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D.T. 1304

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Universidad Pública de Navarra
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March 20, 2013

Abstract

We describe a dynamic macroeconomic model that incorporates firm-level borrowing constraints, competitive CES loan production, and rigidities on both setting prices and wages. The external finance premium (interest-rate spread) is countercyclical with technology and financial shocks, and procyclical with consumption spending shocks. The real effects of financial shocks are significantly amplified when either considering greater rigidities for price/wage setting or a low elasticity of substitution in loan production (banking real rigidities). In the monetary policy analysis, a stabilizing Taylor (1983)-style rule performs slightly better when incorporating a positive and small response coefficient to the external finance premium.

Keywords: financial accelerator, nominal rigidities, real rigidities.

JEL codes: E32, E44.

1 Introduction

During the recent global economic crisis (2007-2012), some countries have experienced GDP stagnation and a steady increase in the rate of unemployment. As one extreme example, the rate of unemployment in Spain tripled by increasing from 8.3% in July 2007 to 25.1% in July

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2012. Other peripheral economies are currently suffering from unemployment soaring as well (Greece, Ireland, Portugal). Why did unemployment rise so sharply in the latest economic recession? The combination of both a rigid labor market and a credit crunch could be one possible answer to this question. Whether coincidently or not, politicians and policy makers across the European Union often mention the need for structural reforms in both the labor market and the banking sector as crucial elements for the economic recovery and the solution to the current Euro sovereign debt crisis.

This paper explores the interaction between market rigidities and financial frictions. The analysis relies on a New-Keynesian model with borrowing constraints, sticky prices and sticky wages, that endogenously provides dynamic equations for output, price inflation, wage inflation, unemployment, and the interest rates on both loans and deposits. The model combines elements of banking intermediation introduced in Christiano *et al.* (2010) or Goodfriend and McCallum (2007), with labor market rigidities that bring unemployment fluctuations as in Casares (2007). Households supply labor to either banks or firms, and decide how to reallocate their stock of wealth between equity and bank deposits. Firms employ labor to produce differentiated consumption goods that sell in a monopolistically competitive market. They require external finance to pay in advance part of their wage bill, which they can borrow from banks.[1] Meanwhile, banks obtain liquidity by issuing deposits and transform them into firm-loans by utilizing a technology that combines labor and collateral in the form of firm-equity. The use of equity (net worth) as collateral brings in the financial accelerator mechanism, which was first described by Bernanke and Gertler (1989) in a model with asymmetric information and agency costs. As two follow-up papers, Carlstrom and Fuerst (1997) find that endogenous principal-agent problems generate a hump-shaped response of output to real shocks, and Kiyotaki and Moore (1997) show that collateral constraints might have a large role in amplifying the effects of economic shocks; including those to the value of collateral.[2]

[1] It is implicitly assumed that there is a delay between the moment in which labor is employed in the production of goods and the moment in which revenue is obtained from selling these goods.

[2] More recently, another strand of literature looks at the relationship between real rigidities in the labor and financial markets: Wasmer and Weil (2004) study the interplay between credit and labor market imperfections in a model with matching frictions, and Petrosky-Nadeau and Wasmer (2011) show that a calibrated model with matching frictions in labor, goods and credit markets does a better job than standard search models at replicating the persistence and volatility of unemployment fluctuations.

Nominal rigidities may also play a crucial role in shaping the persistence and magnitude of the financial accelerator. Bernanke *et al.* (1999), show that in the presence of sticky prices *á la* Calvo (1983), financial frictions account for a 30% greater response of output to technology and demand shocks. Our paper brings a contribution to this literature by studying the interplay between firms' financial constraints and nominal rigidities on both price and wage setting. In addition, a CES technology for loan production is introduced to study the financial accelerator mechanism under different elasticities of substitution in the banking technology (real rigidities).

After presenting a model calibration, the business cycle analysis examine impulse-responses to three types of idiosyncratic shocks: a technology shock, a financial shock and a consumption shock. In particular, we look at the spread between the interest rates on loans and deposits (external finance premium) to check whether financial conditions either propagate or attenuate the effects of shocks. The results are mix. On the one hand, the external finance premium is countercyclical after either technology or financial shocks, when output responds stronger (through the financial accelerator mechanism) as borrowing conditions ease. On the other hand, the external finance premium turns procyclical after demand shocks which reduces the impact on output due to tighter financial conditions.

A credit crunch episode has been reproduced in the model by means of a large adverse financial shock. The model simulations indicate that the magnitude of the real effects of a credit crunch depends on the degrees of both nominal and real rigidities.[3] Hence, sticky wages are crucial to explain a significant decline in output after the credit crunch. If nominal wages were fully flexible, production marginal costs, price inflation and nominal interest rates would drop significantly to wipe away around half of the declining response of output to the negative financial shock. If both prices and wages can be immediately adjusted, the financial shock is mostly absorbed through changes in price and wage inflation, whereas the quantities of either output and employment barely change (by less 10% of what they did under sticky prices and wages). Alternatively, the volume of real rigidities associated with credit market imperfections also plays a significant role on the real effects of financial shocks. Hence, a simulated economy with sticky wages and costly substitutions for equity losses in the banking sector replicates the large decline in unemployment observed in credit crunch episodes.

[3] The real effects of financial shocks and financial intermediation disruption have been recently re-examined by Gertler and Kiyotaki (2010), and Khan and Thomas (2011).

Regarding monetary policy analysis, Cúrdia and Woodford (2010), and Gertler and Karadi (2011) have recently proposed models to study unconventional monetary policy rules that expands central bank credit intermediation to offset a disruption of private financial intermediation. In Cúrdia and Woodford (2010), a monetary policy rule *á la* Taylor (1993) is extended to accommodate a policy reaction to changes in the external finance premium. Following that possibility, we evaluate response coefficients under alternative stabilizing criteria and find robustness on recommending a positive and low coefficient (that does not coincide with Cúrdia and Woodford's prescription for a negative coefficient).

The rest of the paper is organized as follows. Next, Section 2 describes the model. Section 3 is devoted to the calibration of model parameters. In section 4, we carry out the impulse-response analysis to study the effects of technology, financial, and consumption shocks. Sections 5 and 6 show how the model can be applied to describe the real effects of a credit crunch episode and to design a Taylor-type rule extended with a response coefficient to changes in the spread. Section 7 reviews the conclusions.

2 The model

In a closed-economy framework, there are infinitely-lived identical households, who supply labor services and are owners of monopolistically competitive firms and perfectly competitive banks. Each period, these households decide their supply of labor to either banks or heterogeneous firms. They also optimally choose the amount of current consumption, and their allocation of savings in the form of either firms' equity ownership or bank deposits. Simultaneously, firms decide their demand for labor, the demand for banking loans, their production of a differentiated final good to be sold in a monopolistically competitive market and the pricing of that good. As financial intermediaries, banks determine the demand for labor and collateral (firm equity) and the supply of loans. Firms demand loans to finance a fraction of their wage bill. The central bank implements a stabilizing monetary policy rule that sets the nominal interest rate on deposits. Let us examine separately the economic behavior of the economics agents of the model.

2.1 Households

In any period t, the representative household is endowed with the following stock of financial wealth

$$x_t v_t + d_t, \tag{1}$$

where v_t is the aggregate equity value, x_t is the equity share owned by the representative household, and d_t is the stock of deposits held by the representative household. Note that in equilibrium, $x_t = 1$ holds, as the representative household is the single owner of the market portfolio, i.e. of all shares of the existing firms.

Households' preferences are described by the following instantaneous semi-log utility function:

$$U(\varepsilon_t^c, c_t, l_t, m_t) = e^{\varepsilon_t^c} \left(\log c_t - \Psi_l \frac{l_t^{1+\gamma_l}}{1+\gamma_l} - \Psi_m \frac{m_t^{1+\gamma_m}}{1+\gamma_m} \right), \tag{2}$$

where $\Psi_l, \gamma_l, \Psi_m, \gamma_m > 0$, and the arguments are an exogenous utility shock, ε_t^c, the number of Dixit and Stiglitz (1977) baskets of consumption goods, c_t, the bundle of non-banking labor services supplied to firms, l_t, and the amount supplied of banking labor, m_t.

Labor is supplied to both firms and banks. The effective labor income at firms is $w_t l_t (1 - u_t)$, where w_t is the real wage collected per unit of employed labor and u_t is the per-unit rate of unemployment. There is no unemployment in the banking sector, which implies that all its units of labor supply, m_t, receive the banking real wage, w_t^m. Moreover, the representative household collects e_t dividends per equity share x_t of the productive sector, a real interest return r_t^d on holdings of bank deposits, d_t, and a collateral service yield, CSY_t^v, from the holdings of equity, $x_t v_t$. Accordingly, period t real income of the representative household is

$$w_t l_t (1 - u_t) + w_t^m m_t + x_t e_t + CSY_t^v x_t v_t + r_t^d d_t. \tag{3}$$

Income is spent on consumption, on increasing the equity share, and on increasing the stock of deposits. Thus, the household budget constraint in period t is

$$w_t l_t (1 - u_t) + w_t^m m_t + x_t e_t + CSY_t^v x_t v_t + r_t^d d_t \geq c_t + (x_{t+1} - x_t) v_t + d_{t+1} - d_t. \tag{4}$$

The optimizing program consists on maximizing intertemporal utility subject to period budget constraints. Then, households in period t choose the amount of consumption bundles, c_t, the share of mutualized equity they invest for next period, x_{t+1}, the stock of bank deposits for next period, d_{t+1}, and the quantities of non-banking and banking labor they supply , l_t and m_t, so as to maximize intertemporal utility[4]

$$\underset{c_t,x_{t+1},d_{t+1},l_t,m_t}{Max} \quad E_t \sum_{j=0}^{\infty} \beta^j e^{\varepsilon^c_{t+j}} \left(\log c_{t+j} - \Psi_l \frac{l_{t+j}^{1+\gamma_l}}{1+\gamma} - \Psi_m \frac{m_{t+j}^{1+\gamma_m}}{1+\gamma} \right)$$

subject to

$$\begin{aligned} & w_{t+j} l_{t+j} \left(1 - u_{t+j}\right) + w^m_{t+j} m_{t+j} + x_{t+j} e_{t+j} + CSY^v_{t+j} x_{t+j} v_{t+j} + r^d_{t+j} d_{t+j} \\ = \; & c_{t+j} + [x_{t+1+j} - x_{t+j}] v_{t+j} + d_{t+1+j} - d_{t+j}. \end{aligned} \tag{5}$$

The first order conditions of the above maximization problem yield the budget constraint (5) for period t and

$$c_t \; : \; \frac{e^{\varepsilon^c_t}}{c_t} - \lambda_t = 0, \tag{6}$$

$$x_{t+1} \; : \; -\lambda_t v_t + \beta E_t \lambda_{t+1} \left[e_{t+1} + v_{t+1} \left(1 + CSY^v_{t+1}\right)\right] = 0, \tag{7}$$

$$d_{t+1} \; : \; -\lambda_t + \beta E_t \lambda_{t+1} \left(1 + r^d_{t+1}\right) = 0, \tag{8}$$

$$l_t \; : \; -e^{\varepsilon^c_t} \Psi_l l_t^{\gamma_l} + \lambda_t w_t \left(1 - u_t\right) = 0, \tag{9}$$

$$m_t \; : \; -e^{\varepsilon^c_t} \Psi_m m_t^{\gamma_m} + \lambda_t w^m_t = 0, \tag{10}$$

where λ_t denotes the Lagrangian multiplier associated with the budget constraint. Note that, given the above, the equilibrium real interest rate on deposits is reciprocal to the stochastic discount rate,

$$\left(1 + r^d_{t+1}\right)^{-1} = \frac{\beta E_t \lambda_{t+1}}{\lambda_t} = \beta_{t,t+1}, \tag{11}$$

that introduces $\beta_{t,t+1}$ as the stochastic discount factor between current period t and future period $t+1$.

2.2 Banks

In the banking sector, there is a continuum of identical competitive banks. The representative bank issues some amount of real deposits, d_t, which pay a real interest rate, r^d_t and use the proceedings to supply real loans, b_t, to firms that demand liquidity at the competitive real

[4] Simultaneously, households choose the composition of the bundles of consumption and labor supply in relative terms to the ω good variety, $c_t(\omega)$ and $l^s_t(\omega)$. These optimal choices are shown respectively in Sections 1 and 2 of the Technical Appendix.

interest rate on borrowing, r_t^b. The production of loans utilizes a technology that combines the collateral service of equity, v_t, and banking labor, m_t, through the CES specification:[5]

$$b_t = e^{\varepsilon_t^b}[av_t^\theta + (1-a)m_t^\theta]^{\frac{1}{\theta}} \tag{12}$$

where $-\infty < \theta \leq 1$ is the elasticity parameter, $0 < a < 1$ is the equity share coefficient, and ε_t^b is an exogenous financial productivity shock. The elasticity of substitution between collateral, v_t, and labor used to monitor, m_t, is constant at $\frac{1}{1-\theta}$. Note that, in the upper bound, $\theta = 1$, the above production function converges to a linear function with perfect input substitutability (infinite elasticity), whereas as θ approaches to its lower bound the production function turns into a Leontief technology, with no substitutability between the two factors (zero elasticity). As an intermediate case, the Cobb-Douglas technology is particularized by (12) when θ approaches to 0 and there is a unit elasticity of substitution.

As discussed in Section 4 of the Technical Appendix, the banking elasticity of substitution, $\frac{1}{1-\theta}$, inversely determines the size of the response of the marginal cost of producing loans when there is a change in the amount of loan production inputs. Thus, a low elasticity of substitution (θ negative and high) implies that the real marginal cost is severely influenced by relative changes in the use of banking inputs for loan production, which can be interpreted as a real rigidity on banking activities. Following the terminology used by Gopinath and Itskhoki (2010), we take the banking elasticity of substitution, $\frac{1}{1-\theta}$, as one inverse measure of the level of real rigidities in the credit market.[6]

The loan production technology (12) should be interpreted as a reduced form that captures the fact that – in the presence of informational asymmetries – labor-intensive monitoring services and collateral play a crucial role in aligning borrowers' and lenders' incentives so that, other things equal, the amount of loans that a bank is willing to supply increases with both of them.[7]

[5] A log-linear approximation to the loan production technology (12) is derived in Section 3 of the Technical Appendix.

[6] Gopinath and Itskhoki (2010) define real rigidities as "mechanisms that dampen price responses of firms because of factors such as strategic complementarities in price setting, real wage rigidity, the dependence of costs on inputs prices that have yet to adjust, among others.".

[7] As in the tradition of models with credit frictions based on principal-agent problems and agency costs initiated by Bernanke and Gertler (1989).

The optimizing program of the representative bank can be written as follows

$$\underset{m_t, v_t}{Max} \quad \left(r_t^b - r_t^d\right) b_t - w_t^m m_t - CSY_t^v v_t$$

$$s.to \qquad e^{\varepsilon_t^b}[a v_t^\theta + (1-a) m_t^\theta]^{\frac{1}{\theta}} = b_t,$$

which results in the following first order conditions

$$\left(r_t^b - r_t^d\right) \frac{(1-a) m_t^{\theta-1} b_t}{a v_t^\theta + (1-a) m_t^\theta} - w_t^m = 0, \tag{m_t^{foc}}$$

$$\left(r_t^b - r_t^d\right) \frac{a v_t^{\theta-1} b_t}{a v_t^\theta + (1-a) m_t^\theta} - CSY_t^v = 0, \tag{v_t^{foc}}$$

and (m_t^{foc}) leads to the equilibrium borrowing interest rate:

$$r_t^b = r_t^d + \frac{w_t^m m_t}{b_t} \frac{a v_t^\theta + (1-a) m_t^\theta}{(1-a) m_t^\theta}. \tag{13}$$

It should be noticed that r_t^b and r_t^d co-move together one by one, while the banking spread, $r_t^b - r_t^d$, is endogenously determined by the marginal cost of loan production $\frac{w_t^m}{\partial b_t / \partial m_t} = \frac{w_t^m m_t}{b_t} \frac{a v_t^\theta + (1-a) m_t^\theta}{(1-a) m_t^\theta}$. Moreover, the equilibrium return for the collateral service of equity, obtained from (v_t^{foc}), is

$$CSY_t^v = \left(r_t^b - r_t^d\right) \frac{b_t}{v_t} \frac{a v_t^\theta}{a v_t^\theta + (1-a) m_t^\theta},$$

where plugging the expression for the spread, $r_t^b - r_t^d$, obtained above simplifies to

$$CSY_t^v = \frac{w_t^m m_t}{v_t} \frac{a}{(1-a)} \left(\frac{v_t}{m_t}\right)^\theta. \tag{14}$$

2.3 Firms

In period t, each firm specializes in the production of one differentiated consumption good that belongs to the Dixit-Stiglitz consumption bundle. For the representative ω firm, the production technology is given by the following function:

$$y_t(\omega) = e^{\varepsilon_t^z} l_t^d(\omega)^\alpha - \Phi, \tag{15}$$

where $y_t(\omega)$ is output produced by the ω-th firm, $l_t^d(\omega)$ is its labor demand, ε_t^z is an exogenous economy-wide productivity shock, $0 < \alpha < 1$ is a parameter that defines the labor elasticity of output, and $\Phi \geq 0$ is a fixed cost. As in Dixit and Stiglitz (1977), the monopolistically competitive firm ω faces the following market demand constraint,

$$y_t(\omega) = \left(\frac{P_t(\omega)}{P_t}\right)^{-\sigma} y_t, \tag{16}$$

where $\frac{P_t(\omega)}{P_t}$ is the ratio between the price of good produced in the ω-th firm and the aggregate price level, $\sigma > 1$ is the Dixit-Stiglitz constant elasticity of substitution, and y_t is aggregate output.

Introducing a financial friction, we assume that a fraction τ of the real wage bill must be borrowed to meet cash-flow needs of the firm. Let $b_t^d(\omega)$ denote the amount of borrowing (real loans) demanded by firm ω and $\frac{W_t(\omega)}{P_t}$ its firm-specific real wage defined as the ratio of the nominal wage over the aggregate price level. The demand for real loans of firm ω in period t is

$$b_t^d(\omega) = \tau \frac{W_t(\omega)}{P_t} l_t^d(\omega). \tag{17}$$

Real loans, $b_t^d(\omega)$, must be reimbursed to the bank by the end of the period. Firms take as given the real interest on bank loans, r_t^b, which makes the interest payment of firm ω be $r_t^b b_t^d(\omega)$, and its total earnings (profits, or dividends) be equal to:[8]

$$e_t(\omega) = \frac{P_t(\omega)}{P_t} y_t(\omega) - \frac{W_t(\omega)}{P_t} l_t^d(\omega) - r_t^b b_t(\omega). \tag{18}$$

In turn, the optimizing program of firm ω consists of choosing the selling price, $P_t(\omega)$, the labor demand, $l_t^d(\omega)$, and the amount of real loans to borrow from the bank, $b_t(\omega)$, in order to maximize intertemporal earnings:

$$\underset{P_t(\omega), l_t^d(\omega), b_t(\omega)}{Max} \quad \sum_{j=0}^{\infty} E_t \beta_{t,t+j} \left(\left(\frac{P_{t+j}(\omega)}{P_{t+j}} \right)^{1-\sigma} y_{t+j} - \frac{W_{t+j}(\omega)}{P_{t+j}} l_{t+j}^d(\omega) - r_{t+j}^b b_{t+j}(\omega) \right)$$

subject to market and credit constraints

$$e^{\varepsilon_{t+j}^z} l_{t+j}^d(\omega)^\alpha - \Phi = \left(\frac{P_{t+j}(\omega)}{P_{t+j}} \right)^{-\sigma} y_{t+j} \tag{19}$$

$$b_{t+j}(\omega) = \tau \frac{W_{t+j}(\omega)}{P_{t+j}} l_{t+j}^d(\omega) \tag{20}$$

2.4 Price rigidity

Price setting is governed by a Calvo (1983)-type market signal that comes with a constant probability $1 - \eta$. Then, the optimal choices of $P_t(\omega)$, $l_t^d(\omega)$, and $b_t^d(\omega)$ respectively satisfy:

$$\sum_{j=0}^{\infty} \eta^j E_t^\eta \beta_{t,t+j} \left((1-\sigma) \left(\frac{P_t(\omega)}{P_{t+j}} \right)^{-\sigma} \frac{y_{t+j}}{p_{t+j}} + \xi_{t+j}(\omega) \sigma \left(\frac{P_t(\omega)}{P_{t+j}} \right)^{-\sigma-1} \frac{y_{t+j}}{P_{t+j}} \right) = 0, \tag{21}$$

$$-\frac{W_t(\omega)}{P_t} + \xi_t(\omega) \frac{\alpha \left(y_t(\omega) + \Phi \right)}{l_t^d(\omega)} - \varphi_t(\omega) \tau \frac{W_t(\omega)}{P_t} = 0, \tag{22}$$

$$-r_t^b(\omega) + \varphi_t(\omega) = 0. \tag{23}$$

[8] A log-linear version of firm earnings is obtained in Section 5 of the Technical Appendix.

where η is the probability of not being able to set the price in any future period, E_t^η is the rational expectation operator conditional to the lack of optimal pricing in the future, $\xi_t(\omega)$ is the Lagrange multiplier associated with the demand constraint and $\varphi_t(\omega)$ is the Lagrange multiplier associated with the liquidity constraint. Combining the first order conditions on labor demand and loans lead to the following real marginal cost[9]

$$\xi_t(\omega) = \frac{\left(1+\tau r_t^b\right)\frac{W_t(\omega)}{P_t} l_t^d(\omega)}{\alpha(y_t(\omega)+\Phi)}. \tag{24}$$

Solving the first order condition for the optimal price leads to

$$P_t(\omega) = \frac{\sigma}{\sigma-1}\left[\frac{E_t^\eta \sum_{j=0}^{\infty} \eta^j \beta_{t,t+j} \xi_{t+j}(\omega)(P_{t+j})^\sigma y_{t+j}}{E_t^\eta \sum_{j=0}^{\infty} \eta^j \beta_{t,t+j}(P_{t+j})^{\sigma-1} y_{t+j}}\right], \tag{25}$$

Following Walsh (2010, chapter 8), (25) is approximated by the semi-loglinear expression

$$\widehat{P}_t(\omega) = \widehat{P}_t + (1-\beta\eta)\, E_t^\eta \sum_{j=0}^{\infty} \beta^j \eta^j \widehat{\xi}_{t+j}(\omega) + E_t \sum_{j=1}^{\infty} \beta^j \eta^j \pi_{t+j}^p. \tag{26}$$

where we have used the standard notation of hat variables to represent log deviations with respect to their constant steady-state level, e.g. $\widehat{P}_t(\omega) = \log\left(\frac{P_t(\omega)}{P(\omega)}\right)$, and the rate of price inflation in period $t+j$ was defined as the log difference of the price level, $\pi_{t+j}^p = \widehat{P}_{t+j} - \widehat{P}_{t+j-1}$.

Meanwhile, using the log-linearized production function to eliminate labor, the value of $E_t^\eta \widehat{\xi}_{t+j}(\omega)$ consistent with (24) is

$$E_t^\eta \widehat{\xi}_{t+j}(\omega) = \tau r_{t+j}^b + E_t^\eta \widehat{W}_{t+j}(\omega) - \widehat{P}_{t+j} - \tfrac{1}{\alpha}\varepsilon_{t+j}^z + \tfrac{(1-\alpha)}{\alpha(1+\Phi/y)} E_t^\eta \widehat{y}_{t+j}(\omega). \tag{27}$$

Finally, the joint dynamics of firm-level real marginal costs (27) and pricing (26) can be combined to obtain the following inflation equation (New Keynesian Phillips Curve, NKPC)[10]

$$\pi_t^p = \beta E_t \pi_{t+1}^p + \frac{1}{1+\frac{(1-\alpha)\sigma}{\alpha(1+\Phi/y)}}\left(\pi_t^w - \beta E_t \pi_{t+1}^w\right) + \frac{(1-\eta)(1-\beta\eta)}{\eta\left(1+\frac{(1-\alpha)\sigma}{\alpha(1+\Phi/y)}\right)}\widehat{\xi}_t. \tag{28}$$

Price inflation is forward looking and depends on the fluctuations of the aggregate real marginal cost, $\widehat{\xi}_t$, and on the wage inflation effect, $\pi_t^w - \beta E_t \pi_{t+1}^w$. Unlike other NKPC of the literature, the presence of wage inflation in (28) is explained by the reaction of optimal pricing to firm-specific wages through their influence in firm-specific real marginal costs. In addition, financial

[9]It should be noticed that, defining the total cost of the ω firm in period t:
$TC(y(\omega)) = \frac{W_{t+j}(\omega)}{P_{t+j}} l_{t+j}^d(\omega) + r_{t+j}^b \tau \frac{W_{t+j}(\omega)}{P_{t+j}} l_{t+j}^d(\omega)$,
the real marginal cost is:
$\xi_t(\omega) = \frac{\partial TC(y(\omega))}{\partial y(\omega)} = \frac{\partial TC(y(\omega))}{\partial l_t^d(\omega)} \frac{\partial l_t^d(\omega)}{\partial y(\omega)} = \left(1+\tau r_t^b\right) \frac{W_t(\omega)}{P_t} \frac{l_t^d(\omega)}{\alpha y_t(\omega)}$.

[10]The required algebra is shown in Section 6 of the Technical Appendix.

frictions have an indirect influence in the dynamics of the NKPC because the real interest rate on borrowing, r_t^b, is one of the variables that determine fluctuations in the aggregate real marginal cost

$$\widehat{\xi}_t = \tau r_t^b + \widehat{w}_t - \left(\widehat{y}_t - \widehat{l}_t\right),$$

where the constant term has been ignored.[11]

2.5 Wage rigidity

We introduce wage rigidity by assuming that workers and owners of each firm face a constant probability of not being able to revise the labor contract. As in Casares (2007), wage stickiness is subordinated to the same Calvo probability η that governs price stickiness. The Calvo-style lottery generates firm's heterogeneity in the nominal wage, $W_t(\omega)$, and in labor demand, $l_t^d(\omega)$, as well as household's heterogeneity in labor supply, $l_t^s(\omega)$. The nominal wage is revised to make intertemporal labor demand equal to intertemporal labor supply at the firm level. Thus, the revision of the nominal wage at firm ω would be determined as follows

$$E_t^\eta \sum_{j=0}^{\infty} \beta^j \eta^j \left(\widehat{l}_{t+j}^d(\omega) - \widehat{l}_{t+j}^s(\omega)\right) = 0, \tag{29}$$

where η is the Calvo (1983)-type constant probability of not experiencing a labor contract revision, E_t^η is the rational expectation operator conditional on the lack of revisions in the future, and $\left(\widehat{l}_{t+j}^d(\omega) - \widehat{l}_{t+j}^s(\omega)\right)$ represents the log deviation between the type$-\omega$ labor demand and labor supply in period $t+j$ with $j = 0, 1, 2, ..., \infty$. If wage stickiness is eliminated ($\eta = 0.0$), the wage setting condition (31) would determine a perfect matching between labor supply and labor demand, $\widehat{l}_t^d(\omega) = \widehat{l}_t^s(\omega)$.[12] Hence, nominal rigidities on the setting of labor-clearing wages bring about gaps between the amounts of labor supply (workers provided by the household) and labor demand (jobs demanded by the firm).[13]

[11] Concretely, τr^b (where r^b is the steady-state real interest rate on loans) has been dropped from the right hand side of the expression. Ignoring constan terms in dynamic equations imply that all the variables that measure rates (price inflation, wage inflation, nominal interest rates and real interest rates) represent the relative value with respect to the corresponding steady-state rate.

[12] This is the case of an economy with heterogeneous labor and flexible wages, as described in Woodford (2003, chapter 3).

[13] Labor fluctuations are considered at the extensive margin (employment) in order to provide a fundamental interpretation of unemployment. The intensive margin (hours) is shut down by assuming inelastic supply of hours at a fixed constant level.

The value of newly-revised nominal wages depends on how firm-level labor demand and supply enter (29). The optimal labor supply allocation of the representative household implies[14]

$$\widehat{l}_t^s(\omega) = \sigma_w \widetilde{W}_t(\omega) + \widehat{l}_t^s \tag{30}$$

where $\widetilde{W}_t(\omega) = \widehat{W}_t(\omega) - \widehat{W}_t$ is the relative wage, $\sigma_w > 1$ is the Dixit-Stiglitz constant elasticity of substitution across different types of labor supply, and $\widehat{l}_t^s$ is the log fluctuation of aggregate labor supply. Meanwhile, the following relative labor demand is consistent with both the monopolistically competitive demand function and the production technology

$$\widehat{l}_t^d(\omega) = -\frac{\sigma}{\alpha\left(1+\Phi/y\right)} \widetilde{P}_t(\omega) + \widehat{l}_t, \tag{31}$$

where $\widetilde{P}_t(\omega) = \widehat{P}_t(\omega) - \widehat{P}_t$ is the relative price, and $\widehat{l}_t$ is the log fluctuation of aggregate labor demand. Generalizing (30) and (31) for future $t+j$ periods and plugging them into (29) lead to

$$E_t^\eta \sum_{j=0}^{\infty} \beta^j \eta^j \left[-\frac{\sigma}{\alpha\left(1+\Phi/y\right)} \widetilde{P}_{t+j}(\omega) + \widehat{l}_{t+j} - \sigma_w \widetilde{W}_{t+j}(\omega) - \widehat{l}_{t+j}^s \right] = 0. \tag{32}$$

Due to the Calvo-style setting scheme and abstracting from trend inflation, the conditional expectations of relative prices and wages are

$$\begin{aligned} E_t^\eta \widetilde{P}_{t+j}(\omega) &= \tfrac{\eta}{1-\eta}\pi_t^p - E_t \sum_{k=1}^{j} \pi_{t+k}^p, \\ E_t^\eta \widetilde{W}_{t+j}(\omega) &= \tfrac{\eta}{1-\eta}\pi_t^w - E_t \sum_{k=1}^{j} \pi_{t+k}^w. \end{aligned}$$

The substitution of the last results in the wage setting expression (34) gives

$$-\tfrac{\sigma}{\alpha(1+\Phi/y)(1-\beta\eta)} \left(\tfrac{\eta}{(1-\eta)}\pi_t^p - E_t \sum_{j=1}^{\infty} \beta^j \eta^j \pi_{t+j}^p \right) - \tfrac{\sigma_w}{(1-\beta\eta)} \left(\tfrac{\eta}{(1-\eta)}\pi_t^w - E_t \sum_{j=1}^{\infty} \beta^j \eta^j \pi_{t+j}^w \right) = E_t^\eta \sum_{j=0}^{\infty} \beta^j \eta^j u_{t+j}, \tag{33}$$

where the rate of unemployment for any $t+j$ period, $u_{t+j} = \widehat{l}_{t+j}^s - \widehat{l}_{t+j}$, is a log-linear approximation to unemployment as excess supply of labor

$$u_t = \frac{l_t^s - l_t}{l_t^s}.$$

Solving (33) for the current rate of wage inflation, it is obtained

$$\pi_t^w = \tfrac{(1-\eta)}{\eta} E_t \sum_{j=1}^{\infty} \beta^j \eta^j \pi_{t+j}^w - \tfrac{\sigma}{\alpha(1+\Phi/y)\sigma_w} \left(\pi_t^p - \tfrac{(1-\eta)}{\eta} E_t \sum_{j=1}^{\infty} \beta^j \eta^j \pi_{t+j}^p \right) - \tfrac{(1-\eta)(1-\beta\eta)}{\eta\sigma_w} E_t^\eta \sum_{j=0}^{\infty} \beta^j \eta^j u_{t+j},$$

[14] See Section 2 of the Technical Appendix for the proof and further details.

where making the first difference $(\pi_t^w - \beta\eta E_t \pi_{t+1}^w)$ gives (after some algebra)

$$\pi_t^w = \beta E_t \pi_{t+1}^w - \tfrac{\sigma}{\alpha(1+\Phi/y)\sigma_w}\left(\pi_t^p - \beta E_t \pi_{t+1}^p\right) - \tfrac{(1-\eta)(1-\beta\eta)}{\eta\sigma_w} u_t.$$

Finally, using the inflation equation (28) to replace $\left(\pi_t^p - \beta E_t \pi_{t+1}^p\right)$, it is obtained

$$\pi_t^w = \tfrac{(1-\eta)}{\eta} E_t \sum_{j=1}^{\infty} \beta^j \eta^j \pi_{t+j}^w - \tfrac{\sigma}{\alpha(1+\Phi/y)\sigma_w}\left(\pi_t^p - \tfrac{(1-\eta)}{\eta} E_t \sum_{j=1}^{\infty} \beta^j \eta^j \pi_{t+j}^p\right) - \tfrac{(1-\eta)(1-\beta\eta)}{\eta\sigma_w} E_t^\eta \sum_{j=0}^{\infty} \beta^j \eta^j u_{t+j},$$

where making the first difference $(\pi_t^w - \beta\eta E_t \pi_{t+1}^w)$ gives (after some algebra)

$$\pi_t^w = \beta E_t \pi_{t+1}^w - \tfrac{\sigma}{\alpha(1+\Phi/y)\sigma_w}\left(\pi_t^p - \beta E_t \pi_{t+1}^p\right) - \tfrac{(1-\eta)(1-\beta\eta)}{\eta\sigma_w} u_t.$$

Finally, using the inflation equation (28) to replace $\left(\pi_t^p - \beta E_t \pi_{t+1}^p\right)$, it is obtained

$$\pi_t^w = \beta E_t \pi_{t+1}^w - \chi_1 u_t - \chi_2 \widehat{\xi}_t \tag{34}$$

where $\chi_1 = \frac{(1-\eta)(1-\beta\eta)}{\eta\sigma_w}\left(1 + \frac{\frac{\sigma}{\alpha(1+\Phi/y)\sigma_w}}{1+\frac{(1-\alpha)\sigma}{\alpha(1+\Phi/y)}}\right)^{-1}$ and $\chi_2 = \frac{\sigma}{\alpha(1+\Phi/y)\sigma_w}\frac{(1-\eta)(1-\beta\eta)}{\eta\left(1+\frac{(1-\alpha)\sigma}{\alpha(1+\Phi/y)}\right)}\left(1 + \frac{\frac{\sigma}{\alpha\sigma_w}}{1+\frac{(1-\alpha)\sigma}{\alpha(1+\Phi/y)}}\right)^{-1}$. The wage inflation equation (34) somehow resembles an old-fashion Phillips curve in the negative relationship between wage inflation and the rate of unemployment. There is also some negative effect from fluctuations in the real marginal cost as a consequence of the interdependence between wage setting and pricing. An increase in firm-specific marginal costs would raise the relative price and would reduce relative labor demand, which would push nominal wages downwards in (29).

2.6 Monetary policy

The central bank implements a stabilizing monetary policy by adjusting the nominal interest of deposits, R_t^d, in response to changes in the rate of price inflation, changes in economic activity (output), and changes in the cost of external finance. The latter corresponds to unconventional policies that might be desirable in the presence of financial frictions as discussed in Cúrdia and Woodford (2010). The intervention of the central bank is gradual as determined by this partial adjustment Taylor (1993)-type rule extended with responses to credit spreads

$$R_t^d = \mu_R R_{t-1}^d + (1-\mu_R)\left[\mu_\pi \pi_t^p + \mu_y \widehat{y}_t + \mu_{EFP} EFP_t\right], \tag{35}$$

where $0 < \mu_R < 1$, $\mu_\pi, \mu_y > 0$ and $\mu_{EFP} \gtrless 0$ are policy coefficients and EFP_t denotes the external finance premium defined by the spread between the interest rates of loans and deposits

$$EFP_t = r_t^b - r_t^d. \tag{36}$$

3 Calibration

Table 1 describes the baseline calibration of the model for quarterly periods. The intertemporal discount factor is set at $\beta = 0.995$, obtained from a rate of intertemporal preference $\rho = 0.005$ that implies a 2% annualized steady-state rate of return for deposits. The utility function is logarithmic for consumption, and the banking labor curvature is set at $\gamma_l = 4.0$ in order to bring a low labor supply elasticity (0.25). By contrast, it is assumed a unit banking labor supply elasticity, $\gamma_m = 1.0$, to avoid excessive volatility of the banking real wage. The parameters that measure the weight of disutility of either industrial or banking labor take the values that normalizes industrial labor at $l = 1$ in steady state and makes the steady-state real wage be identical across sectors. It requires $\Psi_l = 0.5038$ and $\Psi_m = 103.86$.

The labor elasticity in the production of goods is $\alpha = 0.64$, to bring the standard labor income share assumed in the RBC literature. The fixed cost is set at the level that places firm earnings in steady state at 10% of output.[15] The CES loan production technology is initially parameterized to provide a low substitutability between equity and labor (the elasticity of substitution is $\frac{1}{1-\theta}$ =0.25 with $\theta = -3$) and the labor share for the steady-state production of loans is 0.35, as taken in Goodfriend and McCallum (2007). Moreover, the steady-state value for banking labor is fixed at the number that leaves the external finance premium in steady state at 0.01, i.e. 4% in annualized terms. The resulting value for m in steady-state represents a realistic 0.5% of total labor force. As for the coefficient that determines the borrowing requirement for firms, we assign $\tau = 0.75$ as calibrated in Christiano *et al.* (2010) for a US business cycle model.

[15] In steady-state, $\frac{e}{y} = 1 - \alpha\frac{\sigma-1}{\sigma}\left(1 + \frac{\Phi}{y}\right)$.

Table 1. Calibration.

Parameter description	Value
β, intertemporal discount factor	$\beta = 0.995$
γ_l, industrial labor curvature in utility	$\gamma_l = 4.0$
Ψ_l, industrial labor disutility weight	$\Psi_l = 0.5063$
γ_m, banking labor curvature in utility	$\gamma_m = 1.0$
Ψ_m, banking labor disutility weight	$\Psi_m = 103.86$
θ, input substitutability in loan production	$\theta = -3.0$
a, equity share in loan production	$a = 0.9999$
τ, share of externally financed wage bill	$\tau = 0.75$
σ, Dixit-Stiglitz demand elasticity	$\sigma = 5.0$
σ_w, Dixit-Stiglitz labor supply elasticity	$\sigma_w = 4.0$
η, probability of price/wage rigidity	$\eta = 0.75$
α, labor elasticity in good production	$\alpha = 0.64$
Φ, fixed cost in good production	$\Phi = 0.43$
μ_π, Interest-rate response to inflation	$\mu_\pi = 1.5$
μ_y, Interest-rate response to output	$\mu_y = 0.5/4$
μ_{EFP}, Interest-rate response to EFP	$\mu_{EFP} = 0.43$
μ_R, Interest- rate smoothing	$\mu_R = 0.8$
$\varepsilon_t^z = \rho_z \varepsilon_{t-1}^z + \kappa_t^z$, technology shock	$\rho_z = 0.95$, $\sigma_{\kappa^z} = 0.34\%$
$\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \kappa_t^b$, financial shock	$\rho_b = 0.95$, $\sigma_{\kappa^b} = 1.06\%$
$\varepsilon_t^c = \rho_c \varepsilon_{t-1}^c + \kappa_t^c$, consumption shock	$\rho_c = 0.80$, $\sigma_{\kappa^c} = 1.12\%$

The elasticity of substitution across consumption goods is fixed at the standard value $\sigma = 5.0$, which implies a 25% mark-up in steady state. The Calvo-type probability for both price and wage setting is $\eta = 0.75$ to have the average frequency of optimal setting at one time per year, as quite standard in the New Keynesian literature (Erceg *et al.*, 2000). Finally, the monetary policy rule is designed with the original Taylor (1993) recommended coefficients ($\mu_\pi = 1.5$ and $\mu_y = 0.5/4$), a significant interest-rate inertia ($\mu_R = 0.8$) as suggested by Clarida *et al.* (2000), and the coefficient of response to the external finance premium is set at $\mu_{EFP} = 0.43$ to match the relative volatility of US interest-rate spreads reported in Jin *et al.* (2012).[16]

[16] The standard deviation of the external finance premium (interest-rate spread) is 31% of that of output in

Finally, the stochastic elements of the model are calibrated for their AR(1) processes. Thus, technology shocks on either goods or loan production have strong persistence ($\rho_a = \rho_b = 0.95$), whereas the consumption shock is generated with a more moderate level of persistence ($\rho_c = 0.80$). The standard deviations of the innovations of the shocks are calibrated to satisfy a double criteria. On the one hand, the standard deviation of output fluctuations in the baseline model is 1.38% to match the value obtained in HP-filtered quarterly real GDP in the US over the period 1980:1-2012:2. On the second hand, the long-run variance decomposition reports that 40% of business cycle fluctuations of output are determined by technology shocks on goods production, 40% by demand-side (consumption) shocks, and 20% by financial innovations.[17] In turn, we set $\sigma_{\kappa^a} = 0.34\%$, $\sigma_{\kappa^b} = 1.06\%$, and $\sigma_{\kappa^c} = 1.12\%$.

Solving the non-linear system of equations included in Section 9 of the Technical Appendix results in the following steady-state numerical description:[18]

Table 2. Steady-state numerical solution.

y, output	0.5689
e, earnings	0.0569
v, equity value	11.899
ξ, real marginal cost	0.8000
l, non-banking labor	1.0000
m, banking labor	0.0049
w, real wage	0.2880
b, credit volume	0.2160
r^b, interest rate on loans (%, ann.)	6.0
r^d, interest rate on deposits (%, ann.)	2.0
EFP, spread (%, ann.)	4.0
CSY^v, equity collateral yield (%, ann.)	0.087

quarterly US business cycle data.

[17] These percentages assumed on the output variance decomposition are roughly in line with the results found by Christiano *et al.* (2010) in a DSGE model with financial accelerator estimated with US data and a greater variety of shocks.

[18] It should be recalled that total non-banking labor l has been normalized to 1 in the calibration.

4 Impulse-response analysis

In this section, we examine the dynamic responses of the baseline model to three types of idiosyncratic shocks: a technology shock, a financial shock, and a private spending (consumption) shock. The complete set of twenty dynamic equations of the model is displayed in Section 10 of the Technical Appendix. The size of each shock is one standard deviation of the corresponding innovation, as calibrated above. Figures 1-3 display plots with the responses in percent deviations with respect to the initial steady-state value, except for price inflation, wage inflation, the interest rates and the external finance premium (spread) that are given in direct deviations with respect to their steady-state rates.

4.1 Technology shock

As shown in Figure 1, output reacts with a hump-shaped increase after a positive technology shock, ε_t^a, while unemployment rises due to the reduction in labor demand that accommodates higher productivity. The real marginal cost falls, which explains a decline of price inflation as a result of the mark-up pricing policy of firms. Wage inflation slightly falls at the quarter of the shock and keeps falling gradually in the next quarters, as the combined response of lower labor demand (high unemployment) and a lower real marginal cost in the wage inflation equation (34).

Figure 1 shows a decline in the demand for real loans (borrowing). The need for borrowing is weaker at the firms because the wage bill falls as the number of employed workers decreases. Both earnings and equity rise with lower real marginal costs and lower interest rates, and the amount of banking labor falls beyond the reduction of total loans.

The results indicate that credit frictions act as a financial accelerator, characterized by a countercyclical external finance premium. Since equity rises, the marginal cost of loan production falls and the interest rate on borrowing, r_t^b, turns lower. This would have some additional effect on the decline of the real marginal cost (r_t^b directly affects the firm-specific marginal cost as indicated in equation 24), that would push inflation and interest rates further down, and the economic activity would expand more. Anyway, the drop of the external finance premium is not substantial (7 basis points), which makes the quantitative effects of the financial accelerator be considered rather low.

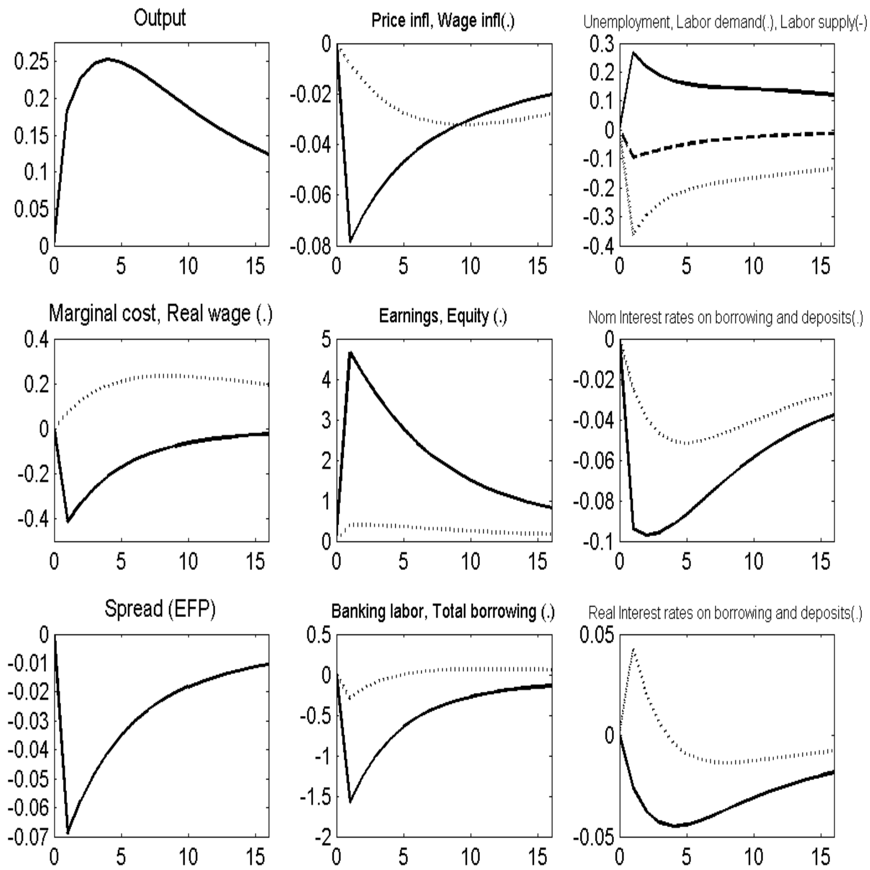

Figure 1: Impulse-response functions. Technology shock, 0.34%.

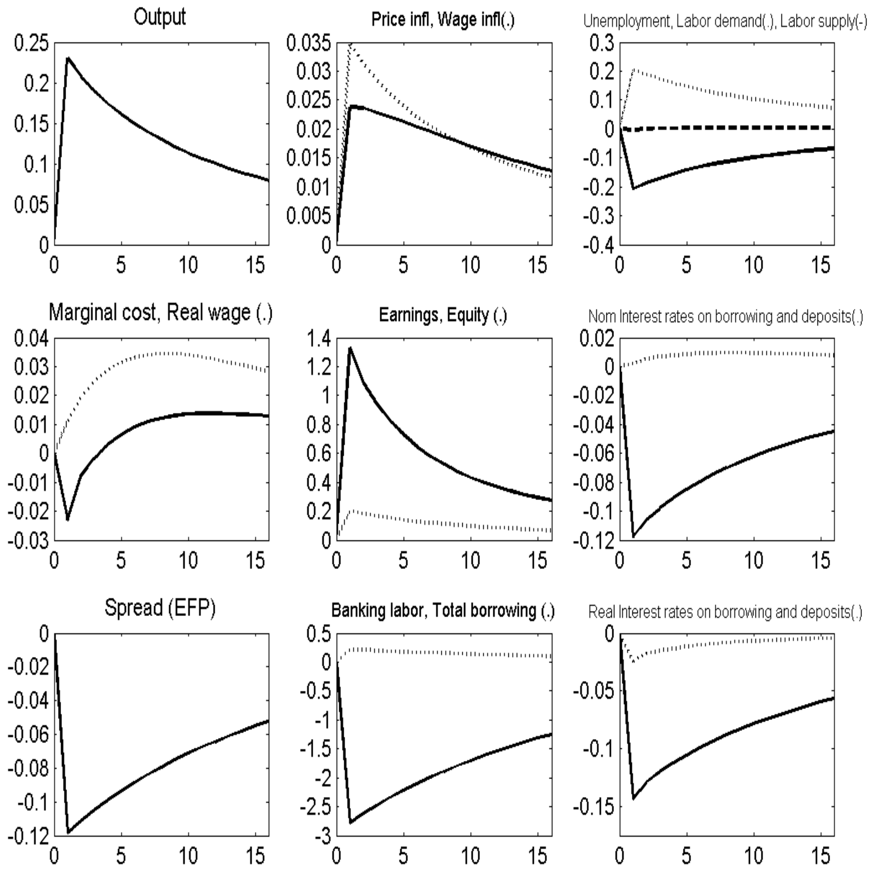

Figure 2: Impulse-response functions. Financial shock, 1.06%.

4.2 Financial shock

The effects of the financial shock, ε_t^b, can be observed in Figure 2. Initially, the marginal cost of loan production comes down as banks can produce a higher amount of real loans using the same quantity of banking inputs. In turn, firms observe a reduction in their cost of borrowing, r_t^b. Such lower external finance cost results in a lower marginal cost for good production and, subsequently, in a lower price inflation when applying mark-up pricing policies. The real interest rate on deposits goes down, and households reduce savings to increase their current consumption in the standard IS-type pattern. The financial shock that started in the banking sector has some expansionary effect in the goods market through higher output produced by firms to meet desired spending on consumption goods.

Labor demand rises to produce the additional amount of consumption goods, and unemployment falls almost proportionally to the increase in labor demand (labor supply barely changes as shown in Figure 2). For the fraction of firms that are able to reset wages, higher labor demand leads to wage revisions upwards. Firm earnings increase with lower real marginal costs, while average equity also rises due to both higher earnings and lower interest rates. The marginal cost of loan production turns lower and there is a countercylical response of the interest-rate spread that brings a financial accelerator mechanism. Finally, banking labor falls significantly to compensate for the increase in banking productivity.

The real-side effects of the financial shock are quantitatively significant (in line with those found by Christiano *et al.*, 2010, in a DSGE-type estimated model): a financial shock that moves down the external finance premium in nearly 12 basis points has an impact on both output and unemployment of a moderate magnitude (between 0.2% and 0.3% deviation with respect to their steady-state levels). The influence on both price and wage inflation is weaker as they both increase only between 2 and 4 basis points.

4.3 Consumption (demand) shock

Figure 3 displays the responses obtained in the model when there is an increase in consumption motivated for a positive innovation in the exogenous shock, ε_t^c. Such demand push is satisfied by firms producing more consumption goods, and charging a higher price when Calvo-type market conditions allow so. Such increase in prices is explained by the fact that the marginal production cost rises with the level of production due to decreasing marginal returns to labor. Thus, both output and price inflation increase. The demand-driven expansion has also immediate effects in the labor market. Labor demand and employment increase, the rate of unemployment falls, and wage inflation rises following the higher nominal wages that can be reset during the period.

In the banking sector, the demand for real loans also increases as a consequence of a higher total wage bill that takes into account the labor market expansion. It explains why banking labor soars while aggregate equity is falling sue to the asset substitution takin place with higher interest rates. Remarkably, the external finance premium is procyclical in this case, which would bring a financial attenuation effect. Since the demand for loans increase, the cost of borrowing is higher and the economic expansion is buffered by the additional increase observed in the marginal cost, the rate of price inflation, and the interest rates. The interest-rate spread peaks by nearly 10 basis points at the time of the shock, which would justify an additional monetary

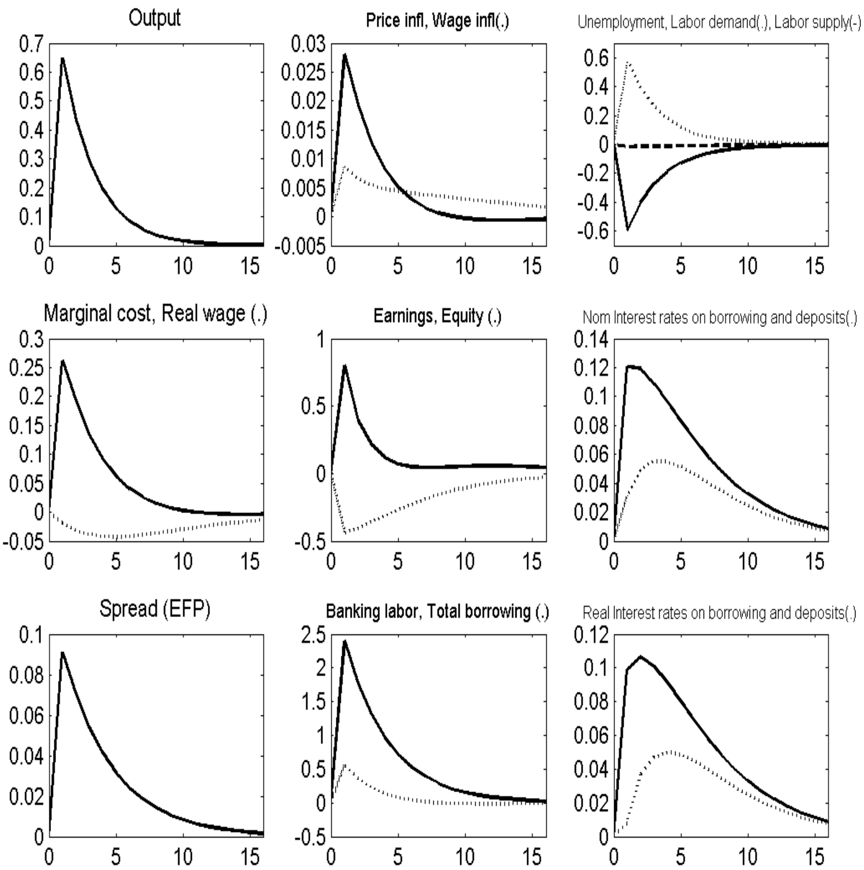

Figure 3: Impulse-response functions. Consumption shock, 1.12%.

policy tightening in Taylor-type rule equivalent to 4.3 basis points (recalling the calibration of $\mu_{EFP} = 0.43$ in 35).

5 Application I. A credit crunch episode

Figure 4 illustrates the responses obtained when there is an exogenous *large* increase in the cost of borrowing.[19] The size of the financial shock is a negative 5% ($\varepsilon_1^b = -5.0$). The economic interpretation of this perturbation can be either a lower productivity in loan production technology (12) or an increase in the external financial requirement of the firms.[20] Anyway, such adverse financial shock raises the external finance premium, $r_t^b - r_t^d$, which reduces significantly the amount of borrowing, and generates a simulated credit crunch episode.

The baseline model with both sticky prices and sticky wages predicts little cuts in nominal wages and prices, while the external finance premium rises nearly 2% in annualized terms. Firms observe how their real marginal cost of production increases, while both earnings and equity value fall to engine the increase of the cost of external finance. Expected deflation and the increase in the real interest rate on deposits explain why the overall demand suffers a contraction as households cut their current spending on consumption goods. In turn, output falls by 1.09% and unemployment rises by nearly 0.98% in the labor market. With falling inflation and output, the central bank gradually cuts the nominal interest rate when applying the Taylor-type rule (35), despite the positive response coefficient on the spread.

In the variant with flexible wages, the effects of the financial shock on the real sector of the economy are cut down significantly (roughly by half, as displayed in Figure 4). As wages fall immediately to restore equilibrium in the labor market (a sharp decline by more than 10% in annualized terms), both the real wage and the real marginal cost decline. Price inflation falls substantially (despite sticky prices) by applying the mark-up policy over decreasing marginal

[19] The responses of the interest rates, price inflation, wage inflation, and the external finance premium have been annualized in the diagrams of Figure 4.

[20] The overall financial constraint, $b_t = \tau \int \frac{W_t(\omega)}{P_t} l_t(\omega)\, dw$, and the loan production technology, $b_t = e^{\varepsilon_t^b}[a v_t^\theta + (1-a) m_t^\theta]^{\frac{1}{\theta}}$, can be combined to obtain

$$[a v_t^\theta + (1-a) m_t^\theta]^{\frac{1}{\theta}} = e^{-\varepsilon_t^b} \tau \int \frac{W_t(\omega)}{P_t} l_t(\omega)\, dw,$$

where an adverse financial shock can also be interpreted as a rising fraction of the wage bill that must be financed by the bank.

costs. In turn, firm equity barely changes and the contractionary effects on output and employment are quite small (they both decline by 0.52% at most), while the rate of unemployment remains constant at the steady-state percentage.

Likewise, Figure 4 shows how the effects of the credit crunch on output and unemployment are also reduced in the model variant with higher substitutability for equity in loan production. The parameterization of $\theta = 0.0$ in (12) would result in a Cobb-Douglas loan production technology with unit elasticity of substitution between equity and labor. It implies multiplying the elasticity by a factor of four from the baseline value of 0.25 with $\theta = -3.0$. A higher substitutability in loan production can be interpreted as lower real rigidities, in the sense that banks can take advantage of a less-costly substitution among loan production inputs. Such strategic complementarity between equity and labor provides a buffering on the cost of borrowing after an adverse financial shock. Thus, Figure 4 displays that the external finance premium only rises by 1% with Cobb-Douglas loans (unit elasticity of substitution), half of the increase observed in the baseline case. In spite of having nominal rigidities on both price and wage settings, the real effects of the adverse financial shocks are of smaller magnitude. Output falls by 0.56% and unemployment rises by 0.58%, also around half of the responses observed with less substitutability of inputs in loan production. The transmission mechanism from the financial shock to real variables is the same as before: raises external finance, increases the real interest rate on both borrowing and deposits, reduces spending on consumption goods, and raises unemployment because of a lower labor demand. The effects are quantitatively smaller than in the baseline model because the initial increase in the cost of external finance is alleviated by a more favorable marginal rate of substitution between the collateral capacity of equity and banking labor.

Finally, the flexible-price, flexible-wage model version brings a mostly-neutralized credit crunch through price/wage adjustments. Both aggregate prices and wages decline sharply (by around 5% annualized) at the quarter of the financial shock. The central bank can implement an aggressive expansionary monetary policy and the nominal interest rate is cut by more than 1%. The real interest rate on deposits barely increases and there is a soft economic contraction.

Volatilities caused by financial shocks

Providing more quantitative results, we have calculated some standard deviations when the

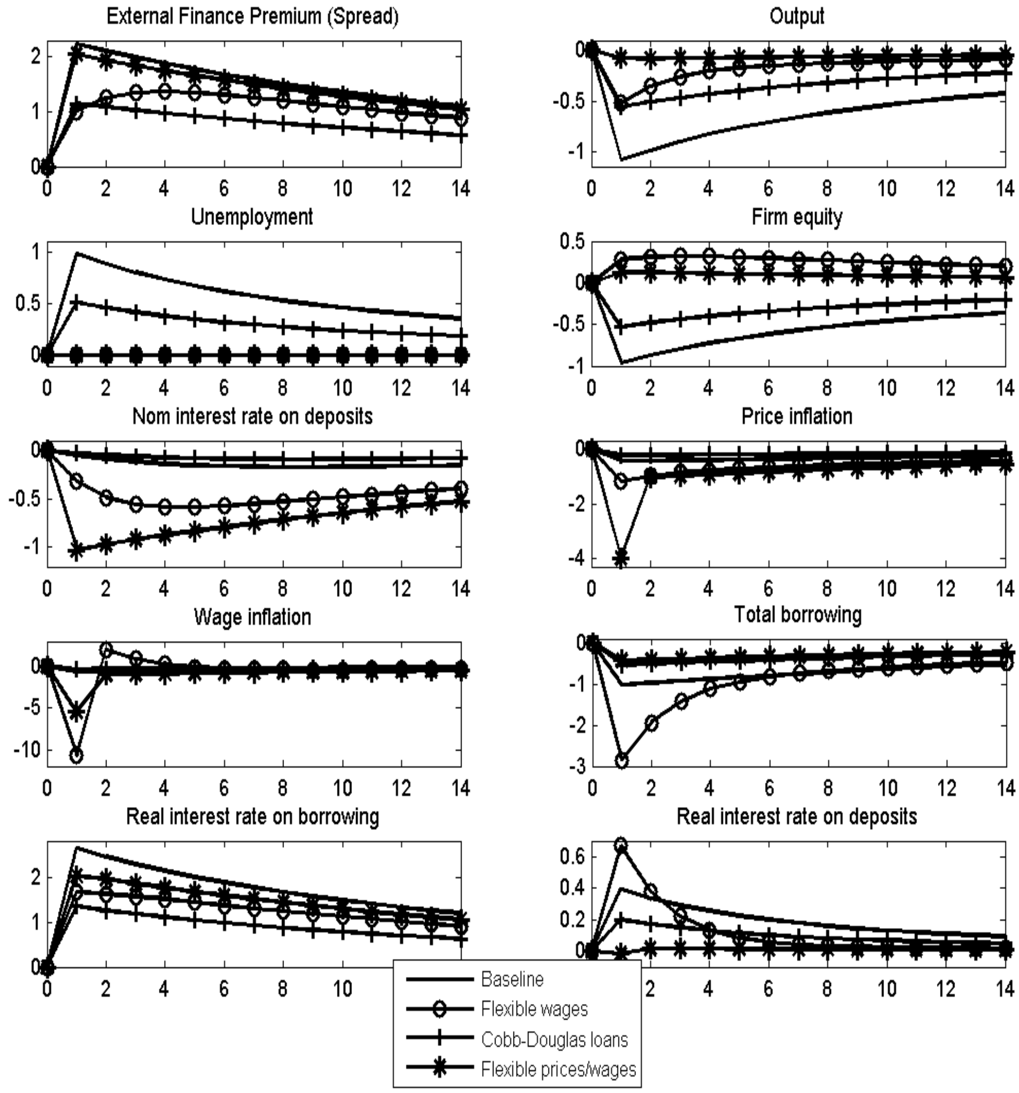

Figure 4: Responses to one adverse financial shock, $\varepsilon_1^b = -5.0\%$.

only source of variability is financial shocks. Thus, innovations of production technology and consumption preferences are shut down to find out the volatilities obtained with the calibrated financial shocks. Table 3 gives the results under alternative scenarios for price/wage setting and real rigidities.

Table 3. Standard deviations conditional to financial shocks (ann., %)

	Spread	Output	Price infl.	Wage infl.	Unem. rate
Sticky prices/wages (baseline)	1.47	0.63	0.33	0.36	0.54
Flexible wages	1.09	0.18	0.63	2.35	0.00
Cobb-Douglas loans	0.77	0.33	0.17	0.19	0.28
Flexible prices/wages	1.38	0.06	1.10	1.32	0.00

With sticky prices and wages (baseline model), the standard deviations of output and unemployment are 0.63% and 0.54% respectively, higher than those of either price inflation and wage inflation. If labor market clears with fully-flexible wages, unemployment has no variability and the real effects of financial shocks are dramatically reduced as the standard deviation of output is only 0.18%. By contrast, wage inflation volatility multiplies by a factor of 6. With Cobb-Douglas loans ($\theta = 0.0$), less-rigid substitutability for equity in loan production brings significant reductions of output and unemployment variability (standard deviations are cut approximately by half in Table 3), while the cyclical fluctuations of price and wage inflation are also weaker than those obtained in the baseline model. If the economy is fully flexible on both price and wage setting, the real effects of financial shocks are quantitatively very low (the standard deviation of output is less than one tenth of that in the baseline model), and most of the adjustment occurs through the strong variability in both price inflation and wage inflation.

Nominal rigidities versus real rigidities

How much of the real effects of the financial shock could be reduced if both prices and wages could be fully adjusted on a quarter-to-quarter basis? And how much of that could be alternatively obtained by having a more flexible substitutability for equity in loan production? We look for answers to these questions by re-measuring the volatility of fluctuations when moving either the nominal rigidity parameter, η, from 0 (fully-flexible prices/wages) to 1 (fixed prices/wages), and the loan production parameter, θ, from -15.0 (highly rigid loan production, *proxy* to Leontief technology) to its upper bound 1.0 (flexible loan production with perfect substitutability

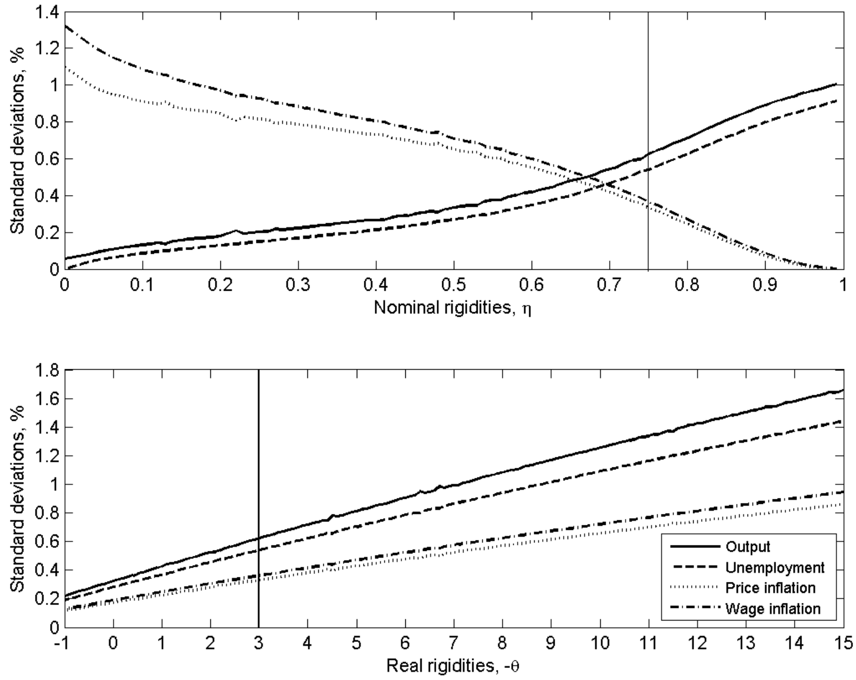

Figure 5: Volatilities and rigidities consitional to financial shocks.

between equity and labor).[21] Figure 5 displays the volatilities (annualized standard deviations) conditional to having financial shocks as the only source of business cycle variability. The vertical lines in Figure 5 represent the parameter setting in the baseline calibration. Notably, nominal and real rigidities have equivalent effects on both output and unemployment variability. Their standard deviations rise from near-zero values when either nominal or real rigidities are eliminated to levels around 1% with either full price/wage rigidity (fixed prices/wages with $\eta = 1$), or highly-rigid loan production conditions (near Leontief technology with $\theta = -10$). As for price/wage inflation, the standard deviations fall with higher nominal rigidities (moving towards zero as η approaches 1), but they rise with higher real rigidities.

In the case with flexible prices/wages ($\eta = 0$ in the top plot of Figure 5), we find that the standard deviation of output is very close to zero (0.06% reported in Table 3). The high volatility of prices/wages nearly absorbs the real effects of financial shocks. And in the case

[21]For the sake of tractability, the real rigidity is measured with $-\theta$ instead of the (negative of the) elasticity of substitution, $-\frac{1}{1-\theta}$, which would quickly approach to minus infinity as the model parameterizes the linear loan production technology ($\theta = 1$).

without real rigidities ($\theta = 1$ in the bottom plot of Figure 5), the perfect substitutability between equity and labor also reduces substantially the real effects of financial shocks (the standard deviation of output is 0.22%), while volatilities of price/wage inflation are very low.

6 Application II. The spread coefficient in Taylor rule

Cúrdia and Woodford (2010) argue that the marginal response of monetary policy to a given increase in the credit spread must be expansionary, i.e. characterized by a $\mu_{EFP} < 0$ coefficient in the extended Taylor-type rule (35). They suggest that monetary policy should be accommodative in the presence of a credit crunch, so that lower interest rates facilitate the economic recovery. Nevertheless, a reduction of R_t^d in response to an increase of the external finance premium expands the interest-rate spread, which might amplify fluctuations through the financial accelerator mechanism.[22] Moreover, a monetary policy rule must sustain the systematic response to the spread under all sources of variability in place (technology shocks, demand shocks and financial shocks in our model), which brings more difficulty on the determination of a desirable value for μ_{EFP}, as recognized in Cúrdia and Woodford (2010).

For a quantitative exercise, we have checked the macroeconomic volatility that results from alternative parameterization of μ_{EFP}. Hence, we have moved the value of μ_{EFP} across the interval between -1 and 1 and calculated the standard deviations obtained in the model for realistic monetary policy targets: variability of price inflation and unemployment. The selection criteria proposed is to minimize a loss function that combines the volatilities of price inflation and unemployment

$$sdv(\pi^p) + \Lambda sdv(u) \tag{37}$$

where $sdv(\pi^p)$ is the annualized percent standard deviation of price inflation, $sdv(u)$ is the percent standard deviation of unemployment, and $\Lambda \geq 0$ is the relative weight assigned to stabilizing unemployment.

Table 4 shows the optimized values of the coefficients of response to credit spreads under three alternative stabilizing preferences of the central bank ($\Lambda = 0.0$, $\Lambda = 0.5$ and $\Lambda = 1.0$):

[22] With further complications in the implementation from the possibility of meeting the lower bound of nominal interest rates (liquidity trap situations).

Table 4. Optimized spread coefficient with alternative stabilizing criteria

Model variant	μ^*_{EFP}		
	$\Lambda = 0.0$	$\Lambda = 0.5$	$\Lambda = 1.0$
Baseline	0.26	0.30	0.32
Flex. wages	0.64	0.64	0.64
Flex. prices/wages, $\eta = 0.0$	0.48	0.48	0.48
Cobb-Douglas loans, $\theta = 0.0$	0.39	0.43	0.50
Quasi-Leontief loans, $\theta = -10.0$	0.16	0.18	0.21
Linear production, $\alpha = 1.0$	0.09	0.15	0.17
No smoothing, $\mu_R = 0.0$	0.08	0.16	0.16

In contrast to the recommendation of Cúrdia and Woodford (2010), the value of μ^*_{EFP} is positive and it belongs to the interval between 0.08 and 0.64 in all cases with combined sources of variability.[23] If the central bank gives equal weights to targeting inflation and unemployment ($\Lambda = 1.0$), the value of μ^*_{EFP} tends to be higher, which implies a deeper responsiveness to credit spreads. With flexible wages, the responses must be more aggressive to stabilize inflation under any criteria. If substitutability between equity and labor in loan production turns easier ($\theta = 0.0$ as in a Cobb-Douglas loan production technology) μ^*_{EFP} increases, whereas the responsiveness of the central bank to the credit spread should be lower with a more rigid loan production (μ^*_{EFP} is lower in the variant with $\theta = -10.0$ relative to the baseline calibration $\theta = -3.0$). In case of a goods production technology with constant labor productivity ($\alpha = 1.0$), the central bank should also be less active in responding to the spread, with lower μ^*_{EFP} under the three stabilizing criteria. And, if monetary policy does not care about interest-rate smoothing ($\mu_R = 0$), the optimized marginal reaction to the spread would be again significantly lower under the three stabilizing criteria.

[23] A positive μ_{EFP} does not imply central bank reactions that raise interest rates during a credit crunch episode. As shown in Figure 4, the nominal interest rate on deposits falls as a joint reaction to inflation, output, and the external finance premium put forth in the extended Taylor-type rule (35).

Table 5. Standard deviations (%) with alternative spread coefficients, $[sdv(y), sdv(\pi^p), sdv(u)]$'

	$\mu_{EFP}= -1.0$	$\mu_{EFP}= 0.0$	$\mu_{EFP}= 1.0$	μ^*_{EFP}
	1.84	1.24	2.00	1.31
Baseline	1.28	0.83	1.08	0.80
	2.12	1.32	1.58	1.23
	4.01	1.60	1.69	1.66
Flex. wages	8.95	1.85	1.43	1.35
	0.00	0.00	0.00	0.00
	1.76	1.76	1.76	1.76
Flex. prices/wages, $\eta = 0.0$	5.80	4.06	4.07	3.79
	0.00	0.00	0.00	0.00
	1.44	1.24	1.46	1.28
Cobb-Douglas loans, $\theta = 0.0$	1.01	0.82	0.85	0.80
	1.65	1.32	1.30	1.26
	2.78	1.24	3.71	1.34
Quasi-Leontief loans, $\theta = -10.0$	1.79	0.85	1.96	0.80
	3.06	1.32	2.97	1.21
	1.82	1.14	1.66	1.12
Linear production, $\alpha = 1.0$	1.03	0.61	0.97	0.60
	1.87	1.12	1.48	1.09
	1.80	1.22	1.97	1.24
No smoothing, $\mu_R = 0.0$	1.37	0.94	1.22	0.91
	2.09	1.36	1.64	1.31

Finally, Table 5 shows the standard deviations of output, price inflation and unemployment obtained under alternative policy responses to the interest-rate spread. Particularly, there is a comparison between the active tightening to spread ($\mu_{EFP}= 1.0$), the active loosening to spread ($\mu_{EFP}= -1.0$), the no reaction policy ($\mu_{EFP}= 0.0$), and the reaction policy that minimizes the loss function (37) with stabilizing preferences marked at $\Lambda = 0.5$ (in the column μ^*_{EFP}). The non-active policy ($\mu_{EFP}= 0.0$) might be considered a benchmark reference as it brings a conventional Taylor-style rule. If the central bank reacts to the spread with one-to-one monetary tightening ($\mu_{EFP}= 1.0$), the stabilizing performance worsens significantly in all cases except for those with flexible wages and/or prices. If the central bank chooses interest-rate cuts in response to an increase in the spread, $\mu_{EFP}= -1.0$, the standard deviations turn generally higher than those with $\mu_{EFP}= 1.0$, and the performance is also worse than with a traditional Taylor-style rule. Finally, the comparison between columns labeled $\mu_{EFP}= 0.0$ and μ^*_{EFP} indicates that the lack of reaction to the spread only slightly worsen off the stabilizing performance. The standard deviations of either inflation or unemployment never increase by more than 10%, when replacing the optimized coefficient μ^*_{EFP} for $\mu_{EFP}= 0$. Thus, a Taylor rule with no spread coefficient keeps most of its stabilizing capacity in this model with financial frictions and endogenous spreads.

7 Conclusions

The introduction of external finance in the optimizing problem of the firm makes the cost of borrowing be one additional component of the marginal cost of production. In turn, the interest rate of loans affects decisions on setting prices, wages, labor demand and the amount of output produced. The external finance premium is obtained endogenously as the marginal cost of loan production in the banking sector. When financial conditions turn tighter and, subsequently, the interest rate of loans rises, the firm faces higher marginal costs and charges higher prices that make a deeper contraction of economic activity. Moreover, the use of firm equity as collateral for the production of loans opens the possibility of a financial accelerator for business cycle fluctuations: an increase in firm equity cuts the cost of loan production and reduces the external finance premium required to firms.

Simulations of the baseline model, with sticky prices and wages, indicate that the borrowing requirement of firms brings about a financial accelerator effect, quantitatively small, when there is a technology shock. Firm earnings and equity rise, the external finance premium falls countercyclically and firms cut prices and increase production further. By contrast, in the presence of a demand-side consumption shock the credit constraint results in some financial attenuation. Equity value falls due to decreasing productivity and higher interest rates, which pushes up the cost of borrowing and the economic activity does not expand as much as initially pushed.

We devoted a special attention to the effects of financial shocks. In the baseline model, a 1%

financial innovation increases output and reduces unemployment by approximately 0.2%. These responses of output and unemployment to financial innovations are quite sensitive to changes in the level of nominal and real rigidities in place. If nominal wages were fully flexible, the real effects of financial shocks would be cut by half. If nominal rigidities are totally eliminated for both price and wage adjustments, the decline of output after an adverse financial shock is less than one tenth of the one observed with sticky prices and wages. Alternatively, a higher elasticity of substitution in loan production (less real rigidities) can also reduce significantly the real effects of financial shocks.

A central bank that is concerned on stabilizing inflation and unemployment by implementing a Taylor-type monetary policy rule should include a coefficient of response to the external finance premium. The effective reaction to any increase in the credit spread should be low and of positive sign. This result is robust to changes in either nominal rigidities on price/wage setting or in real rigidities on banking production. With no reaction to the spread (conventional monetary policy), the stabilizing properties of the Taylor-type rule would slightly worsens off.

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Technical Appendix (Not for publication)

1. Household optimal consumption variety.

Moreover, for a given desired level of aggregate consumption, c_t, the representative household decides $c_t(\omega)$ so to solve the following maximization problem:

$$\underset{c_t(\omega)}{Max} \left[\int_0^F c_t(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}}$$

$$s.to : c_t = \int_0^F \frac{P_t(\omega)}{P_t} c_t(\omega) d\omega$$

The optimal allocation of differentiated consumption goods requires substitution across differentiated consumption goods as determined by the following demand curve[24]

$$\frac{c_t(\omega)}{c_t} = \left(\frac{P_t(\omega)}{P_t} \right)^{-\sigma},$$

where $\sigma > 1$ is the Dixit-Stiglitz constant elasticity of substitution.

2. Household optimal labor supply allocation.

As in Casares (2007), the labor supply bundle is a CES composite of firm-specific amounts of labor supplied

$$l_t^s = \left[\int l_t^s(\omega)^{\frac{1+\sigma_w}{\sigma_w}} d\omega \right]^{\frac{\sigma_w}{1+\sigma_w}},$$

while the aggregate nominal wage is also obtained with a CES aggregation scheme

$$W_t = \left[\int W_t(\omega)^{1+\sigma_w} d\omega \right]^{\frac{1}{1+\sigma_w}}.$$

The optimal allocation of labor supply across firms is determined by solving the problem:

$$\underset{l_t^s(\omega)}{Max} \int W_t(\omega) l_t^s(\omega) d\omega$$

$$s.to : l_t^s = \left[\int l_t^s(\omega)^{\frac{1+\sigma_w}{\sigma_w}} d\omega \right]^{\frac{\sigma_w}{1+\sigma_w}}$$

The first order condition yields

$$W_t(\omega) - \varkappa_t \left(l_t^s\right)^{-\frac{1}{\sigma_w}} l_t^s(\omega)^{\frac{1}{\sigma_w}} = 0,$$

where $\varkappa_t$ is the Lagrangian multiplier. The optimal relative labor supply becomes

$$\frac{l_t^s(\omega)}{l_t^s} = \left(\frac{W_t(\omega)}{\varkappa_t} \right)^{\sigma_w}.$$

[24] Proof available in Walsh (2010, pages 331-332).

As a standard result in monopolistically competitive markets, the Lagrange multiplier $\varkappa_t$ coincides with the aggregate price index. In this case the price index is the nominal wage. Inserting $l_t^s(\omega) = \left(\frac{W_t(\omega)}{\varkappa_t}\right)^{\sigma_w} l_t^s$ in the CES labor supply bundle gives

$$l_t^s = \left[\int \left(\left(\frac{W_t(\omega)}{\varkappa_t}\right)^{\sigma_w} l_t^s\right)^{\frac{1+\sigma_w}{\sigma_w}} d\omega\right]^{\frac{\sigma_w}{1+\sigma_w}} = \frac{l_t^s}{\varkappa_t^{\sigma_w}}\left[\int \left(W_t(\omega)^{\sigma_w}\right)^{\frac{1+\sigma_w}{\sigma_w}} d\omega\right]^{\frac{\sigma_w}{1+\sigma_w}},$$

which can be solved for $\varkappa_t$ as follows

$$\varkappa_t = \left[\int W_t(\omega)^{1+\sigma_w} d\omega\right]^{\frac{1}{1+\sigma_w}} \equiv W_t.$$

Replacing $\varkappa_t$ with W_t in the optimal relative supply equation, it is obtained

$$\frac{l_t^s(\omega)}{l_t^s} = \left(\frac{W_t(\omega)}{W_t}\right)^{\sigma_w}.$$

3. Log-linearized equations for the banking sector

a. CES production function.

Taking the CES production function of external borrowing, $b_t = e^{\varepsilon_t^b}[av_t^\theta + (1-a)m_t^\theta]^{\frac{1}{\theta}}$, and powering to θ gives

$$b_t^\theta = e^{\theta\varepsilon_t^b}[av_t^\theta + (1-a)m_t^\theta],$$

from where we can obtain the loglinear approximation

$$\theta\widehat{b}_t = \theta\varepsilon_t^b + \tfrac{av_{ss}^\theta}{av_{ss}^\theta+(1-a)m_{ss}^\theta}\theta\widehat{v}_t + \tfrac{(1-a)m_{ss}^\theta}{av_{ss}^\theta+(1-a)m_{ss}^\theta}\theta\widehat{m}_t,$$

where variables topped with the hat denote log deviations respect to the steady state level, e.g. $\widehat{b}_t = \log\left(\frac{b_t}{b_{ss}}\right)$, and ss supercripts stands for steady-state levels. Defining $\Omega = \frac{av_{ss}^\theta}{av_{ss}^\theta+(1-a)m_{ss}^\theta}$ as the steady-state contribution of equity for loan production and dropping the θ's leaves the previous expression as follows

$$\widehat{b}_t = \varepsilon_t^b + \Omega\widehat{v}_t + (1-\Omega)\,\widehat{m}_t.$$

b. Spread equation.

Let us define the marginal product of banking labor as

$$g_{m_t} \equiv \frac{\partial b_t}{\partial m_t} = e^{\varepsilon_t^b}(1-a)m_t^{\theta-1}[av_t^\theta + (1-a)m_t^\theta]^{\frac{1}{\theta}-1} = \frac{(1-a)m_t^{\theta-1}b_t}{av_t^\theta + (1-a)m_t^\theta},$$

so that the equilibrium interest rate on borrowing (loans) can be rewritten as

$$r_t^b = r_t^d + \frac{w_t^m}{g_{m_t}}.$$

Multiplying both sides by g_{m_t}, we get

$$(r_t^b - r_t^d) g_{m_t} = w_t^m.$$

(Log)-linearizing, using the approximation $\widehat{x}_t \simeq \frac{x_t - x_{ss}}{x_{ss}}$, and dropping constant terms, it is obtained

$$r_t^b - r_t^d = (r_{ss}^b - r_{ss}^d)\left(\widehat{w}_t^m - \widehat{g}_{m_t}\right).$$

Finally, loglinearizing the marginal product of banking labor results in

$$\widehat{g}_{m_t} = (\theta - 1)\,\widehat{m}_t + \widehat{b}_t - \Omega\theta\widehat{v}_t - (1-\Omega)\theta\widehat{m}_t = \Omega\,(\theta - 1)\,(\widehat{m}_t - \widehat{v}_t) + \varepsilon_t^b,$$

and inserting the result in the spread equation leads to

$$r_t^b = r_t^d + (r_{ss}^b - r_{ss}^d)(\widehat{w}_t^m - \Omega\,(\theta - 1)\,(\widehat{m}_t - \widehat{v}_t) - \varepsilon_t^b).$$

c. Collateral service yield of equity.

Define the marginal return on equity for loan production

$$g_{v_t} \equiv \frac{\partial b_t}{\partial v_t} = e^{\varepsilon_t^b} a v_t^{\theta-1}[a v_t^\theta + (1-a) m_t^\theta]^{\frac{1}{\theta}-1} = \frac{a v_t^{\theta-1} b_t}{a v_t^\theta + (1-a) m_t^\theta},$$

so that the expression relative to CSY_t^v can be rewritten as

$$CSY_t^v = \left(r_t^b - r_t^d\right) g_{v_t}.$$

A semi-loglinear approximation, using $\widehat{x}_t \simeq \frac{x_t - x_{ss}}{x_{ss}}$ and dropping constant terms gives

$$CSY_t^v = \frac{CSY_{ss}^v}{r_{ss}^d - r_{ss}^d}\left(r_t^b - r_t^d\right) + CSY_{ss}^v \widehat{g}_{v_t},$$

where $\widehat{g}_{v_t} = (\theta - 1)\,\widehat{v}_t + \widehat{b}_t - \Omega\theta\widehat{v}_t - (1-\Omega)\theta\widehat{m}_t = (1-\Omega)\,(\theta-1)\,(\widehat{v}_t - \widehat{m}_t) + \varepsilon_t^b$ can be plugged to reach

$$CSY_t^v = \frac{CSY_{ss}^v}{r_{ss}^d - r_{ss}^d}\left(r_t^b - r_t^d\right) + CSY_{ss}^v\,(1-\Omega)\,(\theta-1)\,(\widehat{v}_t - \widehat{m}_t) + \varepsilon_t^b.$$

It can be noticed that the equivalent expression for the collateral yield of equity

$$CSY_t^v = \frac{w_t^m m_t}{v_t} \frac{a}{(1-a)} \left(\frac{v_t}{m_t}\right)^\theta$$

brings the semi-loglinear approximation

$$CSY_t^v = CSY_{ss}^v \left(\widehat{w}_t^m + (\theta - 1) \left(\widehat{v}_t - \widehat{m}_t \right) \right),$$

where replacing $\widehat{w}_t^m$ with the value implied by the expression for $\left(r_t^b - r_t^d\right)$ above, turns into

$$CSY_t^v = CSY_{ss}^v \left(\frac{1}{\left(r_{ss}^b - r_{ss}^d\right)} \left(r_t^b - r_t^d\right) + \Omega \left(\theta - 1\right) \left(\widehat{m}_t - \widehat{v}_t \right) + \varepsilon_t^b + (\theta - 1) \left(\widehat{v}_t - \widehat{m}_t \right) \right),$$

and simplifies to (A17) below.

4. Real rigidities on loan production at the bank.

The total cost of loan production is

$$TC(b_t) = w_t^m m_t + CSY_t^v v_t$$

Equilibrium input prices (from first order conditions on demand for banking labor and collateralizing equity) are:

$$w_t^m = \left(r_t^b - r_t^d\right) \frac{(1-a) m_t^{\theta-1} b_t}{a v_t^\theta + (1-a) m_t^\theta}, \qquad (m_t^{foc})$$

$$CSY_t^v = \left(r_t^b - r_t^d\right) \frac{a v_t^{\theta-1} b_t}{a v_t^\theta + (1-a) m_t^\theta}. \qquad (v_t^{foc})$$

Inserting w_t^m and CSY_t^v in the $TC_t(b)$ function gives

$$TC(b_t) = \left(r_t^b - r_t^d\right) \tfrac{(1-a) m_t^{\theta-1} b_t}{a v_t^\theta + (1-a) m_t^\theta} m_t + \left(r_t^b - r_t^d\right) \tfrac{a v_t^{\theta-1} b_t}{a v_t^\theta + (1-a) m_t^\theta} v_t.$$

Subsequently, the marginal cost of loan production is equal to the spread

$$MC(b_t) = \frac{\partial TC(b_t)}{\partial b_t} = r_t^b - r_t^d.$$

The spread is endogenously determined in the model

$$r_t^b - r_t^d = MC(b_t) = \frac{w_t^m}{\frac{(1-a) m_t^{\theta-1} b_t}{a v_t^\theta + (1-a) m_t^\theta}}.$$

After loglinearizing (and simplifying), log deviations from steady state of the marginal cost of producing a loan at the bank is

$$\widehat{mcb}_t = \widehat{w}_t^m + \Omega \left(1 - \theta\right) \left(\widehat{m}_t - \widehat{v}_t \right),$$

where Ω is the steady-state ratio $\Omega = \frac{a v^\theta}{a v^\theta + (1-a) m^\theta}$.

Let us recall that $\theta \in$ (-$\infty, 1]$ in the loan production technology, and the elasticity of substitution is $\frac{1}{1-\theta}$. Approaching the lower limit of $\theta \rightsquigarrow -\infty$, loan production follows a Leontief technology with a zero constant elasticity of substitution. In the expression for $\widehat{mcb}_t$, a very high and negative θ, ($\theta = -1000$ for example), implies that the change in the relative use of banking inputs $(\widehat{m}_t - \widehat{v}_t)$ has to be very small to avoid high marginal costs of loan production. That might be considered a "real rigidity" for loan production in the sense discussed by Gopinath and Itskhoki (2010).

Alternatively, a Cobb-Douglas type loan production ($\theta = 0$) gives $\widehat{mcb}_t = \widehat{w}_t^m + \Omega\left(\widehat{m}_t - \widehat{v}_t\right)$, allowing for large substitutions $\widehat{m}_t - \widehat{v}_t$ without increasing significantly $\widehat{mcb}_t$ because $0 < \Omega < 1$ by definition. That could be consider quite a"flexible loan production technology".

5. Log-linearized equation for aggregate firm earnings

Firm earnings and the real marginal cost for the ω representative type are, respectively,

$$e_t(\omega) = \frac{P_t(\omega)}{P_t} y_t(\omega) - \left(1 + \tau r_t^b\right) \frac{W_t(\omega)}{P_t} l_t^d(\omega),$$

and

$$\xi_t(\omega) = \frac{\left(1+\tau r_t^b\right) \frac{W_t(\omega)}{P_t} l_t^d(\omega)}{\alpha(y_t(\omega)+\Phi)}.$$

Inserting the value of $\left(1 + \tau r_t^b\right) \frac{W_t(\omega)}{P_t} l_t^d(\omega)$, obtained from the expression of $\xi_t(\omega)$, into the earnings equation results in

$$e_t(\omega) = \frac{P_t(\omega)}{P_t} y_t(\omega) - \xi_t(\omega)\alpha\left(y_t(\omega) + \Phi\right) = y_t(\omega)\left(\frac{P_t(\omega)}{P_t} - \xi_t(\omega)\alpha\right) - \xi_t(\omega)\alpha\Phi.$$

Earnings per output for firm ω yield

$$\frac{e_t(\omega)}{y_t(\omega)} = \left(\frac{P_t(\omega)}{P_t} - \xi_t(\omega)\alpha\right) - \xi_t(\omega)\alpha\frac{\Phi}{y_t(\omega)}.$$

The loglinear approximation for the expression of earning per output gives

$$\widehat{e}_t(\omega) - \widehat{y}_t(\omega) = \frac{1}{1 - \frac{\alpha(\sigma-1)(1+\Phi/y)}{\sigma}}\left(\left(\widehat{P}_t(\omega) - \widehat{P}_t\right) - \frac{\alpha(\sigma-1)}{\sigma}\widehat{\xi}_t(\omega)\right) - \frac{\frac{\alpha(\sigma-1)\Phi/y}{\sigma}}{1 - \frac{\alpha(\sigma-1)(1+\Phi/y)}{\sigma}}\left(\widehat{\xi}_t(\omega) - \widehat{y}_t(\omega)\right),$$

and grouping terms

$$\widehat{e}_t(\omega) = \frac{1}{1 - \frac{\alpha(\sigma-1)(1+\Phi/y)}{\sigma}}\left(\widehat{P}_t(\omega) - \widehat{P}_t\right) - \frac{\frac{\alpha(\sigma-1)(1+\Phi/y)}{\sigma}}{1 - \frac{\alpha(\sigma-1)(1+\Phi/y)}{\sigma}}\widehat{\xi}_t(\omega) + \left(1 + \frac{\frac{\alpha(\sigma-1)\Phi/y}{\sigma}}{1 - \frac{\alpha(\sigma-1)(1+\Phi/y)}{\sigma}}\right)\widehat{y}_t(\omega).$$

The aggregation across all firms $\widehat{e}_t = \int \widehat{e}_t(\omega) d\omega$ leads to

$$\widehat{e}_t = -\frac{\frac{\alpha(\sigma-1)(1+\Phi/y)}{\sigma}}{1 - \frac{\alpha(\sigma-1)(1+\Phi/y)}{\sigma}}\widehat{\xi}_t + \left(1 + \frac{\frac{\alpha(\sigma-1)\Phi/y}{\sigma}}{1 - \frac{\alpha(\sigma-1)(1+\Phi/y)}{\sigma}}\right)\widehat{y}_t,$$

which is equivalent to

$$\widehat{e}_t = -\tfrac{\alpha(\sigma-1)(1+\Phi/y)}{\sigma-\alpha(\sigma-1)(1+\Phi/y)}\widehat{\xi}_t + \left(1 + \tfrac{\alpha(\sigma-1)\Phi/y}{\sigma-\alpha(\sigma-1)(1+\Phi/y)}\right)\widehat{y}_t.$$

6. Price inflation equation (New Keynesian Phillips Curve)

As discussed in Subsection 2.4 of the main text, the optimal choice of pricing at firm ω under Calvo-type rigidities leads to the relative price

$$\widehat{P}_t(\omega) - \widehat{P}_t = (1-\beta\eta)\, E_t^\eta \sum_{j=0}^{\infty} \beta^j \eta^j \widehat{\xi}_{t+j}(\omega) + E_t \sum_{j=1}^{\infty} \beta^j \eta^j \pi_{t+j}^p.$$

The relative real marginal cost for firm ω is defined as its log deviation with respect to the aggregate real marginal cost, $\widetilde{\xi}_{t+j}(\omega) = \widehat{\xi}_{t+j}(\omega) - \widehat{\xi}_{t+j}$. Hence, the log-linearized fluctuation of the firm-level real marginal cost, shown in equation (24) of the main text, implies the following relative real marginal cost

$$\widetilde{\xi}_{t+j}(\omega) = E_t^\eta \widetilde{W}_{t+j}(\omega) + \tfrac{(1-\alpha)}{\alpha(1+\Phi/y)} E_t^\eta \widetilde{y}_{t+j}(\omega) = E_t^\eta \widetilde{W}_{t+j}(\omega) - \tfrac{(1-\alpha)\sigma}{\alpha(1+\Phi/y)} E_t^\eta \widetilde{P}_{t+j}(\omega),$$

where $\widetilde{W}_{t+j}(\omega) = \widehat{W}_{t+j}(\omega) - \widehat{W}_{t+j}$ and $\widetilde{P}_{t+j}(\omega) = \widehat{P}_{t+j}(\omega) - \widehat{P}_{t+j}$ are, respectively, relative wages and prices for firm ω, and the loglinearized demand equation (16) from the main text was also used to introduce relative prices from relative output, $\widehat{y}_{t+j}(\omega) - \widehat{y}_{t+j} = -\sigma\widetilde{P}_{t+j}(\omega)$. Meanwhile, the relative wage conditional to the lack of future wage resetting is

$$E_t^\eta \widetilde{W}_{t+j}(\omega) = \widehat{W}_t(\omega) - \widehat{W}_{t+j} = \widehat{W}_t(\omega) - \widehat{W}_t + \widehat{W}_t - E_t\widehat{W}_{t+j},$$

where using the definition of wage inflation in period $t+j$, $\pi_{t+j}^w = \widehat{W}_{t+j} - \widehat{W}_{t+j-1}$, we can substitute $\sum_{k=1}^{j} \pi_{t+k}^w = \widehat{W}_{t+j} - \widehat{W}_t$ to obtain

$$E_t^\eta \widetilde{W}_{t+j}(\omega) = \widetilde{W}_t(\omega) - E_t \sum_{k=1}^{j} \pi_{t+k}^w,$$

Similarly, the conditional expectation of the relative price depends upon the current relative price and the expected price inflation stream as follows

$$E_t^\eta \widetilde{P}_{t+j}(\omega) = \widetilde{P}_t(\omega) - E_t \sum_{k=1}^{j} \pi_{t+k}^p$$

The last two expressions are inserted in the expression for $\widetilde{\xi}_{t+j}(\omega)$ to obtain

$$\widetilde{\xi}_{t+j}(\omega) = \widetilde{W}_t(\omega) - E_t \sum_{k=1}^{j} \pi_{t+k}^w - \tfrac{(1-\alpha)\sigma}{\alpha(1+\Phi/y)} \left(\widetilde{P}_t(\omega) - E_t \sum_{k=1}^{j} \pi_{t+k}^p\right),$$

and the relative price, $\widehat{P}_t(\omega) - \widehat{P}_t = \widetilde{P}_t(\omega)$, from the above expression becomes

$$\widetilde{P}_t(\omega) = (1-\beta\eta)\, E_t^{\eta} \sum_{j=0}^{\infty} \beta^j \eta^j \left(\widehat{\xi}_{t+j} + \widetilde{W}_t(\omega) - E_t \sum_{k=1}^{j} \pi_{t+k}^{w} - \tfrac{(1-\alpha)\sigma}{\alpha(1+\Phi/y)} \left(\widehat{P}_t(\omega) - E_t \sum_{k=1}^{j} \pi_{t+k}^{p} \right) \right) + E_t \sum_{j=1}^{\infty} \beta^j \eta^j \pi_{t+j}^{p},$$

which simplifies in the following way

$$\left(1 + \tfrac{(1-\alpha)\sigma}{\alpha(1+\Phi/y)}\right) \widetilde{P}_t(\omega) = \widetilde{W}_t(\omega) - E_t \sum_{j=1}^{\infty} \beta^j \eta^j \pi_{t+j}^{w} + \left(1 + \tfrac{(1-\alpha)\sigma}{\alpha(1+\Phi/y)}\right) E_t \sum_{j=1}^{\infty} \beta^j \eta^j \pi_{t+j}^{p} + (1-\beta\eta) \sum_{j=0}^{\infty} \beta^j \eta^j E_t \widehat{\xi}_{t+j}.$$

As a standard result, Calvo-type sticky prices/wages implies a proportional relationship between relative prices/wages in logs and the rate of price/wage inflation

$$\begin{aligned} \widetilde{P}_t(\omega) &= \tfrac{\eta}{1-\eta} \pi_t^{p}, \\ \widetilde{W}_t(\omega) &= \tfrac{\eta}{1-\eta} \pi_t^{w}. \end{aligned}$$

Using both relationships into the previous expression yields

$$\left(1 + \tfrac{(1-\alpha)\sigma}{\alpha(1+\Phi/y)}\right) \tfrac{\eta}{1-\eta} \pi_t^{p} = \tfrac{\eta}{1-\eta} \pi_t^{w} - E_t \sum_{j=1}^{\infty} \beta^j \eta^j \pi_{t+j}^{w} + \left(1 + \tfrac{(1-\alpha)\sigma}{\alpha(1+\Phi/y)}\right) E_t \sum_{j=1}^{\infty} \beta^j \eta^j \pi_{t+j}^{p} + (1-\beta\eta) \sum_{j=0}^{\infty} \beta^j \eta^j E_t \widehat{\xi}_{t+j}$$

or, alternatively,

$$\pi_t^{p} = \left(1 + \tfrac{(1-\alpha)\sigma}{\alpha(1+\Phi/y)}\right)^{-1} \left(\pi_t^{w} - \tfrac{(1-\eta)}{\eta} E_t \sum_{j=1}^{\infty} \beta^j \eta^j \pi_{t+j}^{w} \right) + \tfrac{(1-\eta)}{\eta} E_t \sum_{j=1}^{\infty} \beta^j \eta^j \pi_{t+j}^{p} + \tfrac{(1-\eta)(1-\beta\eta)}{\eta} \left(1 + \tfrac{(1-\alpha)\sigma}{\alpha(1+\Phi/y)}\right)^{-1} \sum_{j=0}^{\infty} \beta^j \eta^j E_t \widehat{\xi}_{t+j}.$$

Taking the difference $(\pi_t^{p} - \beta\eta E_t \pi_{t+1}^{p})$ in the last expression, the following New Keynesian Phillips curve can be obtained

$$\pi_t^{p} = \beta E_t \pi_{t+1}^{p} + \frac{1}{1+\frac{(1-\alpha)\sigma}{\alpha(1+\Phi/y)}} \left(\pi_t^{w} - \beta E_t \pi_{t+1}^{w}\right) + \frac{(1-\eta)(1-\beta\eta)}{\eta\left(1+\frac{(1-\alpha)\sigma}{\alpha(1+\Phi/y)}\right)} \widehat{\xi}_t.$$

8. Log-linearized overall resources constraint

Household budget constraint

$$w_t l_t (1-u_t) + w_t^{m} m_t + x_t e_t + CSY_t^{v} x_t v_t + r_t^{d} d_t = c_t + (x_{t+1} - x_t) v_t + d_{t+1} - d_t. \qquad \text{(H)}$$

Competitive bank zero-profit condition

$$\left(r_t^{b} - r_t^{d}\right) b_t = w_t^{m} m_t + CSY_t^{v} v_t \qquad \text{(B)}$$

Deposit-to-loan equality condition

$$b_t = d_t \tag{D}$$

Portfolio investment equilibrium condition

$$x_t = x_{t+1} = 1 \tag{I}$$

Aggregate earnings

$$\begin{aligned} e_t &= \int e_t(\omega)d\omega = \frac{1}{P_t}\int P_t(\omega)y_t(\omega)d\omega - \frac{1}{P_t}\int W_t(\omega)l_t^d(\omega)d\omega - r_t^b \int b_t(\omega)d\omega. \\ e_t &= y_t - w_t l_t^d - r_t^b b_t, \end{aligned} \tag{F}$$

with aggregate output, $y_t = \frac{1}{P_t}\int P_t(\omega)y_t(\omega)d\omega$, aggregate labor income, $w_t l_t^d = \frac{1}{P_t}\int W_t(\omega)l_t^d(\omega)d\omega$, and aggregate real loans (borrowing), $b_t = \int b_t(\omega)d\omega$.

Inserting (D) in (B), and the result and both (I) and (F) in (H) yields

$$w_t l_t (1 - u_t) + w_t^m m_t + y_t - w_t l_t^d - r_t^b b_t + CSY_t^v v_t + r_t^b b_t - w_t^m m_t - CSY_t^v v_t = c_t + d_{t+1} - d_t,$$

which simplifies to the overall resources constraint

$$y_t = c_t + d_{t+1} - d_t, \tag{ORC}$$

recalling the definition of unemployment to do $l_t (1 - u_t) - l_t^d = 0$. Since the change in deposits is zero in steady state, the loglinearized overall resources constraint (ORC) is

$$\widehat{y}_t = \widehat{c}_t.$$

9. Set of steady-state relationships

$$
\begin{aligned}
r^b &= r^d + \frac{wm}{b}\frac{av^\theta + (1-a)m^\theta}{(1-a)m^\theta} \\
CSY^v &= w\frac{a}{(1-a)}\left(\frac{m}{v}\right)^{1-\theta} \\
v &= \frac{\beta}{1-\beta\left(1+CSY^v\right)}e \\
r^d &= \rho \\
e &= (1-\alpha\xi)\, y \\
\xi &= \frac{\left(1+\tau r^b\right) wl}{\alpha y} \\
\xi &= \frac{\sigma - 1}{\sigma} \\
b &= \tau wl \\
b &= \left[av^\theta + (1-a)m^\theta\right]^{\frac{1}{\theta}} \\
w &= \Psi_l l^{\gamma_l} c/(1-u) \\
w^m &= \Psi_m m^{\gamma_m} c \\
y &= l^\alpha - \Phi \\
y &= c \\
u &= 0
\end{aligned}
$$

We get fourteen non-linear equations that may provide solutions for the fourteen endogenous variables: y, c, l, v, e, w, w^m, u, b, r^b, r^d, CSY^v, ξ and m.

10. Set of log-linear dynamic equations

$$\pi_t^w = \beta E_t \pi_{t+1}^w - \chi_1 u_t - \chi_2 \widehat{\xi}_t \tag{A1}$$

$$\pi_t^p = \beta E_t \pi_{t+1}^p + \frac{1}{1+\frac{(1-\alpha)\sigma}{\alpha(1+\Phi/y)}}\left(\pi_t^w - \beta E_t \pi_{t+1}^w\right) + \frac{(1-\eta)(1-\beta\eta)}{\eta\left(1+\frac{(1-\alpha)\sigma}{\alpha(1+\Phi/y)}\right)}\widehat{\xi}_t \tag{A2}$$

$$\widehat{w}_t = \widehat{w}_{t-1} + \pi_t^w - \pi_t^p \tag{A3}$$

$$u_t = \widehat{l}_t^s - \widehat{l}_t \tag{A4}$$

$$\widehat{y}_t = (1+\Phi/y)\left(\varepsilon_t^z + \alpha\widehat{l}_t\right) \tag{A5}$$

$$\widehat{c}_t = E_t\widehat{c}_{t+1} - \left(R_t^d - E_t\pi_{t+1}^p\right) + (1-\rho_c)\,\varepsilon_t^c \tag{A6}$$

$$\widehat{v}_t = \beta\left(1 + CSY^v_{ss}\right) E_t\widehat{v}_{t+1} + E_t CSY^v_{t+1} + \beta\left(r^d_{ss} + CSY^v_{ss}\right) E_t\widehat{e}_{t+1} - E_t r^d_{t+1} \tag{A7}$$

$$\widehat{e}_t = -\tfrac{\alpha(\sigma-1)(1+\Phi/y)}{\sigma-\alpha(\sigma-1)(1+\Phi/y)}\widehat{mc}_t + \left(1 + \tfrac{\alpha(\sigma-1)\Phi/y}{\sigma-\alpha(\sigma-1)(1+\Phi/y)}\right)\widehat{y}_t \tag{A8}$$

$$\widehat{mc}_t = \tau r^b_t + \widehat{w}_t - (1 + \Phi/y)^{-1}\widehat{y}_t + \widehat{l}_t \tag{A9}$$

$$\widehat{y}_t = \widehat{c}_t \tag{A10}$$

$$R^d_t = \mu_R R^d_{t-1} + (1 - \mu_R)\left[\mu_\pi \pi^p_t + \mu_y \widehat{y}_t + \mu_{EFP} EFP_t\right] \tag{A11}$$

$$r^b_t = r^d_t + (r^b_{ss} - r^d_{ss})(\widehat{w}^m_t - \Omega(\theta - 1)(\widehat{m}_t - \widehat{v}_t) - \varepsilon^b_t) \tag{A12}$$

$$\widehat{b}_t = \widehat{w}_t + \widehat{l}_t \tag{A13}$$

$$\widehat{b}_t = \varepsilon^b_t + \Omega\widehat{v}_t + (1 - \Omega)\widehat{m}_t \tag{A14}$$

$$\widehat{l}^s_t = \frac{1}{\gamma_l}(\widehat{w}_t - \widehat{c}_t - u_t) \tag{A15}$$

$$\widehat{m}_t = \frac{1}{\gamma_m}(\widehat{w}^m_t - \widehat{c}_t) \tag{A16}$$

$$CSY^v_t = CSY^v_{ss}\left(\frac{1}{(r^b_{ss} - r^d_{ss})}\left(r^b_t - r^d_t\right) + (1 - \theta)(1 - \Omega)(\widehat{m}_t - \widehat{v}_t) + \varepsilon^b_t\right) \tag{A17}$$

$$r^b_t = R^b_t - E_t\pi^p_{t+1} \tag{A18}$$

$$EFP_t = r^b_t - r^d_t \tag{A19}$$

$$r^d_t = R^d_t - E_t\pi^p_{t+1} \tag{A20}$$

Endogenous variables (20): π^w_t, π^p_t, r^b_t, r^d_t, R^d_t, R^b_t, CSY^v_t, $\widehat{y}_t$, $\widehat{l}_t$, $\widehat{l}^s_t$, u_t, $\widehat{c}_t$, $\widehat{w}_t$, $\widehat{w}^m_t$, $\widehat{mc}_t$, $\widehat{e}_t$, $\widehat{v}_t$, $\widehat{b}_t$, $\widehat{m}_t$, and EFP_t.

Exogenous variables (3): AR(1) processes determine the evolution of the technology shock, $\varepsilon^z_t = \rho_z\varepsilon^z_{t-1} + \kappa^z_t$, the financial shock, $\varepsilon^b_t = \rho_b\varepsilon^b_{t-1} + \kappa^b_t$, and the private spending (consumption) shock, $\varepsilon^c_t = \rho_c\varepsilon^c_{t-1} + \kappa^c_t$.

11. The non-monetary decentralized economy without nominal rigidities

Taking the baseline model with financial frictions, dropping the central bank interest-rate setting equation, and assuming both flexible prices and flexible wages bring the set of equations:

$$\widehat{l}_t = \frac{1}{\gamma_l}(\widehat{w}_t - \widehat{c}_t - u_t) \tag{A1'}$$

$$\widehat{w}_t = (1 + \Phi/y)^{-1}\widehat{y}_t - \widehat{l}_t \tag{A2'}$$

$$\widehat{y}_t = \widehat{c}_t \tag{A3'}$$

$$\widehat{y}_t = (1 + \Phi/y) \left(\varepsilon_t^z + \alpha \widehat{l}_t \right) \tag{A4'}$$

$$\widehat{c}_t = E_t \widehat{c}_{t+1} - r_t^d + (1 - \rho_c) \varepsilon_t^c \tag{A5'}$$

$$\widehat{v}_t = \beta (1 + CSY_{ss}^v) E_t \widehat{v}_{t+1} + E_t CSY_{t+1}^v + \beta \left(r_{ss}^d + CSY_{ss}^v \right) E_t \widehat{e}_{t+1} - E_t r_{t+1}^d \tag{A6'}$$

$$\widehat{e}_t = \left(1 + \frac{\alpha(\sigma-1)\Phi/y}{\sigma - \alpha(\sigma-1)(1+\Phi/y)} \right) \widehat{y}_t \tag{A7'}$$

$$EFP_t = (r_{ss}^b - r_{ss}^d)(\widehat{w}_t^m - \Omega (\theta - 1) (\widehat{m}_t - \widehat{v}_t) - \varepsilon_t^b) \tag{A8'}$$

$$\widehat{b}_t = \widehat{w}_t + \widehat{l}_t \tag{A9'}$$

$$\widehat{b}_t = \varepsilon_t^b + \Omega \widehat{v}_t + (1 - \Omega) \widehat{m}_t. \tag{A10'}$$

$$\widehat{m}_t = \frac{1}{\gamma_m} (\widehat{w}_t^m - \widehat{c}_t) \tag{A11'}$$

$$CSY_t^v = CSY_{ss}^v \left(\frac{1}{(r_{ss}^b - r_{ss}^d)} EFP_t + (1 - \theta)(1 - \Omega)(\widehat{m}_t - \widehat{v}_t) + \varepsilon_t^b \right), \tag{A12'}$$

$$EFP_t = r_t^b - r_t^d \tag{A13'}$$

There are 13 endogenous variables: r_t^b, r_t^d, CSY_t^v, $\widehat{y}_t$, $\widehat{l}_t$, $\widehat{c}_t$, $\widehat{w}_t$, $\widehat{w}_t^m$, $\widehat{e}_t$, $\widehat{v}_t$, $\widehat{b}_t$, $\widehat{m}_t$, and EFP_t. The model can be solved for real variables, but nominal variables (inflation, nominal interest rates) are undetermined.