Matching up the data on education with economic growth models*

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Abstract

The growth literature has not yet established how data on education should be introduced in theories involving human capital. Early work used enrolment rates as a proxy of human capital whereas more recently it has utilized measures of average educational attainment taking advantage of new data sets. This paper examines alternative specifications of human capital that may match up with the existing data on education. First, we present a standard neoclassical two-sector growth model that adopts a human capital specification proposed in recent papers. In this model the fraction of individual's time endowment in school is viewed as an investment rate. We show that the optimally chosen educational attainment predicted by the calibrated two-sector model is very high and does not correspond to the data. Next, we investigate two growth models with alternative specifications of human capital that are successful in predicting educational attainment levels comparable to those observed in the data.

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1 Introduction

In the growth literature it is not yet well-established how existing data on educational attainment should be entering our models involving human capital. Recent work has employed measures of average years of schooling, primarily from the cross-country data set by Barro and Lee (1993). In this paper we ask the following question: In the context of our theories of economic growth, what is an appropriate specification of human capital to which a measure of average educational attainment successfully corresponds? The primary goal of the paper is to propose human capital specifications that, within a simple calibrated growth model, can successfully produce educational attainment levels comparable to those we observe in the data.

Recent models of economic growth such as Bils and Klenow (1996) and Jones (1997), among others, propose thinking of the fraction of an individual's time endowment spent in school as an investment rate (a "flow" variable). In particular, these papers specify a human capital production function that is based on the "Macro-Mincer" wage equation originated by Bils and Klenow (1996) and empirically examined by Heckman and Klenow (1997).

We first incorporate the specification used in these papers in a standard neoclassical two-sector growth model. We then calibrate the model at steady state and compare its predictions regarding educational attainment to the data. The main finding is that the calibrated model implies an optimally chosen educational attainment level that has a lower bound of more than 30 years, whereas the data supports much smaller levels (i.e. 8-14 years). We interpret this finding as suggesting that the human capital specification used by these papers is too simple.

We extend the basic specification to allow for different elasticities of substitution between skilled and unskilled labor, hoping that assigning appropriate elasticities would produce a specification that works. Our calibration exercises, however, show that educational attainment levels remain very high regardless of the degree of substitutability between raw and skilled labor.

Next, we investigate a model that follows Bils and Klenow (1998). In an updated version of their 1996 paper, Bils and Klenow (1998) (BK thereafter) present a model of schooling and economic growth. The aim of their paper is to examine the causal relationship between growth and schooling, which makes it appropriate to use a finite-lived agent model. This is however outside the standard neoclassical framework. We embed a version of BK's human capital specification that includes experience and a human capital externality in our standard two-sector infinite horizon growth model, and study its steady-state equilibrium predictions. We find that adding both work experience *and* human capital externality in the basic model can generate plausible predictions of the steady-state average educational attainment.

Finally, we present a simple growth model that employs an alternative specification of human capital. The proposed specification does not require additional variables such as work experience or human capital externality to reproduce the evidence. Instead, the new specification provides a map between the average years of schooling and human capital accumulation through the law of motion of the former. A calibration exercise is then performed showing that the calibrated model is successful in predicting educational attainment levels that are similar to those in the data.

The rest of the paper is organized as follows. Section 2 presents a brief discussion regarding the existing data on educational attainment as it relates to growth models with human capital. Section 3 investigates the steady-steady properties of a two-sector growth model following the specification used in recent papers that view years of education as an investment rate. Section 4 incorporates work experience and a human capital externality following BK in a standard infinite horizon problem and investigates its steady-state predictions regarding educational attainment. Section 5 proposes an alternative specification of human capital that is based on a law of motion of years of education. Section 6 concludes.

2 Human Capital Specification, Data, and Growth Models

This section provides a brief discussion on the specification of human capital and the existing data on educational attainment as they relate to growth models. We start by taking a closer look at the definition of human capital.

Is there a universal definition of human capital? Not only there is no universal definition of human capital, but rather there is a considerable amount of confusion in the literature about the meaning of existing definitions. Whereas it is widely accepted that the economics literature provides accurate and clear definitions of physical capital and labor, among other important factors of production, unfortunately this is not true for human capital.

The term "human capital" has been around at least since the seminal work of Becker (1964). Nelson and Phelps (1966) first used human capital in a growth model. Two decades later, Lucas (1988) established human capital as a major contributor to the endogenous growth literature. Lucas

(1988, p. 17) writes: "*The theory of human capital focuses on the fact that the way an individual allocates his time over various activities in the current period affects his productivity, or his human capital level, in future periods.*" As in Lucas (1988), human capital has, for the most part, been perceived as a stock variable that can accumulate. Recall for a moment, the pioneer endogenous growth models that we now refer to as Ak-based models. These models explicitly or implicitly, view human capital (in addition to physical capital) as an accumulable input of production. In fact, the original Ak-based model with production technology $Y = Ak$, delivers its predictions upon the assumption that k is an aggregate form of capital that includes both human and physical capital as perfect substitutes.

The common perception of human capital as a stock variable has been challenged by a number of recent papers arguing that it is better to view human capital as an investment rate (a "flow" variable). The primary reason for the emergence of this alternative interpretation of human capital is data availability constraints. Ideally, we would like to have access to human capital stock measures but unfortunately it is very difficult to construct such numbers. By many accounts the best available dataset used to measure human capital across countries is the educational attainment dataset by Barro and Lee (1993). Bils and Klenow (1996) and Jones (1996, 1997), among others, propose that in growth models the average educational attainment data may best be interpreted as investment rates. The main argument of these papers is that years of schooling must be constant along the balanced-growth path, rather than growing with the economy. According to this view, it is crucial that empirical growth studies use this interpretation of human capital, otherwise the models' predictions may be misleading.[1]

Whether a proponent of the "stock" view or the "flow" view of human capital, it is evident that we ought to search for human capital specifications that within the context of a calibrated model deliver levels of schooling that are comparable to those we observe in the data. In other words, it is important that in the context of our theories of economic growth, we employ appropriate specifications of human capital to which a measure of average educational attainment successfully corresponds.

In an updated version of their 1996 paper, BK present a finite-lived agent model of schooling and economic growth that is capable in producing levels of educational attainment that comply with the data. However, if we interpret average years of schooling as an investment variable, a finite

[1]For an extensive discussion of this argument see Jones (1996).

horizon problem does not seem to be appropriate. If the life span of agents is finite, the rate of return to schooling attains its maximum level at time zero. As the agent grows older, returns to schooling decrease as the number of periods left to work decrease. The fraction of time allocated to education will never remain constant; thus vanishing the main appeal of interpreting average educational attainment as a flow. The models investigated in the remainder of the paper will be in the infinite horizon class. Indeed, one of the models that we will examine in this paper is a modified version of the BK model.

3 The Basic Model

Interpreting an individual's time endowment spent in school as an investment rate seems, at first pass, to be appealing and worthy of further investigation. As such, the aim of this section is to investigate the implications of a standard two-sector growth model, following the human capital specification used in Jones (1997). We focus our attention on the standard neoclassical growth model although it is important to note that the results we obtain hold for the richer in structure R&D-based models.

3.1 Economic Environment

The economy contains households and firms. Each household consists of identical agents that are involved in two types of activities: consumption goods production and human capital investment. Population growth is in this economy exogenous and equal to n. Agents are endowed with one unit of time. They invest a portion of their time to acquire education, and inelastically supply the rest as labor input to the final-goods production. Since the decentralized and centralized problems have the same equilibrium outcomes, we omit further details of the decentralized setup, and consider the central planner's problem.

The model economy is characterized by the following two equations. First, the aggregate production function

$$Y(t) = K(t)^{1-\alpha} \left[A(t)\, H_Y(t)\right]^{\alpha}, \quad 0 < \alpha < 1\,; \tag{1}$$

where $Y(t)$ is output at period t; K is the stock of physical capital; A is the exogenously growing labor-augmenting technology; H_Y is skilled-labor input allocated in final output production; and $(1-\alpha)$ is the share of capital stock.

Second, we have the schooling equation that determines the way by which human capital is formed, and embodied into labor. Skilled-labor input is given as in Jones (1997)

$$H_Y(t) = e^{f(l_h(t))}\, l_y(t)\; L(t)\,; \tag{2}$$

where l_y and l_h are the fractions of the agent's time endowment allocated to final-goods production and human capital formation, respectively; and L is the population size.

Let N be the time endowment of individuals in years. Mapping the population's average years of schooling S into the flow variable l_h requires that

$$l_h^* = \frac{S^*}{N}; \tag{3}$$

where the $(*)$ denotes steady-state values. It follows that the ratio of $f'(l_h)$ to N reflects the returns to schooling estimated in a Mincerian wage regression: an additional year of schooling raises a worker's efficiency by $\frac{f'(l_h)}{N} \times 100$ percent.[2]

3.2 Social planner's problem

Let lower case letters indicate variables normalized by the size of population. Denote by the lower case letter c the amount of per capita consumption. A central planner would choose the path $\{k(t), c(t), l_h(t)\}_{t=0}^{\infty}$ to maximize the lifetime utility of the representative consumer subject to the feasibility constraints of the economy. The problem is stated as follows:

$$\max_{\{k,c,l_h\}} \int_0^\infty e^{-\rho t}\, \frac{c(t)^{1-\theta}-1}{1-\theta}\, dt \tag{4}$$

subject to,

$$y = k^{1-\alpha}\left[A\, e^{f(l_h)}\, l_y\right]^{\alpha} \tag{5}$$

$$\dot{k} = y - c - (n+\delta)\, k \tag{6}$$

$$l_y + l_h = 1 \tag{7}$$

[2]The Mincerian interpretation of human capital is originated by Bils and Klenow (1996) and is based on what Heckman and Klenow (1997) call the "Macro-Mincer" wage equation:

$$\log W_{it}^g = \beta_0 + \beta_1 S_{it} + \varepsilon_{it};$$

where W_{it}^g is the geometric mean wage for country i at time t; S is mean education; and ε is a random error. This interpretation has been adopted by Jones (1996, 1997, 1998), Jovanovic and Rob (1998), and Hall and Jones (1999), among others. Empirical investigation of the effects of human capital on economic growth based on the "Macro-Mincer" earnings equation is carried out by Krueger and Lindahl (1998), and Topel (1999).

$$\dot{L} = n\,L \tag{8}$$

$$L_0,\ k_0,\ \text{given;}$$

where, ρ is the discount factor; θ is the inverse of the intertemporal elasticity of substitution; δ is the depreciation rate of capital; and L_0, k_0, are the initial levels of labor and physical capital per capita, respectively. Equation (6) is the standard law of motion of the stock of per capita physical capital combined with the individual's budget constraint.

The first order conditions (FOCs) for the interior solution obtain the optimal share of time allocated in schooling as

$$\begin{aligned} l_h^* &= 1 - \frac{1}{f'\left(l_h^*\right)}, \quad \text{if } f'\left(l_h^*\right) > 1; \\ &= 0, \quad \text{otherwise.} \end{aligned} \tag{9}$$

At the margin, the cost of investing one more unit of labor in education must equal its benefit, which is the increase in effective labor from additional schooling in the output sector.[3]

The above equation implies that human capital investment will not occur beyond some value of l_h if the returns to schooling are not sufficiently large. Final-good investments, on the other hand, will always take place because its marginal productivity goes to infinity as its allocation falls to zero.[4]

3.3 Calibrating the model at steady state

One way to evaluate a model is to formally calibrate its parameters, simulate its dynamics, and compare its prediction to the data (as suggested by Klenow and Rodríguez-Clare, 1997). When we introduce a human capital technology in a growth model, we hope to at least be able to pin

[3] In a richer model with monopolistic competition and an R&D sector we specify a law of motion for technology, $\dot{A} = \mu A^{\phi}\left(e^{f(l_h)}\,l_A\,L\right)^{\lambda}\left(\frac{A^w}{A}\right)^{\psi}$, where $H_A = e^{f(l_h)}\,l_A\,L$ is the skilled-labor input employed in R&D; A^w is the stock of existing technology in the world, which grows at an exogenous rate g_{A^w}; μ is a parameter that determines the rate by which a new variety arrives; ϕ is a positive externality due to the stock of existing technology; λ is a negative externality due to duplication of effort; and ψ is a technology gap parameter. In this extended model our equation (2) becomes $H_X\left(t\right) = e^{f(l_h(t))} l_X\left(t\right) L\left(t\right)$, where l_X and l_h are the fractions of the agent's time endowment allocated to productive activity X ($\forall\ X = Y,\ A$) and human capital formation, respectively. Solving the optimal problem results in $g_A^* = \frac{\lambda n}{1-\phi}$ as in Jones (1995). Along the balanced-growth path, the amount of time allocated to schooling and R&D is $l_h^* = 1 - \frac{1}{f'\left(l_h^*\right)}$ and $l_A^* = \frac{1-l_h^*}{\frac{1}{\lambda g_A^*}\left[r^* - n - (\phi-\psi)g_A^*\right]+1}$ respectively. As mentioned previously, the R&D-based model's predictions about educational attainment are the same as those of the neoclassical model.

[4] In order to save space, we hereafter omit the corner solution in the expressions determining the optimal share of labor in schooling.

down the human capital measure that corresponds to the data. Accordingly, we examine our basic model's steady-state predictions regarding the schooling variable, l_h^*.

Recall that the Mincerian coefficient (denoted by β) equals $\frac{f'(l_h)}{N}$. The schooling decision given by equation (9) is then exclusively determined by the Mincerian returns to education. To establish whether the model's estimates comply with the data, we need to pick values for β and N. Following Hall and Jones (1999), we employ Psacharopoulos (1994) estimates of Mincerian returns to education:[5]

$\beta_1 = 0.068,$	for $S > 8$
$\beta_2 = 0.101,$	for $S \in [4, 8]$
$\beta_3 = 0.134,$	for $S \in (0, 4)$

Psacharopoulos' (1994) work suggests that $f''(l_h) < 0$. We choose the average "productive" life of workers to be 54 years. This number is obtained from subtracting 6 years (what might be thought of as a pre-schooling period) from 60 years (which is our assumed retirement age). Substituting our chosen values for β and N in equation (9) we obtain the following predictions given in table 1:

Table 1: Estimates of Calibration Exercise in the Basic Model

β	N	l^*_h	S^*
0.068	54	0.73	39.4
0.101	54	0.82	44.3
0.134	54	0.86	46.4

Table 1 reports that for the lower bound of $\beta_1 = 0.068$, $S^* = 39.4$ years. For the world average $\beta_2 = 0.101$, and high bound $\beta_3 = 0.134$ the average years of education S^*, implied by the basic model reach 44.3 years and 46.4 years, respectively. These estimates are implausibly high to believe. In Psacharopoulos (1994), for example, the country with the maximum average educational attainment is the U.S. with $S = 13.6$ years. This simple exercise suggests that the specification of equation (2) is too simple and it fails to predict educational attainment levels consistent with casual observation.

[5]The Mincerian coefficient β_1, corresponds to the OECD. β_2 corresponds to the average of all countries in Psacharopoulos' sample, and is the estimate employed by Jones (1997). β_3 is the average estimate in Sub-Sahara African countries. Bils and Klenow (1996) estimates of the average returns to schooling fall into the $5\% - 15\%$ range. Using Bils and Klenow's bounds do not change our results qualitatively.

3.4 Substitutability between skilled and unskilled labor

In this subsection we extend the human capital technology given by equation (2) in the basic model by relaxing the assumption that raw and skilled labor are perfect substitutes. We then calibrate the model and once again ask whether such a specification works.[6] In particular we examine a nested Constant Elasticity of Substitution (CES) aggregate output specification as follows:

$$Y = K^{1-\alpha}\left[A\left(zL_u^{\rho} + (1-z)\left[e^{f(l_h)}(1-l_h)L_s\right]^{\rho}\right)^{1/\rho}\right]^{\alpha},\ -\infty < \rho < 1; \tag{10}$$

where z is what Arrow et al. (1961) refer to as the *distribution parameter*; L_u is the number of unskilled workers; L_s is the number of skilled workers, who allocate a fraction of their time l_h to schooling; and $\sigma = \frac{1}{1-\rho}$ is the elasticity of substitution between skilled and unskilled labor. In the production function given by equation (10), skilled and unskilled workers are combined into an aggregate by a CES specification. The resulting labor aggregate is then combined with the stock of physical capital by a Cobb-Douglas technology. Dividing both sides of equation (10) by the total number of workers $L = L_u + L_s$ and denoting $\frac{L_u}{L} = l$ (the portion of unskilled-labor in production) gives

$$y = k^{1-\alpha}\left[A\left(zl^{\rho} + (1-z)\left[e^{f(l_h)}(1-l_h)(1-l)\right]^{\rho}\right)^{1/\rho}\right]^{\alpha}. \tag{11}$$

The social planner's problem can then be stated as

$$\max_{\{k,c,l_h,l\}} \int_0^{\infty} e^{-\rho t}\frac{c(t)^{1-\theta}-1}{1-\theta}dt$$

subject to,

$$y = k^{1-\alpha}\left[A\left(zl^{\rho} + (1-z)\left[e^{f(l_h)}(1-l_h)(1-l)\right]^{\rho}\right)^{1/\rho}\right]^{\alpha}$$

$$\dot{k} = y - c - (n+\delta)k$$

$$\dot{L} = nL$$

$$L_0,\ k_0,\ \text{given.}$$

At steady state, the FOCs for the interior solution imply the following optimal allocations:

$$l_h^* = 1 - \frac{1}{f'(l_h^*)} \tag{12}$$

[6]We thank a anonymous referee who suggested this alternative specification.

and

$$l^* = \frac{1}{\left(\frac{z}{1-z}\right)^{\frac{1}{\rho-1}} \left[e^{f(l_h^*)}(1-l_h^*)\right]^{\frac{\rho}{1-\rho}} + 1}. \tag{13}$$

Notice that equation (12) is identical to equation (9). Comparing production functions (11) and (5), we can deduce the reason. Let x be the marginal cost of investing in education. In both cases, the schooling marginal benefit equals $x * f'(l_h)(1-l_h)$. That is, cost and benefit change proportionally when we introduce unskilled labor. Hence, the optimal value of l_h does not vary.

In addition to the aggregate specification of equation (10) we have also experimented with the specification proposed by Stokey (1996). Stokey's aggregate production technology is consistent with the capital-skill complementarity hypothesis advanced by Griliches (1969) and supported by many researchers described in Hamermesh (1986). In particular, we have examined the model's predictions under the specification

$$Y = A\left[\zeta K^\rho + (1-\zeta)L_u^\rho\right]^{\alpha/\rho}\left[e^{f(l_h)}(1-l_h)L_s + \xi L_u\right]^{1-\alpha}; \tag{14}$$

where ζ is the distribution parameter; A is a positive technology parameter; and ξ is the relative efficiency of unskilled labor in supplying mental effort. Interestingly, we find that the optimal share of schooling is once again given by equation $l_h^* = 1 - \frac{1}{f'(l_h^*)}$, which implies implausibly high estimates.

The main finding here is that the predictions of the calibrated CES model regarding educational attainment remain implausibly high.

4 A Modified Version of the BK Model

Bils and Klenow (1998) (BK) proposed a human capital accumulation technology that incorporates two main sources of knowledge acquisition: schooling, and work experience. In their model, individuals live for N periods of which the first s periods are spent in schooling. BK's human capital accumulation problem is equivalent to one in which agents decide the fraction of their life-span to be allocated in schooling. As discussed in section 2, if we interpret average years of schooling as an investment variable, a finite-horizon problem does not seem to be appropriate.

Following BK we incorporate work experience and a human capital externality in the basic model of section 3.[7] At date t, our human capital specification $h(t)$ is now given as

$$h(t) = \overline{h}(t)^{\phi} e^{f(l_h(t))+g(1-l_h(t))}, \; 0 \leq \phi < 1; \tag{15}$$

where $\overline{h}(t)$ is the average human capital level in the economy, which allows for a positive external effect; and $g\,(1 - l_h(t))$ represents the increase in human capital due to work experience. The central planner takes into account the existence of the externality and equalizes $h(t) = \overline{h}(t)$. At the aggregate level, effective labor is then given by

$$H_Y(t) = h(t) l_y(t) L(t) = e^{[f(l_h(t))+g(1-l_h(t))]\frac{1}{1-\phi}} \left(1 - l_h(t)\right) L(t).$$

4.1 Social Planner's Problem

The planner's problem can now be stated as

$$\max_{k,c,l_h,l} \int_0^{\infty} e^{-\rho t} \frac{c(t)^{1-\theta} - 1}{1-\theta} \, dt$$

subject to,

$$y = k^{1-\alpha} \left[A \, e^{[f(l_h)+g(1-l_h)]\frac{1}{1-\phi}} \left(1 - l_h\right) \right]^{\alpha}$$

$$\dot{k} = y - c - (n+\delta)k$$

$$\dot{L} = nL$$

$$L_0, \; k_0, \; \text{given.}$$

The FOCs for the interior solution imply the optimal share of average time endowment in education as

$$l_h^* = 1 - \frac{1-\phi}{f'(l_h^*) - g'(1-l_h^*)}. \tag{16}$$

Notice that the optimal allocation of average time endowment in education, l_h^*, is different from that of the basic model given in equation (9). First, the extra parameter ϕ appears in the numerator of the ratio in equation (16) that is due to the human capital externality. Second, the additional term $g'(1 - l_h^*)$ appears in the denominator that is due to work experience.

[7] In addition, BK allow human capital to enter in the utility function through a positive flow utility from going to school. This assumption is supported by Denison (1963) who argue that an individual is happier from going to school than working. In the model we investigate below we deviate from this assumption because estimates of such flow utility are not well-established in the literature.

4.2 Calibrating the model at steady state

In equation (16), the steady-state allocation to schooling increases with ϕ because the external effect raises the marginal benefit from education. The introduction of an externality in the human capital technology is not therefore sufficient to reproduce the evidence – it generates the opposite effect of what the model needs.

What about if we introduce only experience? Assume that $\phi = 0$. Once again $f'(l_h^*) = \beta N$, and equivalently $g'(1 - l_h^*) = \gamma N$; where γ is the Mincerian return to experience. We assign to γ the BK's estimate of 0.0512.[8] The predicted values of average years in schooling are given in table 2.

Table 2: Estimates of Calibration Exercise in the BK Specification with Experience and $\phi = 0$

$\boldsymbol{\beta}$	**N**	**l^*_h**	**S^***
0.068	54	–	–
0.101	54	0.63	33.9
0.134	54	0.78	41.9

Evidently, including experience in our basic model is not sufficient either to reproduce the evidence. For $\beta = 0.068$ (the Mincerian coefficient for the OECD countries) the predicted values of l_h^* and S^* are negative; clearly an inadmissible result. The reason is that in the most advanced countries the returns to experience are very close to the returns to schooling; investing in education then becomes too costly. For $\beta = 0.101, 0.134$ the predicted values of l_h^* and S^* are implausibly high.

Table 3 reports predictions when in addition to experience we allow a positive human capital externality, $\phi = 0.3$. For $\beta = 0.068$, the predicted value of S is 12.4, which is consistent with the evidence.[9]

We conclude that under the flow approach, the introduction of *both* work experience and human capital externality is necessary to generate plausible predictions of the steady-state average educational attainment. The difference with respect to the cases previously considered is that we

[8] BK's choice for parameter γ reflects the average estimates of the Mincerian returns to experience across 52 countries. For more discussion on these estimates see Bils and Klenow (1998, pp.14-15 and their appendix B).

[9] BK suggest an upper bound of $\phi = 0.23$. Sensitiviy analysis of the paremeter N shows that in the case where $\beta = 0.068$ and $N = 60$ which is also sensible, a value of $\phi = 0.2$ is sufficient to obtain $S^* = 12.4$.

Table 3: Estimates of Calibration Exercise in the BK Specification with Experience and $\phi = 0.3$

β	N	l^*_h	S^*
0.068	54	0.23	12.4
0.101	54	0.74	39.9
0.134	54	0.84	45.5

can now separately vary the marginal cost (through the experience parameter) and the marginal benefit (through the externality parameter) of education investment.

5 An Alternative Model of Growth and Human Capital

In this section, we offer an alternative human capital specification that is consistent with the traditional view that human capital is a stock. It does not admit additional variables, such as work experience, and still delivers educational attainment levels that are consistent with those in the data.

The proposed specification provides a map between educational investment and human capital accumulation through a law of motion of average years of schooling.

Final output production is again given by equation (1). Human capital per capita is now expressed as

$$h(t) = e^{f(S(t))}. \tag{17}$$

The derivative $f'(S(t))$ represents the returns to schooling estimated in a Mincerian wage regression. The law of motion of the average educational attainment is equivalent to the one of physical capital. We must take into account that agents invest a fraction l_h of their time in education, and that the population grows at rate n. At date t, the evolution of S over time is then given as follows:

$$\dot{S}(t) = l_h(t) - n\, S(t). \tag{18}$$

5.1 Social Planner's Problem

The optimal problem is the following:

$$\max_{\{k,c,l_h,S\}} \int_0^\infty e^{-\rho t} \, \frac{c(t)^{1-\theta} - 1}{1-\theta} \, dt$$

subject to,

$$y = k^{1-\alpha} \left[A\, e^{f(S)}\, l_y \right]^{\alpha}$$

$$\dot{k} = y - c - (n + \delta)\, k$$

$$\dot{S} = l_h - n\, S$$

$$l_y + l_h = 1$$

$$\dot{L} = n\, L$$

$$L_0,\ k_0,\ S_0,\ \text{given.}$$

We construct the Hamiltonian, and get the FOCs for the interior solution. After some algebra we show that the Euler equation that characterizes the optimal allocation of labor to human capital investment is

$$f'(S)\,(1 - l_h) + \left[\frac{\dot{y}}{y} + \frac{\dot{l}_h}{1 - l_h} \right] = r; \tag{19}$$

where r is the interest rate that is given by

$$r = (1 - \alpha)\frac{y}{k} - \delta = \rho + \theta\, \frac{\dot{c}}{c} + n. \tag{20}$$

Equation (20) is the standard Euler condition for consumption.

Expression (19) can be interpreted as an arbitrage condition. Notice that the marginal cost of increasing the average educational attainment level equals the marginal productivity of labor in output production, $\frac{\alpha\, y}{1-l_h}$; this is also the shadow price of having additional units of education. The LHS of equation (19) is the returns to sacrificing one unit of output for acquiring schooling. The first term is the dividend due to the increase in effective labor. The term in brackets represents the capital gain/loss, which equals the percentage change in the price of education units. The RHS of equation (19) in turn captures the opportunity cost of schooling investment, given by the interest rate. In equilibrium, both sides must be equalized.

Let g_v be the steady-state growth rate of variable v. Along the balanced growth path $g_y = g_c$, and $\dot{l}_h = 0$. Euler equations (19) and (20) then imply that the optimal share of labor in schooling is

$$l_h^{*} = 1 - \left[\frac{n + \rho + (\theta - 1)\ g_y}{f'(S^{*})} \right]. \tag{21}$$

5.2 Calibrating the model at steady state

We pick standard values for the parameters $g_y = 0.02$, $\rho = 0.04$, $n = 0.016$, and $\theta \in [1, 3.5]$, and use equation (21) to generate values for l_h^* and S^*.[10] Tables 4 and 5 present estimates of l_h^* and S^* for different values of θ and β. The predictions of table 4 do not assume any explicit form for $f(S)$, taking $f'(S) = \beta$ as in previous sections. The estimates of table 5 follow BK and assume that $f(S(t)) = \eta S_t^{\varphi}$, where $\beta > 0$, and $0 < \varphi \leq 1$; three pairs for η and φ, $\{0.18, 0.72\}$, $\{0.32, 0.42\}$ and $\{0.099, 1\}$, are considered.[11]

Table 4: This exercise does not assume an explicit function for $f'(S)$.

β	θ	l^*_h	S^*	β	θ	l^*_h	S^*	β	θ	l^*_h	S^*
0.068	1	0.18	11.0	0.068	2.3	–	–	0.068	3	–	–
0.101	1	0.45	27.9	0.101	2.3	0.19	11.8	0.101	3	0, 05	3.1
0.134	1	0.58	36.4	0.134	2.3	0.39	24.2	0.134	3	0.28	17.8

Table 5: Estimates of Calibration Exercise in the Proposed Model with $f'(S)$ from BK

φ	η	θ	l^*_h	S^*	φ	η	θ	l^*_h	S^*	φ	η	θ	l^*_h	S^*
0.42	0.32	1	0.21	13.3	0.42	0.32	2.1	0.14	8.8	0.42	0.32	3	0.11	6.6
0.72	0.18	1	0.30	18.5	0.72	0.18	2.1	0.17	10.4	0.72	0.18	3	0.10	6.4
1	0.99	1	0.43	27.1	1	0.99	2.1	0.21	13.3	1	0.99	3	0.03	1.9

In the first calibration exercise (table 4) in which we assume that $f'(S) = \beta$, we find for example that for $\beta = 0.068$ and $\theta = 1$ the calibrated share of labor in schooling is $l_h^* = 0.18$, and the resulting mean years of schooling is 11. In the case where $\beta = 0.101$ (the world average value) and $\theta = 2.3$, the model predicts that $l_h^* = 0.19$, and the mean years of schooling is 11.8. These estimates agree with the data.

[10] We set the per capita output growth rate g_y, to 2% which is the approximate post-war per capita output growth rate for the U.S. We also set the growth rate of population n, to match the average labor force growth rate in the U.S. during the period 1950-1980. Estimates of the inverse of the intertemporal elasticity of substitution θ, are taken from Hall (1998), and Attanasio and Weber (1993).

[11] BK regress country estimates of Mincerian returns on country schooling levels in Psacharopoulos (1994) 56-country sample. They obtain the estimate $1 - \varphi = 0.58$ which implies diminishing Mincerian returns to schooling across nations. They also consider two other values; $1 - \varphi = 0.28$ (their point estimate minus two standard errors) and $1 - \varphi = 0$ (no diminishing returns). For each value of φ they set the value η so that the mean of $f'(S) = \eta\varphi S^{\varphi-1}$ equals the mean Mincerian return across Psacharopoulos' sample of 56 countries, which is 0.099.

Turning to the second calibration exercise (table 5) that assumes an explicit form for $f(S)$ following BK, we take advantage of the relationship between l_h^* and S^* provided now by the model. In particular, because at steady state l_h remains constant, the motion equation (18) implies that so does S^*, and that

$$S^* = \frac{l_h^*}{n}. \tag{22}$$

Equation (21) becomes

$$l_h^* = 1 - \left[\frac{n + \rho + (\theta - 1)\, g_y}{f'\left(\frac{l_h^*}{n}\right)} \right]. \tag{23}$$

It is easy to show that equation (23) has a unique root but does not have an analytic solution when $0 < \varphi < 1$; we therefore use numerical approximation methods to obtain solutions. We find that for the value of $\theta = 2.1$ all three pairs of $\{\eta, \varphi\}$ obtain plausible values for l_h^* and S^*. In particular, in the case where $\{\eta = 0.099, \varphi = 1, \theta = 2.1\}$ the calibrated value for S^* is 13.3 which is very close to the value reported in Psacharopoulos (1994) for the U.S. ($S_{U.S.}^* = 13.6$ years). The triple $\{\eta = 0.32, \varphi = 0.42, \theta = 1\}$ also delivers the same value. When we set $\theta = 3$, the predicted values for l_h^* are on the other hand too low.

The main finding here is that our calibrated model is able to deliver levels of educational attainment consistent to those in the data. In other words, for values of $\theta \in [1, 3]$ there is a value of $f'(S)$ (or a pair of η and φ) that provides educational attainment levels that are consistent with those in the data (i.e. $S^* \in [8, 14]$).[12]

The model presented above stays well within the neoclassical growth framework and is simpler than that of the modified BK (presented in the previous section) model that requires work experience and human capital externalities. This makes our specification easier to incorporate in existing theoretical growth models and easier to adopt in growth accounting exercises. In addition, our calibrated model requires only parameter values that are well-established in the literature, such as ρ, θ, g_y, and n.

6 Conclusion

This paper has searched for a specification of human capital that can match existing data on educational attainment with economic growth models. We have first examined a simple two-sector

[12] A sensitivity analysis on the parameters g_y, ρ and n show that our results do not change significantly.

growth model, following the flow interpretation of schooling used in recent papers. Calibrating the basic model reveals several caveats of the human capital specification, and casts doubts in its use in theoretical and empirical work. In particular the model predicts that the optimally chosen educational attainment is between 39 and 46 years, whereas the actual levels we see in the data are between 8 and 14 years. An extended specification that employed a CES specification to aggregate skilled and unskilled labor has been also examined without any success.

We have then followed BK by incorporating work experience in addition to schooling in the basic model. Our main result is that introducing experience alone is not sufficient to reproduce the data. However, by introducing experience together with a human capital externality in the model, successfully generates plausible predictions on educational attainment.

Finally, we have presented an alternative specification of human capital that incorporates an explicit law of motion of the mean years of education. Our model differs from that of BK who, given the scope of their paper, suggest a model of schooling and growth with finite-lived agents. The proposed model is a standard infinite-horizon neoclassical growth model that uses standard parameters found in the literature. Our model differs also from the modified BK model examined in this paper in that it does not admit experience or human capital externalities. Simple calibration exercises of the model at steady state reveal that the proposed specification of human capital is successful in replicating the observed data on educational attainment.

In addition, our alternative human capital specification is more appropriate to perform off-steady-state analysis. It is easy to show that, outside the balanced growth path, enrollment rates and the average educational attainment are negatively correlated. More important, this is actually the relationship that dominates the data (see Pritchett, 1997, pp. 27-30).

The conclusions of the paper for future research are twofold. First, incorporating the proposed human capital specification into an R&D-based model and examining the transitional dynamic properties (rather than the steady-state properties) of the model is a promising next step. Second, using the reduced form equation implied by the modified specification, we can reexamine accounting exercises, such as these of Klenow and Rodríguez-Clare (1997) and Hall and Jones (1999).

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