No financial friction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm value, $v$</td>
<td>2.7850</td>
<td>14.782</td>
<td>2.8172</td>
</tr>
<tr>
<td>Firm dividend, $d$</td>
<td>0.0870</td>
<td>0.0739</td>
<td>0.0867</td>
</tr>
<tr>
<td>Firm return, $d/v$</td>
<td>0.0312</td>
<td>0.0050</td>
<td>0.0308</td>
</tr>
<tr>
<td>Firm entry, $n^e$</td>
<td>0.0256</td>
<td>-</td>
<td>0.0265</td>
</tr>
<tr>
<td>Relative price, $\rho$</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0112</td>
</tr>
<tr>
<td>Total firms, $n$</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0327</td>
</tr>
<tr>
<td>Output, $y$</td>
<td>0.3305</td>
<td>0.2809</td>
<td>0.3402</td>
</tr>
<tr>
<td>Real wage, $w$</td>
<td>0.7368</td>
<td>0.7368</td>
<td>0.7454</td>
</tr>
<tr>
<td>Labor supply, $l^s$</td>
<td>0.3305</td>
<td>0.2809</td>
<td>0.3363</td>
</tr>
<tr>
<td>Consumption, $c$</td>
<td>0.2591</td>
<td>0.2809</td>
<td>0.2656</td>
</tr>
<tr>
<td>Investment, $vn^e$</td>
<td>0.0713</td>
<td>-</td>
<td>0.0746</td>
</tr>
<tr>
<td>Firm output, $py^f$</td>
<td>0.3305</td>
<td>0.2809</td>
<td>0.3292</td>
</tr>
<tr>
<td>Finance cost, $\mu$</td>
<td>0.0178</td>
<td>0.0108</td>
<td>-</td>
</tr>
<tr>
<td>Banking labor, $m$</td>
<td>0.0028</td>
<td>0.0014</td>
<td>-</td>
</tr>
</tbody>
</table>

As shown in Table 2, the market value of firms is significantly higher in the model without firm entry and exit. Hence, firm value is equivalent to 8.43 times quarterly output in the baseline model (2.7850/0.3305) whereas it is more than 50 times in the model without firm entry (14.782/0.2809). Such difference can be explained by applying the never-die condition of the firm, $\delta = 0$, in the equity value equation (18) that makes the level of $v$ soar for any given steady-state dividend $d$. Conversely, the steady-state return of equity is much higher in the models with variable number of firms, which must take into account the rate of firm destruction at $\delta = 2.5\%$ as a sort of equity value depreciation. To compensate for that, the steady-state quarterly return in the baseline model is at $d/v = 3.12\%$ whereas it is at $d/v = 0.5\%$ in the model without firm entry and exit.

Another key difference is that the baseline model shows that a part of income is spent
on equity investment for creating new firms and the rest on purchases of consumption goods. Concretely, consumption takes a share of \( \frac{c}{y} = 0.78 \) and investment the complementary share \( \frac{vnc}{y} = 0.22 \). The model with no firm entry and exit abstracts from equity investment and leaves all the spending for consumption goods. In turn, we observe a contractionary effect on both the labor supply and output. Table 2 documents that they are 15% lower in that model with no firm entry/exit compared to the baseline model \( \left( \frac{2809}{3305} - 1 = -0.15 \right) \). The impossibility of equity investment makes consumption higher and the labor supply shrinks in (27). Finally, both the marginal finance cost and banking labor are significantly lower in the model without firm entry and exit. On the one hand, there is no financial need for creating firms which reduces the demand for loans. On the other hand, the market value of firms is much higher and provides a stronger collateral guarantee for loan production technology that saves banking labor.

Regarding the case with no financial friction, it can be observed in Table 2 that the steady-state numbers are close to those obtained in the baseline model. However, some increase in economic activity is noticeable. The market value of firms rises which encourages firm creation and equity investment. Moreover, labor supply expands as the finance cost of spending disappears from (27). In turn, firm entry, total firms, labor supply, output, consumption and investment increase by percentages between 1.5% and 5%. Meanwhile, firm-level output declines by just 0.4%.

4 Quantitative analysis

This section explores the quantitative implications of endogenous firm creation when external finance is required. In particular, we will assess the steady state effects of variations in the constant levels of labor productivity, banking efficiency and goods substitutability in the three (comparative) scenarios introduced in the previous section.

Productivity

A 10% increase in labor productivity occurs when raising the constant \( A \) from the initially
Figure 1: Steady-state responses to a permanent 10% increase of productivity.

Figure 1 displays the steady-state effects observed on the endogenous variables across model variants. In the model without firm entry/exit, higher productivity results in greater increases on firm-level output, the dividend and the equity value. The absence of new entries facilitates that the constant number of competitors take advantage of higher productivity. In the model variants with endogenous firm creation, the free entry condition (19) determines an increase in the number of firms proportional to the increase in productivity due to falling entry costs.
The model variant that eliminates the financial constraint brings a slightly higher firm value and further firm entry compared to the baseline model. Therefore, the impact of the financial friction is not quantitatively remarkable.

As for the responses of aggregate variables, Figure 1 shows how the increase in aggregate output is stronger in the models with variable number of firms. The entry of new firms clearly offsets the weaker reaction of output per firm and causes aggregate output to rise at a higher rate than the increase in productivity (beyond 13%). The response of the labor supply is quantitatively small as a result of two opposing effects: the higher real wage pushes up labor supply whereas higher consumption reduces the marginal utility and pushes it down. The latter effect slightly dominates over the former as labor supply falls between 0.25% (with firm entry) and 0.10% (without firm entry). The responses of aggregate consumption and the real wage are quite significant and stronger in the models with endogenous firm creation (beyond the percent increase in productivity). Finally, the marginal finance cost rises at similar rates to output due to the increase in the demand for loans to cover the additional expenditures on consumption and investment.

If the financial constraint is dropped, output, consumption and total firms report a slightly higher reaction to the productivity improvement. As there is no marginal finance cost, both labor supply and firm creation grow faster than in the model with a financial constraint.

Banking efficiency

The scale parameter $B$ of the loan production function (3) can measure the efficiency of banking technology. Figure 2 informs on the steady-state reactions observed in the model when $B$ is raised from its calibrated value to a 10% higher level.\textsuperscript{13}

The quantitative implications of the improvement in banking technology are significantly smaller than those observed when labor productivity was raised. However, the results indicate that the dynamics of firm creation and destruction amplify the steady-state effects of an improvement in banking efficiency. Thus, the response of aggregate output is more than three

\textsuperscript{13}The model variant without financial frictions is not included because it does not incorporate loan production.
Figure 2: Steady-state responses to a 10% increase of banking efficiency.
times in the model with firm entry compared to the model without it (0.69% increase in the model with firm entry and 0.22% in the model without it). The transmission mechanism from the banking sector to the real economy takes place through the marginal finance cost, \( \mu \). Figure 1 shows how a 10% increase in banking efficiency cuts the marginal finance cost \( \mu \) by around 25%, which increases firm value between 0.2% and 0.3% in (18). The subsequent firm creation occurs as long as equity value exceeds the entry cost in (19). In turn, the model with firm entry gives nearly a 0.8% increase in the number of firms when banking efficiency rises by 10%. The relative price increases by nearly 0.3% because there are more goods produced in the economy. Meanwhile, firm-level output declines around 0.35% which means that there would be an scenario with more firms of smaller size.

In the model without firm entry/exit, Figure 1 displays how the adjustment fully takes place through the firm-level margin: the lower marginal finance cost gives rise to higher firm-level dividend, equity value and output, keeping the number of firms constant. Among other reasonable results are the positive response of the labor supply (around 0.4% in the baseline model and 0.2% in the model with no firm entry/exit) because the labor supply curve (27) includes the marginal finance cost \( \mu \) as a (negatively-signed) determinant of labor supply. The response of consumption is also higher in the model with endogenous firm creation (0.59% versus 0.22%) as expected from the differences observed in the responses of aggregate income. Finally, the real wage slightly rises in the model with firm entry because it depends on the number of firms, whereas it remains constant in the model without firm entry/exit because it only depends on the (constant) mark-up and the (constant) relative price.

*Elasticity of substitution (mark-up)*

The last exercise consists of increasing the elasticity of substitution \( \sigma \) by 10%. This implies lowering the mark-up, \( \frac{\sigma}{\sigma - 1} \), by 12% due to greater product differentiation.\(^{14}\) Figure 3 provides

\(^{14}\)It should be noticed that a higher \( \sigma \) implies a lower \( \frac{\sigma}{\sigma - 1} \). Using \( \sigma = 3.8 \) and \( \sigma' = (1 + 0.1)3.8 = 4.18 \) leads to a reduction in the mark-up from 35.71% with \( \sigma = 3.8 \) to 31.45% with \( \sigma' = 4.18 \). This is a 12% reduction in percentage terms.
the plots of responses observed in the three model variants. In the cases with firm entry/exit, the firm dividend, $d$, is negatively affected by an increase in the elasticity of substitution, $\sigma$, through a lower mark-up connecting equations (21) and (28). Moreover, the increase in the elasticity of substitution penalizes the steady-state firm value $v$ given in (18), through this lower $d$ and also because of the substantial increase observed in the marginal finance cost, $\mu$. Indeed, our results indicate that a 10% higher elasticity of substitution reduces the steady-state firm value by 1% in the model variants with firm entry/exit. Such fall of equity value slows down the flow of firm entry and the number of goods available for consumption: the number of firms falls by slightly below 12%. These models report a significant expansion in firm-level output (nearly a 10% increase). This result is interesting: if the mark-up falls the market reshapes with less firms that produce more output each in a way that makes it go away from a perfect competition scenario of many-and-small firms. A symmetric change that raised the mark-up would result in net firm entry, higher number of firms and lower production in each firm.

The model variant with no firm entry/exit shows how a lower mark-up gets transmitted into a much higher reduction in firm value, (around 7 times that found in the baseline model). This occurs because equity value is much more sensitive to any change in the dividend in the variant where the firms never die. By contrast, firm-level output clearly reports a more modest increase because there is no reduction in the number of firms.

The impact of higher elasticity of substitution (lower mark-up) on aggregate output is of different sign across models. Thus, both model variants with firm creation and destruction show that aggregate output falls by nearly 4%, whereas the model that does not allow for firm entry reports an opposing increase of aggregate output by 2%. The decline in aggregate output observed in the model with endogenous number of firms is mostly explained by the 12% reduction in the total number of firms entering (23). The lower mark-up eliminates competitors, discouraged when looking at the prospects of lower dividends, and increases the firm-level output. In the model with no financial friction, the responses of output, consumption, total
Figure 3: Steady-state responses to a 10% higher elasticity of substitution (12% lower mark-up).
firms and the labor supply are very similar to the baseline model, which again emphasizes the little quantitative impact of the financial constraint.

Shutting down the possibility of firm entry/exit eliminates the negative impact of lower mark-up on total firms. In contrast, firms cut prices when applying the lower mark-up, which stimulates demand. Hence, the real wage, labor supply and aggregate output rise after a decline in the mark-up in the model without firm entry. Therefore, if firm entry is allowed, a higher mark-up results in an economic expansion in terms of number of firms and aggregate output. If firm entry is not considered, a higher mark-up would rise prices, cut the real wage and reduce aggregate output.

As a summary, Table 3 reproduces percent reactions of some aggregate variables observed across model variants. The numbers reported bring a factor decomposition of aggregate output in terms of supply components, demand components and income shares.

Table 3. Simulation results. % steady-state responses after 10% increase in

<table>
<thead>
<tr>
<th>productivity A</th>
<th>banking efficiency B</th>
<th>elas. of subs. σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>No e/e</td>
<td>No ff</td>
</tr>
<tr>
<td>Aggregate output, $y$</td>
<td>13.35</td>
<td>9.90</td>
</tr>
<tr>
<td>Supply decomposition, $y = npy^f$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total firms, $n$</td>
<td>9.51</td>
<td>0.0</td>
</tr>
<tr>
<td>Firm-level output, $py^f$</td>
<td>3.51</td>
<td>9.90</td>
</tr>
<tr>
<td>Demand decomposition, $y = c + vn^e$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption, $c$</td>
<td>13.42</td>
<td>9.90</td>
</tr>
<tr>
<td>Investment, $vn^e$</td>
<td>13.12</td>
<td>0.0</td>
</tr>
<tr>
<td>Income decomposition, $y = wl^s + dn$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Labor income, $wl^s$</td>
<td>13.35</td>
<td>9.90</td>
</tr>
<tr>
<td>Equity income, $dn$</td>
<td>13.35</td>
<td>9.90</td>
</tr>
</tbody>
</table>

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15 The real wage falls in the model without firm entry because it is required to hold (21) for a constant $\rho$ when there is an increase in the mark-up $\frac{\sigma}{\sigma-1}$.

16 "Base" is Baseline model, "No e/e" is the model variant with no firm entry/exit, and "No ff" is the model variant with no fixed factor inputs.
The contribution of the total number of firms (dynamics of firm creation/destruction) plays a very important role to explain the reaction of output to changes in productivity, banking efficiency and the elasticity of substitution (market power). By contrast, the intensive margin of output (at firm level) shows little reaction after a technology shock and opposing (countercyclical) reactions after changes in either banking efficiency or market power. In the model without firm entry/exit, the responses of aggregate output are fully determined by the responses of firm-level output.

Regarding the decomposition of demand, the baseline model shows responses of both consumption and investment of similar size to those of aggregate output when there is a productivity improvement (slightly higher on investment). They are identical if there is no financial distortion, whereas the effect is fully taken for consumption spending in the model with no entry/exit of firms. By contrast, after a change in banking efficiency and, especially, a change in the elasticity of substitution (mark-up) investment shows a reaction much larger than that of consumption. Thus, investment on creating new firms falls by more than 12% when there is a 10% higher elasticity of substitution that cuts the mark-up by 12%, which is more than 10 times higher than the percent decline observed in purchases of consumption goods (between 1.26% and 1.03%). The model with no entry and exit of firms gives the same 2.11% expansion on output and consumption when there is a decline in the mark-up.

Finally, the income decomposition reported in Table 3 indicates that the increase of output is equally distributed among labor income and equity income when there is an improvement in either productivity or banking efficiency. Nevertheless, the effects of a change in the elasticity of substitution (mark-up) are absorbed quite more significantly in equity income than in labor income, especially in the models with firm entry and exit. The sizeable responses of both the dividend and the number of firms explain why equity income is so sensitive to changes in the mark-up.

variant with no financial friction.
5 Conclusions

This paper has investigated the steady state consequences of combining financial frictions with firm entry-and-exit in a model where the level of economic activity depends both on the number of firms (extensive margin of activity) and on the production of individual firms (intensive margin of activity). In this setting, financial factors have permanent effects on the competitive equilibrium through the influence of the external finance cost upon the market value of firms and labor supply.

The economic analysis results in three main conclusions. First, the financial constraint has contractionary effects on both the equity value and the labor supply of households. In turn, the steady-state levels of firm entry, total firms, labor supply, output, consumption and investment fall by percentages between 1.5% and 5%.

Secondly, firm creation amplifies the impact of an improvement in either labor productivity or bank efficiency on aggregate activity as it collects a procyclical change in the number of firms. In a quantitative comparison, we find that the reaction of aggregate output becomes 35% greater when there is a change in labor productivity and more than 3 times greater when there is a change in banking efficiency.

And thirdly, a higher elasticity of substitution (that implies a decrease of the mark-up) has a negative impact on aggregate output because of a substantial reduction in the number of firms. This last result is reversed in a model without firm entry and exit where both labor supply and output rise with a lower mark-up.
References


Appendix

Technical Appendix 1. Loan production technology

The partial derivatives relating the change in the amount of loans to the change in the factors of loan production are,

\[
\frac{\partial L_t}{\partial m_t} = B (1 - \alpha) (b_{t+1} + \gamma n_{t+1} v_{t+1}) (m_t^d)^{-\alpha} = \frac{(1-\alpha) B}{m_t^d},
\]

\[
\frac{\partial L_t}{\partial b_{t+1}} = B \alpha (b_{t+1} + \gamma n_{t+1} v_{t+1}) (m_t^d)^{1-\alpha} = \frac{\alpha B}{b_{t+1}},
\]

\[
\frac{\partial L_t}{\partial v_{t+1}} = B \alpha (b_{t+1} + \gamma n_{t+1} v_{t+1}) (m_t^d)^{1-\alpha} \gamma = \frac{\alpha B}{b_{t+1} + \gamma n_{t+1} v_{t+1}} \gamma.
\]

Technical Appendix 2. Overall resource constraint.

Combining the household budget constraint,

\[
w_t(l_t^s + m_t^s - m_t^d) + b_t + (d_t + v_t) n_t x_t + g_t = c_t + v_t (n_t + n_t^e) x_{t+1} + (1 + r_t^b)^{-1} b_{t+1},
\]

with the government budget constraint, \( g_t = (1 + r_t^b)^{-1} b_{t+1} - b_t \), the market-clearing condition of firm labor \( l_t^s = n_t l_t^d \), the market-clearing condition for banking labor, \( m_t^s = m_t^d \), and the market-clearing condition for portfolio shares, \( x_t = x_{t+1} = 1 \), it is obtained,

\[
w_t n_t l_t^d + (d_t + v_t) n_t = c_t + v_t (n_t + n_t^e),
\]

where dropping \( v_t n_t \) on both sides, it is equivalent to,

\[
w_t n_t l_t^d + d_t n_t = c_t + v_t n_t^e. \tag{A1}
\]

Under symmetric equilibrium, current dividends are determined as,

\[
d_t = \rho_t y_t^f - \frac{w_t}{A} y_t^f,
\]

where applying the mark-up pricing policy \( \rho_t = \frac{\sigma}{\sigma - 1} \frac{w_t}{A} \) gives,

\[
d_t = \frac{\sigma}{\sigma - 1} \frac{w_t}{A} y_t^f - \frac{w_t}{A} y_t^f = \left( \frac{1}{\sigma - 1} \right) \frac{w_t}{A} y_t^f. \tag{A2}
\]

Substituting (A2) into (A1) leads to,

\[
w_t n_t l_t^d + \left( \frac{1}{\sigma - 1} \right) \frac{w_t}{A} y_t^f n_t = c_t + v_t n_t^e.
\]

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where using the linear production technology under symmetric equilibrium, \( y_t^f = A t^d \), it is obtained,
\[
\frac{w_t}{A} n_t y_t^f + \frac{1}{\sigma - 1} \frac{w_t}{A} y_t^f n_t = c_t + v_t n_t^e.
\]  
(A3)

Finally, putting together terms that come with \( y_t^f \) transforms (A3) as follows,
\[
\frac{w_t}{A} n_t y_t^f = c_t + v_t n_t^e,
\]
where inserting \( w_t = \frac{\sigma - 1}{\sigma} A \rho_t \) from the mark-up definition results in the overall resources constraint,
\[
n_t \rho_t y_t^f = c_t + v_t n_t^e,
\]  
(A4)

The left-hand side of (A4) is total output produced in the economy computed as the product of firm-level output times the relative price times the number of goods-firms, \( y_t = n_t \rho_t y_t^f \).

On the right-hand side of (A4), total spending is the sum of purchases of consumption goods and spending on acquiring newly created firms. Equation (A4) brings, therefore, the overall resources constraint,
\[
y_t = c_t + v_t n_t^e.
\]

Technical Appendix 3. Model without firm entry and exit.

The model with constant number of firms can be considered a particular case of the baseline model. The free entry condition and the firm accumulation equation are ignored while setting \( n = 1 \) and \( n^e = 0 \) instead.

In turn, equation (8) from the main text results in a steady-state firm value that only depends upon dividends and the discount parameter:
\[
v = \left( \frac{\beta}{1 - \beta} \right) d.
\]  
(A5)

As extensive margin fluctuations are shut down, economy-wide output and firm-level output coincide,
\[
y = y^f.
\]  
(A6)
The real wage is fully determined by the elasticity of substitution and labor productivity,

\[ w = \frac{\sigma - 1}{\sigma} A. \]  

(A7)

The lack of investment makes the goods market equilibrium collect only spending consumption goods,

\[ y = c. \]  

(A8)

The equilibrium condition for asset holdings symmetry in steady state brings,

\[ x = 1, \]  

(A9)

while the steady-state marginal finance cost \( \mu \) becomes,

\[ \mu = \frac{wm}{(1-\alpha)c}. \]  

(A10)

The firm-level production function is the linear technology,

\[ y^f = Al, \]  

(A11)

and the labor supply equation in steady state is,

\[ \frac{1 - \phi}{1 - l - m} = w \frac{\phi}{c (1 + \mu)}. \]  

(A12)

Under complete equilibrium symmetry, the steady-state dividend is,

\[ d = \left( 1 - \frac{w}{A} \right) y^f. \]  

(A13)

The loan production technology is,

\[ L/P^c = B (b + \gamma v)^\alpha m^{1-\alpha}, \]  

(A14)

and the stock of government bonds is assumed to be proportional to output in steady state,

\[ b = \Psi y. \]  

(A15)
Finally, the financial constraint only takes into account liquidity requirements for consumption spending,
\[ c = V \frac{L}{Pc}. \]  
(A16)

The steady state solution of the twelve endogenous variables: \( w, c, x, \mu, v, l, y, y^f, d, L/Pc, b, \) and \( m, \) is determined by solving the non-linear system of twelve equations (A5)-(A16).

**Technical Appendix 4. Model without financial frictions**

Dropping the financial constraint from the household optimizing program results in the following steady-state system of equations:

\[ n^e = \frac{\delta}{1-\delta} n, \]  
(A17)

\[ v = \left( \frac{(1-\delta)\beta}{1-(1-\delta)\beta} \right) d, \]  
(A18)

\[ v = f \frac{w}{A}, \]  
(A19)

\[ \rho = n \frac{1}{\sigma-1}, \]  
(A20)

\[ \rho = \frac{\sigma}{\sigma-1} \frac{w}{A}, \]  
(A21)

\[ y = c + n^e v, \]  
(A22)

\[ y = n \rho y^f, \]  
(A23)

\[ x = 1, \]  
(A24)

\[ y^f = Al, \]  
(A25)

\[ \frac{1 - \phi}{1 - nl} = w \frac{\phi}{c}, \]  
(A26)

\[ d = \left( \rho - \frac{w}{A} \right) y^f. \]  
(A27)

The steady state solution of the model provides numerical values to eleven variables: \( n, \rho, w, c, n^e, x, v, l, y, y^f, \) and \( d, \) obtained by solving the above non-linear system of eleven equations, (A17)-(A27), using the calibration of model parameters presented in the next.