Entry and exit in recent US business cycles[*]

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Abstract

I show evidence indicating that the variability of the total number of business units (establishments) has significantly increased in recent US business cycles, accounting for nearly 2/3 of real GDP fluctuations during the 2003-2012 decade. Next, I examine the role of business creation and destruction in an estimated DSGE-style model extended with endogenous entry and exit. Shocks on both entry and, especially, exit have played a crucial role on explaining the latest boom-bust cycle in the US economy. I also find that the estimated innovations of total factor productivity are positive and high in 2010-2012, which might be the consequence of the dramatic increase in the exit rates observed during the recession of 2008-2009.

Keywords: entry and exit, DSGE models, US business cycles.

JEL codes: E20, E32.

1 Introduction

Aggregate fluctuations may be driven by changes in the intensity at which incumbent firms are producing (intensive margin), by the net formation of production units (extensive margin) or by a combination of the two. In Section 2 of this paper, I provide some empirical evidence documenting severe adjustments in the total number of production units over the latest US business cycle (2003-2012). Furthermore, I find a significant role of extensive margin variability for explaining aggregate fluctuations.[1] By contrast, modern dynamic macroeconomic models usually ignore the extensive margin of GDP fluctuations as the number of varieties produced in the economy is assumed to be constant. Well-known examples of these models are Christiano *et al.* (2005) and Smets and Wouters (2003, 2007) that initiated a fashionable literature,

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[1]Bernard *et al.* (2012) and Broda and Weinstein (2010) also show the relevance of business creation and destruction for aggregate fluctuations in the US economy.

the so-called Dynamic Stochastic General Equilibrium (DSGE) paradigm where the number of production units is fixed.

This paper investigates the role of the extensive margin and the influence of entry and exit shocks for aggregate fluctuations using a structural model estimated for the US economy. Hence, Section 3 describes a DSGE-style extended model where there are both entry and exit of goods, endogenously determined to bring period-to-period changes in the total number of varieties. One additional variety is created (entry) when the prospective value of the new production unit is higher than its entry cost. Furthermore, business destruction (exit) takes place if the liquidation value of a production unit exceeds the present value of its expected dividends. Firms are heterogeneous on their productivity, which serves to pin down a productivity threshold that splits up the zones of survival and death in the continuum of varieties of goods. The model also incorporates financial constraints in the optimizing program of both firms and households, and banking activities with a loan production technology that uses equity as collateral. Most of the ingredients of the model are borrowed from Smets and Wouters (2007), Bilbiie *et al.* (2012), Casares and Poutineau (2014), Cavallari (2015), and Lewis and Stevens (2015), and also incorporates original features such as financial frictions for entry, convex costs of entry-exit, endogenous liquidation value and a monetary policy rule that responds to credit spreads.[2]

Using Bayesian econometric techniques, the model has been estimated with US quarterly data from 1993 to 2012. The estimates of the structural parameters are discussed in Section 4 and a comparison between the business cycle regularities of the model and actual data is carried out in Section 5. The business cycle analysis is completed with the discussion of the variance decomposition and the interpretation of some of the estimated impulse-response functions. The recent boom-bust cycle of the US economy is examined in Section 6, where the model shows the importance of technology shocks, entry-exit shocks and financial shocks to explain the financial crisis of 2008-2009 and also the unfinished recovery that came afterwards. In addition, the observed entry and exit rates have a substantial correlation to the estimates of innovations on Total Factor Productivity (TFP). High entry rates tend to bring adverse shocks on TFP, whereas high exit rates typically induce a positive TFP innovation.

2 Empirical motivation

In any period t, aggregate output (y_t) can be decomposed as the product of the number of varieties (n_t) times the average level of output produced of these varieties ($\overline{y}_t$)

$$y_t = n_t \overline{y}_t,$$

and taking logs on both sides brings the additive decomposition

$$\log y_t = \log n_t + \log \overline{y}_t.$$

[2] In addition, Rossi (2015) incorporates entry and exit with inefficient banks in a calibrated model with no capital accumulation and symmetric pricing behavior.

Standard DSGE models ignore the extensive margin variability by fixing the number of goods to a constant value, normalized at $n_t = 1.0$. Is this assumption at odds with the data? The Business Employment Dynamics (BED) database, released by the Bureau of Labor Statistics (BLS), reports quarterly data on the number of entries, exits and total incumbents. There are both establishment-level and firm-level data. I have chosen to look at establishment-level data because many business openings are associated to the same firm, especially in the service sector, that can only be captured through the series of establishment entry.[3] Regarding entry and exit, I use the BED series of number of births (entry) and number of deaths (exit) in order to exclude re-openings, temporary shutdowns and seasonal businesses.[4],[5] The sample period comprises 80 quarterly observations from 1993 to 2012. The starting quarter is determined by the first observation available on Total Private Establishments in the BED release (as well as the numbers of establishment entry and exit). The sample period ends in 2012 because there was an administrative change that occurred in the first quarter of 2013 that brought a discontinuity in the time series.[6]

Taking the total number of US (private) establishments as the indicator for the number of varieties produced, the amount of output per establishment can be obtained by making the ratio between aggregate output and the number of varieties ($\overline{y}_t = y_t/n_t$). Aggregate output can be measured in the data as the amount of real GDP per capita, which is included in the FRED database from the Federal Reserve Bank of St. Louis.[7] After normalizing both output (real GDP per capita) and the number of varieties (total private establishments) at 1.0 in the initial quarter of the sample (1993:1), I have built the series of output per establishment, $\overline{y}_t$, for the whole sample period.[8]

Figure 1 plots the resulting series. The direct visual inspection shows two differentiated periods, roughly split up by the middle of the sample. In the first period (which still corresponds to the so-called *Great Moderation* era), both output and the number of establishments grow steadily, while output per

[3] As defined by the US Census Bureau: "an establishment is a single physical location at which business is conducted or where services or industrial operations are performed. It is not necessarily identical with a company or enterprise, which may consist of one establishment or more. When two or more activities are conducted at a single location under a single ownership, all activities are generally grouped together as a single establishment and classified on the basis of its major activity.".

[4] Births are units with positive third month employment for the first time in the current quarter with no links to the prior quarter, or units with positive third month employment in the current quarter and zero employment in the third month of the previous four quarters. Births are a subset of openings not including re-openings of seasonal businesses.

[5] Deaths are units with no employment or zero employment reported in the third month of four consecutive quarters following the last quarter with positive employment. Deaths are a subset of closings not including temporary shutdowns of seasonal businesses. A unit that closes during the quarter may be a death, but we wait three quarters to determine whether it is a permanent closing or a temporary shutdown. Therefore, there is always a lag of three quarters for the publication of death statistics.

[6] See http://www.bls.gov/bdm/ for the details.

[7] Output is considered in per-capita terms because this is the way that will be taken for the observable series used in the model estimation below.

[8] The distribution of US private establishments, in terms of number of employees, has been stable over the last two decades, which is an indicator for the long-run constant size assumption taken in this paper. The last section of the technical appendix provides empirical support.

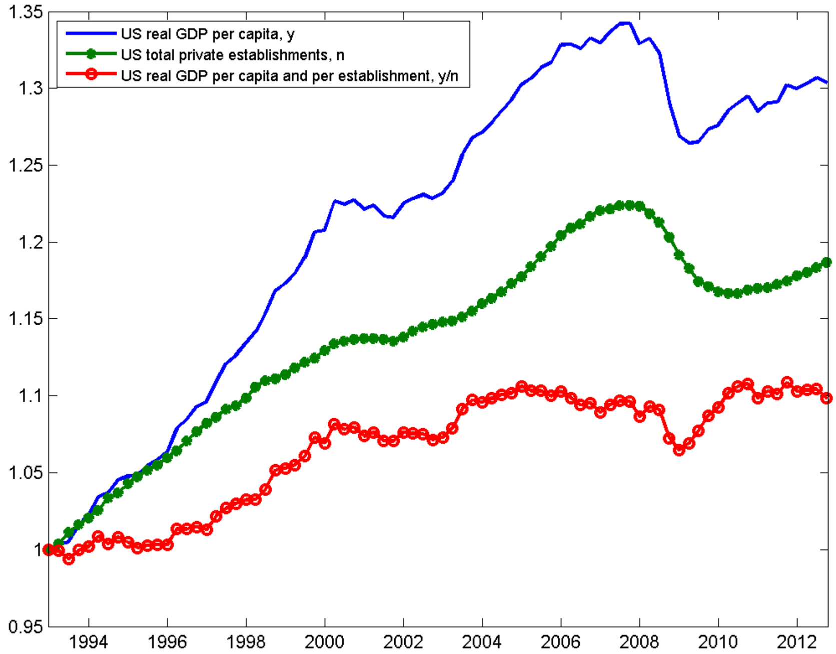

Figure 1: US real GDP per capita and its margins of variability,1993:1-2012:4.

establishment displays more short-run fluctuations along trend. In the 2000's the picture changes. The housing boom and the so-named *Great Recession* that came afterwards are well captured in the graph. Thus, Figure 1 shows how the boom-bust cycle of real GDP is somewhat replicated by the series of total number of establishments, with some lag of one or two quarters. Output per establishment also mirrors the business cycle, but its variability seems to be significantly lower than the one of the number of establishments. In turn, the role of the extensive margin becomes really important for the latest business cycle.

Figure 2 plots the Hoddrick-Prescott filtered series applied to the logs of the original series in order to take a closer look at high frequency changes, e.g. $\widehat{y}^{HP} = 100(\log(y) - \log(y)^{HP})$ is the cyclical component of real GDP per capita in percentage terms. Again, the fluctuations observed in the second half of the sample display remarkable differences relative to the first half. Overall variability of real GDP per capita rises and the extensive margin (number of establishments) provides a strong procyclical behavior. By contrast, the comovement between real GDP and real GDP per establishment is much weaker than in the first half of the sample.

Table 1 reports some numbers to shed more light on this issue by making a cutpoint between periods in the last quarter of 2002. In terms of volatilities, the standard deviations of the Hoddrick-Prescott filtered series are higher in the second subsample, especially for the case of the total number of establishments that multiplies by a factor higher than 3 relative to the first subsample. Cross correlations with output

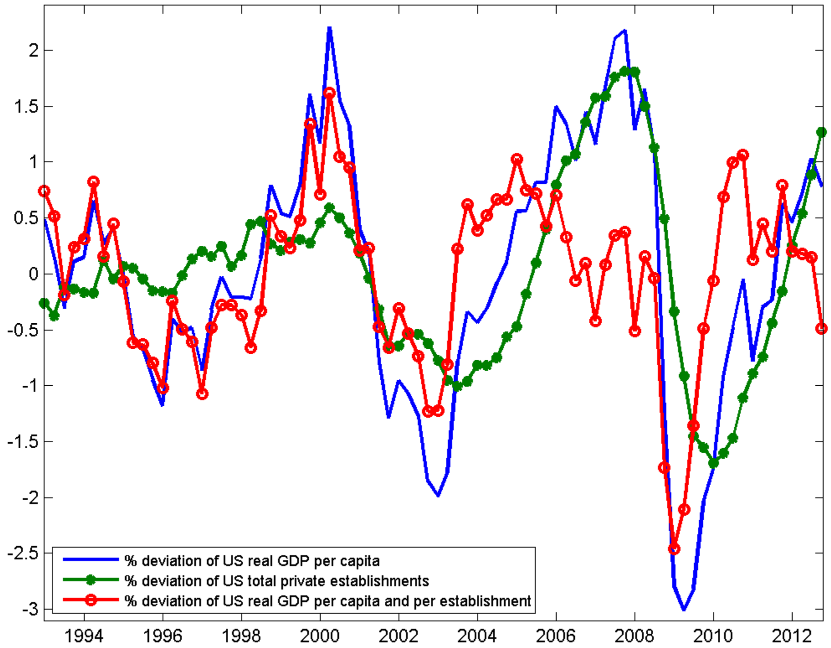

Figure 2: H-P filtered series of US real GDP per capita and its margins of variability, 1993:1-2012:4.

are also reported in Table 1. The fluctuations of the intensive margin are more strongly procyclical in the first subperiod, whereas the extensive margin becomes more procyclical in the second subperiod. The correlation between the intensive and the extensive margins is positive and mild in the first subperiod and turns null in the second subperiod. Using the definition of the variance of a sum of two random variables, I have computed the shares of aggregate output variability that come from the extensive margin, the intensive margin and the covariance. Numbers change dramatically from the first subsample period (1993-2002) to the second period (2003-2012). The extensive margin share rises from 14% to 64%, where the intensive margin share falls from 60% to 36%. Hence, it could be said that output variability is mostly explained by firm-level production adjustments in the 90's whereas net business creation plays the major role in the years of the housing boom, the financial crisis and the *Great Recession* (2003-2012).

Table 1. Extensive and intensive margin variabilities in recent US business cycles.

	First subsample 1993:1-2002:4	Second subsample 2003:1-2012:4	Full sample 1993:1-2012:4
Volatilities:			
std($\widehat{y}^{HP}$), %	0.88	1.38	1.15
std($\widehat{n}^{HP}$), %	0.33	1.11	0.81
std($\widehat{\overline{y}}^{HP}$), %	0.68	0.83	0.75
Cyclical correlations:			
corr($\widehat{y}^{HP}, \widehat{n}^{HP}$)	0.73	0.80	0.75
corr($\widehat{y}^{HP}, \widehat{\overline{y}}^{HP}$)	0.94	0.60	0.71
corr($\widehat{n}^{HP}, \widehat{\overline{y}}^{HP}$)	0.45	-0.00	0.08
Variability decomposition:[9]			
extensive margin share	0.14	0.64	0.50
intensive margin share	0.60	0.36	0.43
covariance share	0.26	-0.00	0.07

The evolution of the total number of establishments is the result of both adding entry and deducting exit with respect to the previous observation. Figure 3 plots the level of US private establishment entry (n^E) and exit (n^X) as defined above. Figure 4 displays both the entry rate, $E = 100n^E/n$, and the exit rate, $X = 100n^X/n$. From 1993 to 2000, entry exceeds exit and this net business creation comes with little variability along trend. In 2001 exit rises to the level of entry which makes the total number of establishments remain basically unchanged for a few quarters. From 2002 to 2007 the level of entries clearly surpasses that of exits and there is substantial business creation that takes place during the housing-driven expansion. In 2007-08, there is a dramatic increase in exit and a similar-size decrease in entry which brings net business destruction of around 80,000 establishments (approximately 1% of total private establishments). As 2009 went by, the number of exits moved down to historically normal levels while entry showed signals of recovery. Over the last quarters of the sample, there is some net business creation (about 10,000 establishments per quarter). Hence, both entry and exit are much more reactive in the latest boom-bust cycle (2003-2012) than in the previous decade. Both variables switch from basically following the long-run growth trend with little variations to mimicking and amplifying the business cycle patterns.

Table 2 provides some numbers that confirm the visual findings of Figures 3 and 4. The standard

[9] The extensive margin share is var($\widehat{n}^{HP}$)/var($\widehat{y}^{HP}$), the intensive margin share is var($\widehat{\overline{y}}^{HP}$)/var($\widehat{y}^{HP}$), and the covariance share is 2cov($\widehat{n}^{HP}, \widehat{\overline{y}}^{HP}$)/var($\widehat{y}^{HP}$).

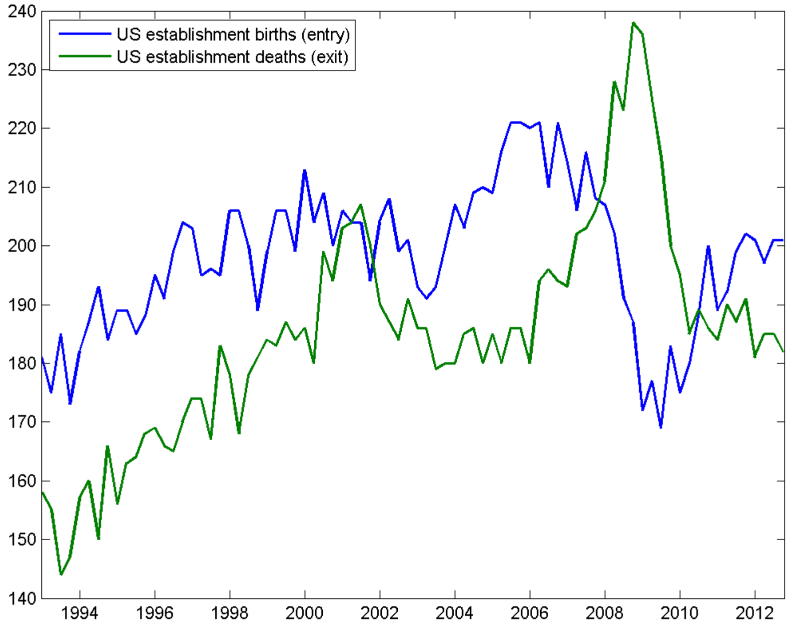

Figure 3: US quarterly entry and exit (thousands), 1993:1-2012:4.

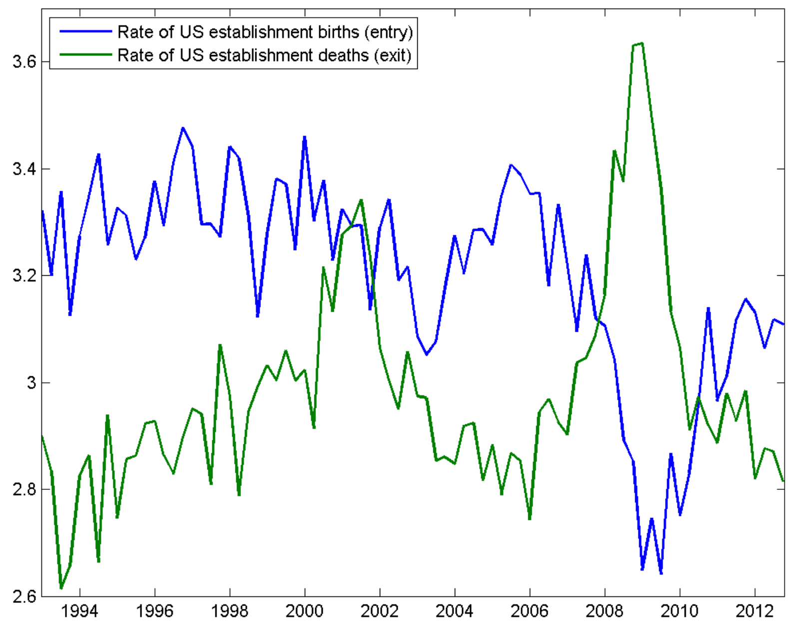

Figure 4: Rates of US quarterly entry and exit, 1993:1-2012:4.

deviation of both entry and exit rates is higher in the second period 2003-2012. The increase is really important for entry rates that report a standard deviation of 0.2% in the 2003-2012 period and only 0.09% in the previous decade. Entry rates become procyclical in the second period, with a positive correlation to real GDP growth at 0.52. In the first subsample, entry was mildly countercyclical (correlation at -0.08). Exit rates are countercyclical throughout the whole period (correlation with $\triangle \log y$ at -0.62), though in the second decade they show a deeper negative response to the cycle (-0.75). Therefore, rates of entry and exit are much more sensitive to business-cycle fluctuations in the period 2003-2012 than in the previous decade.

Table 2. US entry (E) and exit (X) rates in recent US business cycles.

	First subsample 1993:1-2002:4	Second subsample 2003:1-2012:4	Full sample 1993:1-2012:4
Volatilities:			
std(E), %	0.09	0.20	0.19
std(X), %	0.17	0.23	0.20
Cyclical correlations:			
corr($\triangle \log y, E$)	-0.08	0.52	0.43
corr($\triangle \log y, X$)	-0.34	-0.75	-0.62
corr(E, X)	-0.12	-0.71	-0.53

The empirical evidence discussed in this section highlights the role of entry, exit and total number of establishments in the latest US business cycle. Next, I will introduce a DSGE model that endogenously determines the aggregate flows of entry and exit, and allows for a decomposition of aggregate output between the intensive and extensive margins. The model also incorporates credit frictions, banking and financial shocks to be able to capture the sources of variability of the financial turmoil that preceded the 2008-2009 recession. The objective of this modelling is to re-examine the most recent episodes of short-run aggregate fluctuations in the US economy, focusing a special attention on the role of business creation and destruction.

3 A DSGE model with entry and exit

The model represents an economy with households, firms, banks, and the public sector (government and central bank). There are markets for goods, labor, physical capital, government bonds, equity, bank deposits and loans. All the markets are perfectly competitive, except for the goods and labor markets where suppliers have some market power to set, respectively, the nominal price and wage in monopolistic competition. The number of varieties in the goods market changes over time as a result of the flows of both entry and exit of differentiated goods. Several sources of rigidities and frictions are assumed to enhance the empirical fit of the model, following Christiano *et al.* (2005) or Smets and Wouters (2007). The set of real rigidities include consumption habits, adjustment costs on investment, variable capital utilization, time-to-build delays for capital, and adjustment costs of business formation. Regarding nominal rigidities, I consider Calvo (1983)-style stickiness in both price and wage setting. The model also incorporates financial frictions as both households and firms demand loans for covering liquidity constraints. The model description that comes next only focuses on the elements that are not present in conventional DSGE models (entry, exit and financial frictions), and the optimizing programs of both households and firms are incorporated in the technical appendix.

Entry

Following Bilbiie *et al.* (2012), Casares and Poutineau (2014), Cavallari (2015), or Lewis and Stevens (2015), the free-entry decision is based on the comparison between the prospective equity value and the cost of entry. Unlike those papers, the cost of entry in my model is not obtained from the marginal cost of production or from an specific production function. I assume that the cost is a combination of fixed (licence) cost, financial costs and start-up variable costs. In particular, I have this unit cost of entry in real terms

$$\left(1 + r_t^L \tau_h\right) e^{\varepsilon_t^E} \left(f^E + ec_t\right),$$

where f^E is the unit real cost of a license fee required by the government to begin the production of a new variety, ec_t is a variable cost, ε_t^E is an AR(1) exogenous shock, and r_t^L is the real interest of the bank loan needed to finance the τ_h share of the cost of entry financed externally.[10] The variable cost ec_t is meant as a congestion cost for start-ups that therefore increases with the ratio of entries planned, n_t^E, over total number of goods, n_t, as follows[11]

$$ec_t = \varsigma_1^E \left(\frac{n_t^E}{n_t}\right)^{\varsigma_2^E}, \tag{1}$$

where $\varsigma_1^E > 0$ and $\varsigma_2^E > 1$ for convexity. As shown in the technical appendix, combining the first order conditions of the household on number of entries and equity investment, it is obtained the free entry equilibrium condition,

$$\left(1 + r_t^L \tau_h\right) e^{\varepsilon_t^E} \left(f^E + ec_t\right) = \widetilde{v}_t, \tag{2}$$

which equates the marginal cost of entry to the marginal benefit measured by the average equity value of the prospective firms, $\widetilde{v}_t$. Combining log-linear approximations to (1) and (2) to solve out for the dynamic

[10] The financial friction can be justified on the grounds of a mismatch between the flows of income and expenditures. In particular, the household must pay the licence fee and the other costs for entry at the beginning of the period, whereas income may arrive at a constant pace throughout the period.

[11] In the empirical analysis, the congestion cost helps to sluggish the responses of entry to shocks in a way that better replicate actual US data.

equation of log fluctuations of entry results in the dynamic equation

$$\widehat{n}_t^E = \widehat{n}_t + \frac{f^E + ec}{\varsigma_2^E ec} \left(\widehat{\widetilde{v}}_t - \varepsilon_t^E - \tau_h \left(r_t^L - r^L \right) \right),$$

where the '$\widehat{\ }$' label indicates the deviation in natural logarithm with respect to the steady-state balanced-growth path. Households decide to spend on the creation of more new goods when they observe an increase in the average equity value. By contrast, if the exogenous component of the cost of entry increases the number of new goods is going to fall. The third determinant of entry comes from the role of financial frictions on business creation. Since households need external finance for business creation, a high real interest rate of a bank loan, r_t^L, reduces the number of entries. The latter effect can be considered as another channel for observing the financial accelerator mechanism in business cycle fluctuations.[12]

Exit

In a similar vein to Casares and Poutineau (2014), Cavallari (2015) and Rossi (2015), exit is endogenously determined. It is assumed that incumbents produce with a firm-specific productivity dealt from a Pareto distribution. Such time-invariant productivity draw marks the relative position of each firm to reach high or low dividends. At the end of the production period, there is a survival test for each incumbent firm. If the present value of all expected dividends exceeds the liquidation value, the production of the good survives. In the opposite case, the firm shuts down, the production of that variety ends and there is a good destruction (exit). Formally, any firm in period t would face the following choice,

$$\begin{array}{lcll} E_t \sum_{j=1}^{\infty} \beta_{t+j} d_{t+j}(.) & > & lv_t, & \rightarrow \text{ Survive,} \\ E_t \sum_{j=1}^{\infty} \beta_{t+j} d_{t+j}(.) & < & lv_t, & \rightarrow \text{ Exit,} \end{array}$$

where E_t is the rational expectation operator, β_{t+j} is the stochastic discount factor from period t to period $t+j$, $d_{t+j}(.)$ is the firm-level real dividend in period $t+j$, and lv_t is the liquidation value at the end of the current period t.

As standard assumptions in the New-Keynesian literature, firms set prices in monopolistic competition, face a demand constraint *á la* Dixit and Stiglitz (1977), produce under a Cobb-Douglas technology, and demand labor and capital taking the market wage rate and the rental capital rate as given. Firms also must pay back a loan to finance a fraction of their operating costs at the real interest of the loan, r^L. Let me consider that the representative posts the average relative price, $\widetilde{\rho} = \widetilde{P}/P^c$, defined as the ratio between the average producer price and consumption price index.[13] For this firm, there is a critical value of specific

[12] This sequence might be spelled out as follows: economic upturn, higher banking collateral, lower cost of producing loans, lower interest rate of borrowing, higher entries, higher total number of goods, higher total output produced, and further economic upturn.

[13] The relative price of the firm will depend on the Calvo lottery. If the firm sells at the average relative price, it is not being able to set the optimal price.

productivity, z_t^c, at which the dividend stream exactly coincide with the liquidation value. Expanding the real dividends as a function of relative prices and the marginal costs, the critical vale z_t^c is found embedded in the mc_{t+j}^c term that satisfies

$$E_t \sum_{j=1}^{\infty} \beta_{t+j} \left(\left(\widetilde{\rho}_{t+j}\right)^{-\theta_p} y_{t+j} \left[\widetilde{\rho}_{t+j} - mc_{t+j}^c\right] \right) = lv_t. \tag{3}$$

Remarkably, the productivity threshold implied by (3) is time dependent even though firm-specific productivity is time invariant. As shown in the Appendix, the real marginal cost is rises with the real wage, the real rental rate of capital, and the real interest rate of loans, and falls with firm-specific productivity and the economy-wide technology shock.The average real marginal cost is defined at the average productivity $\widetilde{z}$, which implies, $mc_{t+j}^c = \widetilde{mc}_{t+j} \frac{\widetilde{z}}{z_t^c}$, and once inserted in (3) gives,

$$E_t \sum_{j=1}^{\infty} \beta_{t+j} \left(\left(\widetilde{\rho}_{t+j}\right)^{-\theta_p} y_{t+j} \left[\left(\widetilde{\rho}_{t+j}\right) - \widetilde{mc}_{t+j} \frac{\widetilde{z}}{z_t^c} \right] \right) = lv_t. \tag{4}$$

Critical firm-level productivity z_t^c can now be solved in (4). Before doing it, let us introduce the liquidation value as a function that positively depends upon the fraction $(1-\varphi)$ of the licence fee for entry that is reimbursed from the government at exit (nonsunk fixed cost of entry), and negatively on the measure of exit costs, xc_t,

$$lv_t = e^{\varepsilon_t^X} \left((1-\varphi) f^E - xc_t\right), \tag{5}$$

where $0 \leq \varphi \leq 1$ is the sunk part of the license at exit, and ε_t^X brings the AR(1) exogenous component of the liquidation value. The variable cost of exit is convex at the exit rate

$$xc_t = \varsigma_1^x \left(\frac{n_t^X}{n_t}\right)^{\varsigma_2^x}, \tag{6}$$

because it is assumed that $c_1 \geq 0$ and $c_2 > 1$. Accordingly, if the aggregate exit rate is high the liquidation value falls because the cost of exit is higher. This effect may capture real rigidities for exit caused by multiple factors such as workers litigation, legal issues, firing costs, auditing costs, etc. Finally, the characteristics of the Pareto distribution are applied to provide the connection between the critical value of productivity, z_t^c, and the fraction of good varieties that either survive or exit[14]

$$\frac{n_t^A}{n_t} = \left(\frac{z_{\min}}{z_t^c}\right)^{\kappa}, \text{ and} \tag{7a}$$

$$\frac{n_t^X}{n_t} = 1 - \left(\frac{z_{\min}}{z_t^c}\right)^{\kappa}, \tag{7b}$$

where n_t^A/n_t is the survival rate with n_t^A as the number of surviving firms, $z_{\min}$ and κ are coefficients that provide the minimum productivity and the curvature of the Pareto distribution, respectively. After

[14] See Ghironi and Melitz (2005) for more details.

loglinearization of (4), (5), (6), and (7b) the following four equations govern exit dynamics,

$$\begin{aligned}
\widehat{z}_t^c &= \overline{\overline{\beta}} E_t \widehat{z}_{t+1}^c + \left(1-\overline{\overline{\beta}}\right) E_t \widehat{\widetilde{mc}}_{t+1} + \frac{1-\widetilde{mc}\frac{\widetilde{z}}{z^c}}{\widetilde{mc}\frac{\widetilde{z}}{z^c}} \left(\widehat{lv}_t - \overline{\overline{\beta}} E_t \widehat{lv}_{t+1}\right) \\
&\quad - \frac{\left(1-\overline{\overline{\beta}}\right)\left(1-\widetilde{mc}\frac{\widetilde{z}}{z^c}\right)}{\widetilde{mc}\frac{\widetilde{z}}{z^c}} \left(E_t \widehat{\beta}_{t+1} + E_t \widehat{y}_{t+1}\right) - \frac{\left(1-\overline{\overline{\beta}}\right)\left(1-\theta_p\left(1-\widetilde{mc}\frac{\widetilde{z}}{z^c}\right)\right)}{\widetilde{mc}\frac{\widetilde{z}}{z^c}} E_t \widehat{\widetilde{\rho}}_{t+1}, \\
\widehat{lv}_t &= \varepsilon_t^X - \frac{xc}{lv} \widehat{xc}_t, \\
\widehat{xc}_t &= \varsigma_2^X \left(\widehat{n}_t^X - \widehat{n}_t\right), \\
\widehat{n}_t^X &= \widehat{n}_t + \kappa \left(\frac{1-\delta_n}{\delta_n}\right) \widehat{z}_t^c,
\end{aligned}$$

where $\overline{\overline{\beta}} = \beta(n^A/n)$ is the deterministic discount factor adjusted by the steady state survival rate, $\widetilde{mc}$, $\widetilde{z}$, z^c, xc, and lv are the levels in the detrended steady state for the corresponding variables, and $\delta_n = n^X/n$ is the steady-state exit rate. The last relation of the 4-equation exit block explains that the number of exits exceeds that of the change in the number of varieties (exit rate rises) whenever $\widehat{z}_t^c$ is positive. And the observed value of critical productivity $\widehat{z}_t^c$ may rise, according to the first equation, in any of these six events: a higher expected critical productivity, $E_t\widehat{z}_{t+1}^c$, a higher expected marginal costs, $E_t\widehat{\widetilde{mc}}_{t+1}$, a higher current liquidation value relative to its next-period expected value, $\widehat{lv}_t - \overline{\overline{\beta}} E_t \widehat{lv}_{t+1}$, a lower discount factor, $E_t\widehat{\beta}_{t+1}$, a lower expected aggregate demand, $E_t\widehat{y}_{t+1}$, and a lower expected average relative prices, $E_t\widehat{\widetilde{\rho}}_{t+1}$. Therefore, exit depends on both supply-side conditions (the expected real marginal costs, the expected relative prices and the current and expected next-period liquidation values), as well as by elements of the aggregate demand (expected next-period output and the stochastic discount factor). There is also an exogenous component, ε_t^X, that can explain other factors for exit through its influence in the liquidation value.[15] The elasticity of the exit response increases with the shape parameter κ of the productivity Pareto distribution, and falls with the steady-state exit rate δ_n.

Total number of goods-firms

As assumed in Bilbiie *et al.* (2012) and many other papers with business formation, each firm is specialized in the production of a specific consumption good, which makes the number of good varieties coincide with the number of firms. At the beginning of a given period t, there are n_t varieties produced of (consumption) goods. At the end of period t, the productions of n_t^X varieties of goods shut down (exit), while the remaining n_t^A goods survive in the market, such that,

$$n_t = n_t^X + n_t^A. \tag{8}$$

In the meanwhile, n_t^E new goods are created during period t, though their lines of production will begin to operate in $t+1$. At the beginning of period $t+1$, the aggregate number of good varieties is determined

[15] For example, a change in corporate regulations that defends workers or creditors in business closings would be a negative realization of this exit shock. A deeper technological cliff between old and new firms may be another example of a negative shock on liquidation value.

by applying the endogenous survival rate, $\frac{n_t^A}{n_t}$, to both the active lines of production in period t and the newly created entries of goods. In formal terms, the law of motion for the total number of good varieties (or firms) is

$$n_{t+1} = \frac{n_t^A}{n_t}\left(n_t + n_t^E\right). \tag{9}$$

Banks

Loans are competitively supplied by private banks. In period t, the representative bank acts as a financial intermediary that issues an amount of real deposits backed by the central bank, dep_t, and use the proceeds to provide the quantity of real loans, $loan_t$. The production of loans uses a CES technology that combines the collateral service of aggregate equity, v_t, and banking labor, m_t, as follows:

$$loan_t = B[a\left(e^{\varepsilon_t^L}v_t\right)^{\chi} + (1-a)m_t^{\chi}]^{\frac{1}{\chi}}, \tag{10}$$

where $B > 0$ is the scale parameter, $-\infty < \chi \leq 1$ is the elasticity parameter, $0 < a < 1$ is the share coefficient of collateral, and ε_t^L is an exogenous collateral-augmenting AR(1) shock. The elasticity of substitution between banking inputs is constant at $\frac{1}{1-\chi}$.[16] The bank must serve a real interest rate of r_{t-1}^d per unit of real deposit used to finance the loans produced in period t, a collateral service real yield of csy_t per unit of equity used as guarantee, and a market real wage rate w_t per unit of labor hired. Revenues are raised by charging the real interest rate r_t^L per unit of real loan supplied to either households or firms. Hence, the optimizing program of the representative bank is

$$\begin{array}{lll} & \underset{m_t, v_t}{Max} & \left(r_t^L - r_{t-1}^d\right) loan_t - csy_t v_t - w_t m_t \\ s.to & & B\left[a\left(e^{\varepsilon_t^L}v_t\right)^{\chi} + (1-a)m_t^{\chi}\right]^{\frac{1}{\chi}} = loan_t \end{array}$$

which results in the following first order conditions

$$\left(r_t^L - r_{t-1}^d\right)\frac{(1-a)m_t^{\chi-1}loan_t}{a\left(e^{\varepsilon_t^L}v_t\right)^{\chi} + (1-a)m_t^{\chi}} - w_t = 0, \tag{m_t^{foc}}$$

$$\left(r_t^L - r_{t-1}^d\right)\frac{av_t^{\chi-1}e^{\chi\varepsilon_t^l}loan_t}{a\left(e^{\varepsilon_t^L}v_t\right)^{\chi} + (1-a)m_t^{\chi}} - csy_t = 0. \tag{v_t^{foc}}$$

The real interest rate on loans consistent with (m_t^{foc}) is

$$r_t^L = r_{t-1}^d + \frac{w_t m_t}{loan_t}\frac{a\left(e^{\varepsilon_t^L}v_t\right)^{\chi} + (1-a)m_t^{\chi}}{(1-a)m_t^{\chi}}, \tag{11}$$

[16] It should be noticed that in the upper bound, $\chi = 1$, the loan production function (10) converges to a linear function with infinite elasticity of substitution, whereas as χ approaches to its lower bound, the production function turns into a Leontief technology, with no substitutability (zero elasticity). Moreover, the Cobb-Douglas technology is particularized when χ approaches to 0 and there is a unit elasticity of substitution.

which has two components: i) the real interest rate paid for the deposit used to finance the loan, r^d_{t-1}, and ii) the real marginal cost of transforming the deposit into a loan, $\frac{w_t}{\partial loan_t/\partial m_t}$.

The semi-loglinear approximation to (11), using a loglinearized version of (10), is

$$r^L_t - r^L = \left(r^d_{t-1} - r^d\right) + \left(r^L - r^d\right)\left(\widehat{w}_t - \Omega\left(1-\chi\right)\left(\widehat{v}_t + \varepsilon^L_t - \widehat{m}_t\right)\right), \tag{12}$$

where $\Omega = \frac{av^\chi}{av^\chi + (1-a)m^\chi}$ is the steady-state share of collateral (equity) in loan production. The financial accelerator mechanism can be reflected through the inverse relation between fluctuations of equity, $\widehat{v}_t$, and the cost of loans, r^l_t, observed in (12).[17] There are two possible credit spreads in the model. First, the differential between the interest rates of borrowing and saving, $r^l_t - r^d_{t-1}$. Secondly, since households (as equity owners) collect a return from the collateral service of equity, this must be deducted from the direct spread to have a measure of collateralized credit spread. I chose the latter to define the external finance premium

$$efp_t = r^L_t - r^d_{t-1} - \tfrac{csy_t v_t}{loan_t}, \tag{13}$$

which in a semi-loglinear approximation yields

$$efp_t - efp = \left(r^L_t - r^L\right) - \left(r^d_{t-1} - r^d\right) - \tfrac{r^L - r^d - efp}{efp}\left(\tfrac{1}{csy}\left(csy_t - csy\right) + \widehat{v}_t - \widehat{loan}_t\right). \tag{14}$$

Back to the first order conditions of the bank, the collateral service of equity implied by (v^{foc}_t) turns out to be

$$csy_t = \left(r^L_t - r^d_{t-1}\right)\frac{loan_t}{v_t}\frac{a\left(e^{\varepsilon^L_t}v_t\right)^\chi}{a\left(e^{\varepsilon^L_t}v_t\right)^\chi + (1-a)m^\chi_t},$$

where plugging $r^L_t - r^d_{t-1}$ from the first order condition of banking labor derived above simplifies to the equilibrium collateral service yield,

$$csy_t = e^{\chi\varepsilon^L_t}w_t\frac{a}{(1-a)}\left(\frac{m_t}{v_t}\right)^{1-\chi}.$$

Central bank and government

Monetary policy is determined by a Taylor-type (1993) rule, similar to the one used in Smets and Wouters (2007), with the incorporation of unconventional actions in response to credit spreads. Particularly, there is an additional term that captures the negative-sign reaction of the central-bank interest rate to the deviations of the external finance premium with respect to its steady-state level, $efp_t - efp$, as defined in (14). Thus, the central bank adjusts the nominal interest rate to stabilize inflation, the output gap, the change in the output gap, and the external finance premium, with a partial-adjustment pattern that

[17] The reaction of r^l_t to a change in collateral (equity) is measured by the negative coefficient $-(r^l - r^d)\Omega\left(1-\chi\right)$, which implies that the amplifying effect of financial frictions is deeper with higher steady-state spreads, a higher steady-state collateral share, and a lower elasticity of substitution in loan production.

includes lagged nominal interest rate to smooth down monetary policy actions. All leads to the monetary policy rule

$$R_t - R = \mu_R (R_{t-1} - R) + (1-\mu_R) \left[\mu_\pi (\pi_t^c - \pi^c) + \mu_y (\widehat{y}_t - \widehat{y}_t^p) + \mu_{efp} (efp_t - efp)\right] + \mu_{\triangle y} (\triangle \widehat{y}_t - \triangle \widehat{y}_t^p) + \varepsilon_t^R. \tag{15}$$

where R_t is the nominal rate of interest of risk-free assets in period t while R is that in steady state, $(\pi_t^c - \pi^c)$ is the difference between current and steady-state rates of consumer price inflation (to be defined below), $\widehat{y}_t - \widehat{y}_t^p$ is the output gap between the cyclical component of output $(\widehat{y}_t)$ and its potential (natural-rate) realization $(\widehat{y}_t^p)$, $\triangle \widehat{y}_t = \widehat{y}_t - \widehat{y}_{t-1}$ is the change in log fluctuations of output (growth rate), $\triangle \widehat{y}_t^p$ is the same change for potential output, $efp_t - efp$ is the gap between the current external finance premium and its steady-state rate as determined in (14), and ε_t^R is a monetary policy shock.[18] Regarding the policy coefficients, $0 \leq \mu_R < 1$ is the usual policy smoothing parameter, $\mu_\pi > 1.0$ and $\mu_y > 0$ are the Taylor coefficients respectively for inflation and the output gap, $\mu_{efp} < 0$ brings the (unconventional) response of the central bank to fluctuations in the external finance premium, and $\mu_{\triangle y}$ captures the policy reaction to the change in the output gap. The central bank responds to consumer price inflation (CPI) rather than producer inflation to stabilize the real interest rate through the Fisher relation

$$1 + r_t = \frac{1 + R_t}{1 + E_t \pi_{t+1}^c},$$

that incorporates the expected CPI rate, π_{t+1}^c, because the bundles of consumption goods are valued at the consumer price index $P_t^c = \left(\int_0^{n_t} P_t(\omega)^{1-\theta_p} d\omega\right)^{\frac{1}{1-\theta_p}}$.

As for the role of the government, its fiscal policy consists of holding the budget constraint,

$$\varepsilon_t^g = t_t + \left(e^{\varepsilon_t^E} f_t^E n_t^E - e^{\varepsilon_{t-1}^E}(1-\varphi) f^E \left(n_{t-1}^X + \frac{n_{t-1}^X}{n_{t-1}} n_{t-1}^E\right)\right) + \frac{b_{t+1}}{1+r_t} - b_t + (dep_{t+1} - dep_t), \tag{16}$$

which implies that the exogenous spending on consumption goods, ε_t^g, is financed within the period by either collecting lump-sum taxes, t_t, by obtaining net revenues from selling operating licenses and reimbursing a fraction $(1-\varphi)$ to those that suspend their activity, $e^{\varepsilon_t^E} f^E n_t^E - e^{\varepsilon_{t-1}^E}(1-\varphi) f^E \left(n_{t-1}^X + \frac{n_{t-1}^X}{n_{t-1}} n_{t-1}^E\right)$, by issuing bonds b_{t+1} that yield the real return $(1 + r_t)$ in the equilibrium of the bonds market, and by adjusting the central-bank balance sheet to change the level of deposits in the banking system. Following Smets and Wouters (2007), the exogenous spending ε_t^g includes a cross effect provided by the innovations to the technology shock, as a way of capturing variability induced from technology shocks into net exports that cannot be pinned down endogenously in the closed-economy framework at hand.

Inflation dynamics

As also shown in the technical appendix, the optimal pricing a la Calvo (1983) and the aggregation with suboptimal prices that are adjusted with a price indexation rule lead to the following inflation equation

[18] Both government bonds and bank deposits are considered as risk-free assets that provide in equilibrium a nominal interest rate R_t.

(New Keynesian Phillips curve):

$$(\pi_t - \pi) = \frac{\iota_p}{(1+\beta\iota_p)}(\pi_{t-1} - \pi) + \frac{\beta}{(1+\beta\iota_p)}E_t(\pi_{t+1} - \pi) + \frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p(1+\beta\iota_p)}\left(\widehat{\widetilde{mc}}_t - \widehat{\widetilde{\rho}}_t\right) + \frac{(1-\iota_p)}{(1+\beta\iota_p)}\left(\varepsilon_t^p - \beta E_t\varepsilon_{t+1}^p\right), \tag{17}$$

which implies a hybrid response (backward-looking and forward-looking) of producer price inflation to the log difference between the real marginal cost and the relative prices, and also to price-push shocks.[19]

As shown in the technical appendix, the first order conditions of the optimizing program of the representative firm imply the following average real marginal cost

$$\widetilde{mc}_t = \frac{\left(1 + \tau_f r_t^L\right)(w_t)^{(1-\alpha)}\left(r_t^k\right)^\alpha}{(\alpha)^\alpha (1-\alpha)^{(1-\alpha)} e^{\varepsilon_t^a}\widetilde{z}e^{(1-\alpha)\gamma t}},$$

where $0 < \alpha < 1$ is the capital share coefficient of the Cobb-Douglas technology, $0 < \tau_f < 1$ is the fraction of firm variable costs that must be financed from the bank, r_t^k is the real rental rate of capital, ε_t^a is the economy-wide technology shock, and γ is the long-run rate of economic growth. In log-linear terms around the detrended steady state, it is obtained

$$\widehat{\widetilde{mc}}_t = \tau_f\left(r_t^L - r^L\right) + (1-\alpha)\widehat{w}_t + \alpha\widehat{r}_t^k - \varepsilon_t^a. \tag{18}$$

The real marginal cost (18) brings a financial channel on exit dynamics: a higher expected cost of borrowing, $E_t r_{t+1}^L$, raises the expected real marginal cost, $E_t\widehat{\widetilde{mc}}_{t+1}$, and the critical productivity, $\widehat{z}_t^c$, which would result in a higher $\widehat{n}_t^X$ in the exit block discussed above.

Regarding the relative prices, $\widetilde{\rho}_t = \widetilde{P}_t / P_t^c$, the Dixit-Stiglitz price index for the consumption bundle is $P_t^c = \left(\int_0^{n_t} P_t(\omega)^{1-\theta_p} d\omega\right)^{\frac{1}{1-\theta_p}}$ so that, in terms of the average price of the consumption goods is, $P_t^c = \left(n_t \widetilde{P}_t^{1-\theta_p}\right)^{\frac{1}{1-\theta_p}}$. This implies an average relative price $\widetilde{\rho}_t$ that increases with the total number of goods

$$\widetilde{\rho}_t = \frac{\widetilde{P}_t}{P_t^c} = n_t^{\frac{1}{\theta_p - 1}}, \tag{19}$$

which in log fluctuations with respect to steady-state writes

$$\widehat{\widetilde{\rho}}_t = (\theta_p - 1)\,\widehat{n}_t. \tag{20}$$

Remarkably, the dynamic equations for the real marginal cost (18) and the relative prices (20) incorporate unusual elements to the inflation dynamics of the New Keynesian Phillips curve (17). On the one hand, fluctuations of the real marginal cost depend upon the financial conditions. If the interest rate of loans rises the marginal cost of producing goes up and inflation will be higher. Tighter financial conditions are inflationary. On the other hand, (20) means that any positive deviation of the number of varieties with respect to its steady-state level would result in higher relative prices.

[19] Since the log fluctuations of the mark-up are given by $\widehat{\widetilde{\rho}}_t - \widehat{\widetilde{mc}}_t$, any increase (decrease) in the mark-up would bring inflation down (up).

What about consumer price inflation (that is one of the instrumental variables of the monetary policy rule)? Taking logs and the first difference in (19), and then using the definitions of the consumer and producer price inflation, I reach

$$(\pi_t^c - \pi) = (\pi_t - \pi) - (\theta_p - 1)^{-1} (\widehat{n}_t - \widehat{n}_{t-1}) . \tag{21}$$

As indicated in (21), the change in fluctuations of the number of varieties (inversely) explains the gap between the consumer price inflation and the producer price inflation. If there is an increase in the number of varieties relative to the previous period the rate of consumer price inflation will fall below producer price inflation.

Aggregation and general equilibrium

At the representative firm with average pricing, $\widetilde{P}_t$, the average output produced $\widetilde{y}_t$ is determined in the Cobb-Douglas technology,

$$\widetilde{y}_t = e^{\varepsilon_t^a} \widetilde{z} \widetilde{k}_t^{\alpha} \left(e^{\gamma t} \widetilde{l}_t \right)^{1-\alpha} ,$$

where $\widetilde{k}_t$ and $\widetilde{l}_t$ are, respectively, average capital and labor demands.

Inserting (19) in the average demand of intermediate goods, $\widetilde{y}_t = (\widetilde{\rho}_t)^{-\theta_p} y_t$, aggregate output (in terms of bundles of consumption goods) can be related to plant-level production as follows

$$y_t = n_t^{\frac{\theta_p}{\theta_p - 1}} \widetilde{y}_t . \tag{22}$$

Meanwhile, the intensive margin of activity is represented by firm-level average output, expressed in bundles of consumption goods as multiplied by relative prices,

$$\overline{y}_t = \widetilde{\rho}_t \widetilde{y}_t . \tag{23}$$

Combining (22) and (23), and inserting (19) in the result, bring the decomposition of aggregate output used in the empirical analysis of Section 2

$$y_t = n_t \overline{y}_t ,$$

where the extensive margin is the number of varieties of goods, n_t, and the intensive margin is firm-level average output, $\overline{y}_t$. The model closes with the equilibrium conditions for labor, capital, equity, loan, and deposit markets, which respectively, are

$$\begin{aligned}
l_t &= n_t \widetilde{l}_t + m_t, \\
k_t &= n_t \widetilde{k}_t, \\
x_t &= n_t, \\
loan_t &= \tau_f \left(w_t n_t \widetilde{l}_t + r_t^k n_t \widetilde{k}_t \right) + \tau_h n_t^E e^{\varepsilon_t^E} \left(f^E + ec_t \right), \\
dep_t &= loan_t,
\end{aligned}$$

while the equilibrium condition for the market of bundles of consumption goods (the overall resources constraint) is,[20]

$$y_t = c_t + i_t + a_t(u_t)k_t + \varepsilon_t^g + e^{\varepsilon_{t-1}^X} xc_{t-1}\left(n_{t-1}^X + \frac{n_{t-1}^X}{n_{t-1}} n_{t-1}^E\right) + n_t^E e^{\varepsilon_t^E} ec_t.$$

Regarding the exogenous variables of the model, there are seven AR(1) generating processes for the shocks of production technology, household preferences, investment adjustment costs, monetary policy, entry, exit, and banking collateral; two ARMA(1,1) processes for both price-push and wage-push shocks, and the fiscal shock is an AR(1) process with a cross correlation to technology innovations. All of them are listed in the technical appendix. Finally, natural-rate variables are required to implement the Taylor-type monetary policy rule (15). The potential variables are obtained by fully eliminating price and wage stickiness (i.e., by imposing both Calvo probabilities on price and wage setting be equal to zero).

The complete model with entry and exit can be written for short-run fluctuations as the log-linearized set of dynamic equations available in the technical appendix. The non-linear system of equations that determines the balanced-growth solution in steady state is also displayed there.

4 Estimation

Following the popular methodology used by Smets and Wouters (2003, 2007), the model has been estimated through Bayesian econometrics in Dynare. There are ten observable US quarterly series employed in the estimation procedure (to match the number of shocks of the model). The series are the rate of growth of real GDP per capita ($\triangle\widehat{y}$), the rate of growth of real personal consumption expenditures per capita ($\triangle\widehat{c}$), the rate of growth of real fixed private investment per capita ($\triangle\widehat{i}$), the rate of growth of hours of all persons in the nonfarm business sector per capita ($\triangle\widehat{l}$), the Wu and Xia (2014)'s shadow federal funds rate (R) that accommodates unconventional balance-sheet policy actions, the rate of producer price inflation obtained from the GDP price deflator (π), the rate of nominal wage inflation obtained from the index of compensation per hour in the nonfarm business sector (π^W), the spread of commercial and industrial loan rates over intended federal funds rate (efp), the rate of establishment entry (E), and the rate of establishment exit (X).[21] The nominal series were transformed into (constant-price) real series using the Personal Consumption Expenditures (chain-type) Price Index as the price deflator in order to be consistent with the way these series have been defined in the model. Per-capita series were computed dividing by the US civilian labor force adjusted by populational controls as released in the Current Population Survey

[20] Proof available in the Appendix.

[21] Data sources are the FRED database of the St. Louis Fed for real GDP, real consumption, real investment, total hours, price inflation and wage inflation; Wu and Xia (2014) for the nominal interest rate; the Board of Governors of the Federal Reserve System for the external finance premium; and the Business Employment Dynamics report of the BLS for establishment data used to compute entry and exit rates.

of the BLS (2013).[22] The observable series of $\triangle\widehat{y}$, $\triangle\widehat{c}$, and $\triangle\widehat{i}$ have a common constant value in the measurement equation at the estimated steady-state rate of growth γ, whereas $\triangle\widehat{l}$ exhibits no long-run growth. The estimated steady state rate of inflation, π^{ss}, is the long-run component of price inflation, while that component for wage inflation is $\pi^{ss}+\gamma$. The estimated steady-state nominal interest rate is $(1+r^{ss})(1+\gamma)^{-\sigma_c}+\pi^{ss}-1$ and the estimated steady-state external finance premium can be found in the steady-state non-linear system displayed in the technical appendix.[23] The equations that incorporate the counterparts of the effective entry and exit rates in the semi-loglinear model are[24]

$$E_t = E^{ss} + E^{ss}\left(\widehat{n}^E_{t-1} + \widehat{n}^A_{t-1} - \widehat{n}_t - \widehat{n}_{t-1}\right),$$

$$X_t = X^{ss} + X^{ss}\left(\widehat{n}^X_t - \widehat{n}_t\right),$$

where $E^{ss} = X^{ss} = \delta_n$.

The sample period runs from the first quarter of 1993 to the last quarter of 2012, due to data availability on establishment entry and exit (recall discussion in Section 2). Some of the model parameters were calibrated to be fixed during the estimation. In particular the rate of capital depreciation is set at the standard value of $\delta_k = 0.025$ (10% per year), and the steady-state exit rate is at $\delta_n = 0.03$ (12% per year) to match the historical average found in US establishment exit data. Both the scale parameters of the entry and exit cost functions are set at the numerical value that renders a steady-state cost of either entry and exit equivalent to 0.75% of output. It leads to $\varsigma^E_1 = 1.24 * 10^8$ and $\varsigma^X_1 = 6.06 * 10^8$. The value of the share of sunk costs at entry/exit, the parameter τ, is calibrated to imply that the total number of goods in steady state is normalized at $n = 1$, which results in $\varphi = 0.55$. The elasticity of substitution across labor services is fixed at $\theta_w = 4.0$. The external finance premium in steady state is fixed at a value of $efp = 0.005$ (2% per year) to approximate the average observed in the series of credit spread used in the estimation (0.0053). The steady-state ratio of exogenous spending over aggregate output is set at 0.18 as in Smets and Wouters (2007). Finally, in the loan production technology the share parameter a takes the value that corresponds to a steady-state collateral share $\Omega = 0.65$ (65%) as assumed in Goodfriend and McCallum (2007).

The priors and posterior estimates are reported in Tables 3A and 3B. The selection of priors has been done looking at numbers chosen in the related literature. I will only comment on some special cases because their parameterization is either different or new in the field. The hours elasticity in the utility function is assumed to have a high value at the estimation prior ($\sigma_l = 4.0$) to provide a low labor supply elasticity

[22] Quoting BLS (2013): "This research series reflects seasonally adjusted Current Population Survey labor force levels that have been smoothed to minimize the effects of level shifts from population control adjustments in the official series in January 2000 and January of 2003–2015. ".

[23] All the transformed series of observables for the estimation are available upon request.

[24] As a result of applying loglinearizing techniques on the definitions of entry rate $E_t = \frac{\frac{n^A_{t-1}}{n_{t-1}} n^E_{t-1}}{n_t}$ and exit rate $X_t = \frac{n^X}{n_t}$. Hence, the effective entry rate in the model, E_t, only considers the share of desired entries that successfully start operating in the market because they survive in the period of business creation.

as found in most empirical evidence (Chetty, 2012). The (Dixit-Stiglitz) elasticity of substitution takes a prior lower than the number assumed in other studies due to the technical restriction that $\kappa > \theta_p - 1$ to define a constant firm-level average productivity from the Pareto distribution. I assume $\theta_p = 3.8$ as the preferred calibration in Bilbiie *et al.* (2012). In the banking technology for loan production, elasticity of substitution across inputs comes along with an initial value of 0.5 as I set a prior of χ at -1.0. The needs for external finance of both households and firms are determined by the share parameters τ_h and τ_f respectively, which take a median value 0.5 with a moderate standard deviation (0.15). The monetary policy (Taylor-type) rule incorporates the reaction to the external finance premium with a loose coefficient at a prior value of -0.25 with a high standard deviation of 0.15. The elasticities of the variable costs of entry and exit are both assumed with the prior for the estimation at 4.0 and a high standard deviation (1.5) due to parameter uncertainty and lack of previous references. Finally, the parameter that determines the shape of the Pareto distribution (and the elasticity of the exit rate) takes a high prior $\kappa = 5.0$ to meet the technical requirement mentioned above and comes with a high standard deviation to allow for a wide search.

Some of the posterior estimates deviate significantly from the priors. The capital share in loan production is lower with a mean value at 0.17, which might be the consequence of ignoring fixed costs or introducing good varieties. Both Calvo probabilities are close to the upper bound, 0.93 for price stickiness and 0.88 for wage stickiness, reflecting the little volatility of US inflation over the sample period (0.21%, less than one third of that of output growth). The indexation parameters on both price and wage inflation are estimated at small numbers (0.33 on prices and 0.31 on wages) that indicate a predominance of the forward looking pattern on inflation dynamics. The share on the external finance requirement for firms spending is estimated much lower ($\tau_f = 0.24$) than the one for the household spending on business creation ($\tau_h = 0.80$). Both estimated elasticities of the entry and exit costs are higher than their priors at 6.51 and 6.90 respectively. The posterior mean estimate for the parameter that rules the input substitutability in banking is -5.63, which implies a low elasticity of (1/(-5.63-1))=0.15.

Regarding the exogenous processes, the most persistent shocks are the technology shock, the autonomous spending shock, the entry cost shock, the exit cost shock, and the financial shock, with estimated coefficients of autocorrelation around 0.9. These five shocks will take the highest percentages in the model variance decomposition discussed below. The standard deviations of the innovations of the shocks are also reported in Table 3B and their values do not inform well on the relative influence because they enter the dynamic equations of the model multiplied by different coefficients.

Table 3A. Estimates of structural parameters.

	Priors			Posteriors		
	Distr	Mean	Std D.	Mean	5%	95%
α, capital share in production	Normal	0.36	0.10	0.17	0.11	0.24
h, consumption habits	Beta	0.70	0.15	0.68	0.60	0.78
σ_c, consumption elasticity	Normal	1.50	0.50	1.12	0.81	1.40
σ_l, hours elasticity	Normal	4.00	1.00	4.80	3.42	6.41
φ_k, investment adj. costs elast.	Normal	4.00	1.50	6.32	4.42	8.34
$\widetilde{\sigma}_a$, capacity utilization costs elast.	Beta	0.50	0.15	0.87	0.78	0.97
ξ_p, Calvo sticky prices	Beta	0.50	0.15	0.93	0.91	0.96
ξ_w, Calvo sticky wages	Beta	0.50	0.15	0.88	0.85	0.91
ι_p, price indexation	Beta	0.50	0.15	0.33	0.16	0.50
ι_w, wage indexation	Beta	0.50	0.15	0.31	0.11	0.50
θ_p, elasticity of goods substitution	Normal	3.80	0.50	2.67	2.32	3.06
μ_R, interest-rate smoothing	Beta	0.75	0.10	0.79	0.74	0.85
μ_π, inflation response	Normal	1.50	0.25	1.73	1.45	2.00
$\mu_{\widetilde{y}}$, output gap response	Normal	0.12	0.05	0.05	0.03	0.07
$\mu_{\triangle y}$, change in output gap response	Normal	0.12	0.05	0.18	0.14	0.23
μ_{efp}, credit spread response	Normal	-0.25	0.15	-0.24	-0.50	-0.01
χ, banking elasticity	Normal	-1.00	1.50	-5.63	-7.26	-3.78
τ_f, firm financial need	Beta	0.50	0.15	0.24	0.06	0.40
τ_h, household financial need	Beta	0.50	0.15	0.80	0.68	0.91
ς_2^E, entry costs elasticity	Normal	4.00	1.50	6.51	4.73	8.14
κ, shape of Pareto distribution	Normal	5.00	1.50	5.56	3.80	7.26
ς_2^X, exit costs elasticity	Normal	4.00	1.50	6.90	5.50	8.24
π^{ss}, steady-state inflation	Normal	0.50	0.10	0.53	0.36	0.69
$100(\beta^{-1}-1)$, rate of intert. preference	Gamma	0.25	0.10	0.18	0.08	0.28
γ, long-run rate of growth	Normal	0.35	0.10	0.32	0.25	0.39

Table 3B. Estimates of parameters shaping the exogenous processes.

	Priors			Posteriors		
	Distr	Mean	Std D.	Mean	5%	95%
σ_a, std dev technology innovation	Invgamma	0.10	2.00	0.79	0.64	0.95
σ_b, std dev preference innovation	Invgamma	0.10	2.00	1.84	1.23	2.41
σ_R, std dev monetary innovation	Invgamma	0.10	2.00	0.15	0.12	0.17
σ_i, std dev investment innovation	Invgamma	0.10	2.00	0.40	0.30	0.50
σ_g, std dev fiscal/NX innovation	Invgamma	0.10	2.00	2.75	2.33	3.19
σ_P, std dev price-push innovation	Invgamma	0.10	2.00	0.33	0.24	0.44
σ_W, std dev wage-push innovation	Invgamma	0.10	2.00	1.38	0.96	1.85
σ_E, std dev entry cost innovation	Invgamma	0.10	2.00	1.26	0.98	1.52
σ_X, std dev exit cost innovation	Invgamma	0.10	2.00	1.54	0.74	2.37
σ_L, std dev financial innovation	Invgamma	0.10	2.00	6.77	5.76	7.69
ρ_a, autocorrelation of technology shock	Beta	0.50	0.20	0.88	0.85	0.92
ρ_b, autocorrelation of preference shock	Beta	0.50	0.20	0.52	0.33	0.73
ρ_R, autocorrelation of monetary shock	Beta	0.50	0.20	0.15	0.04	0.25
ρ_i, autocorrelation of investment shock	Beta	0.50	0.20	0.69	0.54	0.85
ρ_g, autocorrelation of fiscal/NX shock	Beta	0.50	0.20	0.89	0.80	0.97
ρ_P, autocorrelation of price-push shock	Beta	0.50	0.20	0.69	0.49	0.88
ρ_W, autocorrelation of wage-push shock	Beta	0.50	0.20	0.27	0.07	0.49
ρ_E, autocorrelation of entry cost shock	Beta	0.50	0.20	0.91	0.87	0.94
ρ_X, autocorrelation of exit cost shock	Beta	0.50	0.20	0.91	0.86	0.95
ρ_L, autocorrelation of financial shock	Beta	0.50	0.20	0.90	0.86	0.94
μ_P, moving average of price-push shock	Beta	0.50	0.20	0.45	0.24	0.67
μ_W, moving average of wage-push shock	Beta	0.50	0.20	0.54	0.36	0.73
ρ_{ga}, cross correlation of fiscal/NX shock	Beta	0.50	0.20	0.58	0.29	0.88

5 Business cycle analysis

Since this paper is mostly focused on the role of entry and exit, I will discuss the second-moment statistics of output fluctuations and the entry-exit rates. Table 4 shows the numbers:

Table 4. Second-moment statistics

	$\Delta\widehat{y}$	$\Delta\widehat{\overline{y}}$	$\Delta\widehat{n}$	E	X
U.S. data, 1993:1-2012:4					
Std deviation relative to $\Delta\widehat{y}$	1.0	0.76	0.51	0.28	0.30
Corr. with $\Delta\widehat{y}$	1.0	0.87	0.67	0.43	-0.62
Autocorrelation	0.46	0.13	0.87	0.82	0.83
Estimated model:					
Std deviation relative to $\Delta\widehat{y}$	1.0	0.87	0.50	0.37	0.35
Corr. with $\Delta\widehat{y}$	1.0	0.87	0.69	0.33	-0.44
Autocorrelation	0.38	0.01	0.89	0.91	0.90

The matching between the second-moment statistics of US quarterly data over the sample period 1993-2012 and the numbers obtained in the simulations of the estimated model is reasonably good. Volatilities are measured as the relative standard deviation with respect to the quarterly rate of growth of output, which corresponds to the observable series $\Delta\widehat{y}$ used in the estimation. The model captures a lower variability in the rate of change of the extensive margin ($\Delta\widehat{n}$) than in that of the intensive margin ($\Delta\widehat{\overline{y}}$), with some excessive volatility in the intensive margin. Entry and exit rates report standard deviations of around one third of that of output growth, with numbers slightly higher in the model than in the data. The matching of cross correlations with $\Delta\widehat{y}$ is remarkably precise. The intensive margin is strongly procyclical, with a correlation with changes in aggregate output at 0.87 both in the data and in the model. The extensive margin also shows an important positive comovement between the growth on establishments (goods) and aggregate output growth, with a cross correlation of 0.67 in the data and 0.69 in the model. Meanwhile, entry rates (E) are moderately procyclical (0.43 correlation in the data and 0.33 correlation in the model), and exit rates (X) are somehow more intensively countercyclical in the data (-0.62) than in the model (-0.44). Finally, the persistence is high for entry rates, exit rates and the rates of growth of total number of goods-establishments, whereas aggregate output growth has a low inertia and firm-level output growth virtually none. The estimated model and the data report similar figures on these estimates of autocorrelations (see Table 4).

5.1 Variance decomposition

How are these fluctuations sorted out from the sources of their variability? The estimated DSGE model provides ten innovations to the exogenous processes that can explain the variability of the endogenous variables. I have compiled the long-run variance decomposition in Table 5 for $\Delta\widehat{y}$, $\Delta\widehat{\overline{y}}$, $\Delta\widehat{n}$, E and X, where shocks have been grouped in four categories: supply, demand, entry-exit and financial. The results document that fluctuations of economic activity in the US are the combination of the effects of distinct sources of variability. Thus, technology innovations drive 30% of variability of output growth, price and wage push shocks over 6%, demand-side shocks (especially, fiscal/net exports innovations) take other 30% all together, shocks on entry and exit also nearly explain 30%, and the remaining 4% is due to financial shocks. When it comes to adjustments in the intensive margin, the role of demand shocks is higher, with a variance share at over 36% whereas entry-exit costs are less influential (22%). The opposite is found in the extensive margin, where exogenous variations of entry and exit drive 47% of the variability in the change of the total number of goods and demand shocks only explain 6.3% of it.[25]

Table 5. Variance decomposition, %

	$\Delta\widehat{y}$	$\Delta\widehat{\overline{y}}$	$\Delta\widehat{n}$	E	X
Supply shocks:					
Technology, η^a	30.1	32.2	28.8	20.1	8.1
Price-push, η^P	2.5	3.0	1.8	0.8	5.7
Wage-push, η^W	3.9	2.2	9.6	2.7	8.0
Demand shocks:					
Interest-rate, η^R	7.5	8.9	2.9	6.1	0.2
Preference, η^b	5.0	5.3	1.3	2.3	0.1
Investment, η^i	3.5	5.1	1.1	3.7	0.7
Fiscal/Net exports, η^g	14.2	17.1	1.0	2.1	0.2
Entry-exit shocks					
Entry cost, η^E	8.5	7.5	9.8	46.6	12.1
Exit cost, η^X	20.8	14.8	37.0	0.8	64.0
Financial shocks:					
Collateral, η^L	4.2	3.8	6.9	14.9	1.1

The variability of the entry rate mostly depends on its idiosyncratic shock (46.6%), though technology shocks are responsible for 20% of the variations on E and demand shocks take 14.2% of it. Remarkably, the financial shocks on banking collateral take almost 15% of the variance decomposition of the entry rate, due to its straight influence in the interest rate of loans that determines the financial cost of good creation. As for the exit rate, nearly 2/3 of its overall variability comes explained by the liquidation (exit) cost, whereas shocks on pricing and wage setting jointly take 13.7% of this variance due to its influence on expected dividends. Demand shocks are barely influential in the exit rate (in contrast to their significant role on entry rate variability).

[25] In an estimated model for the *Great Moderation* period, Casares and Poutineau (2014) also find that demand-side shocks are mostly absorbed through the intensive margin, whereas the extensive margin plays a greater role in the responses to supply-side shocks.

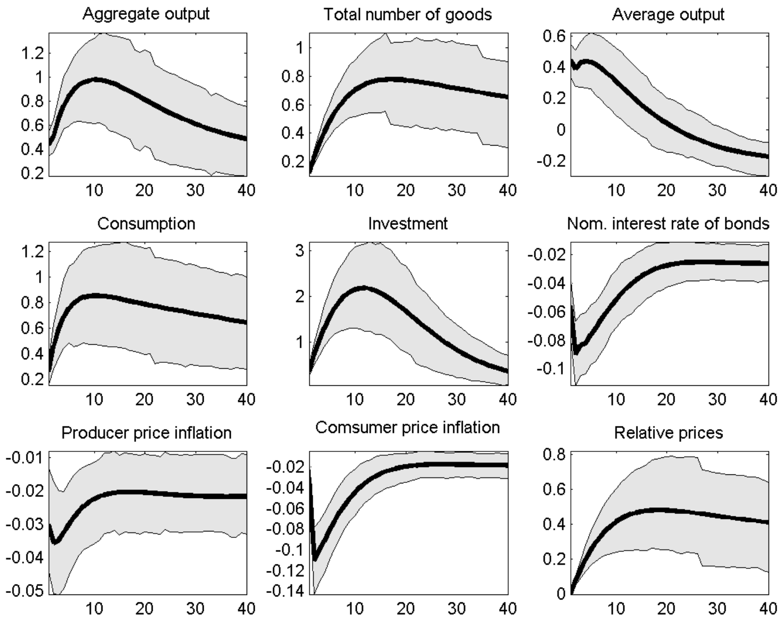

Figure 5: Technology shock, η^a.

5.2 Some impulse responses

Using the latest Dynare version available, I ran Bayesian estimation of impulse-response functions in the model. Four of them are discussed here, whereas the other six are displayed in the technical appendix due to space limitations.

Figures 5-12 exhibit the responses of some of the endogenous variables of the model to a one standard deviation estimated innovation to the technology shock in Figures 5-6, the entry cost shock in Figures 7-8, the liquidation (exit) shock in Figures 9-10, and the banking collateral (financial) shock in Figures 11-12. All the plots show percent deviation for the balanced-growth path in the non-stationary variables and direct difference with respect to the steady-state rate in the stationary variables.

As Figure 5 indicates, a positive technology shock brings net business formation. The total number of goods slowly rises to reach a peak value around 15 quarters after the shock. It makes the aggregate response of output be larger and more persistent. By contrast the average output (the amount produced in a single firm with average productivity) responds immediately after the shock and returns to trend level as the number of varieties of goods produced in the economy picks up. Actually, 20 quarters after the shock the average size of production turns lower than its steady-state level due to the increase in the number of operating firms. The higher productivity makes producer price inflation fall due to lower real marginal costs of production. Consumer price inflation decreases further than producer inflation as a result of a greater number of goods-competitors (see equation 21). Then, the nominal interest rate of bonds (and deposits) moves down when the central bank implements the monetary policy rule (15). Both consumption

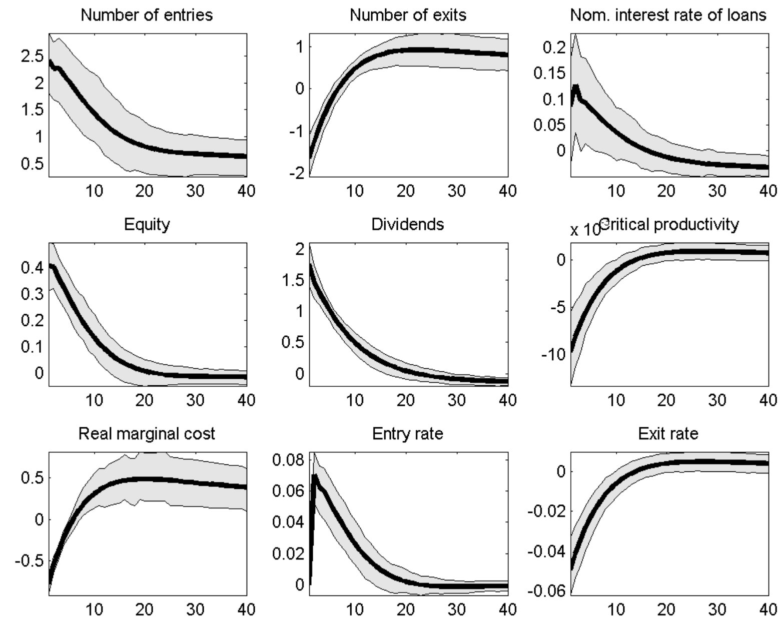

Figure 6: Technology shock, η^a, cont'd.

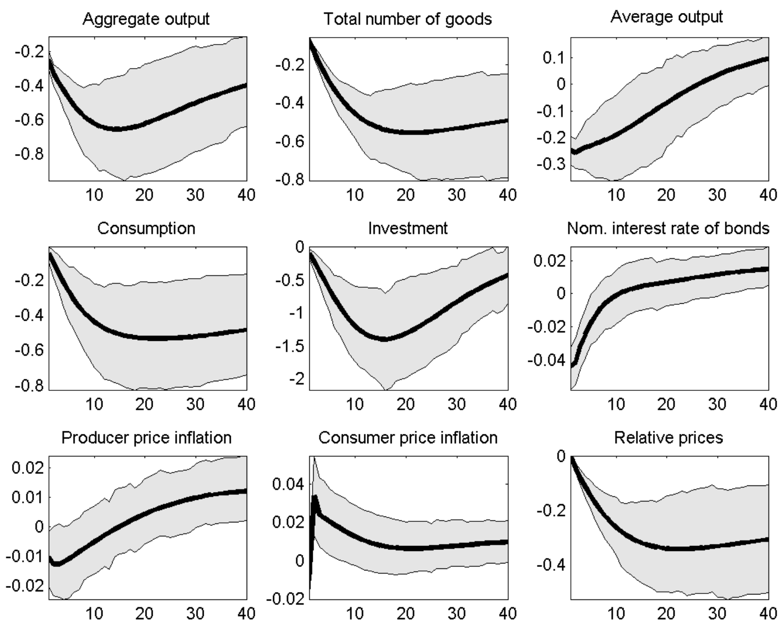

Figure 7: Cost of entry shock, η^E.

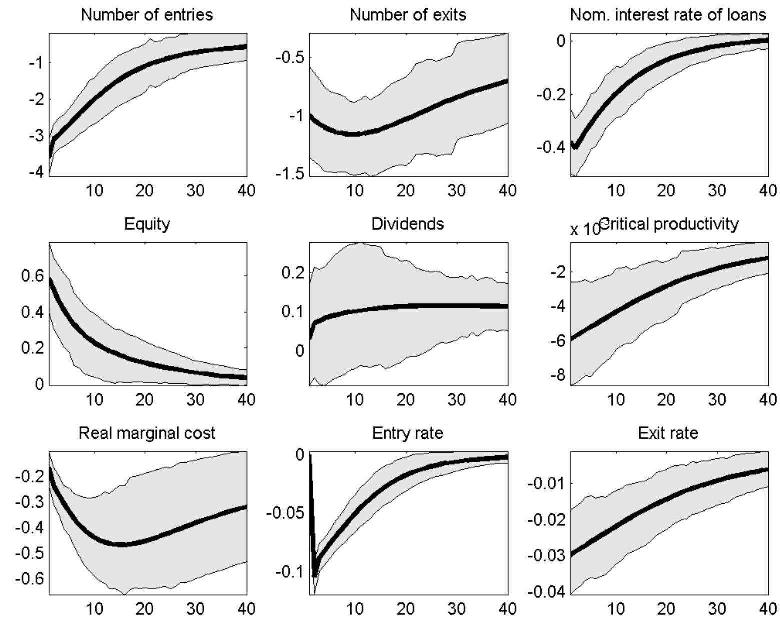

Figure 8: Cost of entry shock, η^E, cont'd.

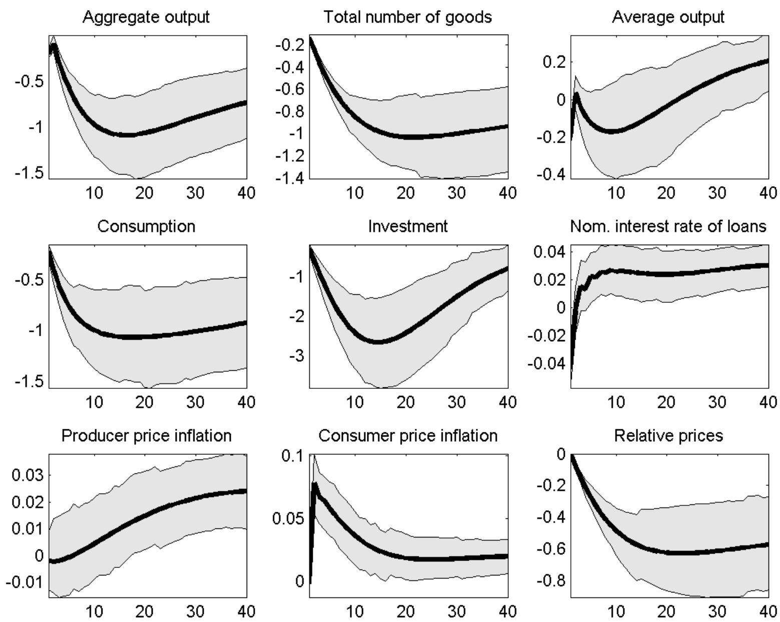

Figure 9: Exit (liquidation value) cost, η^X.

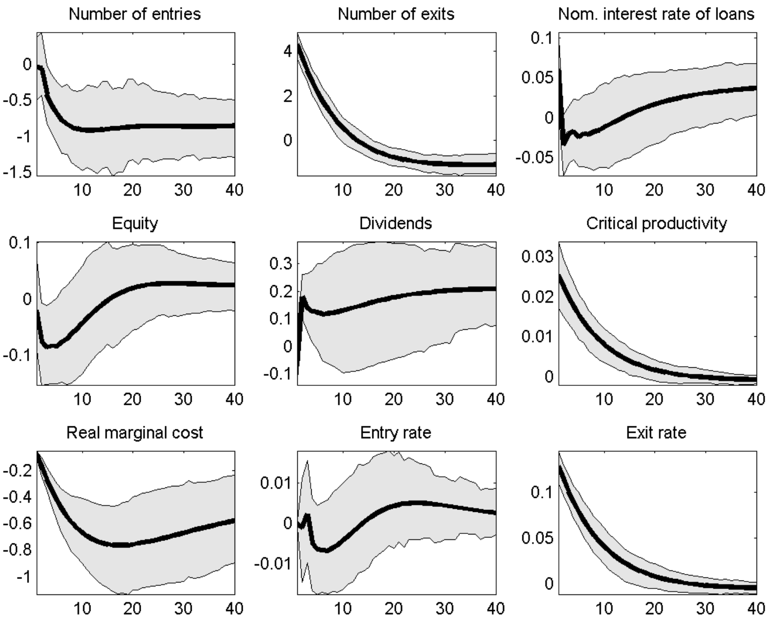

Figure 10: Exit (liquidation value) cost, η^X, cont'd.

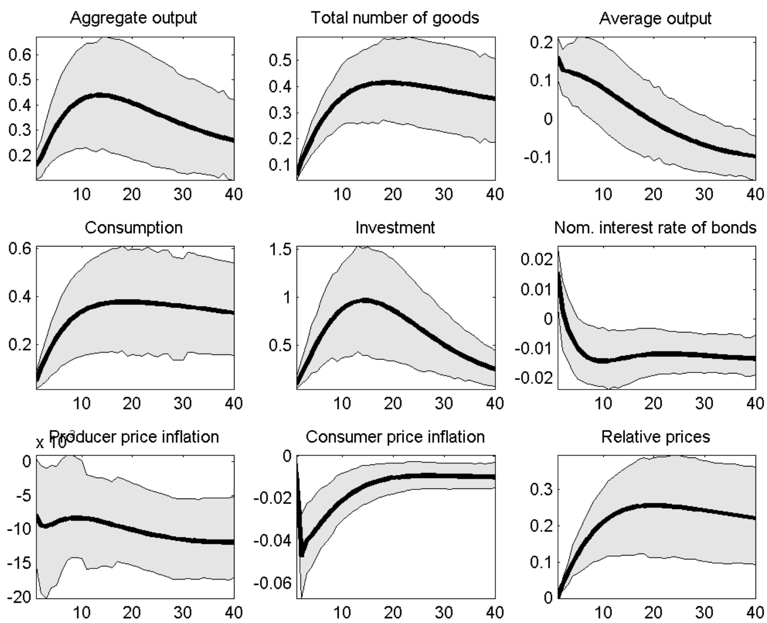

Figure 11: Financial (collateral-augmenting) shock, η^L.

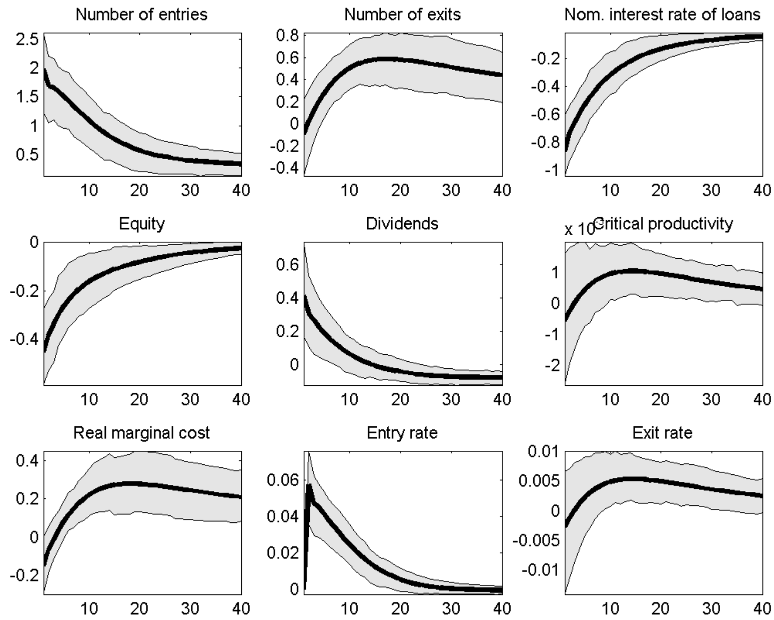

Figure 12: Financial (collateral-augmenting) shock, η^L, cont'd.

and investment spending increase as the real interest rate falls and the marginal product of capital goes up.

Figure 6 displays the way entry and exit respond to the technology shock. The number of new varieties rises because the average equity value is higher with increasing firm dividends and also the discounting interest rate is lower. This result is observed in spite of a higher cost of borrowing that would discourage the spending on creating new goods. The nominal interest rate of loans rises because the demand for credit pushes it up to cover the greater bills on wages, rental capital and entry costs. The fall in the number of exits amplifies the increase in the total number of goods. The persistence of the rise in total factor productivity makes the expected real marginal cost of production fall and the expected aggregate output increase. In turn, the critical productivity threshold gets down and the exit rate turns lower to observe fewer firm closings. Around ten quarters after the shock the real marginal cost turns higher than its trend value and the number of exits switches from negative to positive.

Figures 7 and 8 contain plots of the responses to an unexpected increase in the cost of entry. This brings an aggregate economic contraction mostly explained by the drop in the total number of goods. Average (firm-level) output drops at the quarter of the shock (let me recall that the installation of new firms takes one period in which they do not operate), and slowly returns to its trend level in 15-20 quarters. The economic downturn lowers interest rates and producer price inflation, with an associated fall of input demand and the real marginal cost. Consumer price inflation slightly rises because of the variety effect (equation 21 indicates the way a lower number of goods increases consumer inflation). As Figure 8 shows, the 3% initial fall in the number of entries explains the reduction of the number of goods while the number of exits only falls at a lesser extent (around 1%). Fewer firms shut down because declining demand lowers the expected marginal cost of production and critical productivity moves down. Lower interest rates push up the average equity value to buffer down the fall of entries. The reduction in the nominal interest rate of loans also helps for the recovery in the number of entries. However, the persistence of this shock (autocorrelation at a mean estimate of 0.91) explains that 20-30 quarters after the shock the number of entries is still on the recovery track for its trend level.

As discussed in Section 3, a positive shock on the liquidation value shifts to the same direction both the critical productivity, z^c, and the exit rate, $X = n^X/n$. This is why I may refer to the innovations of the liquidation value shock, ε^X, as an exit cost shock, with the required note that the higher liquidation value in equation (5) is equivalent to a reduction in the exit cost that increases z^c and the number of exits. Actually, there might be elements involved in the decision of firm continuation or liquidation that can be captured by this exogenous component. For examples, a regulatory change that facilitates the closing down of businesses (e.g., lower legal firing costs) or a swing in the sentiment for entrepreneurship due to a worsening in the perception of aggregate risk may bring exogenous exits in the model. Thus, Figure 9 illustrates how a positive exit cost shock has clear contractionary effects on aggregate output, consumption, and investment. Average production of incumbent firms moves erratically with an initial little fall that is quickly eliminated as the reduction in the number of firms kicks in. Despite a decline in the real marginal cost of production, the exit shock is inflationary on both producer and (especially) consumer prices because the lower number of varieties brings down good substitutability and raises inflation. The nominal interest rate of bonds falls at the quarter of the shock but hikes up to the positive side afterwards due to the central bank reaction to inflation.

Unlike what was observed in the case of an entry shock, Figure 10 shows that the exit shock results in opposed reactions of entry and exit, which reinforces the overall fall in the total number of goods. Households decide to invest less in good creation because average equity falls. The effects of the shock are very persistent for the number of exits, aggregate output, the number of goods or consumption due to the high estimated autocorrelation (0.91).

Finally, Figures 11-12 plot the responses of an expansionary financial shock. The amount of banking collateral exogenously rises in the loan production technology and banks can give credit to households and firms at a lower cost. This reduces the interest rate of loans which brings expansionary effects on both demand and supply. Regarding the supply side, the marginal cost of production depends on the cost of borrowing and thus moves down with the financial push. In turn, firms charge lower prices and both producer and consumer price inflation fall. On the demand side, the lower interest rate of loans stimulates spending on good creation and the higher number of entries raises both the total number of goods and aggregate output. Consumption and investment spending also rise. In the estimated impulse-response functions the fall of inflation is quantitatively small due the low estimate of the external finance requirement of the firms ($\tau_f = 0.24$). Exit responds to the easing of financial conditions with a hump-shaped increase. Marginal costs and the productivity threshold move quickly from negative values to the positive side which raises the exit rate. Nevertheless, the effects of the financial shock on entry are much more significant than the responses observed on exits (as documented in the variance decomposition of the entry and exit rates reported in Table 5).

In sum, the analysis of the variance decomposition and the estimated impulse-response functions show the important role of both entry and exit as a propagation channel from idiosyncratic shocks to aggregate fluctuations through the variability in the total number of shocks. In addition, shocks on entry and exit dynamics bring substantial effects on aggregate output, consumption and investment. These results contradict those of Samaniego (2008) where the role of entry and exit for business cycle fluctuations in a US calibrated model with managerial labor is found to be quantitatively very small.

6 The role of entry and exit in the latest boom-bust cycle

Section 2 of this paper provides evidence on how the extensive margin of economic activity (total number of goods-establishments) takes much of the variability observed in aggregate fluctuations of US real GDP from 2003 to 2012. So far, all the results discussed refer to the estimated model over a sample period that begins in 1993 and ends in 2012. In this section, the attention is paid to its second subsample, 2003-2012, a period of increasing macroeconomic volatility and where the role of business creation and destruction is really noticeable in the data.

Figure 13 plots the de-meaned quarterly rate of growth of US real GDP per capita (the observable series, $\Delta\widehat{y}$, minus the estimated long-run rate of growth, γ) and provides vertical colorful bars with the estimated contribution of the idiosyncratic shocks of the model. The aggregate volatility increases from 2007 onwards, when the housing bubble bursts within the turbulences of the financial crisis. I have examined the leading shocks during the quarters of the economic crisis (2007:4-2009:3, when all the observations of $\Delta\widehat{y} - \gamma$ are negative) and the next nine quarters of irregular economic recovery (2009:4-2011:3, when all the quarterly observations of $\Delta\widehat{y} - \gamma$ are positive except for two of them).

Table 6 reports the individual contribution of each shock to the financial crisis and the mean value over the 8-quarter period. The exit shock, the entry shock and the financial shock are the three main contributors to the recession, which explain the big slump in entries (births of US private establishments fall from 216,000 in 2007:3 to 169,000 in 2009:3), the enormous increase in business exits (deaths of US private establishments rise from 193,000 in 2007:1 to 238,000 in 2008:3), and the soaring of the external finance premium (the observed credit spread rises from 1.92% per year in 2007:3 to 3.25% per year in 2009:3). The numbers shown in Table 6 indicate quarterly mean contributions for growth at -0.55%, -0.42% and -0.37% on the exit, entry and financial shocks respectively. The collapse in the housing market is also reflected by a -0.29% average quarterly effect of the investment shock on US growth. Wage-push

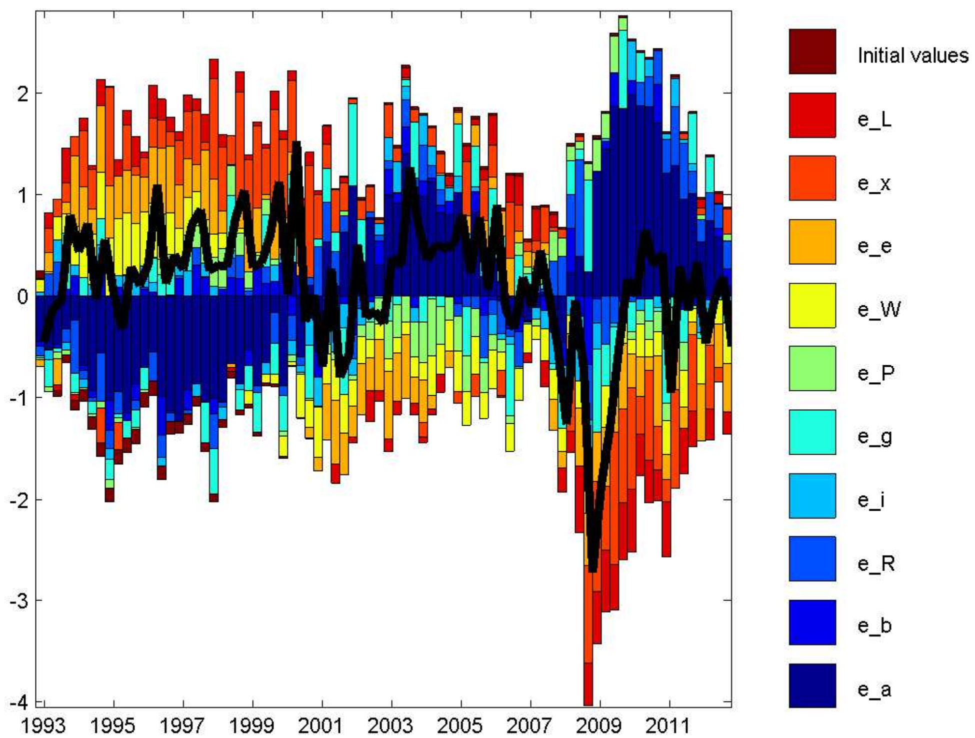

Figure 13: Rate of growth (%) of quarterly US real GDP per capita.

shocks (-0.20%) and fiscal-net exports shocks (-0.13%) also explain the economic recession. By contrast, price-push shocks (+0.22%) and, especially, technology shocks (+0.59%) provide variability that pushes for gains of economic growth. The latter is mostly explained by the contribution of quarters 1 to 3 of 2009 when the innovations on technology shocks contributed for more than 1% quarterly output growth.

Table 6. Shock decomposition during the US economic recession (2007:4-2009:3), %

Innovations	07:4	08:1	08:2	08:3	08:4	09:1	09:2	09:3	Mean	Rank^{-}
Technology, η^a	-0.14	-0.46	0.22	0.38	0.23	1.22	1.44	1.86	0.59	
Preference, η^b	-0.22	-0.23	0.44	-0.20	-0.68	-0.03	0.07	0.33	-0.06	7
Interest-rate, η^R	0.34	0.38	0.33	0.96	-0.54	-0.23	-0.48	-0.27	0.06	
Investment, η^i	-0.31	-0.20	-0.20	-0.64	-0.69	-0.26	0.03	-0.04	-0.29	4
Fiscal/NX, η^g	-0.06	-0.42	0.33	0.10	0.91	-0.82	-0.41	-0.65	-0.13	6
Price-push, η^P	0.29	0.19	0.15	0.08	0.15	0.31	0.25	0.37	0.22	
Wage-push, η^W	-0.29	-0.22	-0.18	-0.26	-0.23	0.02	-0.41	-0.03	-0.20	5
Entry, η^E	-0.31	-0.16	-0.35	-0.58	-0.51	-0.48	-0.58	-0.37	-0.42	2
Exit, η^X	0.03	0.08	-0.61	0.05	-0.96	-1.09	-0.62	-1.29	-0.55	1
Financial, η^L	0.13	-0.24	-0.21	-0.65	-0.42	-0.51	-0.61	-0.46	-0.37	3
Demeaned output growth	-0.50	-1.26	-0.05	-0.74	-2.71	-1.85	-1.29	-0.52	-1.12	

The numbers contained in Table 7 changes significantly with respect to those of Table 6. The economic recovery is mostly explained by technology shocks. The estimated innovations on Total Factor Productivity (TFP) rise during the recession and remain clearly above-trend for the quarters that come after the crisis. The numbers range between 0.94% and 1.97%, with a mean quarterly value at 1.52%. There are only three other model disturbances that contribute positively for the economic recovery: interest-rate shocks, investment shocks, and preference shocks. The quantitative easing of monetary policy explains an average impact on quarterly growth at +0.25%, with large contributions in 2011:2 and 2011:3 (+0.58% and +0.55%, respectively). Both investment shocks and preference shock have minor positive influence on growth with quarterly means at +0.09% and +0.01% respectively, which indicates that private-sector autonomous spending is not leading the recovery. The shocks that explained the economic crisis are still latent in this period, with negative impact on economic growth. Thus, the exit shock brings a -0.61% effect per quarter, the financial shock -0.41% and the entry shock -0.36% in the same magnitudes of quarterly means. Therefore, TFP shocks are really positive and high to compensate for the negative influence of business formation (entry-exit) and the adverse financial shock that raises the cost of borrowing.

Table 7. Shock decomposition during the US economic recovery (2009:4-2011:4), %

	09:4	10:1	10:2	10:3	10:4	11:1	11:2	11:3	11:4	Mean	Rank$^+$
Technology, η^a	1.84	1.97	1.87	1.86	1.70	1.21	1.28	0.94	1.00	1.52	1
Preference, η^b	-0.05	0.14	-0.03	-0.04	0.38	-0.04	-0.02	0.14	-0.10	0.01	4
Interest-rate, η^R	-0.22	-0.06	0.23	0.29	0.33	0.33	0.58	0.55	0.21	0.25	2
Investment, η^i	0.00	0.39	-0.12	0.16	-0.11	0.06	0.27	0.04	0.08	0.09	3
Fiscal/NX, η^g	0.76	-0.30	0.29	-0.22	-0.04	-1.02	-0.10	-0.31	0.49	-0.05	
Price-push, η^P	0.13	-0.07	-0.14	-0.18	-0.20	-0.17	-0.18	-0.28	-0.03	-0.13	
Wage-push, η^W	-0.20	-0.14	-0.32	-0.17	-0.24	-0.34	-0.17	-0.37	-0.13	-0.23	
Entry, η^E	-0.65	-0.33	-0.39	-0.13	-0.70	-0.25	-0.46	-0.21	-0.12	-0.36	
Exit, η^X	-0.91	-1.00	-0.37	-0.88	-0.42	-0.21	-0.96	0.05	-0.75	-0.61	
Financial, η^L	-0.56	-0.62	-0.41	-0.42	-0.31	-0.54	0.02	-0.44	-0.35	-0.41	
Demeaned output growth	0.16	0.00	0.64	0.31	0.41	-0.95	0.28	-0.14	0.32	0.11	

Finally, I will take a close look at the possible linkage between the processes of business entry and exit and the estimated TFP innovations. Jaimovich and Floetotto (2008) find that around 40% of movements in TFP can be attributed to entry and exit dynamics. According to the Schumpeterian idea of creative destruction, an economy with a high exit rate would turn more productive as the least efficient firms would shut down and would be replaced by more efficient businesses (Schumpeter, 1942, chapter 7). Unfortunately, our model is not able to reproduce endogenous innovations that push the production frontier. But the estimates of TFP can be interpreted as the Solow residual that measures empirically the aggregate level of productivity innovations. Figure 14 displays the US establishment entry-exit rates and the estimated TFP innovations from our model during the quarters of the latest US business cycle. There is a visual relation of exit rate and productivity innovations as the red line on the top graph and the blue line on the bottom graph swing together in the quarters of the crisis (they both move up) and the recovery (they both move down). Furthermore, the TFP innovations seem to lag entry rates.

Table 8 documents this delay through the dynamic cross correlations. The contemporaneous cross correlation of exit rates and TFP innovations is moderately high (0.46), and quickly picks up to become 0.80 after three quarters and still 0.79 after one year. Thus, a high exit rate tends to bring about positive TFP innovations that peak in 3-4 quarters later. Such a delay might be accounting for the time period required for innovations to be fully developed and spread out in the markets. Entry rates have the opposite effect of exit rates on TFP innovations. If the entry rate rises TFP innovations move down with a strong negative correlation (between -0-64 and -0.82 depending on the lag). One possible interpretation of this is the fact that many of the new establishments reported in the data are just replicas of successful ones that are installed in different locations (franchise system), which would reduce the overall volume of technological

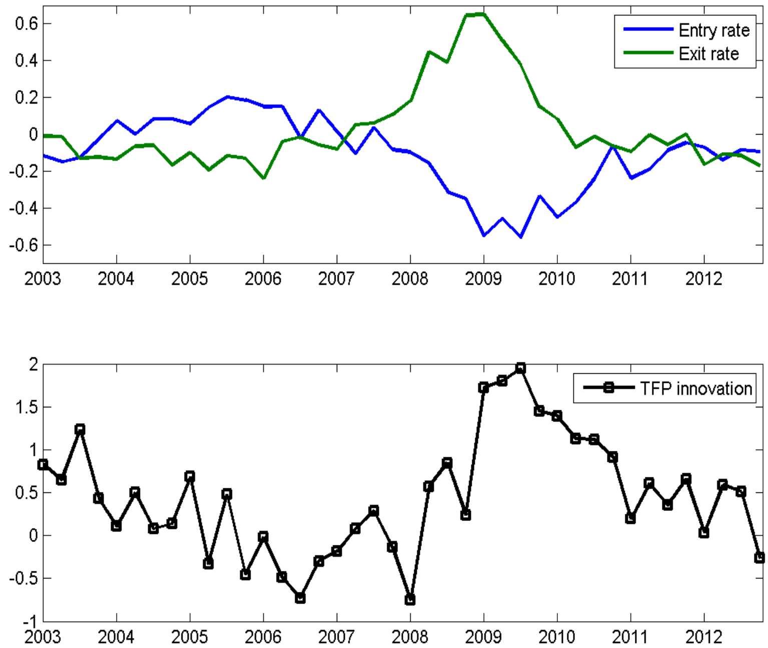

Figure 14: Entry, exit and total factor productivity in the US, 2003-2012.

innovations.[26] The net entry rate (entry minus exit rate) also brings a significant negative impact on TFP innovations to indicate an excess of varieties. It would imply that when the exit rate is higher than the entry rate the impact on TFP innovations is positive and increasing over future quarters.

Table 8. Entry-exit rates and Solow residuals, 2003:1-2012:4.

	j=0	j=1	j=2	j=3	j=4
corr(E_t,η^a_{t+j})	-.75	-.80	-.79	-.71	-.63
corr(X_t,η^a_{t+j})	.46	.62	.73	.80	.79
corr($E_t - X_t$,η^a_{t+j})	-.64	-.76	-.82	-.81	-.77

7 Conclusion

I have found empirical evidence which indicates that the changes in the total number of private establishments have played a significant role on explaining the aggregate fluctuations over the most recent boom-bust US business cycle. This empirical observation motivates the introduction of the extensive margin of variability through both business creation (entry) and destruction (exit) in a DSGE model meant for business cycle analysis. The estimated model confirms that entry and exit shocks originate 30% of US aggregate fluctuations during the quarterly period 1993:1-2012:4. However, technology shocks still explain

[26] As illustrative examples of two big service corporations in the US, Wal-Mart had 4,597 retail stores in US soil on August 31st, 2015, and McDonald's had 14,157 open restaurants in 2012.

30% of overall variability, and demand-side shocks another 30%. Financial shocks also take a 4% share of the variance decomposition and play a significant role on the variability of the entry rate (15% share).

In the analysis of the recent US boom-bust cycle, I obtain that adverse realization of entry-exit shocks, financial shocks and investment shocks are behind the 2008-09 recession. For the recovery, positive technology shocks contribute strongly for growth in tension with the continuation of adverse shocks on entry, exit and financial conditions. Finally, I document an empirical link between the exit rate and the estimated innovations of total factor productivity. A higher exit rate brings positive innovations to productivity, with a lag of 3-4 quarters. High entry rates, by contrast, come together with a reduction of total factor productivity. These results depict the recent US business cycles as a modern example of the Schumpeterian hypothesis of creative destruction.

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TECHNICAL APPENDIX

A. Household optimizing program

The preferences of the j representative household are defined by a utility function, separable between consumption bundles and (the disutility of) labor, which for period t reads

$$e^{\varepsilon_t^b}\left(\frac{(c_t(j)-hc_{t-1})^{1-\sigma_c}}{1-\sigma_c}-\Xi\frac{(l_t(j))^{1+\sigma_l}}{1+\sigma_l}\right),$$

where ε_t^b is an economy-wide AR(1) preference shock, $c_t(j)$ is number of bundles currently consumed, c_{t-1} is aggregate lagged consumption, and $l_t(j)$ is the amount of hours of labor supplied. Regarding the parameters, $\sigma_c > 0$ is the risk aversion coefficient, $0 \leq h < 1$ is an external consumption habit parameter, $\Xi > 0$ is the coefficient of the weight of labor disutility in overall utility, and $\sigma_l > 0$ is the curvature coefficient in disutility of labor. A constant discount factor per period, $\beta < 1$, is used to bring future utility into present time.

The sources of household income are labor and capital earnings, equity return and the interest services of bonds and bank deposits. The representative household possesses market power to set the nominal wage $W_t(j)$ constrained by a labor demand schedule. Labor income is $\frac{W_t(j)}{P_t^c}l_t(j)$, where the real wage is measured in consumption bundles at the price index, P_t^c. Capital income is $r_t^k u_t(j)k_{t-1}(j)$ where r_t^k is the market real rental rate, $u_t(j)$ is the variable capital utilization rate, and $k_{t-1}(j)$ is the stock of capital installed in the previous period. Another source of income is equity ownership. Let $\widetilde{d}_t$ denote the average real dividend and $\widetilde{v}_t$ the average equity (real) value. The representative household gets $\widetilde{d}_t\frac{n_{t-1}^A}{n_{t-1}}x_{t-1}(j)$ as the total dividends from her previous-period share of portfolio investment $x_{t-1}(j)$,obtains a real interest rate of r_{t-1}^d per unit of bank deposits $dep_t(j)$, a collateral service real yield, csy_t, per unit of equity holdings, and the real dividend, $\frac{n_{t-1}^A}{n_{t-1}}\widetilde{d}_t n_{t-1}^E(j)$, from the new varieties created the previous period and that survived the starting period of its life. There is also some revenue from business destruction, which corresponds to both the liquidation value of the exit share, $lv_{t-1}\frac{n_{t-1}^X}{n_{t-1}}x_{t-1}(j)$, where lv_{t-1} is the unit liquidation value, and the liquidation of new goods that shut down even before reaching the first period of life, $lv_{t-1}\frac{n_{t-1}^X}{n_{t-1}}n_{t-1}^E(j)$. The resulting gross income gets deducted in the amount of taxes (expressed in consumption bundles), t_t, and in the interest payments to the bank, $r_t^L loan_t^h(j)$, where r_t^L is the real interest of loans and $loan_t^h(j)$ is the demand for loans of the household in real terms.

Net income is spent on purchases of bundles of consumption goods, $c_t(j)$, on investment on capital goods, $i_t(j)$, on portfolio net investment on incumbent firms, $\widetilde{v}_t\left(x_t(j)-\frac{n_{t-1}^A}{n_{t-1}}x_{t-1}(j)\right)$, on increasing the stock of real bank deposits, $dep_{t+1}-dep_t$, on net purchases of government bonds, $(1+r_t)^{-1}b_{t+1}(j)-b_t(j)$, where $1+r_t$ is the real return of bonds, and $b_{t+1}(j)$ denotes the purchases of real bonds in period t to be reimbursed in $t+1$, and on covering the total cost of entry, $n_t^E(j)e^{\varepsilon_t^E}\left(f^E+ec_t\right)$, where f^E is the unit real cost of the license fee for a new variety, ec_t is a (congestion) entry cost and ε_t^E is an AR(1) exogenous shock. In addition, there is some expenditure required for covering the adjustment cost of variable capital utilization, $a(u_t(j))k_t(j)$ where $a(.)$ is the adjustment cost function described in Smets and Wouters (2007). As a result, the budget constraint of the representative household in period t becomes,

$$\frac{W_t(j)}{P_t^c}l_t\left(j\right)+r_t^k u_t(j)k_{t-1}(j)+\left[\frac{n_{t-1}^A}{n_{t-1}}\left(\widetilde{d}_t+\widetilde{v}_t\left(1+csy_t\right)\right)+\frac{n_{t-1}^X}{n_{t-1}}lv_{t-1}\right]\left(x_{t-1}(j)+n_{t-1}^E(j)\right)$$
$$+r_{t-1}^d dep_t(j)-t_t-r_t^L loan_t^h\left(j\right)=$$
$$c_t(j)+i_t(j)+a(u_t(j))k_t(j)+\widetilde{v}_t x_t(j)+\frac{b_{t+1}(j)}{1+r_t}-b_t(j)+n_t^E(j)e^{\varepsilon_t^E}\left(f^E+ec_t\right)+\left(dep_{t+1}(j)-dep_t(j)\right). \tag{A1}$$

Households face a financial constraint. A constant fraction τ_h of the total cost of creating new goods must be borrowed from the bank. It gives rise to the household demand for real loans

$$loan_t^h\left(j\right)=\tau_h n_t^E(j)e^{\varepsilon_t^E}\left(f^E+ec_t\right), \tag{A2}$$

which can be inserted in (A1) to obtain

$$\frac{W_t(j)}{P_t^c}l_t\left(j\right)+r_t^k u_t(j)k_{t-1}(j)+\left[\frac{n_{t-1}^A}{n_{t-1}}\left(\widetilde{d}_t+\widetilde{v}_t\left(1+csy_t\right)\right)+\frac{n_{t-1}^X}{n_{t-1}}lv_{t-1}\right]\left(x_{t-1}(j)+n_{t-1}^E(j)\right)+r_{t-1}^d dep_t(j)-t_t=$$
$$c_t(j)+i_t(j)+a(u_t(j))k_t(j)+\widetilde{v}_t x_t(j)+\frac{b_{t+1}(j)}{1+r_t}-b_t(j)+\left(1+r_t^L\tau_h\right)n_t^E(j)e^{\varepsilon_t^E}\left(f^E+ec_t\right)+\left(dep_{t+1}(j)-dep_t(j)\right). \tag{A3}$$

Following Smets and Wouters (2007), capital accumulation is costly. Hence, the equation of motion for capital is,

$$k_t(j)=\left(1-\delta_k\right)k_{t-1}(j)+e^{\varepsilon_t^i}\left[1-S\left(\frac{i_t(j)}{i_{t-1}(j)}\right)\right]i_t(j), \tag{A4}$$

where δ_k is the constant rate of capital depreciation rate, $S(.)$ is the investment adjustment cost function with the steady-state properties $S\left(.\right)=S'\left(.\right)=0$ and $S''\left(.\right)=\varphi_k>0$, and ε_t^i is an AR(1) shock to the price of investment relative to consumption goods. The amount of capital that households can effectively supply to the firms, $k_t^s\left(j\right)$, is the product of the utilization rate and the stock of available capital

$$k_t^s\left(j\right)=u_t(j)k_t(j). \tag{A5}$$

Denoting E_t as the rational expectation operator, the representative household seeks to maximize

$$E_t\sum_{j=0}^{\infty}\beta^j e^{\varepsilon_{t+j}^b}\left(\frac{\left(c_{t+j}(j)-hc_{t-1+j}\right)^{1-\sigma_c}}{1-\sigma_c}-\frac{\Xi\left(l_{t+j}(j)\right)^{1+\sigma_l}}{1+\sigma_l}\right)$$

subject to constraints (A3)-(A4) for current period t and the expected expressions in all future periods. The first order conditions computed with respect to the choice variables $c_t(j)$, $u_t(j)$, $k_t(j)$, $b_{t+1}(j)$, $dep_{t+1}(j)$,

$l_t(j)$, $x_t(j)$, and $n_t^E(j)$ are, respectively,

$$\begin{aligned}
e^{\varepsilon_t^b}\left(c_t(j)-hc_{t-1}\right)^{-\sigma_c}-\lambda_t &= 0,\\
r_t^k-a'(u_t(j)) &= 0,\\
\Upsilon_t-\beta E_t\left[\lambda_{t+1}\left(r_{t+1}^k u_{t+1}(j)-a(u_{t+1}(j))\right)+\Upsilon_{t+1}\left(1-\delta_k\right)\right] &= 0,\\
-\lambda_t\left(1+r_t\right)^{-1}+\beta E_t\lambda_{t+1} &= 0,\\
-\lambda_t+\beta E_t\lambda_{t+1}\left(1+r_t^d\right) &= 0,\\
-\Xi\left(l_t(j)\right)^{\sigma_l}+\lambda_t W_t(j)/P_t^c &= 0,\\
-\lambda_t\widetilde{v}_t+\beta E_t\lambda_{t+1}\left[\tfrac{n_t^A}{n_t}\left(\widetilde{d}_{t+1}+\widetilde{v}_{t+1}\left(1+csy_{t+1}\right)\right)+\tfrac{n_t^X}{n_t}lv_t\right] &= 0,\\
-\lambda_t\left(1+r_t^L\tau_h\right)e^{\varepsilon_t^E}\left(f^E+ec_t\right)+\beta E_t\lambda_{t+1}\left[\tfrac{n_t^A}{n_t}\left(\widetilde{d}_{t+1}+\widetilde{v}_{t+1}\left(1+csy_{t+1}\right)\right)+\tfrac{n_t^X}{n_t}lv_t\right] &= 0,
\end{aligned}$$

where λ_t is the Lagrange multiplier of the budget constraint (A3) and Υ_t is the Lagrange multiplier of the capital accumulation constraint (A4).

Remarkably, the equilibrium condition for equity investment obtain from the combination of first order conditions for $b_{t+1}(j)$ and $x_t(j)$ is,

$$\widetilde{v}_t=\frac{1}{1+r_t}\left[\frac{n_t^A}{n_t}E_t\left(\widetilde{d}_{t+1}+\widetilde{v}_{t+1}\left(1+csy_{t+1}\right)\right)+\frac{n_t^X}{n_t}lv_t\right],$$

which implies that the average equity value is the discounted sum of the expected returns from surviving varieties, $\frac{n_t^A}{n_t}E_t\left(\widetilde{d}_{t+1}+\widetilde{v}_{t+1}\left(1+csy_{t+1}\right)\right)$, and the return from dying varieties, $\frac{n_t^X}{n_t}lv_t$. Hence, the equilibrium equity value depends (positively) on the collateral service yield, csy_{t+1}, and on the liquidation value, lv_t, as elements that come from the model extension to respectively incorporate banking and entry-exit. The first order conditions of bonds and deposits bring the interest parity relation

$$r_t=r_t^d.$$

The free entry condition displayed in the text is reached when combining the first order conditions of $n_t^E(j)$ and $x_t(j)$

$$\left(1+r_t^L\tau_h\right)e^{\varepsilon_t^E}\left(f^E+ec_t\right)=\widetilde{v}_t.$$

Wage rigidities and wage inflation dynamics

Households set the nominal wage and supply differentiated labor services with some market power subject to the labor demand constraint,

$$l_t\left(j\right)=\left(\frac{W_t\left(j\right)}{W_t}\right)^{-\theta_w}l_t, \tag{A6}$$

with a constant elasticity of substitution $\theta_w>0$, and where $W_t=\left[\int_0^1 W_t\left(j\right)^{1-\theta_w}dj\right]^{\frac{1}{1-\theta_w}}$ and $l_t=\left[\int_0^1 l_t(j)^{\frac{\theta_w-1}{\theta_w}}dj\right]^{\frac{\theta_w}{\theta_w-1}}$ are respectively aggregate indices of nominal wages and labor allocated across the unit interval. There are wage rigidities *á la* Calvo (1983). Hence, there is a constant probability $0 < \xi_w < 1$ that the household cannot set the optimal wage. If so, the wage would be automatically adjusted by applying a wage indexation rule

$$W_t(.) = W_{t-1}(.) \left[(1+\pi^c_{t-1})^{\iota_w} (1+\pi^c+\varepsilon^W_t)^{1-\iota_w} \right], \tag{A7}$$

in which π^c_{t-1} is the rate of consumer price inflation (CPI) in period $t-1$ computed from the P^c_t price index, $\pi^c_{t-1} = \left(P^c_{t-1}/P^c_{t-2}\right) - 1$, the steady-state CPI rate is π^c, there is an stochastic component introduced through the wage-push ARMA(1,1) shock ε^w_t, and $0 < \iota_w < 1$ is the indexation share that mirrors lagged CPI inflation.

Using the labor demand constraint (A6) in the household budget constraint (A3) gives

$$w_t \left(\frac{W_t(j)}{W_t}\right)^{1-\theta_w} l_t + r^k_t u_t(j) k_{t-1}(j) + \left[\frac{n^A_{t-1}}{n_{t-1}} \left(\widetilde{d}_t + \widetilde{v}_t \left(1+csy_t\right)\right) + \frac{n^X_{t-1}}{n_{t-1}} lv_{t-1}\right] \left(x_{t-1}(j) + n^E_{t-1}(j)\right) + r^d_{t-1} dep_t(j) - t_t$$
$$c_t(j) + i_t(j) + a(u_t(j)) k_t(j) + \widetilde{v}_t x_t(j) + \frac{b_{t+1}(j)}{1+r_t} - b_t(j) + (dep_{t+1}(j) - dep_t(j)) + \left(1 + r^L_t \tau_h\right) n^E_t(j) e^{\varepsilon^E_t} \left(f^E + ec_t\right),$$

where $w_t = \frac{W_t}{P^c_t}$ is the aggregate real wage in terms of bundles of consumption goods. Recalling the Calvo-type stickiness (instrumentalized by the fixed probability of non-optimal wage setting, ξ_w) and the wage indexation rule (A7), the first order condition of the household optimizing program with respect to the nominal wage is

$$E^\xi_t \sum_{j=0}^{\infty} \beta^j \xi^j_w \left(\lambda_{t+j} w_{t+j} (1-\theta_w) \left(\frac{W^*_t \Pi^W_{t,t+j}}{W_{t+j}}\right)^{-\theta_w} \frac{l_{t+j} \Pi^W_{t,t+j}}{W_{t+j}} + \psi_{t+j} \theta_w \left(\frac{W^*_t \Pi^W_{t,t+j}}{W_{t+j}}\right)^{-\theta_w - 1} \frac{l_{t+j} \Pi^w_{t,t+j}}{W_{t+j}} \right) = 0,$$

where E^ξ_t is the rational expectation operator conditional to the lack of future optimal wage setting, W^*_t is the optimal nominal wage chosen by the representative household in period t, $\Pi^w_{t,t+j}$ is the multiplicative indexation factor from period t to period $t+j$, $\Pi^W_{t,t+j} = \prod_{k=0}^{j} \left[(1+\pi^c_{t-1+k})^{\iota_w} (1+\pi^c+\varepsilon^W_{t+k})^{1-\iota_w}\right]$ for $j \geq 1$, λ_{t+j} is the Lagrange multiplier of the budget constraint in period $t+j$, and ψ_{t+j} is the Lagrange multiplier of the labor demand constraint in period $t+j$. Combining the first order conditions of the nominal wage, hours and consumption, and incorporating the definition of the marginal rate of substitution $mrs^*_{t+j} = \frac{e^{\varepsilon^b_{t+j}} (c_{t+j}(j) - hc_{t-1+j})^{-\sigma_c}}{\Xi (l_{t+j}(j))^{\sigma_l}}$ brings the single equation

$$E^\xi_t \sum_{j=0}^{\infty} \beta^j \xi^j_w \left(w_{t+j} (1-\theta_w) \left(\frac{W^*_t \Pi^W_{t,t+j}}{W_{t+j}}\right)^{-\theta_w} \frac{l_{t+j} \Pi^w_{t,t+j}}{W_{t+j}} + mrs^*_{t+j} \theta_w \left(\frac{W^*_t \Pi^W_{t,t+j}}{W_{t+j}}\right)^{-\theta_w - 1} \frac{l_{t+j} \Pi^w_{t,t+j}}{W_{t+j}} \right) = 0.$$

Solving for W^*_t, it is found that the representative household determines the nominal wage as a forward-looking mark-up over the marginal rate of substitution as follows,

$$W^*_t = \frac{\theta_w}{\theta_w - 1} \left[\frac{E^\xi_t \sum_{j=0}^{\infty} \beta^j \xi^j_w mrs^*_{t+j} \left(W_{t+j}\right)^{\theta_w} \left(\Pi^W_{t,t+j}\right)^{-\theta_w} l_{t+j}}{E^\xi_t \sum_{j=0}^{\infty} \beta^j \xi^j_w w_{t+j} \left(W_{t+j}\right)^{\theta_w - 1} \left(\Pi^W_{t,t+j}\right)^{-\theta_w + 1} l_{t+j}} \right].$$

After loglinearizing, I get

$$\widehat{W}_t^* = (1-\beta\xi_w)E_t^{\xi}\sum_{j=0}^{\infty}\beta^j\xi_w^j\left(\widehat{mrs}_{t+j}^* - \widehat{w}_{t+j} + \widehat{W}_{t+j} - \sum_{k=1}^{j}\left(\iota_w\left(\pi_{t-1+k}^c - \pi^c\right) + (1-\iota_w)\,\varepsilon_{t+k}^W\right)\right),$$

where using $\widehat{W}_{t+j} = \widehat{W}_t + \sum_{k=1}^{j}\left(\pi_{t+k}^w - \pi^w\right)$ from the definition of cumulative wage inflation gives

$$\widehat{W}_t^* - \widehat{W}_t =$$
$$(1-\beta\xi_w)E_t^{\xi}\sum_{j=0}^{\infty}\beta^j\xi_w^j\left(\widehat{mrs}_{t+j}^* - \widehat{w}_{t+j} + \sum_{k=1}^{j}\left(\left(\pi_{t+k}^w - \pi^w\right) - \iota_w\left(\pi_{t-1+k}^c - \pi^c\right) - (1-\iota_w)\,\varepsilon_{t+k}^W\right)\right)$$

There is a gap between $\widehat{mrs}_{t+j}^*$ and its aggregate counterpart $E_t^{\xi}\widehat{mrs}_{t+j}$. Following Galí (2008, chapter 6), I use $\widehat{mrs}_{t+j}^* = \widehat{mrs}_{t+j} + \sigma_l\left(\widehat{l}_{t+j}^* - \widehat{l}_{t+j}\right) = \widehat{mrs}_{t+j} - \theta_w\sigma_l\left(\widehat{W}_t^* + \widehat{\Pi}_{t,t+j}^W - \widehat{W}_{t+j}\right) = \widehat{mrs}_{t+j} - \theta_w\sigma_l\left(\widehat{W}_t^* - \widehat{W}_t - \sum_{k=1}^{j}\left(\left(\pi_{t+k}^w - \pi^w\right) - \iota_w\left(\pi_{t-1+k}^c - \pi^c\right) - (1-\iota_w)\,\varepsilon_{t+k}^W\right)\right)$, in the previous result to get

$$(1+\theta_w\sigma_l)\left(\widehat{W}_t^* - \widehat{W}_t\right) =$$
$$(1-\beta\xi_w)\sum_{j=0}^{\infty}\beta^j\xi_w^j\left(\widehat{mrs}_{t+j} - \widehat{w}_{t+j} + \sum_{k=1}^{j}\left(\left(\pi_{t+k}^w - \pi\right)^w - \iota_w\left(\pi_{t-1+k}^c - \pi^c\right) - (1-\iota_w)\,\varepsilon_{t+k}^W\right)\right).$$

Meanwhile, the aggregation scheme for wage setting with Calvo rigidities and the indexation brings the semi-loglinear relation

$$\widehat{W}_t^* - \widehat{W}_t = \frac{\xi_w}{1-\xi_w}\left(\left(\pi_t^w - \pi\right) - \iota_w\left(\pi_{t-1}^c - \pi^c\right) - (1-\iota_w)\,\varepsilon_t^W\right).$$

Combining the last two relations yields

$$\left(\pi_t^w - \pi^w\right) - \iota_w\left(\pi_{t-1}^c - \pi^c\right) - (1-\iota_w)\,\varepsilon_t^W =$$
$$\tfrac{(1-\beta\xi_w)(1-\xi_w)}{\xi_w(1+\theta_w\sigma_l)}\sum_{j=0}^{\infty}\beta^j\xi_w^j\left(\widehat{mrs}_{t+j} - \widehat{w}_{t+j} + \sum_{k=1}^{j}\left(\left(\pi_{t+k}^w - \pi^w\right) - \iota_w\left(\pi_{t-1+k}^c - \pi\right)^c - (1-\iota_w)\,\varepsilon_{t+k}^W\right)\right),$$

and rearranging terms

$$\pi_t^w - \pi^w = \iota_w\left(\pi_{t-1}^c - \pi^c\right) + \beta E_t\left(\pi_{t+1}^w - \pi^w\right) - \beta\iota_w\left(\pi_t^c - \pi^c\right) + \tfrac{(1-\beta\xi_w)(1-\xi_w)}{\xi_w(1+\theta_w\sigma_l)}\left(\widehat{mrs}_t - \widehat{w}_t\right) + (1-\iota_w)\left(\varepsilon_t^W - \beta E_t\varepsilon_{t+1}^W\right)$$

As another possibility, if the wage indexation rule considered nominal wage inflation ($\pi_t^w = W_t/W_{t-1} - 1$), instead of the CPI price inflation, the wage inflation equation would become

$$(1+\beta\iota_w)\left(\pi_t^w - \pi^w\right) = \iota_p\left(\pi_{t-1}^w - \pi^w\right) + \beta E_t\left(\pi_{t+1}^w - \pi^w\right) + \tfrac{(1-\beta\xi_w)(1-\xi_w)}{\xi_w(1+\theta_w\sigma_l)}\left(\widehat{mrs}_t - \widehat{w}_t\right) + (1-\iota_w)\left(\varepsilon_t^W - \beta E_t\varepsilon_{t+1}^W\right).$$

B. Firm optimizing program

There are both intermediate-good and final-good firms in the goods market. Intermediate-good firms combine labor and capital within a production technology to supply varieties of heterogeneous consumption goods that are sold in a monopolistically competitive market to the final-good firm. Intermediate-good producers are price setters constrained by demand conditions and Calvo (1983)-type rigidities. Entry and exit take place in the production of intermediate consumption goods. The final-good firm aggregates all the varieties of intermediate consumption goods to make them available as consumption bundles.

In period t, the intermediate firm of type ω produces a quantity $y_t(\omega)$ of this good using the Cobb-Douglas production function,

$$y_t(\omega) = e^{\varepsilon_t^a} z(\omega) k_t^{\alpha}(\omega) \left(e^{\gamma t} l_t(\omega)\right)^{1-\alpha}, \tag{B1}$$

where $0 < \alpha < 1$ is the capital share parameter, $l_t(\omega)$ and $k_t(\omega)$ are respectively the demand for labor and capital, ε_t^a is a total factor productivity AR(1) shock, $z(\omega)$ is a firm-specific productivity level, and γ is the long-run rate of economic growth. The technology shock ε_t^a is homogeneous to all firms. Nevertheless, there is firm heterogeneity determined by $z(\omega)$, which is obtained as a single draw from the Pareto distribution.[27]

Intermediate-good firms operate in a monopolistically competitive market as in Dixit and Stiglitz (1977). Hence, the amount of firm-specific output, $y_t(\omega)$, is demand-determined in response to its relative price $\frac{P_t(\omega)}{P_t^c}$ and to the aggregate demand for goods, y_t, as follows,

$$y_t(\omega) = \left(\frac{P_t(\omega)}{P_t^c}\right)^{-\theta_p} y_t, \tag{B2}$$

where $\theta_p > 1$ is the constant elasticity of substitution across goods. The discontinuities between flows of revenues and expenditures gives rise to the following firm-level demand for real loans

$$loan_t^f(\omega) = \tau_f \left(w_t l_t(\omega) + r_t^k k_t(\omega)\right), \tag{B3}$$

which implies that a constant fraction $0 < \tau_f < 1$ of real expenditures must be externally financed. As banks charge the real interest rate r_t^L per loan, the real dividend of the representative firm can be written as follows

$$d_t(\omega) = \frac{P_t(\omega)}{P_t^c} y_t(\omega) - w_t l_t(\omega) - r_t^k k_t(\omega) - r_t^L loan_t^f(\omega),$$

where using (B2) and (B3), I get

$$d_t(\omega) = \left(\frac{P_t(\omega)}{P_t^c}\right)^{1-\theta_p} y_t - \left(1 + \tau_f r_t^L\right) w_t l_t(\omega) - \left(1 + \tau_f r_t^L\right) r_t^k k_t(\omega). \tag{B4}$$

Let me assume a Calvo (1983) fixed probability for optimal price setting, $0 < \xi_p < 1$. Hence, firms cannot price optimally in every period under a constant probability ξ_p. In that case, a price indexation rule would be implemented to yield the following price adjustment

$$P_t(.) = P_{t-1}(.) \left[(1 + \pi_{t-1})^{\iota_p} (1 + \pi + \varepsilon_t^P)^{1-\iota_p}\right],$$

[27] The probability distribution function and the cumulative distribution function of $z(\omega)$ are respectively $g(z(\omega)) = \kappa z_{\min}^{\kappa}/z(\omega)^{\kappa+1}$ and $G(z(\omega)) = 1 - (z_{\min}/z(\omega))^{\kappa}$, where $z_{\min} > 0$ is the lower bound and κ is the shape parameter which must be higher than $(\theta_p - 1)$ to have a well-defined average productivity.

in which π_{t-1} is the lagged rate of aggregate producer price inflation, $\pi_{t-1} = \left(\widetilde{P}_{t-1}/\widetilde{P}_{t-1}\right) - 1$, measured at the average firm-level productivity $\widetilde{z}$. Furthermore, π denotes the steady-state producer inflation rate, ε_t^P is an exogenous ARMA(1,1) price-push shock, and $0 < \iota_p < 1$ is the indexation share that responds to lagged inflation.

For optimal pricing, I assume that the representative firm ω operates with average productivity, $\widetilde{z}$. Any variable that refers to average productivity will be labeled with "$\widetilde{}$". Let β_{t+j} denote the stochastic discount factor. In period t, the firm maximizes the expected stream of dividends

$$\sum_{j=0}^{\infty} \beta_{t+j}\xi_p^j \left(\left(\frac{\widetilde{P}_{t+j}}{P_{t+j}^c}\right)^{1-\theta_p} y_{t+j} - \left(1+\tau_f r_{t+j}^L\right) w_{t+j}\widetilde{l}_{t+j} - \left(1+\tau_f r_{t+j}^L\right) r_{t+j}^k \widetilde{k}_{t+j} \right),$$

conditional to the lack of optimal prices in the future, and subject to the expected schedule of Dixit-Stiglitz demand constraints,

$$e^{\varepsilon_{t+j}^a}\widetilde{z}\widetilde{k}_{t+j}^{\alpha}\left(e^{\gamma(t+j)}\widetilde{l}_{t+j}\right)^{1-\alpha} = \left(\frac{\widetilde{P}_{t+j}}{P_{t+j}^c}\right)^{-\theta_p} y_{t+j}, \text{ for } j = 0, 1, 2, ...$$

I use $\widetilde{P}_t^*$ for the optimal price of the representative in period t. Introducing $\frac{\widetilde{P}_t^*}{P_{t+j}^c} = \frac{\widetilde{P}_t^*}{\widetilde{P}_{t+j}}\frac{\widetilde{P}_{t+j}}{P_{t+j}^c} = \frac{\widetilde{P}_t^*}{\widetilde{P}_{t+j}}\widetilde{\rho}_{t+j}$ where $\widetilde{P}_{t+j}$ is the average price across all firms that have the average productivity $\widetilde{z}$ (and they differ due to their specific Calvo pricing histories), and $\widetilde{\rho}_{t+j} = \frac{\widetilde{P}_{t+j}}{P_{t+j}^c}$ is their relative price in period $t+j$ defined as the ratio between the average producer price and the consumer price index, the first order conditions with respect to the optimal price, labor demand and capital demand in period t are,

$$\begin{aligned}
E_t^{\xi}\sum_{j=0}^{\infty}\beta_{t,t+j}\xi_p^j\left((1-\theta_p)\left(\frac{\widetilde{P}_t^*\Pi_{t,t+j}^p\widetilde{\rho}_{t+j}}{\widetilde{P}_{t+j}}\right)^{-\theta_p}\frac{y_{t+j}\Pi_{t,t+j}^p\widetilde{\rho}_{t+j}}{\widetilde{P}_{t+j}} + \widetilde{mc}_{t+j}\theta_p\left(\frac{\widetilde{P}_t^*\Pi_{t,t+j}^p\widetilde{\rho}_{t+j}}{\widetilde{P}_{t+j}}\right)^{-\theta_p-1}\frac{y_{t+j}\Pi_{t,t+j}^p\widetilde{\rho}_{t+j}}{\widetilde{P}_{t+j}}\right) &= 0,\\
-\left(1+\tau_f r_t^L\right) w_t + \widetilde{mc}_t\left(1-\alpha\right)e^{\varepsilon_t^a}\widetilde{z}e^{(1-\alpha)\gamma t}\left(\widetilde{k}_t\left(\omega\right)/\widetilde{l}_t\left(\omega\right)\right)^{\alpha} &= 0,\\
-\left(1+\tau_f r_t^L\right) r_t^k + \widetilde{mc}_t\alpha e^{\varepsilon_t^a}\widetilde{z}e^{(1-\alpha)\gamma t}\left(\widetilde{l}_t\left(\omega\right)/\widetilde{k}_t\left(\omega\right)\right)^{1-\alpha} &= 0,
\end{aligned}$$

where E_t^{ξ} is the rational expectation operator conditional to not receiving the Calvo signal for optimal pricing, $\Pi_{t,t+j}^P = \prod_{k=0}^{j}\left[(1+\pi_{t-1+k})^{\iota_p}(1+\pi++\varepsilon_{t+k}^P)^{1-\iota_p}\right]$ for $j \geq 1$ is the price indexation factor between periods t and $t+j$, and $\widetilde{mc}_{t+j}$ is the Lagrange multiplier of the demand constraint (real marginal cost of firm with average productivity $\widetilde{z}$).The first order condition on price setting implies the optimal price

$$\widetilde{P}_t^* = \frac{\theta_p}{\theta_p - 1}\left[\frac{E_t^{\xi}\sum_{j=0}^{\infty}\beta_{t+j}\xi_p^j\widetilde{mc}_{t+j}\left(\widetilde{P}_{t+j}\right)^{\theta_p}\left(\Pi_{t,t+j}^p\widetilde{\rho}_{t+j}\right)^{-\theta_p} y_{t+j}}{E_t^{\xi}\sum_{j=0}^{\infty}\beta_{t+j}\xi_p^j\left(\widetilde{P}_{t+j}\right)^{\theta_p-1}\left(\Pi_{t,t+j}^p\widetilde{\rho}_{t+j}\right)^{-\theta_p+1} y_{t+j}}\right]. \tag{B5}$$

In loglinear terms, (B5) becomes

$$\widehat{\widetilde{P}}_t^* - \widehat{\widetilde{P}}_t = (1-\beta\xi_p)E_t\sum_{j=0}^{\infty}\beta^j\xi_p^j\left(\widehat{\widetilde{mc}}_{t+j} - \widehat{\widetilde{\rho}}_{t+j} + \sum_{k=1}^{j}\left(\pi_{t+k} - \iota_p\pi_{t-1+k} - (1-\iota_p)\,\varepsilon_{t+k}^p\right)\right). \tag{B6}$$

Froth the Dixit-Stiglitz aggregator with price stickiness and indexation, I can obtain the log-linear expression

$$\widehat{\widetilde{P}}_t^* - \widehat{\widetilde{P}}_t = \frac{\xi_p}{1-\xi_p}\left((\pi_t - \pi) - \iota_p\left(\pi_{t-1} - \pi\right) - (1-\iota_p)\,\varepsilon_t^p\right). \tag{B7}$$

Combining the last two relations, (B6) and (B7), results in the inflation equation

$$
\begin{aligned}
&(\pi_t - \pi) - \iota_p (\pi_{t-1} - \pi) - (1 - \iota_p) \varepsilon_t^p = \\
&\qquad \tfrac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p} \sum_{j=0}^{\infty} \beta^j \xi_p^j \left(\widehat{\widetilde{mc}}_{t+j} - \widehat{\widetilde{\rho}}_{t+j} + \sum_{k=1}^{j} \left((\pi_{t+k} - \pi) - \iota_p (\pi_{t-1+k} - \pi) - (1 - \iota_p) \varepsilon_{t+k}^p \right) \right),
\end{aligned}
$$

which simplifies to the hybrid New Keynesian Phillips curve

$$
(\pi_t - \pi) = \tfrac{\iota_p}{(1+\beta\iota_p)} (\pi_{t-1} - \pi) + \tfrac{\beta}{(1+\beta\iota_p)} E_t (\pi_{t+1} - \pi) + \tfrac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p(1+\beta\iota_p)} \left(\widehat{\widetilde{mc}}_t - \widehat{\widetilde{\rho}}_t \right) + \tfrac{(1-\iota_p)}{(1+\beta\iota_p)} \left(\varepsilon_t^p - \beta E_t \varepsilon_{t+1}^p \right).
$$

Average and critical real marginal cost

The optimality condition of labor demand at the representative firm implies

$$
\left(1 + \tau_f r_t^L\right) w_t = \widetilde{mc}_t (1 - \alpha) e^{\varepsilon_t^a} \widetilde{z} e^{(1-\alpha)\gamma t} \left(\widetilde{k}_t (\omega) / \widetilde{l}_t (\omega) \right)^{\alpha},
$$

where inserting the capital-labor ratio, $\widetilde{k}_t (\omega) / \widetilde{l}_t (\omega)$, consistent with the first order condition of capital, $\left(1 + \tau_f r_t^L\right) r_t^k = \widetilde{mc}_t \alpha e^{\varepsilon_t^a} \widetilde{z} e^{(1-\alpha)\gamma t} \left(\widetilde{l}_t (\omega) / \widetilde{k}_t (\omega) \right)^{1-\alpha}$, gives

$$
\left(1 + \tau_f r_t^L\right) w_t = \widetilde{mc}_t (1 - \alpha) e^{\varepsilon_t^a} \widetilde{z} e^{(1-\alpha)\gamma t} \left(\frac{\widetilde{mc}_t \alpha e^{\varepsilon_t^a} \widetilde{z} e^{(1-\alpha)\gamma t}}{\left(1 + \tau_f r_t^L\right) r_t^k} \right)^{\alpha/(1-\alpha)}.
$$

Terms on $\widetilde{mc}_t$ can be grouped in the previous expression to obtain

$$
(\widetilde{mc}_t)^{1/(1-\alpha)} = \frac{\left(1 + \tau_f r_t^L\right) w_t}{(1 - \alpha) e^{\varepsilon_t^a} \widetilde{z} e^{(1-\alpha)\gamma t} \left(\frac{\alpha e^{\varepsilon_t^a} \widetilde{z} e^{(1-\alpha)\gamma t}}{\left(1 + \tau_f r_t^L\right) r_t^k} \right)^{\alpha/(1-\alpha)}},
$$

and powering both sides to $(1 - \alpha)$ becomes

$$
\widetilde{mc}_t = \frac{\left(\left(1 + \tau_f r_t^L\right) w_t \right)^{(1-\alpha)}}{\left((1 - \alpha) e^{\varepsilon_t^a} \widetilde{z} e^{(1-\alpha)\gamma t} \right)^{(1-\alpha)} \left(\frac{\alpha e^{\varepsilon_t^a} \widetilde{z} e^{(1-\alpha)\gamma t}}{\left(1 + \tau_f r_t^L\right) r_t^k} \right)^{\alpha}},
$$

which simplifies to

$$
\widetilde{mc}_t = \frac{\left(1 + \tau_f r_t^L\right) (w_t)^{(1-\alpha)} \left(r_t^k\right)^{\alpha}}{(\alpha)^{\alpha} (1 - \alpha)^{(1-\alpha)} e^{\varepsilon_t^a} \widetilde{z} e^{(1-\alpha)\gamma t}}.
$$

Finally, the real wage grows in steady state at a constant rate γ, which decomposes the effective real wage between its long-run growth and the detrended real wage, $w_t = e^{\gamma t} w_t^*$.Accounting for this, the average real marginal cost is

$$
\widetilde{mc}_t = \frac{\left(1 + \tau_f r_t^L\right) (w_t^*)^{(1-\alpha)} \left(r_t^k\right)^{\alpha}}{(\alpha)^{\alpha} (1 - \alpha)^{(1-\alpha)} e^{\varepsilon_t^a} \widetilde{z}}.
$$

Meanwhile, the real marginal cost of the firm that produces with the critical productivity level would have all the elements in common with $\widetilde{mc}_t$ except for the firm-specific productivity (it should be noticed that r_t^L, w_t^*, r_t^k and ε_t^a are economy-wide variables)

$$
mc_t^c = \frac{\left(1 + \tau_f r_t^L\right) (w_t^*)^{(1-\alpha)} \left(r_t^k\right)^{\alpha}}{(\alpha)^{\alpha} (1 - \alpha)^{(1-\alpha)} e^{\varepsilon_t^a} z_t^c}.
$$

Comparing the last two expression yields

$$mc_t^c = \widetilde{mc}_t \frac{\widetilde{z}}{z_t^c}.$$

C. Derivation of the overall resources constraint

The household budget constraint, equation (A3) can be used for aggregating across all households to obtain

$$\begin{aligned} w_t l_t + r_t^k u_t k_{t-1} + \left[\tfrac{n_{t-1}^A}{n_{t-1}} \left(\widetilde{d}_t + \widetilde{v}_t \left(1 + csy_t\right)\right) + \tfrac{n_{t-1}^X}{n_{t-1}} lv_{t-1}\right] \left(x_{t-1} + n_{t-1}^E\right) + r_{t-1}^d dep_t - t_t = \\ c_t + i_t + a(u_t) k_t + \widetilde{v}_t x_t + \tfrac{b_{t+1}}{1+r_t} - b_t + \left(dep_{t+1} - dep_t\right) + \left(1 + r_t^L \tau_h\right) n_t^E e^{\varepsilon_t^E} \left(f^E + ec_t\right), \end{aligned}$$

where all the terms that have dropped the j reference now represent the aggregate value, e.g. $k_t = \int_0^1 k_t(j)\, dj$, in the case of labor income I have used the Dixit-Stiglitz property $\int_0^1 W_t(j) l_t(j)\, dj = W_t l_t$, and $w_t = \frac{W_t}{P_t^c}$ denotes the aggregate real wage. Introducing the equilibrium condition for the portfolio shares, $x_{t-1}(j) = n_{t-1}$ and $x_t(j) = n_t$, it is obtained,

$$\begin{aligned} w_t l_t + r_t^k u_t k_{t-1} + \left[\tfrac{n_{t-1}^A}{n_{t-1}} \left(\widetilde{d}_t + \widetilde{v}_t \left(1 + csy_t\right)\right) + \tfrac{n_{t-1}^X}{n_{t-1}} lv_{t-1}\right] \left(n_{t-1} + n_{t-1}^E\right) + r_{t-1}^d dep_t - t_t = \\ c_t + i_t + a(u_t) k_t + \widetilde{v}_t n_t + \tfrac{b_{t+1}}{1+r_t} - b_t + \left(dep_{t+1} - dep_t\right) + \left(1 + r_t^L \tau_h\right) n_t^E e^{\varepsilon_t^E} \left(f^E + ec_t\right). \end{aligned}$$

Next, the law of motion for the number of varieties $n_t = \frac{n_{t-1}^A}{n_{t-1}} \left(n_{t-1} + n_{t-1}^E\right)$ serves to cancel the equity term $\widetilde{v}_t n_t$ in order to yield,

$$\begin{aligned} w_t l_t + r_t^k u_t k_{t-1} + n_t \left(\widetilde{d}_t + \widetilde{v}_t csy_t\right) + lv_{t-1} \left[n_{t-1}^X + \tfrac{n_{t-1}^X}{n_{t-1}} n_{t-1}^E\right] + r_{t-1}^d dep_t - t_t = \\ c_t + i_t + a_t(u_t) k_t + \tfrac{b_{t+1}}{1+r_t} - b_t + \left(dep_{t+1} - dep_t\right) + \left(1 + r_t^L \tau_h\right) n_t^E e^{\varepsilon_t^E} \left(f^E + ec_t\right), \end{aligned}$$

where replacing t_t with the expression implied by the government constraint,

$$\varepsilon_t^g = t_t + \left(e^{\varepsilon_t^E} f_t^E n_t^E - e^{\varepsilon_{t-1}^E} (1 - \varphi) f^E \left(n_{t-1}^X + \tfrac{n_{t-1}^X}{n_{t-1}} n_{t-1}^E\right)\right) + \tfrac{b_{t+1}}{1+r_t} - b_t + \left(dep_{t+1} - dep_t\right),$$

and recalling $lv_{t-1} = (e^{\varepsilon_{t-1}^X} \left((1 - \varphi) f^E - xc_{t-1}\right)$, I reach,

$$\begin{aligned} w_t l_t + r_t^k u_t k_{t-1} + n_t \left(\widetilde{d}_t + \widetilde{v}_t csy_t\right) - e^{\varepsilon_{t-1}^X} xc_{t-1} \left[n_{t-1}^X + \tfrac{n_{t-1}^X}{n_{t-1}} n_{t-1}^E\right] + r_{t-1}^d dep_t = \\ c_t + i_t + a_t(u_t) k_t + \varepsilon_t^g + r_t^L \tau_h n_t^E e^{\varepsilon_t^E} \left(f^E + ec_t\right) + n_t^E e^{\varepsilon_t^E} ec_t, \end{aligned}$$

which introduces the lag of variable exit cost xc_{t-1}. Next, considering the input markets equilibria, $l_t = n_t \widetilde{l}_t + m_t$, and, $u_t k_{t-1} = n_t \widetilde{k}_t$, yields,

$$\begin{aligned} w_t n_t \widetilde{l}_t + w_t m_t + \left[r_t^k n_t \widetilde{k}_t\right] + n_t \left(\widetilde{d}_t + \widetilde{v}_t csy_t\right) - e^{\varepsilon_{t-1}^X} xc_{t-1} \left[n_{t-1}^X + \tfrac{n_{t-1}^X}{n_{t-1}} n_{t-1}^E\right] + r_{t-1}^d dep_t = \\ c_t + i_t + a_t(u_t) k_t + \varepsilon_t^g + r_t^L \tau_h n_t^E e^{\varepsilon_t^E} \left(f^E + ec_t\right) + n_t^E e^{\varepsilon_t^E} ec_t. \end{aligned}$$

The zero profit condition of the competitive bank implies $\left(r_t^L - r_{t-1}^d\right) loan_t = w_t m_t + csy_t n_t \widetilde{v}_t$, which can be used in the previous expression to obtain

$$w_t n_t \widetilde{l}_t + \left[r_t^k n_t \widetilde{k}_t\right] + \left(r_t^L - r_{t-1}^d\right) loan_t + n_t \widetilde{d}_t - e^{\varepsilon_{t-1}^X} xc_{t-1} \left[n_{t-1}^X + \frac{n_{t-1}^X}{n_{t-1}} n_{t-1}^E\right] + r_{t-1}^d dep_t =$$
$$c_t + i_t + a_t(u_t) k_t + \varepsilon_t^g + r_t^L \tau_h n_t^E e^{\varepsilon_t^E} \left(f^E + ec_t\right) + n_t^E e^{\varepsilon_t^E} ec_t,$$

and with the balance sheet relation (ignoring bank reserves) that clears deposits market, $loan_t = dep_t$, I have

$$w_t n_t \widetilde{l}_t + \left[r_t^k n_t \widetilde{k}_t\right] + r_t^L loan_t + n_t \widetilde{d}_t - e^{\varepsilon_{t-1}^X} xc_{t-1} \left[n_{t-1}^X + \frac{n_{t-1}^X}{n_{t-1}} n_{t-1}^E\right] =$$
$$c_t + i_t + a_t(u_t) k_t + \varepsilon_t^g + r_t^L \tau_h n_t^E e^{\varepsilon_t^E} \left(f^E + ec_t\right) + n_t^E e^{\varepsilon_t^E} ec_t.$$

The average dividend of firms that produce intermediate goods, $\widetilde{d}_t = \widetilde{\rho}_t \widetilde{y}_t - w_t \widetilde{l}_t - r_t^k \widetilde{k}_t - r_t^L \widetilde{loan}_t^f$, can be substituted in the previous expression to obtain (after cancelling out terms),

$$r_t^L loan_t + n_t \left(\widetilde{\rho}_t \widetilde{y}_t - r_t^L \widetilde{loan}_t^f\right) - e^{\varepsilon_{t-1}^X} xc_{t-1} \left[n_{t-1}^X + \frac{n_{t-1}^X}{n_{t-1}} n_{t-1}^E\right] =$$
$$c_t + i_t + a_t(u_t) k_t + \varepsilon_t^g + r_t^L \tau_h n_t^E e^{\varepsilon_t^E} \left(f^E + ec_t\right) + n_t^E e^{\varepsilon_t^E} ec_t.$$

where recalling $n_t \widetilde{\rho}_t \widetilde{y}_t = y_t$, $loan_t^h = \tau_h n_t^E e^{\varepsilon_t^E} \left(f^E + ec_t\right)$ and $loan_t = loan_t^h + n_t \widetilde{loan}_t^f$, I get the market-clearing expression for final-good output,

$$y_t = c_t + i_t + a_t(u_t) k_t + \varepsilon_t^g + e^{\varepsilon_{t-1}^X} xc_{t-1} \left[n_{t-1}^X + \frac{n_{t-1}^X}{n_{t-1}} n_{t-1}^E\right] + n_t^E e^{\varepsilon_t^E} ec_t,$$

which in log-linear terms around a detrended steady state with zero inflation is

$$\widehat{y}_t = \frac{c}{y}\widehat{c}_t + \frac{i}{y}\widehat{i}_t + \frac{r^k k}{y}\widehat{u}_t + \frac{g}{y}\varepsilon_t^g + \frac{xc(\delta_n/(1-\delta_n))}{y}\left(\varepsilon_{t-1}^X + \widehat{xc}_{t-1} + \widehat{n}_{t-1}^X + \delta_n\left(\widehat{n}_{t-1}^E - \widehat{n}_{t-1}\right)\right) + \frac{ec(\delta_n/(1-\delta_n))}{y}\left(\widehat{n}_t^E + \varepsilon_t^E + \widehat{ec}_t\right),$$

with

$$\widehat{xc}_{t-1} = \varsigma_2^X \left(\widehat{n}_{t-1}^X - \widehat{n}_{t-1}\right),$$

and

$$\widehat{ec}_t = \varsigma_2^E \left(\widehat{n}_t^E - \widehat{n}_t\right).$$

D. The DSGE model with entry-exit and banking

Set of log-linearized (80) dynamic equations for fluctuations around the detrended steady state:

Law of motion for total number of goods:

$$\widehat{n}_{t+1} = \widehat{n}_t^A + \delta_n \left(\widehat{n}_t^E - \widehat{n}_t\right), \tag{D1}$$

where $\delta_n = \frac{n^X}{n}$ is the steady-state exit rate. Decomposition between surviving and exiting goods:

$$\widehat{n}_t = \left(1 - \delta_n\right) \widehat{n}_t^A + \delta_n \widehat{n}_t^X. \tag{D2}$$

Output decomposition between intensive and extensive margin of fluctuations:

$$\widehat{y}_t = \widehat{n}_t + \widehat{\widetilde{y}}_t, \tag{D3}$$

where the loglinearized definition of the intensive margin yields

$$\widehat{\widetilde{y}}_t = \widehat{\widetilde{\rho}}_t + \widehat{\widetilde{y}}_t. \tag{D4}$$

Entry dynamic equation:

$$\widehat{\widetilde{v}}_t = \varepsilon_t^E + \tau_h \left(r_t^L - r^L\right) + \tfrac{ec}{f^E + ec}\widehat{ec}_t. \tag{D5}$$

Entry (congestion) costs

$$\widehat{ec}_t = \varsigma_2^E \left(\widehat{n}_t^E - \widehat{n}_t\right). \tag{D6}$$

Equity accumulation equation (portfolio investment):

$$\widehat{\widetilde{v}}_t = \overline{\overline{\beta}} v_1 E_t \left(\widehat{\widetilde{v}}_{t+1} + (csy_{t+1} - csy)\right) + \overline{\overline{\beta}} v_2 E_t \widehat{\widetilde{d}}_{t+1} + \overline{\overline{\beta}} \left(v_1 + v_2\right) \widehat{n}_t^A + \overline{\overline{\beta}} v_3 \left(\widehat{n}_t^X + \widehat{lv}_t\right) - (r_t - r) - \widehat{n}_t, \tag{D7}$$

where $v_1 = \frac{\widetilde{v}}{(1-\delta_n)\left(\widetilde{d}+(1+csy)\widetilde{v}\right)+\delta_n lv}$, $v_2 = \frac{\widetilde{d}}{(1-\delta_n)\left(\widetilde{d}+(1+csy)\widetilde{v}\right)+\delta_n lv}$ and $v_3 = \frac{\delta_n lv/(1-\delta_n)}{(1-\delta_n)\left(\widetilde{d}+(1+csy)\widetilde{v}\right)+\delta_n lv}$. Average firm dividend is:

$$\widehat{\widetilde{d}}_t = \widehat{\widetilde{y}}_t + \theta_p \widehat{\widetilde{\rho}}_t - (\theta_p - 1)\widehat{\widetilde{mc}}_t. \tag{D8}$$

The liquidation value is:

$$\widehat{lv}_t = \varepsilon_t^X - \tfrac{xc}{lv}\widehat{xc}_t. \tag{D9}$$

Liquidation (exit) cost:

$$\widehat{xc}_t = \varsigma_2^X \left(\widehat{n}_t^X - \widehat{n}_t\right). \tag{D10}$$

Exit dynamics:

$$\widehat{n}_t^X = \widehat{n}_t + \kappa \left(\tfrac{1-\delta_n}{\delta_n}\right) \widehat{z}_t^c. \tag{D11}$$

Critical productivity for separating exit from surviving:

$$\begin{aligned} \widehat{z}_t^c \;=\; & \overline{\overline{\beta}} E_t \widehat{z}_{t+1}^c + \left(1 - \overline{\overline{\beta}}\right) E_t \widehat{\widetilde{mc}}_{t+1} + \frac{1-\widetilde{mc}\frac{\widetilde{z}}{z^c}}{\widetilde{mc}\frac{\widetilde{z}}{z^c}} \left(\widehat{lv}_t - \overline{\overline{\beta}} E_t \widehat{lv}_{t+1}\right) \\ & - \frac{\left(1-\overline{\overline{\beta}}\right)\left(1-\widetilde{mc}\frac{\widetilde{z}}{z^c}\right)}{\widetilde{mc}\frac{\widetilde{z}}{z^c}} \left(E_t \widehat{y}_{t+1} - \left(R_t - E_t \pi_{t+1}^c - r\right)\right) - \frac{\left(1-\overline{\overline{\beta}}\right)\left(1-\theta\left(1-\widetilde{mc}\frac{\widetilde{z}}{z^c}\right)\right)}{\widetilde{mc}\frac{\widetilde{z}}{z^c}} E_t \widehat{\widetilde{\rho}}_{t+1}. \end{aligned} \tag{D12}$$

Relative prices as a function of number of good varieties:

$$\widehat{\widetilde{\rho}}_t = \tfrac{1}{\theta_p - 1}\widehat{n}_t. \tag{D13}$$

Variety effect from producer price inflation to consumer price inflation:

$$\pi_t^c - \pi^c = (\pi_t - \pi) - \tfrac{1}{\theta_p - 1}\left(\widehat{n}_t - \widehat{n}_{t-1}\right). \tag{D14}$$

New-Keynesian Phillips curve from price-setting with Calvo sticky prices and indexation:

$$(1+\beta\iota_p)(\pi_t-\pi)=\iota_p(\pi_{t-1}-\pi)+\beta E_t(\pi_{t+1}-\pi)+\tfrac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p}\left(\widehat{\widehat{mc}}_t-\widehat{\widehat{\rho}}_t\right)+(1-\iota_p)\left(\varepsilon_t^P-\beta E_t\varepsilon_{t+1}^P\right). \tag{D15}$$

Mark-up definition:

$$\widehat{\widehat{\mu}}_t=\widehat{\widehat{\rho}}_t-\widehat{\widehat{mc}}_t. \tag{D16}$$

Real marginal cost:

$$\widehat{\widehat{mc}}_t=\tau_f\left(r_t^L-r^L\right)+(1-\alpha)\widehat{w}_t+\alpha\widehat{r}_t^k-\varepsilon_t^a. \tag{D17}$$

Consumption equation:

$$\widehat{c}_t=\tfrac{h/(1+\gamma)}{1+h/(1+\gamma)}\widehat{c}_{t-1}+\tfrac{1}{1+h/(1+\gamma)}E_t\widehat{c}_{t+1}-\tfrac{1-h/(1+\gamma)}{\sigma_c(1+h/(1+\gamma))}\left(r_t-r-\left(\varepsilon_t^b-E_t\varepsilon_{t+1}^b\right)\right). \tag{D18}$$

Taylor-type monetary policy rule with unconventional action:

$$R_t-R=\mu_R(R_{t-1}-R)+(1-\mu_R)\left[\mu_\pi(\pi_t^c-\pi^c)+\mu_y(\widehat{y}_t-\widehat{y}_t^p)+\mu_{efp}\widehat{efp}_t\right]+\mu_{\triangle y}(\triangle\widehat{y}_t-\triangle\widehat{y}_t^p)+\varepsilon_t^R. \tag{D19}$$

Goods market equilibrium:

$$\widehat{y}_t=\tfrac{c}{y}\widehat{c}_t+\tfrac{i}{y}\widehat{i}_t+\tfrac{r^kk}{y}\widehat{u}_t+\tfrac{g}{y}\varepsilon_t^g+\tfrac{xc(\delta_n/(1-\delta_n))}{y}\left(\varepsilon_{t-1}^X+\widehat{xc}_{t-1}+\widehat{n}_{t-1}^X+\delta_n\left(\widehat{n}_{t-1}^E-\widehat{n}_{t-1}\right)\right)+\tfrac{ec(\delta_n/(1-\delta_n))}{y}\left(\widehat{n}_t^E+\varepsilon_t^E+\widehat{ec}_t\right), \tag{D20}$$

Production technology for the average-productivity firm:

$$\widehat{\widehat{y}}_t=\alpha\widehat{\widehat{k}}_t+(1-\alpha)\widehat{\widehat{l}}_t+\varepsilon_t^a. \tag{D21}$$

Fisher equation:

$$r_t=R_t-E_t\pi_{t+1}^c. \tag{D22}$$

Wage inflation equation with Calvo-style rigidities and indexation:

$$\begin{aligned}(\pi_t^w-\pi^w)\quad=\quad&\iota_w\left(\pi_{t-1}^c-\pi^c\right)+\beta E_t\left(\pi_{t+1}^w-\pi^w\right)-\beta\iota_w(\pi_t^c-\pi^c)+\tfrac{(1-\beta\xi_w)(1-\xi_w)}{\xi_w(1+\theta_w\sigma_l)}(\widehat{mrs}_t-\widehat{w}_t) \qquad \text{(D23)}\\&+(1-\iota_w)\left(\varepsilon_t^W-\beta E_t\varepsilon_{t+1}^W\right),\end{aligned}$$

where the log-linearized household marginal rate of substitution is,

$$\widehat{mrs}_t=\sigma_l\widehat{l}_t+\sigma_c\left(\tfrac{1}{1-h/(1+\gamma)}\widehat{c}_t-\tfrac{h/(1+\gamma)}{1-h/(1+\gamma)}\widehat{c}_{t-1}\right), \tag{D24}$$

and the real wage dynamics are determined by the log-linear expression implied by its definition ($w_t=W_t/P_t^c$),

$$\widehat{w}_t=\widehat{w}_{t-1}+(\pi_t^w-\pi^w)-(\pi_t^c-\pi^c). \tag{D25}$$

Labor market equilibrium condition:

$$\widehat{l}_t=\tfrac{n\widetilde{l}}{n\widetilde{l}+m}\left(\widehat{n}_t+\widehat{\widehat{l}}_t\right)+\tfrac{m}{n\widetilde{l}+m}\widehat{m}_t. \tag{D26}$$

Capital market equilibrium condition:

$$\widehat{k}_t^s = \widehat{n}_t + \widehat{\widehat{k}}_t. \tag{D27}$$

As in Smets and Wouters (2007), the log-linearized investment equation is,

$$\widehat{i}_t = i_1 \widehat{i}_{t-1} + (1 - i_1) E_t \widehat{i}_{t+1} + i_2 \widehat{q}_t + \varepsilon_t^i, \tag{D28}$$

where $i_1 = \frac{1}{1+\overline{\overline{\beta}}/(1-\delta_n)}$, and $i_2 = \frac{1}{\left(1+\overline{\overline{\beta}}/(1-\delta_n)\right)\gamma^2 \varphi_k}$, and the value of capital goods (Tobin's q) is given, in log-linear terms by the arbitrage condition,

$$\widehat{q}_t = q_1 E_t \widehat{q}_{t+1} + (1 - q_1) E_t \widehat{r}_{t+1}^k - (r_t - r), \tag{D29}$$

where $q_1 = \frac{(1-\delta_k)}{\left(r^k+1-\delta_k\right)}$. Also, following Smets and Wouters (2007), the loglinear expression for capital accumulation is,

$$\widehat{k}_t = k_1 \widehat{k}_{t-1} + (1 - k_1) \widehat{i}_t + k_2 \varepsilon_t^i, \tag{D30}$$

where $k_1 = \frac{1-\delta_k}{1+\gamma}$ and $k_2 = \left(1 - \frac{1-\delta_k}{1+\gamma}\right)\left(1 + \overline{\overline{\beta}}/(1-\delta_n)\right)(1+\gamma)^2 \varphi_k$. Capital can be adjusted in the intensive margin (utilization rate) as well as the extensive margin,

$$\widehat{k}_t^s = \widehat{u}_t + \widehat{k}_{t-1}, \tag{D31}$$

and the log-linearized variable capital utilization rate is,

$$\widehat{u}_t = \left(\frac{1-\widetilde{\sigma}_a}{\widetilde{\sigma}_a}\right) \widehat{r}_t^k. \tag{D32}$$

Demand for capital at the firm level links the rental rate of capital to the marginal product of capital as follows

$$\widehat{r}_t^k = \widehat{w}_t - \left(\widehat{\widehat{k}}_t - \widehat{\widehat{l}}_t\right). \tag{D33}$$

Banking block. The real interest rate of loans:

$$r_t^L - r^L = \left(r_{t-1}^d - r^d\right) + \left(r^L - r^d\right)\left(\widehat{w}_t - \Omega(1-\chi)\left(\widehat{v}_t + \varepsilon_t^L - \widehat{m}_t\right)\right), \tag{D34}$$

total demand for loans

$$\widehat{loan}_t = \frac{\tau_f wn}{loan}\left(\widehat{w}_t + \widehat{n}_t + \widehat{\widehat{l}}_t\right) + \frac{\tau_f r^k nk}{loan}\left(\widehat{r}_t^k + \widehat{n}_t + \widehat{\widehat{k}}_t\right) + \frac{\tau_h n^E \left(f^E + ec\right)}{loan}\left(\varepsilon_t^E + \widehat{n}_t^E\right) + \frac{\tau_h n^E ec}{loan} \widehat{ec}_t, \tag{D35}$$

the loan production technology

$$\widehat{loan}_t = \Omega\left(\widehat{\widehat{v}}_t + \widehat{n}_t + \varepsilon_t^L\right) + (1-\Omega)\widehat{m}_t, \tag{D36}$$

the collateral service yield

$$csy_t - csy = csy\left(\widehat{w}_t + (\chi - 1)\left(\widehat{\widehat{v}}_t + \widehat{n}_t - \widehat{m}_t\right) + \chi \varepsilon_t^L\right), \tag{D37}$$

the external finance premium

$$efp_t - efp = \left(r_t^L - r^L\right) - \left(r_{t-1}^d - r^d\right) - \frac{(csy)(v)}{loan}\left(\frac{1}{csy}\left(csy_t - csy\right) + \widehat{\widehat{v}}_t + \widehat{n}_t - \widehat{loan}_t\right), \tag{D38}$$

the demand for deposits equation

$$r_t^d - r^d = r_t - r, \tag{D39}$$

and the market-clearing condition for the bank deposits market

$$\widehat{dep}_t = \widehat{loan}_t. \tag{D40}$$

Potential (natural-rate) block

Repeat all the equations with p superscript to denote the values reached under no rigidity on both price and wage adjustments, with the exceptions of the New Keynesian Phillips curve (D15) that is replaced by the constant price mark-up condition,

$$\widehat{\widehat{\rho}}_t^p = \widehat{\widehat{mc}}_t^p, \tag{D15^p}$$

and the wage inflation curve (D23) that is replaced by the constant wage mark-up condition,

$$\widehat{mrs}_t^p = \widehat{w}_t^p. \tag{D23^p}$$

Endogenous variables (80):

The following 40 variables: $\widehat{n}_{t+1}, \widehat{n}_t^E, \widehat{n}_t^X, \widehat{n}_t^A, \widehat{z}_t^c$, $\widehat{lv}_t$, $\widehat{xc}_t$, $\widehat{ec}_t$, $\widehat{\widehat{v}}_t$, $\widehat{\widehat{d}}_t$, $\widehat{\widehat{\rho}}_t$, $\widehat{\widehat{y}}_t$, $\widehat{y}_t$, $\widehat{c}_t$, $\widehat{i}_t$, $\widehat{u}_t$, $\widehat{q}_t$, $\widehat{k}_t$, $\widehat{k}_t^s$, $\widehat{l}_t$, $\widehat{\widehat{y}}_t$, $\widehat{\widehat{l}}_t$, $\widehat{\widehat{k}}_t$, $\widehat{\widehat{mc}}_t, \widehat{\widehat{\mu}}_t$, $r_t - r$, $R_t - R$, $\pi_t - \pi$, $\pi_t^c - \pi^c$, $\pi_t^w - \pi^w$, $\widehat{r}_t^k$, $\widehat{w}_t$, $\widehat{mrs}_t$, $r_t^L - r_t^L$, $r_t^d - r^d$, $\widehat{loan}_t$, $\widehat{dep}_t$, $csy_t - csy$, $\widehat{m}_t$ and $efp_t - efp$, and the same set with p superscript to bring the variables corresponding to the potential block.

Exogenous variables (10):

- technology shock: $\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \eta_t^a$ with $\eta_t^a \sim N\left(0, \sigma_{\eta^a}^2\right)$
- preference shock: $\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b$ with $\eta_t^b \sim N\left(0, \sigma_{\eta^b}^2\right)$
- monetary policy shock: $\varepsilon_t^R = \rho_R \varepsilon_{t-1}^R + \eta_t^R$ with $\eta_t^R \sim N\left(0, \sigma_{\eta^R}^2\right)$
- fiscal/NX shock: $\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \rho_{ga}\eta_t^a + \eta_t^g$ with $\eta_t^g \sim N\left(0, \sigma_{\eta^g}^2\right)$
- investment shock: $\varepsilon_t^i = \rho_i \varepsilon_{t-1}^i + \eta_t^i$ with $\eta_t^i \sim N\left(0, \sigma_{\eta^i}^2\right)$
- price-push shock: $\varepsilon_t^P = \rho_P \varepsilon_{t-1}^P - \mu_P \eta_{t-1}^P + \eta_t^P$ with $\eta_t^P \sim N\left(0, \sigma_{\eta^P}^2\right)$
- wage-push shock: $\varepsilon_t^W = \rho_W \varepsilon_{t-1}^W - \mu_W \eta_{t-1}^W + \eta_t^W$ with $\eta_t^W \sim N\left(0, \sigma_{\eta^W}^2\right)$
- cost of entry shock: $\varepsilon_t^E = \rho_E \varepsilon_{t-1}^E + \eta_t^E$ with $\eta_t^E \sim N\left(0, \sigma_{\eta^E}^2\right)$
- liquidation value (exit) shock: $\varepsilon_t^X = \rho_X \varepsilon_{t-1}^X + \eta_t^X$ with $\eta_t^X \sim N\left(0, \sigma_{\eta^X}^2\right)$
- financial collateral shock: $\varepsilon_t^L = \rho_L \varepsilon_{t-1}^L + \eta_t^L$ with $\eta_t^L \sim N\left(0, \sigma_{\eta^L}^2\right)$

Set of non-linear equations that define the detrended steady state

There are 30 endogenous variables: n, n^E, n^X, n^A, r, r^k, $\widetilde{v}$, $\widetilde{d}$, $\widetilde{mc}$, $\widetilde{\rho}$, $\widetilde{y}$, $\widetilde{k}$, $\widetilde{l}$, y, c, i, w, f_E, lv, z^c, $\widetilde{z}$, r^L, r^d, csy, efp, $loan$, dep, ec, xc and m. I normalize $z_{\min} = 1.0$ for the lower bound of firm-level productivity. The steady-state government spending, ε^g, is assumed to be fixed in the calibration at constant 0.18 share with respect to output, $\varepsilon^g = 0.18y$. The capital utilization rate is assumed to be equal to 100% in steady state ($u = 1$) and there are no costs of variable capital utilization ($a(u) = 0$). The household discount rate adjusted for balanced-growth path is $\beta\,(1+\gamma)^{-\sigma_c}$. The value of the share of sunk costs at entry/exit, the parameter τ, is left out to imply that the total number of goods in steady state is normalized at $n = 1$, and the value of the weight of disutility of hours (the parameter Ξ) is set to normalize the hours of labor working in the average-productivity firm at $\widetilde{l} = 1$ in the steady-state wage markup.In the banking sector, a and B are jointly calibrated to bring a collateral share$\frac{a(n\widetilde{v})^{\chi}}{a(n\widetilde{v})^{\chi}+(1-a)m^{\chi}}$ at 0.65 in loan production and a realistic ratio of loans over output, $loan/y$, also in steady state. In turn, the non-linear steady-state system to solve is

$$\widetilde{z} = \left(\tfrac{\kappa}{\kappa-(\theta_p-1)}\right)^{\frac{1}{\theta_p-1}} z_{\min}, \tag{SS1}$$

$$\begin{aligned}
n^E &= \frac{1-\left(\frac{z_{\min}}{z^c}\right)^{\kappa}}{\left(\frac{z_{\min}}{z^c}\right)^{\kappa}}n, && \text{(SS2)}\\
n^A &= \left(\tfrac{z_{\min}}{z^c}\right)^{\kappa} n, && \text{(SS3)}\\
\frac{n^X}{n} &= 1-\left(\tfrac{z_{\min}}{z^c}\right)^{\kappa} = \delta_n, && \text{(SS4)}
\end{aligned}$$

$$\begin{aligned}
r &= \beta^{-1}(1+\gamma)^{\sigma_c}-1, && \text{(SS6)}\\
r^k &= r+\delta_k, && \text{(SS7)}\\
\widetilde{v} &= \frac{\beta(1+\gamma)^{-\sigma_c}\left((1-\delta_n)\widetilde{d}+\delta_n lv\right)}{1-\beta(1+\gamma)^{-\sigma_c}(1-\delta_n)(1+csy)}, && \text{(SS8)}\\
lv &= \frac{\beta(1+\gamma)^{-\sigma_c}(1-\delta_n)}{1-\beta(1+\gamma)^{-\sigma_c}(1-\delta_n)}\left(1-mc\frac{\widetilde{z}}{z^c}\right)\widetilde{y}, && \text{(SS9)}\\
\left(1+r^L\tau_h\right)\left(f^E+ec\right) &= \widetilde{v}, && \text{(SS10)}\\
ec &= \varsigma_1^E\left(\tfrac{n^E}{n}\right)^{\varsigma_2^E} && \text{(SS11)}\\
xc &= \varsigma_1^X\left(\tfrac{n^X}{n}\right)^{\varsigma_2^X} && \text{(SS12)}\\
lv &= (1-\varphi)f_E-xc, && \text{(SS13)}\\
y &= c+i+\varepsilon^g+n^X\tfrac{xc}{1-\delta_n}+n^E ec, && \text{(SS14)}
\end{aligned}$$

$$\begin{aligned}
y &= n\widetilde{\rho}\widetilde{y}, && \text{(SS15)}\\
\widetilde{\rho} &= n^{\frac{\theta_p}{\theta_p-1}}, && \text{(SS16)}
\end{aligned}$$

$$\widetilde{d} = \left(\tfrac{1}{\theta_p}\right)\widetilde{y}, \tag{SS17}$$

$$\widetilde{y} = \widetilde{z}\left(\widetilde{k}\right)^{\alpha}\left(\widetilde{l}\right)^{1-\alpha}, \tag{SS18}$$

$$\frac{\widetilde{mc}}{\widetilde{\rho}} = \frac{\theta_p - 1}{\theta_p}, \tag{SS19}$$

$$\frac{\left(1+\tau_f r^L\right)}{\widetilde{z}}\left(\frac{w}{1-\alpha}\right)^{1-\alpha}\left(\frac{r^k}{\alpha}\right)^{\alpha} = \widetilde{mc}, \tag{SS20}$$

$$w = \frac{\theta_w}{(\theta_w - 1)}\frac{\Xi\left(n\widetilde{l}\right)^{\sigma_l}}{\left(c - h(1+\gamma)^{-1}c\right)^{-\sigma_c}}, \tag{SS21}$$

$$\widetilde{k} = \left(\frac{\alpha\widetilde{mc}\widetilde{z}}{r^k}\right)^{\frac{1}{1-\alpha}}, \tag{SS22}$$

$$i = (\gamma + \delta_k)\, n\widetilde{k}. \tag{SS23}$$

$$r^d = r, \tag{SS24}$$

$$r^L = r^d + \frac{wm}{loan}\frac{a\,(n\widetilde{v})^{\chi} + (1-a)m^{\chi}}{(1-a)m^{\chi}}, \tag{SS25}$$

$$csy = \left(r^L - r^d\right)\frac{loan}{v}\frac{a\,(n\widetilde{v})^{\chi}}{a\,(n\widetilde{v})^{\chi} + (1-a)m^{\chi}}, \tag{SS26}$$

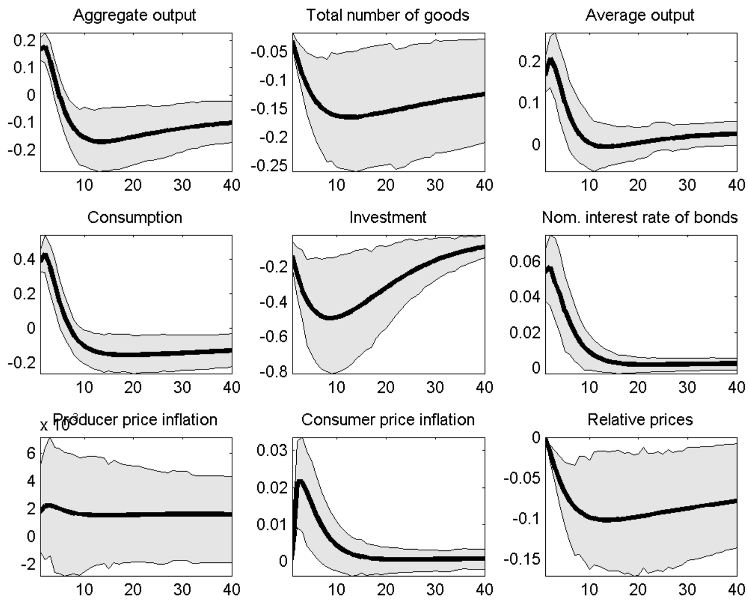
$$loan = B[a\,(n\widetilde{v})^{\chi} + (1-a)m^{\chi}]^{\frac{1}{\chi}}, \tag{SS27}$$

$$dep = loan, \tag{SS28}$$

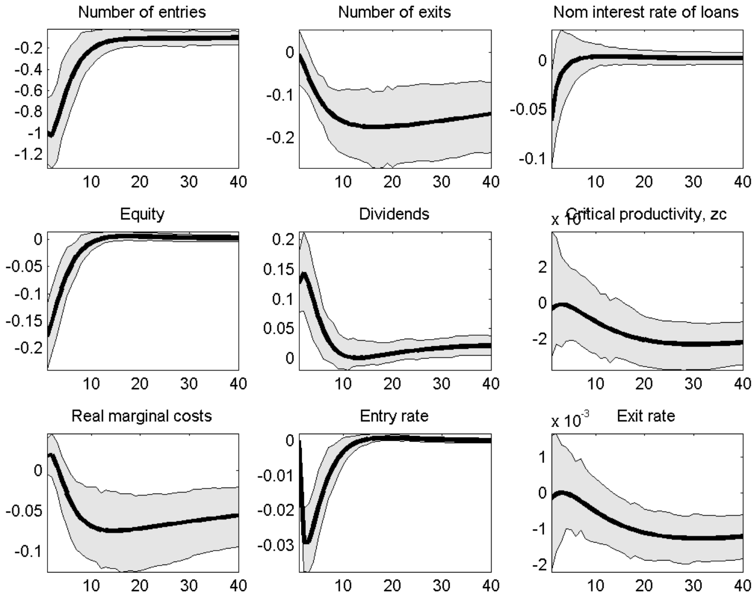
$$loan = \tau_h\left(f^E + ec\right)n^E + \tau_f\left(wn\widetilde{l} + r^k n\widetilde{k}\right), \tag{SS29}$$

$$efp = \left(r^L - r^d\right)\left(1 - \frac{a\,(n\widetilde{v})^{\chi}}{a\,(n\widetilde{v})^{\chi} + (1-a)m^{\chi}}\right) \tag{SS30}$$

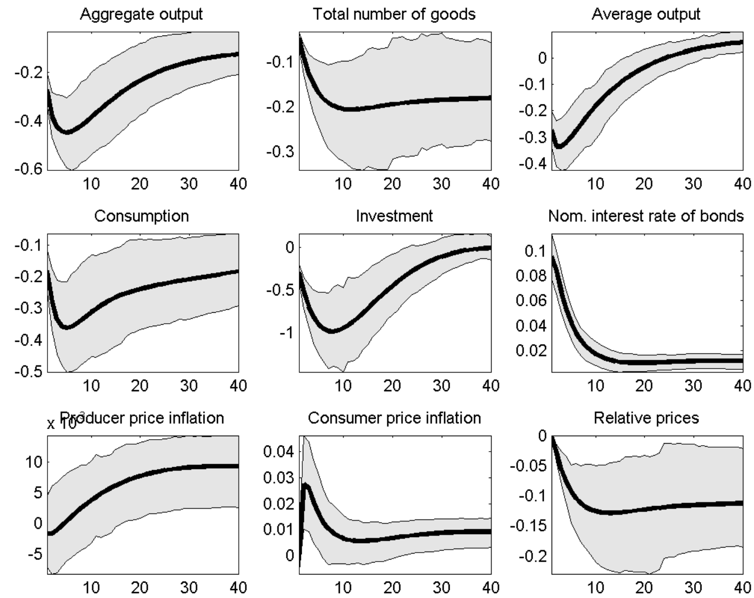
E. Impulse-response functions (that were not displayed in the main text)



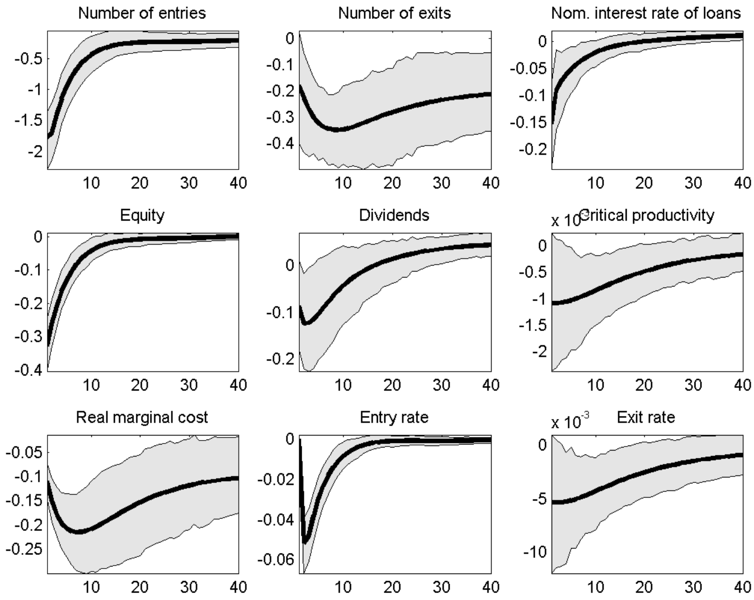
Preference shock, η^b.



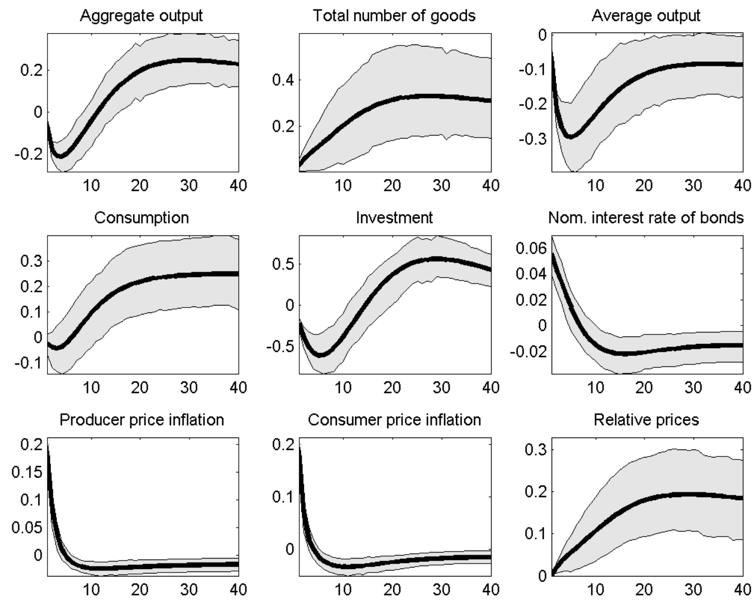
Preference shock, η^b, cont'd.



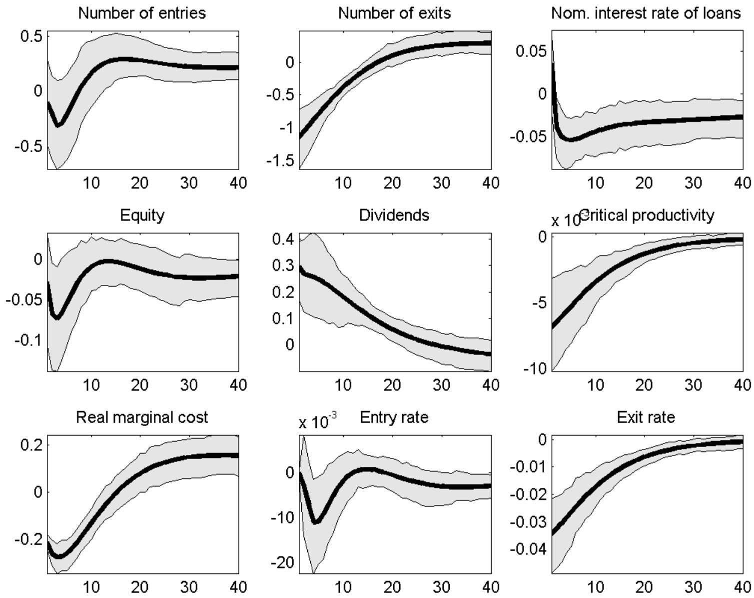
Monetary (interest-rate) shock, η^R.



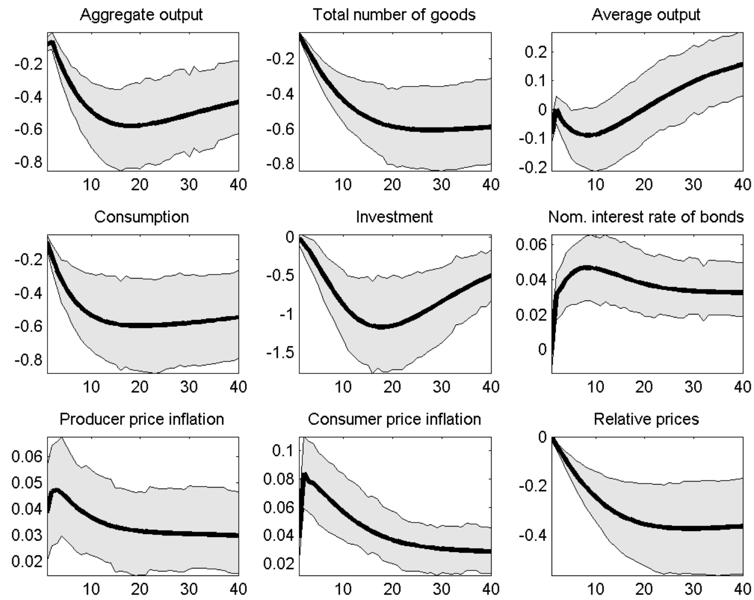
Monetary (interest-rate) shock, η^R, cont'd.



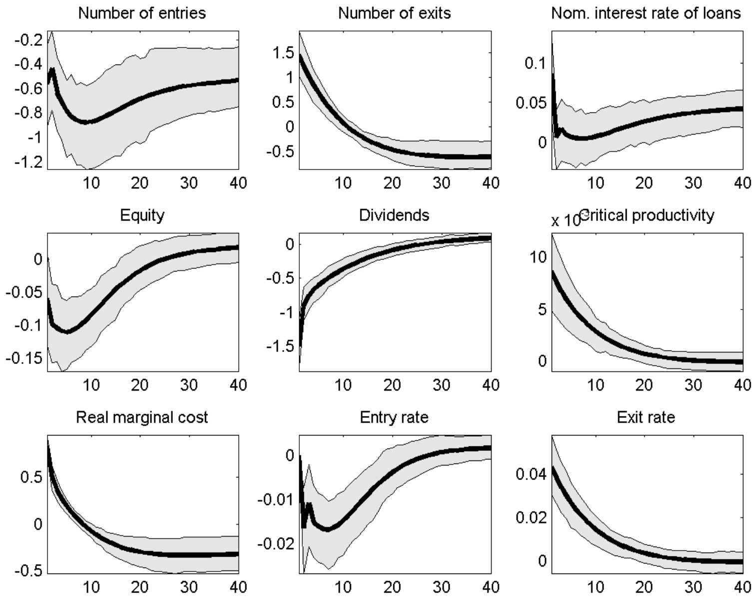
Price-push shock, η^P.



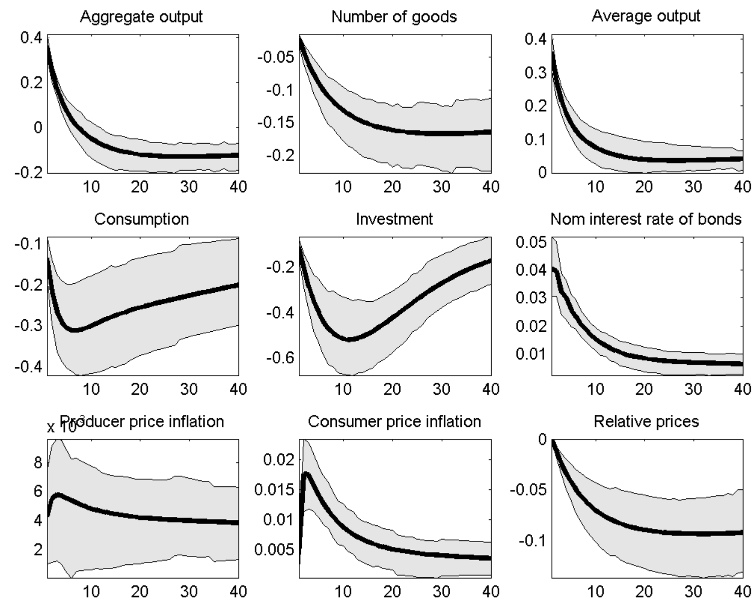
Price-push shock, η^P, cont'd.



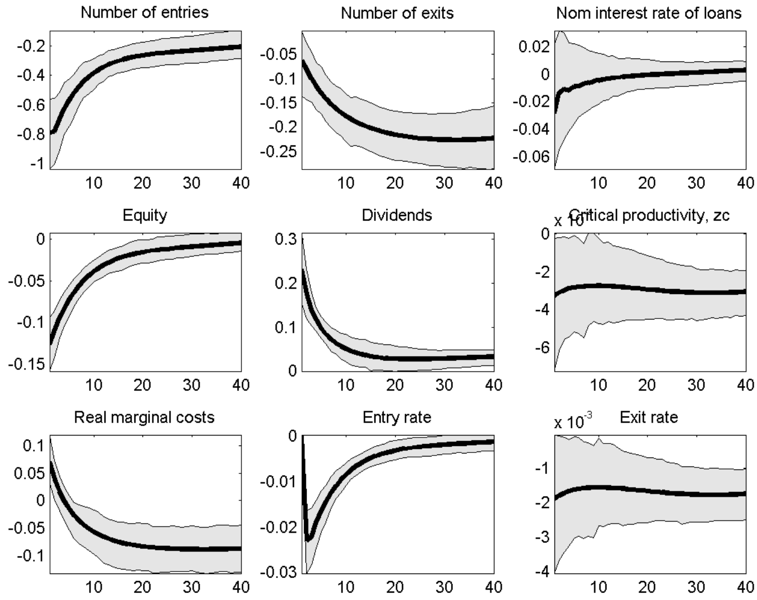
Wage-push shock, η^W.



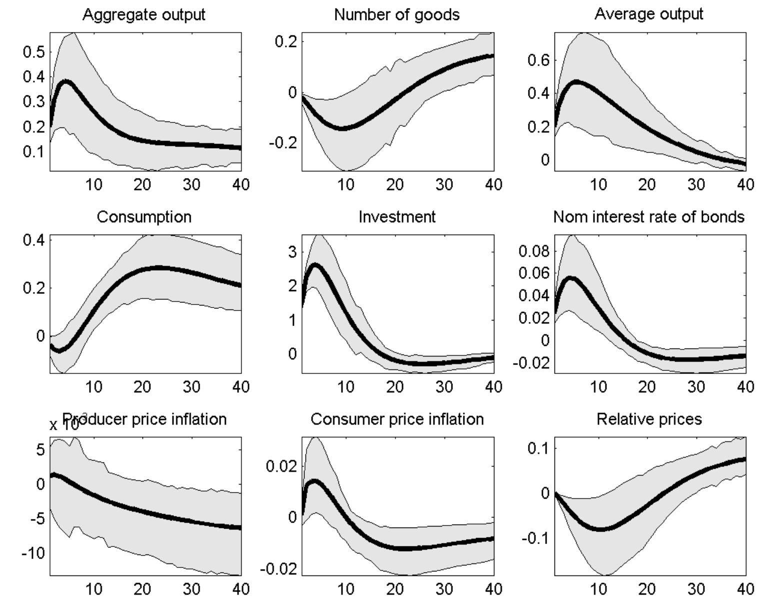
Wage-push shock, η^W, cont'd.



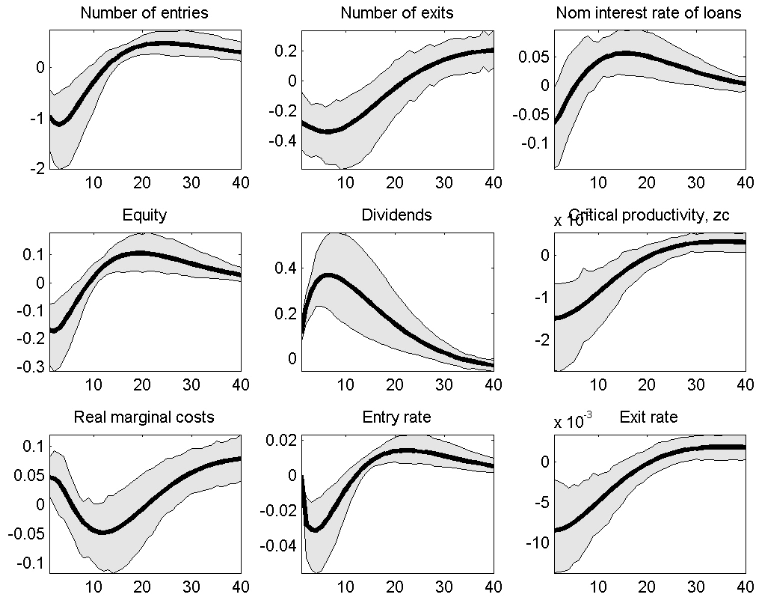
Fiscal/NX shock, η^g.



Fiscal/NX shock, η^g, cont'd.

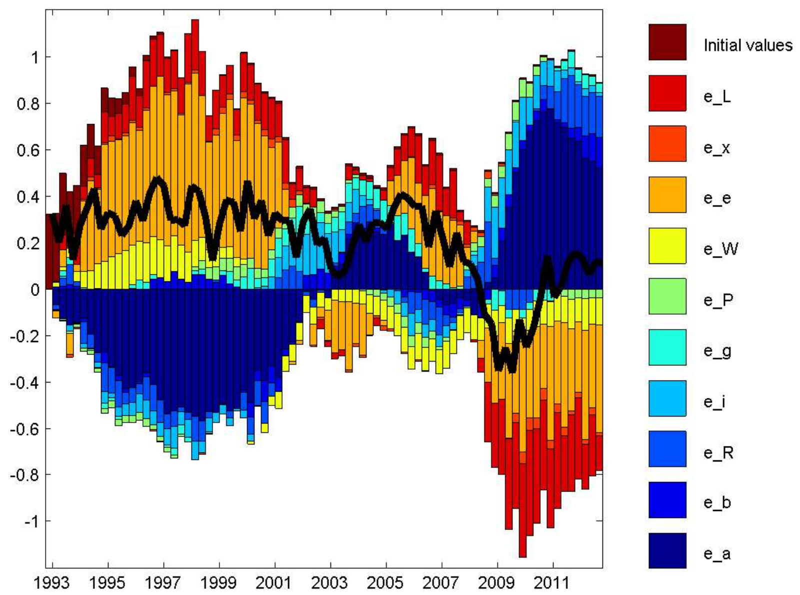


Investment shock, η^i.

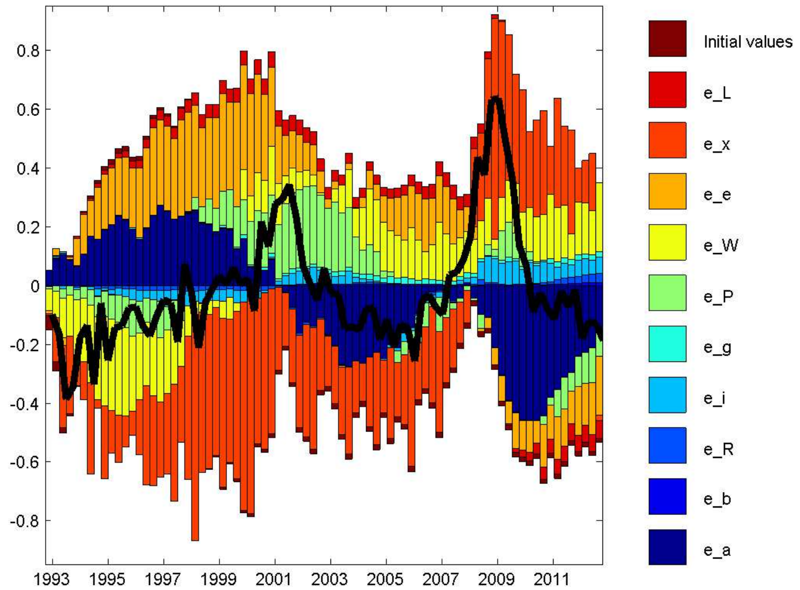


Investment shock, η^i, cont'd.

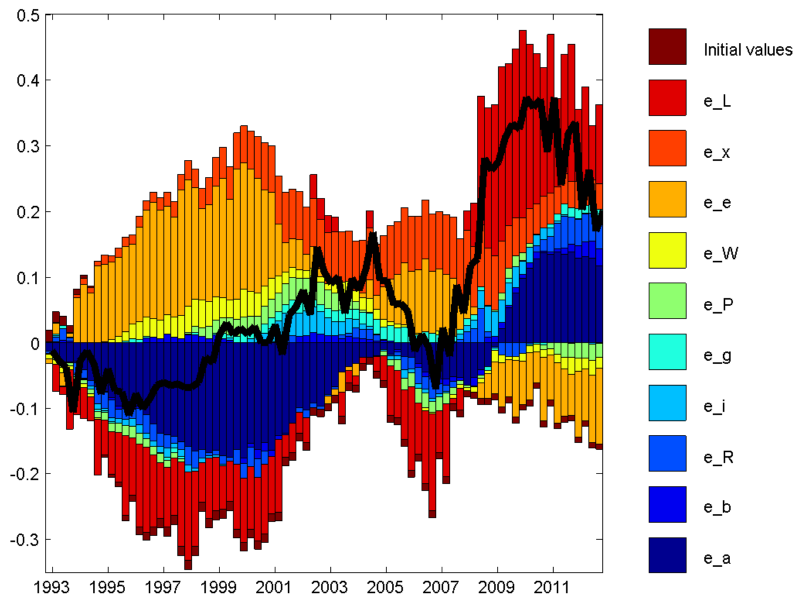
F. Quarterly shock decomposition for entry rates, exit rates and the external finance premium.



US (de-meaned) quarterly rate of entry.



US (de-meaned) quarterly rate of exit.



US quarterly (de-meaned) external finance premium.

G. Distribution of US private establishments according to their size

Shares of US private establishments

	1993	2002	2012
<5 employees	0.488	0.482	0.494
>5 and <49 employees	0.459	0.460	0.449
>50 and <500 employees	0.050	0.055	0.053
>500 employees	0.0026	0.0028	0.0026

Source: US Census Bureau, BDS.