A structural analysis of US entry and exit dynamics

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Abstract

We report empirical evidence indicating that US business formation has recently turned more volatile, procyclical and persistent due to changes in exit dynamics. To study these stylized facts, we estimate a DSGE model with endogenous entry and exit. Business units feature heterogeneous productivity and they shut down if the present value of expected future dividends falls below the current liquidation value. The estimation results imply structural changes in US exit dynamics after 2007: the semi-elasticity of the exit rate to critical productivity has increased and the average plant-level productivity has decreased.

Key words: Endogenous entry and exit, DSGE models, US business cycles.

JEL codes: E20, E32.

1 Introduction

A decline in business formation and entrepreneurial activity has characterized the US economy in recent years (Hathaway and Litan, 2014; Decker, Haltiwanger, Jarmin, and Miranda (2016),

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In this regard, we report two new stylized facts. First, business formation has turned more volatile, procyclical and persistent after the global financial crisis of 2007-08 than what had previously been observed during the Great Moderation period. Second, the dynamics of business destruction (exit) accounts for the previous facts substantially more than business creation (entry).

Following the seminal work of Bilbiie, Ghironi, and Melitz (2008), recent macroeconomic models have incorporated the extensive margin of aggregate fluctuations. The analysis conveyed in our paper belongs to the growing literature on sticky-price models, and extends the analysis of Lewis and Poilly (2012) and Lewis and Stevens (2015) to account for the above mentioned stylized facts. Lewis and Poilly (2012) consider the extensive margin in a Dynamic Stochastic General Equilibrium (DSGE) model to analyze monetary policy and business cycles. In addition, Lewis and Stevens (2015) provide a quantitative exploration of the extensive margin of activity by estimating a DSGE-style model with Bayesian econometrics. However, as a common feature, these papers assume a constant rate of exit which leaves business formation mostly driven by fluctuations in the rate of entry. The variability in the rate of exit observed in the data is, however, at odds with the constant rate of exit rate formulation.

Hence, the first contribution of our paper on the modelling side is to introduce an intertemporal exit rate decision that takes into account the liquidation value of the firm and expected dividends. Remarkably, while the entry of new firms/goods has been widely documented in DSGE models, few papers have proposed an analysis of firms exit. Recently, Cavallari (2015) and Hamano and Zanetti (2017) have proposed endogenous exit of firms motivated by reasons different than ours. These attempts, however, turn out to be not satisfactory in addressing the stylized facts in the post-2008 financial crisis period. In Hamano and Zanetti (2017), for example, the exit decision is based upon current value of profits and it neglects the liquidation cost associated with exit. This feature makes the exit decision intratemporal. In another recent paper, Cavallari (2015) considers scrap values, which are time-invariant and firm specific, in the exit decision. By contrast, we introduce heterogeneity through firm-specific productivity and the liquidation value varies over time. This formulation has the advantage that we can assess how both the heterogeneity of incumbents and changes in the liquidation value can shape the intertemporal exit decision.

In our model, the exit rate depends on the productivity threshold that gives a continuation

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1 There have been also many papers that incorporate business formation in a flexible-price setup such as Devereux, Head and Lapham (1996), Campbell (1998), Samaniego (2008), Bilbiie, Ghironi, and Melitz (2012), Clementi and Palazzo (2016), and Fattal-Jaef (2018).


3 An early working paper by Totzek (2009) developed a model with endogenous business destruction.
value higher than the liquidation value. At the end of the production period, the business unit
remains in the industry if the present value of all expected dividends exceeds the liquidation value.\footnote{Bernard et al. (2010) assume a similar decision making for exit in a model with multi-product firms.}
In the opposite case, the business unit exits the industry, the production of its variety ends and
there is business destruction. We also explore the connections between business formation and
inflation dynamics by incorporating the sticky-price structure \textit{a la} Calvo (1983), instead of using
Rotemberg (1982)’s price adjustment costs traditionally adopted in other papers with extensive
margin.\footnote{Cavallari (2015) also assumes Calvo (1983)-type price rigidities. However, in our setting, we reach a different
expression for the New Keynesian Phillips curve because of differences in the production technology for both existing
and new goods.}

The second contribution on our paper is that we provide a quantitative evaluation of the role
of endogenous entry and exit in DSGE models.\footnote{To our knowledge, this exercise is only conducted in a model with an extensive margin of activity by Lewis and Stevens (2015) but they assume a constant and exogenous exit rate.} Using Bayesian econometrics, we estimate our
model with US quarterly data between 1993 and 2016. The model performs well on replicating
business creation and destruction observed in recent US cycles: the posterior estimates generate
model simulations that provide a good matching on the second-moment statistics of US entry, exit
and net business formation. In the variance decomposition, the sources of fluctuations for entry
and exit are rather different. The entry rate fluctuations are mainly consequence of demand-side
shocks while supply-side shocks, by contrast, have a large impact on the exit rate. Shocks on the
entry cost and on the liquidation value are additional sources of US aggregate fluctuations that
were not accounted in conventional DSGE model. We find that they jointly explain 10% of the
variability of the quarterly rate of growth of US real GDP.

We also estimate the model for the Great Recession period (2006-2013). In line with what
is observed in the data, the exit dynamics of the model changes in the Great Recession. Thus,
we find that the sensitivity of the exit rate to the critical productivity rises substantially. This
makes the aggregate exit rate be more sensitive to the state of the economy as the number of
incumbents shutting down responds more aggressively to changes in market conditions (interest
rates, expected cost of production, expected demand, etc.).

There are microeconomic implications from our estimated macro model due to the hetero-
geney of firm-specific productivity. Actually, we identify the business units of our model (firms)
with establishments that produce a single differentiated good with a specific productivity. The
estimates of the structural parameters (using data on US private establishments) tell us on the
allocation and characteristics of US businesses. Hence, the increase in the exit rate elasticity after
2007 is explained by a higher estimated value of the shape parameter in the Pareto distribution of US establishments. This result indicates a concentration of establishments closer to the minimal productivity, i.e. there is a greater density of low-productivity establishments. We find that the estimated average productivity of US establishments falls from 1.72 in the full sample period to 1.49 during the Great Recession.

The rest of the paper is organized as follows. Section 2 outlines key stylized facts on business formation in the US cyclical fluctuations that motivate empirically our paper. Section 3 presents the baseline model with a special focus on the processes of business creation and destruction. Section 4 introduces the Bayesian estimation strategy and provides the posterior estimates of the structural parameters for the US economy taking the full sample period (1993-2016). Section 5 presents the analysis of second-moment statistics, impulse response functions and variance decomposition to evaluate the business cycle properties of the model. Section 6 carries out a quantitative exploration of the structural changes observed in entry and exit dynamics during the Great Recession (2007-2016). Section 7 concludes with the summary of the most relevant findings of the paper.

2 Empirical motivation

Figure 1 displays the evolution of business formation in the US from 1993 to 2016 through the plots of several quarterly time series obtained from the Business Employment Dynamics database released by the Bureau of Labor Statistics (BLS).\textsuperscript{7} Business units are represented in the data through establishments, which are defined by the BLS as “the physical location of a certain economic activity—for example, a factory, mine, store, or office. A single establishment generally produces a single good or provides a single service.”. Our choice of establishments to measure business formation is due to the fact that other candidates such as firms or enterprises may be the collection of multiple establishments.\textsuperscript{8} The creation or destruction of establishments within multi-establishment firms would have no effect observed in the data.

On the upper-left cell, Figure 1 plots Total Private Establishments (TPE). The quarterly change of TPE is the result of adding up the establishment births (Entry) and subtracting the establishment deaths (Exit).\textsuperscript{9} Taking the mean of TPE change along the whole period, we find

\textsuperscript{7} All the data from the Business Employment Dynamics report are available for free access in https://www.bls.gov/bdm/

\textsuperscript{8} The BLS indicates that "an enterprise (a private firm, government, or nonprofit organization) can consist of a single establishment or multiple establishments".

\textsuperscript{9} The BLS defines establishment births are as those establishments that had positive employment for the first
Figure 1: Quarterly time series from US business formation.
that 25,589 establishments were added on average every quarter, which represents 0.46% of total units 1993. The observations that belong to the 1990’s decade show strong and steady business formation: there was always positive net business creation from 1993 to the end of 2001. The last quarter of 2001 reports the first negative change in the number of establishments with a net business destruction of 2,200 establishments. As Figure 1 shows, the number of exits was catching up the number of entries from the beginning of the millennium and in 2001:4 the number of establishment closings increased by 7,000 units (from 212,000 to 219,000).\textsuperscript{10} During 2002, exit moves substantially down and a period of strong business formation emerges. This business expansion would last until 2007, fueled by low interest rates and the housing bubble, with net business creation of more than 30,000 establishments in many quarters of the housing boom (2003-06). The early stages of the financial crisis in 2007 were characterized by an increasing amount of establishment deaths while births still remained high for a while. The recession hit really hard on business destruction in the first two quarters of 2009, right after the financial turmoil and the credit crunch of the Fall of 2008 (the Lehman Brothers bankruptcy file happened on September 15th, 2008); the number of establishments destroyed in those two quarters were 58,600 and 64,100, respectively. The exit rate skyrocketed to 3.5% of total private establishments in 2008:4 (from 2.7% in 2006) while the entry rate moved downwards from 3.2% at the beginning of 2007 to 2.7% by the end of 2009 (see the lower-right panel of Figure 1). As 2009 went by, the number of exits fell to historically normal levels while entry showed signals of recovery. Over the remaining quarters (between 2010 and 2016), there is some net business creation (about 20,000 establishments per quarter). The lower-left panel of Figure 1 plots the rate of growth of TPE together with real GDP growth, both of them in per-capita terms.\textsuperscript{11} The plots indicate a much clearer co-movement between business formation and economic growth over the Great Recession period (2007-2016) than in earlier periods. Furthermore, net business creation occurs with some time lag (1-2 quarters) with respect to GDP growth.

Table 1 reports descriptive statistics of the rate of growth of TPE per capita, the rate of establishment entry and the rate of establishment exit for the total sample period (1996-2013), and for the subsamples that belong to either the Great Moderation (1996-2006) or the Great time in the third month of the current quarter with no link to the previous three quarters to exclude seasonal reopenings. Establishment deaths are establishments that reported positive employment in the third month of the previous three quarters and zero employment in the current quarter to include only closings that go out of business permanently, excluding seasonal shutdowns.

\textsuperscript{10}The September 11th tragic terrorist attacks occurred in the last quarter of 2001.

\textsuperscript{11}The series have been obtained in per capita terms because the observed series used in the model estimation come in per-capita terms to be consistent with the way macro variables are introduced in the model.
Recession (2007-2016). The rate of entry is calculated as the percent ratio between private sector establishment births and TPE, whereas the rate of exit is the percentage of establishment deaths over TPE. Figure 2 displays a rolling 10-year (40-quarter) window calculation of statistics of volatility (relative standard deviation), procyclicality (correlation with real GDP growth) and persistence (autocorrelation of order 1) for the same three variables reported in Table 1.

Table 1. Descriptive statistics from US business formation

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>Rate of growth of Total Private Establishments per capita</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean, %</td>
<td>0.11</td>
<td>0.21</td>
<td>-0.05</td>
</tr>
<tr>
<td>Std. deviation, %</td>
<td>0.36</td>
<td>0.27</td>
<td>0.43</td>
</tr>
<tr>
<td>Std. deviation wrt real GDP growth</td>
<td>0.60</td>
<td>0.55</td>
<td>0.64</td>
</tr>
<tr>
<td>Corr. with real GDP growth</td>
<td>0.34</td>
<td>0.03</td>
<td>0.42</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>0.37</td>
<td>0.00</td>
<td>0.49</td>
</tr>
</tbody>
</table>

| Rate of Establishment Entry   |                           |                               |                             |
| Mean, %                       | 3.21                      | 3.38                          | 2.97                        |
| Std. deviation, %             | 0.24                      | 0.12                          | 0.15                        |
| Std. deviation wrt real GDP growth | 0.40                      | 0.24                          | 0.23                        |
| Corr. with real GDP growth    | 0.34                      | -0.02                         | 0.03                        |
| Autocorrelation               | 0.87                      | 0.47                          | 0.65                        |

| Rate of Establishment Exit    |                           |                               |                             |
| Mean, %                       | 2.92                      | 2.96                          | 2.85                        |
| Std. deviation, %             | 0.21                      | 0.15                          | 0.26                        |
| Std. deviation wrt real GDP growth | 0.35                      | 0.31                          | 0.40                        |
| Corr. with real GDP growth    | -0.31                     | -0.33                         | -0.56                       |
| Autocorrelation               | 0.82                      | 0.67                          | 0.88                        |

The means of the three variables fall in the Great Recession (2007-2016) compared to the Great Moderation period 1993-2006. In fact, the average value of the quarterly rate of growth of TPE per capita becomes slightly negative during the Great Recession (-0.03%) from a 0.21% value within the expansionary period 1993-2006. The decline of the entry rate strongly contributes to the fall of the average quarterly business formation as its mean falls from 3.38% to 2.97% as a
signal of the decline of entrepreneurial activity in the US economy (Decker et al., 2016). The exit rate also falls, although it does so to a lesser extent from an average of 2.96% in 1993-2006 to 2.85% in the Great Recession.

Figure 2 and Table 1 show two remarkable stylized facts on the second-moment statistics. First, TPE growth turns more volatile, more procyclical and more persistent after the financial crisis of 2007. As the upper cells of Figure 2 show, the standard deviation of TPE growth (in relative terms to real GDP growth), the correlation with real GDP growth and the autocorrelation climb significantly in the 10-year subsample that begins around 1998 which precisely includes the financial crisis. As additional quarters of the Great Recession are included, the fluctuations of TPE growth report similar statistics. Thus, when only considering the Great Recession period (2007-2016) the relative standard deviation of TPE growth is higher than in the whole period. Similarly, the correlation between TPE and real GDP growth is higher in the Great Recession and the autocorrelation also becomes higher. Remarkably, there was no inertia in TPE growth before the financial crisis (see the upper-right cell of Figure 2).

12 As another piece of evidence on this, the average quarterly rate of growth of real GDP per capita within the whole sample period was 0.35%, whereas TPE growth per capita only grew at a quarterly average of 0.11%.
Second, the exit rate turns more volatile, more countercyclical and more persistent after the financial crisis of 2007. Table 1 reports these substantial changes in the standard deviation, the correlation with respect to real GDP growth and the autocorrelation of the exit rate during the Great Recession (2007-2016). Likewise, Figure 2 indicates increasing patterns for the exit relative volatility and persistence, and also a stronger opposite comovement between the entry rate and real GDP growth. Meanwhile, as Figure 2 shows, the business cycle patterns of volatility, cyclicality and inertia of the entry rate are not significantly different across the periods examined (they swing back towards the initial levels at the end of the period). Regarding volatility and autocorrelation, Table 1 reports very similar numbers for the entry rate across samples.\textsuperscript{13}

Taking these empirical findings together, the following conclusions emerge: the evolution of the exit rate in the Great Recession explains why TPE growth becomes more volatile, sensitive to the cycle and persistent, whereas the role of the entry rate in explaining these recent changes for TPE growth is minor. Hence, the exit rate becomes crucial to explain the increasing role of business formation on aggregate fluctuations during the Great Recession. Next, we develop a model with firm turnover that accounts for both endogenous entry and exit rates.

\section{A DSGE model with endogenous business formation}

The model represents an economy populated by households, firms, and the public sector (government and central bank). There are monopolistically competitive markets for goods and labor and perfectly competitive asset markets of capital, equity shares and government bonds. Households purchase bundles of consumption goods, set nominal wages, supply one specific labor service, own assets and decide on business creation and destruction. Remarkably, the number of varieties in the goods market changes over time as a result of flows of entry and exit of differentiated goods. Meanwhile, firms produce and set the price of a differentiated consumption good, and demand bundles of labor services and capital to be used in a technology with firm-specific productivity. Several sources of rigidities and frictions are assumed to enhance the empirical fit of the model following Christiano \textit{et al.} (2005) and Smets and Wouters (2007). The set of real rigidities include consumption habits, adjustment costs on investment, and variable capital utilization. Regarding nominal rigidities, we consider a fixed probability of not being able to optimally adjust both prices and wages as in Calvo (1983).

\textsuperscript{13}It may be noticed that the autocorrelation of the rate of establishment entry is higher in the whole sample period 1993-2016 than in either subsample. This fact can be explained by the downwards trend that the entry rate shows from 2002 to 2008 which splits up between the two subsample periods.
3.1 The extensive margin

Conventional DSGE models (e.g., Smets and Wouters, 2007) assume a constant number of production units and the only way for adjusting aggregate production is through the intensive-margin change in the amount produced by the fixed number of incumbents. We assume that the number of establishments can vary over time. This realistic assumption opens up the extensive margin for aggregate output fluctuations.

Following Bilbiie et al. (2012), each firm is specialized in the production of one specific good within a single location (establishment). It leads to a convenient setup in which the total number of goods, firms and establishments is the same. Formally, there are $n_{t-1}$ varieties of consumption goods (firms, establishments) at the end of period $t-1$. Over the next period $t$, households decide to shut down the production lines of $n^x_t$ goods (firms, establishments) which exit from the market, while the remaining $n^s_t$ goods (firms, establishments) survive and continue operating in the market. Hence, we have

$$n_{t-1} = n^x_t + n^s_t,$$

which implies the following exit rate

$$x_t = \frac{n^x_t}{n_{t-1}},$$

and its complimentary survival rate

$$1 - x_t = \frac{n^s_t}{n_{t-1}}.$$

Households decide how much to invest in the creation of new establishments.\(^{14}\) A one-period time-to-build requirement is assumed in the portfolio choice for establishment creation. Thus, the desired number of entries $n^e_t$ must be decided by households in period $t-1$. At the beginning of period $t$, the firm-specific productivity draws are released for all the new establishments. As we will show later, these draws determine the number of establishments that survive to the birth period, i.e. the number of effective entries $\frac{n^e_t}{n_{t-1}} n^e_t$. Therefore, applying the survival rate, $n^s_t / n_{t-1}$, to both the active lines of production at the end of the previous period and the new desired establishments brings this law of motion

$$n_t = \frac{n^x_t}{n_{t-1}} (n_{t-1} + n^e_t) = n^s_t + \frac{n^e_t}{n_{t-1}} n^x_t,$$

which gives the total number of productive establishments at the beginning of period $t$, $n_t$, as the number of incumbents that remain alive, $n^s_t$, plus the effective number of entries $\frac{n^e_t}{n_{t-1}} n^x_t$. The rate

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\(^{14}\)See the technical appendix for the optimizing programs of both households and firms.
of effective entry is defined as follows
\[ e_t = \frac{n_t^n e_t}{n_{t-1}}. \]  
(4)
The definitions of both \( e_t \) and \( x_t \) imply that the rate of growth of total establishments (net business formation) coincides with the difference between the effective entry rate and the exit rate,
\[ \frac{n_t - n_{t-1}}{n_{t-1}} = e_t - x_t \]
Next, let us describe separately the endogenous determination of the flows of entry and exit.

**Business creation (entry)**

Following Bilbiie et al. (2012), Casares and Poutineau (2014), Cavallari (2015), or Lewis and Stevens (2015), the free-entry decision is based on the comparison between the prospective equity value and the cost of entry. Unlike those papers, the cost of entry is not obtained here from the marginal cost of production or from any specific production function. We assume that the cost is a combination of the licence fee and start-up variable costs. In particular, we have this specification to determine the unit cost of business creation in period \( t \)
\[ \exp(\varepsilon_t^e) f^e + ec_t, \]
where \( \varepsilon_t^e \) is an AR(1) exogenous shock, \( f^e \) is the unit real cost of a license fee required by the government to begin the production of a new variety, and \( ec_t \) is a variable congestion cost for start-ups which increases with the desired entry rate as follows
\[ ec_t = \Theta \left( \frac{n_{t+1}^e}{n_t} \right)^\varsigma, \]  
(5)
setting parameter values \( \Theta > 0 \) and \( \varsigma > 1 \) for convexity.\(^{15}\) In the portfolio choice of the representative household (see technical appendix), the first order conditions of the number of entries and equity investment are, respectively,
\[ \lambda_t v_t = \beta E_t \lambda_{t+1} \left[ \frac{n_{t+1}^e}{n_t} (d_{t+1} + v_{t+1}) + \frac{n_{t+1}^e}{n_t} lv_{t+1} \right], \]  
(6)
\[ \lambda_t (\exp(\varepsilon_t^e) f^e + ec_t) = \beta E_t \lambda_{t+1} \left[ \frac{n_{t+1}^e}{n_t} (d_{t+1} + v_{t+1}) + \frac{n_{t+1}^e}{n_t} lv_{t+1} \right], \]  
(7)
where \( \lambda_t \) is the Lagrange multiplier of the budget constraint, \( v_t \) is the average (expected) equity value, \( \beta \) is the household discount factor, \( d_{t+1} \) is the expected dividend in period \( t + 1 \) if the firm

\(^{15}\)The desired entry rate in period \( t \) is expressed as \( n_{t+1}^e/n_t \) because households decide on business creation one period in advance.
survives and $lv_{t+1}$ is the expected liquidation value in period $t + 1$ if the firm dies. As observed in the right hand side of conditions (6) and (7), the expected marginal benefit combines the returns in the scenarios of survival, $E_t \left( \frac{n_{t+1}}{m_t} (d_{t+1} + v_{t+1}) \right)$, and death, $E_t \left( \frac{n_{t+1}}{m_t} lv_{t+1} \right)$ which are common for both newly created establishments and incumbents. The aggregate survival rate, $\frac{n_s_{t+1}}{n_t}$, and the aggregate exit rate, $\frac{n_x_{t+1}}{n_t}$, are, respectively, the probabilities of remaining and leaving the industry.

Combining (6) and (7) results in the free-entry equilibrium condition

$$\exp(e^e_t) f^e + ec_t = v_t,$$

which equates the marginal cost of entry to its expected marginal benefit. A log-linear approximation to the free entry condition (8) that incorporates the entry cost function (5) brings the following equation for log deviations of entry with respect to its steady state level

$$\hat{n}^e_{t+1} = \hat{n}_t + \frac{1}{\varsigma} (\frac{v}{ec} \hat{v}_t - \frac{f^e}{ec} \hat{e}^e_t),$$

where $v$ and $ec$ are the steady-state levels for average equity and the congestion cost of entry, respectively. Thus, households decide to raise their spending on the creation of new goods when they observe an increase in the average equity value, $\hat{v}_t$. Firm-specific productivity will be observed ex post and the expected return of new establishments is the current average equity value. By contrast, if the exogenous component of the cost of entry, $\hat{e}^e_t$, increases the number of new goods created by the households is going to fall. The elasticity of the congestion entry cost $\varsigma$ modulates the response of log fluctuations of desired entry, $\hat{n}^e_{t+1}$, to both driving factors, $\hat{v}_t$ and $\hat{e}^e_t$.

Using in (6) the first order condition of the government bond,

$$\frac{\lambda_t}{\exp(e^b_t) (1 + r_t)} = \beta E_t \lambda_{t+1},$$

where $r_t$ is the real interest rate of the government bond and $e^b_t$ is an exogenous risk-premium.

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16 The expected return is the same for incumbents and start-ups because our model does not contemplate the role of generating new ideas or investing on R&D as the motivating lines to create new goods or firms. Schumpeterian processes of creative destruction or Romer-type connections between the total number of varieties and possible gains in the stock of knowledge could be introduced as model extensions. In the Schumpeterian case, we could have a link between business churning (replacement of low-productivity firms for other new and more efficient businesses) and the technology shock. Alternatively, the innovations of technology shocks could also be affected by the rise of varieties as suggested by Romer (1990), or even Arrow (1962)'s learning-by-doing could be a link between technological growth and the accumulation of capital goods as a driver for knowledge spillover.

17 Throughout the paper, we follow the standard notation of variables topped with a hat sign “$\hat{}$” to denote log fluctuations with respect to its level in the balanced-growth steady state, whereas variables with no time subscript denote the steady state level along the balanced-growth path.
shock, gives the equilibrium equity value
\[ v_t = \frac{n^{t+1}_t (d_{t+1} + v_{t+1}) + n^{t+1}_x l v_{t+1}}{\exp (\varepsilon^n_t) (1 + r_t)}. \]
and its loglinear approximation is
\[ \hat{v}_t = \beta \gamma v_1 E_t \hat{n}_{t+1} + \beta \gamma v_2 E_t \hat{d}_{t+1} + \beta \gamma (v_1 + v_2) E_t \hat{n}^x_{t+1} + \beta \gamma v_3 E_t \left( \hat{n}^x_{t+1} + \hat{v}_{t+1} \right) - (r_t + \varepsilon^n_t) - \hat{n}_t. \tag{10} \]
The coefficients \( v_1, v_2, v_3 > 0 \) depend on the structural parameters, and \( \gamma > 0 \) is the long-run rate of economic growth.\(^{18}\)

For the entry rate equation, we can take a semi-loglinear approximation to its definition (4),\(^{19}\) and use the lagged version of the business creation dynamics (9) to obtain
\[ e_t - e = \frac{e^n}{\zeta} \left( \frac{\varepsilon^n}{\zeta} e_{t-1} - \frac{\varepsilon^n}{\zeta} e_{t-1} \right) + e \left( \hat{n}^n_t - \hat{n}_{t-1} \right). \tag{11} \]
Hence, equation (11) and a lagged version of equation (10) together govern the entry dynamics of the model.

**Business destruction (exit)**

As one of the main contributions of this paper, exit is endogenously determined from rational behavior. Single-good establishments (firms) produce with a specific productivity dealt from a Pareto distribution. The productivity draw, \( z \), is released after making the decision on entry and it remains for all future periods. The probability density function, \( g(z) \), of the Pareto distribution of productivity draws is
\[ g(z) = \begin{cases} \frac{\kappa z^\kappa}{z^\theta - 1}, & \text{if } z \geq z_{\text{min}} \\ 0, & \text{if } z < z_{\text{min}} \end{cases}, \]
where \( z_{\text{min}} \) is the minimum productivity and \( \kappa \) is a parameter that determines the density or concentration of establishments with low productivity (i.e., close to \( z_{\text{min}} \)). As proved in the technical appendix, the average establishment-level productivity is constant at the value
\[ \bar{z} = z_{\text{min}}^{\kappa} \left( \kappa \right)^{1/(\theta_p - 1)}, \]
which requires the technical constraint \( \kappa > (\theta_p - 1) \).

At the end of the production period, a survival test is faced at each incumbent establishment. Those single goods produced under low-efficiency technologies are at risk of business termination

\(^{18}\)In particular, \( v_1 = \frac{e^n}{(1-\delta_n)(d+e)+\delta_n e} \), \( v_2 = \frac{d}{(1-\delta_n)(d+e)+\delta_n e} \) and \( v_3 = \frac{\delta_n e/(1-\delta_n)}{(1-\delta_n)(d+e)+\delta_n e} \). See the technical appendix for further details.

\(^{19}\)The semi-loglinear approximation to (4) brings \( e_t - e = e \left( \hat{n}^n_t - \hat{n}_{t-1} \right) + e \left( \hat{n}^x_t - \hat{n}_{t-1} \right) \).
due to the lack of profitability over the prospective business cycles. Concretely, if the present value of all expected dividends exceeds the liquidation value, the owner of the establishment (the representative household) will decide continuation for the production line of the single good. In the opposite case, the household will decide to shut down the production of that variety. Hence, the representative establishment \( \omega \) at period \( t \) faces the following choice

\[
E_t \sum_{j=1}^{\infty} \beta_{t,t+j} s_{t,t+j} (\omega) d_{t+j} (\omega) \geq lv_t, \rightarrow \text{Survive},
\]

\[
E_t \sum_{j=1}^{\infty} \beta_{t,t+j} s_{t,t+j} (\omega) d_{t+j} (\omega) < lv_t, \rightarrow \text{Exit},
\]

where \( \beta_{t,t+j} \) is the stochastic discount factor between periods \( t \) and \( t+j \), \( s_{t,t+j} (\omega) \) is the probability of survival for a period that runs from \( t \) to \( t+j \), \( d_{t+j} (\omega) \) is the future dividend in period \( t+j \), and \( lv_t \) is the current liquidation value. At the margin, there would be a critical value of firm-level productivity, \( z^*_{t} (\omega) \), for which the expected dividend stream exactly coincides with the liquidation value,

\[
E_t \sum_{j=1}^{\infty} \beta_{t,t+j} s_{t,t+j} (\omega) d^*_{t+j} (\omega) = lv_t. \tag{12}
\]

The parity equation (12) holds for the representative establishment at one critical value of specific productivity, \( z^*_{t} (\omega) \). Assuming Dixit and Stiglitz (1977)’s monopolistic competition, we can rewrite (12) as follows

\[
E_t \sum_{j=1}^{\infty} \beta_{t,t+j} s_{t,t+j} (\omega) \left( \frac{P_{t+j} (\omega)}{P^c_{t+j}} \right)^{-\nu} y_{t+j} \left[ \frac{P_{t+j} (\omega)}{P^c_{t+j}} - mc^*_{t+j} (\omega) \right] \right) \right) = lv_t, \tag{12'}
\]

where \( \frac{P_{t+j} (\omega)}{P^c_{t+j}} = \rho_{t+j} (\omega) \) is the relative price in terms of consumption bundles, \( y_{t+j} \) is the Dixit-Stiglitz aggregate output, and the critical value \( z^*_{t} (\omega) \) is embedded in the denominator of the real marginal cost, \( mc^*_{t+j} (\omega) \). Even though the firm-specific productivity of the \( \omega \) establishment, \( z (\omega) \), is constant, it is worth noticing that its productivity threshold, \( z^*_{t} (\omega) \), is time dependent due to the variability in expected relative prices, aggregate output, marginal costs, or the liquidation value that enters (12).

As proved in the technical appendix, \( mc^*_{t+j} (\omega) \) is proportional to the real marginal cost for the establishment that produces with the average productivity \( \bar{z} \)

\[
mc^*_{t+j} (\omega) = \frac{\bar{z}}{z^*_{t} (\omega)} \bar{mc}_{t+j}. \tag{13}
\]

Using (13) and loglinearizing (12'), we can obtain the following forward-looking loglinear equation for the fluctuations of \( z^*_{t} (\omega) \):

\[
\dot{z}^{cr} = z_1 \dot{z}^{cr}_{t+1} + z_2 \dot{E}_t \bar{mc}_{t+1} - z_3 E_t \dot{y}_{t+1} + \bar{z} \left( \dot{r} + \xi_t^b \right) - z_4 \dot{E}_t \hat{\dot{\rho}}_{t+1} + z_5 \left( \hat{lv}_t - \beta \gamma s \dot{E}_t \hat{\dot{v}}_{t+1} \right) \tag{14}
\]

\[\text{20}\text{The detailed derivation is available in the technical appendix. It should be noticed that in the original non-}

where \( z_i > 0 \) for \( i = 1, \ldots, 5 \) are coefficients that depend on the structural parameters.\(^{21}\) Equation (14) shows that the value of log fluctuations in the average expected real marginal costs, \( \hat{m}_{t+1} \), raises \( \hat{z}_t \). This occurs because the expected dividends will be lower. By contrast, increases in the expected aggregate demand, \( \hat{y}_{t+1} \), and a higher expected unit revenue from sales, \( \hat{\rho}_{t+1} = \hat{P}_{t+1} - \hat{P}_{t+1}^c \), reduce \( \hat{z}_t \) as they both raise the average expected dividend. If either the real interest rate, \( r_t \), or the exogenous risk premium, \( \varepsilon_b t \), goes up the present value of future dividends turns lower (higher discount factor) and the required critical productivity moves up. Finally, any increase in the current liquidation value relative to its next period’s expected value increases \( \hat{z}_t \) because of the higher return on exit.

The liquidation value is obtained as the non-sunk fraction of the licence fee paid at entry, \( f_e \), adjusted by a liquidation shock, \( \varepsilon_x t \),

\[
lv_t = \exp(\varepsilon_x t) (1 - \tau) f_e,
\]

where \( 0 \leq \tau \leq 1 \) is the sunk fraction of the entry fee. Taking logs in (15) results in this simple expression for log fluctuations of the liquidation value

\[
\hat{lv}_t = \varepsilon_x t,
\]

which leaves the dynamic equation for critical productivity (14) as follows

\[
\hat{z}_t = z_1 E_t \hat{z}_t + z_2 E_t \hat{m}_{t+1} + z_3 (r_t + \varepsilon_t^b) - z_4 E_t \hat{\rho}_{t+1} + z_5 (1 - \rho x \beta \gamma_s) \varepsilon_x t,
\]

introducing the coefficient of autocorrelation of the AR(1) liquidation shock \( |\rho_x| < 1 \). As in Ghironi and Melitz (2005), the critical productivity splits up the fraction of establishments that survive from those that exit at the end of period \( t \), according to the properties of the Pareto distribution. Thus, the exit rate, \( x_t = n_t^e / n_{t-1} \), depends positively on the productivity threshold \( z_t \)

\[
x_t = 1 - \left( \frac{z_{\text{min}}}{z_t} \right)^\kappa,
\]

which, in semi-loglinear terms, implies that exit rate deviations are given by this expression

\[
x_t - x = \kappa (1 - x) \frac{z_t}{z_t} \hat{z}_t \hat{z}_t
\]

where \( \kappa (1 - x) \) defines the semi-elasticity of the exit rate to the critical productivity.

Thus, equations (18) and (17) together govern the exit dynamics of the model.

\(^{21}\)The structural coefficients of (14) are

\[
z_1 = \beta \gamma s + \frac{\kappa (1 - \beta \gamma s) (\rho - \hat{m}_{t+1} \hat{z}_t \hat{z}_t)}{\hat{m}_{t+1} \hat{z}_t \hat{z}_t}, \quad z_2 = (1 - \beta \gamma s), \quad z_3 = \frac{(1 - \beta \gamma s)(\rho - \hat{m}_{t+1} \hat{z}_t \hat{z}_t)}{\hat{m}_{t+1} \hat{z}_t \hat{z}_t}, \quad z_4 = (1 - \beta \gamma s)(\rho - \hat{m}_{t+1} \hat{z}_t \hat{z}_t), \quad z_5 = \frac{(\rho - \hat{m}_{t+1} \hat{z}_t \hat{z}_t)}{\beta \gamma s \hat{m}_{t+1} \hat{z}_t \hat{z}_t}. \]

See the technical appendix for further details.
3.2 Aggregate prices and inflation

The endogenous determination of the number of goods (establishments), \( n_t \), allows for a distinction between the Consumer Price Index (CPI) and the Producer Price Index (PPI). The Dixit-Stiglitz aggregate price level is the CPI that results from the aggregation of prices across the differentiated goods

\[
P^c_t = \left[ \int_0^{n_t} P_t^{1-\theta_p}(\omega) d\omega \right]^{1/(1-\theta_p)}.
\]

Next, let \( \tilde{P}_t \) denote the PPI in period \( t \), obtained as the average price at the establishments producing with the average productivity, \( \tilde{z} \). As shown in the technical appendix, both price indices are related as follows\(^{22}\)

\[
P^c_t = \tilde{P}_t n_t^{1/(1-\theta_p)}.
\]

Hence, the price of the consumption bundle, \( P^c_t \), increases with the producer price index \( P_t \), and decreases with the number of goods \( n_t \). Provided the definition of the average relative price, \( \tilde{\rho}_t = \tilde{P}_t/P^c_t \), relation (19) implies that \( \tilde{\rho}_t \) (unit revenue for the establishment with average productivity) depends positively on the number of goods

\[
\tilde{\rho}_t = n_t^{1/(\theta_p-1)}.
\]

Establishments set prices conditional to a Calvo (1983) nominal rigidity scheme. Hence, there is a fixed probability \( 0 < \xi_p < 1 \) that the establishment cannot set the optimal prices. If that is the case, the establishment will automatically adjust the price by applying an indexation rule that depends on lagged inflation (with a weight \( \iota_p \)), and on the steady state rate of inflation affected by a price-push exogenous shock, \( \varepsilon^p_t \) (with a weight \( 1 - \iota_p \)).

The PPI rate of inflation is \( \pi_t = \tilde{P}_t - \tilde{P}_{t-1} \), evaluated at the establishments that produce with the average productivity \( \tilde{z} \). As proved in the technical appendix, the PPI inflation equation (New Keynesian Phillips curve) of the model is

\[
(\pi_t - \pi) = \frac{\iota_p}{(1+\beta\gamma s\iota_p)} (\pi_{t-1} - \pi) + \frac{\beta\gamma s}{(1+\beta\gamma s\iota_p)} E_t (\pi_{t+1} - \pi) + \frac{(1-\beta\gamma s\iota_p)(1-\xi_p)}{\xi_p s(1+\beta\gamma s\iota_p)} \left( m^c_t - \tilde{m}^c_t \right) + \frac{(1-\iota_p)}{(1+\beta\gamma s\iota_p)} (\varepsilon^p_t - \beta E_t \varepsilon^p_{t+1}),
\]

\(^{22}\)Recalling the definitions of both the CPI and the PPI, equation (19) can be divided by its lagged version to bring a link between the rate of CPI inflation, \( \pi^c_t \), and that of PPI inflation, \( \pi_t \), through the change in the number of goods:

\[
(1 + \pi^c_t) = \left( \frac{n_t}{n_{t-1}} \right)^{1/(1-\theta_p)} (1 + \pi_t).
\]
where $\tilde{mc}_t$ denotes the log deviation of their real marginal cost with respect to the steady-state level, and $\tilde{\rho}_t$ is the log fluctuation of relative prices defined above. In addition, $\beta$ is the household discount factor and $\gamma$ is the balanced-growth rate in steady state.

The inflation dynamics provided by (21) are hybrid between backward-looking due to the indexation rule on lagged inflation and forward-looking due to nominal rigidities on price setting. As in Bilbiie et al. (2008) and Lewis and Stevens (2015), the gap between fluctuations of the real marginal cost and relative prices, $\tilde{mc}_t - \tilde{\rho}_t$, drive inflation variability. However, the slope coefficient is inversely determined by the Calvo probability $\xi_p$, whereas in the cited papers there was a Rotemberg (1982)’s price adjustment cost parameter $\pi$. Moreover, the increase in the number of goods $n_t$ has some deflationary effect because of its relation to relative prices $\rho_t$ (variety effect). Finally, the term on the price-push shock, $\varepsilon_p^t - \beta \gamma s E_t \varepsilon_{t+1}^p$, brings the exogenous source for inflation variability.

### 3.3 Central bank and government

The monetary policy of the model is described by a Taylor (1993)-type rule, of the kind used in standard DSGE models. Thus, we follow Smets and Wouters (2007) and consider that the central bank adjusts the nominal interest rate to stabilize inflation and both the current and the change in the output gap, with a partial-adjustment pattern that includes lagged nominal interest rate to smooth down monetary policy actions,

\[
R_t - R = \mu_R (R_{t-1} - R) + (1 - \mu_R) \left[ \mu_\pi (\pi_t - \pi) + \mu_y (\tilde{y}_t - \tilde{y}_t^p) \right] + \mu_\Delta y \left[ (\tilde{y}_t - \tilde{y}_t^p) - (\tilde{y}_{t-1} - \tilde{y}_{t-1}^p) \right] + \varepsilon_t^R, \tag{22}
\]

where $\mu_\pi > 1.0$, $\mu_y, \mu_\Delta y \geq 0$, and $0 \leq \mu_R < 1$ is the policy smoothing parameter. Both the nominal interest rate and PPI inflation enter (22) as level deviations with respect to their steady state values. The central bank targets PPI inflation $\pi_t$, (and not CPI inflation $\pi_t^c$ that shapes the real interest rate) because optimal monetary policy in a sticky-price framework with business formation must stabilize the producer price level (Bilbiie et al., 2008). As additional ingredients of (22), $(\tilde{y}_t - \tilde{y}_t^p)$ is the output gap between the cyclical component of output $(\tilde{y}_t)$ and its potential (natural-rate) realization $(\tilde{y}_t^p)$, and $\varepsilon_t^R$ is an exogenous monetary policy shock.

Regarding the role of the government, its fiscal policy consists of holding the budget constraint,

\[
\varepsilon_t^g = \varepsilon_t + \exp(\varepsilon_t^f) p_t^c n_t^e - \exp(\varepsilon_t^f) (1 - \tau) f^c (n_t^e + (n_t^e / n_{t-1}) n_{t+1}^e) + \frac{b_t}{\exp(\varepsilon_t^b (1 + \tau))} - b_{t-1}, \tag{23}
\]

which implies that the exogenous public expenditures on consumption goods, $\varepsilon_t^g$, are financed

---

23Potential output is computed assuming fully-flexible prices and nominal wages.
within the period by either collecting lump-sum taxes, \( t_t \), by obtaining net revenues from selling operating licenses, \( f^e \exp(\varepsilon_t^e) n_{t+1}^E - \exp(\varepsilon_t^e) (1 - \tau) f^e (n_t^e + (n_t^e/n_{t-1}) n_t^e) \), and by newly issued bonds \( b_t \) that yield the real return \( \exp(\varepsilon_t) (1 + r_t) \) in the equilibrium of the bonds market.

### 3.4 Aggregate output

Using (20) in the Dixit-Stiglitz demand constraint, \( \tilde{y}_t = (\tilde{\rho}_t)^{\theta_p} y_t \), aggregate output can be related to the average establishment-level production as follows

\[
y_t = \frac{\theta_p}{\theta_p - 1} \tilde{y}_t.
\]

(24)

where noticing \( n_t^{\theta_p - 1} = n_t \left( n_t^{1 - \theta_p} \right) \) and plugging the expression for \( n_t^{\theta_p - 1} \) implied by (20), aggregate output can be decomposed as the number of establishments, \( n_t \), multiplied by firm-level production in terms of consumption bundles, \( \tilde{\rho}_t \tilde{y}_t \),

\[
y_t = n_t \tilde{\rho}_t \tilde{y}_t.
\]

(25)

The extensive margin for aggregate output fluctuations is given by \( n_t \) and the intensive margin is provided by \( \tilde{\rho}_t \tilde{y}_t \), measured at the firms with average productivity. Finally, the equilibrium condition for the market of bundles of consumption goods (overall resources constraint) is,

\[
y_t = c_t + i_t + a(u_t) k_{t-1} + \varepsilon_t^g + e c_t n_{t+1}^e,
\]

(26)

which displays the uses of aggregate income, \( y_t \), on consumption expenditures, \( c_t \), on private investment on capital goods, \( i_t \), on the adjustments costs of variable capital utilization, \( a(u_t) k_{t-1} \), on the exogenous net public spending that results from the fiscal policy, \( \varepsilon_t^g \), and on the total variable cost of planned entries, \( e c_t n_{t+1}^e \).

The complete model with business formation is written for short-run fluctuations as the set of log-linearized set of dynamic equations available in the technical appendix.

### 4 Estimation

The loglinearized model has been estimated in Dynare using the Bayesian estimation routine (Adjemian et al., 2011) with nine exogenous processes that bring model variability: seven AR(1)
shocks on technology, risk-premium, interest rates (monetary policy), investment adjustment costs, fiscal policy, entry cost and liquidation value; and two ARMA(1,1) shocks pushing on prices and nominal wages, respectively. The data used for the estimation consist of nine quarterly time series including the rates of establishment entry and exit obtained in the Business Employment Dynamics report as discussed in Section 2. All the other observable series are comparable to those used in the estimation of the DSGE model by Smets and Wouters (2007) with three differences. First, the CPI has been used to deflate the nominal series to be consistent with the model. Secondly, per-capita series were computed dividing the aggregate series by the US working age population adjusted by populational controls as released in the Current Population Survey of the Bureau of Labor Statistics (2013). And third, we have used the series of shadow nominal interest rate calculated by Wu and Xia (2016) in order to accommodate the effects of the unconventional balance-sheet policy actions in the zero lower bound scenario of the Great Recession.26

The sample period is constrained by data availability on US quarterly entry and exit, which runs from 1993:2 to 2016:2. Thus, we have 93 observations. Some parameters are fixed before the estimation. Following Smets and Wouters (2007), the rate of capital depreciation is set at \( \delta = 0.025 \), the labor elasticity of substitution at \( \theta_w = 3.0 \), and the government spending, \( \varepsilon^g \), is assumed to be 18% of aggregate output in steady-state, \( \varepsilon^g = 0.18y \). In addition, the capital utilization rate is assumed to be equal to 100% in steady state (\( u = 1 \)), when there are no costs of variable capital utilization (\( a(u) = 0 \)). The minimum firm-level productivity is normalized at \( z_{min} = 1.0 \). Finally, the parameter \( \tau \), which determines the share of sunk costs at entry (licence fee), is calibrated to imply that the total number of goods in steady state is normalized at \( n = 1 \). This leads to choosing the value \( \tau = 0.87 \).

Tables 4A and 4B display the priors and estimated posteriors found in the Bayesian estimation. The selection of priors mostly follows Lewis and Stevens (2015) and Smets and Wouters (2007), and we only mention here the few differences with these papers. Hence, the steady-state quarterly rate of exit is set at a prior of \( x = 2.92\% \) (slightly higher than the value assigned by Lewis and

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26Hence, the observed time series obtained for the estimation are the establishment entry rate, the establishment exit rate, the log difference in per-capita real GDP, the log difference in per-capita real Personal Consumption Expenditures, the log difference in per-capita real Private Fixed Investment, the log difference in the real wage obtained from the Average Hourly Earnings in the Nonfarm Business Sector, the rate of change in the GDP price deflator, the Wu and Xia (2016)’s shadow Federal Funds rate, and the log of Hours of per worker obtained as the product of the Average Weekly Hours in the nonfarm business sector and the Civilian Employment in the Nonfarm Business Sector divided by the Civilian Labor Force. The sources to obtain these series are the Business Employment Report from the BLS and the FRED database compiled by the St. Louis Fed. The measurement equations and data definitions are listed in a Table available in the technical appendix.
Stevens, 2015) in order to match the mean value obtained in US data. Regarding the variable cost of goods creation, a quadratic specification is assumed with $\zeta = 2$. Finally, the shape parameter of the Pareto distribution $\kappa$ takes a prior of 5.0, sufficiently high to meet the condition of the Pareto distribution for a well-behaved average productivity.27

27Our prior for $\kappa$ is considerably lower than the value $\kappa = 11.51$ assumed in the calibration of Hamano and Zanetti (2017). The reason is that they choose $\kappa$ to match the average US product destruction rate (6%), which is more than two times higher than the average US establishment exit rate considered here.

<table>
<thead>
<tr>
<th>Table 4A. Priors and estimated posteriors of DSGE model</th>
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<tbody>
<tr>
<td><strong>Priors</strong></td>
</tr>
<tr>
<td><strong>Distr</strong></td>
</tr>
<tr>
<td>$\zeta$: entry cost elasticity</td>
</tr>
<tr>
<td>$\pi^c ec/y$: steady-state entry cost over output</td>
</tr>
<tr>
<td>$K$: exit shape</td>
</tr>
<tr>
<td>$x$: steady-state exit rate</td>
</tr>
<tr>
<td>$\theta_P$: Dixit-Stigitz elasticity</td>
</tr>
<tr>
<td>$\mu_\pi$: inflation in Taylor rule</td>
</tr>
<tr>
<td>$\mu_y$: output gap in Taylor rule</td>
</tr>
<tr>
<td>$\mu_{\Delta y}$: output gap change in Taylor rule</td>
</tr>
<tr>
<td>$\mu_{\Pi}$: inertia in Taylor rule</td>
</tr>
<tr>
<td>$\xi_P$: Calvo price rigidity</td>
</tr>
<tr>
<td>$\xi_w$: Calvo wage rigidity</td>
</tr>
<tr>
<td>$\lambda_P$: price indexation</td>
</tr>
<tr>
<td>$\lambda_w$: wage indexation</td>
</tr>
<tr>
<td>$h$: consumption habits</td>
</tr>
<tr>
<td>$\sigma_c$: risk aversion</td>
</tr>
<tr>
<td>$\sigma_f$: inverse Frisch elasticity</td>
</tr>
<tr>
<td>$\varphi_K$: capital adj. cost elasticity</td>
</tr>
<tr>
<td>$\sigma_a$: capital utilization cost elasticity</td>
</tr>
<tr>
<td>$\alpha$: capital share in production</td>
</tr>
</tbody>
</table>
Table 4B. Priors and estimated posteriors of DSGE model, cont’d

<table>
<thead>
<tr>
<th>Distr</th>
<th>Priors</th>
<th>Mean</th>
<th>Std D.</th>
<th>Mean</th>
<th>90% HPD interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma_{ep}): Std. dev. of technology innov.</td>
<td>Invgamma</td>
<td>0.10</td>
<td>2.00</td>
<td>0.70</td>
<td>[0.60, 0.79]</td>
</tr>
<tr>
<td>(\sigma_{en}): Std dev of risk-premium innov.</td>
<td>Invgamma</td>
<td>0.10</td>
<td>2.00</td>
<td>0.16</td>
<td>[0.13, 0.20]</td>
</tr>
<tr>
<td>(\sigma_{eR}): Std dev of monetary innov.</td>
<td>Invgamma</td>
<td>0.10</td>
<td>2.00</td>
<td>0.13</td>
<td>[0.11, 0.15]</td>
</tr>
<tr>
<td>(\sigma_{ef}): Std dev of fiscal innov.</td>
<td>Invgamma</td>
<td>0.10</td>
<td>2.00</td>
<td>1.99</td>
<td>[1.73, 2.23]</td>
</tr>
<tr>
<td>(\sigma_{ei}): Std dev of investment innov.</td>
<td>Invgamma</td>
<td>0.10</td>
<td>2.00</td>
<td>0.33</td>
<td>[0.26, 0.41]</td>
</tr>
<tr>
<td>(\sigma_{ep}): Std dev of price-push innov.</td>
<td>Invgamma</td>
<td>0.10</td>
<td>2.00</td>
<td>0.71</td>
<td>[0.41, 1.03]</td>
</tr>
<tr>
<td>(\sigma_{ew}): Std of wage-push innov.</td>
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<td>2.00</td>
<td>1.23</td>
<td>[1.03, 1.43]</td>
</tr>
<tr>
<td>(\sigma_{eR}): Std dev of entry cost innov.</td>
<td>Invgamma</td>
<td>0.10</td>
<td>2.00</td>
<td>1.19</td>
<td>[0.67, 1.83]</td>
</tr>
<tr>
<td>(\sigma_{eL}): Std dev of liquidation innov.</td>
<td>Invgamma</td>
<td>0.10</td>
<td>2.00</td>
<td>1.11</td>
<td>[0.46, 1.91]</td>
</tr>
<tr>
<td>(\rho_{a}): Autocorr. of technology shock</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.81</td>
<td>[0.77, 0.85]</td>
</tr>
<tr>
<td>(\rho_{b}): Autocorr. of risk-premium shock</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.94</td>
<td>[0.92, 0.96]</td>
</tr>
<tr>
<td>(\rho_{R}): Autocorr. of monetary shock</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.26</td>
<td>[0.14, 0.39]</td>
</tr>
<tr>
<td>(\rho_{G}): Autocorr. of fiscal shock</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.69</td>
<td>[0.58, 0.82]</td>
</tr>
<tr>
<td>(\rho_{i}): Autocorr. of investment shock</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.65</td>
<td>[0.51, 0.79]</td>
</tr>
<tr>
<td>(\rho_{p}): Autocorr. of price-push shock</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.45</td>
<td>[0.21, 0.72]</td>
</tr>
<tr>
<td>(\rho_{w}): Autocorr. of wage-push shock</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.57</td>
<td>[0.49, 0.65]</td>
</tr>
<tr>
<td>(\rho_{e}): Autocorr. of entry cost shock</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.83</td>
<td>[0.67, 0.97]</td>
</tr>
<tr>
<td>(\rho_{x}): Autocorr. of liquidation shock</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.54</td>
<td>[0.41, 0.69]</td>
</tr>
<tr>
<td>(\mu_{p}): MA(1) of price-push shock</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.49</td>
<td>[0.27, 0.76]</td>
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<tr>
<td>(\mu_{w}): MA(1) of wage-push shock</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.96</td>
<td>[0.94, 0.98]</td>
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<tr>
<td>(\rho_{ga}): cross effect tech.-fiscal</td>
<td>Beta</td>
<td>0.50</td>
<td>0.20</td>
<td>0.76</td>
<td>[0.55, 0.96]</td>
</tr>
</tbody>
</table>

We know discuss some of the results of the posterior estimates, in comparison to standard DSGE models with no business formation (Smets and Wouters, 2007)\(^{28}\):

- The estimate of the coefficient on household risk aversion, \(\sigma_c\), is low (0.79), falling significantly below its prior value (1.5). Subsequently, the elasticity of consumption intertemporal substitution, \(1/\sigma_c\), turns higher due to the procyclicality of business formation.
- The model delivers a lower estimate of price stickiness (\(\xi_p\) at 0.85) relative to that of wage

\(^{28}\)Obviously, we should take into account that the sample period of our estimation does not coincide with the one used in Samets and Wouters (2007).
stickiness ($\xi_w$ at 0.98).\(^{29}\) Therefore, the introduction of endogenous entry and exit results in greater wage rigidity relative to price rigidity. This finding can be theoretically justified on the effect of wage fluctuations on the exit decision through the real marginal cost. Wage adjustments should be moderate to avoid excessive aggregate fluctuations due to exit dynamics and its effect on the extensive margin. Also, the variety effect in the New Keynesian Phillips Curve (21), from the model with business formation, can amplify the variability of the price inflation dynamics and requires a lower Calvo probability to match the volatility of US inflation.

- The posterior mean estimate of the Dixit-Stiglitz demand elasticity is $\theta_p = 2.22$, substantially below its prior and also lower than the numbers typically used in the calibration of DSGE models.\(^{30}\) This result can be explained by the additional role that plays in the model with entry and exit. As implied by (24), the elasticity of aggregate output to variations in the total number of establishments is inversely related to $\theta_p$. The estimation method requires a small value for $\theta_p$ in order to replicate the relative volatility of TPE growth with respect to real GDP growth observed in US data.

- Regarding the parameters that characterize entry and exit dynamics (not present in standard DSGE models), the elasticity of the entry congestion costs is estimated at $\zeta = 2.78$, which is somewhat higher than the prior value that represents the quadratic adjustment costs. The size of the congestion cost of entry is estimated at 1.23% of output in the detrended steady state. The posterior estimate of the shape parameter in the Pareto distribution is substantially below its prior at $\kappa = 2.51$. Such low estimated $\kappa$ is in sharp contrast with the much higher number used in the calibration proposed in Hamano and Zanetti (2017) for a model without an intertemporal approach in the exit decision. Finally, the estimate of the steady-state exit rate is close to the prior rate at $x = 2.85\%$, which provides a good match to the sample mean of the quarterly exit rate of US private establishments. The posterior estimates imply a critical establishment-level productivity in steady state at $z^{cr} = 1.0116$ (1.16% higher than the minimum productivity $z_{\text{min}} = 1.00$), and an average productivity at $\bar{z} = 1.72$.

The parameters of volatility and persistence that characterize the exogenous processes of the model are reported in Table 4B. We are not doing a \emph{vis-à-vis} comparison among them because they do not enter the structural equations of the model with unit coefficients as in the VAR

\(^{29}\)The estimated Calvo probabilities for price and wage stickiness are numbers close to 1.0, which features the inflation moderation observed in recent US business cycles (Casares and Vázquez, 2018).

\(^{30}\)Nevertheless, Smets and Wouters (2007) report a posterior estimate for the steady-state mark-up in their canonical DSGE model of the US economy that corresponds to value of $\theta_p = 2.67$, rather similar to the one we have found in our estimated DSGE model with business formation.
methodology. Their role as sources of variability will be discussed below in the impulse-response and variance decomposition analyses.

5 Business cycle analysis

Table 5 reports the key second-moment statistics of the rate of growth of aggregate output, the rate of growth of the total number of establishments, the entry rate and the exit rate obtained from simulations of the estimated model and their comparison to actual US statistics:

<table>
<thead>
<tr>
<th></th>
<th>(\Delta y_t)</th>
<th>(\Delta n_t)</th>
<th>(e_t)</th>
<th>(x_t)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard deviation (relative to (\Delta y_t))</td>
<td>1.0</td>
<td>0.62</td>
<td>0.50</td>
<td>0.42</td>
</tr>
<tr>
<td>Correlation with (\Delta y_t)</td>
<td>1.0</td>
<td>0.39</td>
<td>0.26</td>
<td>-0.29</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>0.31</td>
<td>0.77</td>
<td>0.87</td>
<td>0.69</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>(\Delta y_t)</th>
<th>(\Delta n_t)</th>
<th>(e_t)</th>
<th>(x_t)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Deviations with respect to US statistics (model-data):</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard deviation (relative to (\Delta y_t))</td>
<td>-</td>
<td>+0.02</td>
<td>+0.10</td>
<td>+0.07</td>
</tr>
<tr>
<td>Correlation with (\Delta y_t)</td>
<td>-</td>
<td>+0.05</td>
<td>-0.08</td>
<td>+0.02</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>-0.05</td>
<td>+0.37</td>
<td>-0.00</td>
<td>-0.13</td>
</tr>
</tbody>
</table>

In the estimated model, the rate of growth of establishments and both the entry and exit rates are less volatile but more persistent than aggregate output growth. Both business formation and entry are procyclical, with a correlation with respect to output growth higher for establishment growth (0.39) than for the entry rate (0.26). Taking lagged output growth the cyclical correlation of both the entry rate and net business formation rise substantially (from 0.26 to 0.50 in the case of the entry rate and from 0.39 to 0.52 for the establishments growth).\(^{31}\) The exit rate is countercyclical with a negative cross correlation with output growth at -0.29.

Table 5 also compares the model statistics to the ones obtained in US data. The matching is fairly good because the quantitative differences are really small. We can mention a excessive

\(^{31}\) The model feature this lagged reaction of entry and business formation because of the time-to-build requirement assumed in the household choice of business creation. In US establishment data, there is also evidence on this. For the sample period 1993:2-2016:2, the cross correlation between TPE growth and lagged real GDP growth is 0.55 (21 basis points higher than their contemporaneous correlation), whereas the cross correlation between the establishment entry rate and lagged real GDP growth is 0.46 (12 basis points higher than the contemporaneous correlation).
persistence on establishment growth and a milder procyclicality of the entry rate as the most significant deviations between the model and the data. Anyway, the estimated model provides a good fit to second-moment statistics of US business formation, entry and exit, which makes it adequate for the analysis of the sources of fluctuations for these variables and its role on generating aggregate fluctuations of US real GDP.

**Impulse-response functions**

We examine the estimated impulse responses to supply-side shocks (Figure 3) and to demand-side shocks (Figure 4). All the shocks have been normalized at the values of the estimated standard deviations reported in Table 3B. In this exercise, we discuss how the flows of establishment entry and exit may affect the propagation of shocks on aggregate output fluctuations.

As Figure 3 shows, a technology shock increases the entry rate and reduces the exit rate, which results in net business formation that pushes aggregate output growth beyond the response observed at the firm level. In other words, the extensive margin amplifies the effect of a technology shock because of the procyclical reaction of net business formation. The rate of entry rises as households invest on business creation because average equity value moves up due to higher expected dividends and lower interest rates. Meanwhile, the exit rate goes downhill as the critical productivity $z^{cr}$ lowers with a declining real marginal cost, a higher expected aggregate output and a higher expected unit revenue at the firms (which depends positively on the number of varieties). As proved in the technical appendix, $z^{cr}$ is proportional to the average productivity of surviving firms, which makes their log fluctuations relative to steady state be identical. In Hamano and Zanetti (2017)’s flexible-price model with endogenous exit, this average productivity rises after a recessionary technology shock. This result can be replicated in our model by reversing the sign of the shock displayed in Figure 3.

The price-push shock corresponds to an unexpected increase in the price indexation factor used by the fraction of firms that cannot adjust the price optimally. In turn, the relative price charged by these firms increase and the critical productivity falls to reduce the exit rate. The rate of growth of establishments moves up in a countercyclical way (aggregate output falls because interest rates hike up when the central bank reacts to higher inflation). The average-productivity establishment reduces the amount produced. Therefore, the price-push shock results in some economic contraction with establishment redistribution, more business units with a lower amount produced at the establishment level.

A wage-push shock comes from an exogenous increase in the wage indexation rule. The quantitative effects are rather small (the lowest response of output growth across all the nine shocks of
Figure 3: Impulse-response functions from estimated supply-side shocks.
the model). The responses are the opposite to the price-push shock: the exit rate increases because the critical productivity goes up as a consequence of the higher costs of production. Meanwhile, the entry rate slightly falls due to the declining equity value.

The shock on the cost of entry moves down the entry rate with a one-period delay as a consequence of the time-to-build constraint in the household portfolio choice. Figure 3 displays the negative impact on aggregate output while the amount produced in the remaining firms increases to offset for the lack of business creation. The shock on the liquidation value increases the exit rate and also brings some reduction of aggregate output. Meanwhile, firm-level output rises as the number of incumbents moves down.

Figure 4 shows the responses obtained after demand-side shocks. Either a risk-premium or
an interest-rate shock shifts the aggregate demand down as household spending on consumption and investment goods turns lower. Such demand contraction results in a lower rate of growth for aggregate output, for the number of firms-establishments and also for the amount of firm-level output produced. The net business destruction is mostly explained by a procyclical reaction of the entry rate because the exit rate barely rises (see Figure 4).\textsuperscript{32} Such significant and persistent fall of business creation comes justified by the lower average equity value due to the combination of higher interest rates and lower expected dividends. Over the first quarters after the shock, the intensive margin absorbs the demand contraction and firms reduce their production with no significant business destruction. However, as time goes by, the number of establishments keeps falling because the entry rate needs around 15 quarters to return to its steady state level. Thus, the model with business formation reports a longer recession after risk-premium shocks in comparison to a conventional DSGE model with constant number of establishments. The variance decomposition examined below will corroborate this long-lasting influence of the risk premium shock (and also interest rate shocks) on aggregate output fluctuations.\textsuperscript{33}

The investment shock displayed in Figure 4 represents an exogenous reduction in the adjustments costs of capital accumulation, which makes it expansionary on both the demand and supply sides of the model. The response of aggregate output is mostly driven by the intensive margin as both firm-level output and aggregate output report similar rates of growth. The rate of establishment growth is mildly procyclical as the exit rate falls beyond the decrease of the entry rate. The lower entry rate is explained by a declining equity value that results from higher interest rates. Meanwhile, the lower exit rate is caused by a reduction in the expected real marginal cost and by a higher expected aggregate demand that together push the critical productivity downwards. Finally, the fiscal shock brings a one-period response of aggregate output that is almost fully absorbed through a higher intensive margin reaction at the existing establishments. The entry rate slightly decreases due to the lower average equity value (the fiscal expansion brings a higher interest rate that crowds out the portfolio investment on equity). Exit and the critical productivity show no significant responses to the fiscal shock.

\textit{Variance decomposition}

The sources of variability of US real GDP growth and establishment entry-exit rates are captured in the variance decomposition of the estimated model. Table 6 reports the percent shares

\textsuperscript{32}Exit does not react considerably because the critical productivity $z_{cr}$ responds in a erratic way as both the real marginal cost and aggregate output are falling to create opposing effects on it.

\textsuperscript{33}Furthermore, this long duration of the effects of risk-premium shocks shows up in the shock decomposition of the estimated model during the quarters that belong to the Great Recession period (2007-2016).
obtained for the contribution of each shock in the long-run variance decomposition (infinite forecasting horizon).\textsuperscript{34}

Demand-side shocks govern most of the fluctuations of the rates of growth of both aggregate and firm-level output, with more than 62% of their variance decomposition. A predominant role is observed for risk-premium shocks with 28.8% of aggregate output growth and 27.9% of firm-level output growth which reflects the large impact of these shocks during the financial crisis and the Great Recession (and their persistence on aggregate fluctuations as discussed in the estimated impulse-response functions). Both monetary and fiscal shocks are also very influential on fluctuations of US real GDP growth (17.2% and 15.6% of the variance decomposition, respectively) to show the active role of the Fed and the government on taking discretionary actions to stimulate or stabilize the economy. Remarkably, the investment adjustment cost shock has a minor influence on the variability of aggregate and firm-level activity (less than 1% share). Technology shocks explain around 24% of fluctuations in aggregate output growth and just 12% on firm-level output growth. The price-push and, especially, the wage-push shocks do not have a significant impact on US aggregate fluctuations. It is particularly striking that wage-push shocks have so little influence on recent US business cycles (less than 1% on aggregate output growth and 1.6 on firm-level output growth) as they were found quite influential over the Great Moderation period (Smets and Wouters, 2007).

The shocks to the entry cost and to the liquidation value have some impact on aggregate fluctuations. Thus, the shock of the entry cost explains 7.3% and 8.5% of variability of the quarterly growth rates of aggregate and firm-level output, respectively. Meanwhile, the liquidation value (exit) shock brings cyclical fluctuations that account for 2.7% of changes in output growth and 2.6% of changes in the establishment-level output growth.

\textsuperscript{34}Moreover, the technical appendix includes figures that display the quarterly shock decomposition for the US real GDP growth, TPE growth, the entry rate and the exit rate.
Table 6. Estimated variance decomposition (%)

<table>
<thead>
<tr>
<th>Output growth decomposition</th>
<th></th>
<th>Entry and exit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Aggregate output, $\Delta \hat{y}$</td>
<td>Establishments, $\Delta \hat{n}$</td>
</tr>
<tr>
<td>Supply-side shocks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Technology, $\eta^a$</td>
<td>23.8</td>
<td>11.6</td>
</tr>
<tr>
<td>Price-push, $\eta^p$</td>
<td>3.2</td>
<td>7.3</td>
</tr>
<tr>
<td>Wage-push, $\eta^w$</td>
<td>0.7</td>
<td>2.2</td>
</tr>
<tr>
<td>Entry cost, $\eta^e$</td>
<td>7.3</td>
<td>30.3</td>
</tr>
<tr>
<td>Liquidation, $\eta^r$</td>
<td>2.7</td>
<td>18.2</td>
</tr>
<tr>
<td>TOTAL</td>
<td>37.7</td>
<td>69.6</td>
</tr>
<tr>
<td>Demand-side shocks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk-premium, $\eta^b$</td>
<td>28.8</td>
<td>17.3</td>
</tr>
<tr>
<td>Interest rate, $\eta^R$</td>
<td>17.2</td>
<td>12.8</td>
</tr>
<tr>
<td>Investment, $\eta^i$</td>
<td>0.7</td>
<td>0.2</td>
</tr>
<tr>
<td>Fiscal, $\eta^g$</td>
<td>15.6</td>
<td>0.1</td>
</tr>
<tr>
<td>TOTAL</td>
<td>62.3</td>
<td>30.4</td>
</tr>
</tbody>
</table>

Supply-side shocks explain slightly more than 2/3 of the short-run variability of the rate of growth of the total number of establishments: technology shocks explain 11.6%, price shocks generate 7.3% of these changes (in a countercyclical pattern as displayed in Figure 3), entry costs take responsibility of 30.3% through its effect on business creation and the liquidation shocks bring 18.2% of variability of establishments growth because of their effect on business destruction. On the demand side, risk-premium and interest rate shocks have a relevant impact on establishment entry that take 17.3% and 12.8% of the total variability of net business formation. The role of investment and fiscal shocks is negligible.

When comparing the determinants of the entry and exit rates, we find that demand-side shocks explain 34.1% of the variability in the entry rate, mostly through the effects of risk-premium and interest rate shocks on equity. The fluctuations of equity value are rather sensitive to risk-premium and interest rate shocks because they have a large quantitative impact on the expected dividends, the expected aggregate demand and the discount factor. The exogenous driver of the entry rate (cost of entry shock) takes a significant 55.3% share of its variance decomposition. Price-push and technology shocks have also influence of entry rate variability with 6.5% and 3.7% shares,
respectively.

Regarding the exit rate variability, supply-side shocks mostly explain it (91.4%) as a combination of the effects from technology shocks (16.3%), price shocks (26.0%) and liquidation shocks (39.1%). We could say that these three shocks are responsible for most of the fluctuations of the critical productivity which drives exit variability. Demand-side shocks altogether only take 8.6% of the overall variance of the exit rate.

6 Exit and entry in the Great Recession

The empirical evidence provided in Section 2 indicates that short-run fluctuations of the quarterly rates of growth of US Total Private Establishments have turned more volatile, more persistent and more correlated with real GDP growth after 2007. Furthermore, the empirical role of exit for US business formation has been more important during the Great Recession than in previous periods as documented in Table 2 and displayed in Figure 2. Next, we carry out an structural analysis of the entry and exit dynamics to discuss the changes observed in the US economy during the Great Recession.\(^{35}\)

Our model with variable establishments determines the evolution in the number of incumbents as a result of the processes of entry and exit. The decisions of both creating and destroying establishments were carefully described in Section 3 and led to the dynamic equations for the entry rate (11) and the exit rate (18). The posterior mean estimates of the structural parameters can be used to calculate the numerical values of the coefficients that define the entry and exit dynamics. Using the numbers reported in Table 3A yields the estimated equations for the Full Sample (FS) period 1993:2-2016:2

\[
\begin{align*}
\hat{e}_t - e &= 0.43\hat{t}_{t-1} - 0.42x_{t-1}^e + 0.0293\left(\hat{n}_{t+1} - \hat{n}_t\right), \\
x_t - x &= 2.44\hat{x}_{t}^{cr},
\end{align*}
\]

which, respectively, imply a semi-elasticity of the change in the entry rate with respect to log fluctuations of lagged average equity at 0.43 and a semi-elasticity of the change in the exit rate with respect to log fluctuations of the critical productivity at 2.44. These elasticities provide a measure of how sensitive entry and exit are to their endogenous driving factors; the average equity

\(^{35}\)The technical appendix contains a section on the quarter-to-quarter shock decomposition of the estimated model, which might be complementary to the analysis of entry and exit conducted here. Furthermore, there is one specific section of that appendix that discussed the sources of fluctuations of US real GDP growth during the Great Recession.
for entry (see equation 10) and the critical productivity for exit (see equation 17). Exogenous factors may directly determine the evolution of the entry and exit rates through their specific shock: the entry cost shock for entry and the liquidation value shock for exit. The estimated model also provides information over the characteristics of these shocks through their coefficients of autocorrelation and the standard deviation of their innovations (Table 4B).

If the model is re-estimated for the Great Recession (GR) period (2007:1-2016:2), keeping the same priors for the structural parameters, the posterior mean estimates obtained lead to the following entry-exit equations

\[ e_t - e = 0.53\hat{e}_{t-1} - 0.54\epsilon_{t-1} + 0.0296 (\hat{n}_{t+1} - \hat{n}_t), \quad \text{(GR\textsuperscript{e})} \]

\[ x_t - x = 3.26\zeta_{t}. \quad \text{(GR\textsuperscript{x})} \]

The comparison between (FS\textsuperscript{e}) and (GR\textsuperscript{e}) indicates that the degree of responsiveness of the entry rate to average equity has slightly increased during the Great Recession relative to the full sample, but the difference is small: the semi-elasticity of the entry rate has slightly increased from 0.43 to 0.53. This change can be confirmed by the reduction observed in the posterior mean estimate of the entry cost elasticity parameter: \( \zeta = 2.78 \) in the full sample falls to \( \zeta = 2.42 \) in the Great Recession.\textsuperscript{36}

Unlike that of the entry rate, the sensitivity of the exit rate to the critical productivity rises quite substantially: it moves up from 2.44 in (FS\textsuperscript{x}) to 3.26 in (GR\textsuperscript{x}). Here the key parameter is the shape of the Pareto distribution of establishment-specific productivity, \( \kappa \). It goes up from 2.51 to 3.32 when cutting the sample down to the Great Recession period. The economic interpretation is the following: a higher estimated \( \kappa \) indicates a greater concentration of establishments closer to the minimum productivity \( z_{\min} \) after 2007. This makes the aggregate exit rate be more sensitive to the state of the economy as the number of establishments shutting down responds more aggressively to changes in the market conditions that determine the critical productivity (expected costs of production, interest rates, expected demand, etc.).

The increased variability and persistence of US business formation might also be caused by a greater role of exogenous volatility in entry and exit during the Great Recession. Let us see if the estimates of the autocorrelation and standard deviation of the shocks that shape the entry-exit dynamics rise in the Great Recession. Table 7 informs on the comparison. Remarkably, the liquidation value (exit) shock has a slightly longer inertia than in the full sample (the coefficient of autocorrelation rises from 0.54 to 0.59), but with a much lower volatility of its innovations

\textsuperscript{36}The technical appendix includes a Table with the posterior estimates of the model parameters in the Great Recession sample period.
The standard deviation falls from 1.11% to 0.47%). The re-estimation of the entry cost shock reports a substantial reduction in both the coefficient of autocorrelation (from 0.83 to 0.52) and the innovations volatility (from 1.19% to 0.47%), which makes it much less influential for business creation during the Great Recession. In the variance decomposition the share of the output growth variability explained by the entry cost shock is just 1.14%, much lower than the 7.35% found in the full sample.

Table 7. Entry and exit dynamics

<table>
<thead>
<tr>
<th></th>
<th>Semi-elasticity</th>
<th>Autocorrelation</th>
<th>Std. deviation, %</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Full sample, 1993:2-2016:2</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Entry</td>
<td>$\varepsilon_{v,ee} = 0.43$</td>
<td>$\rho_e = 0.83$</td>
<td>$\sigma_{\eta e} = 1.19$</td>
</tr>
<tr>
<td>Exit</td>
<td>$\kappa (1-x) = 2.44$</td>
<td>$\rho_x = 0.54$</td>
<td>$\sigma_{\eta x} = 1.11$</td>
</tr>
<tr>
<td><strong>Great Recession, 2007:1-2016:2</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Entry</td>
<td>$\varepsilon_{v,ee} = 0.54$</td>
<td>$\rho_e = 0.52$</td>
<td>$\sigma_{\eta e} = 0.47$</td>
</tr>
<tr>
<td>Exit</td>
<td>$\kappa (1-x) = 3.26$</td>
<td>$\rho_x = 0.59$</td>
<td>$\sigma_{\eta x} = 0.47$</td>
</tr>
</tbody>
</table>

Finally, the Bayesian estimation of the structural parameters determines an empirical value for the average productivity of US establishments according to the Pareto distribution. Its theoretical value is $\bar{z} = z_{min}^\kappa \left( \frac{\kappa}{\kappa-\left(\theta_p-1\right)} \right)^{1/(\theta_p-1)}$ which depends on a calibrated minimal productivity (fixed at $z_{min} = 1.0$) and the estimated values of both $\kappa$ and $\theta_p$. Plugging the values obtained as posterior mean estimates in the full sample brings $\bar{z} = 1.72$. Repeating the estimation exercise for the Great Recession subsample the average plant-level productivity falls to $\tilde{z} = 1.49$. This result might be explained by the negative effect that an increase in the estimated slope coefficient of the Pareto distribution $\kappa$ causes on the average productivity $\bar{z}$. Therefore, we can say that the higher density of establishments with low productivity after the financial crisis has also reduced the average productivity of incumbents. Such productivity slowdown has been recently found in US firm-level data by Decker et al. (2017).

7 Conclusions

This paper introduces a DSGE model with endogenous business creation (entry) and destruction (exit), in which there is a variable number of business units (establishments) that produce a single differentiated good. The main theoretical contribution is the endogenous choice of exit, that results from an intertemporal comparison between the present value of expected future dividends and the
current liquidation value. Establishments shut down if the expected present-value of its future dividends is lower than the liquidation value. A time-varying critical productivity splits up the rates of exit and survival for incumbents. The entry decision is part of the household portfolio choice: a new establishment is created if its prospective equity value is higher than its entry cost.

In the empirical section, we have carried out a Bayesian estimation of the DSGE model including data on establishments entry and exit to discuss the role of business formation on US aggregate fluctuations for the quarterly period 1993:2-2016:2. Hence, the extended model delivers fluctuations of the aggregate GDP explained by either changes at the extensive margin (number of establishments) or at the intensive margin (establishment-level production). The model provides a good matching to the second-moment statistics of US private establishments accumulation, entry and exit.

The simulations of the estimated model indicate that business formation is procyclical after technology and risk-premium shocks, which amplifies the effect of these shocks for aggregate fluctuations. In either price-push or wage-push shocks the number of establishments turn countercyclical while establishment-level production remains procyclical. Fluctuations of the rate of growth of aggregate real GDP and establishment-level output are mostly explained by demand-side shocks whereas supply-side shocks are the main drivers of the establishments growth. The evolution of the entry rate depends upon both demand and supply factors with a prevalence of the former (especially, risk-premium shocks), whereas the exit rate is mainly affected by supply factors (shocks on technology, prices and liquidation value).

The estimation of the structural equations governing entry and exit dynamics show that after 2007 there has been a significant increase in the response of the exit rate to changes in the level of critical productivity. The implication provided by the model is that a larger concentration of low-productivity firms speeds up the exit decisions during the Great Recession (2007-2016). In addition, average productivity across incumbents is estimated to be lower after 2007. Nevertheless, shocks on entry and exit have not strongly contributed to aggregate fluctuations as business formation turned more sensitive to the endogenous determinants.
References


