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Online Appendix for "On the aversion to incomplete preferences"

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This Appendix contains the proofs of Propositions 1, 2, 3 and 4 of independence of the axioms used in Theorems 4.1, 4.2, 5.1 and 5.2 respectively. The proofs are made by showing, for the different axioms, binary relations that satisfy all the axioms in the corresponding theorem except the one whose independence has to be proved. These binary relations are constructed for $\#X = 3$. Without loss of the logical validity of the proofs, in some cases some axioms are satisfied by vacuity. It is also possible to present the proofs of independence in such a way that none of the axioms that are satisfied do it by vacuity. In that case it is necessary to deal with at least four alternatives and, given the combinatorial nature of the problem, the size of the proofs escalates considerably. These proofs are available upon request.

1 Proof of Proposition 1

The axioms in Proposition 1, whose independence has to be proven, are MC, IA, CE, RI and SC.

¬ MC

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Let $X = \{x, y, z\}$ and $\succsim$ defined on $\pi(X)$ as follows:

$\{x, y\} \succ \{y\}$; $\{x, z\} \succ \{z\}$; $\{y, z\} \succ \{z\}$

$\{x\} \sim \{x, y\}$; $\{x\} \sim \{x, z\}$; $\{y\} \sim \{y, z\}$

$\{x, y\} \sim \{x, y, z\}$; $\{y, z\} \sim \{x, y, z\}$; $\{x, z\} \sim \{x, y, z\}$

$\succsim$ satisfies all the axioms but MC because $\{x, y\} \succ \{y\}$,$\{x, z\} \succ \{z\} \nRightarrow \{x, y, z\} \succ \{y, z\}$.

¬ IA

Let $X = \{x, y, z\}$ and $\succsim$ defined on $\pi(X)$ as follows:

$\{x, y\} \succ \{y\}$; $\{x, z\} \succ \{z\}$; $\{y, z\} \succ \{z\}$

$\{x\} \sim \{x, y\}$; $\{x\} \sim \{x, z\}$; $\{y, z\} \sim \{y\}$

$\{x, y\} \sim \{x, y, z\}$; $\{x, y, z\} \succ \{y, z\}$

$\succsim$ satisfies all the axioms but IA because $\{y, z\} \succ \{z\}$,$\{x\} \sim \{x, y\} \nRightarrow \{x, y, z\} \sim \{x, z\}$.

¬ CE

Let $X = \{x, y, z\}$ and let $\succsim$ be a transitive binary relation defined on $\pi(X)$ as follows:

$\{x\} \succ \{y\} \succ \{z\} \succ [\{x, y\} \sim \{y, z\} \sim \{x, z\} \sim \{x, y, z\}]$

$\succsim$ satisfies all the axioms but CE because $\{x\} \succ \{x, z\} \nRightarrow \{x, y\} \sim \{x, y, z\}$.

¬ RI

Let $X = \{x, y, z\}$ and let $\succsim$ be a transitive binary relation defined on $\pi(X)$ as follows:

$\{x, y\} \succ [\{x, y, z\} \sim \{x, z\} \sim \{x\}] \succ \{y\} \succ \{z\} \sim \{z, y\}$

$\succsim$ satisfies all the axioms but RI because $\{z\} \sim \{y, z\} \nRightarrow \{y, z\} \succ \{y\}$.

¬ SC

Let $X = \{x, y, z\}$ and let $\succsim$ be a transitive binary relation defined on $\pi(X)$ as follows:

$\{x\} \sim \{x, y\} \succ \{y\} \succ \{y, z\} \sim \{x, y\} \succ \{x, y, z\}$

$\succsim$ satisfies all the axioms but SC because $\neg\{x, z\} \succsim \{x\}$ and $\neg\{x\} \succ \{x, z\}$.

2 Proof of Proposition 2

The axioms in Proposition 2, whose independence has to be proven, are MR, CR, CC, IC, RI and SC.

¬ MR

Let $X = \{x, y, z\}$ and let $\succsim$ be a transitive binary relation defined on $\pi(X)$ as follows:
$\{z\} \sim \{y, z\} \succ \{y\} \succ \{x\} \succ [\{x, z\} \sim \{x, y\} \sim \{x, y, z\}]$

$\succsim$ satisfies all the axioms but MR because $\neg\{x\} \sim \{x, z\}$ and $\{y, z\} \succ \{y\}; \{y\} \succ \{x, y\} \nRightarrow \{x, y, z\} \succ \{x, y\}$.

$\neg$ CR

Let $X = \{x, y, z\}$ and let $\succsim$ be a transitive binary relation defined on $\pi(X)$ as follows:
$\{x, z\} \succ \{z\}$; $\{x, y, z\} \succ \{y, z\}$; $\{y, z\} \succ \{z\}$
$\{x\} \succ \{x, y\}$; $\{y\} \succ \{x, y\}$
$\{x\} \sim \{x, z\}$; $\{x, y\} \sim \{x, y, z\}$; $\{x, z\} \sim \{x, y, z\}$; $\{y\} \sim \{y, z\}$

$\succsim$ satisfies all the axioms but CR because $\{x\} \succ \{x, y\}, \{x\} \sim \{x, z\} \nRightarrow \{x, z\} \succ \{x, y, z\}$.

$\neg$ CC

Let $X = \{x, y, z\}$ and let $\succsim$ be a binary relation defined on $\pi(X)$ as follows:
$\{x\} \succ \{x, y\}$; $\{y\} \succ \{x, y\}$; $\{x\} \succ \{x, z\}$; $\{z\} \succ \{x, z\}$; $\{y\} \succ \{y, z\}$; $\{z\} \succ \{y, z\}$; $\{y, z\} \sim \{x, y, z\}$

$\succsim$ satisfies all the axioms but CC because $\{y\} \succ \{x, y\}$, $\{z\} \succ \{x, z\} \nRightarrow \{y, z\} \succ \{x, y, z\}$.

$\neg$ IC

Let $X = \{x, y, z\}$ and let $\succsim$ be a binary relation defined on $\pi(X)$ as follows:
$\{x, z\} \succ \{x\}; \{x, y\} \succ \{y\}; \{y, z\} \succ \{z\}$
$\{x\} \sim \{x, y\}; \{y\} \sim \{y, z\}; \{z\} \sim \{x, z\}$
$\{x, y, z\} \succ \{x, z\}; \{x, y, z\} \succ \{x, y\}; \{x, y, z\} \succ \{y, z\}$

$\succsim$ satisfies all the axioms but IC because $\{x\} \sim \{x, y\}$, $\{y\} \sim \{y, z\} \nRightarrow \{x\} \sim \{x, z\}$.

$\neg$ RI

Let $X = \{x, y, z\}$ and let $\succsim$ be a transitive binary relation defined on $\pi(X)$ as follows:
$[\{x, z\} \sim \{x\}] \succ [\{x, y, z\} \sim \{x, y\}] \succ [\{y\} \sim \{y, z\}] \succ \{z\}$

$\succsim$ satisfies all the axioms but RI because $\{x, y\} \succ \{y\} \nRightarrow \{x, y\} \sim \{x\}$.

$\neg$ SC

Let $X = \{x, y, z\}$ and let $\succsim$ be a transitive binary relation defined on $\pi(X)$ as follows:
$[\{y, z\} \sim \{z\}] \succ [\{x, y, z\} \sim \{x, z\}] \succ \{y\} \succ \{x, y\}$

$\succsim$ satisfies all the axioms but SC because $\neg\{x, z\} \succsim \{x\}$ and $\neg\{x\} \succ \{x, z\}$.

3 Proof of Proposition 3

The axioms in Proposition 1, whose independence has to be proven, are MC, IA, CE, WRI and SC. Notice that the only difference between the characterization axioms of Theorem 4.1 (Proposition 1) and those in Theorem 5.1 (Proposition 3) is that RI is replaced by WRI. Given that WRI is clearly a weaker axiom than RI, the examples for the independence of axioms MC, IA, CE and SC that are shown in the proof of Proposition 1 are valid for the corresponding proofs of Proposition 3. As for axiom WRI, consider the same example provided in the proof of Proposition 1 for the independence of RI and notice that example neither satisfies WRI because $\{z\} \sim \{z, y\} \nRightarrow \{z, y\} \succ \{y\}$.

4 Proof of Proposition 4

The axioms in Proposition 4, whose independence has to be proven, are MR, CR, CC, MC, IC, WRI and SC.

$\neg$ **MR**

Let $X = \{x, y, z\}$ and let $\succsim$ be a transitive binary relation defined on $\pi(X)$ as follows:
$[\{z\} \sim \{y, z\}] \succ \{y\} \succ \{x\} \succ \{x, z\} \succ [\{x, y\} \sim \{x, y, z\}]$

$\succsim$ satisfies all the axioms but MR because $\neg\{x\} \sim \{x, z\}$ and $\{z, y\} \succ \{y\}, \{y\} \succ \{x, y\} \nRightarrow \{x, y, z\} \succ \{x, y\}$.

$\neg$ **CR**

Let $X = \{x, y, z\}$ and let $\succsim$ be a transitive binary relation defined on $\pi(X)$ as follows:
$\{x, z\} \succ \{z\}$; $\{x, y, z\} \succ \{y, z\}$; $\{y, z\} \succ \{z\}$
$\{x\} \succ \{x, y\}$; $\{y\} \succ \{x, y\}$
$\{x\} \sim \{x, z\}$; $\{x, y\} \sim \{x, y, z\}$; $\{x, z\} \sim \{x, y, z\}$; $\{y\} \sim \{y, z\}$

$\succsim$ satisfies all the axioms but CR because $\{x\} \succ \{x, y\}, \{x\} \sim \{x, z\} \nRightarrow \{x, z\} \succ \{x, y, z\}$.

$\neg$ **CC**

Let $X = \{x, y, z\}$ and let $\succsim$ be a binary relation defined on $\pi(X)$ as follows:
$\{x\} \succ \{x, y\}$; $\{y\} \succ \{x, y\}$; $\{x\} \succ \{x, z\}$; $\{z\} \succ \{x, z\}$; $\{y\} \succ \{y, z\}$; $\{z\} \succ \{y, z\}$; $\{y, z\} \sim \{x, y, z\}$

$\succsim$ satisfies all the axioms but CC because $\{y\} \succ \{x, y\}$, $\{z\} \succ \{x, z\} \nRightarrow \{y, z\} \succ \{x, y, z\}$.

¬ MC

Let $X = \{x, y, z\}$ and let $\succsim$ be a binary relation defined on $\pi(X)$ as follows:
$\{y, z\} \succ \{y\}$; $\{y, z\} \succ \{z\}$; $\{x, y\} \succ \{y\}$; $\{x, y\} \succ \{x\}$; $\{x, z\} \succ \{z\}$; $\{x, z\} \succ \{x\}$;
$\{x, y, z\} \sim \{y, z\}$

$\succsim$ satisfies all the axioms but MC because $\{x, y\} \succ \{y\}$, $\{x, z\} \succ \{z\} \nRightarrow \{x, y, z\} \succ \{y, z\}$.

¬ IC

Let $X = \{x, y, z\}$ and let $\succsim$ be a binary relation defined on $\pi(X)$ as follows:
$\{x, z\} \succ \{x\}$; $\{x, y\} \succ \{y\}$; $\{y, z\} \succ \{z\}$
$\{x\} \sim \{x, y\}$; $\{y\} \sim \{y, z\}$; $\{z\} \sim \{x, z\}$
$\{x, y, z\} \succ \{x, z\}$; $\{x, y, z\} \succ \{x, y\}$; $\{x, y, z\} \succ \{y, z\}$

$\succsim$ satisfies all the axioms but IC because $\{x\} \sim \{x, y\}, \{z\} \sim \{x, z\} \nRightarrow \{z\} \sim \{y, z\}$.

¬ WRI

Let $X = \{x, y, z\}$ and let $\succsim$ be a binary relation defined on $\pi(X)$ as follows:
$\{x, y\} \succ \{y\}; \{x, z\} \succ \{z\}$
$\{z\} \succ \{y, z\}; \{x\} \succ \{y, x\}$
$\{y\} \sim \{y, z\}; \{x\} \sim \{x, z\}$
$\{x, y, z\} \succ \{y, z\}$
$\{x, z\} \succ \{x, y, z\}$

$\succsim$ satisfies all the axioms but WRI because $\{x, y\} \succ \{y\} \nRightarrow \{x, y\} \succsim \{x\}$.

¬ SC

Let $X = \{x, y, z\}$ and let $\succsim$ be a transitive binary relation defined on $\pi(X)$ as follows:
$[\{y, z\} \sim \{z\}] \succ [\{x, y, z\} \sim \{x, z\}] \succ \{y\} \succ \{x, y\}$

$\succsim$ satisfies all the axioms but SC because $\neg\{x, z\} \succsim \{x\}$ and $\neg\{x\} \succ \{x, z\}$.