ON FREEDOM OF CHOICE, AMBIGUITY, AND THE PREFERENCE FOR EASY CHOICES

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On freedom of choice, ambiguity, and the preference for easy choices *

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Abstract

This paper is devoted to the study of opportunity set comparisons when the characteristics of the options within the sets in question may be ambiguous. We assume that agents display a preference for freedom of choice, but also aversion to the presence of ambiguous options. We propose a suitable environment for approaching this problem, and provide axiomatic characterizations of several rules for ranking sets in such a context.

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WILHELM: *I'm afraid ...*

GRACE: *There's nothing to be afraid of! We've taken all the family's weapons!*

WILHELM: *No, I am afraid of what will happen now. I fear we ain't ready for a completely new way of life. At Manderlay we slaves took supper at 7 ... when do people take supper when they are free? We don't know these things ...*

(From *Manderlay*, Lars von Trier (2005))

1 Introduction

During the last two decades a growing number of works have tried to formalize the idea of *preference for freedom of choice* (see, among many others, Jones and Sugden (1982), Pattanaik and Xu (1990, 1998, 2000), Sen (1991, 1993), Bossert et al. (1994), Puppe (1995, 1996), Bossert (1997, 2000), or Sugden (1998)). In most cases, the central object of analysis is that of an *opportunity set*, which is defined as the set of mutually exclusive options available to the decision maker, from which *he* (and not nature or chance) chooses one. Opportunity sets are then taken as a reference to evaluate the degree of freedom of choice enjoyed by an individual. A common and very natural characteristic of this approach is a *monotonicity* feature, according to which any enlargement of an agent's opportunity set leads to either a strict or weak improvement in terms of freedom of choice.

The main motivation of our work stems from the experience of many situations where, together with a preference for freedom of choice, individuals at some point display an aversion to excessive complexity in the choice problem, and therefore a preference for simpler problems with fewer options. Everyday examples include situations such as ordering a meal from a menu in a restaurant, picking a particular brand of tooth paste from the shelf in a supermarket, deciding which model of car to buy, or employers that hire human resource companies to pre-select job candidates, thus revealing their willingness to pay to have their opportunity sets reduced.

In this paper, we propose an axiomatic model that aims to be consistent with two conflicting assumptions - on the one hand, the preference for freedom of choice, which drives a preference for larger sets, and on the other hand, the preference for ease of choice, which implies fewer options.

The possible motivations for a preference for freedom of choice have been firmly rooted in the spheres of economics and philosophy since Mill (1956) or Nozick (1974), and also in the psychology literature, especially since Lewin (1952). Both fields provide motivation to consider individual freedom of choice as something with a value that transcends the quality of the possible alternatives, having more to do with their actual availability. This is a point of view that we will maintain throughout the paper.[1]

Concerning the roots of the preference for ease of choice, we explain this phenomenon as an aversion to the menu including alternatives that exceed a certain degree of ambiguity. The literature provides at least four explanations that support this hypothesis. The first views ambiguous options as those that raise the computational costs involved in the choice problem. According to the second explanation, the presence of ambiguous options makes it more difficult for the agent to justify his choice to others. The third perceives ambiguous options as *tempting* alternatives that involve self-control costs for the decision maker. Finally, the availability of ambiguous options may also increase the likelihood of *regret*.

With respect to the first explanation, authors such as Simon (1955, 1956), or Krulee (1955) had already introduced in the economic literature the idea

[1] For a survey on the economic roots for the intrinsic value of freedom of choice, see Barberà et al. (2004). For different theories about the psychological benefits of freedom of choice, see, for example, Langer and Rodin (1976), Deci (1985), or Taylor (1989).

that decision processes involve computational costs, and that human beings display a preference for coherence in their decisions (see also Shafir et al. (1993), Montgomery (1983) or Tversky and Kahneman (1986)). This, as Simon (1955) suggests, might motivate individuals to self-restrict the set of available alternatives as a way to simplify the choice problem. Very plausibly, ambiguous options are those that most increase the computational costs that decision procedures need in order to reach minimal standards of internal coherence. This, using Payne's (1982) words, could be interpreted as if ambiguous options would increase the "task complexity" of the choice problem. Therefore, the decision maker, at some point, might prefer to face a choice with fewer, but unambiguous and sufficiently well-known options.

Sometimes the desire for coherence of choice stems from the need to justify choices to others (the spouse, a superior, or a social group to which the agent belongs) (see, for example, Blau (1964), Tetlock (1985), Curley et al. (1986), or Simonson (1989)). In such a situation, the presence of ambiguous options might make it more difficult to provide others with convincing reasons to support a choice that has already been made.

A third possible explanation is associated with the psychological *cost of disappointment* experienced by individuals who have chosen an alternative that turns out to be less satisfactory than expected (see Bell (1985), Loomes and Sugden (1986) or Gul (1991)), and with the models about *self-control costs* associated to the presence in the menu of tempting alternatives (see Gul and Pesendorfer (2001, 2004)). Naturally, the most ambiguous alternatives are those involving higher risk of disappointment.[2] Thus, individuals with

[2] This intuition synchronizes with the measures of potential disappointment that Bell (1985), Loomes and Sugden (1986) or Gul (1991) propose for lotteries in a choice under uncertainty framework.

problems of self-control may prefer to commit to a restricted opportunity set excluding any ambiguous (and tempting) alternatives as a defence mechanism against the potential disappointment they may yield.

Finally, related to the concept of disappointment is that of *regret*. An individual experiences regret when, after choosing an alternative from the menu, he feels that he could have chosen another better one (see Bell (1982), and Loomes and Sugden (1982)). Now, in order to avoid regret, the decision maker might actually prefer to renounce the option to choose ambiguous alternatives, which, again, by their nature, are more likely to produce regret.

Any of the above reasons may, therefore, lead to a disutility due to the mere presence of ambiguous options in the menu. Therefore, the monotonicity assumption, which is referential in the freedom of choice literature, may be violated. Now, the shrinking of an opportunity set may be desirable if the gain in ease of choice is enough to compensate for the preference for freedom. In other words, a trade-off arises between the preference for freedom of choice and the preference for ease of choice, which suggests the convenience of an axiomatic approach.

In order to perform an axiomatic exploration of the formal effects of those two opposite forces in an intuitive way, we introduce a simple qualitative distinction among the feasible alternatives. We split them into a binary partition based on a certain standard of ambiguity, or the decision-maker's degree of knowledge about the alternatives. According to this distinction, some alternatives are sufficiently unambiguous for the decision maker, while the rest do not meet the given standard (for example, some dishes on the menu have been tried previously, while in others the ingredients or mode of preparation are unfamiliar; references are available for some candidates for a job, but not for others, and so on). This rough distinction is enough to show that the informational aspect inevitably introduces a different way to evaluate opportunity sets.

Taking the above mentioned distinction, in the following sections we provide characterizations of several rankings on the basis of very simple axioms. In Section 2 we introduce some notation and definitions. In Section 3 we axiomatically characterize a rule for ranking sets that takes into account only the number of unambiguous options available to the decision maker. By changing one of the axioms we obtain a family of rankings that is based on two numbers only - the number of unambiguous options in the menu, and the number of ambiguous ones, where the former are positively weighted and the latter are negatively weighted. This family is introduced in Section 4, and the axiomatic characterizations of three particular rules belonging to the family are presented in Section 5. We conclude in Section 6 with some final remarks.

2 Basic setup

Let X be a non-empty set of alternatives (finite or infinite) with $|X| \geq 2$, and let $\mathcal{X}$ be the set of all finite subsets of X. We will denote the elements of $\mathcal{X}$ by $A, B \ldots$. We distinguish two categories of alternatives in X - those whose relevant aspects are sufficiently well determined, and the rest, that is, those for which there is insufficient information concerning their relevant characteristics. We will label the former as "unambiguous" options, and the latter as "ambiguous" options.

The standards for this distinction may depend on several factors. For example, it can be sensitive to subjective aspects - some individuals may have very high informational standards in order to feel comfortable when making a choice, in which case they will be more prone to labeling options as ambiguous. The standards may also depend on the significance or triviality of the particular problem of choice, on the degree of time-pressure under which the decision must be made, or other factors. Similarly, according to our terminology, an option could still be labeled as "ambiguous" if the information in possession of the decision maker is abundant but not sufficiently relevant - for example, an employer may have very detailed information about certain aspects of a candidate, such as his private life, but not a clue about his labor skills. In any case, for the purposes of the model, what matters is simply to recognize the fact that such a partition between ambiguous and unambiguous alternatives can be made according to a certain standard. This crisp distinction is sufficient to cover our main concern, namely, that taking into account informational aspects together with the assumption of ambiguity aversion leads naturally to a very different kind of rules for ranking opportunity sets.

As mentioned in the Introduction, and following the line of Pattanaik and Xu's (1990) analysis of the value of freedom of choice, we do not presuppose the existence of a preference relation over the set X of basic alternatives. This is also coherent with Nozick's (1974) defense of the *intrinsic* value of freedom of choice, or Mill's (1956) view that choice is desirable as a means to develop human faculties. It is also supported by the existing theories about the psychological and emotional benefits of the mere ability to choose. Thus, our model is not about the indirect utility a choice situation provides to the decision maker, but about its degrees of freedom and ease of choice, and the trade-offs between these two aspects.

Consequently, in our context the problem can be determined by considering two aspects - the opportunity set the agent enjoys, which is assumed to be an element of $\mathcal{X}$, and the set of alternatives in X, the relevant characteristics of which the decision maker has *enough* information about, that is, the set of unambiguous alternatives. We will call this set the *unambiguous set* or *information set* and assume it to be finite, i.e., this set will also be an element of $\mathcal{X}$. In order to avoid some trivialities, we assume further that the unambiguous set is not empty.

Thus, we are formally interested in the elements of $\mathcal{X} \times \mathcal{X}_{\emptyset}$, where $\mathcal{X}_{\emptyset} := \mathcal{X} \setminus \{\emptyset\}$. We call each $(A, C) \in \mathcal{X} \times \mathcal{X}_{\emptyset}$ a *choice situation* and attach to it the interpretation that the decision maker has the opportunity to choose an option in A while having enough information about the options contained in C. Comparisons of choice situations will be represented by a binary relation $\succsim$ defined on $\mathcal{X} \times \mathcal{X}_{\emptyset}$. For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_{\emptyset}$, $(A, C) \succsim (B, D)$ should be read as "the opportunity to choose from A while knowing about the options in C is weakly preferred to the opportunity to choose from B while knowing about the options in D". The asymmetric and symmetric parts of $\succsim$ will be denoted by $\succ$ and $\sim$, respectively. We want $\succsim$ to capture both the preference for freedom of choice and the aversion to the presence of ambiguous options. Finally, by $\mathcal{P}$ we denote the set of all complete preorders (reflexive, transitive and complete binary relations) defined on $\mathcal{X} \times \mathcal{X}_{\emptyset}$.

An important feature of the model is that the unambiguous set is variable across different choice situations. This reflects the fact that individuals can gather information about the alternatives, and provides a formal framework to analyze the effects on individual welfare of the provision of information, as well as further questions, such as the willingness to pay to obtain information.

3 The unambiguous-options-based rule

We start our analysis by introducing the following four axioms:

Empty choice (EC): For all $C, D \in \mathcal{X}_\emptyset$, $(\emptyset, C) \sim (\emptyset, D)$.

Monotonicity towards unambiguous options (M): For each $C \in \mathcal{X}_\emptyset$ there exists $A \in \mathcal{X} \setminus X$ such that $(A \cup \{x\}, C) \succ (A, C)$ for some $x \in C \setminus A$.

Neutrality towards ambiguous options (N): For each $C \in \mathcal{X}_\emptyset \setminus X$ there exists $A \in \mathcal{X} \setminus X$ such that $(A \cup \{x\}, C) \sim (A, C)$ for some $x \in X \setminus (A \cup C)$.

Independence (IND): For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_\emptyset$, and all $x \in X \setminus A$, $y \in X \setminus B$ with $x \in C \Leftrightarrow y \in D$, $(A, C) \succsim (B, D) \Leftrightarrow (A \cup \{x\}, C) \succsim (B \cup \{y\}, D)$.

According to EC, the decision maker is indifferent between any two situations in which he has no options to choose from, regardless of the amount of information he might have about the (unavailable) alternatives.

Axiom M states that, given any information set, C, we can always find some opportunity set, A, that can be improved by adding *some* new unambiguous option to it. This is a rather weak requirement if one assumes that freedom of choice is valuable to the decision maker. Especially, the axiom becomes very plausible if one thinks of A as the empty set and x as any unambiguous option.

According to N, given any information set, there exists some opportunity set, and some option that does not belong to it, such that the addition of that option to the set does not affect the desirability of the choice situation. It could be interpreted as being equivalent to assuming that the extra freedom incorporated through the addition of the new option is canceled by the aversion felt towards its presence as a result of its ambiguity. For example, this axiom is plausible if one believes that ambiguity only causes aversion when the number of opportunities is great enough. Another possible way to think about N is by assuming that marginal freedom is decreasing with the number of available options while marginal aversion is increasing with it, and at a certain point the two equalize.

IND displays the effect of adding (or dropping) two options that are "of the same type" in the sense that they are either both unambiguous or both ambiguous. The axiom says that the original ranking between any two choice situations is preserved under such a modification. This axiom adapts to our context other similar axioms often found in the literature about freedom of choice.

As we will see in the first theorem, the four axioms introduced so far provide a characterization of the *unambiguous-options-based rule* $\succsim^u \in \mathcal{P}$, which is defined as follows:

For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_\emptyset$,

$$(A, C) \succsim^u (B, D) \text{ iff } |A \cap C| \geq |B \cap D| .$$

Theorem 1 *Let* $\succsim \in \mathcal{P}$*. Then* $\succsim$ *satisfies EC, IND, M, and N if and only if* $\succsim = \succsim^u$*.*

The proof is presented in the appendix.

In order to show the *independence* of the axioms used for the characterization of $\succsim^u$, consider the following four examples. The reader can easily check that each example satisfies all axioms except one:

$\neg$(EC): For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_\emptyset$, $(A, C) \succsim (B, D)$ iff (1) $|C| < |D|$, or (2) $|C| = |D|$ and $|A \cap C| \geq |B \cap D|$.

$\neg$(IND): Let $|X| \geq 3$. For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_\emptyset$, let $\succsim$ be defined as follows: (1) if $|A| \geq 3$ and $|B| \geq 3$, then $(A, C) \sim (B, D)$, (2) if $|A| \geq 3$ and $|B| < 3$, then $(A, C) \succ (B, D)$, (3) if $|A| < 3$ and $|B| < 3$, then $\succsim = \succsim^u$.

$\neg$(M): For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_\emptyset$, $(A, C) \succsim (B, D)$ iff $|A \cap C| \leq$

$|B \cap D|$.

$\neg$(N): For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_\emptyset$, $(A, C) \succsim (B, D)$ iff (1) $|A \cap C| > |B \cap D|$, or (2) $|A \cap C| = |B \cap D|$ and $|A \setminus C| \leq |B \setminus D|$.

4 A family of rules

Axiom N states that, at a certain point, the decision maker will remain indifferent to the addition of a certain ambiguous option. The next axiom shows that it is also always possible to find opportunity sets for which the decision maker will be averse to the addition of a certain ambiguous option.

Aversion towards ambiguous options (AV): For each $C \in \mathcal{X}_\emptyset \setminus X$ there exists $A \in \mathcal{X} \setminus X$ such that $(A \cup \{x\}, C) \prec (A, C)$ for some $x \in X \setminus (A \cup C)$.

In other words, AV expresses the idea that there is some situation where the preference for an easier choice arises. In particular, AV says that, whatever the information set we consider, there exist some opportunity set, A, and some ambiguous option that does not belong to A, such that the former is worsened by the addition of the latter.

It turns out that the replacement of N with AV in the characterization displayed by Theorem 1 does not result in a unique rule for ranking choice situations; it rather generates a family of rules that are based on two numbers only - the number of unambiguous options and the number of ambiguous options, where the former are positively weighted and the latter are negatively weighted.

Theorem 2 *Let* $\succsim \in \mathcal{P}$ *satisfy EC, IND, M, and AV. Then, for all* $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_\emptyset$,

(1) $(|A \cap C| > |B \cap D|$ *and* $|A \setminus C| < |B \setminus D|)$ *implies* $(A, C) \succ (B, D)$,

(2) $(|A \cap C| \geq |B \cap D|$ *and* $|A \setminus C| \leq |B \setminus D|)$ *implies* $(A, C) \succsim (B, D)$.

The proof is presented in the appendix.

5 Trade-offs

In this section we introduce three additional and alternative axioms that, when added to conditions EC, IND, M, and AV, will finally lead to three particular rankings of choice situations that belong to the family described in Theorem 2. Each of these rankings provides a different solution to the trade-off between the preference for unambiguous alternatives and the aversion to ambiguous ones. The first two rules combine these aspects lexicographically, while the third is of an additive nature.

5.1 The unambiguous-options-priority rule

Let us consider a situation where $(A, C) \succ (B, D)$ for some $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_\emptyset$, and where the opportunity set B is included in $A \cap C$, i.e., B contains only options that would be unambiguous in the choice situation (A, C). The idea of robustness of the strict preference displayed by the next axiom requires in this case that the addition of a new option to A does not change the original ranking.

Robustness 1 (ROB1): For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_\emptyset$ with $B \subset (A \cap C)$ and for all $x \in X$, $(A, C) \succ (B, D) \Rightarrow (A \cup \{x\}, C) \succ (B, D)$.

In other words, ROB1 imposes a restriction in a very particular class of comparisons that can be motivated as follows. Assume that, starting from a choice situation, (B, D), there is a transition to another choice situation,

(A, C), where the number of available options has been enlarged, and, moreover, there has been an *improvement* in the information in the sense that the agent has obtained all the relevant information about the options in B that were ambiguous at the beginning. The axiom says that, in such a case, a preference for (A, C) (which is quite plausible) should be robust enough to resist the incorporation of a new option in A, even if the newly added option is an ambiguous one.

We have shown in Section 4 that EC, IND, M, and AV generate a family of rules for ranking choice situations. Adding ROB1 to these axioms results in the characterization of the *unambiguous-options-priority rule* $\succsim^{up} \in \mathcal{P}$, which is defined as follows:

For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_\emptyset$,

$$(A, C) \succsim^{up} (B, D) \text{ iff } \begin{cases} |A \cap C| > |B \cap D|, \\ \text{or} \\ |A \cap C| = |B \cap D| \text{ and } |A \setminus C| \leq |B \setminus D|. \end{cases}$$

Theorem 3 *Let $\succsim \in \mathcal{P}$. Then $\succsim$ satisfies EC, IND, M, AV, and ROB1 if and only if $\succsim = \succsim^{up}$.*

The proof is presented in the appendix.

The *independence* of the axioms used for the characterization of $\succsim^{up}$ can be checked by means of the following five examples:

$\neg$(EC): For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_\emptyset$, $(A, C) \succsim (B, D)$ iff (1) $|C| < |D|$, or (2) $|C| = |D|$ and $(A, C) \succsim^{up} (B, D)$.

$\neg$(IND): Let $|X| \geq 3$. For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_\emptyset$, let $\succsim$ be defined as follows: (1) if $|A| \geq 3$ and $|B| \geq 3$, then $(A, C) \sim (B, D)$, (2) if $|A| \geq 3$ and $|B| < 3$, then $(A, C) \succ (B, D)$, (3) if $|A| < 3$ and $|B| < 3$, then $\succsim = \succsim^{up}$.

$\neg$(M): For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_{\emptyset}$, $(A, C) \succsim (B, D)$ iff (1) $|A \cap C| < |B \cap D|$, or (2) $|A \cap C| = |B \cap D|$ and $|A \setminus C| \leq |B \setminus D|$.

$\neg$(AV): For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_{\emptyset}$, $(A, C) \succsim (B, D)$ iff $|A| \geq |B|$.

$\neg$(ROB1): For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_{\emptyset}$, $(A, C) \succsim (B, D)$ iff (1) $|A \setminus C| < |B \setminus D|$, or (2) $|A \setminus C| = |B \setminus D|$ and $|A \cap C| \geq |B \cap D|$.

5.2 The ambiguous-options-priority rule

Let us now consider another notion of robustness, as shown by the following axiom:

Robustness 2 (ROB2): For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_{\emptyset}$ with $A \subset (B \setminus D)$ and for all $x \in X$, $(A, C) \succ (B, D) \Rightarrow (A, C) \succ (B \cup \{x\}, D)$.

In the case of condition ROB2, whenever the opportunity set A consists only of options that would be ambiguous in the situation (B, D), and if (A, C) is strictly better than (B, D), then the ranking should be preserved when a new option is added to B. In other words, assume now that, in the transition from (B, D) to (A, C) the agent has lost opportunities and that only ambiguous options (in the new situation) remain.[3] Then, a preference for (A, C) over (B, D) should be robust enough as to resist the incorporation to B of a new option, even if it is unambiguous. In other words, the idea behind ROB2 is that, if there is some reason for such a preference for (A, C) to arise, even when A is a subset of the ambiguous options in situation (B, D), this reason should have enough weight as to resist any additional

[3] One might not expect the information to shrink in the transition from one situation to another. However, the informational requirements in the new situation might have changed such that information about the alternatives, that was initially considered sufficient, later becomes insufficient.

slight changes. In some sense, ROB2 is a dual version of ROB1, however, they are not logically compatible.

As shown in our next theorem, the addition of ROB2 to EC, IND, M, and AV results in the characterization of the *ambiguous-options-priority rule* $\succsim^{ap} \in \mathcal{P}$, which is defined as follows:

For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_{\emptyset}$,

$$(A, C) \succsim^{ap} (B, D) \text{ iff } \begin{cases} |A \setminus C| < |B \setminus D|, \\ \text{or} \\ |A \setminus C| = |B \setminus D| \text{ and } |A \cap C| \geq |B \cap D|. \end{cases}$$

Theorem 4 *Let $\succsim \in \mathcal{P}$. Then $\succsim$ satisfies EC, IND, M, AV, and ROB2 if and only if $\succsim = \succsim^{ap}$.*

The proof is presented in the appendix.

In order to show the *independence* of the axioms used for the characterization of $\succsim^{ap}$, consider the following examples:

$\neg$(EC): For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_{\emptyset}$, $(A, C) \succsim (B, D)$ iff (1) $|C| < |D|$, or (2) $|C| = |D|$ and $(A, C) \succsim^{ap} (B, D)$.

$\neg$(IND): Let $|X| \geq 3$. For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_{\emptyset}$, let $\succsim$ be defined as follows: (1) if $|A| \geq 3$ and $|B| \geq 3$, then $(A, C) \sim (B, D)$, (2) if $|A| < 3$ and $|B| \geq 3$, then $(A, C) \succ (B, D)$, (3) if $|A| < 3$ and $|B| < 3$, then $\succsim = \succsim^{ap}$.

$\neg$(M): For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_{\emptyset}$, $(A, C) \succsim (B, D)$ iff $|A \setminus C| \leq |B \setminus D|$.

$\neg$(AV): For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_{\emptyset}$, $(A, C) \succsim (B, D)$ iff (1) $|A \setminus C| > |B \setminus D|$, or (2) $|A \setminus C| = |B \setminus D|$ and $|A \cap C| \geq |B \cap D|$.

$\neg$(ROB2): Take $\succsim = \succsim^{up}$.

5.3 The difference rule

Let us now consider the following axiom.

Dichotomy (DI): For each $C \in \mathcal{X}_\emptyset \setminus X$ there exists $A \in \mathcal{X}_\emptyset \setminus X$ such that $(A \setminus \{x\}, C) \sim (A \cup \{y\}, C)$ for some $x \in A \cap C$, and some $y \in X \setminus (A \cup C)$.

Condition DI is the result of adapting an axiom used by Dimitrov et al. (2004) to our decisional context. The axiom states that, for any information set, there exists some opportunity set for which there is a perfect substitution between unambiguous and ambiguous options - a loss of freedom due to the removal of some unambiguous option from the set is, somehow, equivalent to a loss of ease of choice due to some ambiguous option being added to it. Though it is an independent axiom, the idea suggested by DI is close to that suggested by N. For example, one possible interpretation (but not the only one) is that the loss of freedom resulting from dropping an unambiguous option is decreasing with the number of available alternatives, while the loss of ease of choice caused by adding an ambiguous option is increasing with the number of alternatives. Then the axiom would say that at certain point both values equalize.

We are now ready to present the characterization of the *difference rule*, $\succsim^d \in \mathcal{P}$, defined as follows:

For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_\emptyset$,

$$(A, C) \succsim^d (B, D) \text{ iff } |A \cap C| - |A \setminus C| \geq |B \cap D| - |B \setminus D| .$$

Theorem 5 *Let $\succsim \in \mathcal{P}$. Then $\succsim$ satisfies EC, IND, M, and DI if and only if $\succsim = \succsim^d$.*

The proof is presented in the appendix.

In order to check the *independence* of the axioms used for the characterization of $\succsim^d$, the reader can consider the following examples:

$\neg$(EC): For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_{\emptyset}$, $(A, C) \succsim (B, D)$ iff (1) $|C| < |D|$, or (2) $|C| = |D|$ and $(A, C) \succsim^d (B, D)$.

$\neg$(IND): Let $X = \{x, y\}$, and consider the following ranking on $\mathcal{X} \times \mathcal{X}_{\emptyset}$: $(\{x, y\}, \{x, y\}) \succ (\{x\}, \{x\}) \sim (\{y\}, \{y\}) \sim (\{x\}, \{x, y\}) \sim (\{y\}, \{x, y\}) \succ (\emptyset, \{x\}) \sim (\emptyset, \{y\}) \sim (\emptyset, \{x, y\}) \sim (\{x, y\}, \{x\}) \sim (\{x, y\}, \{y\}) \succ (\{y\}, \{x\}) \succ (\{x\}, \{y\})$.

$\neg$(M): For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_{\emptyset}$, $(A, C) \sim (B, D)$.

$\neg$(DI): For all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_{\emptyset} : (A, C) \succsim (B, D)$ iff $|A| \geq |B|$.

6 Concluding remarks

Our model assumes the preference for freedom of choice, but also the possibility of an aversion to alternatives whose characteristics are sufficiently unknown (what we have called a *preference for easy choices*). Among the different ways in which the decision maker may evaluate opportunity sets containing both "ambiguous" and "unambiguous" options, we propose some plausible solutions derived from a common axiomatic basis. These core axioms are *Empty Choice* (EC), *Independence* (IND), and *Monotonicity towards unambiguous options* (M). Adding *Neutrality towards ambiguous options* (N) to this basis leads to the unambiguous-options-based rule, which establishes that it is the number of unambiguous options that determines the ranking. However, when replacing N by *Aversion towards ambiguous options* (AV), there arises a family of rules that takes into account ease of choice by negatively weighting the number of ambiguous options. Then, starting from EC, IND, M, and AV, by imposing either of the *Robustness* axioms (ROB1 or ROB2), we obtain, respectively, the characterization of two lexicographic rules - the unambiguous-options-priority-rule and the ambiguous-options-priority rule. We have already noted the differences between ROB1 and ROB2. Thus, it is up to the reader to choose between the two ideas of robustness. Finally, if instead of either of the robustness axioms we use *Dichotomy* (DI), then an additive rule that maximizes the difference between the number of unambiguous and ambiguous options is obtained.

Our model performs an axiomatic analysis of the possible effects of the presence of ambiguity, starting with a very elementary partition into ambiguous and unambiguous options, which is enough to obtain plausible ways of comparing opportunity sets that depart qualitatively from the related models or ranking sets in terms of freedom of choice. A natural step forward in this research is to advance in defining the structure of the agent's information about the alternatives. This would presumably also lead to more refined results.

The examples we have used to explain our model are very much based on everyday life choices. However, the trade-off between preference for freedom of choice and aversion to ambiguity could also serve as a basis to explain more complex sociological and anthropological phenomena, such as the reasons why societies or groups adhere to opportunity-restricting institutions (certain religious practices, for example), or even clearly unfair institutions, such as apartheid or slavery, as suggested by the paper's leading quotation. In terms of the general ideas of the paper, we could interpret that such institutions prevail as far as practitioners are sufficiently sure of the consequences of the opportunities they offer, and sufficiently uncertain about the consequences of removing the restrictions. Thus, it is not until an alternative institution is sufficiently well-established, and the consequences of the opportunities it offers are therefore less ambiguous, that it can substitute the old one or coexist alongside it. (See, for example, Chapter 11 in Bowles (2004) for a dynamic model of institutional change of this nature).

As a final thought, it is clear that the idea of "the more freedom of choice the better" prevails in many cultures, especially western ones, in some cases to the extent of becoming a political watchword. However, when it comes to social welfare policy making, one of the potential lessons to be drawn from the general ideas presented in this work is that the idea of freedom should be *qualified*, in the sense that it might only be effective when people are well-informed about their choices. This appeals clearly to educational policies. So that, at some point, it might be better to use public resources on enabling consumers, voters, entrepreneurs, or workers to make well-informed choices, rather than on enlarging their set of options just for the sake of it.

7 Appendix

This section collects the proofs of all theorems that appear in the text. In what follows, for all $S \subseteq X$ and all $k \in \{1, \ldots, |S|\}$, we denote by $(S)_k$ any subset of S with k elements.

Theorem 1 *Let $\succsim \in \mathcal{P}$. Then $\succsim$ satisfies EC, IND, M, and N if and only if $\succsim = \succsim^u$.*

We will first prove the following two lemmas.

Lemma 1 *Let $\succsim \in \mathcal{P}$ satisfy IND and M. Then $(B \cup E, D) \succ (B, D)$ for all $(B, D) \in \mathcal{X} \times \mathcal{X}_\emptyset$ and all $E \subseteq (D \setminus B) \setminus \{\emptyset\}$.*

Proof of Lemma 1. Take $\succsim \in \mathcal{P}$ as above and let $E = \{e_1, \ldots, e_n\}$. By

M, there exists $A \in \mathcal{X} \setminus X$ such that $(A \cup \{x\}, D) \succ (A, D)$ for some $x \in D \setminus A$. Applying IND repeatedly and conveniently this implies that $(\{x\}, D) \succ (\emptyset, D)$ for some $x \in D \setminus A$. By reflexivity, $(\emptyset, D) \sim (\emptyset, D)$ and by IND, $(\{x\}, D) \sim (\{e_1\}, D)$. Thus, by transitivity, $(\{e_1\}, D) \succ (\emptyset, D)$. Now, by IND applied conveniently $(B \cup \{e_1\}, D) \succ (B, D)$. Repeating the same argument with e_2 we obtain $(B \cup \{e_1, e_2\}, D) \succ (B \cup \{e_1\}, D)$, and by the same argument repeated $(n-2)$-times and transitivity, we get $(B \cup E, D) \succ (B, D)$. ■

Lemma 2 *Let* $\succsim \in \mathcal{P}$ *satisfy IND and N. Then* $(B \cup E, D) \sim (B, D)$ *for all* $(B, D) \in \mathcal{X} \times \mathcal{X}_\emptyset$ *and all* $E \subseteq X \setminus (B \cup D)$.

Proof of Lemma 2. If $E = \emptyset$, then Lemma 2 is satisfied by reflexivity. Otherwise, the proof is analogous to the proof of Lemma 1 applying N instead of M. ■

Proof of Theorem 1. Clearly, $\succsim^u$ satisfies the axioms. Suppose now that $\succsim \in \mathcal{P}$ satisfies EC, IND, M, and N. We have to prove that, for all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_\emptyset$,

(1) $|A \cap C| > |B \cap D|$ implies $(A, C) \succ (B, D)$, and

(2) $|A \cap C| = |B \cap D|$ implies $(A, C) \sim (B, D)$.

(1) Let $|A \cap C| > |B \cap D|$. By EC, $(\emptyset, C) \sim (\emptyset, D)$. If $|B \cap D| = 0$ (i.e., $B \cap D = \emptyset$), $(A \cap C, C) \succ (\emptyset, C)$ follows from Lemma 1 with $A \cap C$ in the role of E. By transitivity, $(A \cap C, C) \succ (B \cap D, D)$. If $|B \cap D| = s > 0$, applying IND s-times results in $((A \cap C)_s, C) \sim (B \cap D, D)$. By Lemma 1, with $(A \cap C) \setminus (A \cap C)_s$ in the role of E, we have $(A \cap C, C) \succ ((A \cap C)_s, C)$. This, by transitivity, results in $(A \cap C, C) \succ (B \cap D, D)$. If $A \setminus C = \emptyset$, then, by reflexivity, $(A, C) \sim (A \cap C, C)$. If $A \setminus C \neq \emptyset$, then, by Lemma 2, with $A \setminus (A \cap C)$ in the role of E, we also obtain $(A, C) \sim (A \cap C, C)$. By

an analogous argument, we have $(B, D) \sim (B \cap D, D)$, and by transitivity, $(A, C) \succ (B, D)$.

(2) Let $|A \cap C| = |B \cap D|$. As before, by applying EC and IND the necessary number of times, we get $(A \cap C, C) \sim (B \cap D, D)$. By Lemma 2, with $A \setminus C$ in the role of E, we have $(A, C) \sim (A \cap C, C)$. By the same argument, and with $B \setminus D$ in the role of E, we have $(B, D) \sim (B \cap D, D)$. Thus, by transitivity, $(A, C) \sim (B, D)$. ■

Theorem 2 *Let $\succsim \in \mathcal{P}$ satisfy EC, IND, M, and AV. Then, for all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_\emptyset$,*

(1) *($|A \cap C| > |B \cap D|$ and $|A \setminus C| < |B \setminus D|$) implies $(A, C) \succ (B, D)$,*

(2) *($|A \cap C| \geq |B \cap D|$ and $|A \setminus C| \leq |B \setminus D|$) implies $(A, C) \succsim (B, D)$.*

Note first that the following lemma holds true.

Lemma 3 *Let $\succsim \in \mathcal{P}$ satisfy IND and AV. Then $(B \cup E, D) \prec (B, D)$ for all $(B, D) \in \mathcal{X} \times \mathcal{X}_\emptyset$ and all $E \subseteq (X \setminus (B \cup D)) \setminus \{\emptyset\}$.*

Proof of Lemma 3. The proof is similar to the proof of Lemma 1 except that AV is applied instead of M. ■

Proof of Theorem 2. (1) Let $|A \cap C| |B \cap D|$ and $|A \setminus C| < |B \setminus D|$. As in the first part of the proof of Theorem 1, it can be proved, by using EC, IND and M, that $(A \cap C, C) \succ (B \cap D, D)$. By Lemma 3, we have $(B \cap D, D) \succ (B, D)$. Let $|A \setminus C| = u$. Starting from $(A \cap C, C) \succ (B \cap D, D)$ and applying u-times IND, we obtain $(A, C) \succ ((B \cap D) \cup (B \setminus D)_u, D)$. By Lemma 3, with $(B \setminus D) \setminus (B \setminus D)_u$ in the role of E, $((B \cap D) \cup (B \setminus D)_u, D) \succ (B, D)$. By transitivity, $(A, C) \succ (B, D)$.

(2) The case in which $|A \cap C| > |B \cap D|$ and $|A \setminus C| < |B \setminus D|$ was proved in the previous paragraph. Thus, we will distinguish the three remaining possible cases:

(2.1) $|A \cap C| > |B \cap D|$ and $|A \setminus C| = |B \setminus D|$,

(2.2) $|A \cap C| = |B \cap D|$ and $|A \setminus C| < |B \setminus D|$, and

(2.3) $|A \cap C| = |B \cap D|$ and $|A \setminus C| = |B \setminus D|$.

(2.1) As in the first part of the proof of Theorem 1, it can be proved, by using EC, IND and M, that $(A \cap C, C) \succ (B \cap D, D)$. If $|A \setminus C| = |B \setminus D| = 0$, then it follows directly that $(A, C) \succ (B, D)$. If $|A \setminus C| = |B \setminus D| = u > 0$, by IND repeated u-times, $(A, C) \succ (B, D)$.

(2.2) Let $|A \setminus C| = u$. From EC and applying u-times IND, $(A \setminus C, C) \sim ((B \setminus D)_u, D)$. By Lemma 3, with $(B \setminus D) \setminus (B \setminus D)_u$ in the role of E, $((B \setminus D)_u, D) \succ (B \setminus D, D)$. By transitivity, $(A \setminus C, C) \succ (B \setminus D, D)$. Applying IND $|A \cap C| = |B \cap D|$-times, $(A, C) \succ (B, D)$.

(2.3) From EC and applying IND $|A \cap C| = |B \cap D|$-times, $(A \cap C, C) \sim (B \cap D, D)$. Again by IND applied $|A \setminus C| = |B \setminus D|$-times, $(A, C) \sim (B, D)$.
■

Theorem 3 *Let $\succsim \in \mathcal{P}$. Then $\succsim$ satisfies EC, IND, M, AV, and ROB1 if and only if $\succsim = \succsim^{up}$.*

Proof of Theorem 3. It can be easily checked that $\succsim^{up}$ satisfies the five axioms. Suppose now that $\succsim \in \mathcal{P}$ satisfies EC, IND, M, AV, and ROB1. We have to prove that, for all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_\emptyset$,

(1) $|A \cap C| > |B \cap D|$ implies $(A, C) \succ (B, D)$,

(2) ($|A \cap C| = |B \cap D|$ and $|A \setminus C| < |B \setminus D|$) implies $(A, C) \succ (B, D)$, and

(3) ($|A \cap C| = |B \cap D|$ and $|A \setminus C| = |B \setminus D|$) implies $(A, C) \sim (B, D)$.

(1) Let $|A \cap C| > |B \cap D|$. As in the first part of the proof of Theorem 1, EC, IND, and M together imply $(A \cap C, C) \succ (B \cap D, D)$.

Let us now consider the following partitions of $A \cap C$ and $B \cap D$:

$$\begin{aligned} A \cap C &= (A \cap C)^1 \cup (A \cap C)^2 \cup (A \cap C)^3 \,, \\ B \cap D &= (B \cap D)^1 \cup (B \cap D)^2 \,, \end{aligned}$$

where

$$\begin{aligned} (A \cap C)^1 &= \{x \in A \cap C \mid x \in B \cap D\} \,, \\ (A \cap C)^2 &= \{x \in A \cap C \mid x \in D \setminus (B \cap D)\} \,, \\ (A \cap C)^3 &= \{x \in A \cap C \mid x \in X \setminus D\} \,, \\ (B \cap D)^1 &= \{x \in B \cap D \mid x \in A \cap C\} \,, \\ (B \cap D)^2 &= \{x \in B \cap D \mid x \in X \setminus (A \cap C)\} \,. \end{aligned}$$

By construction $(A \cap C)^1 = (B \cap D)^1$. Hence, $B \cap D = (A \cap C)^1 \cup (B \cap D)^2$. Let $\left|(B \cap D)^2\right| = s_2$. We will consider two cases:

(1.1) $\left|(A \cap C)^2\right| > s_2$, and

(1.2) $\left|(A \cap C)^2\right| \leq s_2$.

(1.1) By Theorem 2,

$$\left((A \cap C)^1 \cup (B \cap D)^2 \,, D\right) \sim \left((A \cap C)^1 \cup (A \cap C)^2_{s_2} \,, D\right).$$

Hence, $(B \cap D, D) \sim \left((A \cap C)^1 \cup (A \cap C)^2_{s_2} \,, D\right)$, and by transitivity,

$$(A \cap C, C) \succ \left((A \cap C)^1 \cup (A \cap C)^2_{s_2} \,, D\right).$$

By ROB1 repeatedly applied, $(A, C) \succ \left((A \cap C)^1 \cup (A \cap C)^2_{s_2} \,, D\right)$. By transitivity, $(A, C) \succ (B \cap D, D)$. If $B \setminus D = \emptyset$, we have that $(A, C) \succ (B, D)$. If $B \setminus D \neq \emptyset$, then, by Lemma 3, $(B \cap D, D) \succ (B, D)$, and by transitivity, $(A, C) \succ (B, D)$.

(1.2) By Theorem 2,

$$\left((A \cap C)^1 \cup (B \cap D)^2 \,, D\right) \sim \left((A \cap C)^1 \cup (A \cap C)^2 \cup (B \cap D)^2_{*} \,, D\right),$$

where $(B \cap D)^2_*$ is any subset of $(B \cap D)^2$ s.t. $\left|(B \cap D)^2_*\right| = \left|(B \cap D)^2\right| - \left|(A \cap C)^2\right|$. Again by Theorem 2,

$$\begin{aligned} & \left((A \cap C)^1 \cup (A \cap C)^2 \cup (B \cap D)^2_*, D\right) \\ \sim \; & \left((A \cap C)^1 \cup (A \cap C)^2 \cup (A \cap C)^3_*, \left(D \cup (A \cap C)^3_*\right) \setminus (B \cap D)^2_*\right), \end{aligned}$$

where $(A \cap C)^3_*$ is any subset of $(A \cap C)^3$ s.t. $\left|(A \cap C)^3_*\right| = \left|(B \cap D)^2\right| - \left|(A \cap C)^2\right|$ (given that $|A \cap C| > |B \cap D|$ by hypothesis, and $|A \cap C|^2 \leq |B \cap D|^2$ such a subset always exists). By transitivity,

$$\begin{aligned} & \left((A \cap C)^1 \cup (B \cap D)^2, D\right) \\ \sim \; & \left((A \cap C)^1 \cup (A \cap C)^2 \cup (A \cap C)^3_*, \left(D \cup (A \cap C)^3_*\right) \setminus (B \cap D)^2_*\right), \end{aligned}$$

that is

$$\begin{aligned} & (B \cap D, D) \\ \sim \; & \left((A \cap C)^1 \cup (A \cap C)^2 \cup (A \cap C)^3_*, \left(D \cup (A \cap C)^3_*\right) \setminus (B \cap D)^2_*\right). \end{aligned}$$

By transitivity,

$$\begin{aligned} & (A \cap C, C) \\ \succ \; & \left((A \cap C)^1 \cup (A \cap C)^2 \cup (A \cap C)^3_*, \left(D \cup (A \cap C)^3_*\right) \setminus (B \cap D)^2_*\right). \end{aligned}$$

By ROB1 repeatedly applied,

$$\begin{aligned} & (A, C) \\ \succ \; & \left((A \cap C)^1 \cup (A \cap C)^2 \cup (A \cap C)^3_*, \left(D \cup (A \cap C)^3_*\right) \setminus (B \cap D)^2_*\right). \end{aligned}$$

By transitivity, $(A, C) \succ (B \cap D, D)$. If $B \setminus D \neq \emptyset$, then, by Lemma 3, $(B \cap D, D) \succ (B, D)$. If $B \setminus D = \emptyset$, then $B \cap D = B$. Again by transitivity, $(A, C) \succ (B, D)$.

(2) Let $|A \cap C| = |B \cap D|$ and $|A \setminus C| < |B \setminus D|$. By Theorem 2, $(A \cap C, C) \sim (B \cap D, D)$. Let $|A \setminus C| = u$. By IND repeated u-times,

$$(A, C) \sim ((B \cap D) \cup (B \setminus D)_u, D).$$

By Lemma 3, with $B \setminus ((B \cap D) \cup (B \setminus D)_u)$ in the role of E, $((B \cap D) \cup (B \setminus D)_u, D) \succ (B, D)$. Thus, by transitivity, $(A, C) \succ (B, D)$.

(3) Let $|A \cap C| = |B \cap D|$ and $|A \setminus C| = |B \setminus D|$. By Theorem 2, $(A, C) \sim (B, D)$. ■

Theorem 4 *Let $\succsim \in \mathcal{P}$. Then $\succsim$ satisfies EC, IND, M, AV, and ROB2 if and only if $\succsim = \succsim^{ap}$.*

Proof of Theorem 4. It is not difficult to check that $\succsim^{ap}$ satisfies the five axioms. Suppose now that $\succsim \in \mathcal{P}$ satisfies EC, IND, M, AV, and ROB2. We have to prove that, for all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_{\emptyset}$,

(1) $|A \setminus C| < |B \setminus D|$ implies $(A, C) \succ (B, D)$,

(2) ($|A \setminus C| = |B \setminus D|$ and $|A \cap C| > |B \cap D|$) implies $(A, C) \succ (B, D)$, and

(3) ($|A \setminus C| = |B \setminus D|$ and $|A \cap C| = |B \cap D|$) implies $(A, C) \sim (B, D)$.

(1) Let $|A \setminus C| = u$ and $|B \setminus D| = v$, with $v > u$. By EC and IND applied u-times, $(A \setminus C, C) \sim ((B \setminus D)_u, D)$. By Lemma 3, with $(B \setminus D) \setminus (B \setminus D)_u$ in the role of E, $((B \setminus D)_u, D) \succ (B \setminus D, D)$. By transitivity, $(A \setminus C, C) \succ (B \setminus D, D)$.

Now, let us consider the following partitions of $A \setminus C$ and $B \setminus D$:

$$\begin{aligned} A \setminus C &= (A \setminus C)^1 \cup (A \setminus C)^2, \\ B \setminus D &= (B \setminus D)^1 \cup (B \setminus D)^2 \cup (B \setminus D)^3, \end{aligned}$$

where

$$\begin{aligned}
(A \setminus C)^1 &= \{x \in A \setminus C \mid x \in B \setminus D\}, \\
(A \setminus C)^2 &= \{x \in A \setminus C \mid x \in X \setminus (B \setminus D)\}, \\
(B \setminus D)^1 &= \{x \in B \setminus D \mid x \in A \setminus C\} = (A \setminus C)^1, \\
(B \setminus D)^2 &= \{x \in B \setminus D \mid x \in X \setminus (A \cup C)\}, \\
(B \setminus D)^3 &= \{x \in B \setminus D \mid x \in C\}.
\end{aligned}$$

Let $(A \setminus C)^1 = \{a_1^-, \ldots, a_{u_1}^-\}$, $(A \setminus C)^2 = \{a_{u_1+1}^-, \ldots, a_u^-\}$, $(B \setminus D)^1 = \{b_1^-, \ldots, b_{u_1}^-\}$, $(B \setminus D)^2 = \{b_{u_1+1}^-, \ldots, b_{v_2}^-\}$, $(B \setminus D)^3 = \{b_{v_2+1}^-, \ldots, b_v^-\}$. Note that, by hypothesis, $\left|(B \setminus D)^2\right| + \left|(B \setminus D)^3\right| > \left|(A \setminus C)^2\right|$. We will consider two cases:

(1.1) $\left|(A \setminus C)^2\right| > \left|(B \setminus D)^2\right|$, and

(1.2) $\left|(A \setminus C)^2\right| \leq \left|(B \setminus D)^2\right|$.

(1.1) Let $\left|(A \setminus C)^2\right| > \left|(B \setminus D)^2\right|$. Consider $\{b_{v_2+1}^-, \ldots, b_u^-\} \subset (B \setminus D)^3$ and let $(A \setminus C)^2_{v_2} = \{a_{u_1+1}^-, \ldots, a_{v_2}^-\}$, $(A \setminus C)^2_v = \{a_{v_2+1}^-, \ldots, a_u^-\}$. By Theorem 2, $\left(\left((A \setminus C) \cup (B \setminus D)^2\right) \setminus (A \setminus C)^2_{v_2}, C\right) \sim (A \setminus C, C)$. Let

$$B^* = \left((A \setminus C) \cup (B \setminus D)^2 \cup \{b_{v_2+1}^-, \ldots, b_u^-\}\right) \setminus \left((A \setminus C)^2_{v_2} \cup (A \setminus C)^2_v\right).$$

Again by Theorem 2, $\left(B^*, \left(C \cup (A \setminus C)^2_v\right) \setminus \{b_{v_2+1}^-, \ldots, b_u^-\}\right) \sim (A \setminus C, C)$. By transitivity, $\left(B^*, \left(C \cup (A \setminus C)^2_v\right) \setminus \{b_{v_2+1}^-, \ldots, b_u^-\}\right) \succ (B \setminus D, D)$. Note that, by construction, $B^* \subset B \setminus D$. Thus, by ROB2 repeatedly applied, $\left(B^*, \left(C \cup (A \setminus C)^2_v\right) \setminus \{b_{v_2+1}^-, \ldots, b_u^-\}\right) \succ ((B \setminus D) \cup (B \cap D), D)$. Given that the first choice situation is indifferent to $(A \setminus C, C)$, by transitivity, $(A \setminus C, C) \succ (B, D)$. By Lemma 1, $(A, C) \succ (A \setminus C, C)$, and again by transitivity, $(A, C) \succ (B, D)$.

(1.2) Let $\left|(A \setminus C)^2\right| \leq \left|(B \setminus D)^2\right|$. Consider $\{b_{u_1+1}^-, \ldots, b_u^-\} \subseteq (B \setminus D)^2$. Then $\left(\left((A \setminus C)^1 \cup \{b_{u_1+1}^-, \ldots, b_u^-\}\right) \setminus (A \setminus C)^2, C\right) \sim (A \setminus C, C)$ by Theorem

2. By transitivity, $\left(\left((A \setminus C)^1 \cup \left\{b^-_{u_1+1}, \ldots, b^-_u\right\}\right) \setminus (A \setminus C)^2, C\right) \succ (B \setminus D, D)$. By ROB2, $\left(\left((A \setminus C)^1 \cup \left\{b^-_{u_1+1}, \ldots, b^-_u\right\}\right) \setminus (A \setminus C)^2, C\right) \succ (B, D)$. By Lemma 1, $(A, C) \succ (A \setminus C, C)$, and by transitivity, $(A, C) \succ (B, D)$.

(2) Let $A \backslash C = \left\{a^-_1, \ldots, a^-_u\right\}$, $B \backslash D = \left\{b^-_1, \ldots, b^-_u\right\}$, $A \cap C = \left\{a^+_1, \ldots, a^+_r\right\}$, and $B \cap D = \left\{b^+_1, \ldots, b^+_s\right\}$, $r > s$. By Theorem 2, $\left((A \setminus C) \cup \left\{a^+_1, \ldots, a^+_s\right\}, C\right) \sim (B, D)$. By Lemma 1, $(A, C) \succ \left((A \setminus C) \cup \left\{a^+_1, \ldots, a^+_s\right\}, C\right)$. Again by transitivity, $(A, C) \succ (B, D)$.

(3) By Theorem 2, $(A, C) \sim (B, D)$. ■

Theorem 5 *Let $\succsim \in \mathcal{P}$. Then $\succsim$ satisfies EC, IND, M, and DI if and only if $\succsim = \succsim^d$.*

We will first prove the following two lemmas.

Lemma 4 *Let $\succsim \in \mathcal{P}$ satisfy IND and DI, and let $(A, C), (B, C) \in \mathcal{X} \times \mathcal{X}_\emptyset$ be such that $B = A \cup E$ with $|E \cap C| = |E \setminus C|$. Then $(A, C) \sim (B, C)$.*

Proof of Lemma 4. Take $\succsim \in \mathcal{P}$ as above and let $E \cap C = \left\{e^+_1, \ldots, e^+_n\right\}$, $E \setminus C = \left\{e^-_1, \ldots, e^-_n\right\}$.

If $C = X$, then $E = \emptyset$ and the Lemma is vacuously satisfied. If $C \subset X$, we will only consider the case $|E \cap C| = |E \setminus C| > 0$, otherwise the Lemma is satisfied by reflexivity. Thus, given that $C \subset X$, by DI, there exists $F \in \mathcal{X}_\emptyset \setminus \{X\}$ such that $(F \setminus \{x\}, C) \sim (F \cup \{y\}, C)$ for some $x \in F \cap C$ and some $y \in X \setminus (F \cup C)$. Applying IND repeatedly, we have $(\emptyset, C) \sim (\{x, y\}, C)$ for some $x \in F \cap C$ and some $y \in X \setminus (F \cup C)$. By reflexivity, $(\emptyset, C) \sim (\emptyset, C)$ and by IND applied twice, $(\{x, y\}, C) \sim (\{e^+_1, e^-_1\}, C)$. Thus, by transitivity, $(\emptyset, C) \sim (\{e^+_1, e^-_1\}, C)$. By IND, $(\{e^+_2, e^-_2\}, C) \sim (\{e^+_1, e^+_2, e^-_1, e^-_2\}, C)$ and by transitivity, $(\emptyset, C) \sim (\{e^+_1, e^+_2, e^-_1, e^-_2\}, C)$. Repeating the same argument $(n-2)$-times and by transitivity, we have $(\emptyset, C) \sim (E, C)$. Thus, by IND, $(A, C) \sim (A \cup E, C)$, i.e., $(A, C) \sim (B, C)$. ■

Lemma 5 *Let $\succsim \in \mathcal{P}$ satisfy IND, M, and DI. Then it also satisfies AV.*

Proof of Lemma 5. Take $\succsim \in \mathcal{P}$ as above and notice that Lemma 1 and Lemma 4 hold. In particular, by Lemma 1 we have that $(\{x, y\}, C) \succ (\{x\}, C)$ for all $x \in X$ and all $y \in C \in \mathcal{X}_\emptyset$ with $y \neq x$. Furthermore, by Lemma 4, we have $(\emptyset, C) \sim (\{x, y\}, C)$ for all $C \in \mathcal{X}_\emptyset$, all $x \in C$, and all $y \in X \setminus C$.

In order to prove that $\succsim$ satisfies AV, we first prove the following

Claim $(\{x\}, C) \succ (\{x, w\}, C)$ *for all $x \in X$, all $C \in \mathcal{X}_\emptyset \backslash X$ and all $w \in X \backslash C$ with $w \neq x$.*

Proof of the Claim. Consider first the case in which $|X| = 2$ and let $X = \{x, w\}$. Notice that, in such a case, we have only to show that $(\{x\}, \{x\}) \succ (\{x, w\}, \{x\})$ and $(\{w\}, \{w\}) \succ (\{x, w\}, \{w\})$. We will demonstrate, without loss of generality, the former implication. By Lemma 1, $(\{x\}, \{x\}) \succ (\emptyset, \{x\})$ and by Lemma 4, $(\emptyset, \{x\}) \sim (\{x, w\}, \{x\})$. Thus, by transitivity, $(\{x\}, \{x\}) \succ (\{x, w\}, \{x\})$.

Suppose next that $|X| \geq 3$, and take x, w as in the Claim. If $C \neq \{x\}$, then there exists $z \in C$, $z \neq x$. By Lemma 4, $(\emptyset, C) \sim (\{z, w\}, C)$, and by Lemma 1 we have $(\{z, w\}, C) \succ (\{w\}, C)$. By transitivity, $(\emptyset, C) \succ (\{w\}, C)$, and by IND, $(\{x\}, C) \succ (\{x, w\}, C)$. If $C = \{x\}$, then, by Lemma 1, $(\{x, w\}, C) \succ (\{w\}, C)$. By IND, $(\{x\}, C) \succ (\emptyset, C)$. On the other hand, by Lemma 4, $(\emptyset, C) \sim (\{x, w\}, C)$. Thus, by transitivity, $(\{x\}, C) \succ (\{x, w\}, C)$, and the claim is proved.

The Claim demonstrates that, for each $C \in \mathcal{X}_\emptyset \backslash X$, there exists $A \in \mathcal{X} \backslash X$ such that $(A \cup \{w\}, C) \prec (A, C)$ for some $w \in X \setminus (A \cup C)$, as required by AV. In particular, A could be any singleton included in C. ■

Corollary 1 *Let $\succsim \in \mathcal{P}$ satisfy IND, M, and DI. Then the statement in*

Lemma 3 holds, that is, $(B \cup E, D) \prec (B, D)$ *for all* $(B, D) \in \mathcal{X} \times \mathcal{X}_\emptyset$ *and all* $E \subseteq (X \setminus (B \cup D)) \setminus \{\emptyset\}$.

Proof of Theorem 5. It can be easily checked that $\succsim^d$ satisfies the four axioms. Suppose now that $\succsim \in \mathcal{P}$ satisfies EC, IND, M, and DI. We have to prove that, for all $(A, C), (B, D) \in \mathcal{X} \times \mathcal{X}_\emptyset$,

(1) $|A \cap C| - |A \setminus C| > |B \cap D| - |B \setminus D|$ implies $(A, C) \succ (B, D)$, and

(2) $|A \cap C| - |A \setminus C| = |B \cap D| - |B \setminus D|$ implies $(A, C) \sim (B, D)$.

Let $|A \cap C| = r$, $|B \cap D| = s$, $|A \setminus C| = u$, $|B \setminus D| = v$.

(1) In this case $r - u > s - v$. We consider the following three possible cases:

(1.1) $r > u$ and $s > v$,

(1.2) $r > u$ and $s \leq v$,

(1.3) $r \leq u$ and $s < v$.

(1.1) Let $r > u$ and $s > v$. By EC, $(\emptyset, C) \sim (\emptyset, D)$. By Lemma 4, $((A \cap C)_u \cup (A \setminus C), C) \sim (\emptyset, C)$. Also by Lemma 4, $((B \cap D)_v \cup (B \setminus D), D) \sim (\emptyset, D)$. Thus, by transitivity, $((A \cap C)_u \cup (A \setminus C), C) \sim ((B \cap D)_v \cup (B \setminus D), D)$. Given that $r - u > s - v$, by IND applied $(s-v)$-times $\big((A \cap C)_{u+s-v} \cup (A \setminus C), C\big) \sim \big((B \cap D)_{v+s-v} \cup (B \setminus D), D\big)$, i.e., $\big((A \cap C)_{u+s-v} \cup (A \setminus C), C\big) \sim (B, D)$. By Lemma 1, $(A, C) \succ \big((A \cap C)_{u+s-v} \cup (A \setminus C), C\big)$, and by transitivity, $(A, C) \succ (B, D)$.

(1.2) Let $r > u$ and $s \leq v$. As in case (1.1), by EC, Lemma 4 and transitivity we get $((A \cap C)_u \cup (A \setminus C), C) \sim ((B \cap D) \cup (B \setminus D)_s, D)$. By Lemma 1, $(A, C) \succ ((A \cap C)_u \cup (A \setminus C), C)$, and, if $s < v$, by Lemma 3,

$$((B \cap D) \cup (B \setminus D)_s, D) \succ (B, D).$$

If $s = v$, $((B \cap D) \cup (B \setminus D)_s, D) = (B, D)$. In any case, by transitivity, $(A, C) \succ (B, D)$.

(1.3) Let $r \leq u$ and $s < v$. As before, by EC, Lemma 4 and transitivity we get $((A \cap C) \cup (A \setminus C)_r, C) \sim ((B \cap D) \cup (B \setminus D)_s, D)$. Since $r - u > s - v$, then $u - r < v - s$. Then we can apply IND $(u - r)$-times obtaining $\left((A \cap C) \cup (A \setminus C)_{r+u-r}, C\right) \sim \left((B \cap D) \cup (B \setminus D)_{s+u-r}, D\right)$. That is, $(A, C) \sim \left((B \cap D) \cup (B \setminus D)_{s+u-r}, D\right)$. By Lemma 3,

$$\left((B \cap D) \cup (B \setminus D)_{s+u-r}, D\right) \succ (B, D).$$

Then, by transitivity, $(A, C) \succ (B, D)$.

(2) In this case $r - u = s - v$. If $r \geq u$ ($s \geq v$), then, as in case (1), by EC, Lemma 4 and transitivity we get

$$((A \cap C)_u \cup (A \setminus C), C) \sim ((B \cap D)_v \cup (B \setminus D), D),$$

and by IND applied $(r - u)(= s - v)$-times, $(A, C) \sim (B, D)$.

If $r < u$ ($s < v$), then, by EC, Lemma 4 and transitivity we get

$$((A \cap C) \cup (A \setminus C)_r, C) \sim ((B \cap D) \cup (B \setminus D)_s, D),$$

and by IND applied $(u - r)(= v \geq s)$-times, $(A, C) \sim (B, D)$. ■

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