

**THE TERM STRUCTURE AS A PREDICTOR
OF REAL ECONOMIC GROWTH:
A general equilibrium approach**

Emilio J. Domínguez
Alfonso Novales

Abstract: Consistent empirical evidence has recently been brought up about the forecasting ability of the term structure of nominal interest rates, relative to future economic activity. However, there has not been much work that would check whether that is a robust property of general equilibrium asset pricing models. We present a theoretical economy, with real and nominal assets issued at different maturities, in which the nominal term structure has, in fact, forecasting power for future real growth. That information content goes beyond the one contained in short-term rates or in monetary policy variables.

Keywords: Term structure, expectations hypothesis, economic fluctuations, business cycles.

JEL Classification codes: E37, E52, F31.

Mailing Address: Alfonso Novales
Dpto. Economía Cuantitativa. Universidad Complutense.
28223 Madrid. Spain.
e-mail: eccua09@sis.ucm.es

Emilio Domínguez
Dpto. de Economía. Universidad Pública de Navarra.
31006 Pamplona. Spain.
e-mail: edominguez@upna.es

1. Introduction

It is a well established empirical fact that the term structure of interest rates has information on future activity. In a recent paper, Plosser and Rouwenhorst (1994) have shown that the slope of the term structure, taken for horizons longer than two years, has information on future economic activity, beyond that already contained in the fluctuations in short term rates. They also show that such a predictive ability is based on something more than predictions of future monetary policy. Even though the slope of the term structure contains expectations of future monetary variables, there is also important information about future real growth that is unrelated to the course of future policy.

Finally, their work shows that foreign term structures may contain information on domestic real growth for some countries. That tends to be true for countries with high and variable inflation rates, which may obscure the information on real activity contained in the domestic term structure. Hence, so long as business cycles show some time coordination across countries, the term structure of those with more stable inflation rates contains information on future real activity of other countries.

Less formalized antecedents on this issue go back to Kessel (1956) and Fama (1986). Laurent (1988) regressed GNP gross growth rates on lagged values of the spread between the US 20-year bond and the federal funds rate. Testing a consumption capital asset pricing model, Harvey (1988) produced evidence that the slope of the term structure was a better predictor of consumption growth than lagged consumption or lagged stock returns. However, predictability extended to just 3 quarters into the future.

The previous finding in Estrella and Hardouvelis (1991) that the information in the term structure that is summarized in its slope helps predict real growth in the United States, was confirmed by Plosser and Rouwenhorst (1994), with quarterly data for 1957-1991. They found the same result for Germany (1960-1991) and Canada (1957-1991), but not much significance for France (1970-1985). For the UK they obtained a significant, positive relation for the slope at short horizons (up to a year), but negative for longer horizons, which is hard to interpret. Similar results were obtained when the slope of the term structure was used to forecast real consumption growth rates. In the UK, which seems to follow a pattern different from other countries, very significant forecasting ability was found for nominal consumption.

These results are somewhat surprising, since there is no clear a priori reason why the slope in the *nominal* term structure should have predictive ability for future *real* economic activity. Theoretically, the slope in the nominal term structure is determined by expected inflation, expected real rates, and risk premiums. From previous research on equilibrium business cycle models [Kydland and Prescott (1982), King, Plosser and Rebelo (1988), among others], we already know that the slope of the real term structure is partly determined by current expectations of growth differentials between future and current consumption, because real interest rates are equal, in equilibrium, to the marginal rates of substitution of consumption. Since the correlation between consumption and output is usually very large in all models, we have correlation between real returns and output growth rates. From the point of view of these theoretical models, whether or not the *nominal* term structure can predict future real growth depends on the behavior of expected inflation, and on its correlation with real interest rates. In particular, unstable behavior of inflation expectations will tend to obscure this predictive power, which will be more evident under stable inflation.

Plosser and Rouwenhorst used continuously compounded annualized k -year growth rates of industrial production. A high level of the shorter spot nominal rate is associated with a lower one-year growth rate of industrial production. The slope of the nominal term structure has predictive ability for future industrial production growth in the US, Germany and the UK, capturing the explanatory power of the short term rate, that becomes non-significant when the slope is included in the regressions that explain growth in industrial production. The coefficients in the slope are positive, indicating that an increase in the spread between long and short-term rates is followed by higher than average real growth.

The slope coefficients decrease with the forecast horizon, but remain significant. The fact that the coefficients in the slope barely change when the short term rate is added to the regression, suggests that the information content in the slope is not just coming from its possible correlation with the short rate. For the US, they find that an increase in the two year interest rate of 100 basis points, holding the one year rate fixed, would be followed by an average increase of 4,5% in each of the following two years on the baseline growth in industrial production. However, the sample mean spread was of just 23 basis points, with a standard deviation of 48 basis points. In addition, a simultaneous reduction in the one and two year interest rates, would keep the slope unchanged, but would be followed by an increase of

0.21% in each of the two years, for each 100 basis points of reduction in interest rates.

To test whether the predictive ability on the compounded growth rates was just due to the ability to forecast short-term growth, Plosser and Rouwenhorst also estimated 'marginal growth' rates regressions, confirming that a notorious short-term forecasting ability was indeed behind the previously discussed results.

They further decomposed the term spread:

$$i_t^k - i_t^1 = [f_{1t}^{k-1} - f_{1t}^1] + [f_{1t}^1 - i_t^1] + i_t^1$$

where the forward rate is defined:

$$f_{1t}^{k-1} = k i_t^k - (k-1) i_t^{k-1}$$

The first term, the *forward spread* is, under the expectations hypothesis, an unbiased predictor of the future nominal rate spread, so it should be expected to capture a good deal of the forecasting ability we have found for that variable. Under the expectations hypothesis, the second term is an unbiased forecast of the next change in the short-term rate. For the US, both terms are significant regressors to explain *cumulative* real growth, while the current short term rate becomes non-significant, confirming that it is the information in the longer end of the term structure which is relevant for forecasting future growth. Future *marginal* growth is, on the other hand, mostly affected by the forward spread and the current short-term rate. In this latter set of regressions, the predictability is greater for the longer horizons.

Lastly, Plosser and Routhenworst checked that neither past nor future money growth turned out to have explanatory power, additional to that in the slope of the term structure, to forecast real growth. This result means that the information in the slope is not just due to reactions to monetary policy, the short rate coming down under loose monetary policy, at the same time the longer rate barely reacts to it. Under that presumption, we would have induced a positive relation between the slope and future growth, so long as monetary expansions have also real effects. However, conditional on monetary variables, the slope would not have additional predictive power for future output growth.

2. A monetary, general equilibrium model of the term structure.

We consider an economy with a representative household, who owns the only firm in the economy. There is a Government, which spends some resources each period, financed through lump sum taxes, money creation and public debt issuing. Government expenditures, G_t , do not play any role in production, nor do they affect household's preferences. The household is made up of a financial intermediary, a worker, a shopper and a firm manager. The shopper, as well as the Government, must pay the consumption good with cash. Investment is a credit good for the firm. At the beginning of each period t , the household holds money, M_t , which is divided between the intermediary and the shopper. Then, the financial intermediary goes to the financial market, the shopper goes to the commodity market, the manager to the firm, and the worker to the labor market.

Financial markets open first, and the intermediary, as well as the Government establish their money demands. In addition, the financial intermediary decides the quantity of Government bonds she wishes. The Government decides at that point on its expenditures and financing mechanism, i.e., on how much to consume as well as on the distribution of its purchasing expenditures between tax collections, and net money and bond creation. Tax revenues are collected at this point.

After closing the financial markets, the labor market opens and production takes place. The firm manager hires some labor and produces output using labor, the stock of physical capital, and inventories as inputs. Afterwards, the market for the consumption/investment good opens and both, shopper and Government purchase consumption good using the money they acquired in the financial market. The firm retains some production to finance its investment and distributes the rest, as dividends, to the household. The commodity market closes for the day.

At the end of the session, the firm pays the worker for the labor he provided, and delivers the household the dividends obtained during the period. All markets are closed, so these funds are retained by the household until markets open next day.

Production: We consider a time to build technology of physical capital accumulation as in Kydland and Prescott(1982). Physical capital is subject to depreciation, and needs J periods to become productive:

$$\begin{aligned} k_{t+1} &= (1-\delta)k_t + S_{1,t} \\ S_{j,t+1} &= S_{j+1,t} \end{aligned} \quad (1)$$

A proportion ϕ_j of each project is paid for during the $j=1,2,\dots,J$ periods until it becomes productive, with $\phi_1+\dots+\phi_J = 1$. $S_{j,t}$ denotes the number of investment units which are, at time t , j periods away from completion, $j=1,2,\dots,J$, and δ is the rate of depreciation of productive capital. Choosing $S_{j,t}$ at time t , the firm is deciding the stock of capital k_{t+J} which will become productive at time $t+J$. The decision on k_t , the stock of capital which is productive at time t , was made at $t-J$ and before. Total investment, I_t is each period:

$$I_t = \sum_{j=1}^J \phi_j S_{j,t} + y_{t+1} - y_t \quad (2)$$

where y_{t+1} is the stock of inventories at the end of t . It is a production factor at time $t+1$.

The production technology for time t output, q_t is again as in Kydland and Prescott(1982):

$$q_t = F(\xi_{2,t}, k_t, n_t, y_t) = \xi_{2,t} n_t^\theta \left[(1-\sigma) k_t^{-\nu} + \sigma y_t^{-\nu} \right]^{-\frac{1-\theta}{\nu}} \quad (3)$$

where n_t denotes hours of employment, $\xi_{2,t}$ is a multiplicative shock in productivity that follows a stationary distribution with expectation one. The shape of the production function guarantees a positive demand for the three production inputs each period.

At each production point, the firm utilizes the stocks of inventories and physical capital accumulated from previous periods. It observes the realization of the random productivity shock, and decides how much labor to hire. When the realization of the shock is known to the firm, the stocks of physical capital and inventories are already given. Once output has been produced, the firm pays the labor factor, makes investment decisions, and distributes dividends D_t . Hence, the firm knows $\Omega_t = \{k_{t+J+1-s}, y_{t+1-s}, n_{t-s}, c_{t-s}, \xi_{2,t+1-s}\}$, $s \geq 1$, when it makes its decisions on labor, n_t , and investment, I_t .

This information scheme is in line with the stochastic structure assumed for the productivity shock in Kydland and Prescott(1982)¹. Our specification implies that the marginal rate of transformation between both types of capital at time t is already known at time $t-1$, since the shock ξ_{2t} , which appears in the productivity of both types of capital, disappears in their ratio:

$$MRT_t^{k,y} = \frac{MP_t^k}{MP_t^y} = \frac{(1-\sigma)}{\sigma} \left(\frac{k_t}{y_t} \right)^{-(1+\nu)} \quad (4)$$

The firm distributes output between salary payments, investment and dividends D_t :

$$\omega_t n_t + D_t + I_t = q_t \quad (5)$$

where all variables, including wages, ω_t , are in real terms, using output as numeraire. They do so to maximize the expected present value of current and future dividends that will be delivered to the household:

$$\begin{aligned} & \underset{\{D_t, n_t, k_{t+j}, y_{t+1}\}}{\text{Max}} & E_0 \sum_{t=0}^{\infty} (\beta^t U_t^c) D_t \\ & \text{given } & y_0, k_{-1}, s_{1,0}, s_{2,0}, \dots, s_{J-1,0} \end{aligned} \quad (6)$$

subject to (1), (2), (3) and (5). The firm discounts future profits using current information² and, in particular, the marginal utility of current consumption, in spite of the fact that dividends will not be used by the consumer until next period.

¹ Theirs is more complex. The productivity shock is split into several components which are sequentially observed by the firm. In that fashion, different decisions are made on the basis of distinct informational specifications, which allows for identifying investment on inventories apart from that on physical capital. We will see in section 5 that our assumption helps in the identification of our model as well.

² Notice that the ratio of two successive discount factors is the marginal rate of substitution of consumption over time. If we started from a generic discount factor μ_t , we would obtain that condition as part of the characterization of equilibrium. In some cases, it is assumed that the discount factor used for the firm incorporates the fact that time t dividends will be used in consumption at time $t+1$. For instance, Christiano (1991) use U_{t+1}^c/P_{t+1} to discount *nominal* time t dividends.

The optimality conditions are:

$$F_t^n = \omega_t \quad (7)$$

$$\phi_J \left(U_t^c - (1-\delta)\beta E_t U_{t+1}^c \right) + \dots + \phi_1 \beta^{J-1} E_t \left(U_{t+J-1}^c - \beta(1-\delta) U_{t+J}^c \right) = \beta^J E_t \left(F_{t+J}^k U_{t+J}^c \right) \quad (8)$$

$$U_t^c = \beta E_t \left((1+F_{t+1}^y) U_{t+1}^c \right) \quad (9)$$

together with the transversality conditions:

$$\lim_{\tau} \beta^{\tau+J} E_{\tau} \left(k_{\tau+J} U_{\tau+J}^c \right) = 0 \quad (10)$$

$$\lim_{\tau} \beta^{\tau+1} E_{\tau} \left(y_{\tau+1} U_{\tau+1}^c \right) = 0 \quad (11)$$

where superindeces indicate partial derivatives and E_t is the expectation conditional on the information set Ω_t .

Along the optimal path, labor is hired each period to the point where its marginal productivity is equal to the real wage. New investment projects are started so that the utility loss of devoting resources to finance all the projects under construction is equal to the expected future utility gain derived from the implied increase in output. Inventories are accumulated to the point where their future marginal product is expected to exactly compensate its owner for the current loss of utility. The transversality conditions select paths along which the expected current value of the terminal stocks of physical capital and inventories are each equal to zero.

The household: The household derives utility from the only consumption good, as well as from leisure. Total available time is normalized to one each period. The utility function is:

$$U(c_t, l_t) = \frac{1}{1-\gamma} \left[c_t^{\xi_t} l_t^{1-\xi_t} - 1 \right]^{1-\gamma} = \frac{1}{1-\gamma} \left[c_t^{\xi_t} (1-n_t)^{1-\xi_t} - 1 \right]^{1-\gamma} \quad (12)$$

$$E[\xi_{1t}] = \alpha \quad \forall t$$

where c_t , l_t , and n_t denote consumption, leisure and working time, respectively. It is a constant relative risk aversion utility function, as in Kydland and Prescott, although with time separability of leisure. It includes a shock ξ_{1t} that makes the marginal rate of substitution between consumption and leisure to randomly evolve over time:

$$\text{MRS}_t^{c,1-n} = \frac{\xi_{1t} (1-n_t)}{1-\xi_{1t} c_t}$$

α indicating the relative importance of consumption and leisure in the utility function.

The household can transfer resources over time buying nominal bonds, B_{t+j}^j , issued by the Government each period t with maturity horizons $j=1,2,\dots,J$. They offer to pay a nominal return i_t^j when they mature at time $t+j$, $j=1,2,\dots,J$. Yields i_t^j on time t bonds are known by investors when they are issued and bought. At time t there is a portfolio of bonds maturing, those issued at time $t-j$ with maturity j , $j=1,2,\dots,J$. The Government also imposes lump-sum taxes T_t on the household to finance its purchasing expenditures.

With our proposed chronological sequence of markets, the household owns at the beginning of each period: 1) a wide portfolio of nominal bonds with maturities at time t , $t+1,\dots, t+J-1$, purchased in previous periods, and 2) $M_t = P_{t-1}w_{t-1}n_{t-1} + P_{t-1}D_{t-1}$ monetary units which brings along as the result, at time $t-1$ prices, P_{t-1} , of the activities of the financial intermediary and the worker at time $t-1$: labor rents plus dividends. Financial markets open and the intermediary materializes his demand for money M_{t+1}^c and bonds, receiving the returns on maturing bonds and paying taxes:

$$\begin{aligned} & \frac{B_{t+1}^1}{P_t} + \dots + \frac{B_{t+1}^J}{P_t} + \frac{M_{t+1}^c}{P_t} - \left(\frac{M_t^c}{P_t} - c_{t-1} \frac{P_{t-1}}{P_t} \right) = \\ & \frac{P_{t-1} \omega_{t-1} n_{t-1} + P_{t-1} D_{t-1}}{P_t} + (1 + i_{t-1}^1) \frac{B_t^1}{P_t} + \dots + (1 + i_{t-1}^J) \frac{B_{t-(J-1)}^J}{P_t} - T_t \end{aligned} \quad (13)$$

After the financial markets close, the worker offers some of his time, output is produced, and the commodity market opens. There, the shopper faces a liquidity constraint that forces her to pay for consumption good with money:

where M_{t+1}^c is the quantity of money she brings from the money market, and P_t is the price of the consumption commodity. So long as nominal interest rates are positive, which will be

$$c_t \leq \frac{M_{t+1}^c}{P_t} \quad (14)$$

the case in equilibrium, this cash-in-advance constraint is satisfied with equality. At the end of the period, the worker receives salary payments and dividends are given to the household, the only owner of the firm.

The household chooses consumption and leisure each period to maximize the expected present value of current and future utility, discounted at rate β , $0 < \beta < 1$, on the basis of the information set $\{n_{t-s}, c_{t-s}, D_{t-s}, \xi_{1,t+1-s}, s \geq 1\}$ and subject to the budget constraint (13) and the cash-in-advance constraint (14):

$$\text{Max}_{[c_t, l_t, M_{t+1}^c, B_{t+1}^1, \dots, B_{t+1}^J]_0^\infty} \beta^t U(c_t, l_t)$$

given initial conditions: $M_0^c, B_0^1, \dots, B_{-(J-1)}^J$

taking as given: $i_t^1, \dots, i_t^J, r_t^1, \dots, r_t^J, T_t, P_t, \omega_t$

leading to the optimality conditions:

$$\frac{U_t^{1-n}}{P_t} = \beta \omega_t E_t \left(\frac{U_{t+1}^c}{P_{t+1}} \right) \quad (15)$$

$$\frac{U_t^c}{P_t} = \beta^j (1 + i_t^j) E_t \left(\frac{U_{t+j}^c}{P_{t+j}} \right), \quad j = 1, 2, \dots, J \quad (16)$$

and transversality conditions:

$$\lim_{\tau \rightarrow \infty} E_\tau \left(\beta^\tau U_\tau^c \frac{M_{\tau+1}}{P_\tau} \right) = 0 \quad (17)$$

$$\lim_{\tau \rightarrow \infty} E_\tau \left(\beta^\tau U_\tau^c \frac{B_{\tau+1}^j}{P_\tau} \right) = 0 \quad j=1, 2, \dots, J$$

Equation (15) has a clear interpretation: working one more hour today raises revenues by the nominal wage: $P_t \omega_t$, at the same time it decreases current utility by U_t^{1-n} . The

proceedings can be used tomorrow to purchase the consumption commodity. The expected increase in tomorrow's utility of an additional unit of currency is given by the conditional expectation in (15). In terms of current utility, we have to discount by β .

Combining (15) and (16) for $j=1$, the optimal consumption/leisure decisions by the household is characterized by:

$$U_t^{1-n} = \frac{\omega_t U_t^c}{1+i_t^1} \quad (18)$$

which can be seen as the supply of labor function. The worker supplies hours of work to the point where the marginal rate of substitution between current consumption and leisure becomes equal to real wages. Labor payments are discounted by the return on one period bonds because they cannot be spent until $t+1$. By definition, the demand for leisure is the complement to one of n_t : $l_t = 1-n_t$.

The quantities demanded of the bonds at different maturities, $b_t^1, b_t^2, \dots, b_t^J$ are not fully identified, since they are all substitutable assets³. Without loss of generality we treat in what follows the whole portfolio as a single one-period bond. That will not preclude us from determining the nominal and real returns of each individual bond, whose characteristics at the distinct horizons, $j=1,2,\dots,J$, will be different in an endogenous way.

The Government: The Government realizes a consumption G_t , which finances raising taxes, T_t , and issuing money and bonds:

$$G_t - T_t = \frac{M_{t+1} - M_t}{P_t} + \frac{B_{t+1}^1 - (1+i_{t-1}^1)B_t^1}{P_t} + \dots + \frac{B_{t+1}^J - (1+i_{t-J}^J)B_{t-(J-1)}^J}{P_t} \quad (20)$$

where $G_t, T_t, M_t, P_t, B_t^1, \dots, B_{t-(J-1)}^J, i_{t-1}^1, \dots, i_{t-J}^J$ represent public consumption at t , lump-sum taxes, money at the end of period $t-1$, time t prices, the volume of bonds issued in period $t-j$,

³ Supply conditions would be needed to characterize them. We can however obtain the total amount of resources devoted to purchasing Government bonds each period, but not its allocation among the different maturities.

$j=1,2,\dots,J$ all maturing at t , and their respective nominal rates of return⁴.

We assume the Government is successful at maintaining its planned expenditure policy, and also that this adopts a very simple form, being constant over time:

$$G_t = G \quad , \quad G > 0 \quad \text{given,} \quad \text{for all } t \quad (21)$$

We keep a simple public expenditure policy because we want to concentrate on the relevance of monetary policy, as well as random production and preferences, in explaining the main characteristics of the term structure.

Public consumption must be paid for with money, so that the government is subject to a cash-in-advance constraint similar to that of the consumer:

$$G = \frac{M_{t+1}^G}{P_t} \quad (22)$$

so that it needs to purchase money in the financial markets at time t .

On the other hand, we assume that the Government has a less than perfect control of the growth rate of money supply:

$$M_{t+1} = (g + \xi_{3t})M_t ; \quad E[\xi_{3t}] = 0 \quad (23)$$

which is subject each period to a random deviation from its target. In real terms:

$$(1 + \pi_t) m_t = (g + \xi_{3,t-1}) m_{t-1} \quad (24)$$

Fiscal policy is defined by a constant level of public consumption and a lump-sum tax each period, which varies over time as a function of the stock of debt, and an equation is needed that determines either the evolution of the stock of bonds, or the tax rule. In the simpler case when there are just one period bonds and the money supply is controlled with no error, we get:

⁴ With bonds paying interest all periods before maturity, this restriction gets more complicated, but our problem does not gain any generality, and results do not change.

$$b_{t+1} = \frac{1+i_t}{1+\pi_t} b_t + \frac{1}{1+\pi_t} (G-T_t) - (g-1) \frac{m_t}{1+\pi_t} \quad (25)$$

where $b_t = B_t/P_t$ and we have used the fact that public consumption is paid for with Government money holdings. A well-known long-run equilibrium condition in the deterministic case which is expected to also hold here is that the gross real rate of interest $(1+r)$ be equal to the inverse of the discount parameter β , and being this less than one, the previous is an explosive first order autoregression in b_t , although the presence of taxes and money growth will tend to stabilize it⁵. To avoid this lack of stationarity, and ignoring the stabilizing effect of money growth, we can fix a tax schedule which responds to the stock of bonds:

$$T_t = T + a b_t \quad (26)$$

where lump-sum taxes have each period a constant component, T , plus a component that depends on the stock of bonds through a . A relationship of this kind guarantees stability of the system⁶ [see Leeper(1991) and Sims(1994)].

To summarize, the Government starts the period by deciding on public consumption, taxes and the amount (positive or negative) of money and bonds that wishes to put in circulation. When financial markets open, the government buys money, pays the consumer/investor the return on the maturing bonds, puts in circulation new money and bonds, and collects taxes. After that, the market for the consumption good opens, and the Government uses its money holdings to purchase the desired commodity units.

⁵ Monetary contractions would tend to destabilize the time evolution of b_t .

⁶ Substituting into (25):

$$b_{t+1} = \frac{1+i_t - a^T}{1+\pi_t} b_t$$

it is easy to see that so long as a^T falls inside the open intervals $(i_t - \pi_t, 1+i_t)$ and $(1+i_t, 2+i_t+\pi_t)$, the resulting autoregressive process will be stationary.

Equilibrium: Given parameter values, including public expenditures G and money growth g , and paths for taxes T_t and bonds $B_{t+1}^1, \dots, B_{t+1}^J$, a *competitive equilibrium* is a set of initial conditions: $P_0, M_0^c, M_0^G, y_0, k_0, s_{1,0}, s_{2,0}, \dots, s_{J-1,0}$ together with real functions defined on $(0, \infty)$: $\{c_t, l_t, n_t^d, k_{t+J}, y_{t+1}, D_t, M_{t+1}^c, M_{t+1}^G, M_{t+1}, i_t^1, \dots, i_t^J, P_t, \omega_t\}$ such that:

- i) given $i_t^1, \dots, i_t^J, P_t, \omega_t, T_t$, and the initial conditions $M_0, B_0^1, \dots, B_{-(J-1)}^J$, the vector of functions $\{c_t, l_t, M_{t+1}^c, B_{t+1}^1, \dots, B_{t+1}^J\}$ solves the utility maximization problem of the consumer,
- ii) given $y_0, k_0, s_{1,0}, s_{2,0}, \dots, s_{J-1,0}, P_t, \omega_t$, the functions: $\{n_t^d, k_{t+J}, y_{t+1}, D_t\}$ solve the maximization problem of the firm,
- iii) $1-l_t = n_t^d$ for all t , which determines equilibrium in the labor market,
- iv) $M_t^c + M_t^G = M_t$ for all t , the aggregate money demand by consumer and Government is equal to the money supply,
- v) the household purchases all bonds of different maturities $j=1,2,\dots,J$, issued by the Government,
- vi) the budget and cash-in-advance constraints (20) and (22) for the Government are satisfied at all periods.

Equilibrium in the labor market implies that the marginal rate of substitution between current consumption and leisure is equal to the marginal product of labor, normalized by short-term nominal rates:

$$F_t^n = RMS_t^{1-n,c} (1 + i_t^1) \quad (27)$$

On the other hand, it is easy to see that Walras' law guarantees that equilibrium holds in the consumption commodity market: First, the budget constraints of household and Government, can be combined into:

$$\frac{M_{t+1}^c}{P_t} + G - \frac{M_{t+1} - M_t}{P_t} = \frac{P_{t-1} \omega_{t-1} n_{t-1} + P_{t-1} D_{t-1}}{P_t} \quad (28)$$

But, since the Government is subject to a cash-in-advance constraint, and the money market is in equilibrium, i.e., $M_t^c + M_t^G = M_t$, then (28) implies:

$$M_t = P_{t-1} \omega_{t-1} n_{t-1} + P_{t-1} D_{t-1} \quad (29)$$

so that the financial flow that the consumer receives at the end of $t-1$ is equal to the total money supply. If we write (29) at time $t+1$, divide through by P_t , and substitute the money market equilibrium condition $iv)$ in the aggregate cash-in-advance constraints of both agents:

$$c_t + G = (g + \xi_{3t}) m_t \quad (30)$$

we get:

$$c_t + G_t = \omega_t n_t + D_t \quad (31)$$

The right hand term in (31) is equal, from the firm's constraint (5), to production, net of investment. That way, we get:

$$c_t + G_t + I_t = F(\xi_{2t}, n_t, K_t, y_t) \quad (32)$$

which implies equilibrium in the market for the consumption commodity, where output is split among private and public consumption, plus investment.

3. Equilibrium analysis

Properties of this model, as well as a full discussion of the technical details needed for its solution, are in Domínguez and Novales (1996). The main difficulty in its analysis resides in the presence of conditional expectations in a nonlinear set of optimality conditions. Competitive equilibrium is characterized as the solution to that set of optimality conditions, together with the laws of motion of state variables, budget constraints for the different agents, monetary and fiscal policy rules, and transversality conditions. With any ad-hoc assumption on expectations, these can be eliminated from the optimality conditions, leaving a set of equilibrium conditions, in the form of a nonlinear system of difference equations in state and decision variables, that could easily be solved on the computer. The solution would be numerical, i.e., in the form of equilibrium time series that would start from a set of initial conditions for the state variables, and that would satisfy all the equilibrium conditions at all time periods.

Solving this type of model under rationality of expectations is much harder, since expectations formulae must be totally consistent with the structure of the whole model. Different approaches have been recently proposed to solve such a class of models: some solve the linear-quadratic model that approximates the theoretical model best. Some others [denHaan and Marcet, 1990] parametrize the expectations of the non-linear expressions that appear in the equilibrium conditions, as an approximation to the true expectations formulae under rationality. A third type of approach, that we follow [Sims, 1989] solves the exact model, without approximating the expectations formulae, either. However, not any numerical solution obtained with this approach is admissible. More to the contrary, almost any solution is explosive, unless some additional stability solutions are imposed. These are found as a standard analysis: they are the eigenvectors corresponding to the unstable eigenvalues of the linear approximation to our system of first order difference equations.

When finding the eigenvalues of the linear approximation, care must be taken to appropriately lag each equation, so that contemporaneous decision variables appear in the full vector of decision, state variables and shocks. The number of stability conditions in the model in section 2 is equal to J , the number of periods needed build productive capital, plus one. In our simulations we take $J = 4$, so that we have five such conditions. They turned out to involve just real sector variables. Variables from the monetary sector: bonds, nominal interest rates, inflation, or financial variables like public expenditures and taxes do not appear in them. Had we not imposed a condition like (26), we would have an additional stability condition, linking the stock of real bonds to state and decision variables. Introducing in the model an 'active' public expenditure policy, that would peg public consumption to output deviations from steady state, for instance, would not alter the number of stability conditions.

Optimality conditions are then written replacing rational expectations of non-linear functions by their realized value plus an expectation error. That involves no approximation, but increases the number of variables to solve for. The idea is that we have enough stability conditions so that starting from realizations for the structural shocks: preferences, technology or money control, an initial conditions on state variables, we can solve the system to obtain equilibrium time series realizations of the whole set of variables, including the expectations errors. In general, one can also take the initial conditions, together with sample realizations for the expectations errors, and obtain equilibrium time series, as well as realizations for the

exogenous shocks. The first alternative, known as 'forward' solving, does not guarantee that the resulting expectations errors will be white noise, which must be, since they are rational expectations errors. The second alternative, 'backwards' solution, might lead to stochastic structures for the structural perturbations very different from the intuition that they are smooth and with a fair deal of permanence.

There are expectations errors of a given function at different horizons. That is the case for the marginal utility of consumption, as well as for its deflated value. Under rationality, each of these two sets must have a MA structure, which we impose, no matter whether we solve forwards or backwards. Finally, it is easy to show that the one-step ahead expectation error in the cross product of the marginal utility of consumption times the marginal productivity of capital, is proportional to that in the product of the marginal utility and the marginal productivity of inventories. That way, we are left with just three one-step ahead independent expectation errors: the forecast error in the marginal utility of consumption, that in its deflated value, and the error in either one of the mentioned cross products. Their number corresponds to the three structural errors. The rest of the expectations errors can be obtained through imposed MA processes. The theoretical model is silent as to the parameter values for those processes.

Bonds and taxes play a residual role in this process. They are unrelated to the rest of variables, and can be solved for from the Government budget constraint (25) and the fiscal rule (26). These equations are not used to analyse stability. As pointed out in Domínguez and Novales (1996), that shows that the Ricardian proposition holds in this economy.

4. Empirical results

In this section, we report on the empirical facts that our asset pricing model is able to reproduce, relative to the ability of the term structure to forecast economic growth. To that extent, we simulate the model to obtain equilibrium realizations under the alternative assumptions that the economy is subject to one or several of the three shocks: on preferences, productivity, or on the control of money growth. That way, we can not only test for the possibility of the effects being significant, but also check on which of the shocks is better able to explain a given empirical observation. We take innovation standard deviations for the

shocks in preferences and productivity equal to 10^{-3} , and AR(1) coefficients equal to 0.9 in both cases. So, both shocks share the same stochastic structure, having the same variance⁷. When used on their own, they produce the same coefficient of variation in output, so that results are comparable. Those variances are then kept unchanged for experiments in which they enter together with other shocks.

The money control shock was described as being white noise in the model, and we take its variance to be equal to that of the 'real' perturbations, in preferences and productivity. Parameter values, as well as a description of the six experiments to which we will be referring, are shown in Table 1. The discount factor is taken to be 0.99, so that we will interpret our results as reflecting quarterly data. The depreciation rate is 2.5% a quarter, close to an annual rate of 10%. Labor takes a share of $2/3$ of output, while the aggregate of physical capital and inventories taking the remaining $1/3$ rd. We suppose that a full year is needed for investment projects to become productive, and also that 25% of the value of a project is financed each of the four quarters needed for completion. The 'elasticity' of consumption in the utility function is $1/3$, being $2/3$ the 'elasticity' of leisure. The coefficient of risk aversion is taken to be 1.5. The two shocks in preferences and productivity are persistent AR(1) processes, with coefficients 0.90 each. Standard deviations in the innovations in both processes are 10^{-3} , while that of the money control error is 10^{-2} .

4.a The term structure slope as a predictor of future growth

Estrella and Hardouvelis (1992) (EH) and Plosser and Rouwenhorst (1994) (PR) have documented a very significant predictive power in the nominal spread over future economic activity in actual economies. EH considered the spread between the annualized returns on 10-year US Government bonds and 3-month T-bills, and quarterly GNP in the US as the measure of economic activity. PR considered monthly data on industrial production as the economic activity variable, and monthly data on one to five-year spot interest rates for the UK, US and

⁷ Coefficients of variation are different, however, since the two shocks have different mean. Their volatility is not the same, in that important respect.

Germany, reproducing the results in EH⁸. They also found in the nominal spread a good deal of predictive ability for future consumption.

EH found significant explanatory power for the slope in these ‘cumulative growth’ regressions, with slope coefficients starting above one and essentially decreasing, together with the explanatory power of the regression, with the forecast horizon. The explanatory power, measured by R-squared statistics, seems to peak up at about 6/7 quarters into the future. They pointed out the interest of this result, specially because lagged GNP growth rates were of no help in forecasting future growth. PR also found substantial explanatory power, but with some differences across countries: a) estimated slope coefficients were below one in countries other than the US, b) there is also substantial ability in the term slope to forecast future consumption growth, and c) the ability to forecast nominal output growth is much lower than for real output growth, except in the UK, which the authors justify on the basis of being the country with the more volatile inflation rate⁹.

Tables 2.a to 2.f present the forecasting ability in the term structure implied by our model, for future economic growth, in all experiments. To that extent, we estimated regressions from cumulative growth rates, i.e., growth between t and $t+k$, on the current spread. Our definition of cumulative, annualized output growth rates: $y(t,k) = (400/k) * [\ln(y_{t+k}) - \ln(y_t)]$ is the same as in PR and EH, but we work with shorter maturities and shorter forecast horizons than these authors. We use the spread between one-year and one-quarter returns, to forecast growth over an interval ranging from 1-quarter to two years. We use the long-short rate spread, with maturities unchanged for different forecast horizons. That is consistent with table 1 in PR and tables 1 and 3 in EH, with which our results are comparable, except for using shorter forecast horizons and spreads. Other results in PR use spreads that cover the same time period over which they forecast output growth.

Our results show substantial forecasting power in the term structure, which is high initially, and then decreases when we try to predict growth over longer horizons. The slope

⁸ We refer in what follows to the only three countries for which PR perform the complete analysis.

⁹ There is a non-trivial choice relative to using averages versus point-in-time interest rates, which is discussed by PR. We cannot address this issue here, for which we would need a continuous time version of our model.

coefficient is estimated above one for the one quarter ahead forecasting exercise, except in one case, and monotonically decreases for longer horizons. Spread coefficients are positive, suggesting that an increase in the spread is followed by higher than average future economic growth. That is, in turn, consistent with the view that the slope of the term structure is high during recessions and low around business cycle peaks. Our estimated coefficients are below those estimated by EH and PR from actual data, so the changes that should occur in the short-long-term spread to suggest a 1% increase in the growth rate of future output are bigger. In our model, an increase of 100 basis points in the spread produces an increase between 0,78% and 1,76% in the annualized rate of growth of output next quarter. If we look at the one-year horizon, we have that a similar increase in the spread produces an increase between 0,08% and 0,36% in the annualized rate of growth over the next year. In spite of getting lower slope estimates in the regression, they are significant in all experiments. Median adjusted R-square values at 1-quarter is 23%, but falls to 7% and 9% at one and 2-years¹⁰. The estimated stimulus on production of a steeper term structure spreads over time, suggesting that it essentially materializes in the very near future.

Due to the overlapping nature of our exercise, the residuals in the regression will have autocorrelation up to 1, 2 or 3 lags, depending on whether we forecast 2, 3, or 4 quarters ahead. We used the Newey-West(1987) correction to compute consistent standard errors, which are provided in the table. There was some evidence of first-order autocorrelation in some experiments, but stronger evidence of fourth-order autocorrelation. Autocorrelation becomes more important as we move to longer forecast horizons, as the theoretical model predicts. There is substantial evidence of ARCH(1) structure in the residuals, specially for longer horizons.

Following EH and PR, we now ask a few questions, relevant to characterize the type of information embedded in the term structure:

- 1) is the predictive power mostly due to the ability to forecast over short horizons or is it quite uniformly distributed over the future?
- 2) does the predictive content in the term structure reflect information other than current and expectations of future monetary policy? There are two ways to test for that:

¹⁰ These are higher initially, but lower at longer horizons, than in EH.

- 2.1) check whether the predictive content in the term structure for long-term growth stems just from movements in the short rate or rather, there are additional sources. From the discussion in the introduction, if monetary policy interventions are the reason for the predictive power we have detected, then short-term rates might be all we need in our forecasting exercise.
- 2.2) money growth rates can be tried as an additional explanatory variable, to test whether they capture the forecasting power found for the term structure slope.

To answer the first question, we ran regressions of the ‘marginal growth rate’ of output, meaning by that the growth rate associated to each individual future quarter. Our definitions of both rates, indicated in the tables, are the same as in EH, but different from PR, since they skip the first year of output growth, so numerical results are comparable to the first and not to the second, even though the same qualitative issues are discussed in both cases. Our forecast horizon is much shorter in both cases. EH found the ability of the slope to forecast ‘marginal’ growth rates to be substantially lower than that to forecast ‘cumulative’ growth rates. Estimated coefficients were lower and turned out to be nonsignificant earlier on. When trying to forecast over longer horizons, PR found that the slope had minor importance concluding, as EH did, that the set of results we have discussed were reflections of the ability of the term structure slope to forecast real growth *over the near future*. Both pieces of work are consistent in this respect, and so are our theoretical results.

4.b Monetary policy and the term structure as a predictor of future economic growth.

To discuss the second issue, about whether current or expected future monetary policy is behind these empirical results, EH as well as PR, added short-term rates to the slope as an explanatory variable in the regressions for output growth rates. PR found that the slope contained, in fact, relevant information which was not already contained in current short-term rates. Even though short rates have in actual data a significant, negative association with future growth when they are used by themselves in a regression, they tend to lose it when the spread is included. These coefficients decrease with the forecast horizon, but remain significant. Their estimated values are almost unrelated to whether or not current short-term rates are included

as an additional regressor, which suggest that both elements contain information which is mostly orthogonal to each other.

EH report results when the real federal funds rate is included as the second explanatory variable in the regression, but claim that very similar results arise when the nominal federal funds rate, or the nominal 3-month T-bill rate is used. They also find that the current real rate is negatively associated to future economic growth. Higher real rates would bring down current investment and hence, future output. More important, they find that the slope continues to have cumulative predictive power for real growth.

Tables 3.a to 3.f show that the short term nominal rate is, by itself, a significant factor explaining future growth rates, with a negative coefficient that decreases over the horizon, but remains significant even for predictions over a two year period. It suggests that the forecasting power in it is essentially over the near future. Estimates for next quarter predictions oscillate between -0.95 and -1.22 , and median R-square values were around 20% for 1-quarter, and 1 and 2-year ahead forecasts. When the slope of the term structure is also included in the regressions, the coefficient in the short-term rate decreases in size, and becomes non-significant when the economy is subject just to real shocks, i.e., shocks on preferences or/and productivity. When there is a monetary shock, the short-term rate retains significant explanatory power, the same result found by EH for the US. PR found that it was just for the UK, the country with more unstable rate of inflation, that short-term rates retained predictive power. That is consistent with our results: monetary shocks make inflation more unstable, obscuring the predictive power in the slope, which then has information both, on future growth and also on expectations of future inflation. There is, in that case, residual information in short-term rates.

When there are monetary shocks, the coefficient in the spread is reduced, relative to the case when it was used as the only explanatory variable, meaning that there is in those cases some information in common between the slope of the term structure and the short-term rate. The opposite arises with shocks on preferences, that give an important amount of information to the spread. It does not even decrease when short-term rates are added to the model.

At one or two-year horizons, the spread becomes non-significant, and takes a negative sign, suggesting that, at those horizons, it is the short-term rate the one that has power,

according to the coefficients we have already reported.

PR found that neither past, nor future money growth, can substitute for term structure variables when trying to forecast future economic growth. They only detected some explanatory power for future money growth rates in the UK, and attributed that to the view that nominal shocks had dominated in that country over their sample period. They found the coefficient in that variable to be fairly stable when moving to longer forecast horizons. Coefficients in the slope for other countries were essentially unaffected by the introduction of the monetary variables in the regression model. They concluded that the information in the term structure seems to be independent of that in the current or future course of monetary policy, specially in those countries with more stable inflation

Our Tables 3.a, 3.e and 3.f show estimates for regressions of future output growth on the current term structure slope, money growth over the last year, and realized monetary growth over the same time horizon as for output growth. In our model, past money growth is irrelevant to forecast future real growth, as it is the case for all countries considered by PR (1994). Percent rejections of the null of lack of significance (under column 'f2' in Tables 2.a, 2.e and 2.f) are very low for past money growth, at a difference of those for future money growth (under 'f1'), which are very large. Future money growth has some explanatory power just over the near future, specially when there is also a shock in preferences. Information in future money growth seems to be additional to that incorporated into the term structure slope, since coefficients in the latter are very similar to those obtained in Tables 2.a to 2.f. Coefficients in money growth are very stable as we enlarge the forecast horizon. These two statistical observations are again analogous to the results found by PR for the UK, the only country in their sample for which monetary variables have some relevance in forecasting future growth.

4.c Term spread decompositions

The term spread was decomposed by PR into the sum of: a) the forward spread, b) the difference between current short term rates and the current one-period hence forward rate, and c) the current short-term interest rate, as indicated in Section 2. The first term is a predictor of the future spread, while the second term is a predictor of next period short rates. The idea

of PR when proposing this decomposition was that the explanatory power found for the current slope might have its reflection on a high explanatory power for the forward spread.

In all experiments, short-term rates retain in this set of regressions the predictive power they had when were used together with the spread. It seems to be the case that the forward spread is correlated with the forward premium, most likely because of the short spread we consider, much shorter than in PR. Part of the predictive power of the slope goes to the forward premium that sometimes, even becomes significant. The most important result is that short-term rates had the same explanatory power as in Table 2, turning out to be significant in economies under monetary shocks.

Tables 4.a to 4.f show that in experiment 1, with just the shock in preferences, the short-term rate maintains its explanatory power for future growth. The estimated coefficient is, in all experiments and horizons, the same as in Tables 2.a to 2.f, and the same happens with the R-squared statistics. That means that the predictive power of the term structure slope decomposes into that of the forward spread and the forward premium. These two variables seem to be highly correlated, so that their respective coefficients are estimated without much precision¹¹.

In short, the term structure in nominal rates, summarized by its slope, has in our model a good deal of forecasting power for future output growth. Short-term nominal rate contain additional predictive power except when the economy experiences just shocks in preferences. When short term rates contain information on future growth, the explanatory power in the slope decreases, meaning that there is some information in common between both variables. In addition, when we add monetary shocks to the model, short-term rates capture some more predictive power, at the same time that the slope loses some. Monetary errors tend to unstabilize inflation, which introduces noise in term structure fluctuations. According to this, in economies with more unstable inflation, we should expect short-term rates to be more important, relative to the slope of the term structure, to predict future growth. That seems to be the case in actual economies [see Plosser and Rouwenhorst (1994)].

¹¹ PR report that the slope in the forward term structure captures much of the predictive power in the spread in nominal rates, but they refer to a different definition of the forward spread, corresponding to time periods wich move with the forecast horizon.

5. Conclusions

This paper examines whether the general equilibrium asset pricing model with real and nominal assets of different maturities is able to reproduce the important forecasting power that has been found in empirical work for the term structure of nominal interest rates. Equilibrium real rates are equal to the marginal rates of substitution between current consumption, and consumption at maturity time, which can be approximated by consumption growth. Output and consumption are highly correlated in most specifications, so that in equilibrium, current real rates can be expected to be correlated with future output growth. It is not at all clear, however, that nominal rates will contain any predictive power for future growth, since they also contain, in addition to possible risk premiums, expectations of future inflation. In a frictionless monetary model, prices and money will move closely together, so that most of the fluctuations in nominal rates will just reflect changes in expectations of future inflation, with almost no information on future output growth.

With leisure in the utility function, the association between real rates and consumption growth becomes less clear, and that between nominal rates and output growth becomes less evident on a priori grounds.

We have considered a monetary economy, with a time-to-build capital accumulation technology, which implies that at each point in time, the consumer/investor chooses a portfolio of real assets with different ex-post rates of return. The Government materializes some consumption, which finances levying lump-sum taxes and issuing bonds. We hence have real and nominal assets at different maturities. Both, consumers and Government are subject to cash-in-advance constraints when purchasing the consumption good. We make the assumption usually accompanying such a constraint, that the worker cannot spend his salary income in consumption until next period. That introduces an inefficiency and pegs the short-term nominal rate to real variables, like consumption, capital and labor. Longer horizon real rates are equal to current expectations of the marginal product of the investment projects that become productive at the corresponding horizon. Long nominal rates are equal to current expectations of real rates, correspondingly deflated.

Our model can reproduce most of the qualitative empirical features documented in Estrella and Hardouvelis (1991) and Plosser and Rouwenhorst (1994), although with some minor differences:

- 1) the information in the term structure of nominal rates, summarized by its slope, contains significant predictive power for future output growth. A steep term structure is followed by output growth higher than average, and the opposite follows for relatively flat term structures,
- 2) the short-term nominal rate is, by itself, in all our examples, a good predictor of future output growth. Low short-term interest rates are followed by output growth above average, and the opposite happens for high rates,
- 3) however, its explanatory power is taken away by the slope in economies subject to real shocks when predicting over the near future. The information in the slope is additional to that contained in the short-term interest rate, specially in economies dominated by real shocks. In them, short-term rates contain significant information just as proxies of the term structure slope, and become non-significant when considered together with the slope. In economies that experience both, real and monetary shocks, there is some information left in the short-term rate, additional to that in the slope. At a difference of the empirical facts, however, the information content in the slope is short lived, so that for longer horizons, the short-term rate keeps its predictive power, while the slope becomes uninformative,
- 4) monetary shocks produce a more unstable inflation, and more volatile inflation expectations tend to obscure the power in the term structure for forecasting real growth. Accordingly, we find the slope of the term structure to be a much better predictor of future output growth rates when the economy is subject just to real shocks than when monetary shocks are also present,
- 5) past money growth rates never have predictive power additional to that in the slope. Having already mentioned that the information in the slope is additional to that in short-term rates, we can conclude that the information in the slope is not just the product of current or past monetary actions. We find predictive power for future money growth rates, in consistency with the empirical observation that in countries experiencing a more volatile inflation, the term structure does not predict output nearly

as well, and money growth contains relevant information.

The forecasting ability in the term structure slope in the model is, however, over the short-term range, and more research is needed to provide with structural facts that may allow to extend that information content in time.

REFERENCES

- Den Haan, W.J. & A. Marcet, 1990, Solving the stochastic growth model by parameterizing expectations, *Journal of Business & Economic Statistics*, 8, 31-34.
- Domínguez, E.J. & A. Novales, 1996, Time varying risk premia in general equilibrium with production, Documento de Trabajo No 9601, ICAE.
- Estrella, A. and G.A. Hardouvelis, 1991, The term structure as a predictor of real economic activity, *Journal of Finance*, 46, 555-576.
- Fama, E.F. 1986, Term premiums and default premiums in money markets, *Journal of Financial Economics*, 12, 529-546.
- Harvey C.R., 1989, The real term structure and consumption growth, *Journal of Financial Economics*, 22, 305-333.
- Kessel, R.A., 1965, The cyclical behavior of the term structure of interest rates, New York: NBER Occasional Paper No 91.
- King, R.G., C.I. Plosser & S.T. Rebelo, 1988, Production, growth and business cycles, *Journal of Monetary Economics*, 21, 309-341.
- Kydland, F.E. & E.C. Prescott. 1982, Time to build and aggregate fluctuations. *Econometrica* 50(6): 1345–1371.
- Laurent, R.D., 1988, An interest rate-based indicator of monetary policy, Federal Reserve Bank of Chicago, *Economic Perspectives*, 12, 3-14.
- Leeper, E.M., 1991, Equilibria under "active" and "passive" monetary and fiscal policies, *Journal of Monetary Economics*, 27, 1129-147.
- Newey, W. & K. West, 1987, A simple, positive definite, heteroskedasticity and autocorrelation consistent covariance matrix, *Econometrica*, 55, 703-708.
- Plosser, C. and K.G.Rouwenhorst, 1994, International term structures and real economic growth, *Journal of Monetary Economics*, 33, 133-155.
- Sims, C.A. 1984, Solving nonlinear stochastic equilibrium models "backwards". Unpublished manuscript. University of Minnesota. Dept. of Economics.
- Sims, C.A. 1989, Solving nonlinear stochastic optimizations and equilibrium problems backwards. Discussion Paper 15. Institute for Empirical Macroeconomics, Federal Reserve Bank, Minneapolis.
- Sims, C.A. 1990, Solving the stochastic growth model by backsolving with a particular nonlinear form for the decision rule. *Journal of Business and Economic Statistics* 8: 45–47.

Table 1	
Parameter values	
Rate of time preference:	$\beta = 0.99$
Depreciation rate:	$\delta = 0.025$
Labor elasticity in production:	0.64
Elasticity of the composed input: physical capital+inventories:	0.36
Weight of physical capital, relative to inventories:	2.57
Number of periods to complete an investment project:	4
Proportions of investment projects financed each period:	$\varphi_1 = \varphi_2 = \varphi_3 = \varphi_4 = 0.25$
Average consumption 'elasticity' in utility function:	1/3
Average 'elasticity' of leisure in utility function:	2/3
Coefficient of relative risk aversion:	1.50
MA parameters in expectation errors in nominal rates:	$\Theta_i = 0.9^i$, $i=1,2,3,4$
AR(1) parameter in technology shock:	0.90
AR(1) parameter in preferences:	0.90
Standard deviation of innovation in technology shock:	0.001
Standard deviation of innovation in preferences:	0.001
Standard deviation of innovation in money growth rate:	0.01
Annual money growth:	0.05
Relation: lump sum taxes / bonds	0.20

Note: Experiment *a* contains three shocks: on preferences, productivity, and money growth. Experiment *b* contains just a shock in preferences and experiment *c* contains just a shock in productivity, while experiment *d* combines both shocks. Experiments *e* and *f* correspond to *b* and *d*, adding a money control shock. The table summarizes the results from 100 regressions with simulated equilibrium series, using the parameter values in Table 1.

Table 2.a
Term spreads as predictors of future real activity

$y(t,k) = \alpha_0 + \alpha_1 (i_t^4 - i_t^1) + \alpha_2 i_t^1 + \varepsilon_{t+k}$											
k	α_1	(s.d.)	α_2	(s.d.)	R^2_{aj}	σ	Q(1)	Q(4)	A(1)	f1	f2
1	1.17	(0.26)			0.22	4.28	2.66	10.48	1.0	0.99	
			-1.06	(0.27)	0.20	4.31	2.10	11.06	0.97		0.99
	0.70	(0.34)	-0.58	(0.32)	0.24	4.22	2.05	10.51	0.95	0.43	0.35
2	0.61	(0.15)			0.19	2.45	6.86	13.19	1.33	0.96	
			-0.57	(0.13)	0.18	2.46	8.23	16.24	1.36		0.98
	0.34	(0.24)	-0.33	(0.20)	0.21	2.41	6.77	13.53	1.19	0.28	0.28
3	0.48	(0.11)			0.21	1.81	15.94	25.02	3.41	0.95	
			-0.45	(0.10)	0.20	1.81	12.71	20.83	2.19		0.98
	0.27	(0.19)	-0.26	(0.16)	0.24	1.77	13.81	22.05	2.48	0.31	0.29
4	0.19	(0.07)			0.06	1.46	24.73	39.74	6.18	0.56	
			-0.28	(0.07)	0.14	1.39	26.74	41.42	6.67		0.92
	-0.09	(0.15)	-0.34	(0.15)	0.15	1.39	27.15	41.97	6.85	0.08	0.68
8	0.13	(0.05)			0.08	0.92	40.20	88.26	14.67	0.71	
			-0.19	(0.05)	0.18	0.87	37.37	78.28	12.86		0.93
	-0.06	(0.11)	-0.24	(0.11)	0.20	0.86	36.25	74.45	12.37	0.13	0.66

Table 3.a
Term spreads, real activity and monetary growth

$y(t,k) = \alpha_0 + \alpha_1 (i_t^4 - i_t^1) + \alpha_2 m(t-4,1) + \alpha_3 m(t,k) + \varepsilon_{t+k}$														
k	α_1	(s.d.)	α_2	(s.d.)	α_3	(s.d.)	R^2_{aj}	σ	Q(1)	Q(4)	A(1)	f1	f2	f3
1	1.39	(0.28)	0.051	(0.045)	0.052	(0.020)	0.28	4.09	2.37	11.76	0.82	0.99	0.07	0.71
2	0.75	(0.15)	0.025	(0.032)	0.058	(0.019)	0.27	2.32	3.43	10.22	0.78	0.99	0.11	0.80
3	0.57	(0.10)	0.016	(0.027)	0.060	(0.021)	0.30	1.69	11.99	19.52	2.30	1.00	0.13	0.80
4	0.23	(0.07)	0.006	(0.021)	0.048	(0.026)	0.16	1.24	22.48	36.87	5.80	0.89	0.06	0.48
8	0.15	(0.05)	0.001	(0.017)	0.043	(0.035)	0.16	0.88	35.35	75.70	12.30	0.87	0.10	0.51

Table 4.a
Components of the term spreads and real activity

$y(t,k) = \alpha_0 + \alpha_1 (f_t^{3,4} - f_t^{1,2}) + \alpha_2 (f_t^{1,2} - i_t^1) + \alpha_3 i_t^1 + \varepsilon_{t+k}$														
k	α_1	(s.d.)	α_2	(s.d.)	α_3	(s.d.)	R^2_{aj}	σ	Q(1)	Q(4)	A(1)	f1	f2	f3
2	0.28	(0.22)	0.25	(0.19)	-0.33	(0.20)	0.21	2.41	7.10	13.74	1.10	0.22	0.28	0.30
3	0.25	(0.18)	0.19	(0.14)	-0.26	(0.16)	0.24	1.77	14.13	23.25	2.40	0.32	0.27	0.30
4	-0.07	(0.14)	-0.07	(0.11)	-0.34	(0.15)	0.15	0.65	27.34	42.04	6.98	0.09	0.07	0.68
8	-0.04	(0.09)	-0.04	(0.08)	-0.24	(0.11)	0.19	0.86	36.48	74.69	12.37	0.13	0.15	0.66

Table 2.b
Term spreads as predictors of future real activity

$y(t,k) = \alpha_0 + \alpha_1 (i_t^4 - i_t^1) + \alpha_2 i_t^1 + \varepsilon_{t+k}$											
k	α_1	(s.d.)	α_2	(s.d.)	R_{aj}^2	σ	Q(1)	Q(4)	A(1)	f1	f2
1	1.76	(0.19)			0.43	2.55	3.27	18.45	1.33	1.00	
			-1.22	(0.21)	0.28	2.85	0.14	6.09	1.12		1.00
	2.07	(0.36)	0.29	(0.26)	0.49	2.55	5.10	22.86	1.39	0.99	0.06
2	0.96	(0.13)			0.42	1.40	1.61	5.01	1.43	1.00	
			-0.65	(0.10)	0.27	1.57	9.82	16.85	2.12		1.00
	1.16	(0.22)	0.20	(0.14)	0.42	1.40	1.27	6.81	1.00	1.00	0.05
3	0.77	(0.06)			0.49	0.97	29.95	39.23	8.73	1.00	
			-0.52	(0.07)	0.32	1.13	32.09	53.51	10.76		1.00
	0.95	(0.13)	0.18	(0.10)	0.50	0.96	29.38	35.50	8.34	1.00	0.04
4	0.36	(0.07)			0.15	1.06	12.65	22.71	2.88	1.00	
			-0.35	(0.06)	0.20	1.03	20.27	38.12	5.56		1.00
	-0.04	(0.20)	-0.40	(0.18)	0.20	1.03	21.42	39.79	6.06	0.04	0.49
8	0.24	(0.04)			0.18	0.63	36.01	92.63	13.12	1.00	
			-0.24	(0.04)	0.28	0.59	37.63	103.01	14.09		1.00
	-0.09	(0.12)	-0.32	(0.11)	0.29	0.59	39.99	108.07	15.06	0.13	0.80

Table 4.b
Components of the term spreads and real activity

$y(t,k) = \alpha_0 + \alpha_1 (f_t^{3,4} - f_t^{1,2}) + \alpha_2 (f_t^{1,2} - i_t^1) + \alpha_3 i_t^1 + \varepsilon_{t+k}$														
k	α_1	(s.d.)	α_2	(s.d.)	α_3	(s.d.)	R_{aj}^2	σ	Q(1)	Q(4)	A(1)	f1	f2	f3
2	1.07	(0.20)	0.90	(0.16)	0.22	(0.14)	0.43	1.38	0.81	4.25	1.02	1.00	1.00	0.10
3	0.93	(0.14)	0.73	(0.11)	0.20	(0.10)	0.53	0.94	24.56	30.69	5.70	1.00	1.00	0.08
4	0.02	(0.17)	-0.03	(0.15)	-0.39	(0.18)	0.20	1.03	20.49	38.55	5.72	0.01	0.04	0.48
8	-0.05	(0.09)	-0.07	(0.09)	-0.32	(0.11)	0.28	0.59	40.03	107.8	14.95	0.05	0.13	0.80

Table 2.c
Term spreads as predictors of future real activity

$y(t,k) = \alpha_0 + \alpha_1 (i_t^4 - i_t^1) + \alpha_2 i_t^1 + \varepsilon_{t+k}$											
k	α_1	(s.d.)	α_2	(s.d.)	R_{aj}^2	σ	Q(1)	Q(4)	A(1)	f1	f2
1	1.40	(0.32)			0.22	3.05	0.37	32.44	0.85	0.98	
			-1.14	(0.28)	0.20	3.10	2.15	38.35	0.86		0.99
	0.92	(0.39)	-0.48	(0.26)	0.22	3.05	0.69	34.78	0.87	0.33	0.02
2	0.66	(0.19)			0.15	1.83	10.94	30.40	1.56	0.94	
			-0.59	(0.16)	0.16	1.82	8.23	39.83	1.28		0.98
	0.28	(0.30)	-0.39	(0.22)	0.17	1.82	8.69	35.14	1.29	0.11	0.12
3	0.53	(0.12)			0.17	1.36	11.46	20.34	1.84	0.99	
			-0.46	(0.12)	0.18	1.36	5.89	17.88	1.07		0.99
	0.25	(0.22)	-0.28	(0.18)	0.18	1.36	7.83	17.76	1.21	0.10	0.12
4	0.15	(0.07)			0.03	0.82	43.95	63.11	18.92	0.40	
			-0.23	(0.06)	0.11	0.92	42.53	59.41	1.39		0.89
	-0.32	(0.19)	-0.47	(0.18)	0.15	0.87	37.88	53.82	13.84	0.32	0.80
8	0.12	(0.04)			0.05	0.64	45.39	94.99	18.93	0.52	
			-0.16	(0.04)	0.11	0.62	40.91	74.39	15.82		0.86
	-0.13	(0.14)	-0.26	(0.13)	0.13	0.61	36.70	62.09	12.76	0.12	0.49

Table 4.c
Components of the term spreads and real activity

$y(t,k) = \alpha_0 + \alpha_1 (f_t^{3,4} - f_t^{1,2}) + \alpha_2 (f_t^{1,2} - i_t^1) + \alpha_3 i_t^1 + \varepsilon_{t+k}$														
k	α_1	(s.d.)	α_2	(s.d.)	α_3	(s.d.)	R_{aj}^2	σ	Q(1)	Q(4)	A(1)	f1	f2	f3
2	0.17	(0.29)	0.21	(0.23)	-0.39	(0.21)	0.17	1.81	8.28	32.91	1.30	0.07	0.11	0.13
3	0.18	(0.20)	0.18	(0.16)	-0.28	(0.18)	0.18	1.35	7.88	17.43	1.23	0.09	0.09	0.12
4	-0.33	(0.17)	-0.24	(0.15)	-0.48	(0.18)	0.16	0.87	37.20	50.13	13.59	0.46	0.38	0.81
8	-0.15	(0.12)	-0.10	(0.11)	-0.26	(0.13)	0.13	0.61	35.66	60.33	12.18	0.17	0.13	0.50

Table 2.d
Term spreads as predictors of future real activity

$y(t,k) = \alpha_0 + \alpha_1 (i_t^4 - i_t^1) + \alpha_2 i_t^1 + \varepsilon_{t+k}$											
k	α_1	(s.d.)	α_2	(s.d.)	R_{aj}^2	σ	Q(1)	Q(4)	A(1)	f1	f2
1	1.61	(0.29)			0.32	3.95	0.94	11.97	1.02	1.00	
			-1.18	(0.26)	0.25	4.16	1.13	9.90	0.91		1.00
	1.48	(0.43)	-0.13	(0.27)	0.32	3.96	0.93	12.03	1.02	0.84	0.00
2	0.79	(0.16)			0.25	2.31	5.08	11.52	1.05	1.00	
			-0.60	(0.13)	0.20	2.39	8.13	16.58	1.45		1.00
	0.66	(0.33)	-0.13	(0.26)	0.25	2.31	5.06	12.05	1.08	0.45	0.08
3	0.65	(0.09)			0.31	1.67	18.93	28.00	3.41	1.00	
			-0.49	(0.10)	0.24	1.75	13.84	23.00	2.45		1.00
	0.59	(0.26)	-0.06	(0.24)	0.31	1.66	17.58	26.57	2.98	0.64	0.09
4	0.27	(0.07)			0.09	1.41	25.21	41.66	6.69	0.88	
			-0.31	(0.08)	0.17	1.35	27.16	43.73	7.60		0.96
	-0.18	(0.22)	-0.44	(0.22)	0.19	1.33	29.89	48.23	0.43	0.12	0.60
8	0.18	(0.04)			0.11	0.89	41.15	92.60	15.54	0.93	
			-0.20	(0.06)	0.20	0.84	37.53	78.94	12.91		0.95
	-0.11	(0.17)	-0.29	(0.17)	0.22	0.83	36.95	75.26	12.89	0.21	0.64

Table 4.d
Components of the term spreads and real activity

$y(t,k) = \alpha_0 + \alpha_1 (f_t^{3,4} - f_t^{1,2}) + \alpha_2 (f_t^{1,2} - i_t^1) + \alpha_3 i_t^1 + \varepsilon_{t+k}$														
k	α_1	(s.d.)	α_2	(s.d.)	α_3	(s.d.)	R_{aj}^2	σ	Q(1)	Q(4)	A(1)	f1	f2	f3
2	0.59	(0.30)	0.51	(0.25)	-0.12	(0.26)	0.25	2.30	5.81	13.10	1.18	0.43	0.46	0.07
3	0.54	(0.23)	0.45	(0.20)	-0.06	(0.24)	0.32	1.66	17.76	28.29	2.87	0.63	0.67	0.08
4	-0.16	(0.17)	-0.14	(0.17)	-0.44	(0.22)	0.18	1.34	30.22	48.37	9.66	0.09	0.12	0.60
8	-0.10	(0.13)	-0.09	(0.13)	-0.29	(0.17)	0.22	0.83	37.02	75.53	13.00	0.19	0.21	0.65

Table 2.e
Term spreads as predictors of future real activity

$y(t,k) = \alpha_0 + \alpha_1 (i_t^4 - i_t^1) + \alpha_2 i_t^1 + \varepsilon_{t+k}$											
k	α_1	(s.d.)	α_2	(s.d.)	R_{aj}^2	σ	Q(1)	Q(4)	A(1)	f1	f2
1	1.05	(0.23)			0.24	2.97	5.89	12.20	1.24	1.00	
			-1.05	(0.22)	0.23	2.99	1.81	7.24	0.79		1.00
	0.63	(0.30)	-0.60	(0.28)	0.27	2.90	3.05	9.00	0.94	0.51	0.39
2	0.54	(0.11)			0.22	1.62	3.06	7.93	1.28	0.99	
			-0.54	(0.12)	0.20	1.64	7.21	11.64	1.55		0.98
	0.34	(0.18)	-0.29	(0.19)	0.25	1.59	3.64	7.85	1.28	0.44	0.37
3	0.45	(0.09)			0.28	1.16	23.45	35.88	6.24	1.00	
			-0.45	(0.08)	0.26	1.18	24.76	39.08	6.67		1.00
	0.28	(0.15)	-0.24	(0.13)	0.32	1.13	23.59	35.05	5.74	0.52	0.37
4	0.21	(0.08)			0.08	1.10	13.70	27.43	2.32	0.73	
			-0.30	(0.07)	0.16	1.05	17.48	32.79	3.36		0.98
	-0.01	(0.14)	-0.31	(0.13)	0.17	1.05	16.80	31.63	2.91	0.10	0.68
8	0.13	(0.04)			0.10	0.64	33.86	82.88	10.58	0.82	
			-0.21	(0.04)	0.22	0.60	33.39	90.95	10.22		0.99
	-0.02	(0.07)	-0.22	(0.07)	0.23	0.60	32.66	88.23	9.71	0.06	0.86

Table 3.e
Term spreads, real activity and monetary growth

$y(t,k) = \alpha_0 + \alpha_1 (i_t^4 - i_t^1) + \alpha_2 m(t-4,1) + \alpha_3 m(t,k) + \varepsilon_{t+k}$														
k	α_1	(s.d.)	α_2	(s.d.)	α_3	(s.d.)	R_{aj}^2	σ	Q(1)	Q(4)	A(1)	f1	f2	f3
1	1.34	(0.24)	0.047	(0.034)	0.044	(0.017)	0.33	2.77	7.05	13.83	1.19	1.00	0.22	0.77
2	0.74	(0.13)	0.026	(0.023)	0.050	(0.016)	0.35	1.48	1.04	4.53	1.07	1.00	0.21	0.91
3	0.59	(0.08)	0.019	(0.020)	0.054	(0.016)	0.43	1.02	13.59	19.86	2.27	1.00	0.26	0.94
4	0.25	(0.07)	0.007	(0.016)	0.041	(0.021)	0.22	0.85	23.11	32.28	5.26	0.99	0.13	0.66
8	0.16	(0.04)	0.002	(0.012)	0.037	(0.026)	0.20	0.60	27.38	66.38	6.22	0.97	0.11	0.51

Table 4.e
Components of the term spreads and real activity

$y(t,k) = \alpha_0 + \alpha_1 (f_t^{3,4} - f_t^{1,2}) + \alpha_2 (f_t^{1,2} - i_t^1) + \alpha_3 i_t^1 + \varepsilon_{t+k}$														
k	α_1	(s.d.)	α_2	(s.d.)	α_3	(s.d.)	R_{aj}^2	σ	Q(1)	Q(4)	A(1)	f1	f2	f3
2	0.30	(0.18)	0.25	(0.13)	-0.30	(0.19)	0.25	1.59	4.18	8.21	1.28	0.45	0.44	0.36
3	0.27	(0.13)	0.20	(0.11)	-0.25	(0.14)	0.32	1.13	23.04	35.77	5.49	0.56	0.47	0.38
4	0.03	(0.13)	-0.01	(0.10)	-0.32	(0.13)	0.17	1.05	16.63	31.29	2.86	0.12	0.08	0.70
8	0.01	(0.07)	-0.02	(0.05)	-0.23	(0.07)	0.23	0.60	32.88	87.51	9.75	0.05	0.06	0.84

Table 2.f
Term spreads as predictors of future real activity

$y(t,k) = \alpha_0 + \alpha_1 (i_t^4 - i_t^1) + \alpha_2 i_t^1 + \varepsilon_{t+k}$											
k	α_1	(s.d.)	α_2	(s.d.)	R_{aj}^2	σ	Q(1)	Q(4)	A(1)	f1	f2
1	0.78	(0.29)			0.11	3.36	3.72	38.01	0.99	0.80	
			-0.95	(0.29)	0.15	3.29	4.96	41.49	1.13		0.96
2	0.30	(0.32)	-0.74	(0.31)	0.15	3.28	4.05	38.67	1.07	0.09	0.53
	0.34	(0.18)			0.07	1.94	9.68	37.31	1.27	0.54	
3			-0.47	(0.15)	0.11	1.89	7.96	40.59	1.20		0.88
	0.06	(0.21)	-0.42	(0.20)	0.12	1.89	7.89	39.10	1.22	0.07	0.51
4			-0.40	(0.12)	0.10	1.48	7.41	21.55	1.05	0.66	
	0.30	(0.13)	-0.34	(0.15)	0.15	1.43	5.39	18.80	1.07	0.09	0.51
8					0.02	0.97	31.32	55.96	13.03	0.23	
	0.08	(0.08)	-0.20	(0.07)	0.09	0.94	39.25	55.51	14.34		0.73
8	-0.09	(0.12)	-0.26	(0.12)	0.11	0.93	36.80	52.05	12.74	0.15	0.53
	0.07	(0.06)			0.03	0.68	39.56	83.95	13.43	0.28	
8			-0.14	(0.05)	0.09	0.66	38.48	72.43	13.11		0.67
	-0.03	(0.10)	-0.15	(0.10)	0.10	0.65	36.51	66.95	11.74	0.12	0.43

Table 3.f
Term spreads, real activity and monetary growth

$y(t,k) = \alpha_0 + \alpha_1 (i_t^4 - i_t^1) + \alpha_2 m(t-4,1) + \alpha_3 m(t,k) + \varepsilon_{t+k}$														
k	α_1	(s.d.)	α_2	(s.d.)	α_3	(s.d.)	R_{aj}^2	σ	Q(1)	Q(4)	A(1)	f1	f2	f3
1	0.97	(0.32)	0.022	(0.042)	0.027	(0.021)	0.15	3.29	2.93	33.32	0.87	0.91	0.04	0.34
2	0.46	(0.18)	0.009	(0.027)	0.030	(0.022)	0.11	1.90	9.75	31.65	1.17	0.75	0.02	0.39
3	0.40	(0.14)	0.006	(0.023)	0.033	(0.023)	0.14	1.44	7.35	19.36	1.24	0.84	0.05	0.37
4	0.12	(0.07)	0.000	(0.018)	0.015	(0.023)	0.06	0.94	16.30	39.84	2.59	0.30	0.10	0.20
8	0.09	(0.05)	-0.002	(0.014)	0.016	(0.030)	0.08	0.66	35.96	74.30	12.01	0.41	0.13	0.29

Table 4.f
Components of the term spreads and real activity

$y(t,k) = \alpha_0 + \alpha_1 (f_t^{3,4} - f_t^{1,2}) + \alpha_2 (f_t^{1,2} - i_t^1) + \alpha_3 i_t^1 + \varepsilon_{t+k}$														
k	α_1	(s.d.)	α_2	(s.d.)	α_3	(s.d.)	R_{aj}^2	σ	Q(1)	Q(4)	A(1)	f1	f2	f3
2	-0.00	(0.22)	0.06	(0.16)	-0.42	(0.20)	0.12	1.89	7.21	36.75	1.34	0.07	0.03	0.49
3	0.03	(0.63)	0.07	(0.12)	-0.33	(0.15)	0.15	1.43	5.32	17.14	1.04	0.06	0.08	0.48
4	-0.10	(0.12)	-0.06	(0.09)	-0.25	(0.12)	0.11	0.93	36.88	50.94	12.86	0.17	0.13	0.46
8	-0.04	(0.09)	-0.02	(0.07)	-0.14	(0.10)	0.10	0.65	36.47	67.10	11.67	0.13	0.09	0.37