Identification of a Ni-vacancy defect in Ni-Mn-Z (Z = Ga, Sn, In):
An experimental and DFT positron-annihilation study

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By means of experimental positron-annihilation-lifetime measurements and theoretical density functional theory (DFT) positron-lifetime calculations, vacancy-type defects in Ni_{50}Mn_{50−x}Sn, (x = 25, 20, 15, 13, 10) and Ni_{50}Mn_{50−x}In, (x = 25, 20, 16, 13) systems are systematically studied. The study is extended to Ni-Mn-Ga systems as well. Experimental results are complemented with electron-positron DFT calculations carried out within the local density approximation and generalized gradient approximation, where five different parametrizations accounting for the γ(r) enhancement factor are analyzed. Theoretical results indicate that the Boronski-Niiminen parametrization of γ(r) is the one that best predicts the experimental results, which ultimately enables us to identify V_{Ni} as the vacancy present in the studied samples. The characteristic positron lifetime related to V_{Ni} ranges between 181 and 191 ps in Ni-Mn-Sn/In systems. Positron-annihilation-lifetime spectroscopy results in these two systems delimit the lower bound of the achievable vacancy concentration, which is much larger compared with the reported values in Ni-Mn-Ga systems. The present work, along with setting the basis for positron simulations in Ni-Mn based Heusler alloys, delimits the effect that the variation of vacancies has in the martensitic transformation in Ni-Mn-Sn systems.

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I. INTRODUCTION

The multifunctional properties that Ni-Mn-Z (Z = Ga, In, Sn, Sb) Heusler alloys exhibit, such as giant magnetoresistance [1,2], the magnetocaloric effect [3,4], large magnetic-field-induced strain [5,6], and the shape-memory effect [7], are directly related to the occurrence of the so-called martensitic transformation (MT). MT is a displacive phase transition which occurs between the highly symmetric austenitic phase and less symmetric martensitic structure [8]. Due to the magnetostructural coupling these alloys exhibit, the structural transition is often accompanied with a magnetic transition. In Ni-Mn based Heusler alloys, the magnetism is mainly based on the indirect exchange between Mn atoms and the magnetic properties depend strongly on Mn-Mn distance [9]. Thus, whereas in the case of Ni-Mn-Ga systems the MT occurs between similar ferromagnetic parent and martensite phases [10], in the so-called metamagnetic shape-memory alloys, Ni-Mn-Z (Z = In, Sn, Sb), the MT often takes place between a ferromagnetic austenite and a weak magnetic or paramagnetic martensite [7]. This magnetization discrepancy ΔM between parent and martensite phases gives rise to additional multifunctional effects such as the inverse magnetic-field-induced shape change [11] and the inverse magnetocaloric effect [12,13].

All in all, the control of the MT plays a key role in the tuning of the aforementioned multifunctional properties. On the one hand, it is well known that the composition is the most critical factor affecting the MT temperature (T_{MT}) [7,14–16]. In addition to composition, recent works have shown that the control of the microstructure becomes crucial to properly tune the MT and the related multifunctional properties [17–19]. Indeed, in Ni-Mn-Sn systems, the retained internal strain at antiphase boundaries promotes the antiferromagnetic coupling between Mn atoms, affecting the magnetostructural properties and ultimately the width of the MT [20]. In addition, the variation of the atomic order has profound effects on the transition temperature, enabling shifts of T_{MT} up to ΔT_{MT} ≈ 100 K [21]. On the other hand, the presence of defects alters the ferromagnetic interactions [18,22], and in the micro and nano regimes, they may inhibit the MT [23].

In that connection, several works have already suggested the potential role that vacancies may play on the MT and...
the magnetic properties of Ni-based Heusler alloys. Zhang et al. [26] suggest that the observed magnetic entropy change in Ni-Mn-Sn ribbons may be a consequence of the enhancement of the atomic order which may be due to the annealing of vacancies. Moreover, Zhenni et al. [27] and Kustov et al. [28,29] point out the potential role that vacancy defects could have on the pinning of the MT, whereas Sánchez-Alarcos et al. [30] and Santamarta et al. [31] suggest different vacancy dynamics as responsible for the observed changes on the Curie temperature $T_C$ and $T_{MT}$ in Ni-Mn-Ga and Ni-Fe-Ga alloys, respectively. In a recent work Hedayati et al. [32] claim that Sn vacancies in Ni-Mn-Sn systems can be used to obtain shifts on $T_{MT}$ up to 80 K. However, none of the previous works contribute any experimental evidence on vacancies.

Thus far, the vast majority of works focused on the effect of vacancies in MT and magnetic properties of Ni-based Heusler alloys have been conducted within the theoretical framework. For instance, from first-principles calculations, Bai et al. [33–35] and Kulkova et al. [36,37] calculated the formation energy of possible antisite and vacancy defects on several Ni-based Heusler compounds. In recent works, by first-principles calculations and Monte Carlo simulations, Kosogor et al. [38] and Wang et al. [39], respectively, link the vacancy concentration ($C_v$) with the ordering process and the effect on the shift of the $T_{MT}$. In this context, Tehrani et al. [40], by means of molecular dynamics simulations, also suggest the potential effect that vacancies may have on $T_{MT}$. Despite the number of investigations suggesting the nontrivial role that vacancies may play in the MT, experimental works have been scarcely reported, the elusive nature of the vacancies’ effect being the reason this is less investigated compared with other physical factors.

One of the most thorough studies has been performed by Merida et al. [41–44]. By positron-annihilation-lifetime spectroscopy (PALS) they experimentally measure the $C_v$ present in Ni-Mn-Ga single crystals and polycrystals. Depending on the composition, they report different equilibrium vacancy concentrations, as well as different migration and formation energies of vacancies. Additionally, they show that with proper annealing treatments [42], $C_v$ can be tuned from $\approx$2000 ppm down to 10 ppm. These works demonstrate the capability of PALS for the study of vacancy dynamics in Ni-based Heusler alloys. In fact, this technique has been widely used over decades and it is a powerful tool for defect characterization in metals [45]. As the positron lifetime is directly related to the electronic density of the material, crystal imperfections such as vacancy defects alter significantly the surrounding electronic density. Thus, when positrons are trapped in such defects the positron lifetime varies, enabling a direct detection of vacancies.

In this work, vacancy-type defects are systematically investigated by PALS in Ni-Mn-In and Ni-Mn-Sn systems. The experimental measurements are complemented with density functional theory (DFT) calculations of the positron lifetime. In order to establish the parametrization of the enhancement factor which best suits the experimental PALS results in Ni-Mn based Heusler alloys, five different widely used parametrizations have been also comparatively studied. Results show that the Boronski-Nieminen parametrization gives the most accurate values for positron lifetimes, which, in turn, enables the identification of the Ni vacancy ($V_{Ni}$) as the vacancy type present in the Ni-Mn-Sn, Ni-Mn-In, and Ni-Mn-Ga systems. Taking advantage of the existing PALS results on Ni-Mn-Ga, theoretical calculations have been carried out in Ni-Mn-Ga to test the agreement between calculations and experimental results. Finally, and contrary to what happens in Ni-Mn-Ga systems, results in Ni-Mn-Sn and in Ni-Mn-In systems show that the irreducible high $C_v$ presents a handicap for its control by means of standard heat treatments.

## II. EXPERIMENTAL METHOD

Ni$_{50}$Mn$_{50-x}$Sn$_x$ ($x$=25, 20, 15, 13, 10) and Ni$_{50}$Mn$_{50-x}$In$_x$ ($x$=25, 20, 16, 13) (Sn$_x$ and In$_x$, respectively) polycrystalline ingots are synthesized from high-purity elements by the arc-melting method under a protective argon atmosphere. Each initial ingot is remelted up to eight times to ensure the homogeneity. Afterwards, Sn$_x$ and In$_x$ samples are homogenized at 1173 K during 24 h. Regarding Sn$_x$ samples, a further annealing of 4 h at 1273 K is performed to avoid dendritic structures [46]. Bulk pieces for the initial characterization are obtained from the center of each ingot by cutting small pieces.

### TABLE I. Composition and characteristic $T_{MT}$ and $T_C$ temperatures obtained from the initial characterization measurements of the studied samples. Crystallographic data employed in AT-SUP calculations of the studied Ni$_{50}$Mn$_{50-x}$Sn$_x$ ($x$ = 25, 20, 15, 13, 10) and Ni$_{50}$Mn$_{50-x}$In$_x$ ($x$ = 25, 20, 16, 13, 10) samples.

<table>
<thead>
<tr>
<th>Composition</th>
<th>Label</th>
<th>$T_{MT}$ (K)</th>
<th>$T_C$ (K)</th>
<th>$a = b = c$ (Å)</th>
<th>Simulated Phase</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ni$<em>{50}$Mn$</em>{50}$Sn$_{20}$</td>
<td>Sn$_{20}$</td>
<td>328</td>
<td>343</td>
<td>6.046</td>
<td>$Fm\bar{3}m$</td>
<td>[14]</td>
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<td>223</td>
<td>313</td>
<td>6.024</td>
<td>$Fm\bar{3}m$</td>
<td>[14]</td>
</tr>
<tr>
<td>Ni$<em>{50}$Mn$</em>{50}$Sn$_{10}$</td>
<td>Sn$_{10}$</td>
<td>310</td>
<td>473</td>
<td>5.995</td>
<td>$P2/m$</td>
<td>[14]</td>
</tr>
<tr>
<td>Ni$<em>{50}$Mn$</em>{50}$Sn$_{13}$</td>
<td>Sn$_{13}$</td>
<td>4.317, 5.621, 6.811</td>
<td>$Pm\bar{m}$</td>
<td>4.361</td>
<td>$Pm\bar{m}$</td>
<td>[24]</td>
</tr>
<tr>
<td>Ni$<em>{50}$Mn$</em>{50}$Sn$_{10}$</td>
<td>Sn$_{10}$</td>
<td>4.333, 5.570, 6.811</td>
<td>$P2/m$</td>
<td>4.281</td>
<td>[14]</td>
<td></td>
</tr>
<tr>
<td>Ni$<em>{50}$Mn$</em>{50}$In$_{25}$</td>
<td>In$_{25}$</td>
<td>310</td>
<td>298</td>
<td>6.071</td>
<td>$Fm\bar{3}m$</td>
<td>[15]</td>
</tr>
<tr>
<td>Ni$<em>{50}$Mn$</em>{50}$In$_{20}$</td>
<td>In$_{20}$</td>
<td>298</td>
<td>225</td>
<td>6.031</td>
<td>$Fm\bar{3}m$</td>
<td>[15]</td>
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<tr>
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<td>In$_{16}$</td>
<td>4.391, 5.620, 6.811</td>
<td>$P2/m$</td>
<td>4.331</td>
<td>$P2/m$</td>
<td>[25]</td>
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<tr>
<td>Ni$<em>{50}$Mn$</em>{50}$In$_{13}$</td>
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<td>4.284, 5.811, 6.401</td>
<td>$P2/m$</td>
<td>4.301</td>
<td>[15,25]</td>
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</table>
using a low-speed diamond saw. The initial characterization consists of energy-dispersive x-ray spectroscopy analysis and differential scanning calorimetric measurements (DSC) carried out in a TA Q100 DSC, by which the composition and the characteristic $T_{\text{MT}}$ and Curie $T_c$ temperatures are determined. The results of the initial characterization are gathered in Table I (the compositional error obtained for all samples lies within 1.5% of the nominal composition), which are consistent with previous experimental reports [14,15].

Subsequently, Sn$_x$ and In$_x$ samples are quenched from 1173 K into iced water (AQ state). In order to ensure thermal equilibrium, samples are kept at 1173 K during 30 min before quenching. The study of the microstructural evolution of the vacancy defects upon postquenching has been performed using the approach employed in Ref. [41]. The samples are subjected to the so-called isochronal annealing cycles (IACs). In essence, the sample is heated up at a constant rate of 10 K/min to a maximum temperature ($T_i$), which in turn is successively increased from 473 K to 873 K in 50 K steps. Afterwards, the sample is cooled down to the initial temperature at the same rate.

After each IAC, PALS measurements are carried out at room temperature (RT) using a fast-fast timing coincidence spectrometer of 250 ps resolution at FWHM. H1949-50 Hamamatsu photomultiplier tubes are equipped with BC-442 plastic scintillators and suited in a collinear geometry. A 15µCi $^{22}$NaCl positron source covered in Kapton is sandwiched by a pair of identical samples. All spectra have been collected with more than 3 × 10$^6$ counts and analyzed with the POSITRONFIT [47] code. The analysis has been performed subtracting the source contribution, which consists of two components. The first is a component with a lifetime ≈1500 ps [48,49] with a measured average intensity of 2.5%. The second component, with an intensity of ≈13%, is related to the positron annihilation in a 7 µm width Kapton foil that wraps the $^{22}$NaCl positron source. The characteristic positron lifetime in Kapton has a very well known value of 382 ps [50,51]. The obtained $\chi^2$ values have been kept below 1.2 in all fittings.

### III. COMPUTATIONAL METHOD

For a better understanding of the experimental results, theoretical calculations of the positron lifetimes have been conducted for the two systems of samples. Using a two-component density functional theory [52,53], the annihilation rate $\lambda$ (i.e., the inverse of the $\tau$ positron lifetime) is evaluated by overlapping the positron density $n_+(r)$ and the electron density of the solid $n_-(r)$ as

$$\lambda = \tau^{-1} = \pi c r_s^2 \int n_+(r)n_-(r)\gamma(r)dr,$$

where $c$ is the speed of light in vacuum, $r_s$ the classical electron radius, and $\gamma(r)$ the so-called enhancement factor that comprises the enhanced electron density due to the Coulombic attraction exerted by $e^+$. The positron lifetime for the perfect (i.e., bulk lifetime) and defected lattice has been calculated with the atomic superposition approximation method (AT-SUP) [54], which provides values in good agreement with measured positron lifetimes in metals and semiconductors [55–58]. The electron density $n_-(r)$ of the crystal and the crystalline Coulomb potential $V_i(r)$ are constructed by adding individual atomic charges $e_n\delta(|\mathbf{r}-\mathbf{R}_i|)$ and $V^{\text{el}}(|\mathbf{r}-\mathbf{R}_i|)$ potentials over the $\mathbf{R}_i$ occupied atomic sites. Finally, the potential felt by the positron $V_+(r)$ is evaluated by adding the positron-electron correlation potential $V_{\text{cor}}[n_-(r)]$ to the Coulombic potential so that

$$V_+(r) = V_i(r) + V_{\text{cor}}[n_-(r)] = \sum_i V^{\text{el}}(|\mathbf{r}-\mathbf{R}_i|) + V_{\text{cor}}[\sum_i e_n\delta(|\mathbf{r}-\mathbf{R}_i|)].$$

The enhancement factor of Eq. (1) and the correlation potential of Eq. (2) have been taken into account within the (i) local density approximation (LDA) and (ii) generalized gradient approximation (GGA) framework. Within the GGA approximation, the $V_{\text{cor}}[n_-(r)]$ has been modeled using the interpolation formula proposed by Boronski and Nieminen [53], which is based on the results of Arponen and Pajanne [59]. For $\gamma(r)$ three different parametrizations have been used. The first is the expression proposed by Boronski and Nieminen [53] which is based on the many-body calculation by Lantto [60] (labeled as LDA-BN),

$$\gamma(r)_{\text{LDA}} = \begin{cases} 1 + 1.23r_s + 0.8295r_s^{1/2} + 1.26r_s^{5/2} + 0.3286r_s^{3/2}, & r_s = (3/4\pi n_-(r))^{1/3}, \\
-0.3455r_s^{5/2} + 1.35r_s & r_s < 0.03. \end{cases}$$

and LDA-AP2,

$$\gamma(r)_{\text{AP2}} = \begin{cases} 1 + 1.23r_s - 0.91657r_s^{3/2} + 1.0564r_s^2 & r_s = (3/4\pi n_-(r))^{1/3}, \\
-0.3455r_s^{5/2} + 1.35r_s & r_s < 0.03. \end{cases}$$

Within the GGA approximation, both correlation potential and the enhancement factor have been taken into account using the expression employed by Barbiellini al. [61] (based on results of Arponen and Pajanne [59]). Regarding $\gamma(r)_{\text{GGA}}$, it is deduced from the enhancement factor obtained within LDA. The effects of the nonuniform electron density are modeled by a parameter $\epsilon = |\Delta \ln n_-(r)|/q_{TF}^2$ which describes the reduction of the screening cloud close to the positron, $q_{TF}$ being the local Thomas-Fermi screening length. Finally, an adjustable parameter $\alpha$ is also introduced, so the corrected enhancement factor then reads

$$\gamma(r)_{\text{GGA}} = 1 + [\gamma(r)_{\text{LDA}} - 1]e^{-\alpha \epsilon}.$$
has not been synthesized and experimentally measured by PALS, this composition has been added to the calculations in order to complete the theoretical study. The crystallographic data used in the calculations are shown in Table I. The simulated structure for each sample has been chosen according to the proper phase at RT. Samples without MT and samples with $T_{MT} < RT$ (Sn$_{25}$, Sn$_{20}$, In$_{25}$, In$_{20}$, and Sn$_{15}$, In$_{16}$) have been simulated by using the cubic L$_2$ austenitic structure. Regarding the calculations of the martensitic phase, most previous works indicate a monoclinic martensite with different degrees of modulation [14,15,24,25]. For the sake of simplicity (especially for vacancy calculations), the martensitic phase has been modeled considering a nonmodulated orthorhombic martensite (using the lattice parameters of the monoclinic structure and fixing $\beta = 90$), which has been proven to give identical results compared to those obtained considering a monoclinic structure.

The annihilation rate $\lambda$ of Eq. (1) is evaluated at both the $\Gamma$ and L points in the Brillouin zone, as well as calculating the mean value of the wave functions from the $\Gamma$ and L points. The simulations are performed using the supercell approach. The supercells corresponding to off-stoichiometric samples have been built starting from a stoichiometric lattice and by substituting Sn/In atoms by Mn atoms until matching the proper composition of each sample. The corresponding supercell is increased in size until a convergence of 0.1 ps is reached in bulk lifetime calculations. Afterwards, in order to overcome the artificial defect-defect interactions caused by periodic boundary conditions, the supercell containing vacancy defects is built with increasing size in order to ensure the convergence of 0.1 ps in lifetime calculations and 0.01 eV in positron binding energies.

The maximum number of atoms used in the vacancy calculations is 500 atoms for the austenitic phase and 512 atoms for the martensitic phase using $5 \times 5 \times 5$ and $4 \times 4 \times 4$ expansions of the primitive unit cell, respectively. A mesh size of 160$^3$ points has been used for both phases. Finally, taking into account the potential felt by the positron $V_p(r)$ of Eq. (2) and the enhancement factor $\gamma(r)$, the Schrödinger equation is solved iteratively at the mesh points of the supercell using a numerical relaxation method [62], from which positron wave functions and their energy eigenvalues are obtained to evaluate the $\lambda$ annihilation rate of Eq. (1).

IV. RESULTS

A. Experimental results

Figure 1 shows the experimental average-positron-lifetime ($\bar{\tau}$) values measured for both Sn$_x$ and In$_x$ series as a function of $T_I$. The first point of PALS measurements corresponds to the AQ $\bar{\tau}$ value. In both sets of samples, the AQ state ranges between 181 and 187 ps. The quenching procedure has been repeated more than three times resulting in identical results. In order to study the vacancy dynamics, AQ samples have been subjected to IAC. In both Sn$_x$ and In$_x$ systems, apart from slight fluctuations of 2–3 ps, $\bar{\tau}$ remains almost constant regardless of $T_I$. Only In$_{16}$ and Sn$_{15}$ samples show a significant drop of $\bar{\tau}$. This drop takes place at 773 K and 573 K, respectively, approaching the $\approx 178$ ps value. However, this drop is rapidly recovered during the next IAC, and finally the $\bar{\tau}$ increases back to its initial AQ’s value. The observed moderate evolution of $\bar{\tau}$ can be understood by the physics governing the positron trapping at defects.

When a positron enters a solid, it thermalizes and diffuses through the lattice, until eventually the positron is annihilated with a surrounding electron. In a defect-free lattice, the positron annihilates from the delocalized state (i.e., Bloch...
state) at an average \( \lambda_b \) rate or with a characteristic \( \tau_b \) lifetime (bulk lifetime). However, when open-volume defects are present they may act as positron-trapping centers. Actually, in the specific case of vacancy defects they act as deep traps for positrons due to the lack of the positive ion. The trapping occurs when a positron turns from the Bloch state into a localized state within a vacancy (i.e., the positron wave function becomes localized at the vacancy). The lack of the ion leaves behind an open volume that in turn is characterized by a lower electron density compared with the perfect lattice. As shown in Eq. (1), the lower the electron density is, the lower is \( \lambda \). Thus, positrons trapped at vacancies annihilate from a localized state at an average rate \( \lambda_e < \lambda_b \), thus exhibiting a longer vacancy-related positron lifetime \( \tau_v \), where \( \tau_v > \tau_b \).

As a result, the moderate variation of \( \tau \) shown by Fig. 1 can be explained considering two different scenarios. On the one hand, the lack of open-volume defects (e.g., vacancies) would imply that all positrons only annihilate from the delocalized state or bulk state. Then, the measured lifetime would reflect the lifetime related to the bulk lifetime, \( \tau_b \). On the other hand, due to a high \( C_e \) retained during the quenching, the contribution of the positrons annihilating from vacancies could become the one dominating \( \tau \), overcoming the contribution of positrons annihilating from the delocalized state (i.e., bulk). In such scenario, the measured lifetime would reflect the characteristic positron lifetime \( \tau_v \) related to the present vacancy defect.

Even though the decomposition of the spectra would provide valuable information about different contributions of \( \tau \), the spectra could not be decomposed in none of the Sn\(_x\) and In\(_x\) samples. As will be discussed further in Sec. V, the saturation-trapping regime of vacancies hinders the decomposition of the decay spectrum of positrons. However, \( \tau \) is the statistically most accurate parameter in PALS experiments, and along with theoretical calculations, it provides information for data interpretation in cases where the experimental data cannot be decomposed [63]. In order to find out whether the measured lifetime is related to the bulk lifetime or to the presence of vacancy defects, experimental results are complemented with theoretical DFT electron-positron calculations.

### B. DFT electron-positron results

Theoretical calculations for the bulk and defected lattice for Sn\(_x\) and In\(_x\) systems have been performed using the AT-SUP method. For each sample the simulated structure has been chosen to match the proper phase at RT; see Table 1. Besides, calculations for the defected lattice have been taken into account considering several types of possible vacancy defects.

Figure 2 illustrates the austenitic and martensitic structures, as well as the nearest neighbors (NNs) of each type of atom. In the stoichiometric \( \text{Fm\text{3}m} \) phase [see Fig. 2(a)], Ni atoms occupy 8c positions, Mn atoms 4a positions, and Z atoms (Ga/Sn/In) atoms 4b positions. However, as the Mn content increases on detriment of Z atoms, the exceeded Mn atoms tend to occupy 4b positions [14,15]. As a result, in off-stoichiometric samples two types of nonequivalent Mn atoms are present. In the studied samples the Ni content has been fixed to 50% and the Mn/Z ratio is the one that is modified throughout the studied samples. It is noteworthy to mention that regardless of the composition, the NNs of both Mn and In/Sn atoms (positions 4a and 4b) are always Ni in both Sn\(_x\) and In\(_x\) systems. Regarding the Ni atom, while in stoichiometric conditions the NNs of Ni are 4 Mn atoms and 4 Z atoms, as soon as the Mn content increases, the probability of Ni atoms to be coordinated with more Mn atoms than Z atoms also does. With respect to the possible vacancies present in the studied samples, apart from Ni, Mn, and Z vacancies (V\(_{\text{Ni}}\), V\(_{\text{Mn}}\), and V\(_{\text{Z}}\), respectively), additional possible vacancies have been also taken into account in off-stoichiometric calculations: V\(_{\text{Mn}}\text{Ni}\), which refers to the vacancy of the antisite Mn atom located at 4b positions, and V\(_{\text{Ni}}\text{Z}\), which refers to the Ni atoms surrounded for different Mn/Z atoms.

The previous description can be extended to the martensitic structure. However, due to the cell contraction caused by the MT, and its consequent symmetry breaking, the NNs of Ni atoms change. As illustrated in Fig. 2(b), in the martensitic structure, contrary to the cubic structure, Ni atoms are not bound to 8 atoms but to 4 atoms (i.e., two Mn atoms and two
Z atoms). As previously pointed out, the exceeded Mn atoms occupy 2\textit{b} positions and the probability of a Ni atom to bound more Mn atoms than Z atoms is non-negligible in samples far away from stoichiometry (i.e., Sn\textsubscript{13}, Sn\textsubscript{10}, In\textsubscript{13}, and In\textsubscript{13}).

In summary, for stoichiometric Sn\textsubscript{25} and In\textsubscript{25} samples, three different types of possible vacancy defects have been considered: \(\text{V}_{\text{Ni}}, \text{V}_{\text{Mn}},\) and \(\text{V}_{\text{Z}}\). Regarding the off-stoichiometric samples, additional \(\text{V}_{\text{Mn}}^{\text{as}}, \text{V}_{\text{Ni}}^{\text{1}}, \text{V}_{\text{Ni}}^{\text{2}}, \text{V}_{\text{Ni}}^{\text{3}},\) and \(\text{V}_{\text{Ni}}^{\text{4}}\) vacancies have been also introduced in the calculations. The Ni vacancy surrounded by 4 Mn and 4 Z atoms has been labeled as \(\text{V}_{\text{Ni}}\), while \(\text{V}_{\text{Ni}}^{\text{1}}\) refers to the Ni vacancy surrounded by an antisite Mn atom, which results in a Ni vacancy defect with 5 Mn atoms and 3 Z at NN. Using the same scheme, the remaining types of Ni vacancies are labeled accordingly, and finally, \(\text{V}_{\text{Ni}}^{\text{4}}\) is assigned to the vacancy of a Ni atom surrounded by 8 Mn atoms. For the martensitic phase, \(\text{V}_{\text{Ni}}^{\text{1}}, \text{V}_{\text{Ni}}^{\text{2}},\) and \(\text{V}_{\text{Ni}}^{\text{3}}\) labels have been used to refer to the Ni vacancies with 3 and 4 Mn atoms at NN, respectively.

The results of the calculations are shown in Fig. 3 for Sn\textsubscript{x} and in Fig. 4 for In\textsubscript{x}. Each figure gathers the calculated positron-lifetime \(\tau\) values for perfect and defected lattices. The different outcomes of the five different parametrizations employed for modeling \(\gamma(r)\) (described in Sec. III) are plotted comparatively as a function of composition. Additionally, the range of experimental \(\tau\) is also plotted by a red-shadowed area.

On the one hand, the lifetime values calculated within the GGA are higher than the LDA ones. Although \(\tau\) values predicted by AP1 and AP2 parametrizations are very similar to one another, the absolute values predicted by these two parametrizations within the GGA and LDA approximations are significantly different, separated by \(\approx 30\) ps offset. Otherwise, \(\gamma(r)_{\text{BN}}^{\text{LDA}}\) parametrizations give intermediate values of the positron lifetimes compared with the values calculated by using \(\gamma(r)_{\text{GGA}}\) (for the two parametrizations) and the two parametrizations \(\gamma(r)_{\text{LDA}}^{\text{AP1}}\) and \(\gamma(r)_{\text{LDA}}^{\text{AP2}}\).
V. DISCUSSION

In order to explain the evolution shown by $\tau$ as a function of $T_i$ (see Fig. 1), in Sec. IV A two possible scenario have been proposed. The first scenario, in which due to a lack of open-volume defects all positrons are annihilated from the delocalized state at a constant rate $\lambda_b = \tau_b^{-1}$, would imply that $\tau \approx \tau_b$. As shown in Fig. 3 and Fig. 4, the calculated bulk lifetime for both systems ranges between 100 and 130 ps. However, experimental $\tau$ values range between 177 and 190 ps. Taking into account that the dispersion of theoretical results between the different parametrizations is $\approx 30$ ps, experimental $\tau$ values are significantly above the calculated bulk lifetimes.

In a second scenario, it is posed that the behavior of $\tau$ may be related to the presence of vacancies. Due to a high $C_v$ retained during quenching, the contribution of the positrons annihilating from vacancies may become the one dominating $\tau$, which would overcome the bulk’s contribution. In such scenario, $\tau$ would approach $\tau_v$. In this connection, theoretically calculated vacancy-related positron lifetimes are more compatible with the experimental results, lying between 170 and 210 ps.

Additionally, the calculated density plots shown in Fig. 5 indicate that the positron density is highly localized within vacancies (i.e., the probability density of the positron is maximum in the vacancy). For the undefected solid the highest position probability density is found at interstitial regions, whereas $|\Psi_+|^2$ vanishes close to atomic nuclei. DFT calculations reveal that regardless of the vacancy type, structure, and composition, the binding energies of positrons to vacancies range $\approx 2–3$ eV. Therefore, positron-lifetime calculations reveal that the studied vacancies act as effective positron traps. Taking into account that $\tau$ values $\approx 180$ ps reported in Ni-Mn-Ga alloys were ascribed to the presence of vacancies [41–44], it is reasonable to assess that vacancies are present in the studied $\text{Sn}_x$ and $\text{In}_x$ samples. As a result, the measured $\tau$ comprehends two different contributions.

Let us define $\eta_1$ as the percentage of total positrons annihilated from the delocalized state with a characteristic lifetime $\tau_b$.
\(\tau_b\). Considering the presence of a single vacancy acting as a positron trap (or an effective vacancy type accounting for the contribution of several vacancy defects with similar \(\tau_v\) \cite{42}), the rest of the positrons (\(\eta_2\)) get trapped at the vacancy and they annihilate from the localized state with a characteristic lifetime \(\tau_v\) related to the defect. As a result, the experimentally measured \(\tau\) comprehend these two contributions, so that \(\tau = \eta_1 \tau_b + \eta_2 \tau_v\). Thus, a larger \(C_v\) implies a higher probability of positrons annihilating from vacancies. In such scenario, the relation between \(C_v\) and \(\tau\) is given by the so-called one-trap model \cite{63},

\[
\tau = \frac{1 + \kappa_v \tau_v}{1 + \kappa_v \tau_b}
\]

or

\[
\kappa_v = \mu_v C_v = \frac{1}{\tau_b} \frac{\tau - \tau_v}{\tau_v - \tau} \rightarrow C_v = \frac{1}{\mu_v} \frac{\tau - \tau_v}{\tau_v - \tau},
\]

where \(\kappa_v\) is the so-called trapping rate and is proportional to \(C_v\) \cite{65}. The parameter \(\mu_v\) is the specific trapping coefficient of the vacancy and it depends on the type of defect and also on the surrounding lattice \cite{66}. The trapping rate \(\kappa_v\) describes how effective is a certain type of defect trapping positrons. As shown in Eq. (7) and Eq. (8), a high value of \(\mu_v\) or \(C_v\) would result on a \(\tau\) value close to \(\tau_v\). In metals, unlike in semiconductors, the specific trapping rate \(\mu_v\) is constant and it does not depend on temperature \cite{66}. In the present work a constant \(\mu_v = 1.5 \times 10^{14}\) s\(^{-1}\) value has been used, which is widely employed for Ni \cite{67,68}.

The lack of variation of experimental \(\tau\) values reflects that the vacancy contribution to \(\tau\) remains almost unaltered as a function of the IAC. Additionally \(\tau\) values are significantly higher than theoretical \(\tau_v\) values, and indeed, \(\tau\) values are closer to the calculated vacancy-related lifetimes. All in all, and taking into account the fact that spectra could not be decomposed, the behavior exhibited by \(\tau\) as a function of \(T_f\) indicates that it lies in a so-called saturation-trapping regime. In such regime, \(|\tau - \tau_v| < 10\) ps \cite{63}, due to a high \(C_v\), the contribution of positrons trapped at vacancies dominates \(\tau\), in such a way that the decomposition becomes unfeasible. Indeed, different fitting procedures have been followed to decompose the spectra without success. It is due to the fact that when \(C_v\) is high enough, the component related to positron vacancy trapping approaches \(\approx 100\%\), making very difficult the spectra decomposition. As a result, in PALS spectra only \(\tau\) can be extracted; in the saturation-trapping regime, \(\tau \approx \tau_v\). Thereby, \(\tau\) values corresponding to AQ states are the ones that approach best the characteristic vacancy-related lifetime.

Figures 3 and 4 show the overlap of theoretical and experimental \(\tau\) results (red-shadowed area). On the one hand, in both Sn\(_i\) and In\(_i\) systems, regardless of the calculated type of defect, GGA seems to overestimate the positron lifetime. Indeed, in the case of V\(_{\text{In}}\) and V\(_{\text{Sn}}\), the predicted lifetimes lie above of 210 ps, 20 ps higher than the experimental \(\tau\) values, which range between 178 and 190 ps. However, it is noteworthy to mention that this fact alone does not imply that \(\gamma(r)\)\(_{\text{GGA}}\) results are not compatible with the experimental results. In principle, according to Eq. (7), a nonsaturated regime of vacancies with a higher characteristic lifetime than the measured \(\tau\) could reproduce a \(\tau_v > \tau > \tau_b\). However, if the values predicted by \(\gamma(r)\)\(_{\text{GGA}}\) were accurate, \(\tau\) would not be in a saturated trapping regime, which would enable the spectra to be decomposed. As previously pointed out the decomposition is not feasible in any of the studied samples, which implies that \(\gamma(r)\)\(_{\text{GGA}}\) gives overestimated values.

On the other hand, within the LDA framework, both \(\gamma(r)\)\(_{\text{LDA}}\) and \(\gamma(r)\)\(_{\text{AP2}}\) systematically underestimate the positron lifetime. Because \(\tau\) must be always \(\tau \leq \tau_v\) (\(\lim_{\kappa_v \rightarrow \infty} \tau = \tau_v\)), the positron-lifetime values predicted by \(\gamma(r)\)\(_{\text{LDA}}\) and \(\gamma(r)\)\(_{\text{AP2}}\) cannot reproduce the experimental \(\tau\) values. As shown in Fig. 4, only in the case of the In\(_{15}\) sample, the characteristic lifetimes calculated by using \(\gamma(r)\)\(_{\text{AP1}}\) and \(\gamma(r)\)\(_{\text{AP2}}\) for V\(_{\text{In}}\) and V\(_{\text{In}}\) vacancies approach the experimental lifetimes. However, the calculated \(\tau\) values do not approach the AQ experimental values (which best approach the defect-related characteristic positron lifetimes), but the minimum experimental \(\tau\) values. As a result, \(\gamma(r)\)\(_{\text{LDA}}\) and \(\gamma(r)\)\(_{\text{AP2}}\) do not predict the appropriate value either.

Regarding the positron-lifetime values calculated by using the \(\gamma(r)\)\(_{\text{BN}}\) parametrization, they lie between those predicted within the OGA and the lifetimes predicted by using \(\gamma(r)\)\(_{\text{AP1}}\) and \(\gamma(r)\)\(_{\text{AP2}}\). In fact, as shown in Fig. 3 and Fig. 4, it seems...
that the most accurate values comparing the experimental results are given by the $\gamma(r)_{\text{LDA}}$ parametrization.

### A. Theoretical calculations on Ni$_2$MnGa

For the sake of validating the accuracy of $\gamma(r)_{\text{BN}}$ parametrization in Sn$_x$ and In$_x$ systems, positron-lifetime calculations have been performed on the Ni$_2$MnGa sample. Taking advantage of the existing PALS results in the literature on Ni$_2$MnGa [43,44], calculated values have been compared with the available experimental results. The crystallographic details, along with the results of theoretical positron calculations on Ni$_2$MnGa, are gathered in Table II. Calculations have been carried out for both austenite ($Fm\bar{3}m$) [69] and tetragonal martensite ($I4/mmm$, No. 139) [70] following the methodology described in Sec. III.

The calculated vacancy-related positron lifetime in Ni$_2$MnGa ranges between 178 and 185 ps, and the lifetime calculated for the martensite phase is $\approx$3–4 ps lower than those calculated in austenite phase. Here, as observed in Sn$_x$ and In$_x$ systems, the calculated vacancy-related positron lifetimes of V$_{\text{Mn}}$ and V$_{\text{Sn}}$ are higher than the one of V$_{\text{Ni}}$. Moreover, either for vacancy-related positron and for bulk lifetimes, the range of the calculated values for Ni$_2$MnGa is in a good agreement, which confirms the validity of the parametrization by using $\gamma(r)_{\text{LDA}}$. For each composition of both Sn$_x$ and In$_x$ alloys, the lowest calculated lifetime values are related to V$_{\text{Ni}}$. Besides, this value decreases with decreasing of the stoichiometry of the samples, from $\approx$190 ps down to 181 ps. Regarding the lifetimes related to V$_{\text{Mn}}$, for a fixed composition, calculations show that as soon as an additional exceeded Mn atom occupies the 4$b$ position, the characteristic lifetime related to V$_{\text{Mn}}$ increases on average $\approx$2 ps; see Table III. Due to the fact that Sn and In carry more electrons (valence and core) than Mn atoms, when an exceeded Mn atom occupies a 4$b$ position, the electronic density surrounding V$_{\text{Mn}}$ decreases. According to Eq. (1), a lower electronic density involves a lower annihilation rate $\lambda$, and therefore, a higher defect-related $\tau$ lifetime.

Finally, as Fig. 3 and Fig. 4 show, the calculated $\tau$ values for V$_{\text{Ni}}$ are the ones that present the best agreement with AQ $\tau$ values, illustrated in the upper part of the red-shadowed area. In the Sn$_x$ set, for Sn$_{25}$, Sn$_{20}$, Sn$_{13}$, and Sn$_{10}$, the measured $\tau$ value and the predicted values match perfectly, while for Sn$_{13}$ the deviation is $\approx$1 ps. The calculated $\tau$ values related to the rest of the vacancies are all 4 ps above. In the case of In$_x$, alloys, again, the best accordance between the measured and calculated lifetime is observed for the V$_{\text{Ni}}$ defect. In the specific case of In$_{16}$ and In$_{13}$ the calculated lifetime matches $\gamma(r)_{\text{LDA}}$ parametrization. Thereby, in the rest of this article the results obtained by means of $\gamma(r)_{\text{LDA}}$ will be analyzed.
with the $\tau$ experimental value and for In$_{20}$ and In$_{25}$ the deviation is about 2 ps. Thus, in light of the experimental and theoretical results for Sn$_x$ and In$_x$ systems, along with the good agreement between $\gamma(r)_{\text{LDA}}$ outcomes and experimental PALS results in the Ni$_2$MnGa system, it is concluded that V$_{Ni}$ is the vacancy type trapping positrons in Ni-Mn-Sn and Ni-Mn-In alloys.

This experimental result is in agreement with previous theoretical predictions. Recently Wang et al. [39] have shown that the most favorable type of defect in Ni-Mn-Ga samples is V$_{Ni}$. Regarding Ni-Mn-Sn and Ni-Mn-In samples, previous theoretical works also point to V$_{Ni}$ as the most favorable defect. Reports of Kulkova et al. [36,37] show that the formation energy $E_f$ of V$_{Ni}$ is the smallest in both the Ni$_2$MnSn and Ni$_2$MnIn systems, with $E_f = 0.4$ eV and $E_f = 0.7$ eV values, respectively, whereas the formation energies of V$_{Mn}$ and V$_{In}$ are $1.04$ eV and $2.59$ eV in Ni$_2$MnSn and $1.58$ eV and $2.29$ eV in Ni$_2$MnIn. Bai et al. [33–35] also suggest V$_{Ni}$ as the most favorable type of defect with $E_f = 0.59$ eV for the Ni$_2$MnIn alloy. Thus, taking into account the theoretical predictions, and the experimental and theoretical PALS results of the present work, it is concluded that in Ni-Mn-Ga, Ni-Mn-Sn, and Ni-Mn-In systems the present vacancy is V$_{Ni}$.

The V$_{Ni}$ concentration ($C_v^{Ni}$) can be estimated by means of Eq. (8). Usually, $\tau_v$ and $\tau_b$ values can be obtained after decomposing $\tau$. However, as discussed before, $\tau$ could not be decomposed in any of the samples of Sn$_x$ or of In$_x$. As an alternative way, and taking into account the good agreement between PALS experiments and DFT calculations, the theoretically calculated $\tau_v$ and $\tau_b$ values of Table III have been used to evaluate Eq. (8). Figure 6(a) shows the generic relation between $C_v$ and $\tau$, according to Eq. (8). In the dilute regime of defects, $\tau$ lies close to $\tau_b$, and in this range the variation of $C_v$ does not influence significantly $\tau$. However, in the midrange, the sensitivity of PALS is maximum, with a linear dependence between $C_v$ and $\tau$. Afterwards, in the saturation-trapping regime, the sensitivity decreases again and $\tau$ approaches $\tau_v$. As illustrated by the thick line in Fig. 6(a), PALS measurements on Sn$_x$ and In$_x$ lie in the saturated regime.

Figures 6(b) and 6(c) gather the estimated $C_v^{Ni}$ for Sn$_x$ and In$_x$ systems, respectively. The markers that delimit the thick line in each curve indicate maximum and minimum experimentally obtained $\tau_v$ values, and their corresponding $C_v^{Ni}$. In both Sn$_x$ and In$_x$ systems the lowest value of the experimentally measured vacancy concentration is $\approx 500$ ppm. The highest $C_v^{Ni}$ value, on the other hand, may go beyond $10^5$ ppm. Particularly, in the case of Sn$_{13}$ the predicted high $C_v^{Ni}$ is a consequence of the lack of sensitivity of the PALS technique in the saturation-trapping regime. However, the lower bound of the calculated $C_v^{Ni}$ lies around hundreds of ppm, a high value compared with those reported in some Ni-Mn-Ga systems [41,42] (around 10 ppm). Regardless of the composition, all samples show saturated trapping features. This result indicates that contrary to what is observed for Ni-Mn-Ga systems, Ni-Mn-Z ($Z =\text{In}, \text{Sn}$) systems present a
high, irreducible $C_v$, which hinders their control by conventional heat treatments.

These results contrast with the ones obtained by Hedayati et al. [32], in which they claim that they were able to tune the $T_{MT}$ as a function of $V_{Sn}$ concentration. It is noteworthy to mention that they do not report any experimental evidence of Sn vacancies. On the one hand, the combination of theoretical and experimental PALS results shows that the actual vacancy present in Ni-Mn-Sn systems is $V_{Sn}$. Additionally, it is demonstrated that Ni-Mn-Sn systems are characterized with a high $C_v$ and that their tuning and control by conventional heat treatments is not straightforward at all.

Indeed, as shown in Fig. 7, the variation of $C^v_{Ni}$ observed in the present work does not produce any $T_{MT}$ shift in the Ni$_{50}$Mn$_{37}$Sn$_{13}$ sample, which is one of the samples investigated by Hedayati et al. The lack of $T_{MT}$ variation is in agreement with the well-known fact that Ni-Mn-Sn systems exhibit a high stability of the L2$_1$ structure, which precludes the variation of the long-range atomic order and its consequent change on $T_{MT}$ [17]. In light of the present results, the capability of $V_{Sn}$ as a parameter to optimize $T_{MT}$ seems to be unfeasible in Ni-Mn-Sn systems.

VI. CONCLUSIONS

PALS measurements in Ni$_{50}$Mn$_{37}$Sn$_{13}$ ($x=25, 20, 15, 10$) and Ni$_{50}$Mn$_{37}$In$_x$ ($x=25, 20, 16, 13$) samples are complemented with DFT electron-positron theoretical calculations. The results confirm that experimentally measured $\gamma$ values correspond to positrons trapped at vacancies. In order to establish the parametrization of $\gamma(r)$ which best suits the experimental PALS results, five different widely used parametrizations of $\gamma(r)$ are analyzed within the LDA and GGA schemes, and compared with experimental data. Results indicate that the $\gamma(r)_{BN}$ LDA parametrization is the one that best predicts PALS values in the studied systems. Additionally, taking advantage of the existing PALS data in Ni-Mn-Ga systems, the accuracy of the $\gamma(r)_{BN}$ LDA parametrization is also tested and confirmed in Ni-Mn-Ga systems. Combining theoretical and experiments, $V_{Ni}$ is identified as the type of vacancy trapping positrons in Ni-Mn-Z ($Z=Ga, Sn, In$) samples. The characteristic $V_{Ni}$-related positron lifetime ranges between 181 and 191 ps. These results show that, unlike what happens in Ni-Mn-Ga alloys, and regardless of the heating treatments, Ni-Mn-Sn and Ni-Mn-In alloys are characterized by a high $C^v_{Ni}$ ($\geqslant 500$ ppm).

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