Growth and Public Support to Research and Imitation: The International Connection

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August, 1999

Abstract

This paper studies technology policy within the context of an R&D-based growth model. We first use cross-country data to document several empirical regularities regarding the relationship between public support to research and imitating, and relative income levels. We then construct a model in which the possibility of copying foreign ideas through a technology that exhibits diminishing imitation opportunities is sufficient to account for the observed average patterns. We also find that policy intervention can produce important welfare benefits; capital accumulation subsidies show a slightly larger contribution to the welfare improvement than R&D support.

Key words: research; imitation; policy; growth; transitional dynamics

JEL Classification Codes: O38; O11

*I wish to thank Emilio Domínguez, Amy J. Glass, Jane E. Ihrig, Aubhik Khan, Thomas Osang, Chris Pappageorgiou, B. Ravikumar, and Alex Wolman for comments. I am also grateful to Robert E. Evenson for kindly supplying the data.

1 Introduction

Governments actively support the accumulation of knowledge. Public research programs, like university research grants and the establishment of industrial research laboratories, try to promote invention and innovation. Public subsidization of technology imitation, on the other hand, facilitates technology diffusion; policy actions now include public education, technical advisory services for potential adopters, and training programs for scientists and engineers. Rustichini and Schmitz (1991) provide numerous examples which illustrate the importance of research and imitation policies in economic growth.

The literature has also recognized the existence of wide technology policy disparities across countries. The extend and determinants of these differences are not however well understood. As Stoneman and Diederen's (1994) study suggests, this is especially true in the case of imitation policy. The need for a deeper analysis of the determinants of imitation policy, and its interaction with public research programs, becomes even more important when we consider that the diffusion of foreign techniques is a main source of productivity growth in both advanced and developing countries (Eaton and Kortum 1996, and Coe et al. 1997). In this paper, we first use cross-country data on research and imitation subsidies to uncover how they vary along the economic development process. We then construct a model, and show that allowing for the possibility of acquiring foreign technology through a technology that exhibits diminishing imitation opportunities is sufficient to reproduce the observed patterns. Our results suggest that foreign ideas are the key driving force behind imitation policy.

We find the following empirical regularities. (i) The ratio of imitation to research support decreases with the level of economic development; Rustichini and Schmitz (1991) also find this puzzling stylized fact, leaving its explanation to future research. (ii) The total technology policy expenditure and research subsidy shares increase along the convergence path. And (iii) the share of government investment in imitation displays a hump shape.

To reproduce the above evidence, we build a non-scale R&D-based growth model.[1] In our framework, technical progress occurs either by imitating foreign ideas or inventing new ones.[2] The two activities are complementary. Both imitation and research are costly, but

[1] See Eicher and Turnovsky (1999) for a detailed discussion on non-scale models of growth.

[2] The imitation of foreign techniques and products by domestic producers is not the only form of technology transfer. There are other channels such as FDI, international licensing agreements, joint ventures, and turnkey projects. What really matters, however, is that the transmission and assimilation of new ideas is not automatic, and is costly, especially in the case of foreign techniques (Pack and Saggi 1997). We focus on imitation because

the former is cheaper (Mansfield et al. 1981). As the technological gap decreases, imitation becomes more expensive, generating a negative externality from current R&D effort to future imitation productivity. We look at economic development as the transition path toward a new steady-state with higher levels of income. We study dynamics using high-degree polynomial approximations; in this, we follow the method proposed by Judd (1992).

What permits the model to replicate the evidence on public support to research and imitation is the interaction between two opposing forces induced by the imitation technology. As a country catches up with the more advanced nations, both the imitation productivity and the negative externality decline. The former effect reduces the incentive to support imitation, whereas the latter encourages both research and copying subsidies.

We also ask the following question: How does technology policy affect economic development? We find that the complete absence of policy intervention can produce important differences with respect to economies that follow the social optimum. At the steady-state, the relative level of output per capita can fall below 60 percent, which implies a loss in consumption of more than 7 percent. The model predicts that the contribution of physical capital accumulation subsidies is slightly larger than the one of technology policy. We obtain this result using a relatively small, empirically-supported mark-up.

Relevant papers for our work are Rustichini and Schmitz (1991), Segerstrom (1991), Jovanovic and MacDonald (1994), Houser (1997), and Davidson and Segerstrom (1998). These papers endogenously determine the optimal allocations both to imitation and research. They are, however, closed-economy models in which imitation targets domestically generated ideas.

Grossman and Helpman (1991), Glass and Saggi (1998), Currie et al. (1999), Van Elkan (1996), and Perez-Sebastian (forthcoming) are general equilibrium open-economy models of growth in which both imitation and innovation are costly activities. As our work, the first three of these studies explore how R&D subsidies affect economic growth, and the last three papers obtain an equilibrium in which imitation and innovation can coexist in the same country. No one, however, analyze the determinants of technology policy, and how it changes as initially poor nations develop and progressively become more integrated in the rest of the world.

We proceed as follows. Section 2 describes the data on government support of research and imitation. Section 3 presents the model. The social planner's problem is stated in section 4. Optimal tax rates are derived in section 5. Section 6 studies transitional dynamics, and in

it captures well this notion, and makes the model tractable.

particular, how the tax rates vary along the development path, and how policy affects welfare. Section 6 concludes.

2 Government Support of Research and Imitation

In this section, we use international data to study the relationship between technology policy and a country's relative income level. The agricultural sector is often the best source of data for issues of technological change. Our case is no different. As far as we know, the only source of data about imitation and research support for a large cross-section of countries is the one constructed by Judd, Boyce and Evenson (1986) for the agricultural sector. It contains data regarding public investment in research and extension (i.e., imitation) as a percentage of the value of the agricultural product. The Penn World Tables, version 5.6, provides annual flows of GDP per worker.

We have data for 78 countries for the years 1962, 1965, 1968, 1971, 1974, 1977 and 1980.[3] Initially, we want to examine what the data say about the functional form relating research and imitation subsidies and a country's level of income. To do this, we compute each variable's average value for every country (see Appendix B), and run the following OLS regression:

$$\ln\left(S_{R,j}\right) = \sum_{i=0}^{N} a_i \left[\ln(y_j)\right]^i + \varepsilon_j; \tag{1}$$

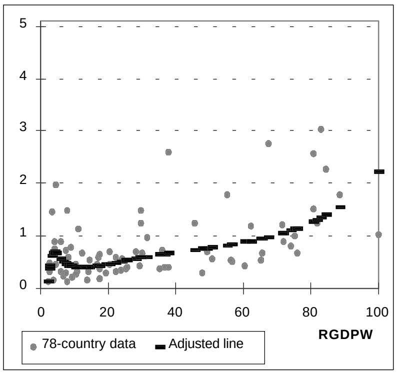
where $S_{R,j}$ is the government support share in output in country j; y_j is relative GDP per worker (RGDPW) as a percentage of the U.S. level; ε_j is the disturbance; and the a_i's are the regression coefficients. We choose the degree of the polynomial N that maximizes the *adjusted-*R^2.[4] In addition, to make sure that added terms to the polynomial provide valuable information, we require that all coefficients must be significant at the 10 percent level. We also analyze the relationship for the Mankiw, Romer and Weil (1992) 22-OECD group. Given the low number of observations in this second sample, we limit to *two* the maximum polynomial degree to avoid overfitting.

Figure 1 presents the results. Next to each chart, p-v is the maximum p-value of the individual significance of the polynomial coefficients; and F is the F-statistic related to the

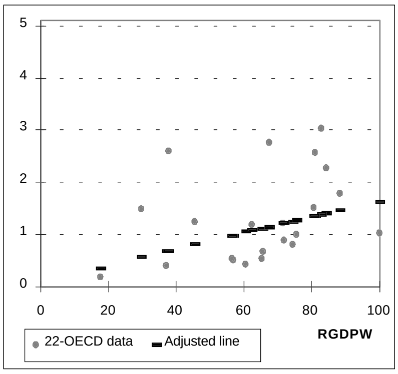
[3] We excluded planned economies (Eastern Europe, Soviet Union, and China) because their government support figures do not reflect market failure. We also get rid of Zambia because its extension and research support shares changed very dramatically for very small variations of its GDP per capita level.

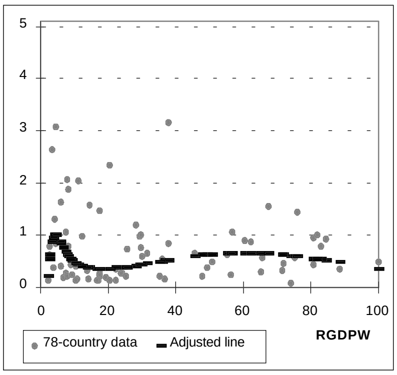
[4] Den Haan and Marcet (1990) establish that these exponenciated polynomials can approximate any function $S_R = f(y)$ by letting N go to infinity. We also run OLS regressions using standard polynomials, but the $\bar{R}^2$ was always lower. The parameter N's upper bound was 10.

Figure 1: Public Sector Research and Extension Expenditures as a Percentage of the Value of Agricultural Product

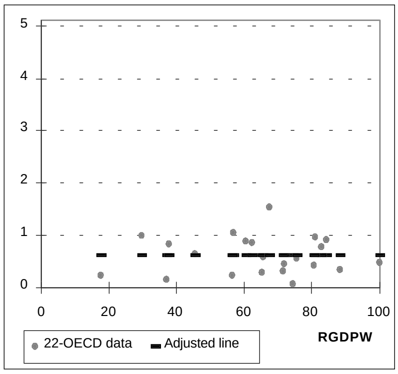


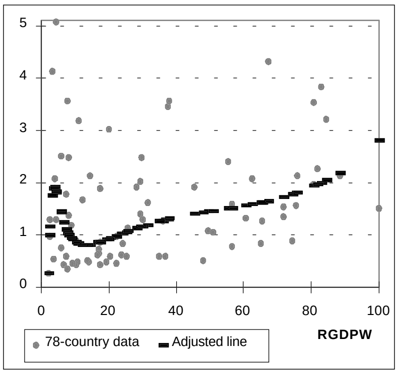
Research Data	
78 Nations	22 OECD
N = 5	N = 1
p-v $\leq$ 0.06	p-v $\leq$ 0.02
$\bar{R}^2$ = 0.32	$\bar{R}^2$ = 0.21
F = 8.3	F = 8.4



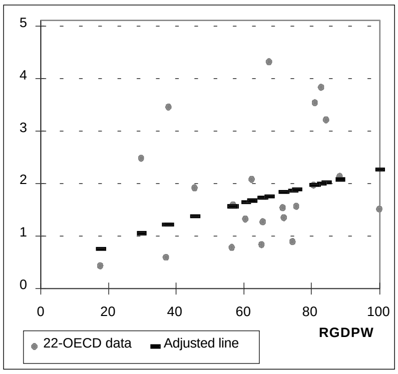


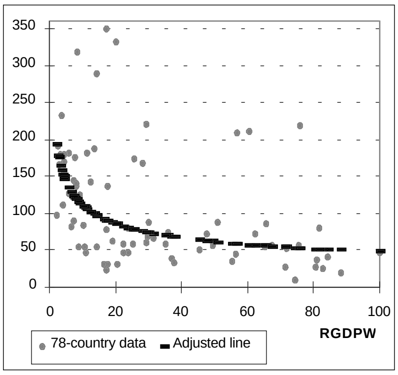
Extension Data	
78 Nations	22 OECD
N = 4	N = 0
p-v $\leq$ 0.01	
$\bar{R}^2$ = 0.08	
F = 2.63	



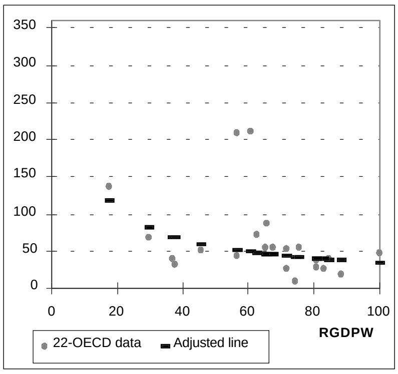


Research plus Extension Data	
78 Nations	22 OECD
N = 5	N = 1
p-v $\leq$ 0.06	p-v $\leq$ 0.05
$\bar{R}^2$ = 0.19	$\bar{R}^2$ = 0.14
F = 4.69	F = 4.30





Extension over Research Data	
78 Nations	22 OECD
N = 1	N = 1
p-v $\leq$ 0.00	p-v $\leq$ 0.06
$\bar{R}^2$ = 0.22	$\bar{R}^2$ = 0.12
F = 21.5	F = 3.82



test of their joint significance. The two charts at the top say that, except for the very less developed countries, there exists a positive relationship between relative income levels and the share of government support to research. For low levels of development, we observe a spike. It can be the result of overfitting the data. But interestingly, as will be seen later, our model's predictions can offer an explanation for this type of pattern.

The data on extension/imitation support do not provide a completely clear picture. The adjusted line shows the same initial investment spike form as the research support case. When we look at the whole 78-country sample, a hump shape behavior arises. Imitation subsidies increase with the level of development, but only up to some point after which they decline. The 22-OECD group chart, on the other hand, says that for the most advanced countries a constant subsidization rate is what best fits the data.

The pattern followed by imitation policy is shown to be quite different from the one displayed by research policy. When we look at the sum of both forms of government support to technical progress, the charts exhibit the same relationship with income levels as research support, which we discussed above. Finally, looking at the bottom of Figure 1, we can see a clear tendency of the ratio of imitation to research support to decrease with the level of economic development in both samples.

For country-groups that differ in their levels of income, and for our two samples, Table 1 presents descriptive statistics: the mean, the minimum and maximum, and the standard deviation. If we use the ratio of the mean value to the standard deviation to assess dispersion, both research and extension statistics say that technology policy varies less among more developed nations. Looking at the *total support* numbers, for example, we see that the most advanced nations have a much higher mean than the 14^{th} percentile country class, but both groups have similar standard deviations. Looking at the figures for the ratio of extension to research support, following the same logic suggests that less advanced countries, on the other hand, exhibit less variation on the weights of imitation and research in technology policy, being imitation the one that dominates.

In addition, Table 1 summarizes the average patterns that we found before. Countries in the 14^{th} percentile are the ones that present the "anomalous" behavior, showing averages numbers that always exceed the low-middle income group (i.e., nations with RGDPW $\in [14.1\%, 28\%]$). For the rest of country groups, the evidence on technology policy shows the following regularities. First, the share of public investment in imitation seems to display a hump shape; starting

Table 1: Summary Statistics: Means, Standard Deviations, Minima and Maxima

RGDPW Range	Research(%)		Extension(%)		TotalSupport(%)		Ext.overRes.(%)	
	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.
	[Min, Max]		[Min, Max]		[Min, Max]		[Min, Max]	
Below 14%	0.62	0.47	0.93	0.83	1.54	1.27	145	60
	[0.13, 1.97]		[0.13, 3.09]		[0.26, 5.06]		[48, 318]	
[14.1%-28%]	0.45	0.14	0.52	0.64	0.97	0.72	111	109
	[0.18, 0.70]		[0.14, 2.33]		[0.42, 3.03]		[24, 350]	
[28.1%-70%]	0.93	0.68	0.78	0.62	1.71	1.00	118	158
	[0.29, 2.77]		[0.16, 3.16]		[0.50, 4.31]		[32, 796]*	
More than 70.1%	1.51	0.76	0.65	0.38	2.16	0.92	54	55
.	[0.67, 3.04]		[0.07, 1.46]		[0.89, 3.83]		[9, 219]	
78-country Sample	0.81	0.64	0.76	0.68	1.56	1.08	115	110
	[0.13, 3.04]		[0.07, 3.16]		[0.26, 5.06]		[9, 796]*	
22-OECD Sample	1.31	0.86	0.63	0.36	1.94	1.10	64	54
.	[0.18, 3.04]		[0.07, 1.55]		[0.42, 4.31]		[9, 211]	

* In Figure 1, the observation that provides the maximum was elimated to raise the size of the shown data.

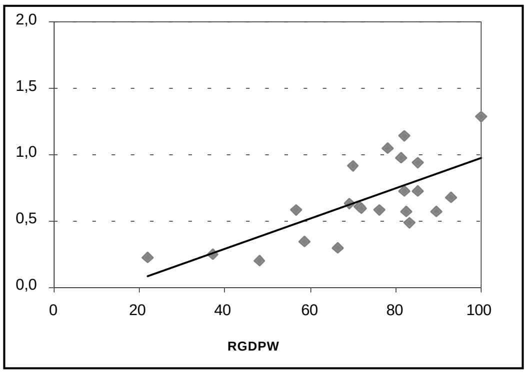
at an average of 0.52 percent for low-middle income countries, raises up to 0.78 for high-middle income economies, and then declines to 0.65 percent for the most advanced group. Second, both public research and total public support increase with income levels. Specifically, the former's average triples along the development path, and the latter more than doubles. Third, the ratio of imitation to research support is inversely related to the level of economic development. We see that this ratio is, on average, almost three times bigger for the less advanced countries than for the most industrialized group.

Although, to the best of our knowledge, good data on imitation and research support for a large cross-section of countries do not exist outside agriculture, Figure 2 shows that the positive relationship between public research effort and income levels also holds at least for the 22-OECD group. The straight line is an OLS regression line which turns out to be very significant, with a p-value for the slope coefficient of 2.4E-4.

3 The Model

We introduce imitation in a version of Jones' (1995) R&D-based growth model. We consider an economy in which the labor force increases at rate n. There are two types of firms: consumption-goods producers, and intermediate-goods manufacturers. The latter invest re-

Figure 2: Gross Domestic R&D Expenditure (GERD) Financed by Government as Percentage of GDP in OECD Countries, 1981-92 Average



Source: Main Science and Technology Indicators, OECD, 1999

sources in R&D to learn new designs for new types of producer durables. When a new design is learned, the firm that absorbs this knowledge acquires a perpetual patent. This allows the firm to manufacture the new variety, and practice monopoly pricing.[5] In this economy, there also exists a government that collects lump-sum taxes, and uses the revenues to tax/subsidize the purchase of intermediate goods by final-goods producers and the R&D activity.

The final-goods sector is made up of a large number of identical firms. At any time period t, they produce an homogeneous output Y_t using labor L_t and a variety of intermediate capital goods x_{it} according to the following CES technology:

$$Y_t = L_t^{\alpha} \left[\int_0^{A_t} x_{it}^{(1-\alpha)\gamma} \, di \right]^{\frac{1}{\gamma}} \quad , \quad 0 < \alpha < 1 \;\; , \;\; \gamma > 0 \; . \tag{2}$$

Intermediate-goods are complementary when $\gamma < 1$; they are substitutes if $\gamma > 1$.

Intermediate-goods manufacturers borrow capital that is allocated to producing existing varieties of producer durables and to R&D. At any point in time t, the R&D activity increases the mass of producer durable types A_t available for final output production according to the following aggregate R&D technology:

$$A_{t+1} - A_t = \mu \; A_t^{\phi} \; \left\{ R_{It}^{\lambda} + \left[R_{Ct} \left(\frac{\eta \, A_t^w}{A_t} \right)^{\beta} \right]^{\lambda} \right\} \; ; \; \lambda \in (0,1) \, ; \; \eta > 0 \; ; \; \phi > 0; \; A_t \leq A_t^w . \tag{3}$$

That is, firms learn new designs either by investing in innovation R_{It} or in imitation R_{Ct}. Copying targets an international knowledge pool of size A_t^w that grows exogenously at rate

[5] Producers are assumed to be unable to circumvent the local monopoly by importing intermediate goods from the rest of the world.

g_{A^w}.[6] This "lab-equipment" form implies that the R&D input is produced with the same technology as consumption goods; capital and labor are just shifted from production of final goods into production of new designs. In equation (3), ϕ captures "intertemporal knowledge spillovers" in learning. The parameter λ controls for the fact that two or more researchers can come up with the same idea either by chance or because of R&D races. Since $0 < \lambda < 1$, a "congestion externality" or, in other words, duplication of effort is present.

In equation (3), the term in brackets captures the effect of imitation in the economy's technical progress. Local firms invest resources in order to absorb the information needed to replicate new products invented abroad. It differs from innovation in that the number of goods that can be copied at any point in time is limited to the finite number that have been discovered elsewhere.[7] The specification incorporates an advantage of "backwardness"; we are assuming that the cost of imitating foreign designs decreases as the worldwide stock gets relatively larger.[8] The R&D technology also allows imitation to become more expensive than innovation. A value of η less than one would imply that some foreign ideas' technical specifications are very complex, and the cost of absorbing them is relatively high compare to the cost of inventing a new one. Since A_t is in the denominator, there exist "diminishing imitation opportunities," which imply a negative externality from current R&D investment to future imitation productivity; higher levels of R&D effort today may decrease the relative size of the international pool of ideas, thus making copying more costly in the future.

For simplicity of exposition, we turn now to examining the social planner's problem. Further information about the decentralized setup will be provided when we study optimal policy. For a detailed description of the decentralized economy, see Appendix A.

[6] The exogeneity of the law of motion of the international pool of ideas implies that we abstract from the fact that domestic innovation might yield new goods that will add up to the international pool of designs. We do it for simplicity. Notice, however, that this effect will be small for low levels of economic development because at this stage innovation investment is relatively low.

[7] A sufficient condition for this to be the case is that $A_t^w > A_{t+1}$ at any t. We then need to assume that η is sufficiently small, but this requirement has no effect on the interior solution optimal allocations. Since $A_t^w \geq A_t$ at each date, $A_t^w(1+g_{A^w}) \geq A_{t+1}$ must also hold. Additionally, at any point in time, the interior solution optimal allocations are a function of η *times* A_t^w. For any optimal sequence $\{A_z\}_{z=t}^{\infty}$, we can then always find a sufficiently large A_t^w and a sufficiently small η for which the above condition holds. For example, redefine A_t^w as $A_t^{w\prime} = A_t^w(1+g_{A^w})(1+\varepsilon)$, and η as $\eta' = \frac{\eta}{(1+g_{A^w})(1+\varepsilon)}$, with $\varepsilon > 0$; they generate the same optimal allocations because $A_t^{w\prime}\eta' = A_t^w\eta$.

[8] See Nelson and Phelps (1966), for example, for a detailed explanation.

4 The Planner's Problem and the Steady State

Policymakers have the power to tax/subsidize firms and consumers. Their goal is to eliminate market imperfections so as to make the model's variables follow their socially optimal time paths. Let K_t and C_t denote the country's physical capital stock and aggregate consumption at date t, respectively. The former equals $\int_0^{A_t} x_{it}\, di$ ($A\bar{x}_t$ in the symmetric equilibrium, with $\bar{x}_t = x_{it}\ \forall i$), and depreciates at rate δ. The central planner chooses the sequence of allocations $\{R_{Ct}, R_{It}, K_t, C_t\}_{t=0}^{\infty}$ to maximize the lifetime utility of the representative consumer subject to the feasibility constraints of the economy

$$\max_{\{C_t, R_{It}, R_{Ct}, K_t\}} \sum_{t=0}^{\infty} \rho^t \left[\frac{\left(\frac{C_t}{L_t}\right)^{1-\sigma} - 1}{1-\sigma} \right]; \tag{4}$$

subject to

$$Y_t = A_t^{\xi} L_t^{\alpha} K_t^{1-\alpha}; \tag{5}$$

$$K_{t+1} - (1-\delta) K_t = Y_t - C_t - R_{It} - R_{Ct}; \tag{6}$$

$$A_{t+1} - A_t = \mu\, A_t^{\phi} \left\{ R_{It}^{\lambda} + \left[R_{Ct} \left(\frac{\eta\, A_t^w}{A_t} \right)^{\beta} \right]^{\lambda} \right\}; \tag{7}$$

$$\frac{A_{t+1}^w}{A_t^w} = 1 + g_{A^w}\,; \tag{8}$$

$$\frac{L_{t+1}}{L_t} = 1 + n\,; \tag{9}$$

$$K_0,\ L_0,\ A_0,\ A_{w0}\ \text{ given;}$$

where ρ is the discount factor. Equation (5) is the well-known Cobb-Douglas form in which production function (2) takes at the aggregate level; where $\xi = \frac{1}{\gamma} - (1-\alpha)$. Y_t can be interpreted as the Gross Domestic Product of the economy at date t. Expression (6) represents the economy's budget constraint as well as the law of motion of the capital stock.

We now restrict our attention to the perfect-foresight equilibrium balanced growth path in which the growth rates of all variables in the model are constant. Let x^* and G_x denote the optimal allocation and the grows growth rate of variable x at steady-state, respectively; and define g_x as $G_x - 1$. Equality (6) implies that $g_Y = g_C = g_K = g_{R_I} = g_{R_C}$. From R&D technology (7), we see immediately that because $g_{R_I} = g_{R_C}$, it must be true that $g_{A^w} = g_A$, otherwise g_A cannot be constant. The value of the ratio $\frac{A^w_t}{A_t}$ then remains invariant along the balanced growth path. Hence, equation (7) implies that

$$G_A^{1-\phi} = G_R^{\lambda}\,. \tag{10}$$

Expressions (5), (9) and (10), in turn, imply that the growth rate of output is given by

$$G_Y = (1+n)^{\frac{1-\phi}{1-\phi-\lambda\frac{\xi}{\alpha}}}\,.[9] \tag{11}$$

The model does not therefore display scale effects on the steady-state growth rates; they are exogenous, exclusively pinned down by the production and R&D technologies. In the log-run, policy only has level effects. Growth effects are however possible along the transition.

5 Optimal Tax Policy

In order to know the government's optimal policies, we need to compare the command optimum allocations to the ones of the decentralized economy. The central planner chooses tax rates so as to equalize private and social returns. In our framework, there are four sources of market failure. First, monopoly pricing of producer durables implies that the amount of intermediate goods rented by final-goods producers is too low from the social planner's viewpoint, thus generating an insufficient stock of capital in the economy. The other three market failure sources affect the R&D allocation. Congestion externalities and diminishing imitation opportunities make the private R&D investment be too high, whereas intertemporal knowledge spillovers have the opposite effect. The social planner will then tax/subsidize the purchase of intermediate-goods, and the R&D activity.

For simplicity, we assume that the government raises revenue through (non-distortionary) lump-sum taxes paid by consumers, and is constrained to maintaining a balanced budget at each date; that is,

$$\tau_{ht} = \tau_{Ct}\,R_{Ct} + \tau_{It}\,R_{It} + \int_0^{A_t} \tau_{xit}\,x_{it}\,di, \text{ for all } t; \tag{12}$$

where τ_{ht} is the lump-sum tax; τ_{xit} is the rate at which the purchase of product i is subsidized at t; and τ_{It} and τ_{Ct} are the rates at which the government subsidizes investment in research and imitation, respectively.

The concavity of the production and R&D technologies guarantees that output will be distributed evenly over all activities. From the planner's problem, the first order condition with respect to physical capital investment gives the intertemporal sequence of socially optimal aggregate consumptions,

$$C_{t+1} = [\rho \ (1 + r_{t+1})]^{\frac{1}{\sigma}} (1+n)^{1-\frac{1}{\sigma}} C_t; \tag{13}$$

where

$$r_t = (1-\alpha) \frac{Y_t}{K_t} - \delta. \tag{14}$$

Condition (13) is standard. It implies that, at the optimum, the gross growth rate of the utility value of consumption per capita must equal the discounted returns to saving, taking into account population growth. That is, individuals must be indifferent between consuming one additional unit of output today and saving it, consuming the proceeds tomorrow. From expression (13), we deduce that the central planner equates the marginal productivity of intermediate goods to their marginal production cost, given by the economy's interest rate (r_t) plus the depreciation rate.

The competitive equilibrium optimality condition for physical capital is different. Assume that one unit of raw capital can be costlessly converted into one unit of any type of producer durable, and that intermediate goods are rented rather than sold. Intermediate-goods producers act as monopolists, taking the final-output manufacturers' inverse demand function as given. The solution to their problem is well known: monopolists charge a mark-up over marginal cost; and in the symmetric equilibrium, assuming that the number of firms is large, the mark-up equals the elasticity of substitution between intermediate capital goods. At time t, the rental price of variety i (p_{it}) is then given by

$$p_{it} = \frac{r_t + \delta}{\gamma (1-\alpha)} = \bar{p}_t \ , \quad \text{for all } i. \tag{15}$$

In order to eliminate this inefficiency, the market price paid by final-goods producers net of subsidies ($p_{it}(1-\tau_{xti})$) must be equalized to the producer-durables marginal cost. Equation (15) implies that the optimal policy is to subsidize the purchase of intermediate goods at rate

$$\tau_{xti} = 1 - \gamma (1-\alpha) = \bar{\tau}_{xt}, \text{ for all } i \in (0, A_t). \tag{16}$$

The central planner will choose τ_{It} and τ_{Ct} so as to make the decentralized economy R&D investment equal the socially optimal amount. Both research and imitation will coexist in equilibrium because of the existence of diminishing returns to R&D effort. From the planner's problem FOCs with respect to the R&D activities, we find that the socially optimal ratio of imitation to research is

$$\frac{R_{Ct}}{R_{It}} = \left(\frac{\eta \, A_t^w}{A_t} \right)^{\frac{\beta\lambda}{1-\lambda}} . \tag{17}$$

It states that the weight of imitation in total R&D investment rises with the technological gap, other things constant.

Let T_t denote the relative size of the modified international pool of designs, $\frac{\eta \, A_t^w}{A_t}$. In terms of $R_t = R_{It} + R_{Ct}$, R&D technology (7) can be written as

$$A_{t+1} - A_t = \mu A_t^{\phi} R_t^{\lambda} \left[1 + T_t^{\frac{\beta\lambda}{1-\lambda}} \right]^{1-\lambda} . \tag{18}$$

Using equations (13), (14), (17), (18), and the FOC with respect to either research or imitation effort, we obtain the Euler equation that governs the dynamics of the socially optimal R&D investment,

$$1+r_{t+1} = \frac{\lambda \, (A_{t+1} - A_t)}{R_t} \left\{ \frac{R_{t+1}}{\lambda \, (A_{t+2} - A_{t+1})} \left[\frac{A_{t+2} - A_{t+1}}{A_{t+1}} \left(\phi - \; \beta\lambda \, \frac{T_{t+1}^{\frac{\beta\,\lambda}{1-\lambda}}}{1 + T_{t+1}^{\frac{\beta\,\lambda}{1-\lambda}}} \right) + 1 \right] + \frac{\xi \, Y_{t+1}}{A_{t+1}} \right\} . \tag{19}$$

At the optimum, the planner must be indifferent between investing one additional unit of output in intermediate-goods production and R&D. The RHS of equation (19) is the social return to R&D. One additional unit of R&D input generates $\frac{\lambda(A_{t+1}-A_t)}{R_t}$ new ideas for new types of producer durables. Each of these new designs will increase next period's output by $\frac{\xi Y_{t+1}}{A_{t+1}}$, and R&D production by $\left[\frac{\lambda(A_{t+2}-A_{t+1})}{R_{t+1}}\right]^{-1}$ *times* $\frac{dA_{t+2}}{dA_{t+1}}$; where $\left[\frac{\lambda(A_{t+2}-A_{t+1})}{R_{t+1}}\right]^{-1}$ gives the shadow price of one additional design, which must equal its marginal (social) cost.

We now determine the competitive equilibrium allocation to R&D. Free entry in the producer-durables sector implies that, at each instant in time, the amount invested in learning must equal the present value of the ideas. We will have then two zero-profit equilibrium conditions, one for research and another for imitation. They can be stated formally as follows:

$$R_{It} \, (1 - \tau_{It}) = V_t \, \mu \; A_t^{\phi} \; R_{It}^{\lambda} \; ; \tag{20}$$

and

$$R_{Ct} \, (1 - \tau_{Ct}) = V_t \; \mu \; A_t^{\phi} \left[R_{Ct} \left(\frac{\eta \, A_t^w}{A_t} \right)^{\beta} \right]^{\lambda} \; ; \tag{21}$$

where V_t is the present value of any patent right at date t – notice that all designs are alike in productivity terms, regardless of whether they are copied or created from scratch. Combining expressions (20) and (21), we obtain the optimal ratio of imitation to research investment in the competitive equilibrium with taxes:

$$\frac{R_{Ct}}{R_{It}} = \left[\left(\frac{1-\tau_{It}}{1-\tau_{Ct}} \right) \left(\frac{\eta\, A_t^w}{A_t} \right)^{\beta\lambda} \right]^{\frac{1}{1-\lambda}} . \tag{22}$$

From equations (17) and (22), we find that to achieve its goal the government must subsidize research and imitation at the same rate; that is,

$$\tau_{Ct} = \tau_{It} \ = \bar{\tau}_{Rt} \ \text{ for all } t. \tag{23}$$

The reason is that both research and imitation contribute in the same way to generate knowledge spillovers. In order words, the externalities depend on the total amount of designs A_t, regardless of how they are learned.

For $\frac{1-\tau_{Rt+1}}{1-\tau_{Rt}}$ sufficiently close to 1, Appendix A shows that the optimal R&D subsidy rate is

$$\bar{\tau}_{Rt} \simeq 1 - \frac{\frac{[1-\gamma(1-\alpha)](1-\alpha)}{(1-\bar{\tau}_{x,t+1})}}{\frac{R_{t+1}}{Y_{t+1}} \left[\phi - \lambda\,\beta \left(\frac{T_{t+1}^{\frac{\beta\,\lambda}{1-\lambda}}}{1+T_{t+1}^{\frac{\beta\,\lambda}{1-\lambda}}} \right) \right] + \lambda\,\xi}. \tag{24}$$

Unlike the social planner, firms do not take into account the existence of diminishing returns in learning due to duplication of effort. They equate marginal costs to average, instead of marginal, R&D productivity. As a consequence, $\bar{\tau}_{Rt}$ may increase with parameter λ. The rate at which R&D must be subsidized declines with the mark-up ratio charged by intermediate-goods manufacturers, $[\gamma\,(1-\alpha)]^{-1}$. This occurs because the mark-up induced by monopoly pricing is irrelevant for the central planner, but it raises the private returns from R&D investment. The terms ξ, ϕ, and $\lambda\beta \left(\frac{T_{t+1}^{\frac{\beta\,\lambda}{1-\lambda}}}{1+T_{t+1}^{\frac{\beta\,\lambda}{1-\lambda}}} \right)$ capture the effect of current R&D on future productivity, which the decentralized economy does not internalize; the third one, in particular, represents the negative externality caused by diminishing imitation opportunities. The R&D share $\frac{R_{t+1}}{Y_{t+1}}$ weights the incidence of the last two external effects because they depend on future investment.[13] Whether the social planner imposes a tax or a subsidy to R&D clearly depends on the model parameters' values.

Because it is optimal to subsidize the two R&D activities at the same rate, public support to imitation and research are given by $\bar{\tau}_{Rt}\,R_{Ct}$ and $\bar{\tau}_{Rt}\,R_{It}$, respectively. Equation (17) then

[13]From equation (19), it is easy to see that the externalities affect the optimal R&D share in the same direction as the optimal R&D tax rates. Substituting for it would not then alter our conclusions, but would make expression (24) cumbersome.

Table 2: Parameter Values Used in the Simulations

Constant Parameters' values		Changing Parameters' values				
		Case	σ	λ	Implied ϕ	Calibrated β
α	0.64	1	1	0.25	0.92	2.03
ρ	0.96	2	1	0.5	0.84	0.83
δ	0.1	3	1	0.75	0.76	0.42
n	0.014	4	2	0.25	0.92	2.3
ξ	0.126	5	2	0.5	0.84	1.03
T^*	0.41	6	2	0.75	0.76	0.58

implies that the ratio of imitation to research public support equals

$$\frac{\bar{\tau}_t \, R_{C2}}{\bar{\tau}_t \, R_{I2}} = \left(\frac{\eta \, A_t^w}{A_t} \right)^{\frac{\beta \lambda}{1-\lambda}} . \tag{25}$$

This result has an important implication. Since in our model the relative size of the international pool of designs is inversely related to the country's level of economic development, we have actually found that the ratio of imitation to research support decreases with the level of economic development. This is consistent with the empirical evidence presented in Section 2. The reason is that the smaller the number of ideas from which a country can choose, the lower the average productivity of imitation.

6 Transitional Dynamics

Next, we dig deeper on how policy varies along the development path, and assess how policy affects welfare. The equilibrium allocations are described by the decision rules, or policy functions. The model however does not deliver closed form solutions for them. Linearizing the system around the steady-state is not useful because our goal is to study the adjustment path for state space points that lie far away from the steady-state. We therefore choose to numerically approximate the solutions. Appendix A provides a detailed description of the method followed. In this section, we first choose values for the different parameters, and then present the dynamics results.

6.1 Calibration

Table 2 shows the parameter values used to carry out our simulations. We assign values of 0.96 to the discount factor (ρ), and 0.1 to the depreciation rate (δ). From Kydland and Prescott

(1991), we take a labor share of 0.64 (α). We set the growth rate of the population (n) to 1.4% per year, the average value for the United States during the period 1950-1980. We assign to the output per capita growth rate the averaged value in the Mankiw, Romer and Weil (1992) intermediate sample, 2.2%. From Domowitz, Hubbard and Petersen (1988), we take a mark-up ratio of 1.35.[14] Estimates of the inverse of the intertemporal elasticity of substitution between present and future consumption go from 1 to 3.5 (Hall 1988, and Attanasio and Weber 1993); we run the experiments for two different values: $\sigma = 1$ and $\sigma = 2$.

The calibration of the R&D technology parameters is more problematic. There are not reliable estimates of λ. Dinopoulos and Thompson (forthcoming) find values of λ as low as 0.17, whereas Jones and Williams (1999) show that 0.5 could be a lower bound. Given that the literature does not provide much guidance, we carry out a sensitivity analysis, and present results for λ equal 0.25, 0.5, and 0.75. Equation (17) says that T_t equals the ratio of imitation to research support. Based on our agricultural data, the average ratio of extension (or imitation) to research support for the industrialized world is very close to 0.41 both in 1970 and 1980.[15] This is the value that we assign to T^*. There are not any empirical estimates of the parameter β. As Parente and Prescott (1994), we pick the value of β for which the planning solution best fits the Japanese output data, taking as given the rest of parameters' values. In particular, we use the fact that Japan' per capita output moved from 19 percent to 74 percent of U.S. output during 1950 to 1980.

Using the steady-state allocations in the command optimum, we directly obtain the remaining model's parameters from the above ones. The implied value of ϕ comes from equation (11). The balanced growth path real interest rate can be obtained from equations (11) and (13); r^* equals 8 percent if $\sigma = 1$, and 10.3 percent when $\sigma = 2$.

6.2 The Imitation of Foreign Ideas and R&D Support

We now show that the existence of diminishing imitation opportunities allows the model to account for the rest of average patterns regarding the relationship between R&D support and economic development presented in section 2.

We look at the equilibrium behavior of the model in response to two simultaneous shocks,

[14] They estimate a producer durable markup ratio using electronic and electric equipment data. Furthermore, they adjust it to separate out fixed costs, which are completely absent in our model.

[15] The exact numbers can be found in Judd et al. (1986, page 86). The industrialized group contains the nations that in 1980 were OECD members, except for Greece, Portugal, Spain, and Turkey.

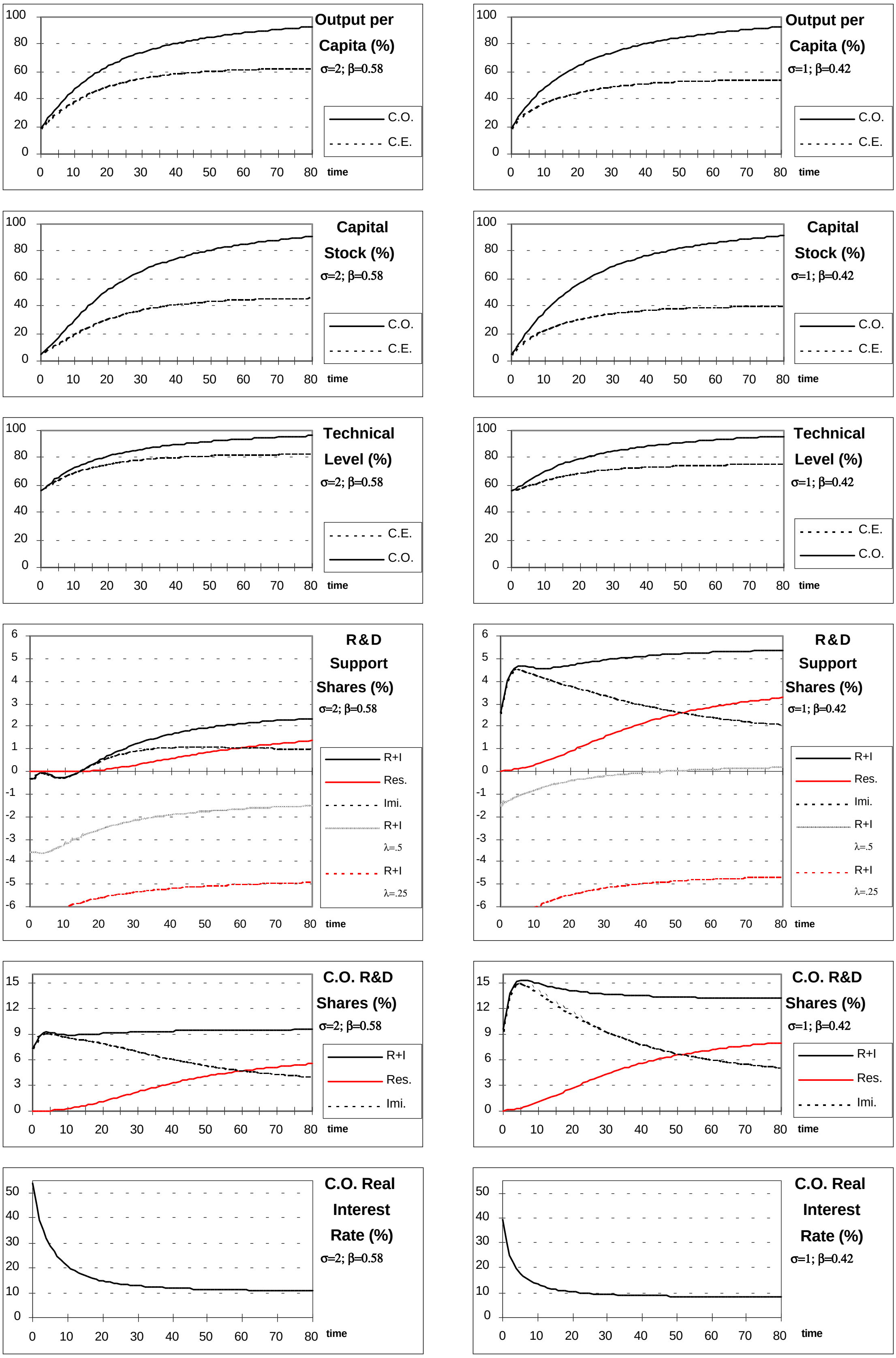
one to each state variable. Given the procedure that we follow to calibrate the parameter β, we use Japanese numbers. Extrapolating from pre-World War II data, Christiano (1989) estimates that the Japanese capital stock in 1946 was only 12 percent of its pre-war steady-state value. In turn, we pick the shock to the relative size of the international knowledge pool so as to make per capita output 19 percent of its steady-state level. Assuming that the U.S. was in steady-state in 1950, this implies initial values for the capital stock and total factor productivity (TFP) of 4.9 percent and 56.3 percent of the U.S. level, respectively. Starting from these initial conditions, we run simulations for six triples (σ, λ, β) that deliver social solutions consistent with Japan's convergence speed (see Table 2).[16]

Figure 3 presents the adjustment paths for selected variables. Starting from the same relative stocks of technology and capital, the command optimum (C.O.) and the zero-tax competitive equilibrium (C.E.) follow very different paths. The discrepancy between the two equilibrium allocations is what induces tax policy. As we saw, equation (16), producer durables are always subsidized at the same rate. R&D policy, on the other hand, is state-dependent. The bottom half of Figure 3 depict the series of the ratios of R&D and R&D support to output. The first thing that we observe is that an R&D subsidy is the optimal policy only if diminishing returns to R&D are sufficiently weak. When λ is small, on the contrary, R&D races generate overinvestment, and an R&D tax becomes optimal.

To understand why public support to R&D declines as σ rises, note that the amount of government giveaways rises both with the subsidization rate and the R&D share. Recall also our approximation, given by equation (24): the subsidization rate declines both with the relative size of the international pool of ideas and β (the negative externality effect). R&D investment in turn declines with its productivity, and with its opportunity cost, that is, the interest rate. A larger σ means a stronger preference for consumption smoothing, which generates lower investment. As we see in Figure 3, the interest rate must then rise to reduce the saving rate and clear the financial market, because for $\sigma > 1$ present and future consumption are complementary; thus reducing the R&D share. In addition, since investment in R&D lowers, the value of β needed to reproduce the Japanese convergence speed rises. Both this stronger diminishing imitation opportunity effect and the larger interest rate reduce the

[16] We do not prove formally that the stable adjustment path is unique. Eicher and Turnovsky (forthcoming), however, establish sufficient conditions for this to be so in a very similar framework. Those conditions are equivalent to requiring that $\phi + \lambda\xi < 1$ and $\alpha > \frac{1}{2}$ in our model. It turns out that the calibrated parameters fulfil these restrictions. Furthermore, during the innumerable simulations, we always found the same stable arm, independent of the initial guess used to approximate the decision rules, and the initial values of the state values.

Figure 3: Adjustment Paths and Optimal Technology Policy, $\lambda = 0.75$ except when noticed



optimal amounts of research and imitation subsidies.

For $\lambda = 0.75$, the model is fully consistent with the patterns suggested by the data. Except for the initial years, the R&D support share (R+I) increases as output moves towards the steady-state. Following the same logic as above, the fast decline in the interest rate is behind the initially rapid increase in R&D support – notice that in equation (24) a larger R&D share also contributes to make the R&D subsidization rate less negative (or more positive). After around 5 periods, the R&D share starts going down, and so does its support share. It is the result of the decline in the relative size of the international pool of designs, which reduces the productivity of imitation. This effect can also be observed in Figure 3 by looking at research investment, which begins increasing significantly only after 5 periods. After 15 periods, interest rates and R&D shares are very close to their steady-state values. The decrease in the negative externality, induced by the shrinking relative size of the international pool of designs, then becomes the dominant force, raising R&D support along the development path.

The interaction between the negative externality, the decreasing imitation productivity, and the declining interest rates is responsible for the ups and downs followed by R&D support at early stages of development. This provides a possible explanation for the behavior of the research and imitation adjusted line within the 14^{th} percentile country-group in Figure 1.

At a more disaggregated level, the model can also reproduce the empirical evidence. This is shown by the fourth row of charts. Support to imitation is the most important component during the first decades, and displays a hump shape because of two opposing forces. First, the decrease along the development path in the size of the negative externality generated by the existence of diminishing imitation opportunities pushes the imitation subsidization rate up. Second, the decline in the relative imitation productivity pushes down the weight of copying in technology policy. The former dominates at low levels of economic development, whereas the latter does later on. We can also see that the public research investment share rises over time – the two forces now push in the same direction. Research and research support eventually overcome imitation and imitation support as the main components of R&D.

6.3 Policy Effects

Finally, we study how policy affects long-run output levels, and welfare. This requires comparing the planning solution to the zero-tax competitive equilibrium. Table 3 presents welfare measures, and the relative levels of output, TFP, and physical capital at the competitive equi-

Table 3: Steady State Relative Below-Trend Levels of Output, Technology, and Capital for the Competitive equilibrium (C.O. equals 100), and Welfare Loss (percentage)

	C.E. Welfare Loss		Relative Output Level		Relative TPF		Relative Capital level	
λ	$\sigma = 1$	$\sigma = 2$	$\sigma = 1$	$\sigma = 2$	$\sigma = 1$	$\sigma = 2$	$\sigma = 1$	$\sigma = 2$
0.25	3.1	3.1	87.4	91.4	102.1	105.2	64.6	67.7
0.5	4.0	2.1	68.1	74.2	87.1	92.0	50.4	55.0
0.75	7.7	3.0	54.9	63.4	75.9	83.2	40.7	46.9

librium steady-state.

The decentralized economy does not take into account diminishing returns to R&D. When these are strong, non-subsidized private firms invest more resources in learning than the central planner, ending up with higher technology stock at the steady-state. As diminishing returns to R&D decline, the social return to learning new ideas overtake the private return, and eventually the opposite scenario emerges.

The competitive equilibrium always generates lower levels of output, and the difference increases with λ. The reason is the low degree of capital accumulation. For example, for a value of λ of 0.5, the levels of output and capital for the decentralized equilibrium are less than 75 and 55 percent of the command optimum steady-state levels, respectively. These figures go down to 64 and 47 percent when λ equals 0.75, which represents a loss of one third of output. This does not however imply that the benefit of correcting the market failure induced by monopoly pricing outweighs the gain from technology policy. Notice that the technological gap is partly responsible for the low levels of capital. In order to assess the contribution of these two types of policies, we compare the social optimum outcome to two different scenarios. In the first one, only optimal R&D taxation is implemented. In the second one, only the purchase of producer durables is subsidized. We carry out this analysis for the $\lambda = 0.75$ case because it is the one that generates positive R&D support rates, which is what we observe in the data.

When the government merely executes R&D policy, technology reaches its social optimum level. From equations (5), (14) and (15), the relative capital stock then equals $[(1-\alpha)\gamma]^{\frac{1}{\alpha}}$. For our parameter values, this expression takes on 62.3 percent, and output is 84.5 percent of the command optimum level. If, on the other hand, only the market failure generated by monopoly pricing is eliminated, equations (5), (14) and (15) force the stock of capital measured in per capita-efficiency units $\left(\frac{K}{A^{\xi/\alpha}L}\right)$ to equal its social optimum. Equation (5) then says that the relative levels of output and capital are the same, and equal relative TFP to the power of $\frac{1}{\alpha}$. The numerical analysis implies that in this scenario relative TFP and output levels are 86.7 and 80.0 percent if $\sigma = 1$, and 93.0 and 89.3 percent if $\sigma = 2$, respectively. Comparing to the numbers in Table 3, we see that the effect on output of government's reaction to monopoly pricing also increases TPF; for example, it goes from 75.9 up to 86.7 percent for $\sigma = 1$. The reason is that a larger $\bar{\tau}_x$ provides incentives to increase R&D investment, reducing the need to subsidize it, as we see in equation (24).

The R&D policy contribution to long-run output, therefore, outweighs the one of producer-durable subsidies when the preference for consumption smoothing is weaker, and the other way round. This occurs because the latter depends only on the mark-up ratio, whereas the former decreases with σ due to the reasons exposed before, agents are willing to sacrifice less present consumption and β need to rise. In the most advanced nations, the share of total public support to research and extension is around 2.5 percent (Figure 1), and the one of research subsidies is about 1 percent (Figure 2). The results for the $\sigma = 2$ case better fit those numbers. Our model then predicts a slightly higher contribution of capital formation policy.

The welfare measure in Table 3 gives the permanent percentage increase in competitive equilibrium consumption necessary to make consumers indifferent between following the socially optimal paths and following the zero-tax decentralized economy sequences. We see that the complete absence of policy intervention produce welfare losses that are always over 2 percent, reaching 7 percent for the low smoothing, weak diminishing returns case. A higher degree of consumption smoothing causes differences between the two equilibria to decline. If we look at the welfare measure, the losses when $\sigma = 2$ are always lower than if $\sigma = 1$, and do not vary much when λ changes. The positive relationship between the loss measure and λ even vanishes when $\sigma = 2$.

7 Summary and Conclusions

Using cross-country data on government research and extension expenditures in the agricultural sector, this paper has documented several empirical regularities regarding the relationship between the level of economic development and public support to research and imitation. Except for countries in the 14^{th} percentile of relative income, which show more complex behaviors, we can summarized the evidence as follows. First, the share of public investment in imitation raises with income across nations that are at early stages of development, and stabilizes (or slightly declines) among the advanced group. Second, both public research and total public support increase with income levels. Third, in the cross-section of countries, the ratio of imitation to research support is inversely related to the level of economic development.

The R&D-based growth model that we have presented incorporates the possibility of copying foreign ideas through a technology that exhibit diminishing imitation opportunities. This allows the transition dynamics of the proposed framework to account for the above average technology policy patterns. As a country catches up with the more advanced nations, both the imitation productivity and the negative externality generated by diminishing imitation opportunities decline. The former effect reduces the incentive to support imitation, whereas the latter encourages research and imitation subsidies. The model also shows that large increases in R&D investment, and therefore in its support share are possible at early stages of development due to the rapid decline of interest rates.

Comparing the social optimum and the competitive equilibrium paths, we find that public intervention can produce important benefits, and that both technology and capital accumulation policies have a very similar contribution to the welfare improvement.

Future research on the determinants of public imitation policy, and its interaction with public research programs, should include the construction of better data sets, especially outside the agricultural sector. Further work is also needed to assess the relative importance of research, imitation, and capital accumulation subsidies along the economic development path, and quantify their contribution to economic growth. Finally, it would be interesting to analyze how the introduction into our model of additional market failure sources, like the ones pointed out by Jones and Williams (1999) and Stoneman and Diederen (1994), affect technology policy.

Appendix A

In order to study dynamics, we first need to state the system of equations that characterize the equilibrium allocations. Regarding the command optimum, the Euler conditions for the control variables, and motion equations for the state variables have already been worked out in the main text. In the next section, we derive the ones for the decentralized economy with taxes.

A.1 The Competitive Equilibrium

In our economy, there is a continuum of identical consumers of size $(1+n)^t L_o$ at date t. They are endowed with one unit of labor in each period. Their preferences are given by the following utility function:

$$U_t = \sum_{j=t}^{\infty} \rho^{j-t} \left(\frac{c_j^{1-\sigma} - 1}{1-\sigma} \right) \; ; \tag{26}$$

where c_t is the amount of consumption per capita in period t.

Inputs are not mobile, and must be exclusively supplied to domestic firms. At each data t, consumers supply their labor inelastically. In return for this service, they receive a wage w_t. We assume the existence of a capital market that supplies the savings of consumers to intermediate-goods producers that issue securities. The equilibrium interest rate r_t clears the market at each point in time. The representative consumer's budget constraint is then given by

$$(1+n)\, a_{t+1} = (1 + r_{t+1})\, (\, a_t + w_t - c_t - \tau_{ht}) \, ; \tag{27}$$

where a_t is the value, in terms of output, of the securities owned by each consumer. Consumers choose the time series of consumption that maximizes (26) subject to (27). The first order condition to this problem gives the Euler equation for aggregate consumption:

$$C_{t+1} = \left[\rho\, (1 + r_{t+1})\right]^{\frac{1}{\sigma}} (1+n)^{1-\frac{1}{\sigma}}\, C_t \, . \tag{28}$$

Final-goods manufacturers are price takers, and earn zero profits in equilibrium. Because intermediate goods are rented rather than sold, equation (2) implies that they solve the following problem:

$$\max_{\{L_t\,,\, x_{it}\,\}} \left\{ L_t^{\alpha} \left[\int_0^{A_t} x_{it}^{(1-\alpha)\gamma}\, di \right]^{\frac{1}{\gamma}} - \omega_t\, L_t - \int_0^{A_t} p_{it}\, (1 - \tau_{xit})\, x_{it}\, di \right\} \; ; \tag{29}$$

where ω_t is the wage rate. For the interior solution to this problem, the first order conditions are

$$\omega_t = \alpha \frac{Y_t}{L_t} \tag{30}$$

$$p_{it} = \left(\frac{1-\alpha}{1-\tau_{xit}}\right) L_t^{\alpha} \left[\int_0^{A_t} x_{jt}^{(1-\alpha)\gamma} \, dj\right]^{\frac{1}{\gamma}-1} x_{it}^{(1-\alpha)\gamma-1} \quad , \ i \in (0, A_t)\,. \tag{31}$$

Equations (30) and (31) represent the inverse demand functions for labor and producer durables, respectively. Given that all intermediate-goods designs provide the same improvement in productivity, we hereafter focus on the symmetric equilibrium in which capital is evenly distributed over all available types; that is, $x_{it} = \bar{x}_t$ for all i.

Equation (16) says that it is optimal to subsidize the purchase of all intermediate goods at the same rate, $\bar{\tau}_{xt}$. Equations (31) and (15) then imply that, if $x_{it} = \bar{x}_t$ for all i, the amount of producer durables of a given type used in the economy is

$$\bar{x}_t = \frac{(1-\alpha)^2 \, \gamma}{(1-\bar{\tau}_{xt}) \, (r_t + \delta)} \left(\frac{Y_t}{A_t}\right). \tag{32}$$

Knowing the gains from discovering new designs, intermediate-goods producers choose how much capital to invest in R&D. Let $R_t = R_{Ct} + R_{It}$. From equations (20), (21) and (22), we can write the zero profit condition for total R&D effort as

$$(1-\tau_{It}) \, R_t^{1-\lambda} = V_t \, \mu \, A_t^{\phi} \left\{1 + \left[\left(\frac{1-\tau_{It}}{1-\tau_{Ct}}\right)\left(\frac{\eta \, A_t^w}{A_t}\right)^{\beta\lambda}\right]^{\frac{1}{1-\lambda}}\right\}^{1-\lambda}. \tag{33}$$

The firms' optimal allocation to R&D across time will be determined by the evolution of the design's value V_t, which is pinned down by the following arbitrage condition:

$$1 + r_{t+1} = \frac{\bar{p}_{t+1} \, \bar{x}_{t+1}}{V_t + \bar{x}_{t+1}} + \frac{V_{t+1} + (1-\delta) \, \bar{x}_{t+1}}{V_t + \bar{x}_{t+1}}. \tag{34}$$

The RHS represents the return to engaging in intermediate-goods manufacturing. Buying a patent right today and manufacturing the products that will be rented tomorrow provides a return that equals the dividend (first summand) plus the capital gain/loss (second summand). The LHS, in turn, gives the gross return from lending to other firms. The above expression says that, in equilibrium, firms must be indifferent between the two alternatives.

From equations (15), (23), (32), (33) and (34), the R&D investment optimal motion in the competitive equilibrium with taxes is given by:

$$(1+r_{t+1})\left(\frac{R_t}{A_{t+1}-A_t}\right)=\frac{[1-\gamma(1-\alpha)](1-\alpha)}{(1-\bar{\tau}_{Rt})\ (1-\bar{\tau}_{x,t+1})}\left(\frac{Y_{t+1}}{A_{t+1}}\right)+\frac{\left(\frac{1-\bar{\tau}_{R,t+1}}{1-\bar{\tau}_{Rt}}\right)R_{t+1}}{A_{t+2}-A_{t+1}}. \tag{35}$$

Comparing expression (35) to equation (19), we see that if $\frac{1-\bar{\tau}_{R,t+1}}{1-\bar{\tau}_{Rt}}=1$, the optimal R&D subsidy rate is given by

$$\bar{\tau}_{Rt}\simeq 1-\frac{\frac{[1-\gamma(1-\alpha)](1-\alpha)}{(1-\bar{\tau}_{x,t+1})}}{\frac{R_{t+1}}{Y_{t+1}}\left[\phi-\lambda\beta\left(\frac{T_{t+1}^{\frac{\beta\lambda}{1-\lambda}}}{1+T_{t+1}^{\frac{\beta\lambda}{1-\lambda}}}\right)\right]+\lambda\xi}. \tag{36}$$

At the aggregate level, market clearing is summarized by the economy's budget constraint. We have seen (i) that households allocate final goods either to consumption or savings; and (ii) that intermediate-goods manufacturers borrow capital to pay labor and the rental rate on producer durables as R&D work is done, and to manufacture the new products. Finally, since trade is not allowed in the model, domestic output must equal domestic expenditure. The economy's budget constraint is then given by

$$Y_t=C_t+I_t+R_t; \tag{37}$$

where I_t is investment at date t. The law of motion of the capital stock is

$$K_{t+1}=(1-\delta)\,K_t+I_t. \tag{38}$$

In the decentralized economy with taxes, the perfect foresight equilibrium is the set of sequences of prices $\{\omega_t,r_t,\bar{p}_t\}_{t=0}^{\infty}$, allocations $\{\bar{x}_t,I_t,R_{Ct},R_{It},C_t\}_{t=0}^{\infty}$, and policies $\{\tau_{ht},\tau_{Ct},\tau_{It},\bar{\tau}_{xt}\}_{t=0}^{\infty}$ such that Euler condition (28) characterizes the consumers' behavior, firms assign resources according to equations (22), (32) and (35), the government balanced its budget given in expression (12), and market clearing condition (37) holds.

A.2 Normalization

In analyzing the equilibrium allocations, it is useful to redefine growing variables such that the resulting normalized ones reach a steady-state. Given the economy's budget constraint, equation (37), we know that consumption, physical capital, and R&D investment will grow at the same rate at steady-state. Production function (5) then implies that output growth will be determined by the effective-labor growth rate. Hence, the appropriate normalization factor for these variables is $A_t^{\frac{\xi}{\alpha}} L_t$. Denote normalized variable D_t by $\hat{D}_t = \frac{D_t}{A_t^{\frac{\xi}{\alpha}} L_t}$. Normalized variables are then measured in per capita-efficiency units. We define

$$\theta = \mu \ (\eta \ A_t^w)^{\phi - 1 + \lambda \frac{\xi}{\alpha}} \ L_t^{\lambda} \ . \tag{39}$$

We saw in section 4 that $g_{A^w} = g_A$. Equation (11) then says that θ is a constant coefficient. In terms of normalized variables, an equilibrium is the set of sequences of prices, normalized allocations $\left\{ \hat{K}_t, \hat{I}_t, \hat{R}_{Ct}, \hat{R}_{It}, \hat{C}_t \right\}_{t=0}^{\infty}$, and policies satisfying the optimality and equilibrium conditions.

Recall that $\tau_{It} = \tau_{Ct} = \bar{\tau}_{Rt}$. The following difference equation system formed by Euler equations (22), (28) and (35), the laws of motion (38) for K_t, and (8) and (18) for T_t, appropriately normalized, describes the decentralized equilibrium with taxes:

$$\hat{C}_{t+1} \, G_{At}^{\frac{\xi}{\alpha}} \, (1+n)^{\frac{1}{\sigma}} = \rho \, \left[1 + \frac{\gamma \, (1-\alpha)^2}{(1 - \bar{\tau}_{xt+1}) \, \hat{K}_{t+1}^{\alpha}} - \delta \right]^{\frac{1}{\sigma}} \hat{C}_t; \tag{40}$$

$$\begin{aligned} \left(\frac{\hat{R}_{t+1}}{1 + T_{t+1}^{\frac{\beta \lambda}{1-\lambda}}} \right)^{1-\lambda} \quad = \quad & \left[1 + \frac{\gamma \, (1-\alpha)^2}{(1 - \bar{\tau}_{xt+1}) \, \hat{K}_{t+1}^{\alpha}} - \delta \right] \left(\frac{1 - \bar{\tau}_{Rt}}{1 - \bar{\tau}_{R,t+1}} \right) G_{At}^{\phi - \frac{\xi}{\alpha}(1-\lambda)} \, * \\ & * \left[\frac{\frac{\hat{R}_t}{(1+n)}}{1 + T_t^{\frac{\beta \lambda}{1-\lambda}}} \right]^{1-\lambda} - \frac{[1 - \gamma(1-\alpha)] \, (1-\alpha)}{(1 - \bar{\tau}_{R,t+1}) \, (1 - \bar{\tau}_{xt+1})} \, \theta \, T_{t+1}^{1 - \phi - \lambda \frac{\xi}{\alpha}} \, \hat{K}_{t+1}^{1-\alpha}; \end{aligned} \tag{41}$$

$$\frac{\hat{R}_{Ct}}{\hat{R}_{It}} = T_t^{\frac{\beta \lambda}{1-\lambda}}; \tag{42}$$

$$\hat{K}_{t+1} \, G_{At}^{\frac{\xi}{\alpha}} \, (1+n) \ = \ \hat{K}_t^{1-\alpha} - \hat{C}_t - \hat{R}_t + (1-\delta) \, \hat{K}_t; \tag{43}$$

$$T_{t+1} = \left(\frac{1 + g_{A^w}}{G_{At}} \right) \, T_t; \tag{44}$$

where

$$G_{At} = 1 + \theta \, T_t^{1 - \phi - \lambda \frac{\xi}{\alpha}} \, \hat{R}_t^{\lambda} \, \left(1 + T_t^{\frac{\beta \lambda}{1-\lambda}} \right)^{1-\lambda}; \tag{45}$$

and

$$\hat{R}_t = \hat{R}_{Ct} + \hat{R}_{It}. \tag{46}$$

The same laws of motion and identities as in the decentralized case, expressions (43), (44), (45) and (46), and Euler conditions (13), (17) and (19), appropriately normalized, define the social planning problem solutions. The second of these Euler equations is equivalent to (42); the other two are given by

$$\hat{C}_{t+1} G_{At}^{\frac{\xi}{\alpha}} (1+n)^{\frac{1}{\sigma}} = \left[\rho \left(1 + \frac{1-\alpha}{\hat{K}_{t+1}^{\alpha}} - \delta \right) \right]^{\frac{1}{\sigma}} \hat{C}_t; \tag{47}$$

and

$$\begin{aligned} \left(\frac{\hat{R}_{t+1}}{1 + T_{t+1}^{\frac{\beta\,\lambda}{1-\lambda}}} \right)^{1-\lambda} + \theta\, T_{t+1}^{1-\phi-\lambda\frac{\xi}{\alpha}}\ \hat{R}_{t+1} \left(\phi - \frac{\beta\,\lambda\, T_{t+1}^{\frac{\beta\,\lambda}{1-\lambda}}}{1 + T_{t+1}^{\frac{\beta\,\lambda}{1-\lambda}}} \right) = \\ = \left(1 + \frac{1-\alpha}{\hat{K}_{t+1}^{\alpha}} - \delta \right) G_{At}^{\phi - \frac{\xi}{\alpha}(1-\lambda)} \left[\frac{\frac{\hat{R}_t}{(1+n)}}{1 + T_t^{\frac{\beta\,\lambda}{1-\lambda}}} \right]^{1-\lambda} - \lambda\, \xi\, \theta\, T_{t+1}^{1-\phi-\lambda\frac{\xi}{\alpha}}\ \hat{K}_{t+1}^{1-\alpha}. \end{aligned} \tag{48}$$

A.3 Numerical Approximation Method

Following Judd (1992), we use high-degree polynomials in the state variables to replicate the policy functions. The parameters of the approximated decision rules are chosen to (approximately) satisfy the Euler equations over a number of points in the state space, using a nonlinear equation solver. A Chebyshev polynomial basis is used to construct the policy functions, and the zeros of the basis form the points at which the system is solved; in other words, we use the method of orthogonal collocation to choose these points. Finally, tensor products of the states variables are employed in the polynomial representations. This method has proven to be highly efficient in similar contexts. For example, for the one-sector growth model, Judd (1992) finds that the approximated values of the control variables disagree with the values delivered by the true policy functions by no more than one part in 10,000.

For most models, however, we cannot directly assess how well the polynomial basis approximates the true solution; but there are indirect measures. For instance, as Judd (1992) argues, we can asses the Euler equation error over a large number of points using the approximated rules. For example, if we employ Euler equation (47), the measure will give the current consumption decision error that agents using the approximated rules make, assuming that the (true) optimal decisions were made in the previous period. The accuracy rises with the degree of the polynomials. To run our simulations, we used polynomials of degree eight. Higher degrees gave more accuracy, but results were almost identical. The policy functions were approximated using the functional form

$$\ln D_t = \Psi_8(\ln T_t, K_t)\ , \quad D_t = C_t, R_t\ ;$$

Table 4: Solution Algorithm Accuracy for Analyzed Cases

				% of Error for $\hat{C}_{t+1}$		% of Error for $\hat{R}_{t+1}$	
	σ	λ	β	Average	Maximum	Average	Maximum
Compe-	1	0.25	2.03	0.02	0.07	0.03	0.14
tive Equi-	1	0.5	0.83	0.02	0.07	0.04	0.18
librium	1	0.75	0.42	0.02	0.07	0.09	0.34
Social	1	0.25	2.03	0.08	0.35	0.10	0.50
Planner's	1	0.5	0.83	0.08	0.35	0.15	0.72
Solution	1	0.75	0.42	0.08	0.35	0.34	1.59
Compe-	2	0.25	2.3	0.02	0.07	0.05	0.25
tive Equi-	2	0.5	1.03	0.02	0.07	0.07	0.35
librium	2	0.75	0.58	0.02	0.07	0.16	0.73
Social	2	0.25	2.3	0.06	0.31	0.16	0.88
Planner's	2	0.5	1.03	0.06	0.31	0.24	1.31
Solution	2	0.75	0.58	0.07	0.33	0.75	3.40

were Ψ_n denotes the n-degree Chebyshev polynomial function. The steps followed were the following. We first approximated the policy functions for the planning problem. Then, these approximations were used to compute optimal state-dependent tax policy rules for the competitive equilibrium solution. The programs were written in gauss-386, and are available upon request. Table 4 reports the maximum and average Euler equation errors found over a grid search of 10,000 state space points for the cases analyzed in the paper.

Appendix B

The following Table contains the data presented in Section 2. Numbers on GDP per worker come from Penn World Tables, Version 5.6, available on line at http://www.nber.org/pwt56.html. For the other variables' sources, see Judd, Boyce and Evenson (1986). We have data for the years 1962, 1965, 1968, 1971, 1974, 1977 and 1980. For each variable, we compute the average value.

Mean Values of GDP per Worker, and of Research and Extension Expenditures as Percentage of the Value of Agricultural Product: Public Sector

Country	Extension Share	Research Share	Extension plus Research Share	Extension over Research (%)	Relative GDP per Worker (U.S.=100)
Argentina	0.391	0.696	1.088	56.280	49.306
Australia	0.965	2.580	3.546	37.411	80.851
Austria	0.900	0.427	1.327	210.612	60.495

Mean Values of GDP per Worker, and of Research and Extension Expenditures as Percentage of the Value of Agricultural Product: Public Sector, cont.

Country	Extension Share	Research Share	Extension plus Research Share	Extension over Research (%)	Relative GDP per Worker (U.S.=100)
Bangladesh	0.159	0.333	0.492	47.547	10.829
Belgium	0.075	0.813	0.888	9.174	74.201
Bolivia	0.141	0.586	0.727	24.096	17.114
Brazil	1.205	0.714	1.919	168.759	28.219
Burundi	0.637	0.333	0.970	191.173	2.651
Canada	0.922	2.282	3.205	40.406	84.378
Chile	0.651	0.970	1.621	67.091	31.416
Colombia	0.210	0.657	0.867	31.979	17.435
Costa Rica	0.421	0.333	0.754	126.249	5.804
Cyprus	0.543	0.728	1.271	74.477	35.927
Denmark	0.586	0.674	1.260	86.936	65.537
Ecuador	0.270	0.565	0.835	47.799	23.986
Egypt	1.580	0.547	2.127	288.671	14.458
El Salvador	0.189	0.298	0.488	63.477	19.190
England	0.865	1.206	2.071	71.761	62.345
Ethiopia	0.127	0.130	0.257	97.643	2.129
Finland	1.069	0.511	1.580	209.014	56.616
France	0.468	0.885	1.353	52.926	71.735
Ghana	0.803	0.587	1.390	136.843	8.084
Greece	0.165	0.419	0.584	39.387	36.836
Guatemala	0.276	0.348	0.624	79.484	23.780
Honduras	0.174	0.313	0.487	55.509	14.130
India	0.194	0.236	0.431	82.179	6.580
Indonesia	0.282	0.310	0.592	90.951	7.393
Iran	0.497	0.560	1.057	88.689	50.918
Ireland	0.645	1.260	1.905	51.216	45.515
Israel	0.627	1.786	2.413	35.084	55.301
Italy	0.302	0.544	0.846	55.519	65.020
Ivory Coast	2.054	1.126	3.181	182.414	11.265
Jamaica	0.142	0.457	0.599	31.182	20.336
Japan	0.843	2.607	3.450	32.320	37.655
Jordan	0.970	0.439	1.409	220.831	29.357
Kenya	1.626	0.890	2.517	182.693	5.770
Korea. Rep.	0.283	0.367	0.650	77.323	17.225
Liberia	0.222	0.126	0.349	175.850	7.737
Madagascar	1.058	0.734	1.792	144.268	7.354
Malawi	2.645	1.471	4.116	179.871	3.333
Malaysia	0.357	0.602	0.960	59.362	22.369
Mali	3.094	1.968	5.062	157.200	4.573

Mean Values of GDP per Worker, and of Research and Extension Expenditures as Percentage of the Value of Agricultural Product: Public Sector, cont.

Country	Extension Share	Research Share	Extension plus Research Share	Extension over Research (%)	Relative GDP per Worker (U.S.=100)
Mexico	0.209	0.291	0.500	72.054	47.796
Morocco	1.468	0.419	1.887	350.133	17.321
Netherlands	0.786	3.041	3.827	25.854	82.937
New Zealand	0.429	1.531	1.961	28.034	80.756
Nicaragua	0.221	0.379	0.599	58.280	25.262
Nigeria	0.993	0.888	1.882	111.810	4.121
Norway	1.547	2.767	4.314	55.915	67.412
Pakistan	0.149	0.272	0.421	54.560	10.444
Panama	0.726	0.419	1.145	173.384	25.605
Paraguay	0.148	0.463	0.611	32.081	16.622
Peru	0.610	0.689	1.299	88.578	30.181
Philippines	0.332	0.176	0.508	188.548	13.574
Portugal	1.008	1.485	2.493	67.889	29.610
Rwanda	0.384	0.165	0.549	233.395	3.659
South Africa	0.766	1.261	2.027	60.790	29.455
Senegal	2.071	1.479	3.550	139.973	7.963
Sierra Leone	0.255	0.203	0.458	125.248	9.166
Spain	0.239	0.539	0.778	44.362	56.454
Sri Lanka	0.979	0.689	1.668	142.119	12.354
Sudan	1.885	0.592	2.477	318.231	8.245
Sweden	0.566	1.012	1.578	55.992	75.308
Switzerland	0.343	1.786	2.129	19.214	88.354
Syria	3.161	0.397	3.558	795.774	37.941
Taiwan	0.147	0.314	0.461	46.956	22.290
Tanzania	0.804	0.485	1.290	165.687	2.755
Thailand	0.403	0.474	0.877	85.071	10.223
Trin. and Toba.	1.458	0.666	2.124	219.086	75.782
Tunisia	2.334	0.701	3.034	333.157	20.239
Turkey	0.244	0.178	0.421	137.158	17.527
Uganda	1.303	0.766	2.069	170.038	4.219
Uruguay	0.223	0.380	0.603	58.592	35.007
U.S.	0.485	1.039	1.524	46.707	100
Venezuela	1.013	1.260	2.273	80.392	81.698
W. Germany	0.327	1.208	1.534	27.055	71.567
Zaire	0.841	0.466	1.308	180.434	4.501
Zimbabwe	0.423	0.776	1.200	54.488	8.811

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Appendix for referees, not for publication

These notes are intended to be helpful to the reader carrying out the refereeing job. They derive some of the analytical expressions contained in the paper.

Equation (5)

Equation (2) says that

$$Y_t = L_t^{\alpha} \left[\int_0^{A_t} x_{it}^{(1-\alpha)\gamma} \, di \right]^{\frac{1}{\gamma}} \quad , \quad 0 < \alpha < 1 \ , \quad \gamma > 0 \, .$$

Given that all intermediate-goods designs provide the same improvement in productivity, we can focus on the symmetric equilibrium in which capital is evenly distributed over all available types; that is, $x_{it} = \bar{x}_t$ for all i. In the symmetric equilibrium, the capital stock equals $K_t = A_t \, \bar{x}_t$. We can then write the above equation as

$$Y_t = L_t^{\alpha} \, A_t^{\frac{1}{\gamma}} \, \bar{x}_t^{1-\alpha} = L_t^{\alpha} \, K_t^{1-\alpha} \, A_t^{\frac{1}{\gamma}-(1-\alpha)} \, .$$

Hence

$$Y_t = A_t^{\xi} L_t^{\alpha} \, K_t^{1-\alpha}; \ \text{ where } \xi = \frac{1}{\gamma} - (1-\alpha) \, .$$

Euler conditions (13)

Using a recursive approach, the Planner's problem at date t can be written as

$$\begin{aligned} W(A_t, K_t) \quad = & \max_{\{I_t, R_{It}, R_{Ct}\}} \ U\left(\frac{A_t^{\xi} L_t^{\alpha} K_t^{1-\alpha} - R_{It} - R_{Ct} - I_t}{L_t} \right) + \\ & + \rho \, W \left\{ A_t + \mu \, A_t^{\phi} \left[R_{It}^{\lambda} + R_{Ct}^{\lambda} \left(\frac{\eta \, A_t^{w}}{A_t} \right)^{\beta\lambda} \right], (1-\delta) \, K_t + I_t \right\}; \end{aligned} \tag{49}$$

where I_t is physical capital investment; and $W(\cdot)$ is a value function. For the interior solution, the first order condition (FOC) with respect to I_t is

$$U' \left(\frac{C_t}{L_t} \right) \frac{1}{L_t} = \rho \, W_K (A_{t+1}, K_{t+1}) \, .$$

Differentiating $W(A_t, K_t)$, equation (49), with respect to K_t at the maximum, we get

$$W_K (A_t, K_t) = U' \left(\frac{C_t}{L_t} \right) \frac{(1-\alpha) \, Y_t}{L_t K_t} + \rho \, W_K (A_{t+1}, K_{t+1}) \, (1-\delta) \, .$$

For our utility function explicit form, and using the last two expressions and equation (9), we obtained condition (13).

Equation (15)

Equation (15) is a standard result in R&D-based models of growth. Let V_{it} be the market value of design i. The evolution of V_{it}, which is pinned down by the following arbitrage condition, determines the firms' optimal allocation to R&D across time:

$$1 + r_{t+1} = \frac{p_{i,t+1}\, x_{i,t+1}}{V_{it} + x_{i,t+1}} + \frac{V_{i,t+1} + (1-\delta)\, x_{i,t+1}}{V_{it} + x_{i,t+1}}. \tag{50}$$

The RHS represents the return to engaging in intermediate-goods manufacturing. Buying a patent right today and manufacturing the products that will be rented tomorrow provides a return that equals the dividend (first summand) plus the capital gain/loss (second summand). The LHS, in turn, gives the gross return from lending to other firms. The above expression says that, in equilibrium, firms must be indifferent between the two alternatives.

To obtain the optimal amount of producer durable i, we must derive condition (50) with respect to $x_{i,t+1}$, taking into account that the price p_{it} is given by the final-output manufacturers' inverse demand function. We get

$$r_{t+1} + \delta = \left(\frac{\partial\, p_{i,t+1}}{\partial\, x_{i,t+1}}\right) x_{i,t+1} + p_{i,t+1}, \tag{51}$$

where p_{it} comes from solving the following final-goods producers' problem:

$$\max_{\{L_t\,,\, x_{it}\,\}} \left\{ L_t^{\alpha} \left[\int_0^{A_t} x_{it}^{(1-\alpha)\gamma}\, di\right]^{\frac{1}{\gamma}} - \omega_t\, L_t - \int_0^{A_t} p_{it}\,(1-\tau_{xit})\, x_{it}\, di \right\};$$

ω_t is the wage rate. Its interior solution gives

$$p_{it} = \left(\frac{1-\alpha}{1-\tau_{xit}}\right) L_t^{\alpha} \left[\int_0^{A_t} x_{jt}^{(1-\alpha)\gamma}\, dj\right]^{\frac{1}{\gamma}-1} x_{it}^{(1-\alpha)\gamma-1}. \tag{52}$$

Equations (51) and (52) imply that in the symmetric equilibrium

$$r_{t+1} + \delta = p_{i,t+1} \left[\left(\frac{1}{\gamma} - 1\right) \frac{\gamma\,(1-\alpha)}{A_t} + \gamma\,(1-\alpha)\right].$$

As the number of firms goes to infinity, so does A_t, and we end up with equation (15).

Equation (17)

For the interior solution to problem (49), the FOCs with respect to R_{It} and R_{Ct} are

$$U'\left(\frac{C_t}{L_t}\right) \frac{1}{L_t} = \rho\, W_A\,(A_{t+1}, K_{t+1})\;\mu\; A_t^{\phi}\, \lambda\, R_{It}^{\lambda-1}, \tag{53}$$

and

$$U'\left(\frac{C_t}{L_t}\right) \frac{1}{L_t} = \rho\, W_A\,(A_{t+1}, K_{t+1})\;\mu\; A_t^{\phi}\, \lambda\, R_{Ct}^{\lambda-1} \left(\frac{\eta\, A_t^{w}}{A_t}\right)^{\beta\lambda}, \tag{54}$$

respectively. The last two conditions generate equation (17).

Euler condition (19)

Substituting (17) into either FOC (53) or (54), and letting $R_t = R_{It} + R_{Ct}$, we get

$$U'\left(\frac{C_t}{L_t}\right)\frac{1}{L_t} = \rho\, W_A\left(A_{t+1}, K_{t+1}\right)\ \mu\ A_t^{\phi}\lambda\left(\frac{R_t}{1+T_t^{\frac{\beta\lambda}{1-\lambda}}}\right)^{\lambda-1}.$$

Differentiating (49) with respect to A_t at the maximum,

$$W_A\left(A_t, K_t\right) = U'\left(\frac{C_t}{L_t}\right)\frac{\xi\, Y_t}{A_t\, L_t} + \rho\, W_A\left(A_{t+1}, K_{t+1}\right)\left\{\mu\ A_t^{\phi-1}\, R_t^{\lambda}\left[\frac{\phi + (\phi-\beta\lambda)\, T_t^{\frac{\beta\lambda}{1-\lambda}}}{\left(1+T_t^{\frac{\beta\lambda}{1-\lambda}}\right)^{\lambda}}\right] + 1\right\}.$$

The last two equations generate condition (19).