EQUALITY OF OPPORTUNITIES: CARDINALITY-BASED CRITERIA*

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Abstract

In this paper we study possible rankings of opportunity profiles. An opportunity profile is a list of sets of alternative opportunities, one set for each agent in the society. We compare such opportunity profiles on the basis of the notion of "equality of opportunities". Our main results show the necessary and sufficient conditions for this comparison to be made using exclusively the information provided by two cardinal measures: the number of common alternatives for all sets in a given profile and/or the difference between the number of alternatives of individual sets. We also show that, under given circumstances, the only way to solve conflicts between these two numbers is to combine them in a lexicographic procedure.

All the proofs, which are quite tedious, are available upon request; corresponding author:

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1. INTRODUCTION.

There have been in recent times several attempts to study the notion of "equality of opportunities" through comparisons of the opportunity sets available to the members of a given society. That is, a social situation is represented by a profile of opportunity sets, one for each individual in that society. Our aim is to find reasonable ways of comparing any two social situations by looking at the two induced opportunity profiles. Our main concern will be the notion of equality of opportunities within this general framework.

The nature of the elements within each set of opportunities is open to a variety of interpretations. By the standards of economic theory, they can be understood as available consumption baskets for the consumer or feasible production plans for the producer. But they may also be interpreted, for example, as different primary goods as in Rawls (1971), different *functionings* as in Sen (1985), or as basic liberties and civil or political rights of the population as in Ok (1997)**.**

We do not adscribe this work to any particular interpretation of the alternatives, but it may be of some help to look upon them as any kind of mutually exclusive opportunities, one of which will be the agent's final choice. This is an interpretation assumed by many works in the literature on "freedom of choice" and "preference for flexibility" (see for example, Arlegi and Nieto (1997), Bossert, Pattanaik and Xu (1994), Gravel (1998), Kreps (1979), Neme, Nieto y Quintas (1996), Pattanaik and Xu (1990), Puppe (1996) or Sen (1991) among others). This interpretation is also present in other works devoted, like this one, to the notion of equality of opportunities, such as Bossert, Fleurbaey and Van de gaer (1996).

Some authors, such as Thomson (1994) or Kranich (1995), propose the study of equality of opportunities on the basis of individual preferences over the alternatives included in opportunity sets. But, on the other hand, there could be a problem of "adaptative preferences" posed by Elster (1982).

Bossert, Fleurbaey and Van de Gaer (1996), and Herrero (1997) start with the assumption of a social ranking, R, on the alternatives, and the goal is to obtain rankings of profiles on the basis of that social ranking R. In Herrero, Iturbe-Ormaetxe and Nieto (1998) this social ranking, R, is induced from a ranking on "uniform profiles"; that is, profiles where every individual has the same opportunity set. In this work we follow the approach of Kranich (1996), Herrero, Iturbe-Ormaetxe and Nieto (1995), and also Ok and Kranich (1998), in which the opportunity sets are evaluated only by the number of alternatives in each set (its cardinality). Our main concern will be the notion of equality of opportunities, leaving aside other considerations such as Pareto efficiency, for instance. This idea will be reflected by the axioms which will be presented as desirable properties of the social preference relation over the opportunity profiles.

Among the rules based on the use of sets cardinality, there are two main principles to evaluate equality in opportunity profiles. One takes into account the difference between the opportunity sets of the richest and the poorest members of society. The second one is based on the similarity between individual opportunity sets. According to the first principle, if the number of opportunities is more or less similar across individuals, then we say that there is more equality of opportunities. According to the second principle, we would like to focus on the number of opportunities commonly shared by all the individuals, and this number would be a reasonable measure of the equality of opportunities of a given profile.

These two principles have been translated into two criteria for ranking opportunity profiles. One seeks to minimize differences between the number of opportunities as in Kranich (1996), and the other to maximize the cardinality of the intersection of all individual opportunity sets as in Herrero, Iturbe-Ormaetxe and Nieto (1995).

Kranich (1996) shows several examples in which the situation improves according to the difference criterion only at the cost of worsening (reducing) the common opportunities. A very relevant question is then how to combine the two cardinality-based criteria. One aim of this paper is to provide an answer to the question of what the reasonable trade-off will be between the two rules.

The plan of the paper goes as follows: In Section 2 we introduce notation and definitions. Section 3 is devoted to the axiomatic structure of the two cardinality-based criteria. In Section 4 we present some examples of paradoxical results from the isolated use of both criteria. We find that, under some given assumptions, the only way to solve such paradoxes is by mixing the two numerical criteria in a lexicographic procedure. All the results corresponding to sections 1 to 4 have been obtained for the two-person case. In Section 5 we present a possible generalization for the *n*-person case, and Section 6 contains some final conclusions

2. NOTATION AND DEFINITIONS.

Let $N=\{1,\cdots,n\}$ denote the set of agents, and let X be an infinite set of *opportunities.* We denote the set of finite subsets of X by L. An *opportunity set* for agent i is an element $A^i \in L$. We consider *profiles of opportunity sets* of the form $\mathbf{A}=[A^1,\cdots,A^n]\in L^n$.

For $\mathbf{A}=[A^1,\cdots,A^n]\in L^n$, $\sigma(\mathbf{A})$ denotes a permutation of $\aleph$ such that $\#A^{\sigma(1)}\geq\#A^{\sigma(2)}\geq\cdots\geq\#A^{\sigma(n)}$. We will also use the following pieces of notation: $\mathbf{A}^{\cup}=\cup_{i=1}^{n}A^i$; $\mathbf{A}^{\cap}=\cap_{i=1}^{n}A^i$. $\mathbf{A}^{-i}$ will denote the $(n-1)$-dimensional profile $\{A^1,\cdots,A^{i-1},A^{i+1},\cdots,A^n\}$.

Let us denote by $\succcurlyeq\subset L^n\times L^n$ any transitive and reflexive binary relation defined on L^n. As we know, $\succ$ defined for $\mathbf{A},\mathbf{B}\in L$ by $\mathbf{A}\succ\mathbf{B} \Leftrightarrow (\mathbf{A}\succcurlyeq\mathbf{B}$ and $\neg(\mathbf{B}\succcurlyeq\mathbf{A}))$, where $\neg$ denotes logical negation, is the asymmetric part of $\succcurlyeq$, and ~ is an equivalence relation. $\geq_L$ will denote the lexicographic order on $\mathbb{R}^n$.

We investigate preferences over profiles of opportunity sets in terms of *equality of opportunities*. In that sense, for $\mathbf{A},\mathbf{B}\in L^n$ we write $\mathbf{A}\succcurlyeq\mathbf{B}$, meaning the profile $\mathbf{A}$ is more equitable than $\mathbf{B}$. $\succ$ and ~ a read as “at least as equitable as” and “as equitable as” respectively. In Section 3 and 4 we investigate rankings on L^2, and in Section 5 we generalize our result for n agents.

3. CARDINAL DIFFERENCE AND NUMBER OF COMMON OPPORTUNITIES AS EQUALITY FACTORS.

In Kranich (1996) the *Cardinality Difference Relation* on profiles of opportunity sets ($\succsim_{CD}$) is defined by: $\forall \mathbf{A},\mathbf{B}\in L^2$, $\mathbf{A} \succsim_{CD} \mathbf{B}$ if $CD(\mathbf{A})\leq CD(\mathbf{B})$, where $\forall \mathbf{A}\in L^2$, $CD(\mathbf{A})=\left|\# A^1-\# A^2\right|$.

This ordering reflects the principle which identifies equality of opportunities with equality of the number of opportunities, and reaches its maximum when all of the agents enjoy the same number of opportunities.

On the other hand, there exists the idea of identifying equality of opportunities with all agents having the same opportunities; this idea finds a natural representation in the criterion *Common Opportunity Relation*, ($\succsim_C$) which appears in Kranich (1996), Herrero, Iturbe-Ormaetxe and Nieto (1998) and in Bossert, Fleurbaey and Van de Gaer (1996), its definition being: $\mathbf{A} \succsim_C \mathbf{B}$ if $\#\mathbf{A}^{\cap}\geq\#\mathbf{B}^{\cap}$ (in the case n=2, if $\#(A^1\cap A^2)\geq\#(B^1\cap B^2)$).

As said before, $\succsim_{CD}$ and $\succsim_C$ are the two orderings with a cardinality basis. In general, they lead to solutions which are different, and, as we will show later, sometimes mutually contradictory. That is why we want first to focus our attention on those rankings of profiles which are consistent with both rules, and for this we propose the following axioms:

Axioms

ANONYMITY (AN)

$\forall \mathbf{A}=[A^1, A^2]\in L^2$, $\mathbf{A}\sim[A^2, A^1]$

MONOTONICITY (MON)

$\forall \mathbf{A}\in L^2$, $\forall x\in X$, $\sigma(\mathbf{A}) \succcurlyeq [A^{\sigma(1)}, A^{\sigma(2)}\backslash\{x\}]$, with strict preference if $x\in(A^1\cap A^2)$.

ASSIMILATION (ASM)

$\forall \mathbf{A}\in L^2$, $\forall x,y_1,y_2\in X$ s.t. $x\in X\backslash\mathbf{A}^{\cup}$, $y_1\in A^1$ and $y_2\in A^2$, $[A^1\cup\{x\}\backslash\{y_1\}, A^2\cup\{x\}\backslash\{y_2\}] \succcurlyeq \mathbf{A}$

INDEPENDENCE OF DISJOINT EXPANSIONS (IDE)

$\forall \mathbf{A}\in L^2$, $\forall x,y\in X\backslash\mathbf{A}^{\cup}$, $x\neq y$, $[A^1\cup\{x\}, A^2\cup\{y\}]\sim\mathbf{A}$

(AN) is a minimal condition in fairness and distributive justice, and says that it is only the distribution of opportunity sets among agents that matters, not their names. (ASM) appears in Kranich (1996), and implies that the substitution of any two opportunities (one for each individual) in a given profile with a common one, will not cause detriment. (MON) requires that dropping an opportunity from the set of the worst-off, will not increase the equality of opportunities (which strictly decreases when such an opportunity was common to both). (IDE) says that increasing the number of opportunities for all (both) agents with new non-common opportunities will not affect the degree of equality. It is very clear that (IDE) goes against the notion of Pareto efficiency, but makes sense from a strictly egalitarian approach. (IDE) implies the property of “Replacing” (REP) used in Herrero, Iturbe–Ormaetxe and Nieto (1998): $\forall \mathbf{A}\in L^2$, $\forall x,y\in X$ such that $x\notin\mathbf{A}^{\cup}$, $y\in A^1\backslash A^2$, then $[A^1\cup\{x\}\backslash\{y\},A^2]\sim\mathbf{A}$.

Proof: Let $z\in X\backslash\mathbf{A}^{\cup}$ s.t. $z\neq x,y$. By (IDE) $[A^1\cup\{x\}\backslash\{y\},A^2]\sim[(A^1\cup\{x\}\backslash\{y\})\cup\{y\},A^2\cup\{z\}]$, and also by (IDE) $\mathbf{A}\sim[A^1\cup\{x\}, A^2\cup\{z\}]$. Then by transitivity $\mathbf{A}\sim[A^1\cup\{x\}\backslash\{y\}, A^2]$.

Notice that the inverse implication is not true: let $\succsim$ defined by: **A** $\succsim$ **B** $\Leftrightarrow$ $\#\mathbf{A}^{\cup}\geq\#\mathbf{B}^{\cup}$. This relation is transitive, and satisfies (REP), but not (IDE).

A Characterization Theorem

THEOREM 1. Let $\succsim$ be a complete ordering defined on L^2; then $\succsim$ satisfies (AN), (MON), (ASM) and (IDE) if and only if:

$\forall \mathbf{A},\mathbf{B}\in L^2$, $[\#\mathbf{A}^{\cap}\geq\#\mathbf{B}^{\cap}$ and $\mathrm{CD}(\mathbf{A})\leq\mathrm{CD}(\mathbf{B})] \rightarrow \mathbf{A} \succsim \mathbf{B}$ and

$[\#\mathbf{A}^{\cap}>\#\mathbf{B}^{\cap}$ and $\mathrm{CD}(\mathbf{A})<\mathrm{CD}(\mathbf{B})] \rightarrow \mathbf{A} \succ \mathbf{B}$

(and hence $[\#\mathbf{A}^{\cap}=\#\mathbf{B}^{\cap}$ and $\mathrm{CD}(\mathbf{A})=\mathrm{CD}(\mathbf{B})] \rightarrow \mathbf{A}\sim\mathbf{B}$).

(Proof of Theorem 1 available upon request).

Theorem 1 states that under the axiomatic conditions, the ranking will always verify the dominance principle described by the two implications of the theorem. In other words, the proposed axiomatic structure constrains the rankings to those based on the common opportunities cardinal and/or the cardinal difference, weighting up the first factor positively and the second one negatively. This implies, for example, that no other factor apart from these two, such as the total sum of opportunities, is determinating in evaluating the equality of opportunities. The class of orderings characterized in Theorem 1 represents a reasonable set of rules in a context of cardinal evaluation of the opportunities.

Indeed, $\succsim_{CD}$ and $\succsim_{\cap}$ are members of this family of rankings and so are several possible combinations of the two (in a lexicographic procedure, for instance). But there are also in the literature many criteria which are not members of the characterized class. For example, in Herrero, Iturbe-Ormaetxe and Nieto (1995,1998), the following orderings over profiles of opportunities are defined:

Lexmin Cardinality Relation ($\succeq_{lm}$): $\forall \mathbf{A},\mathbf{B} \in L^2$, $\mathbf{A} \succeq_{lm} \mathbf{B}$ if $[A^{\sigma(2)},A^{\sigma(1)}] \geq_L [B^{\sigma(2)},B^{\sigma(1)}]$

Maxmin Opportunity Relation ($\succeq_{m}$): $\forall \mathbf{A},\mathbf{B} \in L^2$, $\mathbf{A} \succeq_{m} \mathbf{B}$ if $[\#\mathbf{A}^{\cap},\# A^{\sigma(2)}] \geq_L [\#\mathbf{B}^{\cap},\# B^{\sigma(2)}]$

Lexmin Opportunity Utilitarian Relation ($\succeq_{lu}$): $\forall \mathbf{A},\mathbf{B} \in L^2$, $\mathbf{A} \succeq_{lu} \mathbf{B}$ if $[\#\mathbf{A}^{\cap},\#\mathbf{A}^{\cup}] \geq_L [\#\mathbf{B}^{\cap},\#\mathbf{B}^{\cup}]$

Lexmin Opp. Relation ($\succeq_{lo}$): $\forall \mathbf{A},\mathbf{B} \in L^2$, $\mathbf{A} \succeq_{lo} \mathbf{B}$ if $[\#\mathbf{A}^{\cap},\#A^{\sigma(2)},\#A^{\sigma(1)}] \geq_L [\#\mathbf{B}^{\cap},\#B^{\sigma(2)},\#B^{\sigma(1)}]$

Bossert, Fleuerbay and Van de Gaer (1996) also define the following:

Maxmin Cardinality Relation ($\succeq_{min}$): $\forall \mathbf{A},\mathbf{B} \in L^2$, $\mathbf{A} \succeq_{min} \mathbf{B}$ if $\# A^{\sigma(2)} \geq \# B^{\sigma(2)}$

The above orderings combine in different ways notions of equality of opportunities with some Paretian considerations. It is easy to check that all of these orderings fail precisely in the axiom (IDE).

Other interesting orderings of opportunity profiles defined in Kranich (1996), also violate some of the axioms. For example:

Let, $\forall \mathbf{A} \in L^2$, maxd($\mathbf{A}$)=max{#($A^1 \backslash A^2$), #($A^2 \backslash A^1$)}, and mind($\mathbf{A}$)=min{#($A^1 \backslash A^2$), #($A^2 \backslash A^1$)}, we define:

$\mathbf{A} \succeq_{maxd} \mathbf{B}$ iff maxd($\mathbf{A}$)≤maxd($\mathbf{B}$)

$\mathbf{A} \succeq_{mind} \mathbf{B}$ iff mind($\mathbf{A}$)≥mind($\mathbf{B}$)

$\mathbf{A} \succeq_{mm} \mathbf{B}$ iff (mind($\mathbf{A}$),maxd($\mathbf{B}$)) $\geq_L$ (mind($\mathbf{B}$),maxd($\mathbf{A}$))

4. LEXICOGRAPHIC COMBINATIONS

The two previous criteria for ranking opportunity profiles might be the object of criticism. If we take into account only the Cardinality Difference ordering, then there could be two profiles **A,B** such that

$\#A^1 = \#A^2 = \#B^1 = \#B^2$, but $A^1 \neq A^2$ and $B^1 \equiv B^2$. In this example there are reasons to accept profile **B** as being more egalitarian than **A**, which would suggest the need to measure in some way the opportunities shared by all the agents.

On the other hand, the use of the cardinality of the intersection as the unique criterion by which to rank profiles in terms of equality may also lead to some counterintuitive results. Let **A** be the case $\#A^1 = \#A^2 = 2$ and $A^1 \equiv A^2$, then **A** would be indifferent to a profile **B** such that $\#B^1 = 4$, $\#B^2 = 2$, and $B^2 \subset B^1$. This example would have a satisfactory solution taking into account the Cardinality Difference.

The above examples lead us to consider combinations of the two measures as equality rankings. Now we present some possible *combined rankings*, always under the domain of Theorem 1, that is, verifying the axioms in Section 3:

Let us consider the following rankings, $\succsim_1$ and $\succsim_2$, which are lexicographic combinations of the cardinality-based criteria:

$$\forall \mathbf{A},\mathbf{B} \in L^2,\ \mathbf{A} \succsim_1 \mathbf{B} \text{ iff } [\#\mathbf{A}^{\cap}, -\mathrm{CD}(\mathbf{A})] \geq_{\mathrm{L}} [\#\mathbf{B}^{\cap}, -\mathrm{CD}(\mathbf{B})]$$

$$\mathbf{A} \succsim_2 \mathbf{B} \text{ iff } [-\mathrm{CD}(\mathbf{A}), \#\mathbf{A}^{\cap}] \geq_{\mathrm{L}} [-\mathrm{CD}(\mathbf{B}), \#\mathbf{B}^{\cap}]$$

These criteria solve the conflicts between the two measures in a very similar way to $\succsim_{CD}$ and $\succsim_{\cap}$ respectively, since they always prioritize one of the factors over the other. Hence the kind of objections we may find are quite similar: for example, from a profile where both agents had the same opportunity set, if we were to increase one set with an additional however large, X; and the other with a sole element $x \in X$, this modification would improve the profile according to $\succsim_1$, which seems to be counterintuitive. On the other hand, let us imagine a profile with an empty intersection and a big cardinal difference, and let us suppose that we increase the small set with an additional however large set, X, while increasing the bigger opportunity set with an additional set $X \cup \{x\}$: this would produce a worse profile according to $\succeq_{CD}$ and $\succeq$, which in some contexts could also result counterintuitive.

Other non-lexicographic combinations of the two equality factors could be the following ones:

$$\mathbf{A} \succeq_3 \mathbf{B} \leftrightarrow \frac{c(\mathbf{A})+1}{CD(\mathbf{A})+1} \geq \frac{c(\mathbf{B})+1}{CD(\mathbf{B})+1}$$

Let $a,b \in \Re_+$, $\forall \mathbf{A},\mathbf{B} \in L^2$, $\mathbf{A} \succeq_4 \mathbf{B} \leftrightarrow [a(\#\mathbf{A}^{\cap})-b(CD(\mathbf{A})] \geq [a(\#\mathbf{B}^{\cap})-b(CD(\mathbf{B})]$

The first ranking weights up the number of common opportunities while weighting down the cardinal difference, and also nicely relativizes changes in both values when a conflict arises. However, sometimes this ranking can be arbitrary: for example, if we add to the bigger set a new opportunity which is included in the smaller set, this modification could be better (when the number of common opportunities is smaller than the cardinal difference), or worse (in the contrary case).

$\succeq_4$ represents a class of rankings parametrized by *a* and *b*, and combines both measures as a sum. The values of *a* and *b* could, to some extent, reflect the weight given to each measure, and hence, determine the degree to which one prevails over the other (whenever there is no domination). But this measure might also be arbitrary: when we give "*b*/*a*" additional opportunities to the richer agent -if that number is not entire, we take the first whole after "*b*/*a*"- then, the new situation is equally or more egalitarian.

Apart from $\succeq_3$ and $\succeq_4$ it is also possible to think about other non-lexicographic combinations of the two numerical criteria which also are within the domain of Theorem 1, such as the following:

$\mathbf{A} \succeq_5 \mathbf{B}$ if $\mathbf{A} \succeq_{CD} \mathbf{B}$ when $\#\mathbf{A}^{\cap}, \#\mathbf{B}^{\cap} < k$, and $\mathbf{A} \succeq_{c} \mathbf{B}$ if $\#\mathbf{A}^{\cap}$ or $\#\mathbf{B}^{\cap} \geq k$

$\mathbf{A} \succeq_6 \mathbf{B}$ if $\mathbf{A} \succeq_{c} \mathbf{B}$ when $\#\mathbf{A}^{\cap}, \#\mathbf{B}^{\cap} < k$, and $\mathbf{A} \succeq_{CD} \mathbf{B}$ when $\#\mathbf{A}^{\cap}$ or $\#\mathbf{B}^{\cap} \geq k$

(where k is a fixed natural number).

These solutions could be appropriate in certain contexts but they clearly become arbitrary if we wish to apply them as general rules of equality.

In short, if we want to present a rule which combines both principles, it must provide an answer to such questions as: when is a greater number of common opportunities able to compensate big differences in the number of opportunities; at which point can we say that a similar number of opportunities is more equitable, despite a smaller number of common opportunities; and other similar questions. We have seen how none of the proposed criteria can elude having to look for the solution in a specific direction, and how there is always a degree of arbitrariness in the solution they propose; as a consequence, we can always find an objection to each rule. It is, therefore, worth posing the following question: Is it possible to find an overall, coherent and satisfactory solution to the conflicts between the two cardinal measures? We will now propose two axioms which impose additional properties of consistency in order to come closer to answering this question:

CONSISTENCY (C) $\forall \mathbf{A},\mathbf{B}\in L^2$, $\forall x\in X\backslash(A^1\cup B^1)$ such that:

(i): $x\in A^2$ iff $x\in B^2$; and (ii) $\#A^1\geq\#A^2$ iff $\#B^1\geq\#B^2$

then $[A^1, A^2] \succcurlyeq [B^1, B^2] \leftrightarrow [A^1\cup\{x\}, A^2] \succcurlyeq [B^1\cup\{x\}, B^2]$

NON DECISIVENESS (ND) $\forall \mathbf{A}=[A^1, A^2]\in L^2$, $\#A^1\geq\#A^2$,

$\forall C^1,C^2\in L$ such that $C^1\cap A^1=\varnothing$; $C^2\cap A^2=\varnothing$; and $\#C^1>\#C^2$,

$\forall x\in X\backslash(\mathbf{A}^{\cup}\cup C_1\cup C_2)$; then

$$[A^1\cup C_1, A^2\cup C_2,\cdots,A^n\cup C_n] \;\mathbf{A}\; \succcurlyeq [A^1\cup C_1\cup\{x\}, A^2\cup C_2,\cdots, A^n\cup C_n] \;\mathbf{A}\; \succcurlyeq$$

(C) deals with the rule that certain expansions, which are similar for both profiles, leave the previous preference between them unaffected. (ND) imposes a certain coherence in the ordering on opportunity profiles: if an increase in the cardinality difference is not negative from the social viewpoint, then neither is any additional increase with one new opportunity.

Theorem 1 was to show how, under given conditions, no more than the two numerical measures are necessary and sufficient to evaluate the equality of opportunities. The following theorem shows that, under certain consistency conditions, the only way to combine the two measures is through a lexicographic procedure:

THEOREM 2. Let [illegible] on L^2, [illegible] satisfies (AN), (MON), (ASM), (IDE), (C) and (ND) if and only if $\in \{$ $_{CD}$ [illegible] $\}$

(Proof of Theorem 2 available upon request)

Theorem 2 can be interpreted in the following sense: given the meaning of coherency expressed by axioms (C) and (ND), the only coherent way to combine the two cardinal measures is through a lexicographic procedure. Although Theorem 2 is not formally an impossibility theorem, it could also be interpreted in that sense: If we want to preserve the coherence given by axioms (C) and (ND), we must fall into the arbitrariness which underlies the lexicographic solutions, or we must ignore one of the measures.

5. THE *n*-AGENTS CASE

There are many different ways to generalize the two basic cardinal measures used here. Each of them could be more or less appropriate according to the context in which the equality of opportunities is to be analyzed. In the case of the number of common opportunities, one possible formula is the cardinal of the intersection of the *n* sets in the profile (see Herrero, Iturbe-Ormaetxe and Nieto (1998), and Bossert, Fleuerbaey and Van de Gaer (1996)), however, in contexts where these sets tend to be very different, the intersection may very frequently be empty, and the measure may become non-functional. We could then think of other measures, such as the sum of the number of common opportunities, taken two by two; the maximal (minimal) number of common opportunities between two sets; the intersection between the bigger and the smaller set of opportunities, and others.

The cardinality difference is also susceptible to different generalizations. See, for example, Kranich (1996), who provides a mean of the cardinal differences between sets taken two by two. We could also think of another simpler measure, such as the cardinal difference between the bigger and the smaller set.

Obviously, the type of axiomatic generalization is closely linked to the two respective generalized measures that one decides to adopt. We propose here one possible generalization on the basis of the number of the common opportunities in the n sets of the profile; that is, for $\mathbf{A}\in L^n$, we will consider $\mathbf{A}^{\cap}$. On the other hand, we will use the cardinality difference between the bigger and the smaller set in each profile; that is, $\forall\mathbf{A}\in L^n$, we will consider $CD_n(\mathbf{A})=\max(\#A^i)-\min(\#A^i)$. According to this, the definitions of the criteria described in the preceding sections can immediately be extended to the n-agents case, in particular, we will define the orderings $\succsim_{cg}$, $\succsim_{CDg}$, $\succsim_{1g}$ and $\succsim_{2g}$ respectively by:

$\forall\mathbf{A},\mathbf{B}\in L^n$, $\mathbf{A} \succsim_{cg} \mathbf{B}$ iff $\#\mathbf{A}^{\cap}\geq\#\mathbf{B}^{\cap}$

$\forall\mathbf{A},\mathbf{B}\in L^n$, $\mathbf{A} \succsim_{CDg} \mathbf{B}$ iff $CD_n(\mathbf{A})\leq CD_n(\mathbf{B})$

$\forall\mathbf{A},\mathbf{B}\in L^n$, $\mathbf{A} \succsim_{1g} \mathbf{B}$ iff $[\#\mathbf{A}^{\cap},-CD_n(\mathbf{A})]\geq_L[\#\mathbf{B}^{\cap},-CD_n(\mathbf{B})]$

$\forall\mathbf{A},\mathbf{B}\in L^n$, $\mathbf{A} \succsim_{2g} \mathbf{B}$ iff $[-CD_n(\mathbf{A}),\#\mathbf{A}^{\cap}]\geq_L[-CD_n(\mathbf{B}),\#\mathbf{B}^{\cap}]$

Generalized axioms.

ANONYMITY (AN^g)

$\forall\mathbf{A}\in L^n$, $\mathbf{A}\sim\sigma(\mathbf{A})$.

Applying (AN ^{g}), we can suppose without loss of generality that $\forall \mathbf{A} \in L^n$, it is presented in cardinal order, that is $\mathbf{A}=\sigma(\mathbf{A})$. We will do so from this point on.

MONOTONICITY (MON^g)

$\forall \mathbf{A} \in L^n$, $\forall x \in A^i$ $(i \neq 1)$, $\sigma(\mathbf{A}) \succsim [A^{\sigma(1)},\cdots, A^{\sigma(i)}\backslash\{x\},\cdots, A^{\sigma(n)}]$

with strict preference if $x \in \mathbf{A}^{\cap}$ and $i=n$

ASSIMILATION (ASM^g)

$\forall \mathbf{A} \in L^n$, $\mathbf{A}=[A^1, A^2,\cdots, A^n]$, $\forall x \in X\backslash \mathbf{A}^{\cup}$, $\forall y \in A^1 \mathrm{x} A^2 \mathrm{x} \cdots \mathrm{x} A^n$

$[A^1 \cup \{x\}\backslash\{y_1\}, A^2 \cup \{x\}\backslash\{y_2\},\cdots, A^n \cup \{x\}\backslash\{y_n\}] \succsim \mathbf{A}$

INDEPENDENCE OF DISJOINT EXPANSIONS (IDE^g)

$\forall \mathbf{A} \in L^n$ such that $[\#A^1 \geq \#A^2 \geq \cdots \geq \#A^n]$,

$\forall x \in X^n$ such that: (i) $x_i \neq x_j$ $\forall i,j=1,\cdots,n$; (ii): $x_1 \in A^1 \leftrightarrow x_n \in A^n$;

(iii): $\forall i$, $\#(A^1 \cup \{x_1\}) \geq \#(A^i \cup \{x_i\}) \geq \#(A^n \cup \{x_n\})$; and (iv): $\forall i$, $x_i \notin (\mathbf{A}^{-i})^{\cap}$

then $[A^1 \cup \{x_1\}, A^2 \cup \{x_2\},\cdots, A^n \cup \{x_n\}] \sim \mathbf{A}$

CONSISTENCY (C^g)

$\forall \mathbf{A},\mathbf{B} \in L^n$, $\forall A^j \in \mathbf{A}$, $B^j \in \mathbf{B}$, $\forall x \in X\backslash(A^j \cup B^j)$ such that:

(i): $x \in (\mathbf{A}^{-j})^{\cap}$ iff $x \in (\mathbf{B}^{-j})^{\cap}$; (ii): $\#A^j = \max_{i=1}^{n}(\#A^i)$ iff $\#B^j = \max_{i=1}^{n}(\#B^i)$,

and (iii): $\#A^j \geq \min_{i=1}^{n}(\#A^i)$ iff $\#B^j \geq \min_{i=1}^{n}(\#B^i)$, then

$\mathbf{A} \succsim \mathbf{B} \Leftrightarrow [A^1,\cdots, A^j \cup \{x\},\cdots, A^n] \succsim [B^1,\cdots, B^j \cup \{x\},\cdots, B^n]$

NON-DECISIVENESS (ND^g)

$\forall \mathbf{A} \in L^n$ such that $[\#A^1 \geq \#A^2 \geq \cdots \geq \#A^n]$, $\forall C_i \in L$ $(i=1,\cdots,n)$ such that $A^i \cap C_i = \emptyset$ $\forall i$, and such that $\#C_1 > \#C_n$; and $\forall x \in X \backslash (\mathbf{A}^{\cup} \cup (\cup_{i=1}^{n} C_i))$; then

$[A^1 \cup C_1, A^2 \cup C_2, \cdots, A^n \cup C_n]$ $\mathbf{A}$ [illegible] $\rightarrow$ $[A^1 \cup C_1 \cup \{x\}, A^2 \cup C_2, \cdots, A^n \cup C_n]$ $\mathbf{A}$ [illegible]

All axioms belong to the possible generalizations of the corresponding axioms in the two agents case. As then, it is possible to derive from (IDE^g) the following generalized axiom (REP^g): $\forall \mathbf{A} \in L^n$, $\forall A^i \in \mathbf{A}$, $\forall x,y$ s.t. $x \in X \backslash \mathbf{A}^{\cup}$, $y \in A^i \backslash (\mathbf{A}^{-i})^{\cap}$, $[A^1, \cdots, A^i \cup \{x\} \backslash \{y\}, \cdots, A^n] \sim \mathbf{A}$. The proof of the relation between (IDE^g) and (REP^g) is quite similar to the n=2 case.

THEOREM 1′. Let [illegible] be a complete ordering defined on L^n; then [illegible] satisfies (AN^g), (MON^g), (ASM^g) and (IDE^g) if and only if:

$\forall \mathbf{A},\mathbf{B} \in L^2$, $[\#\mathbf{A}^{\cap} \geq \#\mathbf{B}^{\cap}$ and $CD_n(\mathbf{A}) \leq CD_n(\mathbf{B})] \rightarrow \mathbf{A}$ [illegible] $\mathbf{B}$ and

$[\#\mathbf{A}^{\cap} \geq \#\mathbf{B}^{\cap}$ and $CD_n(\mathbf{A}) < CD_n(\mathbf{B})] \rightarrow \mathbf{A}$ [illegible] $\mathbf{B}$

(and hence $[\#\mathbf{A}^{\cap} = \#\mathbf{B}^{\cap}$ and $CD_n(\mathbf{A}) = CD_n(\mathbf{B})] \rightarrow \mathbf{A} \sim \mathbf{B}$).

THEOREM 2′. Let [illegible] be an ordering on L^n; [illegible] satisfies (AN^g), (MON^g), (ASM^g), (IDE^g), (C^g) and (ND^g) if and only if [illegible] $\in \{$[illegible]$_{CD}$, [illegible]$_{Cg}$, [illegible]$_{1g}$, [illegible]$_{2g}\}$.

(Proof of Theorems 1′ and 2′ available upon request).

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