

**MONOPOLISTIC COMPETITION, STICKY PRICES, AND THE  
MINIMAL MARK-UP IN STEADY STATE**

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# Monopolistic competition, sticky prices, and the minimal mark-up in steady state

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## Abstract

This note reports the rate of inflation that minimizes the mark-up of prices over marginal costs in the steady-state solution of a monopolistic competition model with either Taylor (1980) or Calvo (1983) pricing. The minimal mark-up is always found at a positive and low rate of inflation for any sensible parameter calibration. Actually, the rate of inflation that minimizes the mark-up is very close to ratio between the real rate of discount and the Dixit-Stiglitz elasticity. This result is robust to alternative sticky-price specifications.

*Keywords:* monopolistic competition, sticky prices, minimal mark-up.

*JEL classification:* E12, E31.

## 1 Introduction

The objective of this paper is to calculate the minimal mark-up of prices over marginal costs in economies with monopolistic competition and sticky prices. On that purpose, two types of slow price-adjustment specifications will be introduced in a standard monopolistic competition framework: the Taylor staggered prices (original from Taylor, 1980), and the Calvo partial adjustment based on fixed probabilities (described in Calvo, 1983). Together they represent the bulk of recent literature on optimizing models with sticky prices; the so-called New Keynesian methodology. Examples of papers using the Calvo pricing are Yun (1996), King and Wolman

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(1996), Erceg *et al.* (2000), and Sbordone (2002). The Taylor pricing has been employed in papers such as King and Wolman (1999), Chari *et al.* (2000), and Huang and Liu (2002).

The mark-up is recognized as a source of economic inefficiency that stems from the monopolistic competition structure. It results in certain long-run welfare loss relative to the price-taking behavior of perfect competition as first pointed out by Blanchard and Kiyotaki (1987).<sup>1</sup> Therefore we will search for the rate of inflation that makes the mark-up minimum in steady state to serve as a reference for a long-run monetary policy strategy.

## 2 Monopolistic competition and sticky prices

Let us begin with the monopolistic competition setup described in Dixit and Stiglitz (1977). There is a continuum of firms each of them producing a differentiated good in a monopolistically competitive market. Thus, the firm  $i$  sets the price  $P_t(i)$  in quarter  $t$ , and the amount of output that will sell  $y_t(i)$  is giving by the Dixit-Stiglitz demand equation

$$y_t(i) = \left[ \frac{P_t(i)}{P_t} \right]^{-\theta} y_t,$$

where  $\theta$  is the elasticity of substitution between differentiated goods,  $P_t$  is the aggregate price level, and  $y_t$  is aggregate output. Let us denote the total cost of production for firm  $i$  in quarter  $t$  as  $TC_t(i)$ . Total income of firm  $i$  is  $P_t(i)y_t(i) = P_t(i) \left[ \frac{P_t(i)}{P_t} \right]^{-\theta} y_t$ . Accordingly, the amount of firm  $i$ 's profit in period  $t$  expressed in units of the Dixit-Stiglitz composite good would be  $\left[ \frac{P_t(i)}{P_t} \right]^{1-\theta} y_t - \frac{TC_t(i)}{P_t}$ . Thus, the optimal-price decision in period  $t$  is made by maximizing the intertemporal profit function:

$$\underset{P_t(i)}{\text{Max}} \quad E_t \sum_{j=0}^{\infty} \beta^j \left( \left[ \frac{P_{t+j}(i)}{P_{t+j}} \right]^{1-\theta} y_{t+j} - \frac{TC_{t+j}(i)}{P_{t+j}} \right) \quad (1)$$

where  $E_t$  is the rational expectation operator in period  $t$ , and the discount factor is  $\beta = \frac{1}{1+\rho}$  with  $\rho > 0$  as the real rate of discount.

Now we will introduce price rigidities. Following Calvo (1983), let us assume that there is a

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<sup>1</sup>Other papers that examine this issue are King and Wolman (1996), Khan *et al.* (2003), and Casares (2004).

constant probability  $\eta$  that firms will not be able to change prices. This leads to the following first order condition resulting from (1)

$$E_t \sum_{j=0}^{\infty} \beta^j \eta^j \left( (1 - \theta) \left[ \frac{P_t(i)}{P_{t+j}} \right]^{-\theta} \frac{y_{t+j}}{P_{t+j}} - \frac{\partial TC_{t+j}(i)}{\partial y_{t+j}(i)} \frac{\partial y_{t+j}(i)}{\partial P_t(i)} \frac{1}{P_{t+j}} \right) = 0. \quad (2)$$

Alternatively, it could be assumed that firms can adjust the price with a constant frequency as first proposed by Taylor (1980). In particular, firms can only adjust prices every  $J$  quarters, remaining constant meanwhile. The first order condition resulting from solving (1) becomes then

$$E_t \sum_{j=0}^{J-1} \beta^j \left( (1 - \theta) \left[ \frac{P_t(i)}{P_{t+j}} \right]^{-\theta} \frac{y_{t+j}}{P_{t+j}} - \frac{\partial TC_{t+j}(i)}{\partial y_{t+j}(i)} \frac{\partial y_{t+j}(i)}{\partial P_t(i)} \frac{1}{P_{t+j}} \right) = 0. \quad (3)$$

Let  $\psi_t = \frac{\partial TC_t(i)}{\partial y_t(i)} \frac{1}{P_t}$  denote the real marginal cost in composite-good output units.<sup>2</sup> We are going to insert this definition and the derivative  $\frac{\partial y_{t+j}(i)}{\partial P_t(i)} = -\theta \left[ \frac{P_t(i)}{P_{t+j}} \right]^{-\theta-1} \frac{y_{t+j}}{P_{t+j}}$  obtained from the Dixit-Stiglitz demand equation in the two previous equations to find

$$(1 - \theta) E_t \sum_{j=0}^{\infty} \beta^j \eta^j \left( \left[ \frac{P_t(i)}{P_{t+j}} \right]^{-\theta} \frac{y_{t+j}}{P_{t+j}} \right) = -\theta E_t \sum_{j=0}^{\infty} \beta^j \eta^j \left( \psi_{t+j} \left[ \frac{P_t(i)}{P_{t+j}} \right]^{-\theta-1} \frac{y_{t+j}}{P_{t+j}} \right), \quad (4)$$

$$(1 - \theta) E_t \sum_{j=0}^{J-1} \beta^j \left( \left[ \frac{P_t(i)}{P_{t+j}} \right]^{-\theta} \frac{y_{t+j}}{P_{t+j}} \right) = -\theta E_t \sum_{j=0}^{J-1} \beta^j \left( \psi_{t+j} \left[ \frac{P_t(i)}{P_{t+j}} \right]^{-\theta-1} \frac{y_{t+j}}{P_{t+j}} \right). \quad (5)$$

Expressions (4) and (5) determine the steady-state value of the average mark-up. It can be computed by taking into account a number of steady-state properties of these models: prices rise at a constant rate of inflation ( $\pi$ ), output is constant ( $y$ ), the real marginal cost is also constant ( $\psi$ ), and the rational expectation operators can be dropped. In turn, equations (4)-(5) can be written in steady state as follows

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<sup>2</sup> Assuming a production function homogeneous of degree 1 (which implies constant returns to scale), the real marginal cost is identical across firms. This is the reason why  $\psi_i$  is not firm specific and appears denoted without the  $i$  index.

$$(1 - \theta) \sum_{j=0}^{\infty} \beta^j \eta^j \left( \left[ \frac{P(i)}{(1 + \pi)^j P} \right]^{-\theta} \frac{y}{(1 + \pi)^j P} \right) = -\theta \psi \sum_{j=0}^{\infty} \beta^j \eta^j \left( \left[ \frac{P_t(i)}{(1 + \pi)^j P} \right]^{-\theta-1} \frac{y}{(1 + \pi)^j P} \right) \quad (6)$$

$$(1 - \theta) \sum_{j=0}^{J-1} \beta^j \left( \left[ \frac{P(i)}{(1 + \pi)^j P} \right]^{-\theta} \frac{y}{(1 + \pi)^j P} \right) = -\theta \psi \sum_{j=0}^{J-1} \beta^j \left( \left[ \frac{P(i)}{(1 + \pi)^j P} \right]^{-\theta-1} \frac{y}{(1 + \pi)^j P} \right) \quad (7)$$

The inverse value of the steady-state real marginal cost,  $\psi^{-1}$ , is the ratio of the aggregate price level over the nominal marginal cost in steady state. It represents the average steady-state mark-up of prices over marginal costs. Using the properties of the summation of numbers that decrease at a constant factor, the steady-state solutions for  $\psi^{-1}$  implied by (6) and (7) are

$$\psi^{-1} = \frac{\theta}{\theta - 1} \frac{1 - \beta\eta(1 + \pi)^{(\theta-1)}}{1 - \beta\eta(1 + \pi)^\theta} \left[ \frac{P}{P(i)} \right], \quad (8)$$

$$\psi^{-1} = \frac{\theta}{\theta - 1} \frac{1 - \beta^J(1 + \pi)^{J\theta}}{1 - \beta^J(1 + \pi)^{J(\theta-1)}} \frac{1 - \beta(1 + \pi)^{(\theta-1)}}{1 - \beta(1 + \pi)^\theta} \left[ \frac{P}{P(i)} \right]. \quad (9)$$

The ratio of the aggregate price level over the optimal price in steady state  $P/P(i)$  for the Calvo pricing model is<sup>3</sup>

$$\frac{P}{P(i)} = \left[ \frac{1 - \eta}{1 - \eta(1 + \pi)^{\theta-1}} \right]^{1/(1-\theta)}. \quad (10)$$

Analogously, the steady-state ratio  $P/P(i)$  in the Taylor pricing model is<sup>4</sup>

$$\frac{P}{P(i)} = \left[ \frac{1}{J} \frac{1 - (1 + \pi)^{J(\theta-1)}}{1 - (1 + \pi)^{(\theta-1)}} \right]^{1/(1-\theta)}. \quad (11)$$

Hence, substituting (10) in (8) and (11) in (9) yield the following average steady state mark-up

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<sup>3</sup>This result can be obtained by taking the aggregate price level definition  $P = [(1 - \eta)P(i)^{1-\theta} + \eta P_{-1}^{1-\theta}]^{1/(1-\theta)}$  in steady state with a constant rate of inflation  $\pi$ .

<sup>4</sup>In this case, the aggregate price level is obtained as  $P = \sum_{k=0}^{J-1} [J^{-1} P_{-k}(i)^{1-\theta}]^{1/(1-\theta)}$  where  $P_{-k}(i)$  denotes the optimal price set  $k$  periods ago. Assuming the steady-state condition  $P_{-k}(i) = (1 + \pi)P_{-(k+1)}(i)$  leads to (11).

for the Calvo pricing (12) and the Taylor pricing (13)

$$\psi^{-1} = \frac{\theta}{\theta - 1} \frac{1 - \beta\eta(1 + \pi)^{(\theta-1)}}{1 - \beta\eta(1 + \pi)^\theta} \left[ \frac{1 - \eta}{1 - \eta(1 + \pi)^{\theta-1}} \right]^{1/(1-\theta)}, \quad (12)$$

$$\psi^{-1} = \frac{\theta}{\theta - 1} \frac{1 - \beta^J(1 + \pi)^{J\theta}}{1 - \beta^J(1 + \pi)^{J(\theta-1)}} \frac{1 - \beta(1 + \pi)^{(\theta-1)}}{1 - \beta(1 + \pi)^\theta} \left[ \frac{1}{J} \frac{1 - (1 + \pi)^{J(\theta-1)}}{1 - (1 + \pi)^{(\theta-1)}} \right]^{1/(1-\theta)}. \quad (13)$$

In both cases, the steady-state average mark-up  $\psi^{-1}$  depends on the Dixit-Stiglitz elasticity parameter  $\theta$ , the rate of discount  $\rho$  as determinant of  $\beta = \frac{1}{1+\rho}$ , the level of price rigidities ( $\eta$  under Calvo pricing and  $J$  under Taylor pricing), and the steady-state rate of inflation  $\pi$ .

### 3 Sticky prices and the minimal mark-up in steady state

The market power that firms have in monopolistic competition drives the mark-up of prices over marginal costs above unity. If prices were fully flexible the mark-up would always be constant at  $\psi^{-1} = \frac{\theta}{\theta-1}$ . This result is obtained in the steady-state expressions (12) and (13) when assuming flexible prices ( $\eta = J = 0.0$ ). However, it was shown in the previous section that the presence of sticky prices *à la* Calvo or *à la* Taylor makes the value of the mark-up in steady state depend on the rate of inflation. Based on welfare grounds, it would be desirable to set a long-run target for inflation that resulted in a minimal mark-up. In that case, the long-run deviation of the economy from the (efficient) perfect competition solution would have been reduced as much as possible. With this purpose, we will conduct an exercise of finding the steady-state rates of inflation that minimize  $\psi^{-1}$  for some given model parameters  $\rho$ ,  $\theta$ , and  $\eta$  (in Calvo pricing, equation 12), and  $\rho$ ,  $\theta$ , and  $J$  (in Taylor pricing, equation 13).

For comparative purposes, we will fix the same degree of price stickiness under Calvo and Taylor schemes (parameterized by  $\eta$  and  $J$ , respectively). Let  $Q$  denote the average number of quarters without price adjustment which would represent the level of pricing rigidities. If we notice that in Taylor model  $Q=J$  whereas in Calvo model  $Q=(1 - \eta)^{-1}$ , the calibration  $\eta = [0.5, 0.75, 0.875]$  and  $J = [2, 4, 8]$  provides three specifications for both models in which  $Q$  is two quarters (half a year), four quarters (one year), and eight quarters (two years). These three alternative of price rigidities are going to be examined next.

To begin with, let us assume the following baseline calibration for the other two parameters:

$\rho = 0.005$  (which implies a 2% annual rate of discount), and  $\theta = 10.0$ . Figure 1 displays the plots of  $\psi^{-1}$  related to  $\pi$ . For all the cases depicted, there is a u-shaped pattern representing the steady-state influence of inflation over the markup. In other words, there is a minimal markup at some (optimal) rate of inflation. The left columns of Table 1 provide the numbers. Remarkably, all the sticky-price specifications give a minimal markup at a steady-state rate of inflation very close to 0.2% per year. It means that neither the Chicago rule ( $-400\frac{\rho}{1+\rho} \simeq -2\%$ ) nor the 0% rate of inflation minimize the mark-up. The minimal mark-up is obtained at a low positive rate of inflation, very close to 0.2%. This results is robust to considering the three different levels of price stickiness ( $Q=2$ ,  $Q=4$ , and  $Q=8$ ) in both Calvo pricing as well as Taylor pricing. Therefore, the degree of price stickiness in either Calvo or Taylor models has no influence on determining the rate of inflation in steady state that minimizes the mark-up.<sup>5</sup> The pricing behavior only determines the size of the welfare cost of inflation. With Calvo staggered prices the welfare losses would be clearly larger than with Taylor prices because the markup increases much more rapidly when steady-state inflation moves from its optimal value (compare Calvo and Taylor models in Figure 1).<sup>6</sup> In addition, the longer is the average time without adjusting prices ( $Q$ ), the larger is the welfare cost when inflation deviates from the optimal rate (see Figure 1 within Calvo and Taylor models).

A sensitivity analysis can be conducted by finding rates of inflation that minimizes the mark-up under alternative calibrations for  $\rho$  and  $\theta$ . Results are reported in Table 2. When  $\rho=0.01$  and  $\theta=10.0$ , the mark-up is minimized at a higher rate of inflation, near 0.4%. It implies that a rise of the real interest rate from the baseline  $\rho=0.005$  to  $\rho=0.01$  leads to a higher rate of inflation to minimize the mark-up. This result is obtained with both Calvo and Taylor pricing for any degree of price stickiness. The next calibration reported in Table 2 is  $\rho=0.005$  and  $\theta=6.0$ . This case represents a larger monopolistic power (lower  $\theta$ ) compared to the baseline calibration. Remarkably, the rate of inflation that makes the mark-up minimal in steady state is again higher, close to 0.67%. As a consequence, a higher rate of discount  $\rho$  or a greater monopolistic power (lower  $\theta$ ) will result in a higher rate of inflation needed to minimize the

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<sup>5</sup>This seems somehow surprising because the steady-state relationships (12) and (13) include the price stickiness parameters  $\eta$  and  $J$ .

<sup>6</sup>The same result has been found by Kiley (2002).

mark-up in steady-state. Clearly, this desirable rate of inflation depends on the values assigned to  $\rho$  or  $\theta$  while it did not depend on the pricing behavior. Moreover, it is readily noticeable how this rate of inflation can be fairly well approximated by the ratio  $400\frac{\rho}{\theta}$  (see Table 2). Thus, the ratio of the annualized rate of discount ( $400\rho$ ) over the Dixit-Stiglitz elasticity ( $\theta$ ) provides a very good approximation to the rate of inflation that would minimize the mark-up in steady state.<sup>7</sup>

Returning to the influence of sticky prices in the sensitivity analysis, there is hardly any influence. Once  $\rho$  and  $\theta$  are set, the sticky-price specification (either *à la* Calvo or *à la* Taylor) does not matter for the rate of inflation that minimizes the mark-up. As reported in Table 2, both the Calvo and Taylor pricing provide very similar numbers under any  $Q$ . There is just one minor difference. The Calvo pricing seems to give slightly higher rates of inflation than the Taylor one, especially when there is great price stickiness (see the cases with  $Q=8$ ). However, the difference is quantitatively very small.

Summarizing, the Calvo and Taylor sticky-price specifications have no influence on the determination of the rate of inflation that minimizes the mark-up in steady state. This rate of inflation is a low positive figure, which is not determined by the price-adjustment scheme or the degree of price stickiness. Rather, it is characterized by the model parameters  $\rho$  with a positive influence and  $\theta$  with a negative influence.

## 4 Conclusions

In this paper we have derived the steady-state relationship between the mark-up and the rate of inflation in a monopolistic competition model with two different sticky-price specifications: the Calvo pricing and the Taylor pricing. The minimal mark-up is obtained at a positive and low rate of inflation in both cases. Furthermore, its value is fairly well represented by  $400\frac{\rho}{\theta}$ , which is the ratio between the annual rate of discount and the Dixit-Stiglitz elasticity of a monopolistic competition model.

Regarding the influence of price stickiness, our results show that the rate of inflation that

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<sup>7</sup>The approximation is also very good with many other sensible calibrations of  $\rho$  and  $\theta$  which have been examined, though they are not included in Table 2.



minimizes the mark-up in steady state is nearly identical with the Calvo and Taylor pricing behavior. Moreover, this result can be extended to say this rate of inflation is nearly the same for any extent of price rigidities (ranging from half a year to two years without price adjustment). Therefore, neither the pricing scheme nor the level of price stickiness play any significant role in its determination.

## REFERENCES

- Blanchard, O.J., and N. Kiyotaki, 1987, Monopolistic competition and the effects of aggregate demand, *American Economic Review* 77, 647-666.
- Calvo, G. A., 1983, Staggered pricing in a utility-maximizing framework, *Journal of Monetary Economics* 12, 383-396.
- Casares, M., 2004, Price setting and the steady-state effects of inflation, *Spanish Economic Review* 6, 267-289.
- Chari, V.V., P.J. Kehoe, and E.R. McGrattan, 2000, Sticky price models of the business cycle: Can the contract multiplier solve the persistence problem?, *Econometrica* 68, 1151-1180.
- Dixit, A. K., and J.E. Stiglitz, 1977, Monopolistic competition and optimum product diversity, *American Economic Review* 67, 297-308.
- Erceg, C. J., D. W. Henderson, and A. T. Levin, 2000, Optimal monetary policy with staggered wage and price contracts, *Journal of Monetary Economics* 46, 281-313.
- Huang K. X. D. , and Z. Liu, 2002, Staggered price-setting, staggered wage-setting, and business cycle persistence, *Journal of Monetary Economics* 49, 405-433.
- Khan, A., R. G. King, and A. L. Wolman, 2003, Optimal monetary policy, *Review of Economic Studies* 70, 825-860.
- Kiley, M. T., 2002, Partial adjustment and staggered price setting, *Journal of Money, Credit, and Banking* 34, 283-298.
- King, R. G., and A. L. Wolman, 1996, Inflation targeting in a St. Louis model of the 21st century, *Federal Reserve Bank of St. Louis Review* 78, 83-107.
- King, R. G., and A. L. Wolman, 1999, What should the monetary authority do when prices are sticky?, in *Monetary Policy Rules*, J. B. Taylor, ed., University of Chicago Press.
- Sbordone, A. M., 2002, Prices and unit labor costs: a new test of price stickiness, *Journal of Monetary Economics* 49, 265-292.

Taylor , J. B., 1980, Aggregate dynamics and staggered contracts, *Journal of Political Economy* 88, 1-24.

Yun, T., 1996, Nominal price rigidity, money supply endogeneity, and business cycles, *Journal of Monetary Economics* 37, 345-370.

Table 1. Sticky prices and the rates of inflation that minimize the mark-up in steady state. Baseline calibration ( $\rho=0.005, \theta=10.0$ ).		
Sticky prices <sup>8</sup>	Rate of inflation (annualized, %) that minimizes $\psi^{-1}$	
	Calvo pricing, (12)	Taylor pricing, (13)
Q=2	0.201	0.199
Q=4	0.203	0.199
Q=8	0.206	0.199

Table 2. Sticky prices and the rates of inflation that minimize the mark-up in steady state. Sensitivity analysis.			
$\rho=0.01, \theta=10.0$			
Sticky prices	Rate of inflation (annualized, %) that minimizes $\psi^{-1}$		$400\frac{\rho}{\theta}$
	Calvo pricing, (12)	Taylor pricing, (13)	
Q=2	0.404	0.398	0.40
Q=4	0.411	0.398	0.40
Q=8	0.427	0.398	0.40
$\rho=0.005, \theta=6.0$			
Sticky prices	Rate of inflation (annualized, %) that minimizes $\psi^{-1}$		$400\frac{\rho}{\theta}$
	Calvo pricing, (12)	Taylor pricing, (13)	
Q=2	0.335	0.332	0.33
Q=4	0.338	0.333	0.33
Q=8	0.343	0.333	0.33
$\rho=0.01, \theta=6.0$			
Sticky prices	Rate of inflation (annualized, %) that minimizes $\psi^{-1}$		$400\frac{\rho}{\theta}$
	Calvo pricing, (12)	Taylor pricing, (13)	
Q=2	0.672	0.664	0.67
Q=4	0.684	0.664	0.67
Q=8	0.708	0.664	0.67

<sup>8</sup>Expected number of quarters without price adjustment (Q).

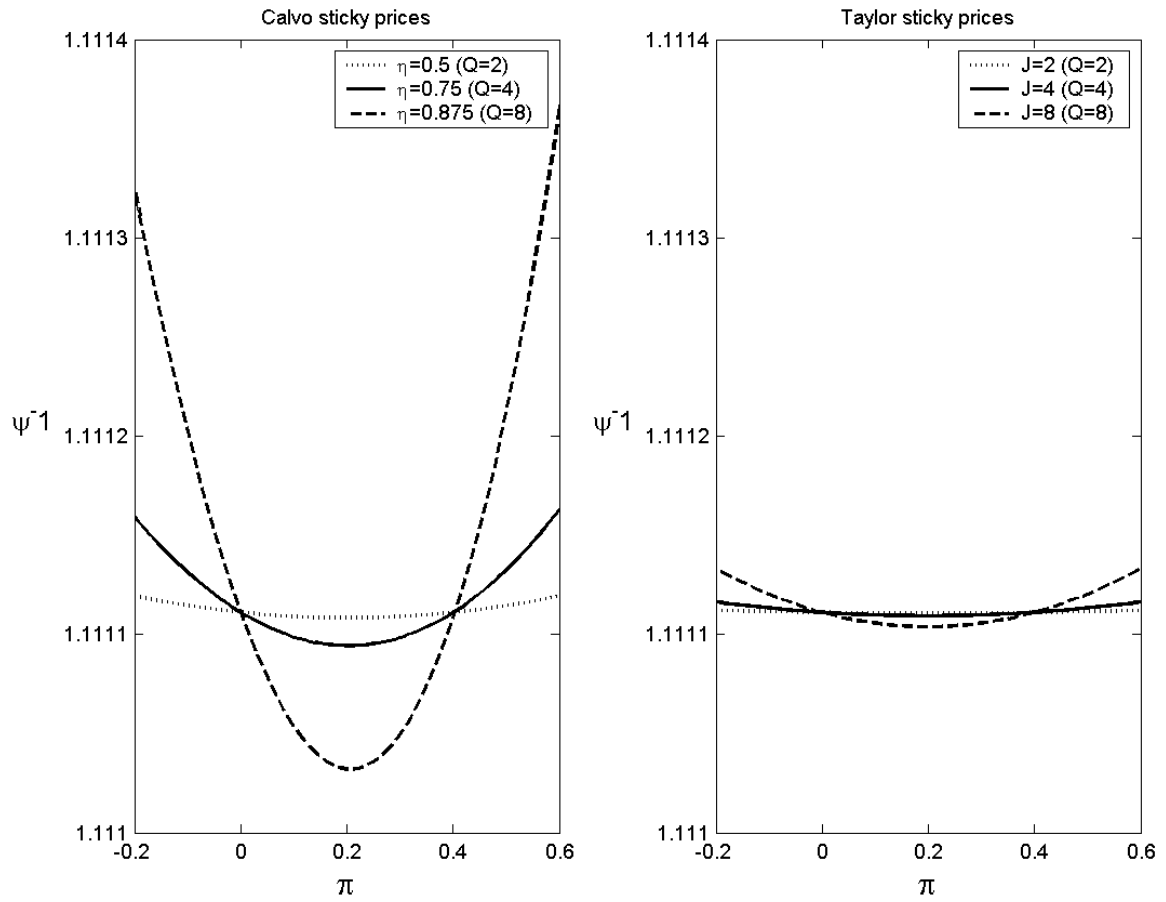


Figure 1: Steady-state relationship between annualized percent inflation ( $\pi$ ) and the average mark-up ( $\psi^{-1}$ ). Baseline calibration,  $\rho = 0.005$  and  $\theta = 10.0$ .