

Electroinductive waves role in left-handed stacked complementary split rings resonators

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Abstract: In this letter it is presented a Left-Handed Metamaterial design route based upon stacked arrays of screens made of complementary split rings resonators under normal incidence in the microwave regime. Computation of the dispersion diagram highlights the possibility to obtain backward waves provided the longitudinal lattice is small enough. The experimental results are in good agreement with the computed ones. The physics underlying the Left-Handed behavior is found to rely on electroinductive waves, playing the mutual capacitive coupling the major role to explain the phenomenon. Our route to Left-Handed metamaterial introduced in this paper based on stacking CSRRs screens can be scaled to millimeter and terahertz for future applications.

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OCIS codes: (160.3918) Metamaterials; (350.3618) Left-handed materials

References and Links

1. V.G. Veselago, "The Electrodynamics of Substances with Simultaneously Negative Values of ϵ and μ ," Sov. Phys. Usp. **10**, 509-514 (1968).
2. J.B. Pendry, A.J. Holden, D.J. Robbins, and W.J. Stewart, "Magnetism from conductors and enhanced nonlinear phenomena," IEEE Trans. Microwave Theory Tech. **47**, 2075-2084 (1999).
3. D.R. Smith, W.J. Padilla, D.C. Vier, S.C. Nemat-Nasser, and S. Schultz, "Composite medium with simultaneously negative permeability and permittivity," Phys. Rev. Lett. **84**, 4184-4187 (2000).
4. R. Marqués, F. Mesa, J. Martel, and F. Medina "Comparative Analysis of Edge- and Broadside- Coupled Split Ring Resonators for Metamaterial Design. Theory and Experiments," IEEE Trans. Antennas Propag. **51**, 2572-2581 (2003).
5. F. Falcone, T. Lopetegi, M. A. G. Laso, J. D. Baena, J. Bonache, M. Beruete, R. Marqués, F. Martín, and M. Sorolla, "Babinet principle applied to metasurface and metamaterial design," Phys. Rev. Lett. **93**, 197401-1-4 (2004).
6. P. Gay-Balmaz, and O. J. F. Martín, "Electromagnetic resonances in individual and coupled split-ring resonators," J. Appl. Phys. **92**, 2929-2936 (2002).
7. B. A. Munk, Frequency Selective Surfaces: Theory and Design, John Wiley, New York, 2000.
8. R. Marqués, F. Medina, and R. R. El-Idrissi, "Role of Bianisotropy in Negative Permeability and Left-Handed Metamaterials," Phys. Rev. B **65**, 144440 1-6 (2002).
9. M. Beruete, M. Sorolla, R. Marqués, J. D. Baena, and M. J. Freire, "Resonance and Cross-Polarization Effects in Conventional and Complementary Split Ring Resonator Periodic Screens," Electromag. **26**, 247-260 (2006).
10. R. Marqués, J. D. Baena, M. Beruete, F. Falcone, T. Lopetegi, M. Sorolla, F. Martín, and J. García, "Ab initio Analysis of Frequency Selective Surfaces Based on Conventional and Complementary Split Ring resonators," J. Opt. A: Pure and Appl. Opt. **7**, S38-S43 (2005).
11. M. Beruete, M. Sorolla, and I. Campillo, "Left-handed extraordinary optical transmission through a photonic crystal of subwavelength hole arrays," Opt. Express **14**, 5445-5455 (2006).
12. S. Zhang, W. Fan, N. C. Panoiu, K. J. Malloy, R. M. Osgood, and S. R. J. Brueck, "Experimental demonstration of near-infrared negative-index metamaterials," Phys. Rev. Lett. **95**, 137404-1-4 (2005).
13. G. Dolling, C. Enkrich, M. Wegener, C. M. Soukoulis, and S. Linden, "Simultaneous negative phase and group velocity of light in a metamaterial," Science **312**, 892-894 (2006).
14. N. Liu, H. Guo, L. Fu, S. Kaiser, H. Schweizer, and H. Giessen, "Three-dimensional photonic metamaterials at optical frequencies," Nat. Mater. **7**, 31-37 (2008).

15. A. J. Hoffman, L. Alekseyev, S. S. Howard, K. J. Franz, D. Wasserman, V. A. Podolskiy, E. E. Narimanov, D.L. Sivo, and C. Gmachl, "Negative refraction in semiconductor Metamaterials," *Nat. Mater.* **6**, 946-950 (2007).
16. M. Beruete, F. Falcone, M. J. Freire, R. Marqués, and J. D. Baena, "Electroinductive Waves in Chains of Complementary Metamaterial Elements," *Appl. Phys. Lett.* **88**, 083503-1-3 (2006).
17. N. Liu, S. Kaiser, and H. Giessen, "Magnetoinductive and Electroinductive Coupling in Plasmonic Metamaterial Molecules," *Adv. Mater.* **20**, 1-5 (2008).
18. J.D. Baena, J. Bonache, F. Martín, R. Marqués, F. Falcone, T. Lopetegi, M. A. G. Laso, J. García, I. Gil, M. Flores, and M. Sorolla, "Equivalent Circuit Models for Split Ring Resonators and Complementary Split Ring Resonators Coupled to Planar Transmision Lines," *IEEE Trans. Microwave Theory Tech.* **53**, 1451-1461, (2005).
19. R. Marqués, F. Martín, and M. Sorolla, *Metamaterials with Negative Parameters: Theory, Design, and Microwave Applications* (John Wiley, New York, 2008).
20. X. Chen, T. M. Grzegorzczak, B. I. Wu, J. Pacheco, Jr., and J. A. Kong, "Robust method to retrieve the constitutive effective parameters of metamaterials," *Phys. Rev. E* **70**, 016608-1-7 (2004).
21. N. Liu, L. Fu, S. Kaiser, H. Schweizer, and H. Giessen, "Plasmonic Building Blocks for Magnetic Molecules in Three-Dimensional Optical Metamaterials," *Adv. Mater.* **20**, 3859-3865 (2008)
22. E. Shamonina, V. A. Kalinin, K. H. Ringhofer, and L. Solymar, "Magneto-inductive waveguide," *Electron. Lett.* **38**, 371-373 (2002).
23. E. Shamonina, V.A. Kalinin, K. H. Ringhofer, and L. Solymar, "Magnetoinductive waves in one, two, and three dimensions," *J. Appl. Phys.* **92**, 6252-6261 (2002).
24. E. Shamonina and L. Solymar, "Magneto-inductive waves supported by metamaterial elements: components for a one-dimensional waveguide," *J. Phys. D* **37**, 362-267 (2004).
25. M. C. K. Wiltshire, E. Shamonina, I. R. Young, and L. Solymar, "Dispersion characteristics of magnetoinductive waves: comparison between theory and experiment," *Electron. Lett.* **39**, 215-217 (2003).
26. M. Aznabet, M. Navarro-Cia, S. A. Kuznetsov, A. V. Gelfand, N. I. Fedorinina, Yu. G. Goncharov, M. Beruete, O. El Mrabet, and M. Sorolla, "Polypropylene-substrate-based SRR- and CSRR-metasurfaces for submillimeter waves," *Opt. Express* **16**, 18312-18319 (2008).

1. Introduction

Left-Handed Metamaterials (LHM) were predicted by Veselago's theoretical study of media having simultaneously negative values of electric permittivity and magnetic permeability [1]. Inversion of Snell's law in the interface between a standard and a left-handed medium, reversal of Doppler and Cerenkov effects, are some of the possible exotic phenomena that take place in these media. The synthesis of a negative electric permittivity was feasible by that time, for example by means of a plasma or a metal at optical frequencies. Negative values of magnetic permeability in contrast were not attainable. It was in 1999 when Pendry [2] proposed a particle which can give a negative magnetic response in a certain frequency range: the Split-Ring Resonator (SRR). The first medium showing simultaneously negative values of permittivity and permeability was developed by Smith *et al.* [3].

Two characteristics make SRRs very interesting in the design of artificial media. The first one is that they are high Q resonators, which produce a strong diamagnetic response above the first resonance, so that they can potentially produce a negative effective μ . The second one is that this first SRR resonance is quasi-static [4], so that SRRs electrical size at such resonance is small enough to allow for a continuous media modelling of the composite.

More recently, the dual configuration or complementary SRR (CSRR), began to be studied as an alternative to SRRs for some applications [5]. The term complementary arises from the well-known Babinet's principle. Similarly to SRRs, CSRRs also behave -at its first resonance- as quasi-static LC resonators. However, due to Babinet's principle, a strong electric dipole generation is the dominant effect at this resonance and cross-polarization effects are also present as in SRRs [6]. The response of CSRRs can be described with the complementary polarizabilities p_z , m_y , and m_x (see Fig. 1 for axes definition) where only p_z and m_y are resonant at the quasi-static frequency [5]

Frequency selective surfaces (FSSs) consisting of two-dimensional arrays of conducting-patches or apertures within a metallic screen are well known in antenna engineering [7]. In these structures, according to diffraction theory, for secondary-grating lobe suppression the size and periodicity of the resonant elements should be smaller than the wavelength of the incident radiation. Otherwise, the resonant properties of the FSS may be related to the coupling between adjacent elements, thus depending on the angle of incidence of the

incoming radiation. The small electrical size of the SRR and CSRR particles make them very attractive in the application to FSSs, called more properly as metasurfaces. These particles have usually been excited by means of axial fields, see [2], but they can also be excited by transversal fields, due to their bianisotropy characteristic [8]. This last feature permits the plane wave excitation of a metasurface constructed with these particles. Besides, a rich variety of cross-polarization effects are observed by changing the angle of incidence [9]. The theory behind these results is based on the simplest homogenization procedure, i.e. on the assumption that the incident field on each SRR coincides with the mean field on the array. At resonance the equivalent SRR admittance goes to infinite and there is total reflection. The same analysis can be reproduced for the CSRR case while for this case there is total transmission [9,10].

A novel route to Left-Handed Metamaterial design has been proposed using subwavelength hole arrays stacked with the proper longitudinal lattice. Prototypes at millimeter wavelengths [11] and, with more fabrication limitations, at optical ones [12,13] have been demonstrated. Recently, an alternative method based upon stacking U-shaped particles has been proposed [14], as well as a solution with stacked layers of semiconductors [15].

In this paper we analyze the effects of axial stacking CSRRs metasurfaces, see Fig. 1, for novel metamaterial design and explain the phenomenon in terms of the so-called Electroinductive Waves (EIWs) [16]. A very interesting conclusion of the present study is that the excitation of EIWs, i.e. by coupling resonators capacitively or electrically, necessarily leads to LH propagation, regardless the configuration of the array (coplanar or stacked). It is worth noting that a similar analysis for just coplanar configurations of complementary U-shaped particles has been reported recently [17].

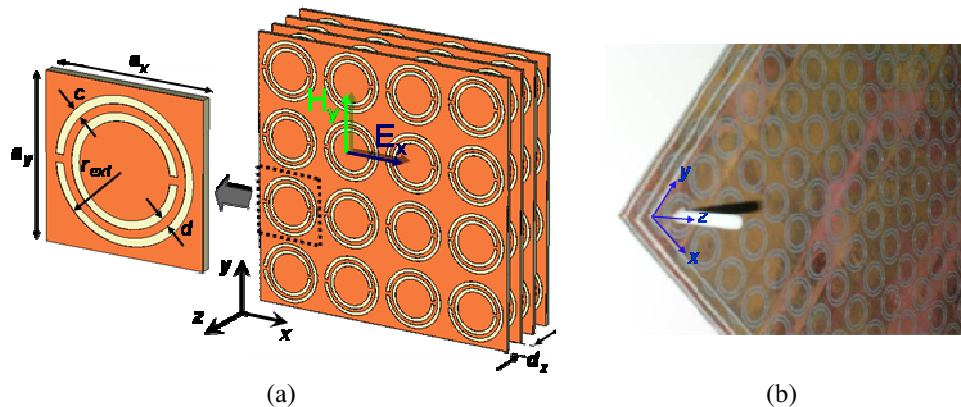


Fig. 1. (a) Schematics of: (left) single CSRR and (right) periodic structure composed of CSRRs metasurfaces. The transverse periodicity are equal $a_x = a_y = 8$ mm and $d_z = 2.145$ mm ($d_z = 0.03\lambda$); the parameters of the unit cell of CSRR are $c = d = 0.4$ mm, $r_{ext} = 3.5$ mm. (b) Picture of the fabricated stacked CSRR-metasurfaces with the aforementioned parameters.

2. Analysis of the metamaterial and electroinductive waves role.

By using the eigenmode solver of CST Microwave StudioTM code, the dispersion diagram of an infinite three dimensional stack of CSRR screens has been computed for four longitudinal lattices, see Fig. 2(a). The dispersion diagram is obtained with a single unit cell and using as boundary conditions magnetic walls at the top and bottom (y-direction), electric walls on the sides (x-direction) and periodic conditions on the front and back planes. The CSRRs are etched on a commercial low loss microwave substrate with relative dielectric permittivity $\epsilon_r = 2.43$ and height $h = 0.49$ mm, coated with a conductive layer of copper of thickness $t = 35$ μ m. The CSRR parameters are (see Fig. 1) $r_{ext} = 3.5$ mm and $c = d = 0.4$ mm, which gives a theoretical frequency of resonance of 4.65 GHz for the CSRR, in accord with the theories developed in [18]. Transversal periodicity is $a = 8$ mm, which is approximately 1/8 of the free

space wavelength at resonance. From the results of Fig. 2(a), it follows that when the longitudinal lattice is very small ($d_z = 2.145 \text{ mm} = 0.03\lambda$ and $d_z = 5 \text{ mm} = 0.08\lambda$, dark and red curves) the signs of phase and group velocities are opposite in the obtained bands. On the other hand, when correlative screens are slightly moved further away ($d_z = 15 \text{ mm} = 0.24\lambda$ and $d_z = 25 \text{ mm} = 0.39\lambda$, blue and green curves), the qualitative behavior changes drastically and the slope becomes positive, accounting for an identical sign of phase and group velocities. From these considerations, it follows that some relevant capacitive effect is present and plays a significant role in the propagative regime inside the stack. This aspect will be discussed later. At this point, the most immediate consequence of this numerical computation is that LHM behavior can be expected for the proposed structure, as shown by the evolution of the electric field E_x in Fig. 2(b) displayed for 15 stacked CSRR-metasurfaces for $d_z = 2.145 \text{ mm}$ (dark curve of Fig. 2(a)) at 5.3 GHz (Supporting video file is attached for a real-time visualization of the E_x evolution along the stack). It is worth noting the peculiar propagation inside the structure across the slits region and its asymmetric behavior between both lateral sides, but symmetric in the yz -plane.

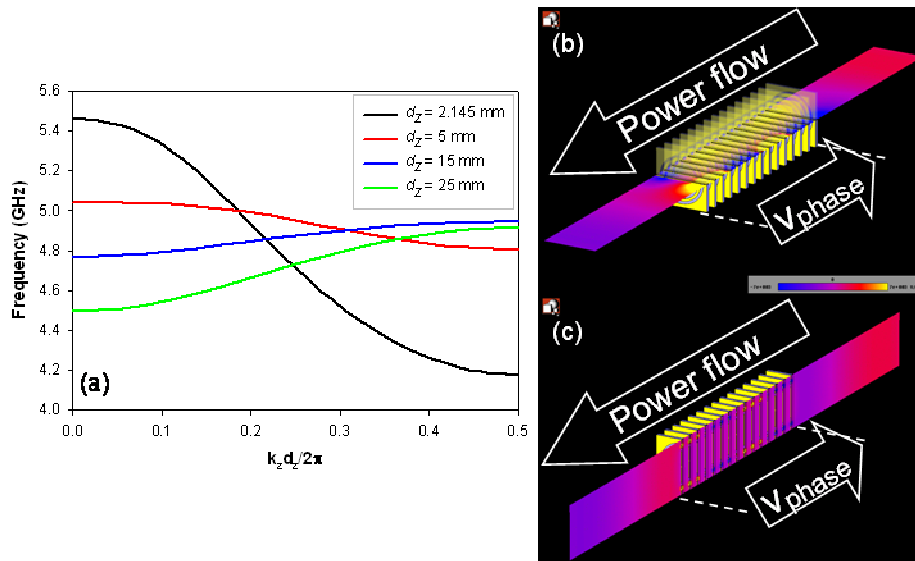


Fig. 2. (a) Infinite structure dispersion diagram for E-field along x -axis and four different longitudinal lattices. Single-frame excerpts from video recordings of the E_x evolution for stacked CSRR-metasurfaces at 5.3 GHz with $d_z = 2.145 \text{ mm}$ in the xz -plane (Media 1) (b) and yz -plane (Media 2) (c).

The symmetric and asymmetric behaviour of the field evolution can be explained in the very nature of the CSRR particle and the way we arrange them in the array. The problem of a plane wave normal incident on a screen made of periodic replication of equal elements, can be reduced to the analysis of a single element surrounded by two mutual perpendicular magnetic walls and electric walls as explained for the dispersion diagram calculation. Note that this case is valid for horizontal polarization.

The symmetric behaviour of the x -component of the electric field, E_x , which appears in the yz -plane follows from the fact that CSRR particle has a single symmetry plane, which is of the magnetic type, in the xz -plane. This plane of symmetry obviously forces symmetry of the electric field above and below, as appears in Fig. 2(c). Conversely, the asymmetric behaviour can be also obtained from the CSRR character. As described in Refs. 5 and 19, a CSRR particle can be described at its first resonance in each face with two resonant dipoles: an electric dipole normal to the plane (p_z in the notation of Fig. 1) and a magnetic dipole tangential to the plane (m_y in the notation of Fig. 1). A careful observation of Fig. 2(b) leads to

the conclusion that the E_x obtained can be related (at least in a first order approach) to that of an electric dipole aligned along z . In this case, there is not a symmetry plane and then, the field can be asymmetric in each half of the space.

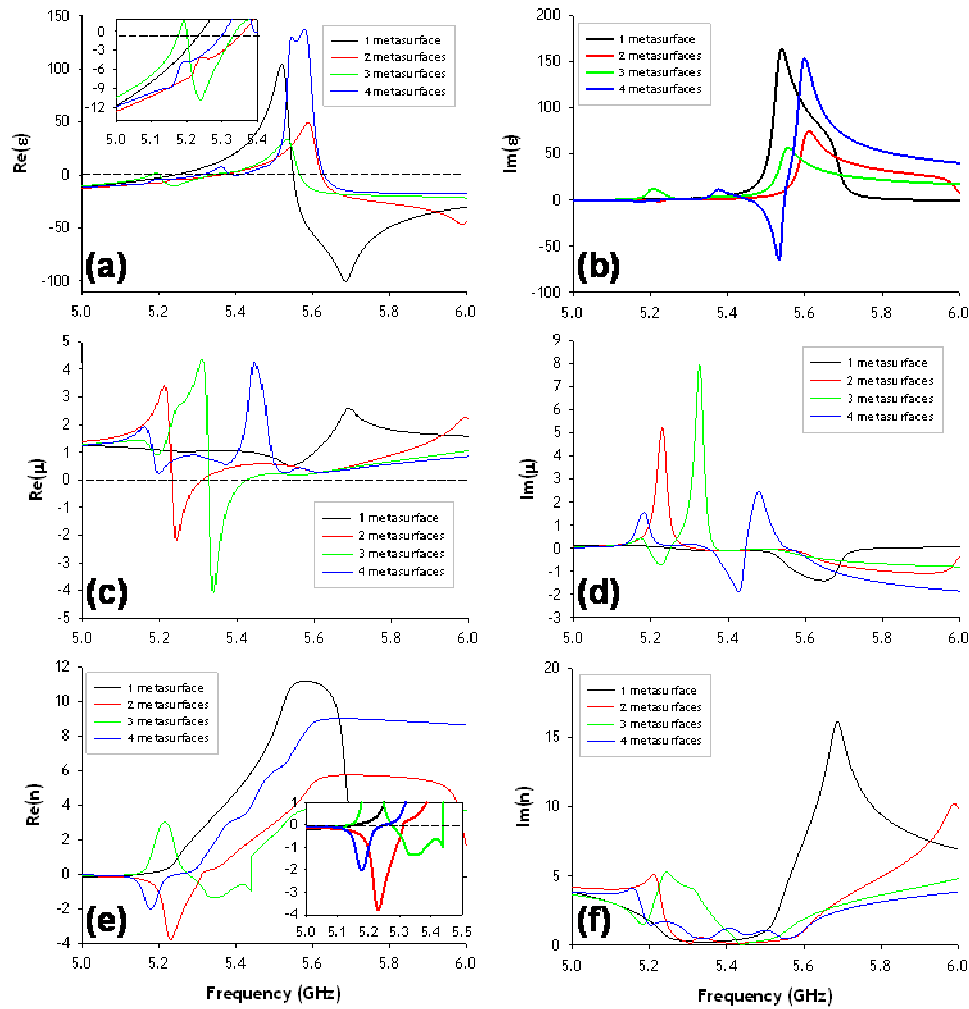


Fig. 3. Retrieved constitutive parameters (a) real part of the dielectric permittivity, (b) imaginary part, (c) real part of the magnetic permeability, (d) imaginary part, (e) real part of the refractive index and (f) imaginary part.

Alternatively, one can demonstrate the existence of a negative index of refraction by using a retrieval method based upon the simulated transmission and reflection coefficients (the retrieval from phase measurements is more sensitive to errors) [20]. The retrieved results are plotted in Fig. 3. Firstly, at the first resonance, retrieved parameters confirm the CSRR resonant behavior: it shows a Lorentzian dispersion for the electric permittivity, whereas magnetic permeability remains positive and close to 1. It is worth mentioning that electric permittivity comes in all cases from negative values since out of resonance, the structure should behave as a diluted metal (Drude dispersion). Besides, when several CSRR-metasurfaces are stacked, noticeable resonances on the magnetic permeability appear. These resonances suffer slight frequency shift due to the loading caused by the addition of new metasurfaces, whereas the one associated to electric permittivity undergoes less shifts once we deal with multilayers. Alike [21], negative sign of the real part of the refractive index is

caused by the contribution of both the real and imaginary parts of the constitutive parameters. This is the reason why the index of refraction is negative within frequency ranges where both real parts of electric permittivity and magnetic permeability are not negative.

A much more elegant explanation of the resulting behavior can be derived, to gain physical insight. Let us recall the concept of Magnetoinductive Waves (MIW) which was proposed in one-, two-, and three dimensional arrays of capacitively loaded metallic rings [22-25]. Nearly magnetostatic waves result from these structures described by Ampère's and Faraday's laws and giving less relevance to the displacement current. MIWs were initially studied in the context of the guiding properties of the SRRs particles forming chains in axial or coplanar configurations. Shamonina *et al.* analyzed both theoretically [22-24] and experimentally [25] their properties and found that these waves owe their existence to the magnetic coupling between adjacent resonators, so they can exist in any periodic structure with magnetic coupling between elements. In that series of papers, Shamonina *et al.* described exhaustively the properties of the waves and the effects of unmatched loading, losses in conductors, tolerances in fabrication, etc, and compared the characteristics of MI waveguides with standard transmission lines.

Complementariness concepts can be used again resulting in electrically coupled resonators. The waves supported by this kind of structure have been termed as Electroinductive Waves (EIW) [16] in analogy with MIW. They can be viewed like a nearly electrostatic counterpart of MIW. Then, the nearly electrostatic effect described by Coulomb's law is dominating.

In fact, the electric field lines follow a complex path but in a first approximation are concentrated in the inner metallic circle of the CSRR (see Fig. 3.29 in Ref. 19) because the CSRR particle is equivalent to a pair of electric dipoles which point normal to the CSRR in opposite directions and two opposite magnetic dipoles parallel to the CSRR as described in [19]. Another source of complexity is the bianisotropy property of the CSRR causing that the magnetic and electric dipoles follow a mixed dependence of the exciting electric and magnetic field components [19].

When a coplanar chain of CSRRs is formed by arraying particles, a mutual capacitance, C_M , arises between the central disks of correlative rings, which gives as a result a transmission line for EIW [16]. The details of the circuit model computation of the individual resonators are reported in [18] and the dispersion Eq. is calculated using the corresponding equivalent circuit shown in Ref. 16. The final Eq. is reproduced here for the sake of clarity,

$$\frac{\omega^2}{\omega_0^2} = \left(1 - \frac{2C_M}{C + 2C_M} \cos(ka) \right)^{-1} \quad (1)$$

where $\omega_0 = [L(C+2C_M)]^{-1/2}$. Note that this resonant frequency is not simply that of the LC-parallel for an isolated resonator. In fact, the capacitance seen by the central conductor is modified by the presence of the two adjacent neighbors and gives as a result the equivalent capacitance for the parallel of the capacitance to ground plane, C , and the two mutual capacitances, $2C_M$. In all the previous Eqs. only the nearest neighbor interaction has been taken into account which is enough to obtain the salient features in this kind of one-dimensional systems. For axial CSRRs arrays almost similar considerations can be done. If the plates are closely stacked, some of the electric field lines can connect correlative screens or, in other words, a mutual capacitance emerges. Obviously, the field distribution is different in coplanar and axial arrays but the qualitative behavior is similar and in a first order approach both configurations can be described with the same equivalent circuit. The above obtained dispersion Eq. shows a great resemblance with that of a MIW guide in the case of nearest neighbor interaction given in [19, 22-25], reproduced here for comparison purposes,

$$\frac{\omega^2}{\omega_0^2} = \left(1 + \frac{2M}{L} \cos(ka) \right)^{-1} \quad (2)$$

where M and L are the mutual inductance and self-inductance of an array of SRRs and $\omega_0 = (LC)^{-1}$ is the resonant frequency of an isolated resonator.

Both Eqs. (1) and (2) coincide if the resonant frequency of the CSRRs and the SRRs is the same and we change:

$$-C_M \rightarrow KM \quad (3)$$

$$C + 2C_M \rightarrow KL \quad (4)$$

where K is an arbitrary constant. If the CSRRs and the SRRs are embedded in an homogeneous isotropic medium, these conditions are fulfilled with $K=4 \cdot (\epsilon/\mu)$ [18].

Similar relations — although with different values of K — arise when both elements are etched on the plane interface between two semi-infinite media [18]. For intermediate situations, this equivalence will be only approximated. In summary, EIWs can be considered as the dual counterparts of MIWs, having a similar electromagnetic behavior in the sense of Babinet's principle [5].

If one analyzes phase and group velocities, it follows from the MIW case, that a backward propagation appears in the coplanar configuration whereas the axial configuration exhibits forward wave propagation. A simple explanation of these findings comes from the fact that the mutual inductance, M , is positive in the axial configuration and negative in the coplanar configuration.

However, for the EIW case it is clear that the mutual capacitance is positive for both coplanar and axial configurations and, logically, backward wave propagation is expected in both configurations. So, the simplified equivalent circuit that serves us to explain EIWs provides a sufficient physical explanation of the left-handed behavior predicted by the dispersion diagram. Moreover, the fields excited at the first CSRR resonance are strongly localized very near the particle. Then, mutual capacitance plays a significant role only when the stack period is very short. This fact explains the strong dependence on the longitudinal lattice of the propagative regime inside the stack. It is then clarified the existence of LH propagation only for longitudinal periods lower than approximately 8 % of the free space wavelength, red curve of Fig 2(a).

3. Experimental results

To check this theoretical result, an experiment has been designed with several planar screens of 24×24 CSRRs (with a total size of 200×200 mm), see Fig. 1(b), with the same parameters employed for the dispersion diagram computation ($d_z = 2.145$ mm). Notice that the measurement is for 2, 3, and 4 stacked screens of 24×24 CSRRs (for comparison purposes it is also given the result for a single plate) whereas the dispersion diagram shown in Fig. 2(a) is calculated for an infinite three dimensional stack, i.e. an infinite number of stacked CSRR screens, each of them having an infinite number of particles. So, we may expect a qualitative agreement in terms of resonance frequency and also bandwidth of the left-handed band.

Free-space amplitude and phase experiments of the stacked metasurfaces have been done illuminating them by a nearly plane wave. The power was transmitted and collected by means of a pair of standard high-gain microwave horn antennas. The normalized amplitude in logarithmic scale, Fig. 4(a), shows a transmission band for a different number of plates centered at 5.3 GHz. This result is in agreement with the calculated dispersion diagram, although a narrower frequency band is observed.

To identify the LHM band we follow the same method used in Ref. 11, and observe the phase difference as the number of stacked metasurfaces increases. After a careful measurement procedure, in the band where the transmission reaches its first maximum, for a fixed frequency, the phase of Fig. 4(b) increases as the number of structure periods increases in contraposition with conventional materials where the phase becomes more negative. This

result reveals that the phase and group velocities are in opposite directions and, therefore, left-handed propagation effects appear inside the structure as it has been discussed in [11].

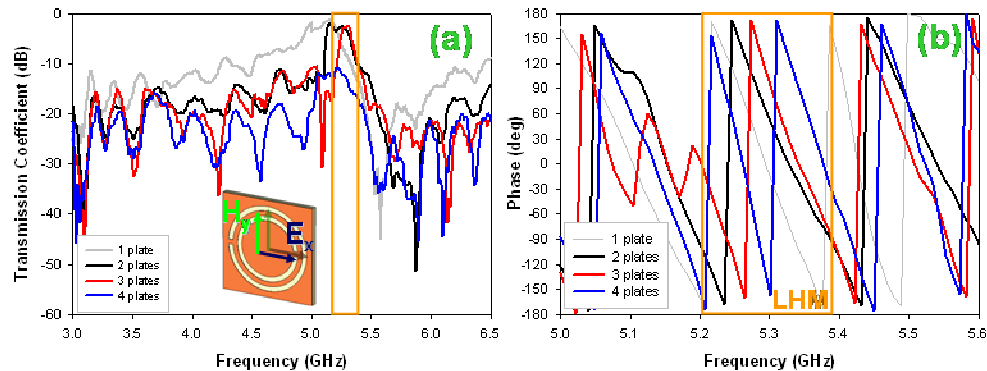


Fig. 4. Measurement for several stacked CSRRs metasurfaces of the transmitted amplitude (a) and phase (b) for the structure described in Fig.1 excited as indicated in the amplitude inset.

4. Conclusions

Summing up, it has been shown that by using stacked CSRRs metasurfaces a LHM can be built. Simulation and experimental results are in moderate agreement. Moreover, the key role of capacitive coupling is explained in terms of EIW. A relevant difference between MIW and EIW has been pointed out: EIW always propagate as backward waves for both axial and coplanar configurations due to the always positive mutual capacitance. By contrast MIW are forward waves in the axial configuration and backward in the coplanar one because the mutual inductance can be positive or negative. Finally, we expect similar behavior at millimeter and THz regimes following the manufacture method explained in Ref. 26, as long as the fabrication difficulty associated with stack alignment at such frequencies is overcome.

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