APPLYING DEA IN THE EVALUATION OF FNP

Efficiency in the Navarra Paddle Circuit

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Abstract: Navarra Paddle Federation (FNP) is experiencing rapid growth in recent years also trying to improve its efficiency and effectiveness. The purpose of this study is to estimate the efficiency of different tournaments in the Navarra paddle circuit since 2010. The methodology applied is based on the non-parametric technique of data envelopment analysis. The results show the tournaments organized in a more efficient manner, and those found to be relatively less effective.

Keywords: Navarra Paddle Federation; Efficiency; Effectiveness; DEA; Linear Programming.

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1. Introduction

With the rise of the paddle in the last years, a very fashionable sport nowadays in Spain, the Navarra Paddle Federation (FNP) has gone through a great transformation recently. Each year, new players federate to play and enjoy several tournaments organized by the federation, which are having a spectacular success in Navarra, due among other factors, to its good organization and implication of FNP members.

Nowadays, there are 3226 federated players in Navarra. This figure puts the FNP as the sixth Federation with more licenses (of the 43 legally constituted) of Navarra and considering that the paddle is a relatively new sport, this is a success. Compared with other autonomous communities, Navarra is the fifth most country licenses. It has to be also considered that Navarra is the community number fifteen on the list the communities with the highest population.

However, although the numbers look very good, there are always things that can be improved. In particular, this paper analyses the efficiency of the the different tournaments in the Navarra paddle circuit. In order to know if each year the FNP is working more efficiently than the previous year, if the FNP is taking full advantage of each tournament, and if not, try to look for possible improvements that could be applied in this organization to become fully efficient. All the relevant variables have been obtained from the database, which provides the exact information since 2010 and that will help to calculate the different efficiencies.

The methodology applied is based on the non-parametric data envelopment analysis (DEA) technique to evaluate efficiency. The empirical application is carried out on a sample of 7 tournaments (male and female) of the Navarra Paddle Circuit between 2010 and 2015.

The study is ordered as follows. This first section describes the problem statement and the outline of thesis. The second section is reviews the theoretical background, to understand the basics concepts about efficiency which are important for understanding the method. The third one is about DEA method. The fourth describes the methodology and description of the model (inputs, outputs, etc…). In the fifth section, the results obtained are showed and finally in the last section, the conclusions of the study, the limitations of the work and future research are presented.

2. Theoretical Background

2.1 Definition of Efficiency

In order to understand the power and limitation of the DEA method which will be describe in the next section, a few basic efficiency concepts are needed.
A simple definition of efficiency could be the ability to do something or produce something (output) without wasting materials, time, money or energy (input). So, the better the performance, the higher the efficiency. In this way, it can be simply defined as the ratio of output to input:

$$efficiency = \frac{output}{input}$$

The more output per unit of input, the greater the efficiency will be. In this case, the absolute of optimum efficiency will be achieved when the greatest possible output per unit of input is achieved.

Definition 1.1 (Efficiency – Extended Pareto-Koopmans Definition). Full (100%) efficiency is attained by any DMU if and only if none of its inputs or outputs can be improved without worsening some of its other inputs or outputs.

In most management or social science applications, the theoretically possible levels of efficiency will not be known. The preceding definition is, therefore, replaced by emphasizing its uses with only the information that is empirically available as in the following definition:

Definition 1.2 (Relative Efficiency). A DMU is to be rated as fully (100%) efficient on the basis of available evidence if and only if the performances of other DMUs does not show that some of its inputs or outputs can be improved without worsening some of its other inputs or outputs.[1]

In the next sections, different ways of measure the efficiency depending on several aspects will be covered.

[1] Cooper, L.M. Seiford and J. Zhu (2011), "Data Envelopment Analysis: Models and Interpretations", Chapter 1, 1-39, in W.W. Cooper, L.M. Seiford and J. Zhu, eds, Handbook on Data Envelopment Analysis, 2nd edition, Springer, New York, 2011, 3.

2.2 Efficiency Measures

2.2.1 Introduction

As it has been introduced before, efficiency could be introduced as the ratio of output to input, and the higher this ratio, the better the performance. This idea is very important it allows managers to evaluate performance without clearly defined preferences. Even though the meaning of efficiency seems simple and intuitive at first, as it will be explained in the next sections, there are many ways to conceptualize this parameter. The most common and classical concepts will be covered in the following sections.
Furthermore, some of these concepts of efficiency can be analysed in different ways, approaching the parameter to the inputs, outputs or why not, both of them.
The aim of this section is to provide some theoretical background as well as describing the most common efficiency concepts.

2.2.2 Setting[2]

If a particular company is chosen and its situation is described in the following way:
The company *k* has used *m* inputs $x^k = (x_1, \dots, x_m) \in \mathbb{R}^m$ to produce *n* outputs $y^k = (y_1, \dots, y_n) \in \mathbb{R}^n$. El set of possible possible production plans of the company *k* is given by the production possibility set T,

$$T = \{(x, y) \in \mathbb{R}^n \text{ x } \mathbb{R}^m \mid x \; can \; produce \; y\}.$$

There are many ways to construct the production plan set *T*. However, in this moment, it does not matter how we estimate *T.* The same efficiency concepts are applicable to productions plan or technologies estimated in different ways.

2.2.3 Efficient production[3]

A production plan will be more efficient if order to produce many outputs or services, the firm or company uses few inputs or resources.

Let consider two companies, (x^1, y^1) and (x^2, y^2). It is said that company 2 dominates company 1 if it produces same or more outputs using less or equal inputs and without being exactly similar to company 1.

Dominance. (x^2, y^2) dominates (x^1, y^1) if and only if $x^2 \leq x^1$, $y^2 \geq y^1$, and $(x^1, y^1) \neq (x^2, y^2)$

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[2] P. Bogetoft and L. Otto (2013), "Benchmarking with DEA, SFA, and R, Springer" NY. Chapter 2, pp. 23-24

[3] P. Bogetoft and L. Otto (2013), "Benchmarking with DEA, SFA, and R, Springer" NY. Chapter 2, pp. 24-25

Therefore, the dominating company has to be strictly better in at least one dimension (to use less of an input or produce more of an output).
For any company, it is really interesting to be more efficient or have a dominant production plan. In this way, firms should focus on increasing their outputs and decreasing their inputs.
In economics, the efficient firms or companies are those that cannot be dominated by other companies. For a given technology or production plan *T,* efficiency is defined in the following way:
Efficiency. (x, y) is efficient in T if and only if there is no $(x', y') \in T$ that can dominate (x, y)
The efficient subset of T, T^E is:

$$T^E = \{(x, y) \in T \mid (x, y) \; is \; efficient \; in \; T\}.$$

The efficient subset T^E of T are the combination of inputs and output that cannot be improved.

However, it is worth remembering that although efficiency is a necessary condition of effectiveness, it is not a sufficient one. It is not sufficient to run fast, it is also important to run in the correct direction and it may be better to run at a moderate speed in the right direction than at full speed.

Until now, the relevance of efficiency and dominance has been described. In the following section measurements efficiency levels will be covered.

2.2.4 Farrell efficiency[4]

Farrell efficiency is probably the most widely used approach to measure efficiency when there are multiples inputs and outputs. Here, two different point views of efficiency are going to be described:

- The *input-based Farrell efficiency* or just *input efficiency* of a plan (x, y) relative to a technology T is defined as:

  $$E = min\{E > 0 | (Ex, y) \in T\}$$

  Is the maximal proportional contraction of all inputs x that allows to produce the same amount of y. For example, $E = 0.7$ indicates that 30 % of the inputs could have been saved and still produced the same amounts of outputs in the ideal efficiency.

[4] P. Bogetoft and L. Otto (2013), "Benchmarking with DEA, SFA, and R, Springer" NY. Chapter 2, pp. 26-30

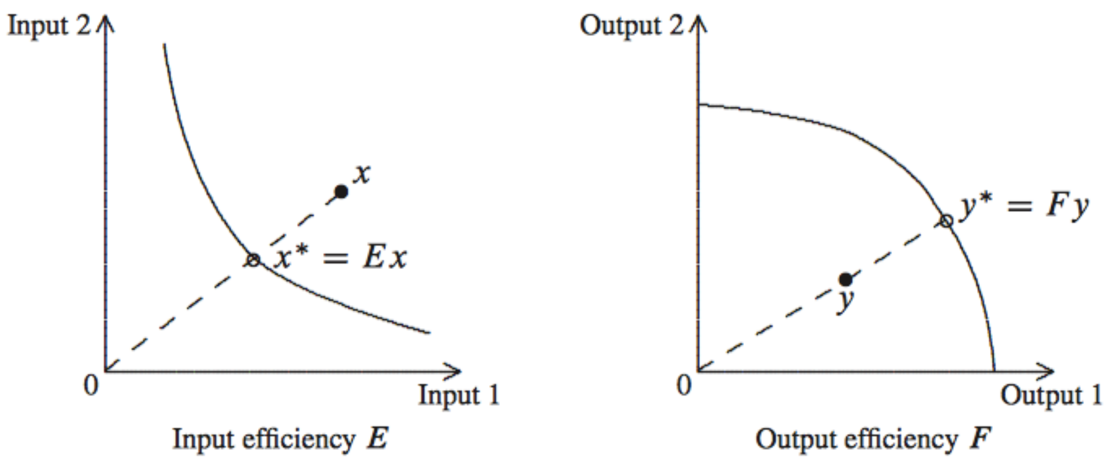
- The *output-based Farrell efficiency* or just *output efficiency* of a plan (x, y) relative to a technology T is defined as:

$$F = max\{F > 0|(x, Fy) \in T\}$$

Is the maximal proportional expansion of all outputs y that is possible to produce the same amount of x. For example, $F = 1.4$ indicates that 40 % more of outputs could have been achieved and using the same amounts of inputs in the ideal efficiency.

Then next figure shows how Farrell efficiency works when there are two inputs and two outputs. In the first picture, it is shown the input isoquant corresponding to the output level y that the firm is producing, and in the second picture, it is shown the output isoquant corresponding to the inputs x that our firm is using.

Proportional reduction and expansion are represented along the dashed. Input efficiency is calculated as the smallest number E that we can multiply on x and remain on or above the isoquant. In the same way, output efficiency is calculated as the largest number F that we can multiply on y and remain below or at the output isoquant. For inputs above and on the input isoquant and outputs below and on the output isoquant curve, we have $E \leq 1$ and $F \geq 1$. The larger E is and the smaller F is, the more efficient the firm is.



Sometimes there are situations in which some inputs or outputs are fixed (as it can be the building of a company or the lands of a farm). In these cases, attention will only be paid to variables (inputs or outputs) that can be controlled. In this way, inputs and outputs are both divided into variable (v) and fixed (f) like this, $(x, y) = (x_v, x_f, y_v, y_f)$ and therefore, the Farrell efficiency will be measured as follows:

$$E^* = min\{E > 0 \middle| (Ex_v, x_f, y_v, y_f) \in T\}$$
$$F^* = max\{F > 0 \middle| (x_v, x_f, Fy_v, y_f) \in T\}$$

where E^* is the factor that represents the proportion in which all variable inputs x_v can be reduced maintaining fixed inputs in the same values (as is logical) and without producing fewer of the outputs $y = (y_v, y_f)$. In the same way, F^* is the factor that represents the proportion in which all variable outputs y_v can be expanded maintaining fixed outputs in the same values (as is logical) and without using more inputs than $x = (x_v, x_f)$.

Farrell efficiency can be used to rank firms. This lists are studied with interest because they often provide rakings with the best firms having the lower F and the worst ones having the highest F, or what is the same, the best firms having the largest E and the worst ones having the lowest E. However, it may be questionable whether Farrell efficiency is enough to rank companies, knowing that this parameter basically only gives us as how each company can improve efficiency by increasing or decreasing inputs or outputs respectively.

Furthermore, some prefer to work with the Shephard measures rather than Farrell measures. It is noted that the distance functions Shephard are simply the inverse of the Farrell. It can be distinguished between input distance function D_i and the output distance function D_o as follows:

$$D_i(x, y) = \max\left\{D > 0 \left| \left(\frac{x}{D}, y\right) \in T \right.\right\} = \frac{1}{E\ (x, y)}$$
$$D_0(x, y) = \min\left\{D > 0 \left| \left(x, \frac{y}{D}\right) \in T \right.\right\} = \frac{1}{F\ (x, y)}$$

2.2.5 Directional efficiency measures[5]

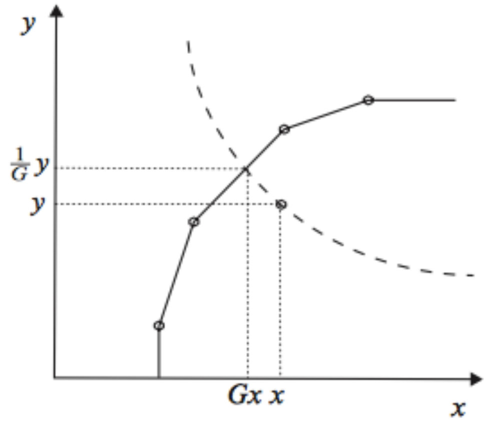
Until now, all inputs and outputs are reduced or expanded by the same factor E or F. In this section, other alternatives efficiency measurements approaches will be covered solve this proportional adjustment.
One suggestion is the *graph hyperbolic measure of technical efficiency* that consider simultaneous improvements on the input and output side by basically combining the Farrell input and output efficiency measures into one measure in the following way:

$$G = min\left\{G > 0 \left| \left(Gx, \frac{1}{G}y\right) \in T \right.\right\}$$

In this function G, it can be observed that the input side is exactly as in the E measure, and the output side has relation with the F measure, when the G is reduced, $1/G$ is expanded and this way is like in the Farrell output efficiency measure.

[5] P. Bogetoft and L. Otto (2013), “Benchmarking with DEA, SFA, and R, Springer” NY. Chapter 2, pp. 31-34

As it is showed in the graphic below (hyperbolic), it is possible to adjust and to measure G at the same time with input Gx and with output $\frac{1}{G}y$.However, as the function G is non-linear, is not so easy to implement it in applications.



Directional distance functions are a more profound alternative of Farrell´s proportional approach. This functions have the objective to determine improvements in a given direction $d \in \mathbb{R}^m$ and to measure how far are to the frontier in d - units. The directional distance function or excess function can be described in the following way:

$$e = e(x, y; T, d) \coloneqq max\{e \in \ \mathbb{R} | (x - ed, y) \in T\}$$

The excess function can be interpreted as the number of times the input x has been used in excess of what is necessary to produce the output y. Therefore, the larger the excess the greater the inefficiency.

It is important to say that directional distances are not comparable across different directions, excess values depend on the direction and on the length of the direction vector. More generally, for arbitrary $\theta \geq 0$:

$$e\big((x, y); \theta d\big) = \frac{1}{\theta} e\big((x, y); d\big)$$

The Farrell approach is a variant of the directional distance function approach, where the firms own inputs are used as the direction vector. In this way:

$$e\big((x, y); x\big) = 1 - E(x, y)$$

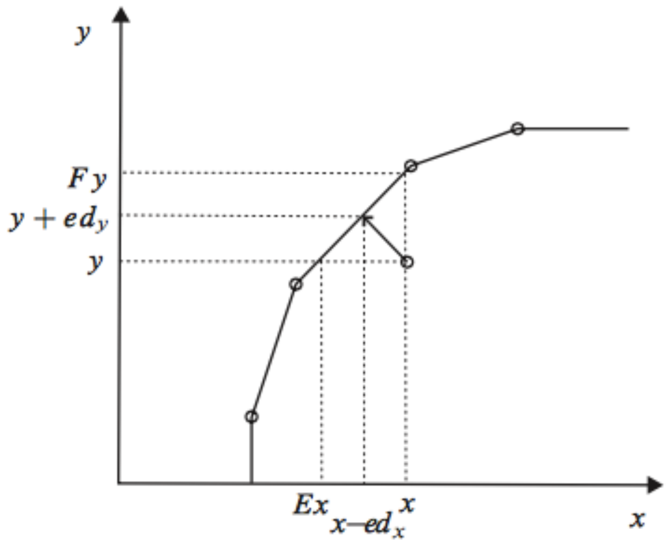
The excess function measures the inefficiency of the the firm. For example, is Farrell efficiency is 85 %, then, the excess is 15%. The excess function can also be expressed in the case of having some fixed inputs or outputs as follows:

$$e\big((x, y); (x_v, 0)\big) = 1 - E^*\big(x_v, x_f, y\big)$$

Now, instead of just working with variable and fixed inputs and outputs, the directional distance function approach allows us to work with different grades of discretion. Some dimensions can be controlled or changed more easily and others. In this way, we can combine input and output efficiency perspectives using the direction distance approach, examine whether it is possible to produce more output and use fewer inputs and look for changes in direction $(d_x, d_y) \in \mathbb{R}^m \times \mathbb{R}^n$. The directional excess function is defined as follows:

$$e = max\{e > 0 | (x - ed_x, y + ed_y) \in T\}$$

With only one input, one output and the direction $(d_x, d_y) = (1,1)$ we have the following graph, where the arrow indicates the direction. The correct direction in practice usually depend on the application.

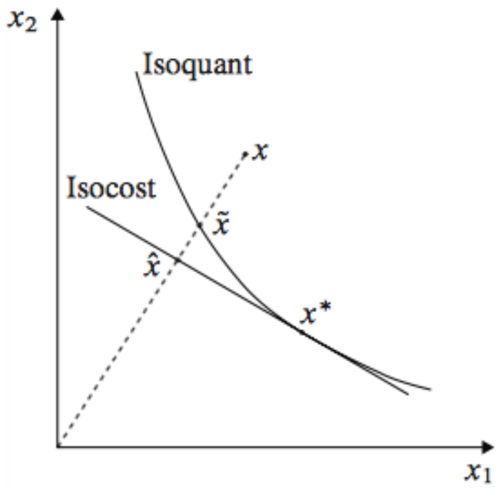


2.2.6 Efficiency measures with prices[6]

Until now, relative importance of inputs and outputs have not been assumed. In this section, the relative weights, prices or priorities that can naturally have the different inputs and outputs are going to be taken into account. This fact, will be useful to decompose the efficiency into technical efficiency (associated with the use of optimal procedures) and allocative efficiency (associated with the use or production of optimal combinations of inputs or outputs).

If prices associated with the inputs are considered, w will be the n-vector of input prices, $w \in \mathbb{R}^n$ and wx will be the costs of the production plan (x, y) and in this way, and being $c = wx$, the production plan will be (c, y).

[6] P. Bogetoft and L. Otto (2013), "Benchmarking with DEA, SFA, and R, Springer" NY. Chapter 2, pp. 35-41



Cost and input allocative efficiency

Assuming that a company has used x inputs and without taking into account by the moment the price, the *technical input efficiency* can be measured as follows:

$$TE = \frac{||\bar{x}||}{||x||}$$

where $\bar{x}$ is the point in the isoquant obtained proportionally from x along the dashed line.
Now, and knowing that $x^*$is the optimal cost input combination, the *cost-efficiency* CE can be defined like this:

$$CE = \frac{wx^*}{wx}$$

where x^* is found by solving the cost minimization problem, min $w'x$ subject to $(x', y) \in T$. This point is where the the isocost line is tangent to the isoquant.
The technical efficiency can be rewritten in the following way:

$$TE = \frac{w\bar{x}}{wx}$$

If it is possible to save 15% of all inputs from x to $\bar{x}$, it is also possible to save 15% of the cost. Now, comparing the cost of $\bar{x}$ and x^*, the difference is the cost of choosing the technical efficient plan $\bar{x}$ instead of a cheaper input mix x^*. Here, the *allocative efficiency* is presented as:

$$AE = \frac{wx^*}{w\bar{x}}$$

as it can be easy deduced, AE is always lower or equal than 1. For example, AE=0.75 means that 25% could have been saved by better allocating of the inputs toward a cheaper input mix.
Summing up, it can be distinguished between technical efficiency TE, cost efficiency CE and allocative efficiency AE, and these can be related as:

$$CE = \frac{wx^*}{wx} = \frac{wx^*}{w\bar{x}}\frac{w\bar{x}}{wx} = AE \cdot TE$$

So, for a company to be cost–efficient, it must use the right resources and it must use them in the right way.

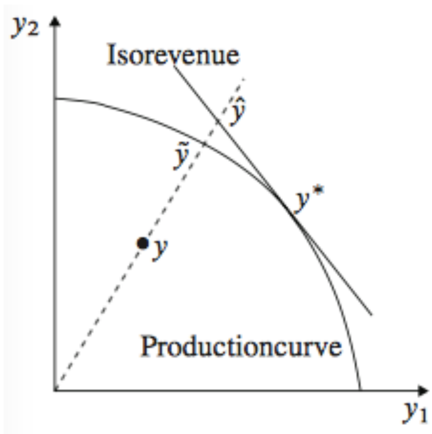
Finally, defining $\hat{x}$ as the point with the same cost as x^* and in the dotted contraction line, the following relationships between CE, AE and TE by comparing the length of x, $\bar{x}$ and $\hat{x}$ can be done:

$$TE = \frac{\|\bar{x}\|}{\|x\|}, AE = \frac{\|\hat{x}\|}{\|\bar{x}\|}, CE = \frac{\|\hat{x}\|}{\|x\|}$$

Revenue and output allocative efficiency

In the same way, the output side is going to be described. Now, the aim is to look whether the output mix is optimal in terms of maximizing revenue for a given input. Defining the output prices $p \in \mathbb{R}^m$, y as the observed output and y^* as the optimal revenue output obtained for the revenue-optimizing problem max py' subject to $(x, y') \in T$, the revenue efficiency can be defined as follows:

$$RE = \frac{py^*}{py}$$



The revenue efficiency can be rewritten also like this:

$$RE = \frac{py^*}{py} = \frac{py^*}{p\bar{y}}\frac{p\bar{y}}{py} = \frac{py^*}{p\bar{y}}F = AF \cdot F$$

where F is the Farrell output technical efficiency, and $\bar{y} = Fy$. So, $\bar{y}$ is the technically efficient point obtained with the expansion of y along the dotted line. AF is the output oriented allocative efficiency, which is the revenue obtained by choosing the best mix of output relative to the revenue.

In the same way as in the input side, for company to be fully revenue efficient, it must perform both full output technical and full output allocative efficiency. In other words, it must produce without losing any resource and also produce the proper mix of services.

The decomposition of revenue efficiency can be described like in the input side by comparing the length of vectors on the dotted line as well. In this case, $\hat{y}$ is the point expanded on the dotted line that has the same revenue as y^*.

$$F = \frac{||\bar{y}||}{||y||}, AF = \frac{||\hat{y}||}{||\bar{y}||}, RE = \frac{||\hat{y}||}{||y||}$$

Profit efficiency

Now, having prices w and p of inputs and outputs respectively, the *profit efficiency* can be defined as follows:

$$PE = \frac{py - wx}{py^* - wx^*}$$

where (x, y) is the observed production plan and (x^*, y^*) is the profit maximizing production plan obtained from the solution of max $py' - wx'$ subject to $(x', y') \in T$.

All company will be interested in having this parameter as high as possible logically, for that, they should have the ability t get the most out of given resources, select a cost-minimal input mix and select a revenue maximizing output mix.

Again, it is possible to decompose the inefficiency into different parts related to technical inefficiency, input allocative efficiency and output allocative efficiency.

2.2.7 Dynamic efficiency[7]

Performance of firms often change with time, for this reason, it is essential to have measures that capture this changes. Technological progress should be taken also into account, due to many of the behaviours changes in companies are due to it.

For example, two periods of a firm can be described, period s and period t, and for each period there are different technologies also. In the benchmarking literature, the

[7] P. Bogetoft and L. Otto (2013), "Benchmarking with DEA, SFA, and R, Springer" NY. Chapter 2, pp. 41-44

most used approach to dynamic evaluations is the *Malmquist index*. Let $E^i(s,t)$ be a measure of the performance of a firm i in period s agaisnt the technology in period t. Now, the technology and the production data depending on the period are distinguished.

To measure the improvement in a firm i from period s to period t, one technology is fixed (in this case the s technology) and then look at the changes in efficiency compared to a fixed one.

$$M^s = \frac{E(t,s)}{E(s,s)}$$

if the firm has improved form period s to period t, then $E(t,s) > E(s,s)$ and of course, $M^s > 1$. Therefore, $M^s > 1$ if the company improve over time and $M^s < 1$ if the company is worse over time (if it moves away from the frontier). For example, if a firm is 30% efficient in period s and 90% efficient in period t, the it has improved by a factor of 3. In real problems is more difficult beacuase in addition to changing technology will also change the input and output mix.

In the same way, but in this case fixing the technology , M^t can be expressed as follows:

$$M^t = \frac{E(t,t)}{E(s,t)}$$

And, because there is no need to fixe one or the other, the Malmquist index is:

$$M(s,t) = \sqrt{M^s M^t} = \sqrt{\frac{E(t,s)}{E(s,s)}\frac{E(t,t)}{E(s,t)}}$$

This parameter measures how much the firm changes between periods s and t. This change in performance may be due to two factor, the technological progress and special initiatives. In this way, the Malmquist measure is decomposed as follows:

$$M(s,t) = \sqrt{\frac{E(t,s)}{E(t,t)}\frac{E(s,s)}{E(s,t)}}\frac{E(t,t)}{E(s,s)} = TC(s,t)EC(s,t)$$

where, TC is the *technical change* and EC is the *efficiency change.* Values over 1 in the TC represents technological progress in the sense that more output can be produce using fewer inputs. EC measure how actualize is the technology against the present technology.

Every firm should try to improve not only compared to itself over the time, but also with respect to other firms that have also benefited from general technological progress.

Lastly, one should be careful interpreting results from different periods, one cannot simply accumulate the changes unless a fixed-based technology is used.

2.2.8 Structural and network efficiency[8]

In this section, efficiency measurements and evaluations for a collection of firms to see whether the industry structure is efficient or not are going to be analysed.

First, consider the possible impact of merging firms 1 and 2, which have used similar inputs to produce similar outputs , and that those firms have (x^1, y^1) and (x^2, y^2) production plans respectively.

If both companies become integrated but continue to operate as two independent entities they will transform the vector of inputs $x^1 + x^2$ into the vector of outputs $y^1 + y^2$.
Using the Farrell input approach, it is possible to measure the potential gains from merging companies 1 and 2:

$$E^{1+2} = min\{E \in \mathbb{R} \,|(E(x^1 + x^2), y^1 + y^2) \in T\}$$

In $E^{1+2} > 1$, the merger is costly and if $E^{1+2} < 1$, it is possible to save via merger. $E^{1+2} = 0.7$ would suggest than 30% of all inputs could be saved by integrating both firms. In the same way, $E^{1+2} = 1.1$ means that 10% more of all resources would be needed.

2.2.9 Choice between efficiency measures[9]

This section some key concerns related to the choice between alternative efficiency measurements are going to be discussed.

Controllability is a very important factor, as it has been explained before, there are some variable that can be controlled and other that it is impossible to control. The *time perspective* is also relevant because usually the longer the time the more factors variable there are. For example, a hospital may no have much control over demand, and as a result, input-based evaluations may be more relevant, while a farmer may have many fixed resources (land, etc.) and, therefore, should be evaluated more in terms of the output.

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[8] P. Bogetoft and L. Otto (2013), "Benchmarking with DEA, SFA, and R, Springer" NY. Chapter 2, pp. 45-47

[9] P. Bogetoft and L. Otto (2013), "Benchmarking with DEA, SFA, and R, Springer" NY. Chapter 2, pp. 48-49

The *level in a hierarchy* is also significant. It is not the same analyse how efficient a boss is than to analyse the efficiency of a receptionist.

Other relevant point is the *intended use* of the efficiency score. In a learning experience, the exact efficiency measurement is less important than the ability to find relevant peers taking into account the firms own preference, strategies, etc.

Data availability and computations ease are elements that help evaluations to be more specific. For example, prices for inputs allow calculation of cost efficiency analyses that decompose efficiency into allocative and technical efficiency, which will provide more information than a pure technical efficiency analysis. Likewise, using data from several years allows more robust evaluations and may possibly allow us to separately consider general productivity shifts and catch-up effects.

It is also important to keep the rational ideal model in mind when considering indices of technical efficiency. Ideally, efficiency should reflect utility effectiveness because efficiency is a sort of proxy for *utility effectiveness*. Although efficiency provides a useful filter, efficiency is not a sufficient condition for firm effectiveness.

Finally, when thinking about improvements, small improvements of the right type may be more valuable than large improvements to less important aspects.

3. DEA method

3.1 Introduction

In 1978, **Data Envelopment Analysis** (DEA) was developed by Charnes, Cooper and Rhodes[10]. DEA is considered as one of the most dominant techniques in the evaluation of the performance of production units. It uses linear programming to evaluate the comparative efficiency of operational units that employ similar production processes.

In brief, DEA aims at converting multiple input and output measures into a single efficiency score for each of the decision making units (DMUs) in the set under evaluation. The efficiency is defined as the ratio of the weighted sum of outputs to the weighted sum of inputs. This efficiency score forms a comprehensive measure of their performance, Charnes et al. (1978). Each DMU is evaluated by comparing its efficiency score with that of other DMUs or with the hypothetical performance of composite DMUs. A DMU is considered to be inefficient when another DMU, can produce more outputs with the same inputs or can produce the same outputs with fewer inputs. Otherwise, the DMU is considered to be efficient. An efficient DMU achieves a score of 1 (or 100%) while inefficient DMUs receive lower scores. One of the the main advantages of the DEA method is that it provides information about the reference set of each non-efficient DMU. This reference set provides the necessary adjustments that must be made by the non-efficient DMU in order to became efficient.[11]

It seems that the use must have expertise on linear programming in order to execute the DEA models and interpret the DEA outcomes. However, shortly after 2000, there is no need to have a good background in linear programming thanks to Microsoft Excel software, which includes DEA algorithms.

Thus, DEA became a new and easy tool in operational research for measuring technical efficiency. Up to now, this benchmarking technique has been successfully applied to evaluate, compare and suggest ways to improve in many organizations, including educational departments, health care, prisons, agricultural production, banking, armed forces, sports, market research, transportation, courts, benchmarking, index number construction and many other applications.

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[10] Charnes, A., W.W. Cooper and E. Rhodes (1978), "Measuring the efficiency of decision making units", European Journal of Operational Research, 2, pp. 429-444.

[11] S. Dimitris, A. Karakitsiou, O. Mitsopoulou, "Applying QE-DEA in the evaluation of the Greek public sector: The case of the agencies for aliens and immigration of the Decentralized Administration of Macedonia and Thrace.",p. 3

3.2 What does DEA do?

DEA, compared with other methods, identifies information and relationships not identifiable with alternative techniques (usually more complicated) that are commonly used.

- DEA compares different production units considering all inputs and outputs, and identifies one the one hand the most efficient units (branches, departments, individuals) and on the other hand the inefficient units in which real efficiency improvements are possible. In brief, DEA is a very powerful benchmarking technique.

- This powerful benchmarking technique calculates the amount of cost and resource savings that can be achieved in case that each inefficient unit becomes as efficient as the most efficient units.

- Changes in the inefficient service units are identified, then management can implement this specific changes to achieve potential savings located with DEA. In addition, DEA estimates the amount of additional service an inefficient unit can provide without the need to use additional resources.

- Management receives information about performance of service units that can be used to help transfer system and managerial expertise from better-managed, relatively efficient units to the inefficient ones. This has resulted in improving the productivity of the inefficient units, reducing operating costs and increasing profitability. [12]

3.3 DEA legend

The following letters will be used to help describe the method:

DMU → Decision making unit
n → Number of $DMUs$ being compared in the DEA analysis
i → Number of outputs used by the $DMUs$
j → Number of inputs generated by the $DMUs$
y_{iq} → Amount of output i generated by DMU_q
x_{jq} → Amount of input j used by DMU_q
u_i → Weights assigned by DEA to output i
v_j → Weights assigned by DEA to input j
θ → Efficiency rating of the DMU being evaluated by DEA[13]

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[12] H. David Sherman and Joe Zhu, "Service Productivity Management" Improving Service Performance using Data Envelopment Analysis (DEA), pp. 50-51

[13] H. David Sherman and Joe Zhu, "Service Productivity Management" Improving Service Performance using Data Envelopment Analysis (DEA), p 63.

3.4 Productive unit efficiency

Before attempting to improve the performance of a production unit in order to be competitive, measuring the efficiency of these units and identifying sources of inefficiency is necessary. Overall, the production unit term refers to a unit of production of certain outputs by spending certain inputs. All inputs and outputs have an impact on efficient operation of such units, even though some are considered more or less important.

The most frequent method used to measure efficiency is based on ratios. Their handicap is that they reflect only a few of the factors having an impact on the overall efficiency of a productive unit.

Considering that there is a set of n productive units $(DMU_1, \ldots, DMU_n)$ and that each unit uses m inputs to produce s outputs, the input matrix will be $X = \left[x_{ij}, i = 1, \ldots, m, j = 1, \ldots, n\right]$ and the output matrix $Y = \left[y_{ij}, i = 1, \ldots, s\ j = 1, \ldots, n\right]$. For example the q-th line X_q and Y_q that shows the inputs and outputs of unit DMU_q. In this case, the efficiency rate of such a unit can the be generally expressed as:

$$\frac{weighted\ sum\ of\ outputs}{weighted\ sum\ of\ inputs} = \frac{\sum_{i=1}^{s} u_i y_{iq}}{\sum_{j=1}^{m} v_j x_{jq}}$$

where:

$u_i, i = 1, \ldots, s$, are weights assigned to i-th output

$v_j, j = 1, \ldots, m$, are weights assigned to j-th input

There are different ways to evaluate the efficiency rate as defined above, namely *multicriterial decision methods* and *data envelopment analyses* (DEA). These approaches differ in how they obtain input and output weights.

- Multicriterial decision methods usually expect the user to define the weights p_i and w_j upfront, therefore, the user determines the significance of individual inputs and outputs in the analysis. Such an analysis yields the rate of utility of given units. It reflects the relative importance of inputs and outputs represented by their respective weights. Based on this analysis units can be ranked from the worst to the best performer.
- DEA models calculate input and output weights by means of an optimising calculation. Based on that, units can be classified into efficient and inefficient. In inefficient units, target values of inputs and outputs are given which would lead to efficiency. [14]

[14] Ing. Kristína Vincová, Technical University Kosice (2005), *Using DEA models to measure efficiency*, BIATEC, Volume XIII, 8/2005, pp. 24-25.

3.5 Basic DEA models, CCR and BCC

DEA models evaluate n productive units DMU_s, where each DMU takes m different inputs to produce s different outputs. The essence of DEA models in measuring the efficiency of productive unit DMU_q lies in maximising its efficiency rate. However, subject to the condition that the efficiency rate of any other units in the population must not be greater than 1. All variables must be considered, so, weights of all inputs and outputs must be greater than 0. This model is defined as a linear divisive programming model:

$$maximize \qquad \frac{\sum_i u_i y_{iq}}{\sum_j v_j x_{jq}} \qquad (1)$$

$$subject\ to:$$

$$\frac{\sum_i u_i y_{ik}}{\sum_j v_j x_{jk}} \leq 1, k = 1, \dots, n$$

$$u_i \geq \in, i = 1, \dots, s$$

$$v_j \leq \in, j = 1, \dots, m$$

Now, this model is converted into a linear programming model and transformed into a matrix as follows:

$$maximize \qquad z = u^T Y_q \qquad (2)$$

$$subject\ to:$$

$$v^{\,T} X_q = 1$$

$$u^T Y - v^T X \ \leq 0$$

$$u \geq \in, v \leq \in$$

Model (2) is called primary CCR model *(Charnes, Cooper, Rhodes).* The dual model to this can be stated as follows:

$$minimize \qquad f = \theta - \in (e^T s^+ + e^T s^-) \qquad (3)$$

$$subject\ to:$$

$$Y\lambda - s^{\,+} = Y_q$$

$$X\lambda + s^{\,-} = \theta X_q$$

$$\lambda, s^+, s^- \geq 0$$

where $\lambda = (\lambda_{1,}, \dots, \lambda_n), \lambda \geq 0$ is a vector assigned to individual productive units, s^+ and s^- are vectors of addition input and output variables, $e^T = (1, \dots, 1)$ and ∈ is a constant (Economic reasoning: for any amount of output we must use at least a minimum quantity of every input. If any of the inputs is 0, the total output is 0 as well) greater than 0, normally 10^{-6} or 10^{-8}. In evaluating the efficiency of unit DMU_q, model (3) looks for a virtual unit characterised by inputs $X\lambda$ and outputs $Y\lambda$, which are linear combinations of inputs and outputs of other units of the population and which are better that the inputs and outputs of unit DMU_q. In this way, $X\lambda \leq X_q$ and $Y\lambda \geq Y_q$. So, unit DMU_q is identical to the virtual unit, $X\lambda = X_q$ and $Y\lambda = Y_q$, then is rated efficient.

If unit DMU_q is CCR efficient, then:

- The value of variable θ is equal to 0.
- The values of additional variable s^+ and s^- are equal to 0.

Therefore, DMU_q is CCR efficient if the optimum value of the model (3) equals to 1. In any other case, it will be rate as inefficient. The optimum value of the objective function f^* determines the efficiency rate of the unit concerned. In inefficient units $\theta < 1$. So, in order to make DMU_q efficient, a proportional reduction of inputs should be done, and DEA advises the user how should the unit works to reach the perfect efficiency.

Models (2) and (3) are input oriented, they advise the user how much should decrease the inputs to become efficient. Output oriented model are defined in the following way:

$$maximize \qquad g = \Phi + \in (e^T s^+ + e^T s^-) \qquad (4)$$

$$subject\ to:$$

$$Y\lambda - s^+ = \Phi Y_q$$

$$X\lambda + s^- = X_q$$

$$\lambda, s^+, s^- \geq 0$$

Now, unit DMU_q is CCR efficient if $g^* = 1$. If the value of the unit is greater than 1, then, the unit is inefficient. The variable Φ indicates that more output is needed to achieve the efficiency.

Models (2), (3) and (4) assume constant returns to scale, for instance, a double increase in inputs leading to a double increase in outputs. In this case, models (3) and (4) need to be rewritten to include the condition of convexity $e^T\lambda = 1$. Afterwards, they are referred to as BCC models *(Banker, Charnes, Cooper).*

The main goal of DEA analysis is to find target values for inputs X'_q and outputs Y'_q for an inefficient unit to become efficient. Target values are calculated:

- By means of productive unit vectors:

  $$X'_q = X\lambda^*$$

  $$Y'_q = Y\lambda^*$$

  where λ^* is the vector of optimal variable values.

- By means of efficiency rate and values of additional variables s^- and s^+:

  Input oriented CCR model:

  $$X'_q = \theta X_q - s^-, Y'_q = Y_q + s^+$$

  Output oriented CCR model:

  $$X'_q = X_q - s^-, Y'_q = \Phi Y_q + s^+$$

  where θ is the efficiency rate in the input-oriented model and Φ is the efficiency rate in the output oriented model.[15]

3.6 Observations

- DEA gives the ''benefit of the doubt" to each unit being evaluated trying to make it look as efficient as possible in comparison with the other units. This means that the inefficiencies noted would tend to understate the actual inefficiencies that may be present.

- When a DEA analysis is determined to be complete in terms of using appropriate inputs and outputs, it offers paths to achieve real improvements in performance. The amount of the improvements that are technically available would be at least as great as the amount identified with DEA. Indeed, the conservative nature can occasionally result in all or almost all the units being assigned an efficiency rating of one.

- The weights are assigned by DEA to make each service unit look as efficient as possible. If the user tries to substitute another set of weights that are believed to be more reflective of the market than the weights assigned by DEA, the inefficiency will be greater and the potential benefits of improving

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[15] Ing. Kristína Vincová, Technical University Kosice (2005), *Using DEA models to measure efficiency*, BIATEC, Volume XIII, 8/2005, pp. 25-26.

the inefficient units to approach the best practices will be greater than estimated with the first model.

- Relative weights: The weights assigned to the inputs and outputs have managerial and analytic value. In DEA models, influencing these weights allows the manager to substantially increase the DEA insights about ways to improve performance.[16]

3.7 Interpretation of the Results

Basically, the interpretation of DEA results tends to proceed in the following way:

- The efficiency ratings are generated. Unit that are efficient ($\theta = 1$) are relatively, and not strictly efficient. That is, no other unit is clearly operating more efficiently than these units, but it is possible that all units, including these relatively efficient units, can be operated more efficiently.
- Inefficient units are identified by an efficiency rating of $\theta < 1$.
- The efficiency reference set (ERS) indicates the relatively efficient units against which the inefficient units were most clearly determined to be inefficient.
- Management can also use DEA to identify other methods or combinations of methods to improve the efficiency of inefficient units. [17]

[16] H. David Sherman and Joe Zhu, "Service Productivity Management" Improving Service Performance using Data Envelopment Analysis (DEA), pp. 66-68.

[17] H. David Sherman and Joe Zhu, "Service Productivity Management" Improving Service Performance using Data Envelopment Analysis (DEA), pp. 62-63.

4. Methodology and description of the model

4.1 Methodology

The efficiency of the different DMU_s will be estimated using the non-parametric methodology of DEA (Charnes et al., 1978, 1981), which has been defined in Chapter 3 in detail.

DEA is a model that evaluates the relative efficiency of different homogeneous DMU_s, based on linear programming techniques. A DMU converts similar inputs into similar outputs. In this paper, DEA is applied to paddle tournaments. Generally speaking, DEA is an extension of the traditional ratios analysis that identifies a DMU as efficient when no other DMU is capable of producing a higher output from the same input (output orientated) or, alternatively, of producing the same output from less input (input-orientated). DEA allows the use of multiple inputs and outputs. [18]

4.2 Description of the model

The application is carried out on a sample of 7 tournaments (male and female) of the Navarra Paddle Circuit between 2010 and 2015. Every year these tournaments are disputes under the same conditions as they are scoring championships in the Navarre ranking paddle, which orders all the pairs of circuit depending on the points earned in each tournament. This circuit has this fixed structure since 2010, as this apparently is giving very good results. Thus, it will be analyzed separately each year in different tables, being the seven tournaments present in each table, with the correspondent data.

All the relevant variables have been obtained from the database, which provides the exact information since 2010 and that will help to calculate the different efficiencies.

The model specification chosen in this study is comprised of two inputs and two outputs.

The inputs used are:

1- The expenses (in Euros), which indicates the amount of money needed to organize each tournament, including money to pay referees, rental facilities, sports equipment, awards …
2- Total federated players, which indicates the number of players who are federated in the federation and therefore they can play tournaments and have other advantages and facilities.

[18] Ricardo Sellers-Rubio and Francisco Mas-Ruiz, "Economic efficiency in supermarkets:evidences in Spain", pp. 160.

While the measurement of the outputs are:

1- Total inscription, which are the most important variable for the FNP. This is the number of players who sign in each tournament, including both boys and girls. Like any sports federation, this has to be the most important parameter to maximize, as the success of a federation depends on the people who enjoy their tournaments and activities.
2- The incomes (in Euros), which indicates the amount of money that the FNP collects both from the inscriptions and the sponsors.

All the data are presented in the following tables, from year 2010 until year 2015.

Table 1 Inputs and outputs of 2010 model

	Inputs		**Outputs**	
	Expenses in €	Total federated	Total inscriptions	Incomes in €
DMU	X_1	X_2	Y_1	Y_2
1	22 912,33	1 312	716	21 131,62
2	22 024,04	1 352	742	21 351,66
3	18 012,23	1 462	774	17 968,44
4	16 245,12	1 515	666	15 912,02
5	14 452,10	1 597	708	14 906,77
6	17 926,58	1 650	766	16 001,52
7	17 512,54	1 721	792	16 319,15

Table 2 Inputs and outputs of 2011 model

	Inputs		**Outputs**	
	Expenses in €	Total federated	Total inscriptions	Incomes in €
DMU	X_1	X_2	Y_1	Y_2
1	24 412,33	1735	830	23 999,41
2	24 671,84	1792	810	23 624,72
3	18 124,92	1812	798	17 925,31
4	18 245,73	1899	710	16 501,09
5	15 252,10	1921	606	14 906,77
6	19 122,46	1985	806	16 906,56
7	19 325,41	2062	800	17 214,99

Table 3 Inputs and outputs of 2012 model

	Inputs		Outputs	
	Expenses in €	Total federated	Total inscriptions	Incomes in €
DMU	X_1	X_2	Y_1	Y_2
1	28 512,41	2075	996	28 617,31
2	28 301,75	2123	890	27 533,64
3	19 134,41	2184	888	18 125,31
4	19 045,41	2239	754	18 451,23
5	16 002,98	2297	688	14 127,62
6	20 226,31	2351	906	16 236,32
7	20 245,33	2412	924	16 126,99

Table 4 Inputs and outputs of 2013 model

	Inputs		Outputs	
	Expenses in €	Total federated	Total inscriptions	Incomes in €
DMU	X_1	X_2	Y_1	Y_2
1	31 022,33	2435	1022	30 227,94
2	31 188,05	2498	1034	28 453,76
3	20 220,70	2517	996	19 991,50
4	20 111,35	2555	862	19 366,91
5	17 091,34	2612	720	15 123,66
6	22 746,77	2691	906	18 922,43
7	22 892,44	2748	1002	18 194,73

Table 5 Inputs and outputs of 2014 model

	Inputs		Outputs	
	Expenses in €	Total federated	Total inscriptions	Incomes in €
DMU	X_1	X_2	Y_1	Y_2
1	33 393,02	2 761	1070	32 935,14
2	33 274,99	2 788	1096	28 738,40
3	21 964,91	2 819	1060	21 411,90
4	21 456,14	2 855	1000	20 466,91
5	17 864,88	2 873	788	15 709,13
6	23 871,99	2 915	1000	19 976,98
7	24 136,74	2 950	1010	19 986,98

Table 6 Inputs and outputs of 2015 model

	Inputs		Outputs	
	Expenses in €	Total federated	Total inscriptions	Incomes in €
DMU	X_1	X_2	Y_1	Y_2
1	34 796,66	2961	1052	31 423,22
2	35 245,86	3022	1046	29 156,34
3	24 079,59	3059	1032	21 955,70
4	23 273,46	3111	1004	21 556,71
5	20 854,23	3178	722	16 813,63
6	25 757,45	3202	922	20 571,36
7	25 946,56	3226	972	20 166,98

5. Results and discussions

First of all, the efficiencies of the tournaments in their respective years is represented in the following tables, also pointing which of them has high efficiency (HE) and which of them has low efficiency (LE).

In order to obtain the efficiency of the different DMU a spreadsheet in Microsoft excel was modelled properly. The (BCC) was solved using the Excel's Solver add-in. The process was carried out DMU by DMU, changing only a cell to select the desired DMU in the Excel sheet. Then, using the solver, the efficiency score of the selected DMU is calculated in a very fast way. This process was followed for each tournament every year.

Table 7 2010 DMUs classification

DMU	Efficiency Score	Classification
1	1,0104	LE
2	**1,0000**	**HE**
3	**1,0000**	**HE**
4	1,0337	LE
5	**1,0000**	**HE**
6	1,0315	LE
7	**1,0000**	**HE**

As Table 7 indicates, DMU_2, DMU_3, DMU_5 and DMU_7 in the year 2010 obtained an efficiency score equal to 1 (or efficiency rating 100%), which means that they used the exact amount of their inputs to produce their outputs. Therefore, they are also considered to be the reference units of the analysis in 2010.

Table 8 2011 DMUs classification

DMU	Efficiency Score	Classification
1	**1,0000**	**HE**
2	1,0159	LE
3	**1,0000**	**HE**
4	1,0934	LE
5	**1,0000**	**HE**
6	**1,0000**	**HE**
7	1,0083	LE

As Table 8 indicates, DMU_1, DMU_3, DMU_5 and DMU_6 in the year 2011 obtained an efficiency score equal to 1 (or efficiency rating 100%), which means that they used the exact amount of their inputs to produce their outputs. Therefore, they are also considered to be the reference units of the analysis in 2011.

Table 9 2012 DMUs classification

DMU	Efficiency Score	Classification
1	**1,0000**	**HE**
2	1,0311	LE
3	**1,0000**	**HE**
4	**1,0000**	**HE**
5	**1,0000**	**HE**
6	1,0131	LE
7	**1,0000**	**HE**

As Table 9 indicates, DMU_1, DMU_3, DMU_4 and DMU_7 in the year 2012 obtained an efficiency score equal to 1 (or efficiency rating 100%), which means that they used the exact amount of their inputs to produce their outputs. Therefore, they are also considered to be the reference units of the analysis in 2012.

Table 10 2013 DMUs classification

DMU	Efficiency Score	Classification
1	**1,0000**	**HE**
2	**1,0000**	**HE**
3	**1,0000**	**HE**
4	1,0235	LE
5	**1,0000**	**HE**
6	1,1090	LE
7	1,0033	LE

As Table 10 indicates, DMU_1, DMU_2, DMU_3 and DMU_5 in the year 2013 obtained an efficiency score equal to 1 (or efficiency rating 100%), which means that they used the exact amount of their inputs to produce their outputs. Therefore, they are also considered to be the reference units of the analysis in 2013.

Table 11 2014 DMUs classification

DMU	Efficiency Score	Classification
1	**1,0000**	**HE**
2	**1,0000**	**HE**
3	**1,0000**	**HE**
4	1,0116	LE
5	**1,0000**	**HE**
6	1,0661	LE
6	1,0563	LE

As Table 11 indicates, DMU_1, DMU_2, DMU_3 and DMU_5 in the year 2014 obtained an efficiency score equal to 1 (or efficiency rating 100%), which means that they used the exact amount of their inputs to produce their outputs. Therefore, they are also considered to be the reference units of the analysis in 2014.

Table 12 2015 DMUs classification

DMU	Efficiency Score	Classification
1	**1,0000**	**HE**
2	1,0057	LE
3	**1,0000**	**HE**
4	**1,0000**	**HE**
5	**1,0000**	**HE**
6	1,1227	LE
7	1,0653	LE

As Table 12 indicates, DMU_1, DMU_3, DMU_4 and DMU_5 in the year 2015 obtained an efficiency score equal to 1 (or efficiency rating 100%), which means that they used the exact amount of their inputs to produce their outputs. Therefore, they are also considered to be the reference units of the analysis in 2015.

Table 13 Efficiency scores

	DMU 1	DMU 2	**DMU 3**	DMU 4	**DMU 5**	DMU 6	DMU 7
2010	1,0104	1,0000	**1,0000**	1,0337	**1,0000**	1,0315	1,0000
2011	1,0000	1,0159	**1,0000**	1,0934	**1,0000**	1,0000	1,0083
2012	1,0000	1,0311	**1,0000**	1,0000	**1,0000**	1,0131	1,0000
2013	1,0000	1,0000	**1,0000**	1,0235	**1,0000**	1,1090	1,0033
2014	1,0000	1,0000	**1,0000**	1,0116	**1,0000**	1,0661	1,0563
2015	1,0000	1,0057	**1,0000**	1,0000	**1,0000**	1,1227	1,0653
Average	1,0017	1,0088	**1,0000**	1,0270	**1,0000**	1,0571	1,0222

Table 13 report the efficiency of each tournament of the analysis (from 2010 until 2015). The average efficiency indicates which tournament is working better over the years and therefore gives us a very important and valuable information at the time of improving in those championships where the average is worse.

According to the Table 13, DMU_3 and DMU_5 are the best tournaments in terms of organization in the Navarra Paddle Circuit as they were in the frontier for the whole time (EF = 1,0000). Meanwhile, DMU_6 was the worst performer, due to its efficiency scores was always quite far from the frontier and just in 2011 was in the frontier.

One of the strengths of DMU_3 and DMU_5 are to be played in the period of Easter and in late August or early September, time in which people in general, or at least students, are on holidays and is easier to enjoy the tournaments paddle with less worries. The FNP should to focus more on those tournaments where efficiency is worse in general. To do this, they should try to get better sponsors or sign better contracts with them, offer more attractive prizes for more people to sign up to those tournaments or try to reduce the cost generated in the organization of tournaments, making agreements with clubs.

However, in general, all efficiencies are fairly close to the frontier (values close to 1) and there is no unit that is far away from this frontier, which is good and indicates that the federation is working well building a consistent and efficient circuit.

6. Conclusion and future research

The aim of the project has been to analyze the efficiency of 7 tournaments of the Navarra Paddle Circuit between 2010 and 2015. The methodology employed is based on the estimation of efficiency through the non-parametric technique of DEA and the data for the empirical study was extracted from the FNP's database.

Given the fact that some other parameter could be included in a deeper analysis, the conclusion of the study should be made with caution. For this reason, the scope of the results obtained in this study should only be considered at the level of the sample used.

The finding shows the existence of high levels of efficiency, with two highest average scores of 1,0000 in the third and fifth tournament of the circuit. Moreover, the lowest efficiency score was obtained in the sixth tournament of 2015 with a rate of 1,1227 which is not very far from the frontier, it is quite close actually, which indicates that none of the tournament had a bad performance. In this way, we can confirm that the FNP is using its inputs in a proper way in order to produce their outputs.

Finally, as a future line of research, it could start studying the other circuits within the FNP, like the Navarra Minors Circuit, the South Navarra Circuit or the Navarra Clubs Circuit. These circuits are becoming more consistent in the recent years and also would be important to analyze them. In addition, it should also be interesting to analyze the young technification plan, which is being very successful in recent years and so to see how to continue improving.

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