Sense and sensibility: using a model to examine the relationship between public pre-school places and fertility

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Abstract

This paper presents a stochastic dynamic mathematical model, in which a Family Policy Index (*XFPI*) is included to measure and compare two different models of provision of resources to support families with children from 0 to 3 years old. The main variables in this model are the *XFPI*, fertility, mortality, emigration and immigration rates. This mathematical model was validated in two different countries, Spain and Norway, during the 2007-2015 period. A sensitivity analysis was applied to simulate the future trend (2016-2030), examining the influence of providing public pre-school services (0 to 3 years) on (*XISF*). The results obtained show that these services may indeed have an influence on fertility rates, as long as they are developed extensively.

Key words: public childcare service; fertility; *XFPI*; Spain; Norway; family policies; demography; dynamic mathematical model.

1. Introduction: mathematical models, fertility and childcare

The aim of this paper is to elaborate, validate and apply a stochastic dynamic mathematical model, defined by sex and age, to improve the prediction of fertility using previous mathematical models. Mathematical models are very useful to outline projections on the behavior of

populations and they are also appropriate to help design public policies, since they anticipate, with a certain degree of reliability, the impact that specific measures may have on citizens' lives. The mathematical model used in this article is based on the design of a series of mathematical formulas that help to calculate the evolution of the population using two types of variables:

a) Demographic variables, in particular those that regulate population growth: births, deaths, emigration and immigration;

b) Variables related to policies for families with children aged 0 to 3 years: these policies are grouped into three categories: the so-called "educational" or "services" policies, which refer to the number of public childcare places offered to children between 0 and 3 years of age; parental leave, that is, the time assigned in each country to fathers and mothers to take care of their children after birth; and finally, monetary transfers, the money that the State grants to families with children (0-3 years) to support their care and upbringing.

In this research, the model was developed and validated, firstly, for the 2007-2015 period, so that the simulated data obtained using the model could be validated by temporal and age cohorts fitted regarding demographic behavior in the past. Once the model has been validated, demonstrating a high level of fit to reality, it is considered valid to make projections about population behavior in the future.

The initial hypothesis of this paper is that policies which are designed to support families have an influence on fertility behavior, as these are resources citizens bear in mind when considering having children (Thévenon and Gauthier, 2011). Based on this hypothesis, a mathematical model was developed to predict the impact of family policies on fertility. In this regard, the public childcare services from 0 to 3 years were selected to test the model, as these are fundamental for achieving a satisfactory work-life balance and enhancing gender equality. In this sense, this paper aims to expand the existing sociological tools to analyze the impact of certain measures on

demographic phenomena, providing effective instruments to evaluate the implementation of future family policies.

The mathematical model constructed in this paper is composed of socio-demographic variables defined by age and sex, and is based on a previous model, the von Foerster-Mckendrick model, which was originally designed to predict population dynamics, by sex and age, in a generic human population (Micó, Caselles, Soler, Sanz & Martínez, 2008). This model was improved through the development of the *Family Policy Index (XFPI)*, composed of sociodemographic variables that facilitate the prediction of fertility behavior. The initial hypothesis can be represented by a Forrester Diagram (1961). This diagram is used in System Dynamics to connect all the variables included in the model (Figure 1¹). Likewise, it is a Causal Diagram which allows writing the model equations in the computer with a particular terminology, where the model can be validated, observing the temporal evolution of the variables and performing a sensitivity analysis.

Figure 1: Forrester Diagram of the demographic subsystem and the link with Family Policy

Index

¹ The specific nomenclature used in this model is explained in detail in Annex I.

From a mathematical perspective one of the goals of this paper is, along the same lines as previous research, to incorporate an index within the Von Foerster-McKendrick model. Caselles, Soler, Sanz & Micó (2014) proposed a stochastic demographic model defined by age and sex in which the Human Development Hybrid Index proposed in the Human Development Reports (UNDP, 2010) is included in order to consider life expectancy at birth by gender in a future scenario. Sanz, Micó, Caselles & Soler (2014) present a demographic model defined by sex, in which the Gender-Related Development Index (UNDP, 2010) is introduced with the aim of improving the future well-being of a country, as well as reaching a stable population. Sanz, Caselles, Micó & Soler (2016) designed an index, following the methodology presented in the Human Development Reports (UNDP, 2010), that makes it possible to measure environmental quality using CO₂ emissions and the consumption of renewable and nuclear energies. Likewise, Soler, Sanz, Caselles & Micó (2018) introduced a demographic model defined by sex, which incorporates three other indexes defined by UNDP (2010), namely the Human Development Index, the Gender Development Index and the Gender Empowerment Index, to study, using a sensitivity analysis, which environmental quality variables are correlated to deaths, births and quality of life. Finally, Sanz, Caselles, Micó & Soler (2018) proposed a new index to measure the happiness of a country through five dimensions: development, freedom, solidarity, justice and peace, and introduced it into a demographic model that provides strategies to maximize happiness.

Furthermore, from a sociological perspective, the paper aims to clarify the relationship between family policies and fertility rates in different countries, contributing to understand the social outcomes that result from the implementation of particular public policies.

Conceptual aspects: the relationship between family policies and fertility

Family policies in general, and in particular those aimed at caring for children under three years

of age, have received considerable attention in academic, political and social arenas in recent decades, since they are perceived as a necessary resource to alleviate tensions that arise between public and private life, most particularly, between work and the care of children. Likewise, family policies are a suitable vehicle for modifying existing childcare attitudes and behaviors, both at the individual and social level. Indeed, childcare services, especially when these are public and universal, are considered the most important resource available when it comes to reconciling work and family responsibilities, promoting gender equality (Korpi, 2000; Rindfuss, Guilkey, Morgan, Kravdal & Guzzo, 2007). Regarding the purposes of this article, it is relevant to recall the theories linked to the second demographic transition, referred to the transformation of the patterns of constitution and reproduction of the families after the Second World War. Their postulates are found on the argument that the work-family conflicts emerged as a consequence of the changing gender roles, and also on the idea that the impact of these conflicts on lower fertility can be overcome by developing family support policies and promote fathers involvement in child care (Ferragina and Seeleib-Kaiser, 2015; Goldscheider, Bernhardt & Lappegard, 2015, Navarro & Clua Losada, 2013; Thévenon & Gauthier, 2011).

In this research, the influence of pre-school childcare services on fertility rates was compared in two countries, Spain and Norway, which maintain two different approaches in regards of family policies. While it is true that in recent decades the enrollment rates have increased in both countries, the differences are still very significant. Norway has facilitated childcare services through a public² and universal offer. These public services have been promoted as a fundamental element to support the incorporation of women into the labor market and are supported by a strong consensus in civil society and in the political arena (Rostgaard, 2014). In Spain, however, private initiatives prevail. Although the evolution of enrollment rates in public centers has

² in Norway the public services coexist with privately subsidized childcare centers, but they are assimilated to public provisions in regards of price and services.

increased in recent years, the supply of public places is insufficient to meet the existing demand. Previous studies have identified that public coverage in Spain, for the population aged 0 to 3 years, barely reaches 20% (Elizalde-San Miguel, Díaz Gandasegui & Sanz, 2018), a constraint that forces families to use alternative childcare resources: private services for those families who can afford it and family care (nuclear or extensive) for those who have no access to private services or prefer this traditional resource.

As mentioned above, Norway's commitment to promoting universal childcare services is part of a broader and more explicit objective: to support and facilitate female employment (Hegewisch & Gornick, 2011). This is a strategy that has proved effective in recent years, reaching female employment rates over 70% (Labor Force Survey, Eurostat³), and has been combined with relatively high fertility rates, ranging between 1.7 and 1.9 children per woman, in a context of low European fertility (Kvande, 2009; Læppegård, 2010), confirming that countries with highest female employment rates are also those with the highest fertility rates (Daly, 2000; Esping-Andersen, 2002; Læppegård, 2010; Thevenon, 2011). Spain represents the opposite example. Spain has one of the lowest female employment rates in Europe (it only recently exceeded 50%⁴) and is also the European country with the lowest fertility rate, between 1.1 and 1.4 children (Eurostat: fertility indicators⁵).

Certainly, this relationship between female employment, fertility and family policies has been a very prolific line of analysis in recent years in Social Sciences from different perspectives (Billingsley, 2014; Björklund, 2006; D'Addio & d'Ercole, 2005; Gauthier, 2013; Gauthier & Philipov, 2008; Hoem, 2008; Kalwij, 2010; Luci-Greulich & Thévenon, 2013; McDonald, 2001;

³ Employment rates. Labour Force Survey Series:EU_MAIN_TREE.data.popul.labour.employ.lfsa.lfsa_emprt'. Data extracted on May 25th 2018.

⁴ Idem.

⁵ Fertility indicators: (demo_find): data extracted on May 25th, 2018

Figure 2: Forrester Diagram of the Family Policy Index subsystem and the link with Demographic Subsystem.

Starting from this table of relations between variables, together with the variables introduced in Figure 1, the mathematical model was elaborated and validated.

2. Methods and Materials

2.1. The Demographic Model

The mathematical model developed for this paper is composed of the basic equations of the Von Foerster-McKendrick model, incorporating the Family Policy Index to calculate the structure of the population.

The fundamental variables of the model are the population densities of men and women per unit of age, that is, the distribution of the population by sex and age cohorts. In this paper, this structure has been maintained, but the equations have been written in finite differences, following the classical structure of the equations for a System Dynamics model. The main contribution of this research is the introduction of the Family Policy Index, which allows an in-depth study of fertility considering not only demographic but also sociological variables, which have a significant influence on fertility behavior (Läppegard, 2010).

Below, the generic equations of the demographic model used are presented. The starting point of this demographic model is that by Micó et al. (2008), defined by sex, time and age cohorts. In its continuous form it is constituted by the following equations:

$$\frac{\partial w_i(t, x)}{\partial t} + c \frac{\partial w_i(t, x)}{\partial x} = -b_i(t, x)w_i(t, x) + m_i(t, x), \quad (1)$$

$$w_i(t, 0) = (\int_0^\infty a_i(t, x)w_2(t, x)), \quad (2)$$

$$w_i(t_0, x) = u_i(x) \quad (3)$$

Where $i = 1$ represents men and $i = 2$ women.

Equation (1) from von Foerster-McKendrick determines the dynamic of the population density $w_i(t, x)$ ($POPL_i$ in Figure 1), where $b_i(t, x)$ represents the specific rate of mortality ($RDEA_i$ in Figure 1) and $m_i(t, x)$ the migratory balance (immigrations minus emigrations, $YNMI_i$ y $EMIG_i$ in Figure 1). From Equation 1 we obtain, therefore, the distribution of the population by age, sex and year.

Equation (2) represents the boundary condition, that is, the population that is 0 years old, $x=0$, which equals the births ($\int_0^\infty a_i(t, x)w_2(t, x)$, $a_i(t, x)$ being the specific fertility rate by age of the mother and defined by the sex of the newborn).

Equation (3) is the initial condition, that is, the population density at the beginning of the period ($t=t_0$), $u_i(x)$ or the distribution of the population considering the age in the initial year.

From this model, as indicated above, it was assumed that the specific fertility rate by age of the mother and defined by the sex of the newborn ($a_i(t, x)$) ($RFER_i$ in the Figure 1) can be calculated through the Family Policy Index. This will therefore be considered to be dependent on age and $XFPI$ (4),

$$a_i(t, x) = a_i(XFPI(t), x), i = 1, 2, \quad (4)$$

On the other hand, other hypotheses have also been made about migratory balance and deaths, following what Caselles, Soler, Sanz & Micó (2014) proposed in their research. Firstly, given that the migratory balance is defined by the difference between immigration and emigration (5), this will be defined as the product of migrations by age cohort in the initial year ($f_i(x)$ y $g_i(x)$)

and population density, (6) and (7). We should clarify that this simplification in the migratory balance is accepted, in the current research, by the satisfactory results obtained in the model validation.

$$m_i(t, x) = ynmi_i(t, x) - emig_i(t, x) \quad (5)$$

$$ynmi_i(t, x) = f_i(x) \cdot w_i(t, x) \quad (6)$$

$$emig_i(t, x) = g_i(x) \cdot w_i(t, x) \quad (7)$$

Finally, deaths are defined as the product of the specific mortality rate by age and sex in the initial year ($b_i(x)$) and population density (8)

$$b_i(t, x)w_i(t, x) = b_i(x)w_i(t, x) \quad (8)$$

With these considerations concerning the initial model, the following equations are obtained,

$$\frac{\partial w_i(t, x)}{\partial t} + c \frac{\partial w_i(t, x)}{\partial x} = (-b_i(x) + f_i(x) - g_i(x))w_i(t, x), \quad (9)$$

$$w_i(t, 0) = \left(\int_0^\infty a_i(XFPI(t), x)w_2(t, x) \right), \quad (10)$$

$$w_i(t_0, x) = u_i(x) \quad (11)$$

The solutions were calculated by the Euler approximation in both variables, following the classical structure of the equations for a System Dynamics model, in which complex phenomena can be modelled and analyzed considering the feedback process among different variables. This method of approximation is considered the optimum in previous research, such as Djidjeli, Price, Temarel & Twizell (1998) and Christopher (2004).

2.2.Obtaining the Demographic Rates and the *XFPI*

Once the basic equations of the model (9), (10) and (11) had been defined, we proceeded to design the functions that allow us to calculate, in a fitted way, the values of all the demographic rates.

The historical demographic data for Spain and Norway in the 2007-2015 period regarding population, deaths, births, emigration and immigration, by sex and age, were obtained from Eurostat. Regarding the variable $XFPI$, the data for its calculation, in both countries, were obtained from various sources, as specified in Annex I.

The details of the equations, which fit the historical data for both countries, are explained below. Annex I shows the determination coefficients, as well as the randomness of residues and the graph fitted to the real data and the data calculated by the equation.

- **Specific fertility rate by age of the mother and defined by sex of the child**

$a_i(XFPI(t), x)$. The data used correspond to the initial year, 2007. The information obtained represent the division of men and women, the births by the sex of the newborn and the age of the mother divided by women of that age, considering the range 14-50 years, this being the fertile age.

The fitted function is the linear combination of two Gaussian equations with positive parameters:

$$a_i(XFPI(t), x) = \alpha_{i0} + \alpha_{i1} e^{-\frac{(x \cdot XFPI(t) - \mu_{i1})^2}{2\beta_{i1}}} + \alpha_{i2} e^{-\frac{(x \cdot XFPI(t) - \mu_{i2})^2}{2\beta_{i2}}} \quad (12)$$

- **Specific mortality rates by age and sex $b_i(x)$.** The data used correspond to the initial year, 2007. These rates have been obtained with the distribution of deaths by sex and age between the population density.

The function considered to fit the real data is a function defined by parts:

$$b_i(x) = \begin{cases} \alpha_{i0} + \alpha_{i1} e^{-\frac{x}{\beta_{i1}}} + \sum_{j=2}^3 \alpha_{ij} e^{-\frac{(x-\mu_{ij})^2}{2\beta_{ij}}} & 0 \leq x \leq 60 \\ \alpha_{i4} + \alpha_{i5} e^{-\frac{x-\mu_{i5}}{\beta_{i5}}} + \sum_{j=6}^7 \alpha_{ij} e^{-\frac{(x-\mu_{ij})^2}{2\beta_{ij}}} & 61 \leq x \leq 100 \end{cases} \quad (13)$$

It should be noted that in this case there has not been a fitted like the one presented by Micó et al. (2008), since the data trend of the first age cohorts do not follow the same pattern as the last cohorts, and therefore more parameters have been introduced.

- **Specific rates of Immigration and Emigration by age and sex** ($f_i(x)$, $g_i(x)$, respectively). The data used correspond to the initial year, 2007. The information obtained represents the distribution of emigrants and immigrants by sex and the receiving population, considering sex.

The following function was considered to fit this demographic data:

$$f_i(x) \text{ ó } g_i(x) = \alpha_{i0} + \sum_{j=1}^4 \alpha_{ij} e^{-\frac{(x-\mu_{ij})^2}{2\beta_{ij}}} \quad (14)$$

The rates constructed are valid for both countries (see detail in Annex III), since the fitted degree between the values simulated by the designed functions and the real data is very high. It is therefore understood that these formulas can be extrapolated to any country with sufficient demographic information.

- **Family Policy Index, $XFPI(t)$.** This is calculated through 16 input variables (Elizalde-San Miguel, Díaz Gandasegui and Sanz, 2018) which were fitted according to the period considered (1999-2015 period). Table 1 shows those variables that can be fitted, since their real data is not constant in the period when the information is available. As the model is defined in a continuous way, these input variables must be fitted over time by logistical functions that depend on these trends (Marchetti, Meyer, & Ausubel, 1996). The graphic adjustment of these functions, the coefficient of determination obtained and the detail of the functions for each variable can be found in Annex I.

$$input_{variable} = \alpha_0 + \frac{1}{1 + \alpha_1 e^{-\frac{x-\mu_1}{\beta_1}}} \quad (15)$$

Table 1. Input variables of the Demographic Model

Acronyms	Temporary adjustment	
	Spain	Norway
<i>DFAL</i>	Yes	Yes
<i>DMAL</i>	NO (constant)	NO (constant)
<i>DPLS</i>	NO (constant)	NO (constant)
<i>ECCB</i>	NO (constant)	NO (constant)
<i>OVLP</i>	NO (constant)	NO (constant)
<i>TDFA</i>	NO (constant)	NO (constant)
<i>TICB</i>	NO (constant)	NO (constant)
<i>TICC</i>	NO (constant)	NO (constant)
<i>TPAR</i>	NO (constant)	NO (constant)
<i>XCCB</i>	Yes	NO (constant)
<i>XPRI</i>	NO (data not available)	Yes
<i>XPUB</i>	Yes	Yes
<i>ECCC</i>	NO (constant)	NO (constant)
<i>ECBG</i>	NO (constant)	NO (constant)
<i>XCBG</i>	NO (constant)	NO (constant)
<i>PPPD</i>	Yes	NO (constant)

It should be noted that the adjustment of the historical data to the mathematical structures for each country was performed using Mathematica 11.00 with the NonLinearModelFit package and with the fitter function Regint (Caselles, 1998, 2008).

3. Results

3.1. Validation of the Model

As mentioned above, the historical data necessary to validate the model was obtained from different statistical bases, in the 2007-2015 period. The software used for validation was SIGEM (Caselles, 1998, 2008).

The model was designed as a set of differential and functional equations. The solutions were calculated with the Euler Method following Djidjeli et al. (1998), which explains that the Euler Method is the most adequate one to solve such equations. This approach results in a set of finite differential equations that were programmed in Visual Basic 6.0 and executed using the software SIGEM.

The validation was performed numerically by calculating the determination coefficients and the random residue tests. In addition, the degree of overlap of the results obtained for each year and the actual historical data are graphically represented. The validation process was considered successful for the period under examination in both countries, because the coefficients of determination, R^2 , are very high, and the maximum relative error does not exceed 2.3%. This was tested with the specific fertility rate by age of the mother and defined by the sex of the newborn in Norway in the first and last simulated year (2008 and 2015, respectively).

Figure 3: Specific fertility rate by age of the mother and defined by sex of the newborn ($\cdot 100$). Values for male newborns in Norway in the year 2008. $R^2=0.9977$. Real data (points) and simulated data (line).

Figure 4: Specific fertility rate by age of the mother and defined by sex of the newborn ($\cdot 100$). Values for male newborns in Norway in the year 2015. $R^2=0.9727$. Real data (points) and simulated data (line).

Figure 5: Specific fertility rate by age of the mother and defined by sex of the newborn ($\cdot 100$). Values for female newborns in Norway in the year 2008. $R^2=0.9974$. Real data (points) and simulated data (line).

Figure 6: Specific fertility rate by age of the mother and defined by sex of the newborn ($\cdot 100$). Values for female newborns in Norway in the year 2015. $R^2=0.9711$. Real data (points) and simulated data (line).

3.2. Application: Sensitivity Analysis

Once the mathematical model had been validated, demonstrating its usefulness for calculating the fertility behavior using demographic variables and family policies, a sensitivity analysis was carried out, reflecting the input variables of the model, which have a higher incidence on a defined objective variable.

The sensitivity analysis applied in this research measures the influence that a minor change in an input variable, in this case the provision of public places in nursery schools (*XPUB*), may have on an output variable, the Synthetic Index of Fertility (*XISF*). As mentioned earlier, this model will be used to examine whether the increase in public investment in pre-school places has an impact on the synthetic fertility index of a given country, in this case Spain and Norway, in a simulated future trend (2016- 2030).

This output variable could also be affected by the remaining input variables. Therefore, in order to observe the real effect of the *XPUB* variable in the given output variable *XISF*, other input variables must be considered as a constant or a random sample of all the possible combined values should be examined. Otherwise, as in the present case, the analysis would be valid only for the specific situation in question. An example of this approach has been presented by Caselles, Ferrer, Martínez de Lejarza, Pla, & Temre (1999).

Some methodological observations:

- It is assumed that the input variables are independent.
- If the real data do not proceed from a random sample taken within the range of all possible values for each input variable, the conclusions will only be valid for the particular situation considered.
- The future trend was simulated using the stochastic model, simulations were carried out to find the best direct relationship between each input variable and the selected output variable.
- These data were fitted by linear and/or quadratic functions because with these functions it is easier to interpret their coefficients in relation to the sensitivity of the dependent variable with respect to the independent variable. In the linear function, $y = m \cdot x + n$, represents the increasing or decreasing rate of y in relation to x . In the quadratic function

$y = a \cdot x^2 + b \cdot x + c$, this increasing or decreasing rate is determined by the derivative function, $2 \cdot a \cdot x + b$. The aforementioned rate is what determines the degree of sensitivity of y in relation to x ; that is, the increase or decrease of y for each unit of increase in x in general (for a linear fitted) or at each point (for the parabolic type fitted).

- The determination coefficient, R^2 , determines the fraction of variability of the output variable, which is explained by the input variable with the considered function.

It should be noted that the $XISF$ was calculated by adding the $RFER$ (Specific Rates of Fertility by Age) of the different age groups (15-49 years), and is expressed as one unit,

$$XISF(t) = \sum_{j=15}^{49} \frac{RFER}{1000} \quad (16)$$

Consequently, a sensitivity analysis of the real data will be carried out and the results will be extrapolated to the future, with a sensitivity analysis in the 2016-2030 period.

3.2.1. Sensitivity analysis of the past

The sensitivity of the past was performed using historical real data, for Spain during the 1999-2014 period and for Norway during the 1999-2015 period.

The results for Spain are as follows:

Figure 7: Spain. Sensitivity analysis of the past period. *XISF*, Synthetic Fertility Index (Y) against *XPUB*, Public Places in Public Schools (X). From 1999 to 2014.

$$xisf(xpub) = 1.02176250 + 0.0000049126xpub - 1.5686 \times 10^{-11}xpub^2$$

In Figure 7 it can be observed that the *XPUB* variable is related to *XISF* ($R^2 = 0.8579$), in a quadratic relationship.

The resulting parabola from the adjustment of the simulated data can be interpreted as follows:

1 - The potential impact associated with a possible variation in *XPUB* is calculated using the derived function,

$$2 \cdot (-1.5686 \cdot 10^{-11}) \cdot xpub + 0.0000049.$$

2 - The maximum value of *XISF* (approximately 1.45) is associated with a value of *XPUB*,

$$XPUB = -0.0000049 / (2 \cdot (-1.5686 \cdot 10^{-11})) = 156190.233$$

$$XISF = -1.5686 \cdot 10^{-11} \cdot 42616.95^2 + 0.0000049 \cdot 42616.95 + 1.02176 = 1.404$$

3 - As the *XPUB* values during the 1999-2014 period are between 90000 and 248000, an increase of 10000 seats at its lowest value is associated with the variation in *XISF*,

$$(2 \cdot (-1.5686 \cdot 10^{-11}) \cdot 90000 + 0.0000049) \cdot 10000 = 0.0207652$$

While the same increase in its highest value is associated with the variation in *XISF*:

$$(2 \cdot (-1.5686 \cdot 10^{-11}) \cdot 248000 + 0.0000049) \cdot 10000 = -0.0288026,$$

That is, the sensitivity of the *XISF* value in relation to *XPUB* is low, being positive near the lower limit and negative near the upper limit.

4 - Using the same procedure, it is possible to obtain any intermediate value.

Based on the values obtained, the sensitivity analysis applied to the past shows a quadratic evolution in the relationship between fertility in Spain and the provision of public places in pre-school services from 0 to 3 years old. This implies that the increase in the number of public places contributes, only up to a certain point, to raise fertility. These results must be contextualized considering that at the moment of maximum expansion of public pre-school services in Spain, in the first decade of the twenty-first century (the previous stage to our model), the offer increased from 40000 places (42440 in 2000) to 125000 in 2007, an increment of nearly 300%. In that period the fertility also evolved from values that reached a historical minimum (1.13 children per woman in 1998) to the maximum of the last decades (1.45 in 2008). Indeed, it is pertinent to remark that this was a period of economic expansion, development of significant public investment in care services and with a considerable increase of migrant population; consequently, these reasons may have affected the evolution of fertility rates during those years. Hence, the model demonstrates that the investment in public childcare services has an important influence on fertility during the first period (until 2008), which can be named the moment of impact, but, from that year on, the number of public places offered in Spain has been rising at a slower pace and, consequently, there is a significant decline in impact produced in the synthetic fertility index. In this sense, it is noteworthy that the number of children between 0 and 3 years, in 2015, was 1305000 children, for whom 215000 public places were provided, which represents only 16.5% of coverage, a provision which does not encourage families to have children, and explains, at least partially, the decline in fertility. Certainly, even in the period of maximum expansion the services were insufficient, transmitting the lack of commitment of the State towards the families with children.

The results for Norway are as follows:

Figure 8: Norway. Sensitivity analysis of the past period. *XISF*, Synthetic Fertility Index (Y) against *XPUB*, Public Places in Public Schools (X). From 1999-2015.

$$xisf(xpub) = 1.56543 + 0.00000731426xpub - 4.21143 \times 10^{-11}xpub^2$$

In the Figure 8 it can be observed that the *XPUB* variable is related to *XISF* ($R^2 = 0.196846$), in a quadratic relation.

The resulting parabola from the adjustment of the simulated data can be interpreted as follows:

1 - The potential impact associated with a possible variation in *XPUB* is calculated using the derived function,

$$2 \cdot (-4.21143 \cdot 10^{-11}) \cdot x_{pub} + 0.00000731.$$

2 - The maximum value of *XISF* (approximately 1.88) is associated with a value of *XPUB*,

$$XPUB = -0.00000731 / (2 \cdot (-4.21143 \cdot 10^{-11})) = 86838$$

$$XISF = -4.21143 \cdot 10^{-11} \cdot 43766.39^2 + 0.00000731 \cdot 43766.39 + 1.56543 = 1.88301$$

3 - As the *XPUB* values during the 1999-2014 period are between 45000 and 103000, an increase of 10000 places at its lowest value is associated with the variation in *XISF*,

$$(2 \cdot (-4.21143 \cdot 10^{-11}) \cdot 45000 + 0.00000731) \cdot 10000 = 0.0352398$$

While the same increase in its highest value is associated with the variation in *XISF*:

$$(2 \cdot (-4.21143 \cdot 10^{-11}) \cdot 103000 + 0.00000731) \cdot 10000 = -0.01362,$$

That is, the sensitivity of the *XISF* value in relation to *XPUB* is high, being positive near the lower limit and negative near the upper limit.

4 - Using the same procedure, it is possible to obtain any intermediate value.

In Norway, fertility is significantly higher than in Spain; indeed, in recent years the fertility has reached values of 1.8-1.9 children per woman due to the strong development of family policies in the country (Rindfuss et al., 2007).

The relationship between fertility and the investment in public places in pre-school services is similar to that observed in Spain, i.e. a quadratic relationship, which indicates that the increase in public places coincides with a growth in fertility, although the intensity of this relationship points to a different explanation in the Scandinavian country. The maximum values of fertility in Norway, which are around 1.9 children per woman, concur with a provision of approximately 90000 places, which represent almost universal coverage (since the population of children from

0 to 3 years old is around 100000). With this offer of public places, fertility acquires values that range between 1.7 and 1.9, figures that are notably higher than in Spain and with an oscillation which responds to other factors. Therefore, family policies and, in particular, the provision of pre-school services, are an incentive for fertility but, once universal coverage is guaranteed, the impact of this service on fertility obviously disappears.

3.2.2. Sensitivity analysis of the future

Once the relationship between these two variables has been verified in the past, the mathematical model designed provides information about the foreseeable future, specifically in the 2016-2030 period. Figure 9 shows the results for Spain.

Figure 9: Spain. Sensitivity analysis for the future period. *XISF*, Synthetic fertility index (Y) against *XPUB*, Public Places in Public Schools (X). From 2016 to 2030.

$$xisf(xpub) = -13042.735 + 0.11013xpub - 2.32467 \times 10^{-7}xpub^2$$

In the Figure 9 it can be observed that the *XPUB* variable is related to *XISF* ($R^2 = 0.881$), in a quadratic relation.

The resulting parabola from the adjustment of the simulated data can be interpreted as follows:

1 - The potential impact associated with a possible variation in $XPUB$ is calculated using the derived function,

$$2 \cdot (2.325 \cdot 10^{-7}) \cdot xpub + 0.1101.$$

2 - The maximum value of $XISF$ (approximately 0.675) is associated with a value of $XPUB$,

$$XPUB = -0.1101 / (2 \cdot 2.325 \cdot 10^{-7}) = 236872.303$$

$$XISF = -2.325 \cdot 10^{-7} \cdot 236872.303^2 + 0.1101 \cdot 236872.303 - 13042.735 = 1.36$$

3 - As the $XPUB$ values during the 2016-2030 period are between 236500 and 238000, an increase of 1000 places at its lowest value is associated with the variation in $XISF$,

$$(2 \cdot (2.325 \cdot 10^{-7}) \cdot 236500 + 0.1101) \cdot 1000 = 0.173$$

While the same increase in its highest value is associated with the variation in $XISF$:

$$(2 \cdot (2.325 \cdot 10^{-7}) \cdot 238000 + 0.1101) \cdot 1000 = -0.524$$

That is, the sensitivity of the $XISF$ value in relation to $XPUB$ is low, being positive near the lower limit and negative near the upper limit.

4 - Using the same procedure, it is possible to obtain any intermediate value.

The application of the model developed in this research to the prediction of fertility in Spain, assuming that family policies will maintain their current trend (characterized by their underdevelopment), indicates that the provision of public pre-school childcare services will grow modestly, oscillating between 236500 and 238000 in the simulated period (see Figure 10). As a result, the fertility will continue the gradual fall experienced in the recent past.

Figure 10: Comparison of the sensitivity analysis of Spain, past (black) and future (blue) period for $XISF$, Synthetic Fertility Index (Y) against $XPUB$, Public Places in Public Schools (X). From 2016 to 2030.

The future projection of the current situation in Spain demonstrate that the insufficient provision of public pre-school childcare services, around 20% of the coverage (Elizalde-San Miguel, Díaz Gandasegui & Sanz, 2018), is not perceived as a stimulating incentive for families to have children. Certainly, the architecture of the childcare services is unsatisfactory and does not influence the decision to have children positively. The provision of pre-school services in the Mediterranean country has not been used to increase fertility rates and as a result the families are presently aware that the lack of public pre-school services may produce economic constraints and work-family conflicts. However, the situation in Norway has been different. The predicted results for Norway are as follows:

Figure 11: Norway. Sensitivity analysis for the future period. *XISF*, Synthetic Fertility Index (Y) against *XPUB*, Public Places in Public Schools (X). From 2016 to 2030.

$$xfpi(xpub) = 355329 - 6.900985 \cdot xpub + 0.0000335069xpub^2$$

In Figure 11 it can be observed that the *XPUB* variable is related to *XISF* ($R^2 = 0.668664$), in a quadratic relationship.

The resulting parabola from the adjustment of the simulated data can be interpreted as follows:

1 - The potential impact associated with a possible variation in *XISF* is calculated using the derived function,

$$2 \cdot (0.0000335069) \cdot xpub - 6.900985.$$

2 - The maximum value of *XISF* (approximately 1.984) is associated with a value of *XPUB*,

$$XPUB = 6.900985 / (2 \cdot (0.0000335069)) = 102979$$

$$XISF = 0.0000335069 \cdot 102979^2 - 6.900985 \cdot 102979 + 355329 = 1.95817$$

3 - As the *XPUB* values during the 2016-2030 period are between 102860 and 103000, an increase of 50 places at its lowest value is associated with the variation in *XISF*

$$(2 \cdot (0.0000335069) \cdot 102860 - 6.900985) \cdot 50 = -0.39727$$

While the same increase in its highest value is associated with the variation in *XISF*:

$$(2 \cdot (0.0000335069) \cdot 102860 - 6.900985) \cdot 50 = 0.07182,$$

That is, the sensitivity of the *XISF* value in relation to *XPUB* is low, being negative near the lower limit and positive near the upper limit.

4 - Using the same procedure, it is possible to obtain any intermediate value.

In Norway, as mentioned earlier, the provision of public places in the pre-school stage recently attained almost universal coverage and, consequently, the quadratic relationship indicates that once the offer has reached a certain value (above 80%) the influence of this variable on fertility decreases (so the simulated data are consistent with the historical data). The mathematical model designed ratifies this approach: in Norway, the public places offered for children under three years old cover the educational demand of the population and, correspondingly, the effect of this variable on fertility has become irrelevant, and the fluctuations in the resulting rates can be explained by other factors (see Figure 12).

Figure 12: Comparison of the sensitivity analysis of Norway, past (black) and future (blue) period for *XISF* Synthetic Fertility Index (Y) against *XPUB*, Public Places in Public Schools (X). From 2016 to 2030.

The fertility in Norway, one of the highest rates in Europe, is mostly related to the recent investment in children's education, as it can be observed in the sensitivity analysis covering the past. The pre-school services have been developed considering the existing demand, this is, the maximum level socially required, and consequently provide the perfect scenario to test its impact on fertility rates, which has proved to be positive as it has contributed to increase the fertility rates (within a European context of low fertility rates). Therefore, the case of Norway indicates that pre-school services may be an effective tool to increase fertility rates and (at least partly) explains the current fertility rates in Spain, considering the underdevelopment of these resources in the Mediterranean country.

4. Conclusions

In this paper, we focus on the possible relationship between family policies and fertility taking as a reference two countries, Spain and Norway, which represent two different models of public childcare policies and, at the same time, opposite extremes in regards to fertility in Europe. The results of this research confirm, using a mathematical model, the existence of a quadratic

relationship between the two variables analyzed: the provision of public places in pre-school services and the synthetic fertility index. Although the quadratic relationship exists in both countries, its interpretation, due to the values reached in the synthetic fertility index and the coverage of public places, is very different. In Spain, the supply of public places barely covers 20% of the demand and, consequently, its capacity to transform the reproductive patterns of the population is very limited. Certainly, its modest development in recent times sporadically influenced fertility, when the measures adopted to increase the offer of public services led to a significant growth in fertility (that could have also been due to other factors). However, after that upward momentum the figures dropped again because the impact on the population was very limited. Thus, the effectiveness of this tool in Spain has been limited because its development has been insufficient. In this sense, the sensitivity analysis of the future in Spain suggests that a family policy model that maintains the current patterns of underdevelopment will not produce an increment in fertility, and Spain will remain at the bottom of Europe in this regard.

In Norway, the quadratic relationship has a different meaning, since the relationship between both variables started to be negative when the coverage of public places was guaranteed and, subsequently, the ‘agency capacity’ of the educational variable over fertility faded away. For this reason the slight predicted variations in fertility up 2030 (between 1.7 and 1.9) cannot be attributed to public investment in childcare (if the current investment is maintained in the future).

Considering this situation, it seems advisable that governments should contemplate different intervention strategies in this area, evaluating different scenarios and actions regarding the development and application of family policies and their influence on the fertility and the well-being of their citizens. Hence, according to the results obtained, pre-school services may contribute to increase fertility rates, but only when they are developed sufficiently.

Nonetheless, it is important to highlight that family policies do not have or should not have the

exclusive goal of increasing fertility. Certainly, family policies are a tool to encourage fertility through significant universal investments, but this objective should not be dissociated from other purposes that are probably interrelated with it, and which constitute fundamental instruments to advance and produce favorable social changes, such as promoting gender equality and increasing female employment rates, facilitating the well-being of families, reducing conflicts between public and private life and attaining equality and social cohesion through the provision of public care services.

Family policies are an essential element to transform social reality and, with regards to fertility, they constitute a facilitating element to aid families to have the number of children desired. Our mathematical model constitutes a valid tool to support the design of political measures able to modify the existing social reality. In this regard, it is intended to continue this research line analyzing the influence of other family-support public policies on fertility and extrapolating this mathematical model to other countries. This will be possible because the generic demographic model designed considers the entry and exit of people and, at the same time, the variable which measures the family policies is obtained through equations that incorporates the public policies which support families. Besides, the mathematical model, which is continually under construction and open to progress, allows to be extended in future researches as the current entry variables may be calculated through other variables if we introduce, for example, a family economic subsystem.

In this sense, beyond the provision of pre-school places, which constitute a single public policy designed to support families, families decide having children considering the entire scenario of resources (and constraints). Thus, this mathematical model has the potential to contribute to the existing knowledge in two different ways: a) applying a sensibility analysis to the full scope of family policies (also including parental leaves and monetary transfers) to better understand how, and up to what extent, the overall support provided by public policies, in an specific country,

influence fertility rates; and b) to build future strategies and scenarios, considering the variability of the input variables, to increase the Family Policy Index. In this sense, other tools may also be useful, such as a genetic algorithm, which generate the best situation without providing previous hypothesis.

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