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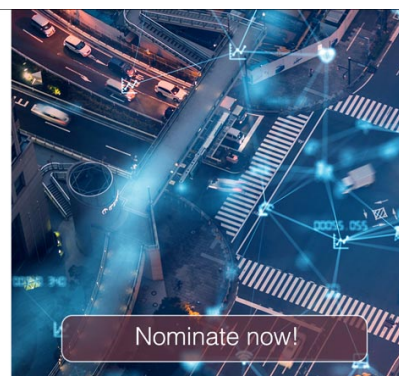


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Optimal strain gauge configurations for the estimation of mechanical loads in the main shaft of HAWT

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Abstract. In Structural Health Monitoring of wind turbines, measuring the mechanical loads is a key issue. The customary techniques for this task use a full-bridge strain gauge configuration to measure each of the six load components exerted on the shaft. However, using only six strain gauges should be sufficient to estimate the six load components if a one-to-one correspondence was achieved. In this paper a different approach to mechanical loads estimation is presented where, measuring the strain of individual gauges in quarter-bridge configurations, it is possible to estimate all the load components from a single set of gauges. The configurations are optimally determined making use of the D-optimality criterion, which maximises the observability of the estimated components. The approach also provides configurations where the apparent strain related to temperature variations is automatically compensated. Results show several optimal configurations for different measuring conditions and shows that six strain gauges are enough to estimate all the load components. The new approach also opens the possibility to obtain configurations with more strain gauges as well as configurations that have to meet other requirements.

1. Introduction

Strain gauges are common sensors in mechanical engineering due to their accuracy and low unit cost. They are bonded on the parts at locations where the strain is expected to be the largest. While gauges provide information of strain, usually the objective is not to know the strain itself, but other properties as stress, loads or deflections, which may not be directly measurable with a sensor. In order to estimate the applied load, they are commonly mounted in full Wheatstone bridges obtaining a voltage measurement that is proportional to the desired load component [1].

When measuring loads in circular cross-section shafts, there are commonly accepted configurations of gauges devoted to measure bending, shear, torsion and axial loads [2]. These configurations share a property: the remaining loads have no influence on the measured load component. This property is very useful because the existence of any other load component does not influence the measurement of the desired one. Moreover, while the temperature variations provoke *apparent strain*, these configurations prevent the load measurement to be altered by this fact, as the apparent strain of the gauges cancel to each other.



Despite these desirable properties, these configurations need for four strain gauges to measure each load. In other words, four sensors are needed to measure one quantity while, in theory, a one to one relation could be found between the number of sensors and the number of measured quantities. This statement encourages to search for a procedure to determine strain gauge configurations in order to simultaneously estimate all the mechanical loads present in a shaft, six load components, with the minimum amount of gauges which should also be six. The criterion chosen to get the optimum configuration will be based on the variance of the estimation of the loads, which is desired to be minimum.

While full-bridges measure a single voltage related to the strain of four gauges in order to estimate a single load component, in this approach the voltage of each strain gauge will be measured in a quarter-bridge and the estimate of each load component will be calculated as a different linear combination of the measured strains. Therefore, when using full Wheatstone bridge, the sum of strains is done by the physics of the electric circuit, while when using quarter bridges these sums are done mathematically. The advantage of the later case is that each strain gauge can contribute to the estimation of all the load components.

This paper is organised as follows. Section 2 is devoted to write an analytical expression of the strain of a single gauge in terms of its location and the *wrench* (three force and three moment components of the mechanical loads) exerted on the shaft. The analytical expression of the strain of the gauge happens to be linear in the wrench components. In Sections 3 and 4 a linear regression model and an optimisation procedure are built in order to determine the configuration of a set of gauges that minimises the variance of the estimated wrench components. In Section 5 the obtained results are presented while the conclusions are drawn in Section 7.

2. Determination of the strain of an arbitrarily located gauge

This section describes the determination of the strain of a single gauge upon the action of an external wrench. Using a set of bases and coordinates, stress tensor at the location of the strain gauge is calculated first and then an explicit expression of the strain of the gauge is obtained as a function of components of the external wrench.

2.1. Measuring approach

As mentioned in the introduction, a key aspect of this work is that, instead of dedicating a set of gauges for each measured load, a bigger set of gauges is dedicated to measure a whole set of loads. In the typical configurations, the strain gauges are assembled in a full- or half-bridge circuit configuration where a single voltage is measured.

The use of Wheatstone full-bridges have some advantages over using half- or quarter-bridges. For example, as all the strain gauges are bonded to the part, all of them are exposed to the same temperature. Since the Wheatstone bridge circuit electrically sums the resistance of two of the gauges and subtracts the resistance of the other two, any temperature effect is automatically compensated and temperature variations have no influence on the measurement of the loads.

Using quarter-bridges it is possible to reach the same result. If each of the 4 strain gauges of a full Wheatstone bridge was connected in a different quarter-bridge circuit, adding and subtracting the contribution of each of them mathematically (and not electrically as in the Wheatstone circuit) the same load results and sensitivities of the full-bridge would be achieved. This approach has an interesting advantage: the voltage drop measurement of a gauge in a quarter-bridge can be used in different loads estimation. That leads to a powerful outcome which is key in this paper: for a set of gauges, each of them in a quarter bridge, several different loads can be calculated as different linear combinations of the voltage drops in the output of the quarter bridges.

2.2. Geometric model

In this section the geometric model for the shaft and the strain gauges is developed. The shaft is supposed to be a solid or hollow cylinder with a circular cross-section. The strain gauges are bonded on the perimeter of a single cross-section of the shaft and the wrench to be estimated is the one that the shaft is experiencing in that precise section. The moment loads are calculated with respect to the point where the axis of the shaft and the cross section intersect, *i.e.*, point O in 1. The geometry of the cross-section is supposed to be constant in its vicinity and Saint Venant's principle is supposed to be met. Shafts material is supposed to be linear, elastic and isotropic.

The position and orientation of the strain gauges are determined in terms of two parameters. Projection bases and points are also defined in order to write the wrench and tensor components.

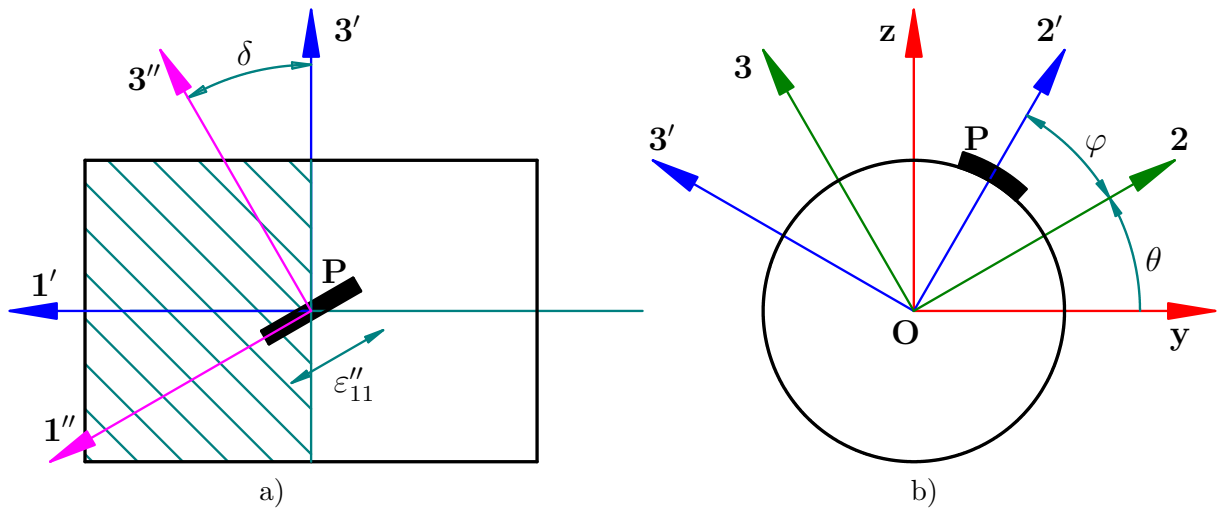


Figure 1: Location and orientation of a strain gauge. Figure a) is seen from $2'$.

As shown in Fig. 1, four bases are defined: xyz is fixed to the nacelle; base 123 is fixed to the shaft and rotates an angle θ with respect to the nacelle; base $1'2'3'$ is also fixed to the shaft but points towards a strain gauge, determining its position in the perimeter of the shaft in terms of the angle φ ; finally, base $1''2''3''$ determines the orientation of the strain gauge with respect to the axial direction of the shaft in terms of angle δ . The location and orientation of the gauges (actual values of φ and δ) are not known *a priori* and will be determined as the result of an optimisation procedure.

As it can be useful to obtain the coordinates of any vector or tensor in any of the mentioned projection bases, proper transformation matrices are defined. In this context, if $\{\mathbf{v}\}_{123}$ are the components of vector \mathbf{v} in base 123 and $[\mathbf{R}]_{xyz}^{123}$ is the coordinate transformation matrix from base 123 to xyz , then the components of \mathbf{v} in base xyz can be expressed as follows:

$$\{\mathbf{v}\}_{xyz} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix}_{xyz}^{123} \{\mathbf{v}\}_{123} = [\mathbf{R}]_{xyz}^{123} \{\mathbf{v}\}_{123} \quad (1)$$

Equivalent coordinate transformation matrices can be defined in order to write the components of any vector or tensor in the other bases.

2.3. Stress tensor components

The wrench that is to be estimated is the one that the right half of the shaft in 1a exerts on the left half. The components of the force and moment of this wrench in bases 123 and 1'2'3' are related to each other as:

$$\begin{Bmatrix} F'_1 \\ F'_2 \\ F'_3 \end{Bmatrix}_{1'2'3'} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \varphi & \sin \varphi \\ 0 & -\sin \varphi & \cos \varphi \end{bmatrix}_{1'2'3'}^{123} \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}_{123} = \begin{Bmatrix} F_1 \\ F_2 \cos \varphi + F_3 \sin \varphi \\ -F_2 \sin \varphi + F_3 \cos \varphi \end{Bmatrix}_{1'2'3'} \quad (2)$$

$$\begin{Bmatrix} M'_1 \\ M'_2 \\ M'_3 \end{Bmatrix}_{1'2'3'} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \varphi & \sin \varphi \\ 0 & -\sin \varphi & \cos \varphi \end{bmatrix}_{1'2'3'}^{123} \begin{Bmatrix} M_1 \\ M_2 \\ M_3 \end{Bmatrix}_{123} = \begin{Bmatrix} M_1 \\ M_2 \cos \varphi + M_3 \sin \varphi \\ -M_2 \sin \varphi + M_3 \cos \varphi \end{Bmatrix}_{1'2'3'} \quad (3)$$

where F_1 denotes axial force, F_2 and F_3 denote shear forces, M_1 denotes torsion moment and M_2 and M_3 denote bending moments with respect to point O . Despite the objective is to get the components of the wrench in base 123, the procedure will deal first with its components in base 1'2'3' because the calculation of the stress tensor components in this base is straight forward.

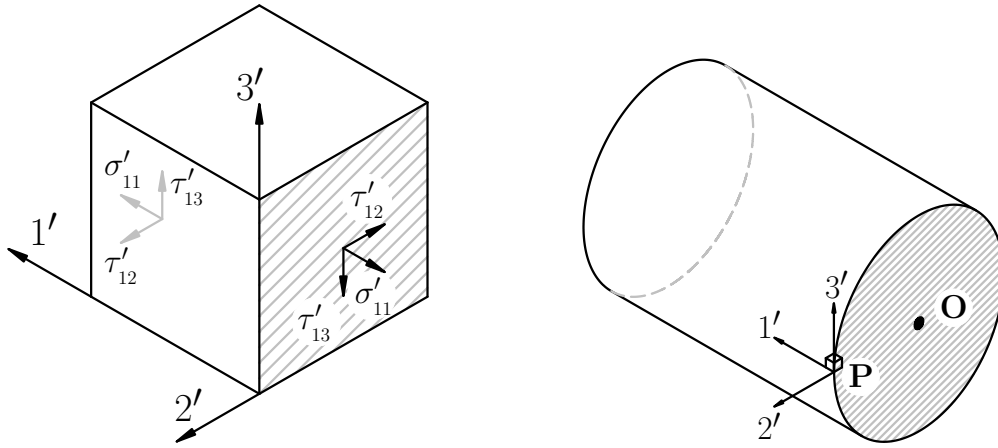


Figure 2: Solid differential at point P .

The first step is to write the stress tensor of the shaft at point P in terms of the wrench components exerted on the shaft at O . If the convention shown in 2 is met, the stress tensor at P can be expressed as follows:

$$[\bar{\sigma}(\mathbf{t})]_{1'2'3'} = \begin{bmatrix} -\frac{F'_1}{A} + \frac{M'_3}{w} & 0 & -\frac{M'_1 R}{I_P} - \frac{F'_3}{kA} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{1'2'3'} = \begin{bmatrix} \sigma'_{11} & \tau'_{12} & \tau'_{13} \\ \tau'_{12} & \sigma'_{22} & \tau'_{23} \\ \tau'_{13} & \tau'_{23} & \sigma'_{33} \end{bmatrix}_{1'2'3'} \quad (4)$$

where $\mathbf{t}' = (F'_1, F'_2, F'_3, M'_1, M'_2, M'_3)^T$ gathers the wrench components in base 1'2'3', A is the area of the cross section, w is the *sectional modulus* and I_P is the *polar moment of inertia*. The *shear coefficient* k represents the ratio of the average shear stress on a section to the shear stress at the centroid. For hollow circle cross sections k takes the following expression [3]:

$$k = \frac{6(1+\nu)(1+m^2)^2}{(7+6\nu)(1+m^2)^2 + (20+12\nu)m^2} \quad (5)$$

where ν is the Poisson modulus, $m = \frac{r}{R}$, and r and R are the inner and outer radii, respectively. This expression is also valid for full circle cross sections making $r = 0$ and thin-walled round tube sections making $r = R$.

2.4. Strain of a single gauge

In order to determine the components of the strain tensor $\bar{\varepsilon}$ in terms of the stress tensor $\bar{\sigma}$, their components are written in vector form:

$$\begin{aligned}\bar{\sigma} &= (\sigma'_{11}, \sigma'_{22}, \sigma'_{33}, \tau'_{12}, \tau'_{23}, \tau'_{13})^T \\ \bar{\varepsilon} &= (\varepsilon'_{11}, \varepsilon'_{22}, \varepsilon'_{33}, \varepsilon'_{12}, \varepsilon'_{23}, \varepsilon'_{13})^T\end{aligned}\quad (6)$$

where the components of the symmetric strain tensor are:

$$[\bar{\varepsilon}]_{1'2'3'} = \begin{bmatrix} \varepsilon'_{11} & \varepsilon'_{12} & \varepsilon'_{13} \\ & \varepsilon'_{22} & \varepsilon'_{23} \\ & & \varepsilon'_{33} \end{bmatrix}_{1'2'3'} \quad (7)$$

In the vector form, the following relation between strain and stress holds:

$$\bar{\sigma} = \mathbf{D}\bar{\varepsilon} \quad (8)$$

and if the material is linear, elastic and isotropic, \mathbf{D} is written as

$$\mathbf{D} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ & 1-\nu & \nu & 0 & 0 & 0 \\ & & 1-\nu & 0 & 0 & 0 \\ & & & 1-2\nu & 0 & 0 \\ & & & & 1-2\nu & 0 \\ & & & & & 1-2\nu \end{bmatrix} \quad (9)$$

where constants E and ν are the Young's and Poisson's moduli, respectively.

Since the strain gauge is aligned with axis $1''$, the strain tensor will be expressed in base $1''2''3''$ as follows:

$$[\bar{\varepsilon}]_{1''2''3''} = [\mathbf{R}]_{1''2''3''}^{1'2'3'} [\bar{\varepsilon}]_{1'2'3'} [\mathbf{R}]_{1'2'3'}^{1''2''3''} \quad (10)$$

Taking the $(1, 1)$ component, the strain of the gauge, ε , can be obtained.

Defining the wrench vector $\mathbf{t} = (F_1, F_2, F_3, M_1, M_2, M_3)^T$ and the row vector $\mathbf{w} = \frac{\partial \varepsilon}{\partial \mathbf{t}}$, as ε is linear in \mathbf{t} , the explicit expression for the strain of the gauge can be written as

$$\varepsilon = \mathbf{w}\mathbf{t} \quad (11)$$

where the explicit expression of the row vector \mathbf{w} is:

$$\mathbf{w}(\varphi, \delta) = \left\{ \begin{array}{l} \frac{(1+\nu)\sin^2\delta-1}{EA} \\ -\frac{2(\nu+1)\cos\delta\sin\delta\sin\varphi}{kEA} \\ \frac{2(\nu+1)\cos\delta\sin\delta\cos\varphi}{kEA} \\ \frac{R(\nu+1)\sin 2\delta}{EI_p} \\ \frac{((1+\nu)\sin^2\delta-1)\sin\varphi}{Ew} \\ -\frac{((1+\nu)\sin^2\delta-1)\cos\varphi}{Ew} \end{array} \right\}^T \quad (12)$$

For a gauge located at an arbitrary point defined by φ and δ , equation (11) provides the strain of the gauge related to an arbitrary wrench \mathbf{t} . Next sections show the way the optimal values of φ and δ are calculated.

3. Linear regression model

In this section it will be shown how to use equation (12) in order to obtain the estimation of the wrench components in terms of the strain measurements of a set of gauges.

The wrench vector has $p = 6$ components (3 forces and 3 moments) and therefore a minimum of p strain gauges will be necessary for the wrench estimation. Let us suppose that a set of $n \geq p$ gauges is bonded in the perimeter of a cross section of the shaft. Denoting the strain of the i^{th} gauge as ε_i ($i = 1, \dots, n$) and being φ_i and δ_i the angles that determine its position and orientation, a linear system of equations can be built evaluating equation (12) for the n strain gauges as:

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{Bmatrix} = \begin{bmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \\ \vdots \\ \mathbf{w}_n \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ M_1 \\ M_2 \\ M_3 \end{Bmatrix} \quad (13)$$

where $\mathbf{w}_i = \mathbf{w}(\varphi_i, \delta_i)$. Gathering the n mechanical strains in vector $\boldsymbol{\varepsilon}$ and the n row vectors \mathbf{w}_i in \mathbf{W} , equation (13) can be rewritten as:

$$\boldsymbol{\varepsilon} = \mathbf{W}(\boldsymbol{\vartheta}) \mathbf{t} \quad (14)$$

where $\boldsymbol{\varphi} = (\varphi_1, \varphi_2, \dots, \varphi_n)$, $\boldsymbol{\delta} = (\delta_1, \delta_2, \dots, \delta_n)$ and $\boldsymbol{\vartheta} = (\boldsymbol{\varphi}, \boldsymbol{\delta})$. Matrix $\mathbf{W} = \mathbf{W}(\boldsymbol{\vartheta})$ represents the $n \times p$ observation matrix which has all the information of the location of the gauges. Measuring the strain of the gauges for an instant and writing \mathbf{W} for the configuration of the gauges, one could solve the linear system of equation (14) to calculate \mathbf{t} in terms of $\boldsymbol{\varepsilon}$, assuming \mathbf{W} has full rank.

In an statistical framework, it is more convenient to approach equation (14) as an estimation problem whenever vector $\boldsymbol{\varepsilon}$ is measured with certain noise. Therefore, this equation can be rewritten in terms of the *measured* strains, $\boldsymbol{\varepsilon}_m$, and the measurement errors, \mathbf{e} , as:

$$\boldsymbol{\varepsilon}_m = \mathbf{W}\mathbf{t} + \mathbf{e} \quad (15)$$

where both $\boldsymbol{\varepsilon}_m$ and \mathbf{e} are random variables [4, 5] and \mathbf{W} is supposed to be deterministic, *i.e.* known without uncertainty. In this context, solving the system of equations is equivalent to calculate an *estimate* of the wrench. Thus, the *Maximum Likelihood Estimator* of \mathbf{t} can be calculated as [5, 6, 7, 8]:

$$\hat{\mathbf{t}} = (\mathbf{W}^T \boldsymbol{\Sigma}^{-1} \mathbf{W})^{-1} \mathbf{W}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\varepsilon}_m \quad (16)$$

where $\boldsymbol{\Sigma} = E[\mathbf{e}\mathbf{e}^T]$ is the covariance matrix of \mathbf{e} . As $\boldsymbol{\varepsilon}_m$ is a random variable, the estimation of \mathbf{t} , $\hat{\mathbf{t}}$, will also be a random variable. The estimate in equation (16) is also known as the *Markov Estimate* or the *Best Linear Unbiased Estimate* [5, 9].

If all the strain gauges are of the same type, it can be assumed that the variance of their measurement error will be the same. Therefore,

$$\boldsymbol{\Sigma} = \text{var}(\mathbf{e}) = \text{var}(\boldsymbol{\varepsilon}_m) = \text{var}(\varepsilon_m) \mathbf{I}_n \quad (17)$$

where $\text{var}(\varepsilon_m)$ is the variance of the individual strain gauges and \mathbf{I}_n is the $n \times n$ identity matrix. Then, the estimator of \mathbf{t} given in equation (16) simplifies to:

$$\hat{\mathbf{t}} = (\mathbf{W}^T \mathbf{W})^{-1} \mathbf{W}^T \boldsymbol{\varepsilon}_m \quad (18)$$

and its variance can be written as [6]:

$$\text{var}(\hat{\mathbf{t}}) = \text{var}(\varepsilon_m) (\mathbf{W}^T \mathbf{W})^{-1} \quad (19)$$

4. Optimal estimation of the wrench components

4.1. Optimality criterion

From equation (18) it can be derived that for any configuration, provided $\mathbf{W}^T \mathbf{W}$ has full rank, an estimation of \mathbf{t} can be obtained from the strain measurements vector $\boldsymbol{\varepsilon}_m$. However, equation (19) shows that the variance of this estimation depends on the actual values of the elements of $\mathbf{W}(\boldsymbol{\vartheta})$. In this context, the objective of this paper is to determine the configuration of the strain gauges that minimises the variance of \mathbf{t} . However, as $\text{var}(\hat{\mathbf{t}})$ is a matrix, an alternative optimisation criterion has to be used to determine the optimal configuration.

In this context, D-optimality is the best index for maximising the observability of the wrench [10]. Moreover, this criterion is *scaling invariant*, so it is insensitive to the different dimensions of the components of the wrench (forces and moments). Assuming equation (17) holds, and dropping the scaling factor $\text{var}(\boldsymbol{\varepsilon}_m)$, the cost function \mathcal{F} of the D-optimality criterion can be written as [5, 8]:

$$\mathcal{F}(\mathbf{W}) = -\log(\det(\mathbf{W}^T \mathbf{W})) \quad (20)$$

Thus, in order to determine the optimal location and orientation of the strain gauges, $\boldsymbol{\vartheta}^{opt}$, it would suffice to solve the following minimisation problem:

$$\boldsymbol{\vartheta}^{opt} = \arg_{\boldsymbol{\vartheta}} \min(\mathcal{F}(\mathbf{W}(\boldsymbol{\vartheta}))), \quad \text{subject to } \mathbf{c}(\boldsymbol{\vartheta}) = \mathbf{0} \quad (21)$$

where $\mathbf{c}(\boldsymbol{\vartheta}) = \mathbf{0}$ are optional constraints.

4.2. Temperature compensation

If a measurement is performed when the temperature differs from the one present during the calibration, a temperature induced strain will occur. This temperature response is reversible and the effects disappear when the gauges are at the calibration temperature. In the literature, this temperature response is often called *apparent strain* [2].

One efficient way to compensate the temperature effects relies on using various strain gauges. Choosing a suitable configuration, the mechanical loads can be calculated making the temperature response of the different gauges compensate each other.

In order to follow this approach, let us suppose that the gauges will sense certain strain related to the temperature variations, $\boldsymbol{\varepsilon}_T$. This strain will be indistinguishable from the strain related to mechanical loads, $\boldsymbol{\varepsilon}$. In order to take into account the temperature variations, the measured strains, $\boldsymbol{\varepsilon}_m$, should be written as:

$$\boldsymbol{\varepsilon}_m = \boldsymbol{\varepsilon} + \boldsymbol{\varepsilon}_T + \mathbf{e}. \quad (22)$$

Assuming that any possible temperature effect affects the measurement of all the strain gauges in the same way, the temperature strain could be written as $\boldsymbol{\varepsilon}_T = \mathbf{1}\varepsilon_T$, where $\mathbf{1}$ represents an $n \times 1$ vector of ones and ε_T is an unknown scalar. Therefore, the previous equation can be rewritten as:

$$\boldsymbol{\varepsilon}_m = [\mathbf{W} \quad \mathbf{1}] \begin{Bmatrix} \mathbf{t} \\ \varepsilon_T \end{Bmatrix} + \mathbf{e}. \quad (23)$$

We could refer to equation (23) as the *extended system of equations* where the temperature strain has been taken into account. This linear system is analogous to equation (15) where matrix \mathbf{W} and vector \mathbf{t} have been extended with a column and a row, respectively. Therefore, the estimation and optimisation procedures presented before can also be applied to the extended system if configurations which provide temperature compensation are desired.

5. Results and discussion

The procedure described in the previous sections has been applied in order to determine the optimal configurations of a set of gauges that makes it possible to estimate the six components of the wrench with the minimum possible variance.

5.1. Solution for the minimum number of gauges

For the first configuration $n = 6$ gauges have been used with no requirement for temperature compensation. The optimal solution is observed to be slightly dependent on the specific value of ν . For the ratio of the steel, $\nu = 0.3$, the solution is:

$$\begin{aligned}\varphi^{opt} &= (0^\circ, 0^\circ, 120^\circ, 120^\circ, 240^\circ, 240^\circ) + (\varphi_a, \varphi_b, \varphi_a, \varphi_b, \varphi_a, \varphi_b) \\ \delta^{opt} &= (27.1, -27.1, 27.1, -27.1, 27.1, -27.1)\end{aligned}\quad (24)$$

where φ_a and φ_b can take arbitrary values. A convenient solution can be achieved making $\varphi_a = \varphi_b = 0$, leading to a solution where the six gauges can be gathered in three couples that share the same value of φ . Moreover, the couples are uniformly distributed on the perimeter of the shaft and the δ angles of each couple of gauges are opposite to each other.

5.2. Solution using rosettes

As it can be interesting to build configurations based on rosettes [2], the procedure is applied to a configuration with $n = 6$ gauges gathered in three rosettes. In this context, the gauges of a rosette will be located at the same φ and the angle between their orientation (*shift angle*) will be constant. For a shift angle of 60° , the following solution is obtained:

$$\begin{aligned}\varphi^{opt} &= (0^\circ, 0^\circ, 120^\circ, 120^\circ, 240^\circ, 240^\circ) \\ \delta^{opt} &= (30^\circ, -30^\circ, 30^\circ, -30^\circ, 30^\circ, -30^\circ)\end{aligned}\quad (25)$$

This constrained optimal configuration is very close to the unconstrained optimum, as can be directly derived comparing Eqs. (24) and (25). However, compared to the previous one, this can be implemented using rosettes, which reduces the number of bonding operations and reduces the uncertainty for the actual angular position of the gauges.

Applying the procedure for a shift angle of 90° , the following solution is obtained:

$$\begin{aligned}\varphi^{opt} &= (0^\circ, 0^\circ, 120^\circ, 120^\circ, 240^\circ, 240^\circ) \\ \delta^{opt} &= (45^\circ, -45^\circ, 45^\circ, -45^\circ, 45^\circ, -45^\circ)\end{aligned}\quad (26)$$

This configuration provides worse performance compared to the previous one, as the optimality criterion takes a bigger value.

The configurations of Eqs. (25) and (26) are represented in 3.

5.3. Solutions for temperature compensation

When the temperature effects have to be compensated, additional strain gauges are needed. In this case $n = 8$ strain gauges are used for the configuration, the observation matrix of equation (23) is used in the optimisation procedure and the gauges are required to be gathered in rosettes. For a shift angle of 60° , the optimal solution is observed to slightly depend on ν . For $\nu = 0.3$:

$$\begin{aligned}\varphi^{opt} &= (0^\circ, 0^\circ, 90^\circ, 90^\circ, 180^\circ, 180^\circ, 270^\circ, 270^\circ) \\ \delta^{opt} &= (-10.1^\circ, 49.9^\circ, 10.1^\circ, -49.9^\circ, -10.1^\circ, 49.9^\circ, 10.1^\circ, -49.9^\circ)\end{aligned}\quad (27)$$

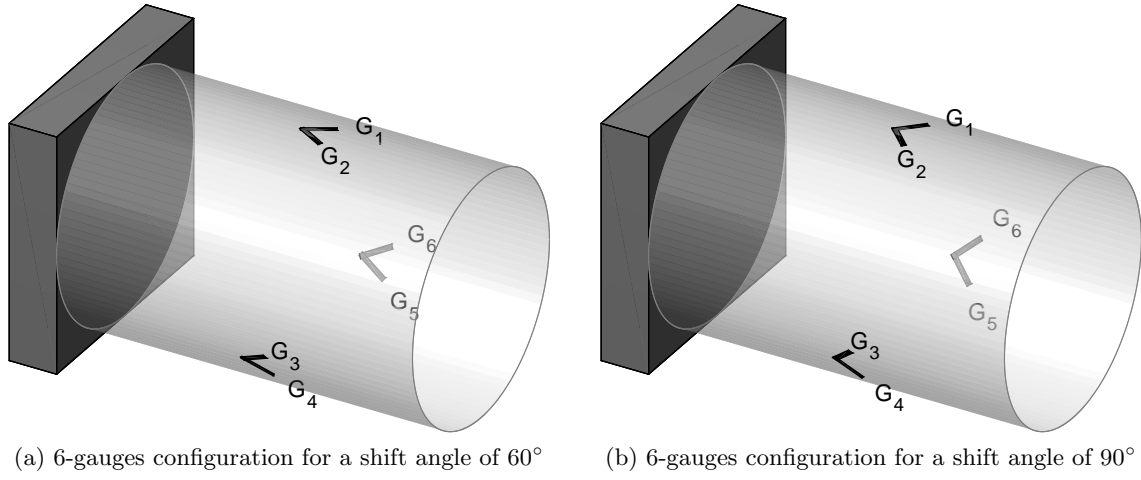


Figure 3: Optimal 6-gauges constrained configurations for the estimation of \mathbf{t} .

Running the optimisation procedure for a shift angle of 90° , the resulting optimal configuration is obtained which does not depend on the value of ν :

$$\begin{aligned}\varphi^{opt} &= (0^\circ, 0^\circ, 90^\circ, 90^\circ, 180^\circ, 180^\circ, 270^\circ, 270^\circ) \\ \delta^{opt} &= (60^\circ, -30^\circ, 30^\circ, -60^\circ, 60^\circ, -30^\circ, 30^\circ, -60^\circ)\end{aligned}\quad (28)$$

The configurations of Eqs. (27) and (28) are represented in 4.

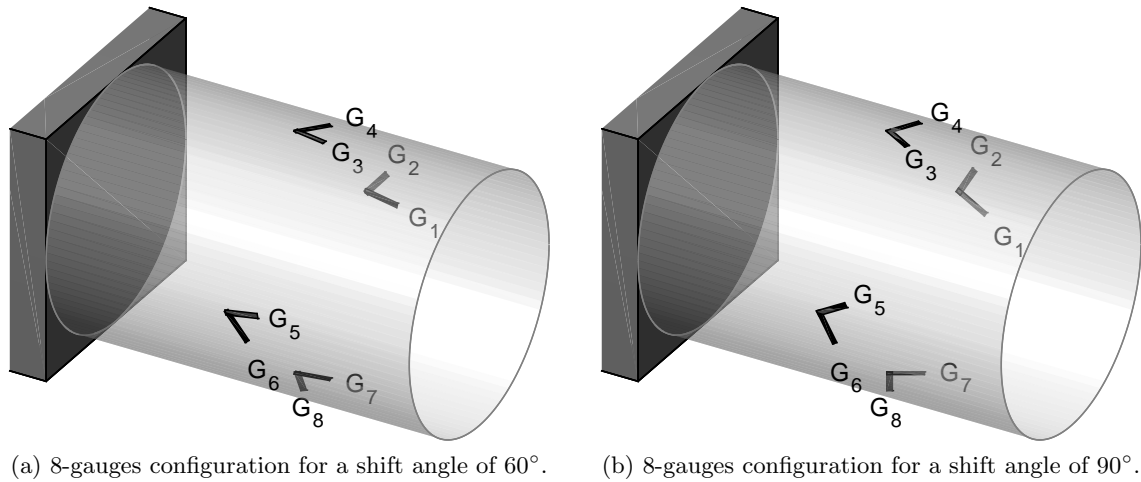


Figure 4: Optimal 8-gauges constrained configurations for the estimation of \mathbf{t} .

6. Considerations for a practical implementation

In the previous sections a procedure has been developed in order to get optimal strain gauge configurations to measure the wrench components with the minimum possible variance, which has been the objective of this research. However, the obtained results are far from being practically implementable as-is and further research would be necessary in the way towards designing a fully functional wrench measuring device.

One reasonable concern for the practical implementation can be not to know which is the sensitivity of the approach to the uncertainty of the parameters and measurement noise. In fact, performing this analysis is mandatory to know *a priori* the range of uncertainty in the estimation of the wrench components.

For a complete validation of the approach, at least for some of the optimal configurations obtained in the results, a calibration procedure should also be designed in order to be able to correct the measurement errors to an acceptable level. As in this approach all the strain gauges contribute to measure all the wrench component, a dedicated calibration procedure should be designed taking into account the particular characteristics of the measuring device.

A practical implementation of an experimental set up would certainly bring to light any other practical inconveniences that might compromise the application of the approach.

Being these issues fundamental for the implementation of a full functionality measuring device, they have not been tackled in this paper and they remain as future work.

7. Conclusions

In this paper a new approach to the measurement of mechanical loads in circular cross-section shafts is presented. Compared to the state of the art, this approach measures the strain of gauges in quarter-bridge configurations so that the load components are estimated as different linear combinations of the strain of the individual gauges.

The first step has been to write an expression for the strain of an arbitrarily located gauge in terms of the mechanical loads (forces and moments) exerted on the shaft. Setting this relation for a set of gauges, a linear regression model has been developed where the load components are estimated in terms of the measured strains of the gauges. The paper shows that 6 strain gauges are enough to estimate the 6 load components, while the state of the art uses 24 (4 gauges for each load component).

The procedure has been extended so that rosettes can be used instead of individual gauges. As a result, couples of gauges are located virtually on the same location and a calibrated angle between their orientations is given. Two optimal configurations have been obtained for shift angles of 60° and 90° .

In order to address the apparent strains devoted to temperature variations in the presented approach, the procedure has been extended so that the configurations are insensitive to temperature variations by using more than 6 strain gauges. Therefore, two more optimal configurations have been obtained applying the procedure to a set of 8 strain gauges. Applying the shift angle constraints, these configurations provide the estimation of the 6 components of the mechanical load in circular cross-section shafts, with temperature compensation and with a very reduced number of strain gauges.

Finally it is important to note that the approach proposed in this paper opens the possibility to obtain optimal solutions with an bigger number of strain gauges. Moreover, it is also possible to further extend the presented procedures in order to address other load measuring problems with different characteristics.

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