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ON FIRM-LEVEL, INDUSTRY-LEVEL, AND AGGREGATE EMPLOYMENT FLUCTUATIONS

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On firm-level, industry-level, and aggregate employment fluctuations

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Abstract

Employment fluctuations are examined, at different levels of aggregation, in a dynamic model that provides firm-specific hiring decisions due to search frictions and sticky pricing. The results indicate that firm-level employment dispersion rises with higher price stickiness and higher demand elasticity, whereas it falls with more convexity of search costs and with a higher labor supply elasticity. Industry-level employment is more volatile and less procyclical than aggregate employment, and a larger industry size reduces volatility and raises co-movement with output. The calibrated model is able to match the volatility, autocorrelation and cyclical correlation of US industry-level employment when incorporating firm-specific technology shocks.

Keywords: employment fluctuations, search frictions, sticky prices, firm-specific shocks.

JEL codes: E3, J2, J3, and J4.

1 Introduction

Search frictions and unemployment have been recently introduced in dynamic macroeconomic models that assume homogeneous employment in the labor market (Walsh, 2005; Trigari, 2009; Blanchard and Galí, 2010). Such representative-agent models do not seem to be compatible with a search-and-matching theory of unemployment *a la* Mortensen and Pissarides (1994). Firm differentiation might

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be required to explain why workers become eventually fired while others are able to find a new job by filling a vacancy posting.[1] In other words, the endogenous determination of employment dynamics ought to be firm specific.

Hence, this paper describes a model with search frictions and unemployment that contemplates heterogeneous employment.[2] The key model ingredients are monopolistically competitive firms as in Dixit and Stiglitz (1977), sticky prices *a la* Calvo (1983), and a labor market with search frictions of the Mortensen-Pissarides style. Firm-level employment dynamics can be determined as a combined response to price rigidities and search frictions when pricing and hiring decisions are connected at the optimizing program of the firm. In turn, firms differentiate in many dimensions: they have a specific selling price, they have a different number of employees, they offer a particular number of vacancies, they produce a different quantity of output, they organize different shifts of hours at work, and they pay a different nominal wage. Industry-level employment fluctuations are obtained as the average across the set of firms that belong to one industry. Finally, aggregate employment fluctuations are governed by a dynamic equation similar to the one derived in the models with search frictions and unemployment cited above.

The empirical evidence indicate that firm-level and industry-level fluctuations are more volatile than aggregate fluctuations (Comin and Phillipon, 2005; Davis *et al.*, 2007). Moreover, this volatility divergence has risen in the last decades (Comin and Mulani, 2006).[3] This paper contributes to explain the determinants of higher employment volatility at disaggregated levels. In the calibrated model, employment dispersion across firms rises with the degree of price rigidity, and the elasticity of demand for consumption goods, and falls with an increase in the elasticity of the search cost function, and the elasticity of substitution in the labor supply. In model simulations with alternative industry sizes, business cycle statistics show an intense reduction of variability in the aggregation from firm-level employment to industry-level employment. This reduction is much higher with a larger industry

[1] Quoting Davis et al. (2007), page 113: "Theories of unemployment based on search and matching frictions (Mortensen and Pissarides (1999) and Pissarides (2000)) rely on idiosyncratic shocks to drive job destruction and match dissolution.".

[2] Other recent papers that describe a model with firm-specific employment are Sveen and Weinke (2008) and Thomas (2011). The model of this paper differs from these in the wage setting assumptions.

[3] Using recent US data released by the Bureau of Labor Statistics, I report in the Appendix A that the standard deviation of industry-level employment in the US has been 38.7% higher than the one of fluctuations of total private employment.

size. The results of this paper also show that industry-level employment volatility is more volatile and less procyclical than aggregate employment in both the calibrated model and US data.

However, the baseline model is not able to replicate the volatility gap because the standard deviation of industry-level employment is less than 10% higher than the standard deviation of aggregate employment, even in the case of a small industry size. Subsequently, the baseline model will be modified to incorporate firm-specific technology shocks as another source of firm heterogeneity. The simulation results will show that the calibrated model with idiosyncratic shocks is able to replicate the volatility, cyclical correlation and inertia observed in US industry-level employment fluctuations.

The rest of the paper contains four more sections. Section 2 describes the details and derivation of the baseline model and offers a calibration of its parameters. Section 3 examines the determinants of the volatility observed in firm-level employment fluctuations. Industry-level employment is defined in Section 4 and the effects of the industrial size on the second-moment statistics of employment fluctuations are examined in the baseline model and in one variant that incorporates firm-specific technology shocks. Section 5 reviews the main results.

2 A search-and-matching model with firm-specific employment

The supply-side of the economy is formed by monopolistically competitive firms of the type described in Dixit and Stiglitz (1977). Thus, firms may set a specific price while the amount of output produced is determined at the demand curve

$$y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta_p} y_t, \tag{1}$$

where $y_t(i)$ is output produced at the representative firm i, $P_t(i)/P_t$ is the ratio of price set by firm i over the aggregate price level, y_t is aggregate output, and $\theta_p > 0.0$ is a constant elasticity parameter. In their production technology, firms have two forms of varying labor input: at the extensive margin (number of workers employed, $n_t(i)$), and in the intensive margin (number of hours per worker demanded, $h_t^d(i)$). Assuming constant capital, and the same labor productivity in both margins, the production function of the i firm is

$$y_t(i) = \left(\exp(z_t) h_t^d(i) n_t(i)\right)^{1-\alpha}, \tag{2}$$

where $0 < \alpha < 1$, and z_t denotes the economy-wide technology shock. After substituting (2) into (1), the demand constraint faced by the i-th firm is

$$\left(\exp(z_t) h_t^d(i) n_t(i)\right)^{1-\alpha} = \left(\frac{P_t(i)}{P_t}\right)^{-\theta_p} y_t. \tag{3}$$

Wages are adjusted by the firm to equate total hours of labor supply and labor demand.[4]. Hence, the nominal wage is the hourly rate that equates the willingness of workers to spend time at the firm (supply of total hours) with the need of workhours for the firm (demand for total hours).[5] For the specific i firm, the demand for total hours is obtained by turning (3) around to yield

$$h_t^d(i) n_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\theta_p}{1-\alpha}} \frac{y_t^{\frac{1}{1-\alpha}}}{\exp(z_t)}. \tag{4}$$

Meanwhile, the supply of total hours is determined by solving the household optimizing program. It is assumed that there is a representative large household as in Merz (1995). The members of the household who are working pool their labor income to be split up evenly in a way that conveys consumption insurance for the unemployed members. The representative household demands bundles of differentiated consumption goods, c_t, and supplies bundles of total hours of labor services, $h_t^s n_t$. Assuming constant elasticity of substitution *á la* Dixit-Stiglitz, the optimal allocation of total hours supplied to the i-th firm is positively related to the relative wage, $W_t(i)/W_t$, as follows

$$h_t^s(i) n_t(i) = \left(\frac{W_t(i)}{W_t}\right)^{\theta_w} h_t^s n_t, \tag{5}$$

where $\theta_w > 0$ is the labor supply elasticity of substitution.[6] Using (4) and (5) to equate total hours supplied and demanded at firm i, $h_t^d(i) n_t(i) = h_t^s(i) n_t(i)$, and solving for the labor-clearing nominal wage, $W_t(i)$, it is obtained[7]

$$W_t(i) = \frac{P_t}{c_t^{-\sigma}} \left(\left(\frac{P_t(i)}{P_t}\right)^{-\frac{\theta_p}{1-\alpha}} \frac{y_t^{\frac{1}{1-\alpha}}}{\exp(z_t)}\right)^{\frac{1}{\theta_w}}, \tag{6}$$

[4] Alternatively, wages are defined in a Nash-style bargaining setup in many papers of the Mortensen-Pissarides literature (Walsh, 2005; Krause and Lubik, 2007; Christoffel and Kuester, 2008; and Trigari, 2009).

[5] The impossibility of instantaneous hiring obliges the firm to modify the amount of hours per worker, $h_t^d(i)$, when output must be adjusted to meet current demand conditions. The other inputs of the production function (2) cannot be used to adjust the level of production because they are either exogenous (the technology shock, z_t) or predetermined (employment, $n_t(i)$).

[6] See Casares (2007) for more details.

[7] Casares (2008) defines the labor-clearing nominal wage for hours per worker, $h_t^d(i) = h_t^s(i)$, using a similar model outline.

which reveals that the nominal wage depends on the firm-specific price, affecting it negatively due to the reduction in the demand for total hours (4). The firm i will take into account this relationship between $W_t(i)$ and $P_t(i)$ in the optimizing program introduced below.

Next, let us describe the hiring decision. As in the search and matching literature (Mortensen and Pissarides, 1994), hiring workers is costly for the firm. In particular, the firm must post a vacancy in the market and wait for a matching of that vacancy with some unemployed worker. The search cost of the i–th firm, $c(v_t(i))$, is an increasing function of its number of vacancy postings, $v_t(i)$,

$$c(v_t(i)) = c_0 \left(v_t(i)\right)^{1+c_1},$$

where $c_0 > 0$ is a scale parameter and $c_1 \geq 0$ is the elasticity of the marginal cost of posting a vacancy with respect to the number of vacancies. The hiring process requires one period to fill the vacancy. Meanwhile, job destruction is determined by the constant separation rate, $0 < s < 1$.[8] As a result, the employment accumulation equation for the i-th firm becomes

$$(1-s)n_t(i) + v_t(i)q_t = n_{t+1}(i), \tag{7}$$

which implies that next period's employment is the predetermined sum of the jobs that remain after current period, $(1-s)n_t(i)$, plus the number of new hirings, $v_t(i)q_t$, obtained as the product of the number of vacancies posted by the probability q_t of filling them with a match. The matching probability is defined by the aggregate matching rate for vacancies

$$q_t = \frac{m_t}{v_t},$$

where m_t is the total number of matchings. The matching technology, strictly bounded between 0 and 1, as recommended by Den Haan *et al.* (2000) and Petrongolo and Pissarides (2001), determines these matchings as follows

$$m_t = \frac{u_t v_t}{\left(u_t^{\xi} + v_t^{\xi}\right)^{1/\xi}},$$

where $\xi > 0$ and $u_t = 1 - n_t$ is the rate of unemployment.

The price setting decision of the firm is not separated from the hiring decision; both the price and next-period employment are jointly determined in the solution of its optimizing program. Thus,

[8] Hall (2005) and Shimer (2005) claim that the separation rate is quite stable in the US and has little effect on employment fluctuations.

the representative i firm seeks to maximize the intertemporal profit function

$$E_t \sum_{k=0}^{\infty} \beta_{t,t+k} \left[\left(\frac{P_{t+k}(i)}{P_{t+k}} \right)^{1-\theta_p} y_{t+k} - \frac{W_{t+k}(i)}{P_{t+k}} h^d_{t+k}(i) n_{t+k}(i) - c_0 \left(v_{t+k}(i) \right)^{1+c_1} \right]$$

subject to constraints (3) and (7) in period t and future periods. Future profits are discounted at the stochastic discount factor $\beta_{t,t+k}$ for $k = 1, 2, ..., \infty$. The first order condition regarding the choice of next-period employment $n_{t+1}(i)$ is

$$-E_t \beta_{t,t+1} \frac{W_{t+1}(i)}{P_{t+1}} h^d_{t+1}(i) + E_t \beta_{t,t+1} \frac{\partial y_{t+1}(i)}{\partial n_{t+1}(i)} \psi_{t+1}(i) - \varphi_t(i) + E_t \beta_{t,t+1} \left[(1-s) \varphi_{t+1}(i) \right] = 0, \quad (8)$$

where $\psi_{t+1}(i)$ and $\varphi_t(i)$ are the Lagrange multipliers respectively attached to constraints (3) in period t and (7) in their respective periods. The optimality condition on the demand for hours per worker, $h^d_t(i)$, is

$$-\left[\frac{W_t(i)}{P_t} n_t(i) + \frac{\partial W_t(i)}{\partial h^d_t(i)} \frac{1}{P_t} n_t(i) \right] + \frac{\partial y_t(i)}{\partial h^d_t(i)} \psi_t(i) = 0,$$

where the partial derivative $\frac{\partial W_t(i)}{\partial h^d_t(i)} = \frac{\partial W_t(i)}{\partial P_t(i)} \frac{\partial P_t(i)}{\partial y_t(i)} \frac{\partial y_t(i)}{\partial h^d_t(i)} = \frac{1}{\theta_w} \frac{W_t(i)}{P_t} n_t(i)$ can be inserted in the last expression to identify $\psi_t(i)$ as the real marginal cost

$$\psi_t(i) = \frac{\left(1 + \frac{1}{\theta_w}\right) \frac{W_t(i)}{P_t} n_t(i)}{\frac{\partial y_t(i)}{\partial h^d_t(i)}} = \frac{\left(1 + \frac{1}{\theta_w}\right) \frac{W_t(i)}{P_t} n_t(i) h^d_t(i)}{(1-\alpha)\, y_t(i)}. \quad (9)$$

Moving (9) one period ahead and using the result in (8) leads to

$$\frac{1}{\theta_w} E_t \beta_{t,t+1} \left[h^d_{t+1}(i) \frac{W_{t+1}(i)}{P_{t+1}} \right] = \varphi_t(i) - E_t \beta_{t,t+1} \left[(1-s) \varphi_{t+1}(i) \right]. \quad (10)$$

The interpretation of (10) can be done in microeconomic terms: the marginal benefit expected for a new job on the left-hand side (measured as the expected net saving of work hours to accommodate the new employee) must be equal to the marginal cost of creating a new job on the right-hand side. The shadow value of a job for the firm, $\varphi_t(i)$, can be extracted from the first order condition on the number of vacancies posted in period t, $v_t(i)$, which says

$$-c_0 (1 + c_1) \left(v_t(i) \right)^{c_1} + \varphi_t(i) q_t = 0,$$

defining $\varphi_t(i)$, as the marginal cost of vacancy posting divided by the probability of making a match

$$\varphi_t(i) = \frac{c_0 (1 + c_1) \left(v_t(i) \right)^{c_1}}{q_t}. \quad (11)$$

Prices are sticky. Following Calvo (1983), there is a constant probability $0 \leq \eta \leq 1$ that the firm is not able to set the optimal price. Supposing that the i firm receives the Calvo-type signal to price optimally, the first order condition that must satisfy is

$$E_t^{\eta} \sum_{k=0}^{\infty} \beta_{t,t+k} \eta^k \left[(1-\theta_p) \left(\frac{P_t^*(i)}{P_{t+k}} \right)^{-\theta_p} \frac{y_{t+k}}{P_{t+k}} - \frac{\partial W_{t+k}(i)}{\partial P_t^*(i)} \frac{h_{t+k}^d(i) n_{t+k}(i)}{P_{t+k}} + \theta_p \psi_{t+k}(i) \left(\frac{P_t^*(i)}{P_{t+k}} \right)^{-\theta_p - 1} \frac{y_{t+k}}{P_{t+k}} \right] = 0,$$

where E_t^{η} is the rational expectation operator conditional to the lack of optimal pricing in future periods, and $P_t^*(i)$ is the optimal price set in period t. The partial derivative implied by (6) $\frac{\partial W_{t+k}(i)}{\partial P_t^*(i)} = -\frac{\theta_p}{\theta_w(1-\alpha)} \frac{W_{t+k}(i)}{P_t^*(i)}$, the definition of the real marginal cost (9), and the demand constraint (1) can be used in the previous expression to reach

$$E_t^{\eta} \sum_{k=0}^{\infty} \beta_{t,t+k} \eta^k \left[(1-\theta_p) \left(\frac{P_t^*(i)}{P_{t+k}} \right)^{-\theta_p} \frac{y_{t+k}}{P_{t+k}} + \theta_p \left(1 + \frac{1}{\theta_w} \right) \left(\frac{P_t^*(i)}{P_{t+k}} \right)^{-\theta_p - 1} \frac{y_{t+k}}{P_{t+k}} \psi_{t+k}(i) \right] = 0, \tag{12}$$

Equations (8) and (12) jointly determine the dynamic behavior of prices and employment. Log-linearizing techniques can be used to find linear approximations that explain the period-to-period evolution of these variables. Borrowing the standard notation, the hat symbol on top of a variable refers to the log deviation of that variable from its steady-state level. For example, $\widehat{n}_t = \log\left(\frac{n_t}{n}\right)$ represents the log deviation of current aggregate employment, n_t, from its steady state level, n. In addition, tilde-topped variables denote relative variables measured as log deviations with respect to the aggregate variable: the relative employment of firm i in period t is written as $\widetilde{n}_t(i) = \log\left(\frac{n_t(i)}{n_t}\right)$. As shown in Appendix B, next period's relative employment, $\widetilde{n}_{t+1}(i)$, is inversely related to firm's expected relative price, $\widetilde{P}_t(i)$. In addition, search costs justify some inertial pattern for employment accumulation that makes next period's hiring depend upon the current level of employment. Thus, relative employment dynamics are determined by the following log-linear expression

$$\widetilde{n}_{t+1}(i) = \tau_1 \widetilde{n}_t(i) - \tau_2 \widetilde{P}_t(i), \tag{13}$$

where the analytical solutions for τ_1 and τ_2 are

$$\tau_1 = \frac{\frac{(1+\rho)c_1(1-s)}{(\rho+s)s}}{1+\frac{(1+\rho)c_1}{(\rho+s)s}+\frac{(1-s)^2 c_1}{(\rho+s)s}-\tau_1\frac{(1-s)c_1}{(\rho+s)s}}, \tag{14a}$$

$$\tau_2 = \frac{\eta\left(\frac{(1+\theta_w^{-1})\theta_p}{1-\alpha}+\tau_2\frac{(1-s)c_1}{(\rho+s)s}\right)}{1+\frac{(1+\rho)c_1}{(\rho+s)s}+\frac{(1-s)^2 c_1}{(\rho+s)s}-\tau_1\frac{(1-s)c_1}{(\rho+s)s}}. \tag{14b}$$

Meanwhile, the dynamic equation for aggregate employment fluctuations becomes[9]

$$\left(\tfrac{(1+\rho)c_1}{(\rho+s)s}+\tfrac{(1-s)^2c_1}{(\rho+s)s}\right)\widehat{n}_{t+1}=\tfrac{(1+\rho)c_1(1-s)}{(\rho+s)s}\widehat{n}_t+\tfrac{(1-s)c_1}{(\rho+s)s}E_t\widehat{n}_{t+2}$$
$$+E_t\widehat{h}_{t+1}+E_t\widehat{w}_{t+1}+\tfrac{(1+\rho)}{\rho+s}E_t\widehat{\beta}_{t,t+1}+\tfrac{(1+\rho)(1+c_1)}{\rho+s}\widehat{q}_t-\tfrac{(1-s)(1+c_1)}{\rho+s}E_t\widehat{q}_{t+1}. \quad (15)$$

As a consequence of convex search costs, the fluctuation of next-period aggregation employment, $\widehat{n}_{t+1}$, depends on both its lag, $\widehat{n}_t$, and its expected lead, $E_t\widehat{n}_{t+2}$. It gives both backward-looking and forward-looking dynamics that smooth employment fluctuations. The stochastic discount factor, $E_t\widehat{\beta}_{t,t+1}$, expected hours per worker, $E_t\widehat{h}_{t+1}$, and the expected real wage, $E_t\widehat{w}_{t+1}$, have also a positive influence on next-period employment; these three variables increase the expected marginal return of hirings at the firm level (see equation 10). In addition, the probability of a successful hiring, $\widehat{q}_t$, has a positive effect on next-period employment. By contrast, next-period expected matching probability, $E_t\widehat{q}_{t+1}$, has a negative impact on $\widehat{n}_{t+1}$ because firms find less costly to postpone hirings for the future.

Also applying loglinearizing techniques, the pricing equation (12) leads to the inflation equation (proof in Appendix B)

$$\pi_t=\beta E_t\pi_{t+1}+\frac{(1-\beta\eta)(1-\eta)}{\eta\left(1+\frac{\theta_p(\theta_w^{-1}+\alpha)}{1-\alpha}\right)}\widehat{\psi}_t, \quad (16)$$

which shows that inflation dynamics are driven by fluctuations of the real marginal cost, $\widehat{\psi}_t$, with the standard forward-looking pattern of canonical New Keynesian models (Woodford, 2003, chapter 3).

The rest of the model is completed with a household instantaneous utility function, separable between consumption and total hours

$$U(c_t,h_t^s n_t)=\frac{c_t^{1-\sigma}}{1-\sigma}-\Psi\frac{(h_t^s n_t)^{1+\gamma}}{1+\gamma},$$

and a standard household budget constraint as described in the optimizing program of Casares (2007). The labor supply equation is obtained when substituting the first order condition of consumption into the first order condition of the supply of total hours

$$h_t^s n_t=\left(\frac{w_t c_t^{-\sigma}}{\Psi}\right)^{\frac{1}{\gamma}},$$

[9] Proof also available in Appendix B.

where $\frac{1}{\gamma}$ is the Frisch labor supply elasticity. In log-linear terms, the equilibrium real wage consistent with the labor supply schedule is

$$\widehat{w}_t = \gamma \widehat{h}_t + \gamma \widehat{n}_t + \sigma \widehat{c}_t. \tag{17}$$

Combining first order conditions of consumption and bonds leads to the standard semi-loglinear IS curve

$$\widehat{c}_t = E_t \widehat{c}_{t+1} - \tfrac{1}{\sigma}(R_t - E_t \pi_{t+1}). \tag{18}$$

In addition, a Taylor-type monetary policy rule provides interest-rate reactions of the monetary authority to the current rate of inflation, $\mu_\pi > 1.0$, to the log change in output (proxy of the output gap), $\mu_y > 0$, featuring a component of interest-rate smoothing, $0 < \mu_R < 1$ as follows

$$R_t = (1 - \mu_R) \left[\mu_\pi \pi_t + \mu_y \left(\widehat{y}_t - \widehat{y}_{t-1} \right) \right] + \mu_R R_{t-1}. \tag{19}$$

Other dynamic equations have been introduced above: log fluctuations of the aggregate real marginal cost from the aggregation of (9) across firms

$$\widehat{\psi}_t = \widehat{w}_t + \widehat{n}_t - \widehat{y}_t + \widehat{h}_t, \tag{20}$$

log fluctuations of output around the steady-state level implied by the aggregation across firms of the production function (2)

$$\widehat{y}_t = (1 - \alpha) \left(\widehat{n}_t + \widehat{h}_t + z_t \right), \tag{21}$$

log fluctuations of unemployment from the loglinearization of $u_t = 1 - n_t$

$$\widehat{u}_t = -\tfrac{n}{u} \widehat{n}_t, \tag{22}$$

where $\frac{n}{u}$ is the employment-to-unemployment ratio in steady state; log fluctuations of aggregate vacancies obtained from the aggregation across firms of the loglinear version of (7)

$$\widehat{v}_t = \tfrac{1}{s} \widehat{n}_{t+1} - \tfrac{1-s}{s} \widehat{n}_t - \widehat{q}_t, \tag{23}$$

log fluctuations of matchings obtained by taking logs and aggregating across firms on the matching function, $m_t = \frac{u_t v_t}{\left(u_t^\xi + v_t^\xi \right)^{1/\xi}}$,

$$\widehat{m}_t = \tfrac{v^\xi}{u^\xi + v^\xi} \widehat{u}_t + \tfrac{u^\xi}{u^\xi + v^\xi} \widehat{v}_t, \tag{24}$$

with the steady-state weights $\frac{v^\xi}{u^\xi + v^\xi}$ and $\frac{u^\xi}{u^\xi + v^\xi}$; the loglinear probability of posting a successful vacancy obtained from taking logs and aggregating across firms in the definition $q_t = m_t / v_t$

$$\widehat{q}_t = \widehat{m}_t - \widehat{v}_t, \tag{25}$$

the log-linearized overall resources constraint that includes the cost of vacancy postings

$$\widehat{y}_t = \tfrac{c}{y}\widehat{c}_t + \tfrac{c_0(v)^{1+c_1}}{y}(1 + c_1)\widehat{v}_t, \tag{26}$$

where $\frac{c}{y}$ and $\frac{c_0(v)^{1+c_1}}{y}$ are respectively the steady-state shares of consumption and search costs relative to output; and the definition of log deviations of the intertemporal discount factor

$$E_t\widehat{\beta}_{t,t+1} = -(R_t - E_t\pi_{t+1}). \tag{27}$$

In summary, the macroeconomic model consists of thirteen equations, the set (15)-(27), that may provide solution paths for its thirteen endogenous variables: $\widehat{y}_t$, $\widehat{c}_t$, R_t, π_t, $\widehat{\psi}_t$, $\widehat{w}_t$, $\widehat{n}_{t+1}$, $\widehat{h}_t$, $\widehat{u}_t$, $\widehat{v}_t$, $\widehat{m}_t$, $\widehat{q}_t$, and $\widehat{\beta}_{t,t+1}$.

2.1 Baseline calibration

Table 1 provides a baseline quarterly calibration for the parameters of the model. Regarding search frictions technology, I follow the empirical evidence reported by Yashiv (2007) to set a 5% separation rate per quarter, $s = 0.05$. The elasticity of the marginal cost of posting a new vacancy is $c_1 = 0.05$ to have it close to the usual linear technology (Walsh 2005, Christoffel and Kuester, 2008). As for the scale parameter, I set the value $c_0 = 0.36$ because it implies that search costs take 3% of total output in steady state. The matching technology coefficient is $\xi = 8.15$ in order to reproduce the 62% relative volatility of US aggregate employment with respect to aggregate output fluctuations as reported in Thomas (2011).[10]

[10] It turns out that the relative volatility of aggregate employment is quite sensitive to the specification of ξ.

Table 1. Baseline calibration of model parameters.

Separation rate	$s = 0.05$
Matching technology	$\xi = 8.15$
Search cost elasticity	$c_1 = 0.05$
Search cost scale	$c_0 = 0.36$
Consumption utility curvature	$\sigma = 1.39$
Labor utility curvature	$\gamma = 2.0$
Steady-state discount factor	$\beta = 0.995$
Production technology	$\alpha = 0.36$
Dixit-Stiglitz demand elasticity	$\theta_p = 11.0$
Dixit-Stiglitz labor supply elasticity	$\theta_w = 20.0$
Sticky-price probability	$\eta = 2/3$
Monetary policy rule	$\mu_\pi = 1.5$, $\mu_y = 0.5/4$ and $\mu_R = 0.8$
Technology shocks	$z_t = 0.95 z_{t-1} + \varepsilon_t$
Std. deviation of innovations	$std(\varepsilon_t) = 1.52\%$

Household preferences are parameterized with a risk aversion coefficient at $\sigma = 1.39$ as estimated by Smets and Wouters (2007) in a DSGE model of the US economy. Meanwhile, the curvature parameter on the disutility of labor is set at $\gamma = 2.0$ to result in a low Frisch labor supply elasticity ($\gamma^{-1} = 1/2 = 0.5$) as suggested by numerous empirical studies (Altonji, 1986; Card, 1994; Blundell and Macurdy, 1999). The steady-state quarterly discount factor $\beta = 0.995$, which implies a 2% annualized real interest rate in steady state.

The production function (2) takes the usual capital-share coefficient, $\alpha = 0.36$, while the labor-augmenting technology shock, z_t, is randomly generated by an AR(1) stochastic process with a 95% serial correlation. The standard deviation of the technological innovations is 1.52% to replicate the variability observed in recent fluctuations of US aggregate private employment.[11]

Price stickiness is defined by the Calvo probability of non-optimal pricing $\eta = 2/3$, so as to have an average frequency of setting optimal prices equal to three quarters, as recently observed in data reported by Nakamura and Steinsson (2009).[12] The Dixit-Stiglitz demand elasticity is $\theta_p = 11.0$ to

[11] As documented in Appendix A, the standard deviation of the quarterly series of HP-filtered US Total Private Employment is 1.37% over the sample period 1994:1-2010:4.

[12] Bils and Klenow (2004) founnd significantly shorter price durations of around 5 months.

imply a 10% mark-up in steady state as suggested by the empirical evidence found by Basu and Fernald (1997). The Dixit-Stiglitz labor supply elasticity is $\theta_w = 20.0$ to render a 5% participation of search costs in the steady-state real marginal cost as in Krause *et al.* (2008).[13] Finally, the Taylor-type monetary policy rule is implemented with the original coefficients suggested by Taylor (1993), $\mu_\pi = 1.5$ and $\mu_y = 0.5/4$, together with a rather high interest-rate smoothing coefficient, $\mu_R = 0.8$.

2.2 Technology shocks and aggregate fluctuations

Solid lines of Figure 1 show the responses of the calibrated model to a one standard deviation positive technology innovation (1.52%). Dashed lines indicate the responses under fully-flexible pricing ($\eta = 0.0$). A positive productivity shock brings higher output, lower unemployment and a decline in both inflation and the nominal interest rate. The responses of output and inflation are larger and more immediate under flexible prices. The adjustment of labor to the productivity shock brings mixed effects: the extensive margin (employment) increases substantially whereas the use of the intensive margin (hours per worker) falls by a much lower extent. Firms take advantage of the higher marginal benefit of hiring to increase the number of workers. The number of job matchings rises as a result of the increase in vacancy postings. Vacancies, matchings and employment respond more rapidly and aggressively in the model variant with flexible prices. Finally, the real wage is much less procyclical with sticky prices because there is a slight decline at the quarter of the shock that is quickly reversed to have a positive response in the next quarters.

3 Determinants of firm-level employment dispersion

The employment dispersion across firms is determined in equation (13) where next period's relative employment $\widetilde{n}_{t+1}(i)$ depends upon current inflation and the relative price with constant elasticities τ_1 and $-\tau_2$. Solving out (14a) and (14b) for the proposed calibration in Table 1 gives rise to the following numbers

$$\tau_1 = 0.7842, \text{ and } \tau_2 = 1.1327,$$

[13] Following Krause *et al.* (2008), I can write the steady-state relation $mc = s(1 + x)$, where mc is the real marginal cost, s is the real unit labor cost and x is the search cost relative effect. After calibration, $x = 0.05$.

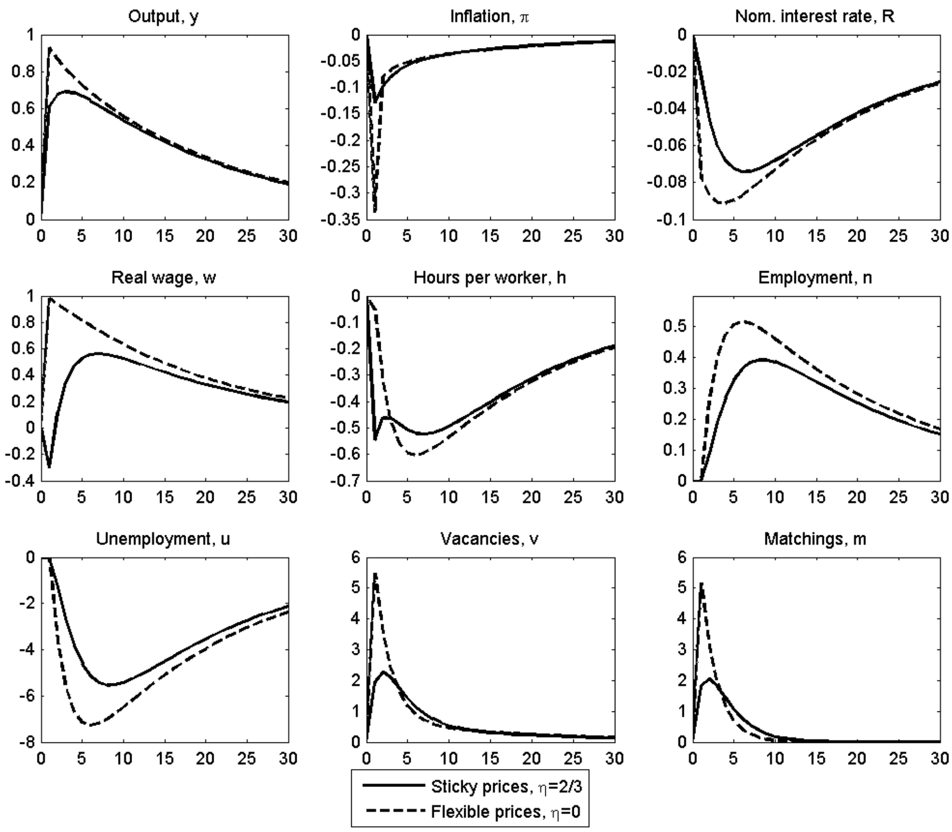

Figure 1: Impulse-response functions from a one standard deviation technology shock.

which bring a moderate employment inertia ($\tau_1 = 0.7842$), and a substantial dependence of relative prices on next period's employment ($\tau_2 = 1.1327$).

The model can be used to look for the structural determinants of employment dispersion. The unconditional standard deviation of relative employment fluctuations, $std(\widetilde{n}(i))$, provides a direct measure of firm-level employment dispersion. Recalling (13) and Calvo-type staggered pricing, the expression that obtains $std(\widetilde{n}(i))$ is[14]

$$std(\widetilde{n}(i)) = \sqrt{\frac{\tau_2^2+2\tau_1\tau_2\frac{\eta\tau_2}{1-\eta\tau_1}}{1-\tau_1^2}}\, std\left(\widetilde{P}(i)\right). \tag{28}$$

Using the results shown in Woodford's (2003, pages 694-696), the standard deviation of relative prices can be approximated by the following expression

$$std\left(\widetilde{P}(i)\right) = \sqrt{\frac{\eta}{(1-\eta)^2}}\, std\,(\pi)\,, \tag{29}$$

where $std\,(\pi)$ is the standard deviation of economy-wide inflation. Plugging (29) in (28) yields

$$std(\widetilde{n}(i)) = \sqrt{\frac{\left(\tau_2^2+2\tau_1\tau_2\frac{\eta\tau_2}{1-\eta\tau_1}\right)\eta}{\left(1-\tau_1^2\right)(1-\eta)^2}}\, std\,(\pi)\,. \tag{30}$$

Under the proposed calibration, I get $std(\widetilde{n}(i)) = 1.24\%$. What are the factors behind the firm-specific employment variability? To answer this question, exercises of sensitive analysis can show the effects of altering the parameters of the baseline calibration to see the impact on employment dispersion across firms, measured by $std(\widetilde{n}(i))$. In particular, the following four parameters are examined: the Calvo probability η which represents the degree of price stickiness, the Dixit-Stiglitz labor supply elasticity θ_w, the convexity of search costs c_1, and the Dixit-Stiglitz demand elasticity, θ_p. Figure 2 displays the results.

In the top-left plot, it can be observed how firm-level employment dispersion is significantly influenced by the degree of price rigidity. As Calvo probability rises, the standard deviation of firm relative employment increases significantly, moving from less than 0.5% when $\eta = 0.5$ to levels close to 10% reached when η is at 0.9. More slowly price adjustments (higher Calvo probability η) result in wider price dispersion across firms (see 29); those firms that have not been able to set the optimal price for many periods must face a larger discrepancy with respect to optimal pricing. In turn, firm-specific employment variability rises with higher price stickiness. If price stickiness turns very

[14] See Appendix C for the proof.

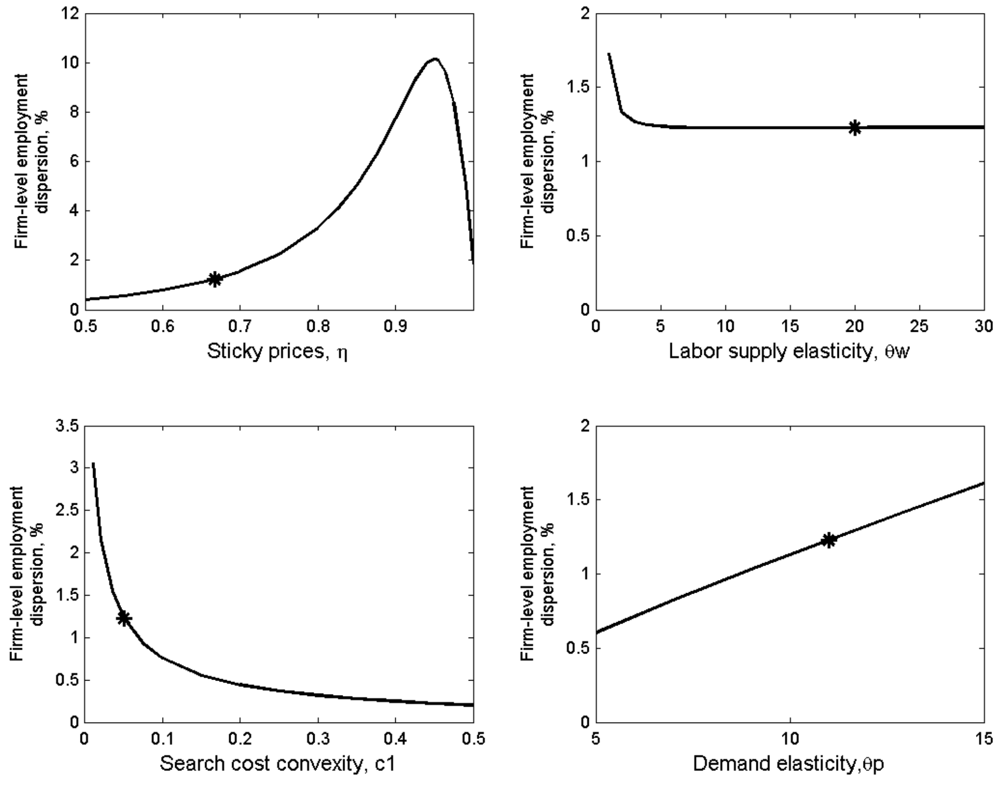

Figure 2: Determinants of firm-level employment dispersion, $std(\widetilde{n}(i))$. Results under baseline calibration are marked with '*'.

severe (as η approaches 1.0) firm-level employment dispersion falls dramatically because $std(\pi)$ is close to 0 in (30) as very few prices change.

The elasticity of substitution for labor supply allocation, θ_w, affects employment dispersion in the opposite direction: if labor supply elasticity is higher any excess demand for firm-specific hours will be absorbed with a more moderate wage rise which buffers the reaction of employment. A rise in labor supply elasticity implies a reduction of τ_2 (with a lower value for θ_w in 14b) that has a negative impact on $std(\widetilde{n}(i))$ in (30). Such reduction of firm-level employment dispersion is only noticeable for low elasticities. Figure 2 indicates that $std(\widetilde{n}(i))$ rapidly declines from 1.75% to 1.25% if the elasticity θ_w is raised from 1.0 to 3.0.

The convexity of search costs is quite influential on employment variability across firms. Figure 2 shows, in the bottom-left plot, a reduction in the value of $std(\widetilde{n}(i))$ from more than 3% to values close to 0% when the curvature of the search cost function is raised from 0.01 to 0.5. Thus, a more costly search process (higher c_1) results in less firm-level employment dispersion: if vacancy posting is more expensive the firm will slow down job creation.

Finally, the bottom-right plot of Figure 2 indicates that if demand elasticity rises the variability of firm-specific employment fluctuations increases in a moderate way. This effect is found in the model because a higher Dixit-Stiglitz demand elasticity would increase the dispersion of firms on demand-determined output (1), total hours (4) and the nominal wage (6). In turn, the sensitivity of relative employment to changes in the relative price is higher. Formally, τ_2 depends positively on θ_p as indicated in (14b) and a higher τ_2 raises the measure of employment dispersion, $std(\widetilde{n}(i))$, in (30). In the interval [5,15] for values assigned to θ_p, the observed $std(\widetilde{n}(i))$ goes up from 0.6% to 1.6% as displayed in Figure 2.

4 Industry-level employment fluctuations

Let us define one industry as a group of firms. For simplicity, all industries have the same size in our model. There are I industries formed by S firms each, that cover the total number of firms N, which implies $N = S * I$. Relative employment in the j industry, $\widetilde{in}_t(j) = \log\left(\frac{in_t(i)}{n_t}\right)$, is defined as the average across the firms that belong to that industry[15]

$$\widetilde{in}_t(j) = \int_{1+S(j-1)}^{Sj} \widetilde{n}_t(i)di, \tag{31}$$

[15]In the detrended steady-state, all firms share the same constant level of employment.

for $j = 1, 2, ..., I$. Firms are ranked from number 1 to number N, the first set of S firms belong to industry number 1, firms from number $1 + S$ to number $2S$ belong to industry 2, firms from number $1 + 2S$ to number $3S$ belongs to industry 3, and so forth.

By model assumption, industry-level employment is one-period predetermined alike firm-level employment. Then, relative industry-level employment for period $t + 1$ is determined in period t by rewriting (31) one period ahead, where inserting the firm-level relationship (13) gives

$$\widetilde{in}_{t+1}(j) = \int_{1+S(j-1)}^{Sj} \left(\tau_1 \widetilde{n}_t(i) - \tau_2 \widetilde{P}_t(i)\right) di. \tag{32}$$

Relative employment in the j-th industry for next period is explained by the average of current relative employment and current relative prices, across the set of firms that belong to that j industry. Hence, the outcome of Calvo lotteries determines relative prices and relative employment. If a firm i that belongs to the j industry could set the optimal price, its relative price $\widetilde{P}_t(i)$ would be the log difference between the optimal price and the aggregate price level, i.e. $\widetilde{P}_t(i) = \widetilde{P}_t^*(i)$. As shown in Appendix B, the relative optimal price $\widetilde{P}_t^*(i)$ is determined by a forward-looking equation that includes both the expected real marginal costs and expected inflation

$$\widetilde{P}_t^*(i) = \frac{1-\beta\eta}{1+\frac{\theta_p(\theta_w^{-1}+\alpha)}{1-\alpha}} E_t \sum_{k=0}^{\infty} \beta^k \eta^k \widehat{\psi}_{t+k} + E_t \sum_{k=1}^{\infty} \beta^k \eta^k \pi_{t+k}. \tag{33a}$$

If the Calvo-type signal for the i firm that belongs to the j industry did not allow optimal pricing, the firm would have its price stuck to the value from the previous period with the relative price[16]

$$\widetilde{P}_t(i) = \widetilde{P}_{t-1}(i) - \pi_t. \tag{33b}$$

Therefore, Calvo lotteries determine relative price dynamics through either (33a) or (33b). Such relative pricing across firms that belong to the j-th industry can be used in (32). Finally, the dynamic equation for the log fluctuation of industry-level employment is obtained from inserting (32) in the definition $\widetilde{in}_{t+1}(j) = \widehat{in}_{t+1}(j) - \widehat{n}_{t+1}$ and solving out for $\widehat{in}_{t+1}(j)$ to reach

$$\widehat{in}_{t+1}(j) = \int_{1+S(j-1)}^{Sj} \left(\tau_1 \widetilde{n}_t(i) - \tau_2 \widetilde{P}_t(i)\right) di + \widehat{n}_{t+1}, \tag{34}$$

where $\widehat{n}_{t+1}$ is the log fluctuation of aggregate employment governed by equation (15) of the model.

The business cycle properties of industry-level employment are examined in the model by computing second-moment statistics of volatility, correlation or autocorrelation from simulation exercises.

[16] Noticing that when the price of firm i cannot be adjusted $\widetilde{P}_t(i) \equiv \log P_t(i) - \log P_t = \log P_{t-1}(i) - \log P_t = \log P_{t-1}(i) - \log P_{t-1} + \log P_{t-1} - \log P_t = \widetilde{P}_{t-1}(i) - \pi_t$.

Artificial series are generated from random draws of a normal distribution with a standard deviation fixed at the calibrated value of 1.52%. These draws provide white-noise innovations, ε_t, for the AR(1) technology shocks, $z_t = 0.95 z_{t-1} + \varepsilon_t$. Total number of firms is set at $N = 10,000$ and I assume several (alternative) industrial sizes: $S = 1$, $S = 10$, $S = 100$, $S = 1,000$ and $S = 10,000$. This is a wide range of industrial size from having single-firm industries ($S = 1$) to the case in which all firms belong to the same industry ($S = N = 10,000$) and industry-level employment coincides with aggregate employment. As discussed above, firms (and industries) are differentiated due to their history of Calvo lotteries. Therefore, 10,000 independent signals are also randomly generated from a $[0, 1]$ uniform distribution for every sample period. Recalling the baseline Calvo probability ($\eta = 2/3$), if the number drawn is higher than 2/3, the firm could price optimally following (33a). Otherwise, the relative price would directly be (33b). Industry-level employment is then computed in (34) with the calibrated coefficients $\tau_1 = 0.7842$ and $\tau_2 = 1.1327$. The artificial samples contain 200 observations, and the first 20 observations are discarded to guarantee a random start. Second-moment statistics of volatility (standard deviation), cyclical correlation (coefficient of correlation with aggregate output) and serial correlation (coefficient of autocorrelation of order 1) are calculated for industry-level employment at the alternative levels of industrial size, S. This exercise was repeated 5,000 times and the average values are reported next in Table 2:

Table 2. Employment fluctuations. Second-moment statistics.

Baseline model	Std. dev, %	Corr. with output	Autocorrelation
Firm-level employment	2.29	0.54	0.96
Industry-level employment ($S = 10$ firms)	1.49	0.81	0.98
Industry-level employment ($S = 100$ firms)	1.38	0.87	0.99
Industry-level employment ($S = 1,000$ firms)	1.37	0.88	0.99
Aggregate employment	1.37	0.88	0.99
US data:	Std. dev, %	Corr. with output	Autocorrelation
Industry-level employment	1.90	0.45	0.91
Aggregate employment	1.37	0.81	0.95

It can be observed that the size reduces the standard deviation of industry-level employment. As more firms are included in each industry, the averaging (smoothing) effect on firm-level employment fluctuations makes industry-level employment less volatile. By contrast, the co-movement between industry-level employment and aggregate employment is more intense with a higher industrial size, S.[17] However, even in the limit case of only one firm per industry ($S = 1$), the correlation coefficient between fluctuations of industry-level employment and output is still moderate at 0.54. Finally, industry-level employment is very persistent at all levels of industrial size. As reported in Table 2, the coefficient of autocorrelation increases slightly with larger industrial size, and it is close to the 1.0 upper bound in all the cases.

US data of industry-level and aggregate employment fluctuations are collected in Appendix A. The bottom rows of Table 2 provide their second-moment statistics. Comparing with the simulation results of the model, the volatility of US industry-level employment (standard deviation at 1.90%) is greater than in the model except in the case of single-firm industries (standard deviation at 2.29%). Meanwhile, the cyclical correlation of industry-level employment is lower in US data (0.45) than in the model with any industrial size S. Regarding the serial correlation, the strong persistence in industry-level employment (with any S) of the model is slightly above the 0.91 value of coefficient of autocorrelation found in US data.

Figure 3 illustrates these results by displaying the statistics of volatility and cyclical correlation obtained in one model simulation. The standard deviation of firm-level employment is between 1% and 5% and it diminishes between 0.8% and 1.8% in 10-firm industry-level employment ($S = 10$). A larger size of industries ($S = 100$ or $S = 100$) brings standard deviations much closer to that of aggregate employment. As for the correlation with aggregate output, the simulations with small industries give a wide range of coefficients of correlation. Some numbers even turn negative when $S = 1$, whereas industry-level employment shows a strong procyclical co-movement with a large S.

Summarizing, the baseline model is only able to produce industry-level employment volatility observed in recent US data if the number of firms per industry is between 1 and 10. This low volatility might be connected to the fact there is only one source for firm-level employment differentiation: the history of Calvo-style lotteries received by the firms. Next, a second source of differentiation across firms is incorporated to the model: firm-specific technology shocks, with the objective of increasing variability of industry-level employment fluctuations to provide a better fit with the observed volatility in US data.

[17] This result can be explained by the relative effect of those firms that do not receive the Calvo-type signal and make countercyclical hiring decisions as they must keep prices unchanged.

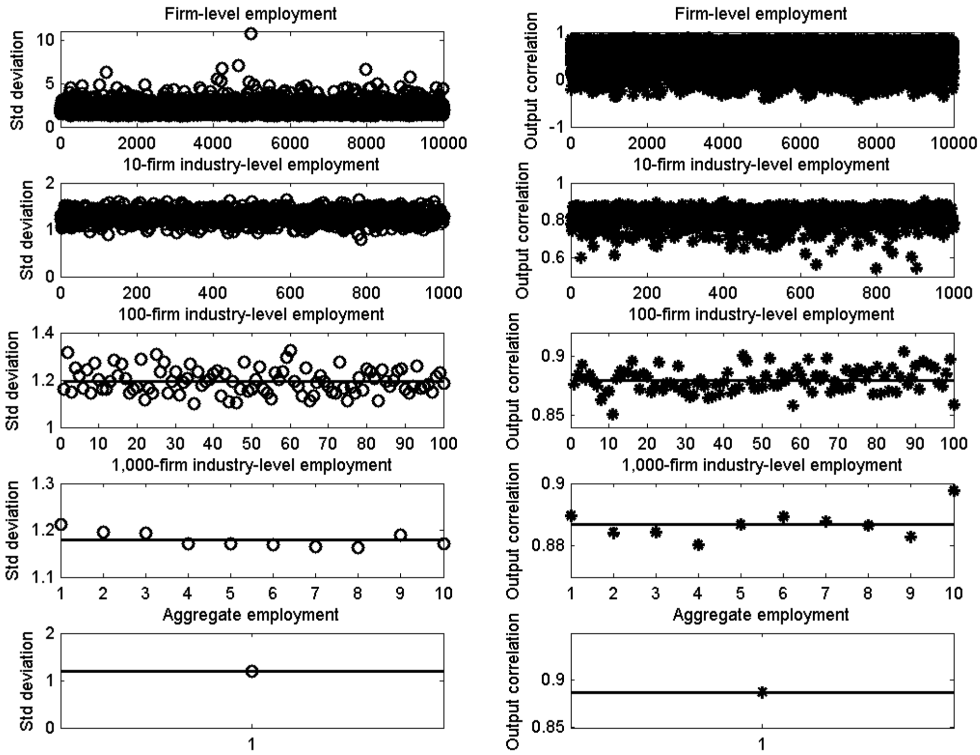

Figure 3: Percent standard deviations (o, left) and output correlations (*, right) of employment fluctuations in one baseline model simulation. Horizontal lines indicate average values.

4.1 Introducing firm-specific technology shocks

The production function of the representative i-th firm (2) can be slightly modified to accommodate a firm-specific labor-augmenting technology shock, $z_t(i)$,

$$y_t(i) = \left(\exp(z_t(i))h_t^d(i)n_t(i)\right)^{1-\alpha}. \tag{2'}$$

The influence of $z_t(i)$ on the production capabilities of the firm is transmitted to the demand for total hours, the hours-clearing nominal wage, the quantity of output produced, and the pricing decision. Hence, combining (2') with the Dixit-Stiglitz demand curve (1), the demand for total hours at the i-th firm would change to

$$h_t^d(i)n_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\theta_p}{1-\alpha}} \frac{y_t^{\frac{1}{1-\alpha}}}{\exp(z_t(i))}, \tag{4'}$$

while the hours-clearing nominal wage, $W_t(i)$, would also incorporate $z_t(i)$ to be

$$W_t(i) = \frac{P_t}{c_t^{-\sigma}} \left(\left(\frac{P_t(i)}{P_t}\right)^{-\frac{\theta_p}{1-\alpha}} \frac{y_t^{\frac{1}{1-\alpha}}}{\exp(z_t(i))}\right)^{\frac{1}{\theta_w}}. \tag{6'}$$

Both the demand for total hours (4') and the nominal wage (6') will have influence in the hiring decision of the firm. A positive relative technology shock raises relative labor productivity, lowers the relative demand for hours (as indicated in 4'), and cuts the relative hours-clearing nominal wage (as indicated in 6'). As shown in the Appendix D, the relative employment dynamics of the representative i-th firm is governed by the following equation

$$\widetilde{n}_{t+1}(i) = \tau_1\widetilde{n}_t(i) - \tau_2\widetilde{P}_t(i) + \tau_3\widetilde{z}_t(i), \tag{13'}$$

where $\widetilde{z}_t(i) = z_t(i) - z_t$ is the relative technology shock, τ_1 and τ_2 have identical analytical solutions as in the baseline model, τ_3 depends upon the structural parameters as follows:

$$\tau_3 = \frac{\left[\tau_3\frac{(1-s)c_1}{(\rho+s)s} + \tau_4(1-\eta)\left(\frac{(1+\theta_w^{-1})\theta_p}{1-\alpha} + \tau_2\frac{(1-s)c_1}{(\rho+s)s}\right) - (1+\theta_w^{-1})\right]\rho_z}{1 + \frac{(1+\rho)c_1}{(\rho+s)s} + \frac{(1-s)^2c_1}{(\rho+s)s} - \tau_1\frac{(1-s)c_1}{(\rho+s)s}}, \tag{35}$$

and ρ_z is the coefficient of autocorrelation of firm-specific technology shocks (defined below).

Unlike the baseline model, the presence of firm-specific technology shocks makes the optimal pricing decision be firm specific. Thus, the optimal price will be lower than the average optimal price if the firm-specific technology shock is above the average technology shock. Put differently, a positive firm-specific technology shock reduces the relative wage in (6') and lowers the marginal cost of production which makes the optimal price move downwards. In formal terms, the negative relationship between relative optimal prices and firm-specific technology shocks is collected at the following loglinear expression

$$\widetilde{P}_t^*(i) = \widetilde{P}_t^* - \tau_4 \widetilde{z}_t(i), \tag{36}$$

where $\widetilde{P}_t^*(i) = \log\left(\frac{P_t^*(i)}{P_t}\right)$ and $\widetilde{P}_t^* = \int_1^N \log P_t^*(i)di$. Recalling that $\widetilde{P}_t^* = \frac{1-\beta\eta}{1+\frac{\theta_p(\theta_w^{-1}+\alpha)}{1-\alpha}} E_t \sum_{k=0}^{\infty} \beta^k \eta^k \widehat{\psi}_{t+k} + E_t \sum_{k=1}^{\infty} \beta^k \eta^k \pi_{t+k}$, (36) becomes

$$\widetilde{P}_t^*(i) = \frac{1-\beta\eta}{1+\frac{\theta_p(\theta_w^{-1}+\alpha)}{1-\alpha}} E_t \sum_{k=0}^{\infty} \beta^k \eta^k \widehat{\psi}_{t+k} + E_t \sum_{k=1}^{\infty} \beta^k \eta^k \pi_{t+k} - \tau_4 \widetilde{z}_t(i), \tag{37}$$

which is comparable to equation (33a) in the baseline model. As also shown in the Appendix D, the analytical solution for τ_4 is

$$\tau_4 = \frac{(1+\theta_w^{-1})(1-\beta\eta)}{(1-\beta\eta\rho_z)\left(1+\frac{\theta_p(\theta_w^{-1}+\alpha)}{1-\alpha}\right)}. \tag{38}$$

Inserting in (35) and (38) the numerical values of the parameters defined at the baseline calibration (Table 1) and also $\rho_z = 0.95$, it is obtained, $\tau_3 = 0.0728$ and $\tau_4 = 0.1188$; which, as postulated above, indicate a positive elasticity of firm-level employment to its specific technology shock and a negative elasticity of optimal pricing in response to the specific technology shock.

The introduction of firm-specific shocks is made through a generalization of the baseline model described above. For the representative i-th firm, let us take the AR(1) generating process

$$z_t(i) = \rho_z z_{t-1}(i) + \varepsilon_t(i), \tag{39}$$

where $\varepsilon_t(i)$ is defined as one linear combination between the firm-specific innovation $\chi_t(i)$ and the economy-wide average innovation[18]

$$\varepsilon_t(i) = \phi\chi_t(i) + (1-\phi)\int_1^N \chi_t(i)\,di,$$

with $0 < \phi < 1$. Redefining the aggregate technology innovation as the average across firm-specific innovations, $\varepsilon_t = \int_1^N \chi_t(i)\,di$, the relative technology shock, $\widetilde{z}_t(i) = z_t(i) - z_t$, that results from (39) is

$$\widetilde{z}_t(i) = \rho_z \widetilde{z}_{t-1}(i) + \phi\left(\chi_t(i) - \varepsilon_t\right). \tag{40}$$

[18] Pesaran and Xu (2013) assume a similar combination between idiosyncratic and economy-wide exogenous perturbations for technology shocks.

Setting $\phi = 0$ reduces the model to the case of one economy where all firms receive the same technology innovation, ε_t, which recovers the setup described in Section 2 with economy-wide technology shocks. Obviously, there is a connection between firm-specific shocks and aggregate shocks. Since all firm-level innovations are i.i.d. draws, the standard deviation of the economy-wide technology innovation is related to that of the firm-specific innovation as follows

$$std(\varepsilon_t) = \frac{1}{\sqrt{N}} std(\varepsilon_t(i)),$$

which, as pointed out by Gabaix (2011), induces a very rapid reduction in aggregate volatility when the number of firms increases. For example, if $N = 10,000$ firms and the calibrated volatility of the aggregate innovations is $std(\varepsilon_t) = 1.52\%$ the required standard deviation of firm-specific innovations is $std(\varepsilon_t(i)) = \sqrt{N} std(\varepsilon_t) = 152\%$.

The value assigned to ϕ is aimed at fixing the problem of low volatility of industry-level employment in the model. Comin and Philippon (2005) observe that the volatility of firm-level sales growth have been in the US between 5 and 15 times higher than aggregate real GDP volatility.[19] They also report international evidence on the relative volatility of firm-level employment: many countries concentrate around a 10-time factor between firm-level and aggregate employment volatilities. After some preliminary testing, I set $\phi = 0.03$ to obtain average standard deviations of firm-level employment and output that are not far from 10 times higher than the standard deviations of their respective aggregate fluctuations.[20]

Table 3 informs that volatility of both firm-level and industry-level employment rises dramatically with the introduction of calibrated firm-specific technology innovations ($\phi = 0.03$). The average standard deviation of firm-level employment is 10.9%, around 5 times the number found in the model without firm-specific shocks (go to Table 2 for the comparison). If industries are represented by groups of 10 firms the standard deviation of employment fluctuations is also higher at 3.69%. If the industrial size is $S = 100$ firms, the model nearly replicates the standard deviation of average US industry-level employment fluctuations (1.76% in the model and 1.90% in US data). The case with large industries ($S = 1,000$) still keeps a substantial firm-level employment variability at 1.47%. Regarding the correlation with output, firm-specific shocks lower the coefficients of cyclical correlation obtained in the baseline model. A moderate industrial size (some number between $S = 10$ and

[19] Comin and Mulani (2006) provide empirical evidence for a trend increase in sales growth volatility in US firms.

[20] Concretely, $std(y(i))/std(y) = 11.59$ and $std(n(i))/std(n) = 7.85$.

$S = 100$) would give the correlation between industry-level employment and output found in US data (0.45). As reported in Table 3, the cyclical correlation of industry-level employment in the model is 0.32 with $S = 10$ and 0.68 with $S = 100$.[21] Finally, the autocorrelation of employment fluctuations is very high (0.99) with any industrial sizes.

Table 3. Employment fluctuations. Second-moment statistics with firm-specific shocks ($\phi = 0.03$).

Model with sticky prices ($\eta = 2/3$)	Std. dev, %	Corr. with output	Autocorrelation
Firm-level employment	10.9	0.11	0.99
Industry-level employment ($S = 10$ firms)	3.69	0.32	0.99
Industry-level employment ($S = 100$ firms)	1.76	0.68	0.99
Industry-level employment ($S = 1,000$ firms)	1.47	0.83	0.99
Aggregate employment	1.39	0.88	0.99
Model with flexible prices ($\eta = 0$)			
Firm-level employment	11.4	0.13	0.99
Industry-level employment ($S = 10$ firms)	3.95	0.37	0.99
Industry-level employment ($S = 100$ firms)	2.06	0.73	0.98
Industry-level employment ($S = 1,000$ firms)	1.80	0.84	0.98
Aggregate employment	1.73	0.88	0.98
US data:	Std. dev, %	Corr. with output	Autocorrelation
Industry-level employment	1.90	0.45	0.91
Aggregate employment	1.37	0.81	0.95

Figure 4 shows the results obtained in one simulated example for a comparison to the baseline model without firm-specific shocks displayed in Figure 3. It is confirmed that both firm-level and industry-level fluctuations of employment are much more volatile and co-move much less in the model with firm-specific shocks.

After the model extension, the two sources of firm-level employment heterogeneity are price rigidities (history of Calvo-type signals) and the realization of firm-specific shocks. How would the results

[21] Alternatively, the current setup might be transformed to accommodate higher comovement between industry-level and aggregate employment fluctuations with small-size industries. For example, Long and Plosser (1983) give cross-sectoral cost shares by industries. Or as in a recent paper by Gabaix (2011), a fat-tailed distribution of firms may reduce significantly the diversification effects and gain co-movement through the influence of idiosyncratic shocks of large firms on aggregate fluctuations.

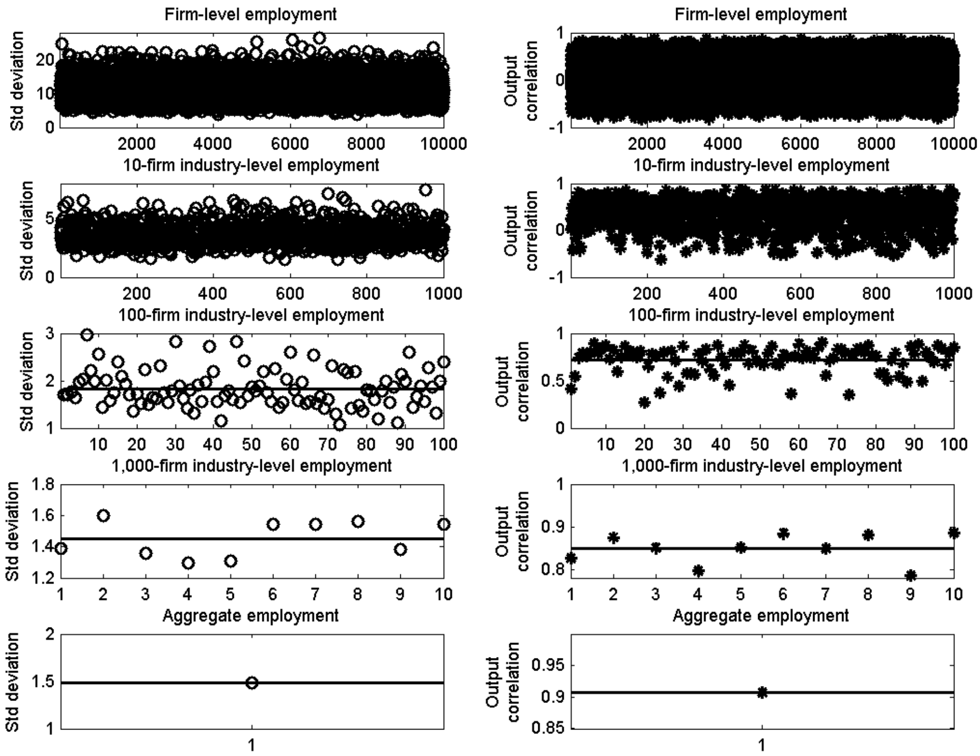

Figure 4: Percent standard deviations (o, left) and output correlations (*, right) of employment fluctuations in model simulation with firm-specific technology shocks and $\phi = 0.03$. Horizontal lines indicate average values.

change if prices would be freely adjusted by all firms (and the sticky-price channel would remain shut down)? I did the experiment of recalculating second-moment statistics of firm-level, industry-level and aggregate employment fluctuations in the model with firm-specific shocks ($\phi = 0.03$) and without sticky prices *(*$\eta = 0$*)*. The central rows of Table 3 show the results. Remarkably, numbers do not differ substantially from the sticky-price case: both volatility and cyclical correlation are just slightly higher with flexible prices. Therefore, price rigidities play a minor role for disaggregated unemployment fluctuations with technology shocks. By contrast, firm-specific productivity innovations are crucial to shape volatilities or cyclical correlation of both industry-level and firm-level fluctuations of employment.

5 Conclusions

In recent US business cycles, both volatility and cyclical correlation of employment fluctuations decline when aggregating from industry-level to aggregate data. The average standard deviation across industry-level employment has been 1.90%, whereas the standard deviation of aggregate employment is 1.37%. In addition, the correlation between aggregate employment and real GDP is 0.81, while that correlation for industry-level employment falls to 0.45. This paper has presented a calibrated model with heterogeneous labor across firms to examine employment fluctuations at different levels of aggregation.

In the structural analysis, the determinants of firm-level employment dispersion are price stickiness, convexity of search costs for job creation, a low elasticity of substitution in the labor supply, and a high elasticity of substitution in the demand for goods. These factors explain why firm-level volatility of employment is significantly higher than aggregate employment fluctuations. Industry-level employment is obtained as the average employment in a group of firms. After calibration of parameters, industry-level employment fluctuations are found to be less volatile and more procyclical than in US employment data. Thus, the model is modified to incorporate firm-specific technology shocks in order to increase the average volatility of industry-level employment fluctuations. Simulations with a medium industrial size give a good model fit of volatility, cyclical correlation and persistence of US industry-level employment. Even in the model variant with flexible prices, firm-specific technology shocks bring realistic second-moment patterns of industry-level employment fluctuations, which leaves price stickiness with a minor role for the empirical fit.

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Appendix A. *US industrial employment data.*

Second moments statistics of HP-filtered US industry-level employment, 1994:1-2010:4.

	Weight	Standard deviation, %	Correlation with output	Auto correlation
Mining (21)	.0060	3.70	0.41	0.90
Construction of buildings (236)	.0160	3.65	0.79	0.95
Heavy and civil engineering construction (237)	.0093	3.07	0.70	0.93
Specialty trade contractors (238)	.0422	3.54	0.78	0.95
Wood products (321)	.0056	4.05	0.87	0.93
Nonmetallic mineral products (327)	.0052	2.90	0.83	0.93
Primary metals (331)	.0055	3.77	0.77	0.91
Fabricated metal products (332)	.0164	3.34	0.75	0.93
Machinery (333)	.0134	3.55	0.59	0.93
Computer and electronic products (334)	.0158	3.46	0.51	0.94
Electrical equipment and appliances (335)	.0053	2.80	0.66	0.93
Motor vehicles and parts (3361,2,3)	.0116	4.45	0.84	0.87
Furniture and related products (337)	.0060	3.67	0.89	0.94
Food manufacturing (311)	.0159	0.48	0.05	0.70
Apparel (315)	.0045	2.41	0.56	0.85
Paper and paper products (322)	.0056	1.38	0.71	0.90
Printing and related support activities (323)	.0074	2.18	0.72	0.94
Chemicals (325)	.0096	1.02	0.63	0.93
Plastic and rubber products (326)	.0087	2.50	0.83	0.88
Wholesale trade of durable goods (423)	.0315	2.17	0.77	0.96
Wholesale trade of nondurable goods (424)	.0210	0.94	0.76	0.91
Electronic markets and agents and brokers (425)	.0071	1.77	0.67	0.95

Source: Bureau of Labor Statistics (BLS).[22]

[22] The Bureau of Labor Statistics (BLS) provides a wide range of monthly employment data, with several levels of disaggregation. I obtained quarterly series by making three-month average values in industries classified by the BLS, excluding small industries with less than 400,000 workers in 1994. The sample covers 67 industries that account for approximately 89% of the series of Total Private Employment (TPE) also reported by the BLS. The short-run

Second moments statistics of HP-filtered US industrial employment, 1994:1-2010:4 (cont'd.).

	Weight	Standard deviation, %	Correlation with output	Auto correlation
Motor vehicle and parts dealers (441)	.0187	1.80	0.74	0.89
Furniture and home furnishings stores (442)	.0054	2.90	0.85	0.92
Electronics and appliance stores (443)	.0053	2.62	0.71	0.92
Building material and garden supply stores (444)	.0120	1.78	0.69	0.91
Food and beverage stores (445)	.0301	0.74	0.41	0.93
Health and personal care stores (446)	.0096	0.99	0.35	0.91
Gasoline stations (447)	.0094	0.80	0.36	0.90
Clothing and clothing accessories stores (448)	.0140	2.13	0.79	0.91
Sporting goods, hobby, book, and music stores (451)	.0067	1.77	0.53	0.82
Department stores (4521)	.0169	1.57	0.43	0.85
Air transportation (481)	.0055	2.82	0.45	0.88
Truck transportation (484)	.0140	2.35	0.79	0.93
Support activities for transportation (488)	.0054	2.18	0.69	0.92
Couriers and messengers (492)	.0058	2.67	0.37	0.62
Warehousing and storage (493)	.0057	2.02	0.73	0.94
Utilities (22)	.0062	0.97	-0.30	0.94
Publishing industries except Internet (511)	.0096	2.06	0.64	0.95
Motion picture and sound recording industries (512)	.0038	1.92	0.38	0.70
Telecommunications (517)	.0117	3.57	0.30	0.96
Credit intermediation and related activities (522)	.0274	1.71	0.45	0.93
Securities, commodity contracts, investments (523)	.0078	3.17	0.57	0.96
Insurance carriers and related activities (524)	.0232	0.97	0.37	0.93
Real estate (531)	.0141	1.18	0.66	0.92

components of these series were obtained by taking the natural logarithms and running the HP filter. For the cyclical correlation with output, I also used the HP-filtered component of the series of US Real GDP available at St. Louis Fed website. In the Table, the number in parenthesis after each industry indicates its North American Industry Classification System (NAICS) code. The column labeled "Weight" provides the coefficient used for the computation of the weighted averages, defined as the ratio of the sample mean of industrial employment over TPE.

Second moments statistics of HP-filtered US industrial employment, 1994:1-2010:4 (cont'd.).

	Weight	Standard deviation, %	Correlation with output	Auto correlation
Rental and leasing services (532)	.0064	1.83	0.76	0.93
Legal services (5411)	.0113	0.83	0.51	0.91
Accounting and bookkeeping services (5412)	.0087	2.68	0.46	0.91
Architectural and engineering services (5413)	.0127	2.67	0.58	0.96
Computer systems design and related services (5415)	.0116	4.45	0.53	0.96
Management and technical consulting services (5416)	.0076	2.51	0.36	0.93
Management of companies and enterprises (55)	.0185	1.62	0.59	0.96
Employment services (5613)	.0330	6.16	0.91	0.91
Business support services (5614)	.0079	1.91	0.29	0.92
Services to buildings and dwellings (5617)	.0167	1.53	0.74	0.91
Educational services (61)	.0268	0.80	-0.36	0.74
Offices of physicians (6211)	.0202	0.34	0.08	0.86
Home health care services (6216)	.0080	3.33	-0.33	0.95
Hospitals (622)	.0436	0.51	-0.36	0.92
Nursing and residential care facilities (623)	.0282	0.51	-0.73	0.92
Social assistance (624)	.0210	0.94	0.27	0.84
Arts, entertainment, and recreation (71)	.0184	1.37	0.56	0.80
Accommodation (721)	.0186	1.82	0.74	0.89
Food services and drinking places (722)	.0888	0.86	0.60	0.92
Repair and maintenance (811)	.0125	1.30	0.59	0.91
Personal and laundry services (812)	.0129	0.62	0.40	0.83
Membership associations and organizations (813)	.0285	0.89	-0.18	0.88
Weighted average for industrial employment:		1.90	0.45	0.91
Total Private Employment (TPE)		1.37	0.81	0.95

Appendix B. *Derivation of the employment and inflation equations in the baseline model.*

The employment equation

Optimality in firm-level hiring decisions is determined by equations (10) and (11), subject to the employment accumulation constraint (7). The substitution of both (11) and the equation correspondent to (11) for period $t+1$ in equation (10) yields

$$\frac{1}{\theta_w} E_t \beta_{t,t+1} \left[h_{t+1}^d(i) \frac{W_{t+1}(i)}{P_{t+1}} \right] = \frac{c_0(1+c_1)(v_t(i))^{c_1}}{q_t} - E_t \beta_{t,t+1} \left[\frac{(1-s)c_0(1+c_1)(v_{t+1}(i))^{c_1}}{q_{t+1}} \right], \tag{B1}$$

which can be loglinearized to reach

$$E_t \widehat{h}_{t+1}^d(i) + E_t \left(\widehat{W}_{t+1}(i) - \widehat{W}_{t+1} \right) + E_t \widehat{w}_{t+1} = \frac{1+\rho}{\rho+s} \left[c_1 \widehat{v}_t(i) - \widehat{q}_t - E_t \widehat{\beta}_{t+1} \right] - \frac{1-s}{\rho+s} \left[c_1 E_t \widehat{v}_{t+1}(i) - E_t \widehat{q}_{t+1} \right], \tag{B2}$$

where $\widehat{w}_{t+1} = \widehat{W}_{t+1} - \widehat{P}_{t+1}$. Meanwhile, taking logs in (6) and subtracting the log of the aggregate nominal wage yields

$$\widetilde{W}_t(i) = -\frac{\theta_p}{\theta_w(1-\alpha)} \widetilde{P}_t(i), \tag{B3}$$

where $\widetilde{W}_t(i) = \log\left(\frac{W_t(i)}{W_t}\right)$ and $\widetilde{P}_t(i) = \log\left(\frac{P_t(i)}{P_t}\right)$ respectively are relative wages and prices. Similarly, taking logs in (4) and subtracting the log of aggregate total hours leads to the following (log of) demand for hours depending on relative employment and relative prices

$$\widehat{h}_t^d(i) = -\widetilde{n}_t(i) - \frac{\theta_p}{1-\alpha} \widetilde{P}_t(i) + \widehat{h}_t. \tag{B4}$$

Moving (B4) and (B3) one period forward leads to expressions for $E_t \widehat{h}_{t+1}^d(i)$ and $E_t \widetilde{W}_{t+1}(i)$ such as

$$\begin{aligned} E_t \widehat{h}_{t+1}^d(i) &= -\widetilde{n}_{t+1}(i) - \frac{\theta_p}{1-\alpha} E_t \widetilde{P}_{t+1}(i) + E_t \widehat{h}_{t+1}, \text{ and} \\ E_t \left(\widehat{W}_{t+1}(i) - \widehat{W}_{t+1} \right) &= -\frac{\theta_p}{\theta_w(1-\alpha)} E_t \widetilde{P}_{t+1}(i), \end{aligned}$$

which can be inserted in the loglinear optimality condition (B2) to obtain

$$\begin{aligned} &- E_t \widetilde{n}_{t+1}(i) - \frac{(1+\theta_w^{-1})\theta_p}{1-\alpha} E_t \widetilde{P}_{t+1}(i) + E_t \widehat{h}_{t+1} + E_t \widehat{w}_{t+1} = \\ &\qquad \frac{1+\rho}{\rho+s} \left[c_1 \widehat{v}_t(i) - \widehat{q}_t - E_t \widehat{\beta}_{t+1} \right] - \frac{1-s}{\rho+s} \left[c_1 E_t \widehat{v}_{t+1}(i) - E_t \widehat{q}_{t+1} \right]. \end{aligned} \tag{B5}$$

Log deviations of firm-specific current vacancies, $\widehat{v}_t(i)$, can be obtained by loglinearizing (7) rearranging terms, as follows

$$\widehat{v}_t(i) = \frac{1}{s} \widehat{n}_{t+1}(i) - \frac{1-s}{s} \widehat{n}_t(i) - \widehat{q}_t = \frac{1}{s} \left(\widetilde{n}_{t+1}(i) + \widehat{n}_{t+1} \right) - \frac{1-s}{s} \left(\widetilde{n}_t(i) + \widehat{n}_t \right) - \widehat{q}_t, \tag{B6}$$

Both (B6) and its corresponding expression one period ahead for $E_t\widehat{v}_{t+1}(i)$ are substituted in (B5) to obtain

$$
\begin{aligned}
&-E_t\widetilde{n}_{t+1}(i)-\tfrac{(1+\theta_w^{-1})\theta_p}{1-\alpha}E_t\widetilde{P}_{t+1}(i)+E_t\widehat{h}_{t+1}+E_t\widehat{w}_{t+1}=\\
&\qquad\tfrac{1+\rho}{\rho+s}\left[c_1\left(\frac{1}{s}\left(\widetilde{n}_{t+1}(i)+\widehat{n}_{t+1}\right)-\frac{1-s}{s}\left(\widetilde{n}_t(i)+\widehat{n}_t\right)-\widehat{q}_t\right)-\widehat{q}_t-E_t\widehat{\beta}_{t+1}\right]\\
&\qquad\quad-\tfrac{1-s}{\rho+s}\left[c_1E_t\left(\frac{1}{s}\left(\widetilde{n}_{t+2}(i)+\widehat{n}_{t+2}\right)-\frac{1-s}{s}\left(\widetilde{n}_{t+1}(i)+\widehat{n}_{t+1}\right)-\widehat{q}_{t+1}\right)-E_t\widehat{q}_{t+1}\right],
\end{aligned}
$$

which is equivalent to

$$
\begin{aligned}
&-\left[1+\tfrac{(1+\rho)c_1}{(\rho+s)s}+\tfrac{(1-s)^2c_1}{(\rho+s)s}\right]E_t\widetilde{n}_{t+1}(i)+\tfrac{(1+\rho)c_1(1-s)}{(\rho+s)s}\widetilde{n}_t(i)+\tfrac{(1-s)c_1}{(\rho+s)s}E_t\widetilde{n}_{t+2}(i)=\\
&\left(\tfrac{(1+\rho)c_1}{(\rho+s)s}+\tfrac{(1-s)^2c_1}{(\rho+s)s}\right)\widehat{n}_{t+1}-\tfrac{(1+\rho)c_1(1-s)}{(\rho+s)s}\widehat{n}_t-\tfrac{(1-s)c_1}{(\rho+s)s}E_t\widehat{n}_{t+2}+\tfrac{(1+\theta_w^{-1})\theta_p}{1-\alpha}E_t\widetilde{P}_{t+1}(i)-E_t\widehat{h}_{t+1}-E_t\widehat{w}_{t+1}\\
&\qquad-\tfrac{(1+\rho)}{\rho+s}E_t\widehat{\beta}_{t+1}-\tfrac{(1+\rho)(1+c_1)}{\rho+s}\widehat{q}_t+\tfrac{(1-s)(1+c_1)}{\rho+s}E_t\widehat{q}_{t+1}. \qquad \text{(B7)}
\end{aligned}
$$

The result obtained in (B7) implies a certain relationship between firm-specific employment and pricing of the following kind

$$\widetilde{n}_{t+1}(i)=\tau_1\widetilde{n}_t(i)-\tau_2\widetilde{P}_t(i), \qquad \text{(B8)}$$

where τ_1 and τ_2 are undetermined coefficients to be found below. Using (B8) to infer $E_t\widetilde{n}_{t+2}(i)$, it is obtained

$$E_t\widetilde{n}_{t+2}(i)=\tau_1\widetilde{n}_{t+1}(i)-\tau_2E_t\widetilde{P}_{t+1}(i),$$

which after being plugged in (B7) results in

$$
\begin{aligned}
&-\left[1+\tfrac{(1+\rho)c_1}{(\rho+s)s}+\tfrac{(1-s)^2c_1}{(\rho+s)s}-\tau_1\tfrac{(1-s)c_1}{(\rho+s)s}\right]E_t\widetilde{n}_{t+1}(i)+\tfrac{(1+\rho)c_1(1-s)}{(\rho+s)s}\widetilde{n}_t(i)=\\
&\left(\tfrac{(1+\rho)c_1}{(\rho+s)s}+\tfrac{(1-s)^2c_1}{(\rho+s)s}\right)\widehat{n}_{t+1}-\tfrac{(1+\rho)c_1(1-s)}{(\rho+s)s}\widehat{n}_t-\tfrac{(1-s)c_1}{(\rho+s)s}E_t\widehat{n}_{t+2}+\left(\tfrac{(1+\theta_w^{-1})\theta_p}{1-\alpha}+\tau_2\tfrac{(1-s)c_1}{(\rho+s)s}\right)E_t\widetilde{P}_{t+1}(i)-E_t\widehat{h}_{t+1}-E_t\widehat{w}_{t+1}\\
&\qquad-\tfrac{(1+\rho)}{\rho+s}E_t\widehat{\beta}_{t+1}-\tfrac{(1+\rho)(1+c_1)}{\rho+s}\widehat{q}_t+\tfrac{(1-s)(1+c_1)}{\rho+s}E_t\widehat{q}_{t+1}. \qquad \text{(B9)}
\end{aligned}
$$

Moreover, the Calvo-style pricing scheme implies that the expected relative price $E_t\widetilde{P}_{t+1}(i)$ is calculated as a weighted average between the current price and the expected optimal price

$$E_t\widetilde{P}_{t+1}(i)=\eta\left(\log P_t(i)-E_t\log P_{t+1}\right)+(1-\eta)E_t\widetilde{P}^*_{t+1}(i). \qquad \text{(B10)}$$

Calvo pricing also implies $\widetilde{P}^*_t(i)=\frac{\eta}{1-\eta}\pi_t$ and, subsequently, $E_t\widetilde{P}^*_{t+1}(i)=\frac{\eta}{1-\eta}E_t\pi_{t+1}$ that can be used in (B10) to yield

$$E_t\widetilde{P}_{t+1}(i)=\eta\widetilde{P}_t(i),$$

which it is inserted in (B9) to obtain

$$-\left[1+\tfrac{(1+\rho)c_1}{(\rho+s)s}+\tfrac{(1-s)^2c_1}{(\rho+s)s}-\tau_1\tfrac{(1-s)c_1}{(\rho+s)s}\right]E_t\widetilde{n}_{t+1}(i)+\tfrac{(1+\rho)c_1(1-s)}{(\rho+s)s}\widetilde{n}_t(i)=$$
$$\left(\tfrac{(1+\rho)c_1}{(\rho+s)s}+\tfrac{(1-s)^2c_1}{(\rho+s)s}\right)\widehat{n}_{t+1}-\tfrac{(1+\rho)c_1(1-s)}{(\rho+s)s}\widehat{n}_t-\tfrac{(1-s)c_1}{(\rho+s)s}E_t\widehat{n}_{t+2}+\eta\left(\tfrac{(1+\theta_w^{-1})\theta_p}{1-\alpha}+\tau_2\tfrac{(1-s)c_1}{(\rho+s)s}\right)\widetilde{P}_t(i)-E_t\widehat{h}_{t+1}-E_t\widehat{w}_{t+1}$$
$$-\tfrac{(1+\rho)}{\rho+s}E_t\widehat{\beta}_{t+1}-\tfrac{(1+\rho)(1+c_1)}{\rho+s}\widehat{q}_t+\tfrac{(1-s)(1+c_1)}{\rho+s}E_t\widehat{q}_{t+1}. \quad \text{(B11)}$$

The aggregation of (B11) over the continuum of firms leads to the macro relationship that determines employment fluctuations

$$\left(\tfrac{(1+\rho)c_1}{(\rho+s)s}+\tfrac{(1-s)^2c_1}{(\rho+s)s}\right)\widehat{n}_{t+1}=E_t\widehat{h}_{t+1}+E_t\widehat{w}_{t+1}+\tfrac{(1+\rho)c_1(1-s)}{(\rho+s)s}\widehat{n}_t$$
$$+\tfrac{(1-s)c_1}{(\rho+s)s}E_t\widehat{n}_{t+2}+\tfrac{(1+\rho)}{\rho+s}E_t\widehat{\beta}_{t+1}+\tfrac{(1+\rho)(1+c_1)}{\rho+s}\widehat{q}_t-\tfrac{(1-s)(1+c_1)}{\rho+s}E_t\widehat{q}_{t+1} \quad (15)$$

which is equation (15) in the main text. Another consequence of (B11) is that the analytical expressions for the undetermined coefficients τ_1 and τ_2, consistent with the assumed relationship (B8), are

$$\tau_1 = \frac{\frac{(1+\rho)c_1(1-s)}{(\rho+s)s}}{1+\frac{(1+\rho)c_1}{(\rho+s)s}+\frac{(1-s)^2c_1}{(\rho+s)s}-\tau_1\frac{(1-s)c_1}{(\rho+s)s}}, \text{ and}$$
$$\tau_2 = \frac{\eta\left(\frac{(1+\theta_w^{-1})\theta_p}{1-\alpha}+\tau_2\frac{(1-s)c_1}{(\rho+s)s}\right)}{1+\frac{(1+\rho)c_1}{(\rho+s)s}+\frac{(1-s)^2c_1}{(\rho+s)s}-\tau_1\frac{(1-s)c_1}{(\rho+s)s}},$$

that respectively become expressions (14a) and (14b) in the main text.

The inflation equation

I start by making a log-linear approximation to the price setting equation (12) that renders

$$\widehat{P}_t^*(i)=(1-\beta\eta)E_t^\eta\sum_{k=0}^{\infty}\beta^k\eta^k\left(\widehat{P}_{t+k}+\widehat{\psi}_{t+k}(i)\right), \quad \text{(B12)}$$

where log deviations from steady state of firm-specific real marginal costs can be obtained from (9) as follows

$$\widehat{\psi}_{t+k}(i)=\widehat{W}_{t+k}(i)-\widehat{P}_{t+k}+\widehat{n}_{t+k}(i)-\widehat{y}_{t+k}(i)+\widehat{h}_{t+k}^d(i). \quad \text{(B13)}$$

Subtracting log deviations of the aggregate real marginal cost, $\widehat{\psi}_{t+k}=\int_1^N\widehat{\psi}_{t+k}(i)di$, from (B13) yields

$$\widehat{\psi}_{t+k}(i)=\widehat{\psi}_{t+k}+\widetilde{W}_{t+k}(i)+\widetilde{n}_{t+k}(i)-\widetilde{y}_{t+k}(i)+\widetilde{h}_{t+k}^d(i),$$

where using (B3) for $\widetilde{W}_{t+k}(i)$, the log-linear version of (1) for $\widetilde{y}_{t+k}(i)$, and (B4) for $\widetilde{h}_{t+k}^d(i)$, I get

$$\widehat{\psi}_{t+k}(i)=\widehat{\psi}_{t+k}-\tfrac{\theta_p(\theta_w^{-1}+\alpha)}{1-\alpha}\widetilde{P}_{t+k}(i). \quad \text{(B14)}$$

Inserting (B14) in (B12), it is obtained

$$\widehat{P}_t^*(i) = (1-\beta\eta)E_t^\eta \sum_{j=0}^{\infty} \beta^j \eta^j \left(\widehat{P}_{t+j} + \widehat{\psi}_{t+j} - \tfrac{\theta_p(\gamma+\alpha)}{1-\alpha} \widetilde{P}_{t+j}(i) \right),$$

where subtracting the log of the aggregate price level, $\widehat{P}_t$, on both sides of the equation, I reach

$$\widetilde{P}_t^*(i) = (1-\beta\eta)E_t^\eta \sum_{k=0}^{\infty} \beta^k \eta^k \left(\widehat{\psi}_{t+k} - \tfrac{\theta_p(\theta_w^{-1}+\alpha)}{1-\alpha} \widetilde{P}_{t+k}(i) + \sum_{x=1}^{k} \pi_{t+x} \right). \tag{B15}$$

The rational expectation of future relative prices, conditional to optimal pricing in t and the lack of optimal price adjustments in the future, is $E_t^\eta \widetilde{P}_{t+k}(i) = \widehat{P}_t^*(i) - E_t\widehat{P}_{t+k} = \widehat{P}_t^*(i) - \widehat{P}_t + \widehat{P}_t - E_t\widehat{P}_{t+k} = \widetilde{P}_t^*(i) + E_t \sum_{x=1}^{k} \pi_{t+x}$. Using this result, (B15) becomes

$$\left(1 + \tfrac{\theta_p(\theta_w^{-1}+\alpha)}{1-\alpha}\right) \widetilde{P}_t^*(i) = (1-\beta\eta)E_t^\eta \sum_{k=0}^{\infty} \beta^k \eta^k \left(\widehat{\psi}_{t+k} + \left(1 + \tfrac{\theta_p(\theta_w^{-1}+\alpha)}{1-\alpha}\right) E_t \sum_{x=1}^{k} \pi_{t+x} \right),$$

which is equivalent to

$$\left(1 + \tfrac{\theta_p(\theta_w^{-1}+\alpha)}{1-\alpha}\right) \widetilde{P}_t^*(i) = (1-\beta\eta)E_t^\eta \sum_{k=0}^{\infty} \beta^k \eta^k \widehat{\psi}_{t+k} + \left(1 + \tfrac{\theta_p(\theta_w^{-1}+\alpha)}{1-\alpha}\right) E_t \sum_{k=1}^{\infty} \beta^k \eta^k \pi_{t+k},$$

or alternatively

$$\widetilde{P}_t^*(i) = \frac{1-\beta\eta}{1 + \frac{\theta_p(\theta_w^{-1}+\alpha)}{1-\alpha}} E_t \sum_{k=0}^{\infty} \beta^k \eta^k \widehat{\psi}_{t+k} + E_t \sum_{k=1}^{\infty} \beta^k \eta^k \pi_{t+k}. \tag{B16}$$

Combining (B16) with $\widetilde{P}_t^*(i) = \frac{\eta}{1-\eta}\pi_t$ from the Calvo pricing scheme leads to

$$\pi_t = \frac{(1-\eta)(1-\beta\eta)}{\eta\left(1+\frac{\theta_p(\theta_w^{-1}+\alpha)}{1-\alpha}\right)} E_t \sum_{k=0}^{\infty} \beta^k \eta^k \widehat{\psi}_{t+k} + \tfrac{1-\eta}{\eta} E_t \sum_{k=1}^{\infty} \beta^k \eta^k \pi_{t+k},$$

where one can do $\pi_t - \beta\eta E_t \pi_{t+1}$ to reach the New Keynesian Phillips curve (16) that governs inflation dynamics

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\beta\eta)(1-\eta)}{\eta\left(1+\frac{\theta_p(\theta_w^{-1}+\alpha)}{1-\alpha}\right)} \widehat{\psi}_t.$$

Appendix C. *Derivation of the standard deviation of firm-specific relative employment.*

From the dynamic evolution of relative employment, $\widetilde{n}_{t+1}(i) = \tau_1 \widetilde{n}_t(i) - \tau_2 \widetilde{P}_t(i)$, the variance of relative employment is

$$var(\widetilde{n}(i)) = (1-\tau_1^2)^{-1} \left[\tau_2^2 var(\widetilde{P}(i)) - 2\tau_1\tau_2 cov(\widetilde{n}(i), \widetilde{P}(i)) \right].$$

The covariance between relative employment and the relative price is the expected product of relative employment times the relative price

$$cov(\widetilde{n}(i), \widetilde{P}(i)) = E\left[\widetilde{n}(i) \cdot \widetilde{P}(i) \right]$$

where the relative price is obtained as a weighted average between lagged relative prices (adjusted by current inflation) and the optimal relative price

$$\widetilde{P}(i) = \eta(\widetilde{P}_{-1}(i) - \pi) + (1-\eta)\widetilde{P}^*(i).$$

Combining the last two expressions and recalling the Calvo-style pricing to use $\widetilde{P}_t^*(i) = \frac{\eta}{1-\eta}\pi_t$, I have

$$cov(\widetilde{n}(i), \widetilde{P}(i)) = E\left[\widetilde{n}(i) \left(\eta(\widetilde{P}_{-1}(i) - \pi) + (1-\eta)\left(\tfrac{\eta}{1-\eta}\pi \right) \right) \right] = \eta E\left[\widetilde{n}(i) \cdot \widetilde{P}_{-1}(i) \right].$$

Lagging the dynamic equation on relative employment, $\widetilde{n}_t(i) = \tau_1 \widetilde{n}_{t-1}(i) - \tau_2 \widetilde{P}_{t-1}(i)$, and using again $\widetilde{P}_t^* = \frac{\eta}{1-\eta}\pi_t$, it is obtained

$$cov(\widetilde{n}(i), \widetilde{P}(i)) = \eta E\left[\widetilde{n}(i) \cdot \widetilde{P}_{-1}(i) \right] = \eta E\left[\left(\tau_1 \widetilde{n}_{-1}(i) - \tau_2 \widetilde{P}_{-1}(i) \right) \cdot \widetilde{P}_{-1}(i) \right],$$

where I can use $E\left[\left(\widetilde{P}_{-1}(i) \right)^2 \right] = var\left(\widetilde{P}(i) \right)$, and $E\left[\widetilde{n}_{-1}(i) \cdot \widetilde{P}_{-1}(i) \right] = cov(\widetilde{n}(i), \widetilde{P}(i))$ to reach

$$cov(\widetilde{n}(i), \widetilde{P}(i)) = \eta\tau_1 cov(\widetilde{n}(i), \widetilde{P}(i)) - \eta\tau_2 var\left(\widetilde{P}(i) \right).$$

Solving the last expression for $cov(\widetilde{n}(i), \widetilde{P}(i))$, it is obtained

$$cov(\widetilde{n}(i), \widetilde{P}(i)) = -\tfrac{\eta\tau_2}{1-\eta\tau_1} var\left(\widetilde{P}(i) \right),$$

and substituting the result in the expression for $var(\widetilde{n}(i))$ that is displayed above gives

$$var(\widetilde{n}(i)) = (1-\tau_1^2)^{-1} \left[\tau_2^2 var(\widetilde{P}(i)) + 2\tau_1\tau_2 \tfrac{\eta\tau_2}{1-\eta\tau_1} var\left(\widetilde{P}(i) \right) \right] = \frac{\tau_2^2 + 2\tau_1\tau_2 \frac{\eta\tau_2}{1-\eta\tau_1}}{1-\tau_1^2} var\left(\widetilde{P}(i) \right),$$

where taking the square root leads to the expression for the standard deviation of relative employment

$$std(\widetilde{n}(i)) = \sqrt{\tfrac{\tau_2^2 + 2\tau_1\tau_2 \frac{\eta\tau_2}{1-\eta\tau_1}}{1-\tau_1^2}}\, std\left(\widetilde{P}(i) \right),$$

that is used in Section 3 of the text.

Appendix D. *Derivation of the employment and inflation equations in the model with firm-specific technology shocks.*

Firm-specific employment dynamics are governed by the loglinearized equation that determines optimal hiring, which was displayed above as equation (A2)

$$E_t\widehat{h}^d_{t+1}(i) + E_t\widetilde{W}_{t+1}(i) + E_t\widehat{w}_{t+1} = \tfrac{1+\rho}{\rho+s}\left[c_1\widehat{v}_t(i) - \widehat{q}_t - E_t\widehat{\beta}_{t+1}\right] - \tfrac{1-s}{\rho+s}\left[c_1 E_t\widehat{v}_{t+1}(i) - E_t\widehat{q}_{t+1}\right]. \quad \text{(D1)}$$

Remarkably, the introduction of firm-specific technology shocks has an influence on firm-specific employment through the impact on both expected wages and hours of the firm. Taking logs in (4') and subtracting the log of aggregate demand for total hours results in the following expression for the relative demand for hours in the i-th firm

$$\widehat{h}^d_t(i) - \widehat{h}_t = -\widetilde{n}_t(i) - \tfrac{\theta_p}{1-\alpha}\widetilde{P}_t(i) - \widetilde{z}_t(i), \quad \text{(D2)}$$

where $\widetilde{z}_t(i) = z_t(i) - z_t$. The labor-clearing relative nominal wage, $\widetilde{W}_t(i)$, would also incorporate $\widetilde{z}_t(i)$ from a log-linear approximation to (6') that gives

$$\widetilde{W}_t(i) = -\tfrac{\theta_p}{\theta_w(1-\alpha)}\widetilde{P}_t(i) - \theta_w^{-1}\widetilde{z}_t(i). \quad \text{(D3)}$$

Both (D2) and (D3) can be written for period $t+1$ and substituted in (D1) to obtain

$$-E_t\widetilde{n}_{t+1}(i) - \tfrac{(1+\theta_w^{-1})\theta_p}{1-\alpha}E_t\widetilde{P}_{t+1}(i) - (1+\theta_w^{-1})\rho_z\widetilde{z}_t(i) + E_t\widehat{h}_{t+1} + E_t\widehat{w}_{t+1} =$$
$$\tfrac{1+\rho}{\rho+s}\left[c_1\widehat{v}_t(i) - \widehat{q}_t - E_t\widehat{\beta}_{t+1}\right] - \tfrac{1-s}{\rho+s}\left[c_1 E_t\widehat{v}_{t+1}(i) - E_t\widehat{q}_{t+1}\right], \quad \text{(D4)}$$

where ρ_z is the coefficient of autocorrelation of the AR(1) technology shocks. Using in (D4), $\widehat{v}_t(i) = \frac{1}{s(1-\xi)}\left(\widetilde{n}_{t+1}(i) + \widehat{n}_{t+1}\right) - \frac{1-s}{s(1-\xi)}\left(\widetilde{n}_t(i) + \widehat{n}_t\right) - \frac{(1-s)\xi}{s(1-\xi)}\widehat{u}_t$, (already derived in Appendix B), and the corresponding expression for $\widehat{v}_{t+1}(i)$ yields

$$-\left[1 + \tfrac{(1+\rho)c_1}{(\rho+s)s} + \tfrac{(1-s)^2 c_1}{(\rho+s)s}\right]E_t\widetilde{n}_{t+1}(i) + \tfrac{(1+\rho)c_1(1-s)}{(\rho+s)s}\widetilde{n}_t(i) + \tfrac{(1-s)c_1}{(\rho+s)s}E_t\widetilde{n}_{t+2}(i) =$$
$$\left(\tfrac{(1+\rho)c_1}{(\rho+s)s} + \tfrac{(1-s)^2 c_1}{(\rho+s)s}\right)\widehat{n}_{t+1} - \tfrac{(1+\rho)c_1(1-s)}{(\rho+s)s}\widehat{n}_t - \tfrac{(1-s)c_1}{(\rho+s)s}E_t\widehat{n}_{t+2} + \tfrac{(1+\theta_w^{-1})\theta_p}{1-\alpha}E_t\widetilde{P}_{t+1}(i) + (1+\theta_w^{-1})\rho_z\widetilde{z}_t(i)$$
$$- E_t\widehat{h}_{t+1} - E_t\widehat{w}_{t+1} - \tfrac{(1+\rho)}{\rho+s}E_t\widehat{\beta}_{t+1} - \tfrac{(1+\rho)(1+c_1)}{\rho+s}\widehat{q}_t + \tfrac{(1-s)(1+c_1)}{\rho+s}E_t\widehat{q}_{t+1}. \quad \text{(D5)}$$

The relationship between firm-specific employment, firm-specific prices and firm-specific technology shocks is assumed to be of this type

$$\widetilde{n}_{t+1}(i) = \tau_1\widetilde{n}_t(i) - \tau_2\widetilde{P}_t(i) + \tau_3\widetilde{z}_t(i). \quad \text{(D6)}$$

Using (D6) to infer $E_t\widetilde{n}_{t+2}(i)$, it is obtained

$$E_t\widetilde{n}_{t+2}(i) = \tau_1\widetilde{n}_{t+1}(i) - \tau_2 E_t\widetilde{P}_{t+1}(i) + \tau_3\rho_z\widetilde{z}_t(i),$$

which after being plugged in (D5) results in

$$\begin{aligned}
&-\left[1+\tfrac{(1+\rho)c_1}{(\rho+s)s}+\tfrac{(1-s)^2c_1}{(\rho+s)s}-\tau_1\tfrac{(1-s)c_1}{(\rho+s)s}\right]E_t\widetilde{n}_{t+1}(i)+\tfrac{(1+\rho)c_1(1-s)}{(\rho+s)s}\widetilde{n}_t(i)=\\
&\left(\tfrac{(1+\rho)c_1}{(\rho+s)s}+\tfrac{(1-s)^2c_1}{(\rho+s)s}\right)\widehat{n}_{t+1}-\tfrac{(1+\rho)c_1(1-s)}{(\rho+s)s}\widehat{n}_t-\tfrac{(1-s)c_1}{(\rho+s)s}E_t\widehat{n}_{t+2}+\left(\tfrac{(1+\theta_w^{-1})\theta_p}{1-\alpha}+\tau_2\tfrac{(1-s)c_1}{(\rho+s)s}\right)E_t\widetilde{P}_{t+1}(i)\\
&+\left[(1+\theta_w^{-1})-\tau_3\tfrac{(1-s)c_1}{(\rho+s)s}\right]\rho_z\widetilde{z}_t(i)-E_t\widehat{h}_{t+1}-E_t\widehat{w}_{t+1}-\tfrac{(1+\rho)}{\rho+s}E_t\widehat{\beta}_{t+1}-\tfrac{(1+\rho)(1+c_1)}{\rho+s}\widehat{q}_t+\tfrac{(1-s)(1+c_1)}{\rho+s}E_t\widehat{q}_{t+1}.
\end{aligned} \tag{D7}$$

The optimal price is firm-specific (and therefore it needs aggregation for computing the average) due to different technology shocks received at each firm. In particular, it is initially assumed that relative optimal pricing is negatively relative to technology shocks through their impact on wages (see D3) and marginal costs

$$\widetilde{P}_t^*(i) = \widetilde{P}_t^* - \tau_4\widetilde{z}_t(i), \tag{D8}$$

which implies that the relative optimal price expected for next period is $E_t\widetilde{P}^*_{t+1}(i) = E_t\widetilde{P}^*_{t+1} - \tau_4\rho_z\widetilde{z}_t(i)$ and the expected relative price $E_t\widetilde{P}_{t+1}(i)$ that appears in (D7) is

$$\begin{aligned}
E_t\widetilde{P}_{t+1}(i) &= \eta\left(\log P_t(i) - E_t\log P_{t+1}\right) + (1-\eta)E_t\widetilde{P}^*_{t+1}(i) =\\
&\eta\left(\widetilde{P}_t(i) - E_t\pi_{t+1}\right) + (1-\eta)\left(E_t\widetilde{P}^*_{t+1} - \tau_4\rho_z\widetilde{z}_t(i)\right),
\end{aligned} \tag{D9}$$

where $E_t\pi_{t+1} = E_t\log P_{t+1} - \log P_t$ is expected next period's inflation. Calvo pricing implies $\widetilde{P}_t^* = \frac{\eta}{1-\eta}\pi_t$ and, subsequently, $E_t\widetilde{P}^*_{t+1} = \frac{\eta}{1-\eta}E_t\pi_{t+1}$ that can be used in (D9) to yield

$$E_t\widetilde{P}_{t+1}(i) = \eta\widetilde{P}_t(i) - \tau_4\rho_z(1-\eta)\widetilde{z}_t(i),$$

which it is inserted in (D7) to obtain

$$\begin{aligned}
&-\left[1+\tfrac{(1+\rho)c_1}{(\rho+s)s}+\tfrac{(1-s)^2c_1}{(\rho+s)s}-\tau_1\tfrac{(1-s)c_1}{(\rho+s)s}\right]E_t\widetilde{n}_{t+1}(i)+\tfrac{(1+\rho)c_1(1-s)}{(\rho+s)s}\widetilde{n}_t(i)=\\
&\left(\tfrac{(1+\rho)c_1}{(\rho+s)s}+\tfrac{(1-s)^2c_1}{(\rho+s)s}\right)\widehat{n}_{t+1}-\tfrac{(1+\rho)c_1(1-s)}{(\rho+s)s}\widehat{n}_t-\tfrac{(1-s)c_1}{(\rho+s)s}E_t\widehat{n}_{t+2}+\left(\tfrac{(1+\theta_w^{-1})\theta_p}{1-\alpha}+\tau_2\tfrac{(1-s)c_1}{(\rho+s)s}\right)E_t\widetilde{P}_{t+1}(i)\\
&+\left[(1+\theta_w^{-1})-\tau_3\tfrac{(1-s)c_1}{(\rho+s)s}-\tau_4(1-\eta)\left(\tfrac{(1+\gamma)\theta_p}{1-\alpha}+\tau_2\tfrac{(1-s)c_1}{(\rho+s)s}\right)\right]\rho_z\widetilde{z}_t(i)-E_t\widehat{h}_{t+1}-E_t\widehat{w}_{t+1}\\
&-\tfrac{(1+\rho)}{\rho+s}E_t\widehat{\beta}_{t+1}-\tfrac{(1+\rho)(1+c_1)}{\rho+s}\widehat{q}_t+\tfrac{(1-s)(1+c_1)}{\rho+s}E_t\widehat{q}_{t+1}.
\end{aligned} \tag{D10}$$

The aggregation of (D10) over the continuum of firms leads to two interesting results. First, firm-specific employment dynamics are determined by the linear relationship conjectured as (D6) with the following analytical solutions for the τ_1, τ_2 and τ_3 coefficients

$$\begin{aligned}
\tau_1 &= \frac{\frac{(1+\rho)c_1(1-s)}{(\rho+s)s}}{1+\frac{(1+\rho)c_1}{(\rho+s)s}+\frac{(1-s)^2c_1}{(\rho+s)s}-\tau_1\frac{(1-s)c_1}{(\rho+s)s}}, \\
\tau_2 &= \frac{\eta\left(\frac{(1+\theta_w^{-1})\theta_p}{1-\alpha}+\tau_2\frac{(1-s)c_1}{(\rho+s)s}\right)}{1+\frac{(1+\rho)c_1}{(\rho+s)s}+\frac{(1-s)^2c_1}{(\rho+s)s}-\tau_1\frac{(1-s)c_1}{(\rho+s)s}}, \text{ and} \\
\tau_3 &= \frac{\left[\tau_3\frac{(1-s)c_1}{(\rho+s)s}+\tau_4(1-\eta)\left(\frac{(1+\theta_w^{-1})\theta_p}{1-\alpha}+\tau_2\frac{(1-s)c_1}{(\rho+s)s}\right)-(1+\theta_w^{-1})\right]\rho_z}{1+\frac{(1+\rho)c_1}{(\rho+s)s}+\frac{(1-s)^2c_1}{(\rho+s)s}-\tau_1\frac{(1-s)c_1}{(\rho+s)s}}.
\end{aligned}$$

Secondly, the expression for aggregate employment dynamics is identical to the model with economy-wide technology shocks

$$\begin{aligned}
\left(\frac{(1+\rho)c_1}{(\rho+s)s}+\frac{(1-s)^2c_1}{(\rho+s)s}\right)\widehat{n}_{t+1} &= E_t\widehat{h}_{t+1}+E_t\widehat{w}_{t+1}+\frac{(1+\rho)c_1(1-s)}{(\rho+s)s}\widehat{n}_t+\frac{(1-s)c_1}{(\rho+s)s}E_t\widehat{n}_{t+2} \\
&\quad +\frac{(1+\rho)}{\rho+s}E_t\widehat{\beta}_{t+1}+\frac{(1+\rho)(1+c_1)}{\rho+s}\widehat{q}_t-\frac{(1-s)(1+c_1)}{\rho+s}E_t\widehat{q}_{t+1}.
\end{aligned} \tag{D11}$$

For pricing dynamics with firm-specific technology shocks, let us recall expression (B12)

$$\widehat{P}_t^*(i)=(1-\beta\eta)E_t^\eta\sum_{k=0}^{\infty}\beta^k\eta^k\left(\widehat{P}_{t+k}+\widehat{\psi}_{t+k}-\frac{\theta_p(\theta_w^{-1}+\alpha)}{1-\alpha}\widetilde{P}_{t+k}(i)\right)-\frac{(1+\theta_w^{-1})(1-\beta\eta)}{1-\beta\eta\rho_z}\widetilde{z}_t(i). \tag{D12}$$

and log fluctuations of the relative marginal cost

$$\widetilde{\psi}_t(i)=\widetilde{W}_t(i)+\widetilde{n}_t(i)-\widetilde{y}_t(i)+\widetilde{h}_t^d(i),$$

where I can use (D2), (D3) and the Dixit-Stiglitz curve $\widetilde{y}_t(i)=-\theta_p\widetilde{P}_t(i)$, and later generalize the result for any $t+k$ period to reach

$$\widetilde{\psi}_{t+k}(i)=-\frac{\theta_p(\theta_w^{-1}+\alpha)}{1-\alpha}\widetilde{P}_{t+k}(i)-(1+\theta_w^{-1})\widetilde{z}_{t+k}(i). \tag{D13}$$

Inserting $\widehat{\psi}_{t+k}(i)=\widetilde{\psi}_{t+k}(i)+\widehat{\psi}_{t+k}$ and (D13) in (D12) and subtracting the log of the aggregate price level, $\widehat{P}_t$, on both sides of the equation, I reach

$$\widetilde{P}_t^*(i)=(1-\beta\eta)E_t^\eta\sum_{k=0}^{\infty}\beta^k\eta^k\left(\widehat{\psi}_{t+k}-\gamma\widetilde{n}_{t+k}(i)-\frac{\theta_p(\theta_w^{-1}+\alpha)}{1-\alpha}\widetilde{P}_{t+k}(i)+\sum_{x=1}^{k}\pi_{t+x}\right)-\frac{(1+\theta_w^{-1})(1-\beta\eta)}{1-\beta\eta\rho_z}\widetilde{z}_t(i) \tag{D14}$$

The rational expectation of the steam of future relative prices, conditional to optimal pricing in t and the lack of optimal price adjustments in the future, is $E_t^\eta\widetilde{P}_{t+k}(i)=\widehat{P}_t^*(i)-E_t\widehat{P}_{t+k}=\widehat{P}_t^*(i)-$

$\widehat{P}_t + \widehat{P}_t - E_t\widehat{P}_{t+k} = \widetilde{P}_t^*(i) - E_t \sum_{x=1}^{k} \pi_{t+x} = \widetilde{P}_t^*(i) - E_t \sum_{k=1}^{j} \pi_{t+k}$. Using this result, (D14) becomes

$$\left(1 + \tfrac{\theta_p(\theta_w^{-1}+\alpha)}{1-\alpha}\right) \widetilde{P}_t^*(i) = (1-\beta\eta) E_t^{\eta} \sum_{k=0}^{\infty} \beta^k \eta^k \left(\widehat{\psi}_{t+k} + \left(1 + \tfrac{\theta_p(\theta_w^{-1}+\alpha)}{1-\alpha}\right) E_t \sum_{x=1}^{k} \pi_{t+x} \right) - \tfrac{(1+\theta_w^{-1})(1-\beta\eta)}{1-\beta\eta\rho_z} \widetilde{z}_t(i).$$

Recalling the proposed relation for optimal price dynamics, $\widetilde{P}_t^*(i) = \widetilde{P}_t^* - \tau_4 \widetilde{z}_t(i)$, the analytical solutions for τ_4 consistent with the last equation is

$$\tau_4 = \frac{(1+\theta_w^{-1})(1-\beta\eta)}{(1-\beta\eta\rho_z)\left(1+\frac{\theta_p(\theta_w^{-1}+\alpha)}{1-\alpha}\right)},$$

whereas average optimal prices are

$$\widetilde{P}_t^* = \frac{1-\beta\eta}{1 + \frac{\theta_p(\theta_w^{-1}+\alpha)}{1-\alpha}} E_t \sum_{k=0}^{\infty} \beta^k \eta^k \widehat{\psi}_{t+k} + E_t \sum_{k=1}^{\infty} \beta^j \eta^j \pi_{t+k}. \tag{D15}$$

Combining (D15) with $\widetilde{P}_t^* = \frac{\eta}{1-\eta}\pi_t$ from the Calvo scheme leads to

$$\pi_t = \frac{(1-\eta)(1-\beta\eta)}{\eta\left(1+\frac{\theta_p(\theta_w^{-1}+\alpha)}{1-\alpha}\right)} E_t \sum_{k=0}^{\infty} \beta^k \eta^k \widehat{\psi}_{t+k} + \frac{1-\eta}{\eta} E_t \sum_{k=1}^{\infty} \beta^k \eta^k \pi_{t+k},$$

where one can do $\pi_t - \beta\eta E_t \pi_{t+1}$ to reach the identical New Keynesian Phillips curve to that obtained in the model with economy-wide technology shocks

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\beta\eta)(1-\eta)}{\eta\left(1+\frac{\theta_p(\theta_w^{-1}+\alpha)}{1-\alpha}\right)} \widehat{\psi}_t.$$